Addiction in networks*

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Abstract

This paper presents a dynamical model of addiction on networks, in which consumers are influenced by the consumption levels of their friends. We study the long-term consequence of peer influence by characterizing steady-state consumption as a function of the position of consumers on the network and as a function of forward-looking attitudes. We also study the impact of the social network on welfare. Last, we examine the impact of various public policies on the demand of addictive good, including a key-player policy, representing a rehabilitation program targeting one individual, that takes into account the interpersonal influences on the network.

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1 Introduction

How does the consumption of friends influence the demand for addictive goods? How does the structure of social networks affect both the demand and the effectiveness of public policies? These questions are particularly relevant in highly interconnected societies, especially among young people, given the widespread prevalence of addiction worldwide. According to a 2018 report from the World Health Organization (WHO), alcohol abuse contributed to more than 3 million deaths in 2016, accounting for one in every 20 deaths worldwide. Similarly, WHO estimates that tobacco use led to more than 8 million deaths annually in 2019, including approximately 1.2 million deaths due to second-hand smoke. Additionally, many indicators point to a rise in illicit drug use over the past decade, especially in Europe, with particular increases in the use of cocaine and methamphetamine. These alarming statistics highlight the importance of understanding addiction and its social dimensions. For instance, it is well-documented that the consumption of alcohol, tobacco, and drugs is influenced by peer behavior. Studies, such as those by Kremer and Levy (2008), Kremer and Levy (2007), and Powell et al. (2005), have shown that peer consumption plays a significant role in fostering these behaviors.

While the general mechanism of peer influence on private consumption is well-established, its specific impact on the demand for addictive goods raises additional questions, especially in relation to addiction. Addictive behaviors accumulate over time and the influence of peers can have lasting consequences on individual health. Despite the significant implications, the role of peer influence within social networks on addiction has received limited attention in the literature.

This paper addresses this gap by proposing a dynamic model of addiction, in which individuals are influenced by the consumption behaviors of their peers. We introduce a social network framework, where individual consumption is reinforced by the consumption of friends. The contemporaneous influence of peers not only affects current consumption but also shapes future consumption patterns. The extent of this impact depends on whether consumers are myopic or whether they consider the long-term consequences of their current consumption on future addiction. Specifically, we explore two key questions: how the structure of social networks influences the long-term demand for addictive goods, and how public policies can account for these network effects.

To explore these questions, we develop a model featuring a single addictive good, where consumers are influenced by the aggregate consumption of their social contacts. We specify linear-quadratic utility functions and multiplicative synergies among peers. Addiction is modeled according to modern economic theories, with two key settings: one where consumers are myopic, and addiction functions as habit formation—where past consumption increases the return on current consumption; and another in line with the rational addiction model, where consumers account for the negative consequences of their current consumption on their future health. Additionally, we incorporate time-inconsistency through hyperbolic discounting.

We present our results in several stages. First, we establish the impact of network position on long-run consumption. Whatever the behavior of consumers, Bonacich centrality shapes steady state consumption. Through that centrality measure, a consumer is affected not only by her neighbors, but also by all consumers in the network. We also compare the impact of forward-looking behavior, and we find a sharp result holding on any network: consumption is lower under more pronounced forward-looking attitude when the future benefits from higher consumption by peers is outweighed by the negative consequences on health.

We then undertake some comparative statics related to addiction characteristics and peer influence. The statics are not immediate, because those parameters affect not only the individual characteristics but also the intensity of interaction shaping the long-run demand for addictive good. We are then able to sign the overall effect on consumption levels, taking into account the change in sensitiveness to others' consumption at equilibrium.

We then turn to welfare considerations, in a utilitarian perspective. We pin down the efficient consumption as a function of the position on the network, and give conditions allowing to compare efficient and steady-state consumption, depending here again on the forces shaping the benefits from addiction and health prejudice. In particular, focusing on regular networks, we find that the efficient consumption can be higher than the equilibrium consumption when network density is sufficiently low.

Last, we consider public policies, aiming at decreasing the aggregate consumption levels. We first consider an homogeneous price increase, typically in the context of legal drugs like cigarettes, and we find that more central agents are more responsive to price variation. We then consider alternative policies, that can be oriented to legal or illegal drugs, by examining rehabilitation programs focused on a subset of consumers. Assuming limited budget, and given the networked influence, this results in a key-player type of policy, in which the treated agent does not necessarily fully stops her consumption. In particular, a rehabilitation program aiming at reducing the addiction characteristics of consumers leads to target an agent maximizing a specific centrality index; interestingly, this centrality index depends on the budget level, and it is highly sensitive to network structure.

Overall, our paper stresses how behaviors are impacted by network structure, and how policy intervention should take this into account.

Literature. This paper contributes to the literature on the demand for addictive goods, focusing on how social networks and peer influences affect individual consumption decisions. Several studies have documented the role of peers in influencing addiction-related behaviors.

For instance, Kremer and Levy (2008) find that students who are randomly assigned roommates who drank heavily before university significantly increase their own alcohol consumption. This effect persists even after the first year, suggesting that early exposure to heavy drinking can have long-lasting impacts on individual behavior. Similarly, Kremer and Levy (2007) show that the probability of an adolescent starting to smoke increases significantly if their friends smoke. Their estimates suggest a peer elasticity of 0.5, meaning that a 10 percent increase in smoking within the peer group leads to a 5 percent increase in individual consumption. In the same vein, Mir et al. (2011) demonstrates that marijuana use among high school students is strongly influenced by their close friends, with network effects amplifying the spread of risky behaviors.

Beyond the direct influence of peers, several economic models have examined how addiction is shaped by individual behavior over time, considering factors such as habits, rational addiction, and time-inconsistency. Pollak (1970) introduces the concept of habit formation, modeling how consumption decisions are influenced not only by current preferences or income but also by past consumption. This work suggests that people develop habits, which in turn affect their demand for goods over time. Becker and Murphy (1988) develop the theory of rational addiction, where individuals make consumption decisions about addictive goods based on a forward-looking utility maximization framework. According to this model, individuals weigh both the immediate satisfaction and the future costs of their addictive behaviors. Gruber and Kőszegi (2007) further extend this idea by introducing time-inconsistent preferences into the model. They argue that individuals often underestimate the future costs of addictive behaviors, which has important implications for public policy, particularly concerning the taxation of addictive goods.[1]

These models primarily focus on how inter-temporal decisions related to addiction are influenced by rational or forward-looking behavior. Our paper builds on this literature by incorporating social networks into the analysis of addiction. We introduce a detailed framework in which individuals' consumption decisions are influenced not only by their own preferences but also by the consumption behaviors of their peers, thereby capturing the social diffusion of addiction. This network-based approach complements existing models by adding a social dimension to the understanding of addiction[2].

Another strand of literature addresses the impact of health policies on group behaviors[3]. For example, Cutler and Gleaser (2010) show that when tobacco taxes

[1] Becker et al. (1994) provide evidence in favor of rationality in the context of cigarette addiction, but Gruber and Kőszegi (2007) argue that it is difficult for empirical studies to distinguish rationality from alternative models such as hyperbolic discounting.

[2] Reif (2019) models group influence on addictive behavior with closely related modeling assumptions. The main differences with our setup are that consumers are influenced by the mean consumption of the addictive good by other consumers in her reference group; i.e. there is no network in their analysis.

[3] There is also a literature about taxes and advertising restriction; see Chaloupka (1991), Carpenter and Cook (2008), Cawley and Ruhm (2012).

increase, consumption declines not only among the smokers directly affected but also among their non-smoking friends, indicating a social diffusion effect. This suggests that public policies targeting individual behaviors can have broader impacts on social networks. Building on this, our paper introduces a novel policy tool: key-player policies, which link the effectiveness of interventions to the structure of social networks. By identifying key players within a network, policymakers can target individuals whose behavior will have the most significant ripple effect on others, thereby optimizing the impact of public health interventions.

Last, our paper adds to the literature on network games. That literature has been initiated by Ballester et al. (2006), and pursued by Bramoullé et al. (2014). In that literature, few papers are more closely related to ours. First, Boucher et al. (2006) consider network games applied to risky behaviors, including cigarette and alcohol goods, and explore non linear best-responses. With respect to their work we provide micro-foundations to addictive behaviors on networks, by bridging the literature on dynamic addiction and the literature on network games. Regarding key-player policies, Ballester et al. (2006) provide the first analysis, in which the target is dropped out of the network. Belhaj and Deroïan (2018) extend the analysis to a setup with contracts, and where the contract affects the action of the target without necessarily inducing a drop out.

The remainder of the paper is organized as follows. We introduce our framework in Section 2. The equilibrium in consumption for addictive good under myopia is analyzed in Section 3, while Section 4 focuses on forward-looking consumers. Section 5 studies the efficient consumption levels and examines the trade-off between equilibrium and efficient consumption. We examine several public interventions in Section 6. Section 7 concludes the paper. All proofs are relegated to Section 8.

2 Model

We introduce a network of peers in a standard model of addiction, in a model that integrates rational addiction and time-inconsistent preferences. We consider a dynamical setting with an infinite number of discrete periods $t \in \{0, 1, 2, \cdots\}$, where a society of infinitely lived agents choose an individual consumption level of an addictive good at each period, and maximize the flow of their instantaneous utilities over all periods. Social peers influence agent's consumption at every period.

