Labor Market Power and Collusive Behavior

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Abstract

This paper develops a theory of collusion in the presence of labor market power. In an oligopoly-oligopsony setting, a firm needs to increase its wage offers to recruit more workers and expand production, which dampens incentives to deviate from a collusive outcome. As a result, labor market power increases firms' ability to collude, and collusion harms consumers and workers, underlining the need for antitrust authorities to monitor collusive behavior also in labor markets. However, if only wage collusion is monitored, or is prevented by enforcing a minimum wage, firms fiercely collude on prices, leaving consumers worse off than under unconstrained collusion. By creating externalities across independent product markets, labor market power also engenders cross-market collusion and implies that conglomerate mergers produce anticompetitive multimarket-contact effects. No-poaching and non-compete agreements, preventing a firm from hiring its rivals' workers, act as facilitating practices; pay-equity regulations similarly discourage deviations and facilitate collusion.

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Masters are always and everywhere in a sort of tacit, but constant and uniform combination, not to raise the wages of labor above their actual rate.

— Adam Smith, *The Wealth of Nations* (1776, p. 67)

1 Introduction

Across many industries, large firms enjoy significant and increasing degrees of oligopsony power in labor markets.[1] This raises concerns about anticompetitive labor market practices, particularly collusive behavior.[2] In July 2021, a White House executive order encouraged "the FTC and DOJ to strengthen antitrust guidance to prevent employers from collaborating to suppress wages".[3] Accordingly, the DOJ has recently brought several criminal cases against naked wage-fixing agreements, the sharing of relevant wage information, and no-poaching and non-compete agreements.[4] In 2023, the FTC has signed a new agreement with the DOL to bolster efforts to protect workers by promoting competitive labor markets, identifying collusive behavior as the first area of mutual interest for the two agencies.[5]

Concerns over collusive behavior in labor markets add to those over collusion in product markets, and the two may often coexist. Indeed, in many industries, due to skill specialization or the local nature of labor and product markets, large firms that recruit from the same labor markets also operate in the same product markets, and enjoy market power in both.[6] Therefore, they can collude both to suppress wages and increase prices. For instance, the chicken-processing companies involved in one of the major wage collusion cases investigated to date have also been sued for price-fixing, with the two cases having factual overlap.[7] Similar concerns have recently arisen in the health care industry.[8]

This paper aims to develop *a theory of collusion in the presence of labor market power*, guiding antitrust enforcement and regulatory interventions. How does oligopsony power affect collusive strategies and their sustainability? Are antitrust authorities' interventions to monitor collusion in labor markets aligned with consumer protection, or do they necessarily rely on a *broad mandate* that includes worker protection? How do labor market regulations, such

[1] Berger et al. (2022) develop a general equilibrium oligopsony model of the labor market and find that it is quantitatively consistent with documented empirical regularities suggestive of oligopsony. Several other studies find low estimates for firm-level labor supply elasticities across many different sectors (see, e.g., Manning, 2021).

[2] For empirical evidence of employer collusion, see Sharma (2024).

[3] See the Fact Sheet at `https://www.presidency.ucsb.edu/documents`.

[4] See `https://www.antitrustalert.com/tag/wage-fixing/` for an overview of some recent cases.

[5] See the Press Release at `https://www.ftc.gov/news-events/news/press-releases/2023/09/`. Similarly, in the EU, a recent policy statement from the European Commission treats wage-fixing and no-poaching agreements as *restriction of competition by object*: see `https://competition-policy.ec.europa.eu/document/`.

[6] Using plant-level data, Tortarolo and Zarate (2018) find significant price mark-ups and wage mark-downs.

[7] Because plants cluster in the areas in which chickens are raised by farmers, there are multiple plants in relatively small areas in which they compete for workers. In both wage- and price-fixing cases, plaintiffs argue that the defendants exchanged information through various intermediaries, including a company called Agri Stats. The top three chicken industry firms named in the DOJ's Agri Stats lawsuit (Tyson, Pilgrim's Pride, and the recently-merged Sanderson-Wayne Farms) have already been subject to at least $698 million in settlements: see `https://accountable.us/report`.

[8] Revelations about a data analytics firm's role in determining medical payments have raised concerns about potential price fixing in health care, prompting calls for a federal investigation. See `https://www.nytimes.com/2024/05/01/us/multiplan-health-insurance-price-fixing.html`.

as minimum wage and pay-equity provisions, and the antitrust treatment of no-poaching and non-compete agreements impact collusion and, ultimately, consumer and worker welfare?

To address these questions, Section 2 introduces labor market power, deriving from workers' idiosyncratic preferences for different employers,[9] in a barebone model of collusion in oligopoly: the Bertrand supergame — i.e., the infinitely repeated Bertrand oligopoly game with homogeneous products. At each period, firms simultaneously make wage offers to workers and set product prices. These choices determine the labor force and the consumers' demand for each firm. Firms also employ other variable factors (e.g., flexible capital inputs, such as materials), traded in competitive markets, to produce the demanded output;[10] the production function exhibits constant returns to scale. Within this framework, Section 3 characterizes the cartel outcome — i.e., the industrywide-profit-maximizing wage and price levels, which would arise if cartels were legal and contractually enforceable — and the collusion outcome — i.e., the most profitable (stationary and symmetric) subgame-perfect equilibrium of the supergame. It presents two main sets of results.

First, *labor market power facilitates collusion*: The critical discount factor above which the cartel outcome is sustainable under collusion is lower than in perfectly competitive labor markets. This is because a deviating firm cannot capture the entire cartel profit in the presence of oligopsony power. While it can still attract all its rivals' consumers by slightly undercutting the monopoly price, expanding production to satisfy their demand requires hiring additional workers. Since the (residual) labor supply is upward-sloping, this entails paying higher wages. It is, therefore, impossible for a deviating firm to capture the whole cartel revenue without increasing its average production costs, which weakens deviation incentives, ultimately fostering collusion.

Second, the analysis reveals the *interplay between collusion in labor and product markets*: The best collusive scheme involves both sub-competitive wages and supra-competitive prices. This outcome equalizes the ratio of the marginal profit from collusion to the marginal profit from deviation across labor and product markets: In simple terms, firms exploit *multimarket contact* to allocate their collusive power across both markets, so they simultaneously raise prices and lower wage offers.

These results have relevant policy implications, which are examined in Section 4. In the presence of labor market power, collusion is easier to sustain and harms consumers and workers alike, so competition authorities need to intensify their enforcement activities. Moreover, they must actively monitor collusion also in labor markets, even if they follow a pure consumer surplus standard. The reason is that, if only price collusion is monitored, firms can still collude in the labor market:[11] By coordinating to reduce their wage offers, they end up hiring fewer workers,

[9]See, e.g., Azar et al. (2022), who conclude that job differentiation gives employers market power, allowing them to pay workers less than their marginal productivity.

[10]As in standard Bertrand models, a firm is always committed to satisfying all consumers' demand at its posted price. In this model, it can do so for any labor force at its disposal by adjusting its endowment of flexible capital. The *no rationing* assumption ensures the existence of a zero-profit static Nash Equilibrium, which implies that restricting attention to subgame-perfect equilibria in grim-trigger strategies is without loss of generality, but is otherwise inessential to the results.

[11]Collusive behavior often takes place through price- and wage-fixing agreements, requiring communication (though, being illegal, hence not enforceable in court, these agreements must be incentive-compatible) — see, e.g., Harrington (2006). If antitrust authorities prevent firms from agreeing on prices (e.g., by monitoring commu-

making it individually optimal to produce less than under competitive wage-setting behavior. As a result, even if firms' price-setting behavior is non-cooperative due to the monitoring of price collusion, equilibrium output is reduced, to the detriment of consumers. However, shifting antitrust authorities' (limited) budget devoted to monitoring collusive behavior from product to labor markets would lead to the worst outcome from consumers' standpoint: Pure price collusion yields even higher prices than a multimarket collusive arrangement. The reason is that when wage collusion is monitored, competitive behavior in the labor market leads to higher wages, which in turn raises production costs, making higher prices incentive-compatible.[12]

When collusive behavior cannot be monitored, the introduction of a binding minimum wage, or an increase thereof, may similarly lead to an increase in equilibrium employment and prices. In the presence of oligopsony power, such a pass-through of an increased regulatory minimum wage to consumers would not occur under competitive behavior: A binding minimum wage deprives oligopsonists of incentives to hire fewer workers in order to pay lower wages; under competitive behavior, this would give them incentives to expand production, i.e., to set lower prices. However, this effect may be outweighed by the strengthening of price collusion, which is triggered by the inability of colluding in wages. Thus, consumer harm from an increase in the minimum wage, often found in the empirical literature since Card and Krueger (1994), is not inconsistent with the presence of labor market power; instead, it is suggestive of collusive behavior.

The result that oligopsony power fosters collusion also implies that firms capture most of the benefits from their labor market power: The pass-through to consumer prices of wage markdowns due to firms' market power *vis-à-vis* workers is more limited than under competitive behavior. As a result, policies aimed at limiting the extent of firms' labor market power by protecting workers' bargaining power — e.g., strengthening trade unions — benefit workers mostly at the expense of firms rather than consumers.

Labor market power, deriving from the local nature of many (especially low-skilled) labor markets (e.g., Marinescu and Rathelot, 2018), also has implications for standard policy measures aimed at enhancing the competitiveness of product markets. First, promoting product market globalization (e.g., through free trade agreements), whereby firms in different geographic markets compete for consumers in a global product market, may fail to lead to competitive prices. The opportunity to serve consumers in other markets does not necessarily strengthen firms' incentives to deviate from a collusive outcome, due to the difficulty to recruit workers from those corresponding labor markets in order to expand production. This finding is consistent with the evidence on mark-ups in De Loecker and Eeckhout (2018) and underscores the irreplaceable role of antitrust authorities' monitoring of collusion.