Networks. Let $\mathcal{N} = \{1, 2, \cdots, n\}$ represent a society with a finite number of agents. Agents interact through a network. The network is defined by its agency matrix $\mathbf{G}$, a binary and symmetric matrix representing social relationships. That is, the entry $g_{ij} = g_{ji} = 1$ if agent i and j are connected in the network, and $g_{ij} = g_{ji} = 0$ if agent i and j are not connected. By convention $g_{ii} = 0$ for all i. We denote by $N_i = \{j : g_{ij} = 1\}$ the set of agents linked to i in network $\mathbf{G}$. To avoid trivialities, we assume that the network is connected (no agent is isolated).

Instantaneous utilities. Let $c_{i,t}$ represent the consumption of addictive good by agent i at time t. Agent i derives instantaneous utility from consuming the addictive good at time t. This individual utility depends on the present consumption of the good $c_{i,t}$, the stock of past consumption $A_{i,t}$, and the consumption of the good by the neighbors in the network $\bar{c}_{-i,t} = \sum_{j \in \mathcal{N}} g_{ij} c_{j,t}$. That is, the instantaneous utility of agent i at time t can be expressed as

$$u_{i,t} = u(c_{i,t}, A_{i,t}, \bar{c}_{-i,t}), \tag{1}$$

with

$$A_{i,t} = (1-\gamma)A_{t-1} + c_{t-1}.$$

Note that the stock of past consumption is equivalently written as the discounted sum of all past consumption of the good, i.e., $A_{i,t} = \sum_{s=1}^{t}(1-\gamma)^{s-1}c_{i,t-s}$. The discounting of the sum reflects the idea that past consumption has less impact on present utility the further back in time it occurred. If the discounting parameter $\gamma \in]0,1]$ is large, it means that the consumption of addictive good will matter less in the long run.

Agents are influenced by the sum of consumption of her neighbors Following the literature on addiction[4], we specify an instantaneous utility function of linear-quadratic form:

$$u_{i,t}(c_{i,t}) = \alpha_c c_{i,t} - \frac{1}{2}\alpha_{cc}c_{i,t}^2 + \alpha_{cA}c_{i,t}A_{i,t} - \frac{1}{2}\alpha_{AA}A_{i,t}^2 - \frac{1}{2}\alpha_p(c_{i,t} - \bar{c}_{-i,t})^2. \quad (2)$$

with $\alpha_c > 0$, $\alpha_{cc} > 0$, $\alpha_{cA} > 0$, $\alpha_{AA} > 0$, $\alpha_p > 0$. This utility function can be decomposed into four parts. The first part is the utility directly associated with the consumption of the addictive good. It is increasing and concave. The second part represents the addictivity of the good. By $\alpha_{cA} > 0$, the more one has consumed the good in the past, the higher the marginal utility of the present consumption of the good. The third part represents the disutility generated by the past consumption of the good. When the addictive good is harmful, if consumption has been high in the past, this can deteriorate health and then generate disutility.

[4]See Becker and Murphy (1988), Becker et al. (1994), Chaloupka (1991), and Gruber and Köszegi (2001), Reif (2019), Piccoli and Tiezzi (2021).

Finally, the last part of the utility represents the peer effects. We use a conformity specification here, where consumer's utility depends negatively on the distance between their consumption and their neighbors' consumption.

Dynamical problem. Consumers maximize the discounted sum of their utilities over time (as in Becker and Murphy (1988)), in a rational addiction setting encompassing time-inconsistency (as in Laibson (1997) or Gruber and Köszegi (2001)).

$$\max_{c_{i,t}\geq 0}\{u_{i,t}+\beta\sum_{s=1}^{\infty}\delta^{s}u_{i,t+s}\}.$$

The sum of future utilities is discounted by two factors which play different roles. Parameter $\delta \in [0,1[$ is the classical preference for the present. It is the relative preference between two consecutive periods in time. Parameter $\beta \in [0,1]$ is the bias toward the present period. It is the relative preference between the present period and any future period.

Note that if $\beta = 0$ or $\delta = 0$ then consumers are myopic and only consider their current utility. If $\beta = 1$ and $\delta > 0$, then consumers are rational and their consumption choices in each period will be consistent with their past and future decisions. If $\beta \in]0,1[$ and $\delta > 0$, then the agents are biased toward the present.

We develop our analysis in Four stages. First, we elicit best-responses in the general case. Second, we study the long run behaviors under peer influence under myopic attitudes. Third, we analyze how farsightedness affects behaviors. Last, we explore public policies.

3 Myopic agents

In this section, we consider the myopic case, that is we assume $\beta = \delta = 0$. A key implication of peer influence is to induce complementarities in contemporaneous addictive consumption. Then, we present our main characterization result, by identifying the long run behaviors under networked peer influence. Last, we undertake comparative statics with respect to the main parameters of the model.

With this specification, the first-order condition given in Proposition 4 becomes:

Proposition 1. *Consider myopic agents. The best-response addictive good consumption of agent i in period t can be written as:*

$$c_{i,t} = \frac{1}{\alpha_{cc} + \alpha_p}\left(\alpha_c + \alpha_{cA}A_{i,t} + \alpha_p \bar{c}_{-i,t}\right). \tag{3}$$

By Proposition 1, the best-response is increasing in past consumption through the addictive stock: a higher past consumption increases the addictive stock, which pushes toward more current period consumption. Moreover, the best-response is increasing in neighbors' current consumption by conformism, i.e. the consumption of peers amplifies addiction.

This formulation has an exact counterpart on equilibrium path. Indeed, exploiting the linear relationship between current addictive stock and one-period lagged addictive stock, the best-response consumption of agent i in period t can be written as a function of current and preceding consumption profile as follows:

Corollary 1. *Consider myopic agents. The best-response addictive good consumption of agent i in period t can be written as:*

$$c_{i,t} = \frac{\gamma\alpha_c}{\alpha_{cc} + \alpha_p} + \left(1 - \gamma + \frac{\alpha_{cA}}{\alpha_{cc} + \alpha_p}\right)c_{i,t-1} + \alpha_p\left(\bar{c}_{i,t} - (1-\gamma)\bar{c}_{i,t-1}\right). \tag{4}$$

By Corollary 1, the entire history of past consumption levels condenses into the dependence of the last period's consumption profile on the best-response path. Not surprisingly, lagged consumption tends to foster current consumption through an additional induced addiction. Peer influence is now represented by the discounted difference between current and lagged peer consumption. One implication is that the relationship between peer consumption path and current consumption stays positive when all the consumption trends are increasing for every agent in the society, for all value of parameter γ.

We turn to the characterization of the consumption of addictive good at the steady state. Let $\lambda(\mathbf{G})$ denote the maximal eigenvalue of network $\mathbf{G}$. The next assumption guarantees the convergence of the dynamical system under myopia:

Assumption 1.

$$\lambda(\mathbf{G}) < \frac{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}}{\alpha_p}.$$

Proposition 2. *Consider myopic agents. Under Assumption 1, the vector of consumption of addictive good converges to*

$$\mathbf{c}_\infty = \kappa(\mathbf{I} - \mu\mathbf{G})^{-1}\mathbf{1}, \tag{5}$$

where

$$\kappa = \frac{\alpha_c}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}}, \mu = \frac{\alpha_p}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}}.$$

Proposition 2 has several implications. First, consumption levels are related to a centrality index, the Bonacich centrality; meaning that more central agents in the sense of that centrality measure have a greater propensity to consume. This is due to networked complementarities in consumption. When peers' influence is shaped by the average neighbors' consumption, it is well-known that heterogeneous characteristics (typically $\alpha_{c,i}$) is necessary to produce differentiated consumption.

Second, the impact of peers on long run consumption is amplified by addiction through parameter γ. That parameter affects not only the constant of the long run interaction system, but also the intensity of interaction.

It is instructive to examine the impact of the network of peers with respect to the no-peer case.

Comparative statics. We undertake comparative statics with respect to $\alpha_{cA}, \gamma, \alpha_p$. For any parameter of interest say α, the derivative of the steady state consumption profile with respect to a parameter α is given by

$$\frac{\partial \mathbf{c}_\infty}{\partial \alpha} = \frac{\partial \kappa}{\partial \alpha}\mathbf{b} + \kappa \frac{\partial \mu}{\partial \alpha}\frac{\partial \mathbf{b}}{\partial \mu}. \tag{6}$$

The network affects the derivative of steady state consumption twice: first, centrality acts as a multiplier effect of the derivative of the constant of the linear interaction system; second the centrality is itself affected through the variation of the intensity of interaction. It is therefore crucial to understand how centralities vary with the intensity of interaction. We find:

Lemma 1.

$$\frac{\partial \mathbf{b}}{\partial \mu} = \frac{1}{\mu}(\mathbf{b}_\mathbf{b} - \mathbf{b}). \tag{7}$$

Lemma 1 provides a simple formulae that links the marginal increase of Bonacich centrality with intensity of interaction to the difference between the weighted Bonacich centrality and the simple Bonacich centrality. Injecting (7) into (6), we get

$$\frac{\partial \mathbf{c}_\infty}{\partial \alpha} = \frac{\partial \kappa}{\partial \alpha}\mathbf{b} + \frac{\kappa}{\mu}\frac{\partial \mu}{\partial \alpha}(\mathbf{b}_\mathbf{b} - \mathbf{b}). \tag{8}$$

This allows us to sign the statics:

Proposition 3 (Comparative statics)**.** *The steady state addictive consumption is increasing in α_{cA}, decreasing in γ, and increasing in α_p.*

That the marginal effect of parameter α_{cA} on consumption is positive is the direct consequence of addiction. As well, lowering parameter γ increases the long run impact of the stock of past consumptions, thus increasing the steady state consumption. Last, a larger peer pressure, through increased parameter α_p, fosters long-run consumption irrespective of the position on the network.