Second, in local labor markets it is often the case that the same workers can be hired by

nication among pricing managers), firms can still engage in wage-fixing agreements (e.g., through communication among HR directors); in these cases, provided they offer the agreed wage, firms can set any price level without triggering a punishment, which results in competitive (i.e., static-Nash) price-setting behavior taking wages set at the collusive level. The same *semi-collusive* outcome arises if antitrust authorities can only infer whether the prevailing price is supra-competitive given the observed firms' labor force.

[12]Monitoring of wage collusion is thus complementary to, rather than a substitute for, monitoring of price collusion: The welfare gain to consumers from the monitoring of price collusion is higher when wage collusion is monitored.

firms selling different products. In these circumstances, oligopsony power introduces cross-market externalities through the labor market: Higher wage offers by any firm also harm firms selling in independent product markets by increasing the wage they need to offer to recruit a given number of workers. As a result, firms selling in independent product markets can collude together (on wages and price levels) to internalize these externalities, and *conglomerate mergers* — i.e., mergers across firms in independent product markets — facilitate such *cross-market collusion*, thereby leading to higher prices in all markets, consistent with empirical evidence (e.g., Ciliberto and Williams, 2014). These findings advocate for a strict conglomerate merger policy when the merging parties recruit workers in the same labor markets.

Finally, extending the model to long-term employment contracts allows for the identification of labor-market-specific facilitating devices. Considering an overlapping generations model with identical cohorts of workers, Section 5 shows that no-poaching agreements (NPA), prohibiting firms from making offers to each others' current employees, can be used as facilitating practices. The impossibility of making poaching offers makes the (residual) labor supply steeper for a deviating firm that wants to expand its labor force, discouraging deviations from a collusive equilibrium. The reason is that, for any wage offer above the (candidate) equilibrium one, a deviating firm can expand its labor force to a greater extent when it can make the same offer also to its rivals' workers, as some of them would accept it. Thus, signing binding NPA enables firms to sustain more collusive outcomes. This result provides an anticompetitive rationale for the widespread use of NPA also in low-skilled labor markets (e.g., Krueger and Ashenfelter, 2022) and justifies their *per se illegality* antitrust status in these markets. Non-compete agreements (NCA), whereby a worker commits with the current employer to not work for a competitor in the future, and which are also widely used in low-skilled labor markets (Starr et al., 2021), can similarly dampen deviation incentives.[13]

Through a different mechanism, also pay-equity regulations, which require firms not to wage-discriminate among workers ("equal pay for equal work"), facilitate collusion: For a firm contemplating an increase in its wage offers to recruit more newcomers and (absent NPA or NCA) rivals' incumbent workers, pay-equity regulations imply the obligation to correspondingly increase the remuneration of its current employees, which reduces the profitability of such deviation. This result may explain why the introduction of pay-transparency rules, which help enforce pay-equity regulations by revealing eventual pay disparities among coworkers performing similar work within a firm, often leads to a reduction in average wages (Cullen, 2024).

Section 6 concludes. All proofs are in Appendix A. Appendix B contains additional material.

Related literature. Starting from Friedman (1971), an extensive literature has analyzed collusion in oligopoly supergames. Previous studies have examined how the sustainability of collusion depends on firms' asymmetries or product differentiation, capacity constraints, market transparency, business cycles or demand fluctuations, *inter alia*, and how firms can employ facilitating practices such as joint venture agreements and resale price maintenance: see Ivaldi et al. (2003) for an excellent overview. All these models, however, assume that firms are price takers in the input markets and produce at a constant marginal cost. Departing from these

[13] Workers do not expect to receive attractive poaching offers from other employers along any stationary equilibrium path, so they are willing to sign a non-compete clause without asking for compensation.

assumptions, this paper shows that labor market power facilitates collusion.

The mechanism underlying this facilitating effect is that a deviating firm faces increasing marginal costs when expanding production because of the need to raise its wage offers to attract more workers. This result holds whenever each firm faces an upward-sloping labor supply, no matter whether the oligopolists also interact in the same labor market. Diseconomies of scale indeed make it possible to sustain supra-competitive equilibrium prices even absent repeated interactions, as shown by Dastidar (1995) in his analysis of the Bertrand game with convex costs. Unlike in Dastidar (1995) and follow-up work (see Vives, 1999, for an overview), by microfounding the cost function through the modeling of the labor market, this paper also examines how firms that simultaneously interact in the same labor and product markets can collude in both markets.

In the latter respect, this work relates to the literature on multimarket contact. Bernheim and Whinston (1990) were the first to formalize the insight that multimarket contact can facilitate collusion. In their framework with multiple independent product markets, multimarket contact pools the incentive-compatibility constraints for collusion across markets, which can relax binding constraints if markets are asymmetric. This insight has been extended to interdependent product markets by Spagnolo (1999) and Choi and Gerlach (2013). Considering vertically related markets,[14] this paper unveils novel mechanisms through which multimarket contact facilitates collusion, driven by the profitability of a joint deviation (in output and input markets). The paper also extends the analysis to multiple product markets, showing that, unlike in Bernheim and Whinston (1990), conglomerate mergers can have anticompetitive multimarket-contact effects even when (product) markets are completely symmetric. This is because such mergers reduce the profitability of a joint deviation across product markets, as expanding production in one market entails higher recruitment costs also in the other market.[15]

Moreover, the present paper is the first to model wage collusion. The long-standing literature on monopsony or oligopsony power in labor markets has abstracted from product market interactions and collusion. Borrowing the terminology from Manning (2021), two main approaches have been taken in this literature to model employers' market power: The "new classical" approach posits employer differentiation deriving from heterogeneity in tastes among workers, and borrows standard industrial organization models of imperfect competition (e.g., Bhaskar and To, 1999); the "modern" approach is instead based on frictions in the labor market deriving from a search-and-matching process (e.g., Burdett and Mortensen, 1998). This model follows the first approach, but its insights are robust to the case where labor market power derives from search frictions (see Appendix B.1). The few papers examining non-cooperative (Tong and Ornaghi, 2021) or cooperative (Gonzaga et al., 2014) oligopoly-oligopsony models have followed the same approach, but have relied on static analyses, thereby neglecting collusive behavior.

Finally, some recent papers have examined the trade-off between reduced worker mobility and increased employee training entailed by no-poaching and non-compete agreements (e.g.,

[14]Previous works on collusion in vertically related markets (e.g., Nocke and White, 2007, Piccolo and Miklos-Thal, 2012, and Normann et al., 2015) have focused on models of vertical supply chains.

[15]The industrial organization literature on conglomerate mergers — i.e., mergers of firms selling in independent product markets — is rather sparse. In a recent paper, Chen and Rey (2023) have examined the welfare effects of these mergers in the presence of heterogeneous "consumption synergies" deriving from bundling of independent products. In this paper, independent product markets are instead linked through the labor market.

Martins and Thomas, 2023, and Shi, 2023, respectively). Mukherjee and Vasconcelos (2012) have similarly considered these agreements as alternative means for employers to avoid engaging in wage wars for hiring each others' "star workers". In all these models, the rationale for these clauses and their competitive and welfare effects only apply to high-skilled labor markets, where workers' training or individual performance are relevant. By showing how these agreements can help firms to sustain more collusive outcomes, this paper provides a novel anticompetitive rationale for their use also in low-skilled labor markets, which is robust to, but does not rely on, firms also interacting in the same product market. The mechanism is similar in spirit to Aghion and Bolton (1987) and Rasmusen et al. (1991), where contractual restrictions to mobility discourage potential competition (i.e., entry) rather than the competitive behavior (i.e., deviations) of incumbents.

2 Model

This section describes the baseline model and examines the benchmark scenarios where firms have no (individual) labor market power.

2.1 Set-up

Consider a product market with n symmetric firms. To produce, they need to employ workers, whom they hire from the same labor market, and other (variable) production factors.

Product market. Firms produce perfect substitutes and compete *à la* Bertrand. That is, each firm $i \in \mathcal{N} \equiv \{1, ..., n\}$ posts a price p_i at which it is committed to serving all the resulting demand (*no rationing of consumers*). Perfect substitutability implies that consumers only buy from the lowest-priced firm(s): Consumers' demand is a downward-sloping function $Q(p)$ of the lowest available price $p \equiv \min_{i \in \mathcal{N}} p_i$; if several firms charge p, they equally split $Q(p)$. Formally, firm i sells

$$q_i^d \equiv \mathbb{1}[p_i = p] \frac{Q(p)}{\#\{i : p_i = p\}}, \tag{1}$$

where $\#\{i : p_i = p\}$ is the number of firms that charge the lowest price in the market.

Production function. Each firm i produces with a constant returns to scale (hereafter, CRS) production function: Its output is given by $q_i^s \equiv F(\ell_i, k_i)$, where $F(\cdot)$ is (positively) homogeneous of degree one, ℓ_i is the labor force at its disposal, and k_i represents the amount of another variable factor, which will be referred to as *flexible capital*, it employs for production. Flexible capital is traded in a competitive market at rate r.