4 Forward looking agents

In this section, we study forward looking behaviors. We focus on naïve version of rational addiction theory with possible time-inconsistency. That is, a naïve agent maximizes her intertemporal utility with a psychological attitude that makes her unaware of the fact that her future selves will revise her plans.

In that model, individual best-responses in period t can be expressed as follows:

Proposition 4. *Consider naïve forward-looking agents. The best-response addictive good consumption of agent i in period t can be written as:*

$$c_{i,t} = \tau_k + \tau_c^- c_{i,t-1} + \tau_c^+ c_{i,t+1} + \tau_p \bar{c}_{i,t} + \tau_p^- \bar{c}_{i,t-1} + \tau_p^+ \bar{c}_{i,t+1}, \tag{9}$$

With

$$\tau_k = \frac{\gamma(1-\delta(1-\gamma))\alpha_c}{\alpha_{cA}(\delta(1-\gamma))(\beta+1)+(\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1)-\beta\delta\alpha_{AA}},$$

$$\tau_c^- = \frac{\alpha_{cA}+(1-\gamma)(\alpha_{cc}+\alpha_p)}{\alpha_{cA}(\delta(1-\gamma))(\beta+1)+(\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1)-\beta\delta\alpha_{AA}},$$

$$\tau_c^+ = \frac{\delta(1-\gamma)(\alpha_{cc}+\alpha_p)+\beta\delta\alpha_{cA}}{\alpha_{cA}(\delta(1-\gamma))(\beta+1)+(\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1)-\beta\delta\alpha_{AA}},$$

$$\tau_p = \frac{\alpha_p(\delta(1-\gamma)^2+1)}{\alpha_{cA}(\delta(1-\gamma))(\beta+1)+(\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1)-\beta\delta\alpha_{AA}},$$

$$\tau_p^- = \frac{-\alpha_p(1-\gamma)}{\alpha_{cA}(\delta(1-\gamma))(\beta+1)+(\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1)-\beta\delta\alpha_{AA}},$$

$$\tau_p^+ = \frac{-\alpha_p\delta(1-\gamma)}{\alpha_{cA}(\delta(1-\gamma))(\beta+1)+(\alpha_{cc}+\alpha_p)(\delta(1-\gamma)^2+1)-\beta\delta\alpha_{AA}}.$$

Conform to the literature on forward-looking addiction, Proposition 4 shows that current consumption is positively related to future consumption, and it is negatively related to the future consumption of peers. We pursue with steady state characterization. The next assumption guarantees the convergence of the dynamical system under naïve forward looking behavior:

Assumption 2.

$$\lambda(\mathbf{G}) < \frac{\alpha_p+\alpha_{cc}-\frac{\alpha_{cA}}{\gamma}}{\alpha_p} + \beta\,\frac{\delta(\alpha_{AA}-\gamma\alpha_{cA})}{\gamma(1-\delta(1-\gamma))\alpha_p}.$$

Interestingly, the impact of parameter β on the convergence condition can be either positive or negative, depending on whether the ratio $\frac{\alpha_{AA}}{\alpha_{cA}}$ is lower or higher than $\frac{1}{\gamma}$. Convergence condition is more demanding under prevalence the health degradation dimension (through α_{AA}) than over addiction (through α_{cA}): indeed, under high ratio $\frac{\alpha_{AA}}{\alpha_{cA}}$, a forward-looking agent, anticipating more serious future health problems, will refrain his current consumption level, which relaxes the convergence condition. In opposite, when the ratio $\frac{\alpha_{AA}}{\alpha_{cA}}$ is low, the same forward-looking agent will rather privilege the future extra utility derived from additional current consumption.

Proposition 5. *Consider naïve forward-looking agents. Under Assumption 2, the steady state profile of addictive good consumption converges to:*

$$\mathbf{c}_\infty = \kappa(I - \mu\mathbf{G})^{-1}\mathbf{1}, \tag{10}$$

with

$$\kappa = \frac{\alpha_c}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\beta\delta}{\gamma}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}}, \mu = \frac{\alpha_p}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\beta\delta}{\gamma}\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}}.$$

We explore the comparative statics with respect to forward-looking behavior, by envisaging an increase in parameter δ, the time preference for the present, and an increase in parameter β, the time-inconsistent attitude.

Proposition 6. *The steady state consumption is always monotonic in parameters δ and β. It is decreasing in parameters δ, β if and only if*

$$\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma. \tag{11}$$

By Proposition 6, the statics, taking parameters separately, are always monotonic. The sign of those statics does not depend on the position on the network.

Regarding both preference for the present and the time-inconsistent attitude, when the ratio $\frac{\alpha_{AA}}{\alpha_{cA}} < \gamma$, higher parameter β or δ, i.e. less time-inconsistency or less preference for the present, fosters the consumption of addictive good. Hence, to be able to compare the consumption of two individuals such that one is more forward-looking and more time-inconsistent than another, it is key to understand how consumers perceive the impact of addiction on their health (through the ratio $\frac{\alpha_{AA}}{\alpha_{cA}}$).

In particular, the network characteristics does not affect the statics. Hence, forward looking behavior leads to lower long-run consumption than under myopia when inequality (11) holds, irrespective of the network structure.

5 Efficient consumption

We take a standard utilitarian approach, and we focus on long run behaviors. We specify the social welfare of profile $\mathbf{c}$ as

$$W(\mathbf{c}; \mathbf{G}) = \sum_{i \in \mathcal{N}} u_i(\mathbf{c}; \mathbf{G}).$$

The following proposition expresses how peer network affects the efficient long-run consumption

Proposition 7. *The vector of efficient consumption of addictive good is given by*

$$\hat{\mathbf{c}} = \kappa_E \Big(\mathbf{I} - \mu_E \mathbf{G}(\mathbf{I} - \frac{1}{2}\mathbf{G})\Big)^{-1} \mathbf{1}, \tag{12}$$

where

$$\kappa_E = \frac{\alpha_c}{\alpha_{cc} + \alpha_p + \frac{\alpha_{AA}}{\gamma^2} - \frac{2\alpha_{cA}}{\gamma}}, \mu_E = \frac{2\alpha_p}{\alpha_{cc} + \alpha_p + \frac{\alpha_{AA}}{\gamma^2} - \frac{2\alpha_{cA}}{\gamma}}.$$

By Proposition 7, the impact of the network structure is given by matrix $\mathbf{G}(\mathbf{I} - \frac{1}{2}\mathbf{G})$. The qualifying effect by the term $\mathbf{G}^2$ is inherent to the conformist component in utilities, and the efficient allocation takes care of the health degradation of agents' neighbors.

We turn to the comparison between steady state consumption and efficient consumption. The general analysis is cumbersome, still some partial insights are given here:

Corollary 2. *When $\frac{\alpha_{AA}}{\alpha_{cA}} < \gamma$, the efficient consumption is higher than the steady state consumption when α_p is sufficiently low.*

When $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$, the efficient consumption is lower than the steady state consumption when α_p is sufficiently low.

When $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$ and $\frac{\alpha_{AA}}{\alpha_p + \alpha_{cc}} > \gamma^2$, the efficient consumption is lower than the steady state consumption.

By Corollary 2, when health damage are sufficiently low as compared to the positive externalities stemming from the addiction, the efficient consumption exceeds the steady state consumption, since agents do not fully internalize the net positive externalities of their consumption. However, When $\frac{\alpha_{AA}}{\alpha_{cA}} < \gamma$, i.e. when health damages are sufficiently high, agents over-consume, as they don't fully internalize the consequence of addiction on health. For intermediary damage, the efficient consumption is lower than the steady state consumption under low peer effect, while high peer effect can lead to magnify the positive externalities from addiction and revert the order. Finally, for sufficiently low damages, the efficient consumption is higher than the steady state consumption under low peer effect.

The analysis of regular networks is instructive. Let

$$k_c(\beta,\delta) = \frac{1}{2} + \frac{1}{2\gamma\sqrt{\alpha_p}}\sqrt{\gamma^2\alpha_p - 4\Big(1 - \frac{\beta\delta\gamma}{1-\delta(1-\gamma)}\Big)(\alpha_{AA} - \gamma\alpha_{cA})}.$$

To proceed we consider parameters such that both steady state and long-run efficient consumption levels are finite for all regular networks. This is ensured when (i) $\alpha_p < \frac{1}{\gamma(n-2)}\Big(\gamma\alpha_{cc} - \alpha_{cA} + \beta\delta\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}\Big)$ and (ii) $\alpha_{cc} - \frac{2\alpha_{cA}}{\gamma} + \frac{\alpha_{AA}}{\gamma^2} > 0$. We then find:

Proposition 8. *Consider a regular network of degree k and assume (i) and (ii). When $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$, the efficient consumption is lower than the steady state consumption for all $k = \{0, \cdots, n-1\}$. When $\frac{\alpha_{AA}}{\alpha_{cA}} < \gamma$, the efficient consumption is higher than the steady state consumption when $k < k_c(\beta,\delta)$.*

Note that increasing the intensity of peer effects α_p can only increase the density threshold k_c. Hence, the higher intensity of peer effects α_p, the smaller the maximal network density below which the efficient allocation exceeds the steady state.