Labor market. In the labor market, there is a measure J of ex-ante identical workers, indexed by $j \in [0, J]$, where J is (finite but) sufficiently large, so that workers are always in *excess supply* for firms $i \in \mathcal{N}$. Each worker j inelastically supplies one unit of labor when hired by a firm i in exchange for a wage $w_{i,j}$. All workers have the same outside option w_0, whose value is common knowledge, say the *competitive wage* they would earn if hired outside of the considered industry. Worker j's utility from accepting firm i's offer is given by $w_{i,j} + \xi_{i,j}$, where $\xi_{i,j}$ are i.i.d.

draws from a continuous c.d.f. $\Xi(\cdot)$ with bounded support $[\underline{\xi}, \overline{\xi}]$, and represent j's non-monetary extra-utility from working in i rather than in the outside competitive sector. The realizations of $\xi_{i,j}$ are worker j's private information, and workers are *anonymous* from firms' standpoint.

Firms can (costlessly) make personalized wage offers to as many workers as they want; each worker observes the available offers and decides which one to accept (if any). Each firm i is committed to paying $w_{i,j}$ to workers j who have accepted its offer no matter how much it produces — i.e., no new hires or dismissals are possible after consumers' demand realizes (*short-term rigid labor force*).[16] Therefore, firm i's labor force is[17]

$$\ell_i \equiv \int_{j\in[0,J]} \mathbb{1}\left[w_{i,j} + \xi_{i,j} \geq \max\left\{w_0, \max_{i'\in\mathcal{N}\setminus\{i\}} (w_{i',j} + \xi_{i',j})\right\}\right] \mathrm{d}j, \tag{2}$$

with the convention that $w_{i,j} = -\infty$ for any worker j to whom firm i does not make an offer.

Timing and solution concept. Each firm i sets its product price p_i and makes offers $\{w_{i,j}\}_{j\in[0,J]}$ to workers. These choices are simultaneous and determine the amount of output each firm must produce, $q_i = q_i^d$ given in Eqn. (1), and each firm's labor force, ℓ_i given in Eqn. (2), respectively. Under the no-rationing and short-term rigid labor force assumptions, (q_i, ℓ_i) pin down the amount of flexible capital $K(\ell_i, q_i) \equiv F^{-1,k}(\ell_i, q_i)$ (with $F^{-1,k}(\cdot)$ denoting the inverse of the production function with respect to k) firm i needs to employ.

This paper considers the infinitely repeated version of this stage game, with perfect monitoring, long-lived firms and short-lived workers.[18] That is, time is discrete and indexed by $t = 0, 1, ...$, identical cohorts of one-period-lived workers enter the labor market at each t, and firms discount profits at a common rate $\delta \in (0, 1)$. The solution concept is *stationary symmetric Subgame Perfect Nash Equilibrium in pure strategies*, hereafter referred to as SPNE, with *no wage-discrimination* — i.e., along the equilibrium path, all firms charge the same price ($p_i \equiv p$) and offer the same wage to all workers over time ($w_{i,j} \equiv w$); no restrictions are imposed off-equilibrium path.

Discussion and assumption. This paper aims to understand the impact of labor market power on collusive behavior. As price- and wage-fixing cartels are *per se illegal*, cartel provisions are not enforceable in court, and colluding firms are constrained to choose self-sustainable price and wage levels. Throughout the paper, *cartel outcomes* refer to price and wage levels that would prevail if cartels were enforceable and *collusive outcomes* to those arising in the most profitable SPNE of the supergame.[19]

The first building block of the framework is a barebone model of collusion in oligopoly, namely, the Bertrand supergame. This model is particularly tractable because Nash reversion yields a discounted profit of zero, which, by the results in Abreu (1988), implies that restricting

[16]No-rationing and short-term rigid labor force are assumed even if a firm makes negative profits (*no exit*).

[17]Ξ being continuous, workers' tie-breaking rule is immaterial to the analysis.

[18]As flexible capital is not a strategic choice, the stage game is the simultaneous-choice wage- and price-setting game. As firms can only post uniform prices, provided they sell a non-durable good, whether consumers are short- or long-lived is immaterial to the analysis.

[19]The distinction between *explicit* and *tacit* collusion is immaterial to the analysis, except in Section 4.1, where I examine antitrust authorities' monitoring of collusion, which is only possible for explicit collusion.

attention to grim-trigger strategies (Friedman, 1971) is without loss of generality. This conclusion will still be valid in my model, which significantly simplifies the analysis. Formally, a stationary action profile $(p_i, \{w_{i,j}\}_{j\in[0,J]})_{i\in\mathcal{N}}$ is a SPNE in grim-trigger strategies of the supergame defined above if, at any given t, firms play the specified actions if and only if no firm has played differently at any $t' < t$; else, a Nash Equilibrium of the stage game (hereafter, *static NE*) is played.

The Bertrand game assumes that firms are committed to satisfying all consumers' demand at their posted prices, which is plausible in cases where there are high costs of turning consumers away (see Vives, 1999, for a discussion). For this to be feasible also off-path, firms must be able to adjust their production capacity after demand is realized. In this model, firms produce using labor and flexible capital and,[20] in line with the literature on multimarket contact (Bernheim and Whinston, 1990), I consider simultaneous firms' choices in labor and product markets to rule out commitment effects. Then, flexible capital must be eventually adjusted after consumers express their demand to ensure no-rationing. The assumption that capital inputs, such as materials, lack adjustment costs and monopsony power aligns with empirical evidence (Yeh et al., 2022).