Proposition 8 provides two main messages. First, whether we get over or under consumption with respect to the efficient level is shaped by two conflicting forces. On the one hand, the positive externalities induced by peer effects, and that are not internalized by individuals, leads to under-consumption with respect to the efficient level. On the other hand, the health concern induced by addiction leads to over-consumption. In total, the efficient consumption gets lower than the equilibrium consumption when peer effects are low enough, which means a sufficiently low network density.

Second, the impact of forward looking behavior depends on the ratio $\frac{\alpha_{AA}}{\alpha_{cA}}$. For any network density: the network density above which the long-run consumption

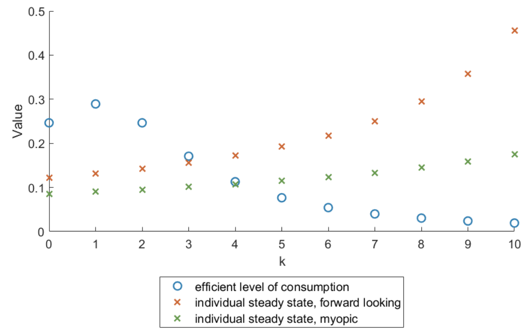

Figure 1: Efficient and steady state consumption levels as a function of network density k on regular networks; Blue curve represents the efficient consumption, the Yellow curve represents the myopic consumption, the Red curve represents the forward-looking consumption.

exceeds the efficient consumption is increasing with forward looking behavior (i.e. in parameters β, δ) whenever increased forward-looking attitude reduces consumption; that condition is related to whether inequality (11) holds, irrespective of network characteristics.

6 Public policy

In the face of harmful addictions, public intervention is justified when it generates external costs.[5] When agents are myopic, government intervention can for instance be justified because consumers don't take into account the future consequence on their health. This is no longer the case under rational addiction,

[5]Public intervention can also be justified under ignored internal costs associated with these behaviors.

because consumers then fully internalize these consequences. However, time-inconsistency restores the interest of public intervention (Gruber and Köszegi (2001)).

This being said, irrespective of whether consumers take the future into account in their choice, there are other forms of externalities. For instance, addiction affects relatives, friends, families (see Manning et al. (1991)). Moreover, the network of peers induces synergies in addiction, that can be an additional matter for the health of consumers. That is, the presence of peers in itself justifies public intervention.

In this section, we consider two policy interventions. We will assume throughout the section that $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$, meaning that the impact of the consumption of addictive good on health is a real concern leading the government to intervene in order to reduce the consumption of addictive good. We will first examine a variation of the price of a legal addictive good, like cigarette or alcohol. Then, we will explore a network-based key-player public policy, in the context of either legal or illegal addictive good, consisting in a rehabilitation program through adequate medicine[6].

[6] Assessing the performance of rehabilitation programs is a complex task. In that regard, relapse is often considered a part of the recovery process, and various factors, including the type of substance used, duration of use, and individual circumstances, can influence relapse rates. Comprehensive treatment programs that include medical, psychological, and social support components have been shown to improve outcomes and reduce the likelihood of relapse. There are few statistics on relapse rates and the effectiveness of rehabilitation programs. For instance, the National Institute on Drug Abuse (NIDA) reports that relapse rates for substance use disorders are between 40 percent and 60 percent, which is comparable to relapse rates for other chronic diseases such as hypertension or asthma. Additionally, McPheeters et al. (2023) found that approximately 50 percent of individuals with alcohol use disorders relapse within the first year following treatment. This study highlights the challenges in maintaining long-term sobriety and underscores the

Price variation. Harmful addictions being sensitive to price, the government can discourage these behaviors by taxation. Suppose that the price of addictive good contains a tax component that is chosen by the government. We examine the impact of peer networks for legal addictive goods like cigarette, by studying how price variation affects the demand. Define by p the unit price of the addictive good (considered fixed in time). The instantaneous individual utility then becomes

$$u_{i,t}(c_{i,t}) = (\alpha_c - p)c_{i,t} - \frac{1}{2}\alpha_{cc}c_{i,t}^2 + \alpha_{cA}c_{i,t}A_{i,t} - \frac{1}{2}\alpha_{AA}A_{i,t}^2 - \frac{1}{2}\alpha_p(c_{i,t} - \bar{c}_{-i,t})^2.$$

That is, price affects the steady state consumption through the constant of the system of interaction. Hence, considering forward-looking agents, under Assumption 2, the steady state profile of addictive good consumption is equal to

$$\mathbf{c}_\infty = \kappa(p)\mathbf{b}(\mu), \tag{13}$$

with $\mathbf{b}(\mu) = (I - \mu\mathbf{G})^{-1}\mathbf{1}$ the Bonacich centrality with decay parameter μ, and with

$$\kappa(p) = \frac{\alpha_c - p}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\beta\delta}{\gamma}\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)}}, \mu = \frac{\alpha_p}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\beta\delta}{\gamma}\frac{\alpha_{AA} - \gamma\alpha_{cA}}{1-\delta(1-\gamma)}}.$$

The price affecting the constant κ and not the decay parameter μ, clearly increasing the unit price entails a decrease in all steady state consumption levels, and individual consumption change is proportional to the agent's Bonacich centrality. Therefore:

Proposition 9. *Consider a legal addictive good. More central agents, in the sense of Bonacich centrality measure, are more responsive to price variation.*

need for ongoing support and intervention.

Proposition 9 proposes a peer-effect based micro-foundation to Chaloupka (1991), who estimates that more addicted (myopic) individuals are found to respond more to price, in the long run, than less addicted (myopic) individuals.

Proposition 9 has important practical insights regarding policy efficiency. First, not only more central agents consume more, but also they are more responsive to price increase. Thus, in opposite to alternative psychological mechanisms favoring the inertia of big consumers, the message is that the part of consumption that comes from peer effect is sensitive to a price change policy. Second, since steady state consumption is shaped by Bonacich centrality, the level of consumption predicts the sensitivity to price change. Hence, the policymaker does not need to observe the network to understand the sensitivity of a consumer to price change. Third, that simple proposition has potentially testable implication; roughly speaking, controlling for individual characteristics, a positive relationship between current consumption and consumption change may help tracking peer effects.

Key-player policy. Consider a key-player policy consisting in reducing the consumption of a given consumer by an exogenous amount. For instance, this can be the result of a rehabilitation program. By the presence of peer effects, this will affect the consumption of other consumers. Given peer effects, reduced individual consumption entails a reduction in consumption of peers. The optimal targeting policy may thus be a function of the structure of the network of peers.

To have a clue on how peer networks affect targeting, consider a policymaker with a budget to spend in the rehabilitation of one consumer, in the aim of of decreasing the aggregate addictive good consumption. This is a key-player policy. Depending on the budget, the reduction can be partial or total. Assume that with budget $\omega > 0$ spent to care about consumer i. We consider two possible rehabilitation program, one leading to a reduction of the private benefit of consuming the good, one corresponding to a reduction of addiction. In both cases, we assume a limited budget inducing partial rehabilitation for any treated consumer i. Recall that $\mathbf{M} = (I - \mu \mathbf{G})^{-1}$, and call the diagonal entry (i, i) of matrix $\mathbf{M}$, which is m_{ii}, the self-loop centrality of agent i.

We start with a policymaker providing a rehabilitation program to agent i, that entails a reduction in the private benefit α_c for agent i. Suppose that with budget ω the program leads to a private benefit α'_c such that $\kappa' = \kappa - f_i(\omega)$. Next proposition shows which agent to select and the impact on aggregate consumption.

Proposition 10. *Consider a policymaker undertaking a private benefit-oriented key-player policy, with a limited budget ω. Targeting agent i reduces the aggregate consumption to*

$$f_i(\omega) \cdot b_i. \tag{14}$$

Hence, the optimal policy consists in targeting the agent i maximizing $f_i(\omega) \cdot b_i$.

Proposition 10 shows that the optimal target maximizes Bonacich centrality when all consumers are equally sensitive to the rehabilitation program. Moreover, the optimal target does not depend on the budget level. More generally, the impact on aggregate consumption is the product of the individual consumption reduction in isolation (i.e. considering $\alpha_p = 0$ by the network centrality of the target).

We pursue with a policymaker providing a rehabilitation program focused on agent i, entailing a reduction in her addiction parameter α_{cA} (a very similar exercise can be done with a decrease in parameter γ). To simplify, we consider myopic agents. Suppose that with budget ω the program leads to a reduction of addiction through a change of parameter α_{cA} to α'_{cA}, such that $\alpha'_{cA} = \alpha_{cA} - \Delta_i(\omega)$, with function f_i increasing, and assuming $\lim_{\omega\to\infty} \Delta_i(\omega) \leq \alpha_{cA}$; so that $\alpha'_{cA} \geq 0$. Contemplating the system of linear interaction characterizing the steady state consumption, this entails a change in both constant and intensity of interaction in line i. The next proposition exploits this limited change. Let

$$f_i(\omega) = \frac{\alpha_p \Delta_i(\omega)}{\gamma} \cdot \frac{1}{(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma})(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\Delta_i(\omega)}{\gamma})}.$$

Then:

Proposition 11. *Consider a policymaker undertaking a private addiction-oriented key-player policy, with a limited budget ω leading to change α_{cA} to $\alpha_{cA} - \Delta_i(\omega) \geq 0$. The optimal key-player policy consists in choosing consumer i maximizing*

$$\mathbf{1}^T \mathbf{c}' - \mathbf{1}^T \mathbf{c} = -\kappa f_i(\omega) \cdot \frac{b_i^2}{\mu + f_i(\omega)(m_{ii} - 1)}. \tag{15}$$

Proposition 11 shows that the optimal target maximizes an index depending on both Bonacich centrality, self-loop centrality and budget. When all consumers are equally sensitive to the rehabilitation program, i.e. $f_i(\omega) = f(\omega)$ for all i, the optimal target maximizes the index

$$\frac{b_i^2}{\mu + f(\omega)(m_{ii} - 1)}.$$

Interestingly, the budget affects the optimal target. For low budget, the agent with highest centrality b_i is selected. For large budget, such that $\mu << f(\omega)(m_{ii} - 1)$, the optimal target maximizes the index $\frac{b_i^2}{(m_{ii}-1)}$, which is close to the so called inter-centrality index $\frac{bi^2}{m_{ii}}$. Which corresponds to a problem of dropping an agent out of the network (see Ballester et al. (2006)).

To illustrate, Figure 2 presents a network with 11 agents and three type of agents, respectively represented by agents 1, 2 and 3.

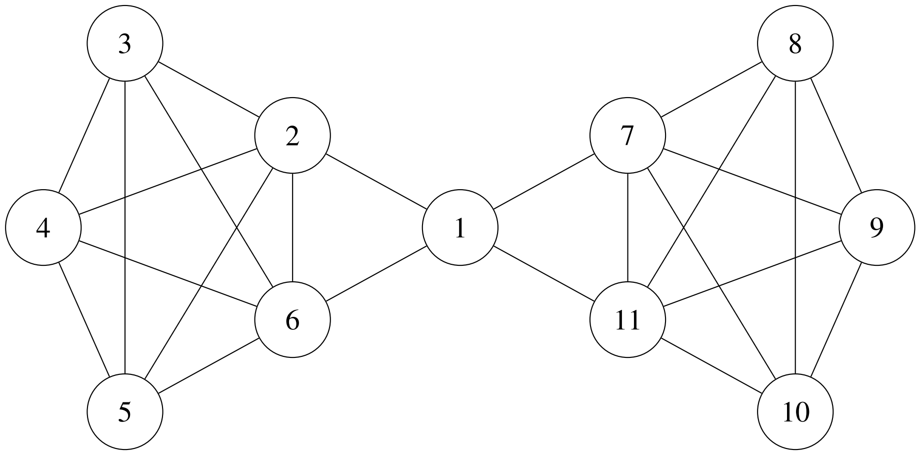

Figure 2:

Figure 3 represents the variation of aggregate consumption after a rehabilitation program reducing parameter α_{cA} for one agent in the network, as a function of the budget. The blue (resp. red, green) curve represents the effect of the policy applied to agent 1 (resp. 2, 3). For each budget level, the key player is the agent with the lowest curve. This is agent 2 if the budget is low, and this is agent 1 if the budget is high.

To confirm the illustration, we examine the star network:

Corollary 3. *Consider the star network. The key player is the central agent when* $\omega < \omega_c := f^{-1}(2 + \mu n)$ *while the key player is a peripheral agent if* $\omega > \omega_c$*.*

The proof rests on the fact that the Bonacich centrality is higher for the central for any intensity of interaction, while the ratio $\frac{b_i^2}{m_{ii}-1}$ is always higher for a peripheral agent (see the proof of Corollary 3). Tuning the budget from 0 to sufficiently large level, the result follows.

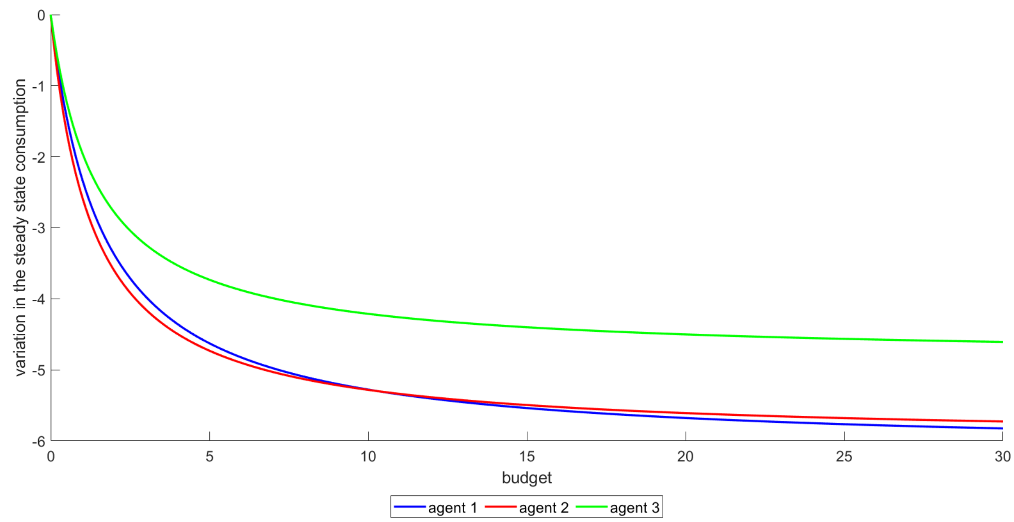

Figure 3: Variation of aggregate consumption after a rehabilitation program reducing parameter α_{cA} for one agent in the network, as a function of the budget. The blue (resp. red, green) curve represents the effects of the policy to agent 1 (resp 2, 3).

7 Conclusion

This paper has addressed the impact of a network of peers on the demand for addictive good. We modeled peer influence as a linear-in-sum model. The analysis shows the role of the Bonacich centrality in shaping steady state consumption levels, under both myopic and forward-looking attitudes. The analysis also stressed how network characteristics affect the trade off between equilibrium consumption and efficient consumption. Last, a public policy intervention aiming at reducing addictive good consumption should take care of network effects, either in a context of addictive good taxation, or in a context of rehabilitation program.

It would be useful to understand deeper how peer influence varies with the duration and the level of consumption in addictive good. In that respect, it would be challenging exploring further the endogenous network formation of peer networks, when incentives to form links are closely related to the addiction consumption levels of involved partners. On the empirical ground, Boucher et al. (2006) find that for addictive behavior like drinking, conformism is a reasonable specification, and that people tend to rely on less active agents rather than mean. It would thus be useful to bring data to the testable implications of this model.

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8 Proofs

8.1 Proofs under myopia

Proof of Corollary 1. By Corollary 1, the best-response consumption levels of agent i in period t and $t-1$ satisfy:

$$\begin{cases} \alpha_c - \alpha_c c c_{i,t} + \alpha_{cA} A_t - \alpha_p c_{i,t} + \alpha_p \bar{c}_{i,t} = 0, \\ \alpha_c - \alpha_c c c_{i,t-1} + \alpha_{cA} A_{t-1} - \alpha_p c_{i,t-1} + \alpha_p \bar{c}_{i,t-1} = 0. \end{cases}$$

Taking the first equation minus $(1-\gamma)$ times the second equation, we get:

$$\gamma\alpha_c - (\alpha_{cc} + \alpha_p)(c_{i,t} - (1-\gamma)c_{i,t-1}) + \alpha_{cA}(A_t - (1-\gamma)A_{t-1}) + \alpha_p(\bar{c}_{i,t} - (1-\gamma)\bar{c}_{i,t-1}) = 0.$$

Observing that $A_t - (1-\gamma)A_{t-1} = c_{t-1}$ and rearranging, we find :

$$c_{i,t} = \frac{\gamma\alpha_c}{\alpha_{cc} + \alpha_p} + \Big(1 - \gamma + \frac{\alpha_{cA}}{\alpha_{cc} + \alpha_p}\Big)c_{i,t-1} + \alpha_p\Big(\bar{c}_{i,t} - (1-\gamma)\bar{c}_{i,t-1}\Big).$$

□

Proof of proposition 2. First, the consumption of addictive good converges if $\lambda(\mathbf{G}) < \frac{1}{\mu}$, i.e.

$$\lambda(\mathbf{G}) < \frac{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}}{\alpha_p}.$$

This is guaranteed by Assumption 1.

The computation of the steady state consumption is straightforward.