To avoid the results of the paper be driven by the no-rationing assumption, the following will be assumed throughout:

(A) *Serving all consumers' demand would be optimal for a firm deviating from the cartel outcome, even if it could not adjust its wage offers and had the option to target fewer consumers.*

Assumption (A) is formalized in Appendix A, together with other technical assumptions guaranteeing that all problems considered in the analysis are well-behaved.

Following the "new-classical approach" to labor market power (Manning, 2021), firms enjoy labor market power because workers are in excess supply and have idiosyncratic preferences for different employers. The considered model, under a Type I Extreme Value specification for $\Xi(\cdot)$, is equivalent to a logit model, which has often been estimated in empirical works (e.g., Card et al., 2018, Tortarolo and Zarate, 2018).[21] The assumptions that workers are short-lived and can only be employed by firms selling in the same product market will be later relaxed. The robustness of the main results with respect to the other modeling choices is discussed in Appendix B.2.

2.2 Benchmarks: No individual-firm labor market power

Before proceeding with the analysis, this section shows that the considered framework is a straightforward extension of the analysis of collusion in the Bertrand oligopoly supergame whenever firms $i \in \mathcal{N}$ are identical from workers' viewpoint and so, at least individually, have no labor market power.

[20] If labor were the only (variable) production factor, wage choices alone would determine firms' capacity (at least in the short-run), and output would always be sold at the market-clearing price (Kreps and Scheinkman, 1983). Then, collusion in the labor or product market would be equivalent. Constant returns to scale (with respect to $\{\ell_i, k_i\}$), while being in line with empirical evidence in many manufacturing industries (e.g., Berger et al., 2022), are mainly assumed for consistency with the standard Bertrand supergame; same as for firm symmetry.

[21] The considered firms may be able to mark-down the competitive salary w_0 which, as standard in partial equilibrium analysis, is taken as exogenous (this is always the case if, e.g., $\underline{\xi} \geq 0$). However, the results of the paper only rely on each firm $i \in \mathcal{N}$ facing an upward-sloping (residual) labor supply function.

Perfectly competitive labor market. Suppose that the labor market is perfectly competitive: $\xi_{i,j} \equiv 0$ for all i and j, so that *(i)* firms need to offer at least w_0 to hire any worker, and *(ii)* workers being in *excess supply*, firms can hire as many workers as they want at w_0. Therefore, each firm i only chooses how many offers to make, determining its labor force ℓ_i,[22] and firms' cost-minimization problems are independent of each other.

For any anticipated quantity q_i to sell, the optimal *labor demand* $\ell^*(q_i)$ of firm i is obtained by minimizing the production cost $w_0\ell_i + rK(\ell_i, q_i)$. As the production function is CRS and all factors' prices are constant, standard arguments imply that *(i)* the optimal labor demand is such that $\ell^*(q_i) = q_i\ell^*(1)$, and *(ii)* firms can produce any amount of output at a constant marginal, or average, cost: Formally, the minimized average cost function is

$$C_0(q) \equiv \frac{1}{q}[w_0\ell^*(q) + rK(\ell^*(q), q)] = w_0\ell^*(1) + r\,K(\ell^*(1), 1) \equiv c_0 \qquad \forall q > 0. \tag{3}$$

As a result, the analysis unravels as in a standard Bertrand supergame with constant marginal costs (Friedman, 1971): Firms always choose the individually optimal labor demand in equilibrium, and a firm deviating from a candidate equilibrium price p can appropriate the whole industry profit by slightly undercutting p (so to capture all consumers' demand $Q(p)$ instead of its share $1/n$) and optimally expanding its labor demand (from $\ell^*(Q(p)/n)$ to $\ell^*(Q(p))$) in order to produce $Q(p)$ at the same marginal cost c_0.

No individual-firm labor market power. Suppose now that all firms $i \in \mathcal{N}$ are still identical from workers' viewpoint, but workers value differently working in the considered industry compared to the outside competitive sector: Formally, for any worker j, $\xi_{i,j} \equiv \xi_j \sim \Xi$ for all i.