□

Proof of lemma 1. Let's pose $\mathbf{M} = (I - \mu\mathbf{S})^{-1}$ and $\mathbf{b} = \mathbf{M}\mathbf{1}$. Notice that $\mathbf{M}$ is the limit of a Neumann series :

$$(I - \mu\mathbf{S})^{-1} = \sum_{k=0}^{\infty} (\mu\mathbf{S})^k.$$

So,

$$\begin{aligned}\frac{\partial \mathbf{b}}{\partial \mu} &= \sum_{k=1}^{\infty} k\mu^{k-1}\mathbf{S}^k\mathbf{1} \\ &= S\sum_{k=1}^{\infty} k\mu^{k-1}\mathbf{S}^{k-1}\mathbf{1} \\ &= S\sum_{j=0}^{\infty} (j+1)\mu^{j}\mathbf{S}^{j}\mathbf{1}.\end{aligned}$$

If we denote $\mathbf{Z} = \sum_{j=0}^{\infty} j\mu^j\mathbf{S}^j$, we have :

$$\frac{\partial \mathbf{b}}{\partial \mu} = \mathbf{S}(\mathbf{Z}+\mathbf{M})\mathbf{1}.$$

We can now write $\mathbf{Z}$ as in function of M:

$$\begin{aligned}\mathbf{Z} &= \sum_{j=1}^{\infty} j\mu^j\mathbf{S}^j \\ &= \sum_{j=1}^{\infty} \mu^j\mathbf{S}^j + \mu\mathbf{S}\sum_{j=1}^{\infty}(j-1)\mu^{j-1}\mathbf{S}^{j-1} \\ &= \sum_{j=1}^{\infty} \mu^j\mathbf{S}^j + \mu\mathbf{S}\mathbf{Z} \\ &= (\mathbf{M}-\mathbf{I}) + \mu\mathbf{S}\mathbf{Z}.\end{aligned}$$

Which is equivalent to :

$$\begin{aligned}\mathbf{Z} &= (\mathbf{I}-\mu\mathbf{S})^{-1}(\mathbf{M}-\mathbf{I}) \\ &= \mathbf{M}(\mathbf{M}-\mathbf{I}).\end{aligned}$$

It follows that $\frac{\partial \mathbf{b}}{\partial \mu} = \mathbf{SM}^2$. Yet,

$$\begin{aligned}\mathbf{SM} &= \mathbf{S}\sum_{k=0}^{\infty}(\mu S)^k \\ &= \frac{1}{\mu}\sum_{k=0}^{\infty}(\mu \mathbf{S})^{k+1} \\ &= \frac{1}{\mu}(\sum_{k=0}^{\infty}(\mu \mathbf{S})^k - I) \\ &= \frac{\mathbf{M}-\mathbf{I}}{\mu}.\end{aligned}$$

So,

$$\begin{aligned}\frac{\partial \mathbf{b}}{\partial \mu} &= \frac{1}{\mu}(\mathbf{M}-\mathbf{I})\mathbf{M}\mathbf{1} \\ &= \frac{1}{\mu}(\mathbf{M}^2-\mathbf{M})\mathbf{1}.\end{aligned}$$

Or, denoting $\mathbf{M}^2\mathbf{1} = \mathbf{b_b}$[7]:

$$\frac{\partial \mathbf{b}}{\partial \mu} = \frac{1}{\mu}(\mathbf{b_b} - \mathbf{b}).$$

□

Proof of proposition 3. Derivative of the steady state consumption with respect to α_{cA}. A direct check shows $\frac{\partial \kappa}{\partial \alpha_{cA}} > 0, \frac{\partial \mu}{\partial \alpha_{cA}}$. We then deduce from (8) that $\frac{\partial \mathbf{c}_\infty}{\partial \alpha_{cA}} > 0$.

Derivative of the steady state consumption with respect to γ. A direct check shows $\frac{\partial \kappa}{\partial \gamma} < 0, \frac{\partial \mu}{\partial \gamma} < 0$. We then deduce from (8) that $\frac{\partial \mathbf{c}_\infty}{\partial \gamma} < 0$.

Derivative of the steady state consumption with respect to α_p.

[7] If $\mathbf{b_a} = \sum_j m_{ij} a_j$ represents the Bonacich profile weighted by vector $\mathbf{a}$, $\mathbf{b_b}$ represents the Bonacich vector weighted by the unweighted Bonacich vector.

Define $\phi = \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}$. Direct computation entails that $\frac{\partial \mathbf{c}_\infty}{\partial \alpha_p} > 0$ if and only if

$$\frac{\phi}{\alpha_p + \phi}\mathbf{b}_\mathbf{b} > \mathbf{b}. \tag{16}$$

Given that

$$\frac{\phi}{\alpha_p + \phi} = \frac{\phi}{\alpha_p}\mu = (\frac{1}{\mu} - 1)\mu = 1 - \mu,$$

equation (16) is also written

$$(1 - \mu)\mathbf{b}_\mathbf{b} > \mathbf{b},$$

i.e., letting $\mathbf{0}$ denote the n-dimensional vector of zeros,

$$(1 - \mu)\mathbf{M}^2\mathbf{1} > \mathbf{M1},$$

i.e.,

$$\mathbf{M}^2\mathbf{1} - \mathbf{M1} - \mu\mathbf{M}^2\mathbf{1} > \mathbf{0},$$

i.e.,

$$\mathbf{M}(\mathbf{M} - \mathbf{I})\mathbf{1} - \mu\mathbf{M}^2\mathbf{1} > \mathbf{0},$$

i.e., given that $\mathbf{M} - \mathbf{I} = \mu\mathbf{GM}$,

$$\mu(\mathbf{G} - \mathbf{I})\mathbf{M}^2\mathbf{1} > \mathbf{0},$$

i.e., given that $\mathbf{GM} = \mathbf{MG}$ on undirected networks,

$$\mathbf{M}^2(\mathbf{G} - \mathbf{I})\mathbf{1} > \mathbf{0}, \tag{17}$$

Now, letting vector $\mathbf{D} = \mathbf{G1}$ represent the profile of degrees,

$$(\mathbf{G} - \mathbf{I})\mathbf{1} = \mathbf{D} - \mathbf{1} \geq 0,$$

as we assume that there is no isolated agent. As $\mathrm{M}^2 > \mathbf{0}$, it follows that inequality (17) holds.

□

8.2 Proofs under rational addiction

Proof of proposition 4. First, we can calculate instantaneous utility as a function of past and present consumption of addictive goods by replacing $A_{i,t}$ by its expression as a function of past consumption $U_t(c_{i,t},..,c_{i,0})$. As a reminder we have :

$$A_{i,t} = \sum_{j=0}^{t-1} (1-\gamma)^{t-1-j} c_j.$$

So,

$$U_t(c_{i,t},..,c_{i,0}) = \alpha_c c_{i,t} + \frac{1}{2}\alpha_{cc} c_{i,t}^2 + \alpha_{cA} c_{i,t} \sum_{j=0}^{t-1} (1-\gamma)^{t-1-j} c_j + \alpha_{AA} \sum_{j=0}^{t-1} (1-\gamma)^{t-1-j} c_j + \alpha_p c_{i,t} \bar{c}_{-i,t}.$$

Let's denote :

$$\begin{aligned} U(c_{i,t},...,c_{i,0}) &= U_t(c_{i,t},...,c_{i,0}) + \beta \textstyle\sum_{j=1}^{\infty} \delta^j U_{t+j}(c_{i,t+j},...,c_{i,t},...,c_{i,0}), \\ U_{c_{i,t}} &= \tfrac{\partial U_t}{\partial c_{i,t}}, \\ U_{A_{i,t+j}} &= \tfrac{\partial U_{t+j}}{\partial c_{i,t}}. \end{aligned}$$

At the individual optimum, at time t, individual i chooses $c_{i,t}$ such that :

$$\begin{aligned} & \tfrac{\partial U}{\partial c_{i,t}} = 0, \\ \iff \quad & U_{c_{i,t}} = -\beta \textstyle\sum_{j=1}^{\infty} \delta^j U_{A_{i,t+j}}. \end{aligned}$$

Noting that :

$$U_{A_{i,t+j}} = (1-\gamma)^{j-1}(\alpha_{cA} c_{i,t+j} - \alpha_{AA} \sum_{s=0}^{t+j+1} (1-\gamma)^{t+j-s-1} c_s).$$

We can compute the two following differences :

$$\delta(1-\gamma)U_{c_{i,t}} - U_{c_{i,t-1}} = \beta\delta(\alpha_{cA}c_{i,t} - \alpha_{AA}\sum_{s=0}^{t+1}(1-\gamma)^{t-s-1}c_s), \tag{18}$$

$$\delta(1-\gamma)U_{c_{i,t+1}} - U_{c_{i,t}} = \beta\delta(\alpha_{cA}c_{i,t+1} - \alpha_{AA}\sum_{s=0}^{t+2}(1-\gamma)^{t-s}c_s)). \tag{19}$$

The next step is to compute the difference (19) $-(1-\gamma)$(18). The aim of this operation is to eliminate the discounted sum of all past consumption.

$$(5) - (1-\gamma)(4),$$

$$\implies \delta(1-\gamma)U_{c_{i,t+1}} - (\delta(1-\gamma)^2+1)U_{c_{i,t}} + (1-\gamma)U_{c_{i,t-1}} = \beta\delta\Big((\alpha_{cA}c_{i,t+1} - \alpha_{AA}\sum_{s=0}^{t+2}(1-\gamma)^{t-s}c_s) - (1-\gamma)(\alpha_{cA}c_{i,t} - \alpha_{AA}\sum_{s=0}^{t+1}(1-\gamma)^{t-s-1}c_s)\Big).$$

With :

$$U_{c_{i,t}} = \alpha_c + \alpha_{cc}c_{i,t} + \alpha_{cA}\sum_{j=0}^{t-1}(1-\gamma)^{t-1-j}c_j - \alpha_p(c_{i,t} - \bar{c}_{-i,t}).$$

We then replace $U_{c_{i,t}}$ by its expression above :

$$\alpha_c\phi_c + \alpha_{cc}\phi_{cc} + \alpha_{cA}(\phi_{cA1} + \phi_{cA2}) + \alpha_p\phi_p + \alpha_{AA}\phi_{AA}.$$