Then, the cost-efficient way for a cartel to produce a quantity q is defined as

$$C_0(q) \equiv \frac{1}{q}\min_{w}\left\{wL_0(w) + rK(L_0(w), q)\right\}, \tag{4}$$

where $L_0(w) \equiv J[1 - \Xi(w_0 - w)]$ is the industrywide labor supply when all firms $i \in \mathcal{N}$ offer $w_{i,j} = w$ to all workers $j \in [0, J]$. Slightly abusing notation, denote by $p = c_0$ the solution to $p = C_0(Q(p))$. For any $p > c_0$, a firm deviating from a candidate equilibrium (p, w) can appropriate the whole industry profit. In fact, it can capture all consumers' demand $Q(p)$ by slightly undercutting the candidate equilibrium price p; and it can hire all workers $L_0(w)$ by slightly overcutting the candidate equilibrium wage w.[23]

Results and implications. The previous arguments imply that, in both the considered scenarios, the following results hold:

Proposition 0 (No individual-firm labor market power). *If the labor market is perfectly competitive ($\xi_{i,j} \equiv 0$) or firms have no individual labor market power ($\xi_{i,j} \equiv \xi_j \sim \Xi$), then:*

[22] Formally, each firm i offers w_0 to all workers with probability ρ_i such that it ends up hiring ℓ_i workers in equilibrium — e.g., with two firms and workers breaking ties with equal probability across them, $J\rho_i\left(\rho_{i'}\frac{1}{2} + (1 - \rho_{i'})\right) = \ell_i$.

[23] This argument also implies that, in any candidate static NE, firms produce at the optimized average cost.

- *For all $\delta < \delta_0^M \equiv (n-1)/n$, the unique SPNE is the repetition of the static NE, in which firms price at the minimized average cost and make zero profits;*

- *For all $\delta \geq \delta_0^M$, the cartel outcome — i.e., the price and labor demand or wage offers that maximize industry profits — is a SPNE.*

The results of Proposition 0 show that, provided firms have no individual labor market power, the analysis yields the standard results of collusion in the Bertrand oligopoly supergame, whose main features can be summarized as follows:

1. *Competitive static equilibrium*: The static game admits a unique (symmetric) NE, where firms price at the optimized average cost ($p = c_0$) and make zero profits. Starting from any candidate NE with $p > c_0$, where firms make positive profits $\pi = (p - c_0)Q(p)/n$, each firm can profitably deviate by slightly undercutting this price to serve all consumers' demand $Q(p)$; this output is produced at the same average cost c_0 because the deviating firm recruits the extra workers at the same wage.

2. *"Bang-bang" collusion*: For any price $p \in (c_0, p_0^M]$, where $p_0^M \equiv \arg\max_p (p - C_0(Q(p)))Q(p)$ denotes the cartel price, the deviation examined above allows the deviating firm to reap the whole industry profit in the deviation period. However, it triggers Nash reversion and so a profit of zero forever after. As a result, no collusion at all is sustainable (i.e., $p = c_0$ for all t) if $\delta < \delta_0^M$, whereas the cartel outcome is sustainable for all $\delta \geq \delta_0^M$.

3. *Pure price collusion*: The cartel outcome can be implemented by colluding firms by only coordinating on setting price p_0^M. Firms do not need to also coordinate their behavior in the labor market: Each firm always acts in its individual best interest in the labor market, to produce its share of the monopoly output in the most cost-efficient way (i.e., at the optimized average cost defined above). Put it differently, once firms have agreed on *how much* to produce (i.e., on the product price p), there is no need for colluding also on *how* to produce (i.e., on how many workers to hire or on wage offers to make).[24]

To sum up, in the absence of (individual-firm) labor market power, antitrust authorities only need to monitor firms' price-setting behavior in industries featuring relatively few competitors to ensure that consumers have access to competitively-priced products, and efficient employment and wage levels. The analysis of Section 3 will show that none of these results holds in the presence of individual-firm labor market power.

3 Equilibrium analysis

When workers derive firm-specific non-monetary utility from working in the considered industry ($\xi_{i,j} \sim^{\text{i.i.d.}} \Xi$ for all i, j), each firm has labor market power: It can offer a lower wage relative to its competitors, and still hire any worker with positive probability. This section contains the equilibrium analysis in this scenario. Section 3.1 characterizes the Nash Equilibria of the static

[24]Conversely, coordination only on labor market behavior (i.e., agreeing to restrict the number of offers or to offer the wage prevailing in the cartel outcome) would not suffice to implement the cartel outcome, because firms would have incentives to undercut p_0^M even holding fixed the number of workers they hire in the cartel outcome.

game. Section 3.2 analyzes the supergame, first characterizing the conditions under which the cartel outcome can be sustained as a SPNE, and then deriving the collusive outcome when it cannot.

3.1 Static game

This section characterizes firm behavior in the static game to grasp the first insights on the impact of labor market power and to show that grim-trigger strategies are without loss of generality in the supergame. To simplify the exposition and without loss of insights, I restrict attention to symmetric NEs.