With :

$$
\begin{aligned}
\phi_c &= \delta(1-\gamma) - \delta(1-\gamma)^2 - 1 + 1 - \gamma \\
&= \delta(1-\gamma)(1-1+\gamma) - \gamma \\
&= \gamma(\delta(1-\gamma) - 1), \\
\phi_{cc} &= -\delta(1-\gamma)c_{i,t+1} + (\delta(1-\gamma)^2+1)c_t - (1-\gamma)c_{t-1} \\
\phi_{cA1} &= \delta(1-\gamma)\sum_{s=0}^{t}(1-\gamma)^{t-s}c_s - (\delta(1-\gamma)^2+1)\sum_{s=0}^{t-1}(1-\gamma)^{t-1-s}c_s + (1-\gamma)\sum_{s=0}^{t-2}(1-\gamma)^{t-2-s}c_s, \\
&= \delta(1-\gamma)(c_t + (1-\gamma)c_{t-1} + \sum_{s=0}^{t-2}(1-\gamma)^{t-s}c_s) - (\delta(1-\gamma)^2+1)(c_{t-1} + \sum_{s=0}^{t-2}(1-\gamma)^{t-1-s}c_s) + (\\
&= \delta(1-\gamma)c_t + (\delta(1-\gamma)^2 - \delta(1-\gamma)^2 - 1)c_{t-1} + (\delta(1-\gamma)^3 - (\delta(1-\gamma)^3 + (1-\gamma)) + (1-\gamma)) \\
&= \delta(1-\gamma)c_t - c_{t-1}, \\
\phi_{cA2} &= -\beta\delta(c_{t+1} - (1-\gamma)c_t), \\
\phi_p &= (\delta(1-\gamma)^2+1)(c_t - \bar{c}_t) - \delta(1-\gamma)(c_{t+1} - \bar{c}_{t+1}) - (1-\gamma)(c_{t-1} - \bar{c}_{t-1}), \\
\phi_{AA} &= -\beta\delta(\sum_{s=0}^{t}(1-\gamma)^{t-s}c_s - (1-\gamma)\sum_{s=0}^{t-1}(1-\gamma)^{t-s-1}c_s) \\
&= -\beta\delta c_t.
\end{aligned}
$$

We then regroup the terms in c_t,$c_{t-1}$and c_{t+1} to find equation (1).

□

Proof of proposition 5. First, the consumption of addictive good converges if $\lambda(\mathbf{G}) < \frac{1}{\mu}$:

$$
\lambda(\mathbf{G}) < \frac{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \beta\frac{\delta(\alpha_{AA} - \gamma\alpha_{cA})}{\gamma(1-\delta(1-\gamma))}}{\alpha_p}.
$$

This is guaranteed by Assumption 2.

Then, we compute the steady state by setting $c_{i,t} = c_{i,t-1} = c_{i,t-2} = c_{i,\infty}$ and $\bar{c}_{-i,t} = \bar{c}_{-i,t-1} = \bar{c}_{-i,t-2} = \bar{c}_{-i,\infty}$. We find the individual steady state :

$$c_{i,\infty} = \kappa + \mu \bar{c}_{-i,\infty}, \tag{20}$$

with

$$\kappa = \frac{\tau_k}{\tau_c^+ - \tau_c + \tau_c^-},$$
$$\mu = -\frac{\tau_p^+ + \tau_p + \tau_p^-}{\tau_c^+ - \tau_c + \tau_c^-}.$$

The equilibrium vector of consumption is found by writing the system of n equations of individual steady state in matrix form :

$$\mathbf{c}_\infty = \kappa \mathbf{1} + \mu \mathbf{G} \mathbf{c}_\infty$$
$$\Leftrightarrow \mathbf{c}_\infty = \kappa (I - \mu \mathbf{G})^{-1} \mathbf{1}.$$

□

8.3 Proofs efficiency

Proof of Proposition 7. We consider the long run utilities, i.e. utilities at the steady state.

$$u_i(\mathbf{c}, \mathbf{G}) = \alpha_c c_i - \frac{\alpha_{cc}}{2} c_i^2 + \alpha_{cA} c_i A_i - \frac{\alpha_{AA}}{2} A_i^2 - \frac{1}{2} \alpha_p (c_i - \bar{c}_i)^2.$$

Let $\psi = \alpha_{cc} + \alpha_p + \frac{\alpha_{AA}}{\gamma^2} - \frac{2\alpha_{cA}}{\gamma}$.Taking $A_i = \frac{c_i}{\gamma}$ in the long run, and aggregating, we get

$$W(\mathbf{c}_\infty, \mathbf{G}) = \alpha_c \sum_i c_{\infty,i} - \frac{1}{2} \psi \sum_i c_{\infty,i}^2 + \alpha_p \sum_i c_{\infty,i} \bar{c}_{\infty,i} - \frac{\alpha_p}{2} \sum_i \bar{c}_{\infty,i}^2.$$

Let $g_{ij}^{(2)} = \sum\limits_{k} g_{ik}g_{kj}$ be the entry (i,j) of matrix $\mathbf{G}^2$. Then, $\frac{\partial W}{\partial c_i} = 0$ if and only if

$$\psi c_{\infty,i} - 2\alpha_p \sum_j \left(g_{ij} - \frac{1}{2} g_{ij}^{(2)} \right) c_{\infty,j} = \alpha_c.$$

□

Proof of Corollary 2. Assume $\frac{\alpha_{AA}}{\alpha_{cA}} \leq \gamma$. Then,

$$\alpha_{cc} + \alpha_p + \frac{\alpha_{AA}}{\gamma^2} - \frac{2\alpha_{cA}}{\gamma} = \alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{1}{\gamma}\left(\frac{\alpha_{AA}}{\gamma} - \alpha_{cA}\right) \leq \alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma}.$$

This entails that $\kappa_E \geq \kappa$. Furthermore, $\mu_E > \frac{\alpha_p}{\psi} \geq \mu$.

Assume now $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$. Then, $\kappa_E < \kappa$. This implies that $\hat{\mathbf{c}} < \mathbf{c}_\infty$ when α_p tends to 0.

Last, suppose when $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma$ and $\frac{\alpha_{AA}}{\alpha_p + \alpha_{cc}} > \gamma^2$. Then, $\kappa_E < \kappa$ and $\mu_E < \mu$. This implies $\hat{\mathbf{c}} < \mathbf{c}_\infty$ for any $\alpha_p > 0$.

□

Proof of Proposition 8. The proof is given for any value of parameters β, δ (whereas the corollary is stated for $\beta = \delta = 0$).

Consider a regular network $\mathbf{G}$ of degree $k \in \{0, 1, \cdots, n-1\}$. Fix β, δ. The individual steady state consumption is given by

$$c_\infty(k) = \frac{\alpha_c}{\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} - \alpha_p k + \beta\delta \frac{\alpha_{AA} - \gamma\alpha_{cA}}{\gamma(1-\delta(1-\gamma))}}.$$

As well, noticing that $\mathbf{G}^2\mathbf{1} = k^2\mathbf{1}$, few computation show that the individual efficient consumption is given by

$$\hat{c}(k) = \frac{\alpha_c}{\alpha_{cc} + \alpha_p - \frac{\alpha_{cA}}{\gamma} + \frac{1}{\gamma}(\frac{\alpha_{AA}}{\gamma} - \alpha_{cA}) - 2\alpha_p k + \alpha_p k^2}.$$

Assume first $k = 0$ (empty network). Then, $c_\infty(k) < \hat{c}(k)$ if and only if $\frac{\alpha_{AA}}{\gamma} - \alpha_{cA} < \beta\delta\frac{\alpha_{AA}-\gamma\alpha_{cA}}{1-\delta(1-\gamma)}$; that is

$$(\alpha_{AA} - \gamma\alpha_{cA})\Big(1 - \delta(1 - \gamma(1 - \beta))\Big) < 0.$$

And given that the second bracket is positive, we get that $c_\infty(k) < \hat{c}(k)$ if and only if $\alpha_{AA} < \gamma\alpha_{cA}$.

Assume then $k \geq 1$. Then, $c_\infty(k) < \hat{c}(k)$ if and only if

$$k^2 - k + \frac{(1 - \frac{\beta\delta\gamma}{1-\delta(1-\gamma)})(\alpha_{AA} - \gamma\alpha_{cA})}{\gamma^2\alpha_p} < 0.$$

The discriminant of this second-order polynomial is negative whenever the ratio $\frac{\alpha_{AA}}{\alpha_{cA}} > \gamma + \frac{\gamma^2\alpha_p}{4\alpha_{cA}} \cdot \frac{1-\delta(1-\gamma)}{1-\delta(1-\gamma)-\beta\delta\gamma}$; that latter condition thus implies $c_\infty(k) > \hat{c}(k)$. Otherwise, we get two roots $k' < \frac{1}{2}$ and

$$k'' = \frac{1}{2} + \frac{1}{2\gamma\sqrt{\alpha_p}}\sqrt{\gamma^2\alpha_p - 4\Big(1 - \frac{\beta\delta\gamma}{1 - \delta(1-\gamma)}\Big)(\alpha_{AA} - \gamma\alpha_{cA})}.$$

We then get $c_\infty(k) < \hat{c}(k)$ if and only if $k < k''$. Now, observe that $k" < 1$, meaning $c_\infty(k) > \hat{c}(k)$, if and only if

$$\frac{4}{\gamma^2\alpha_p}\Big(1 - \frac{\beta\delta\gamma}{1 - \delta(1-\gamma)}\Big)(\alpha_{AA} - \gamma\alpha_{cA}) > 0$$

That is $\alpha_{AA} - \gamma\alpha_{cA} > 0$.

□

8.4 Proofs Public policy

Proof of Proposition 10. Let consumer i have a reduction of its private benefit following the rehabilitation program. Then, in the linear system of interaction defining steady state consumption, $\kappa' = \kappa - f_i(\omega)$ for agent i only, all other constants being fixed. Let $\mathbf{1}_i = (0, \cdots, 0, 1, 0, \cdots, 0)^T$ be the vector with entry 1 in position i, zero otherwise. Then

$$(\mathbf{I} - \mu\mathbf{G})\mathbf{x} = \kappa\mathbf{1} - f_i(\omega)\mathbf{1}_i,$$

that is, recalling $\mathbf{b} = \mathbf{M}\mathbf{1}$,

$$\mathbf{x} = \kappa\mathbf{b} - f_i(\omega)\mathbf{M}\mathbf{1}_i.$$

Hence,

$$\mathbf{1}^T\mathbf{x} - \kappa\mathbf{1}^T\mathbf{b} = f_i(\omega)\mathbf{b}_i.$$

□

Proof of Proposition 11. Let consumer i face a reduction of its addiction parameter α_{cA} following the rehabilitation program. First note that $\lim\limits_{\omega\to\infty} \Delta_i(\omega) \leq \alpha_{cA}$ guarantees a positive steady state consumption after the rehabilitation program. Then, line i in the linear system of interaction defining steady state consumption is modified as follows: for agent i (only), μ is modified by $\mu' = \mu - f_i(\omega)$ and κ is modified by $\kappa' = \kappa - h_i(\omega)$, with

$$f_i(\omega) = \frac{\alpha_p\Delta(\omega)}{\gamma} \cdot \frac{1}{(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma})(\alpha_p + \alpha_{cc} - \frac{\alpha_{cA}}{\gamma} + \frac{\Delta_i(\omega)}{\gamma})},$$

and $h_i(\omega) = \frac{\alpha_c}{\alpha_p} f_i(\omega)$. Then, denoting the initial consumption $\mathbf{c} = \kappa\mathbf{M}\mathbf{1}$, and the modified inverse linear matrix of the modified system $\mathbf{M}' = \mathbf{M} - f_i(\omega)\mathbf{W}$, and the modified consumption profile $\mathbf{c}' = \mathbf{M}'(\kappa\mathbf{1} - h_i(\omega)\mathbf{1}_i)$, we find that

$$\mathbf{c}' = (\mathbf{M} - f_i(\omega)\mathbf{W})\Big(\kappa\mathbf{1} - \frac{\alpha_c}{\alpha_p} f_i(\omega)\mathbf{1}_i\Big),$$

so that

$$\mathbf{1}^T\mathbf{c}' - \mathbf{1}^T\mathbf{c} = -\kappa f_i(\omega)\mathbf{1}^T\mathbf{W}\mathbf{1} - \frac{\alpha_c}{\alpha_p} f_i(\omega)\mathbf{1}^T\mathbf{M}\mathbf{1}_i + \frac{\alpha_c}{\alpha_p} f_i(\omega)^2\mathbf{1}^T\mathbf{W}\mathbf{1}_i. \quad (21)$$

We need to identify matrix $\mathbf{W}$. To proceed, we use the Sherman-Morrison formulae, that states the following property: Suppose $\mathbf{Q}$ is an invertible n-square matrix with real entries and $\mathbf{r}, \mathbf{s} \in \mathbb{R}^n$ are column vectors. Then $\mathbf{Q} + \mathbf{r}\mathbf{s}^T$ is invertible if and only if $1 + \mathbf{s}^T\mathbf{Q}^{-1}\mathbf{r} \neq 0$. If $\mathbf{Q} + \mathbf{r}\mathbf{s}^T$ is invertible, its inverse is given by

$$(\mathbf{Q} + \mathbf{r}\mathbf{s}^T)^{-1} = \mathbf{Q}^{-1} - \frac{\mathbf{Q}^{-1}\mathbf{r}\mathbf{s}^T\mathbf{Q}^{-1}}{1 + \mathbf{s}^T\mathbf{Q}^{-1}\mathbf{r}}.$$

Let vector $\mathbf{g}_i = (g_{i1}, \cdots, g_{in})^T$ be the i's column of matrix $\mathbf{G}$; let matrix $\mathbf{G}_i$ be the n-square matrix with row i equal to row i in matrix $\mathbf{G}$ and all other rows with zero entries. We apply this formula with $\mathbf{Q} = \mathbf{I} - \mu\mathbf{G}$, $\mathbf{r} = f_i(\omega)\mathbf{1}_i$ and $\mathbf{s} = \mathbf{g}_i$. We then find $\mathbf{M}' = \mathbf{M} - f_i(\omega)\mathbf{W}$ with

$$\mathbf{W} = \frac{\mathbf{M}\mathbf{G}_i\mathbf{M}}{1 + f_i(\omega)\sum_k g_{ik}m_{ki}}.$$

That is, recalling that $\mu\mathbf{G}\mathbf{M} = \mathbf{M} - \mathbf{I}$,

$$\mathbf{M}' = \mathbf{M} - f_i(\omega)\mu\frac{\mathbf{M}\mathbf{G}_i\mathbf{M}}{\mu + f_i(\omega)(m_{ii} - 1)},$$

and after simplification of the numerator, we get

$$\mathbf{m}'_{kl} = \mathbf{m}_{kl} - f_i(\omega)\frac{m_{ki}(m_{il} - 1_{l=i})}{\mu + f_i(\omega)(m_{ii} - 1)},$$

where $1_{l=i}$ means 1 if $l = i$, 0 otherwise. Plugging that expression into the aggregate steady state consumption, we find

$$\mathbf{1}^T\mathbf{W}\mathbf{1} = \frac{b_i(b_i - 1)}{\mu + f_i(\omega)(m_{ii} - 1)}. \quad (22)$$

Also,

$$\mathbf{1}^T\mathbf{W}\mathbf{1}_i = \frac{b_i(m_{ii}-1)}{\mu + f_i(\omega)(m_{ii}-1)}. \tag{23}$$

Noticing that $\mathbf{1}^T\mathbf{M}\mathbf{1}_i = b_i$ and plugging (22) and (23) into (21), and recalling that $\frac{\alpha_c}{\alpha_p}\mu = \kappa$, we find after rearrangement

$$\mathbf{1}^T\mathbf{c}' - \mathbf{1}^T\mathbf{c} = -\kappa f_i(\omega)\cdot\frac{b_i^2}{\mu + f_i(\omega)(m_{ii}-1)}.$$

□

Proof of Corollary 3. Let $\mathbf{G}_s$ represent the adjacency matrix of the star network. Let subscript c stand for central agent, p for peripheral agent. We first show that $\frac{b_c^2}{m_{cc}-1} < \frac{b_p^2}{m_{pp}-1}$ for all $\mu < \bar{\mu} = \frac{1}{\sqrt{n-1}}$ (recalling that the maximal eigenvalue of $\mathbf{G}_s$ is equal to $\sqrt{n-1}$), then we prove the proposition.

Let $\mathbf{M} = (\mathbf{I} - \mu\mathbf{G}_s)^{-1}$. A few computations implies

$$\begin{cases} b_c = \frac{1+(n-1)\mu}{1-(n-1)\mu^2}, \\ m_{cc} = \frac{1}{1-(n-1)\mu^2}, \\ b_p = \frac{1+\mu}{1-(n-1)\mu^2}, \\ m_{pp} = \frac{1-(n-2)\mu^2}{1-(n-1)\mu^2}. \end{cases}$$

Then, $\frac{b_c^2}{m_{cc}-1} < \frac{b_p^2}{m_{pp}-1}$ whenever

$$\frac{(1+(n-1)\mu)^2}{(1-(n-1)\mu^2)(n-1)\mu^2} < \frac{(1+\mu)^2}{(1-(n-1)\mu^2)\mu^2}.$$

That is, after simplification,

$$(n-2)((n-1)\mu^2 - 1) < 0,$$

or,

$$\delta < \bar{\mu}.$$

Hence, for all $\mu < \bar{\mu}$, $\frac{b_c^2}{m_{cc}-1} < \frac{b_p^2}{m_{pp}-1}$.

Let us now prove that $\frac{b_c^2}{\mu+f(\omega)(m_{cc}-1)} < \frac{b_p^2}{\mu+f(\omega)(m_{pp}-1)}$ whenever $f(\omega) > 2+\mu n$. Indeed,

$$\frac{b_c^2}{\mu + f(\omega)(m_{cc} - 1)} < \frac{b_p^2}{\mu + f(\omega)(m_{pp} - 1)},$$

means

$$\frac{(1 + (n-1)\mu)^2}{\mu + f(\omega)\frac{(n-1)\mu^2}{1-(n-1)\mu^2}} < \frac{(1 + \mu)^2}{\mu + f(\omega)\frac{\mu^2}{1-(n-1)\mu^2}}.$$

Or, after simplification,

$$(n-2)\mu\Bigg((2 + n\mu)(1 - (n-1)\mu^2) + f(\omega)((n-1)\mu^2 - 1)\Bigg) < 0.$$

That is,

$$(1 - (n-1)\mu^2)(2 + n\mu - f(\omega)) < 0.$$

And given that $1 - (n-1)\mu^2 = det(\mathbf{I} - \mu\mathbf{G}_s)^{-1} > 0$, we get

$$f(\omega) > 2 + n\delta.$$

□