Wage-offer game. The next lemma describes firm behavior in the personalized-offers game *vis-à-vis* workers:

Lemma 1 (**wage offers**). *For each firm i, making non-discriminatory wage offers — i.e., offering $w_{i,j} \equiv w_i \ \forall j \in [0, J]$ — is a dominant strategy in the static game. Then, if $w_{i'} \equiv w \ \forall i' \in \mathcal{N} \setminus \{i\}$, firm i hires a measure $\ell_i \equiv L(w_i, w)$ of workers, where the function $L(\cdot)$ is such that*

$$\frac{\partial L(w_i, w)}{\partial w_i} > 0 > \frac{\partial L(w_i, w)}{\partial w}, \quad \textit{and} \quad \left(\frac{\partial L(w_i, w)}{\partial w_i} + \frac{\partial L(w_i, w)}{\partial w} \right) \bigg|_{w_i = w} > 0.$$

As making offers is costless and workers are anonymous from firms' viewpoint, the minimal-cost way for a firm i to hire any labor force ℓ_i is, irrespective of its rivals' behavior, to offer the same wage to all workers $j \in [0, J]$. Because, in the case of a non-degenerate distribution of offers, the higher ones are accepted with larger probability, differentiating offers across workers (and so, *a fortiori*, not approaching some of the workers) would increase the average wage a firm ends up paying to recruit any given measure ℓ_i of workers, as i would recruit mostly workers to whom it has offered high wages.[25]

As a result, in a static setting, this game of personalized offers is equivalent to a standard wage-competition model with differentiated employers, where each firm posts a wage and is committed to hiring all workers who accept this wage. Each firm then faces a labor supply function obtained from Eqn. (2) for $w_{i,j} \equiv w_i$ for all $j \in [0, J]$, which is upward-sloping in its offer and downward-sloping in the competitors' offers: Labor market power thus introduces oligopsonistic competition for workers.

Diseconomies of scale. How does oligopsonistic competition affect firms' cost structure? Defining firm i's optimized average cost function, for any given rivals' symmetric offers $w_{i'} \equiv w \ \forall i' \in \mathcal{N} \setminus \{i\}$, as

$$C(q; w) \equiv \frac{1}{q} \min_{w_i} \left[w_i L(w_i, w) + r\, K(L(w_i, w), q) \right], \tag{5}$$

[25] The logic of this result is the same behind the optimality of uniform pricing for a monopolist facing consumers with unit demand (Riley and Zeckhauser, 1983). As workers are anonymous, from each firm i's viewpoint any set of rivals' wage offers $\{w_{i',j}\}_{i' \in \mathcal{N} \setminus \{i\}, j \in [0,J]}$ just translates into a different distribution of workers' best alternative option, which is immaterial to the optimality of a uniform wage offer.

the following result holds:

Lemma 2 (**diseconomies of scale**). *For any symmetric rivals' offer w, firm i's optimized average production cost is increasing in output: $\partial C(q;w)/\partial q > 0$.*

Lemma 2 states the well-known result that a firm operating under a CRS production function and enjoying monopsony power in some input markets faces an increasing average cost function (see, e.g., Gelles and Mitchell, 1996). In the present setting, where firms recruit from the same labor market, this result applies, holding fixed rivals' wage offers, if firm i behaves as a monopsonist given its upward-sloping residual labor supply function, which must be the case in any static NE.

Equilibrium characterization. For any candidate equilibrium price p, each firm i expects to sell $q_i = Q(p)/n$, and accordingly chooses what wage w_i to offer workers in order to minimize its production cost — i.e., for given rivals' offers w, it produces at the optimized average cost $C(Q(p)/n;w)$. The equilibrium wage offers are thus a fixed point of firms' cost-minimization problems:

Lemma 3 (**competitive wage offers**). *For any candidate NE price p, there is a unique symmetric equilibrium wage offer $W^*(p)$, and it is decreasing in p.*

As in the benchmark models examined in Section 2.2, in a candidate equilibrium where firms charge a higher price, each anticipates a lower consumer demand and thereby finds it optimal to hire fewer workers; hence, firms optimally reduce their wage offers.

Each firm's profit in the candidate equilibrium with price p can be written as

$$\pi(p) \equiv \left[p - C\left(\frac{Q(p)}{n}; W^*(p)\right)\right]\frac{Q(p)}{n}. \tag{6}$$

A firm's best deviation consists in slightly undercutting the candidate equilibrium price p, attracting all consumers' demand $Q(p)$,[26] and optimally increasing its wage offers to hire more workers, in order to minimize the corresponding production cost, thus obtaining a profit

$$\pi^D(p) \equiv [p - C\left(Q(p); W^*(p)\right)]\, Q(p). \tag{7}$$

Any price p such that $\pi(p) \geq \max\{\pi^D(p), 0\}$, together with the corresponding wage offer $w = W^*(p)$, is a static NE.

Proposition 1 (**static NE**). *The static game admits a continuum of NEs: There exist prices $\underline{p}^N < \overline{p}^N$ such that $(p, W^*(p))$ set by all firms is a NE for all $p \in [\underline{p}^N, \overline{p}^N]$; firms' equilibrium profit is zero for $p = \underline{p}^N$ and is strictly increasing in p.*

The diseconomies of scale effect (Lemma 2) implies that if a firm undercuts the candidate equilibrium price p and so serves all the demand $Q(p)$ alone, its (optimized) average production cost increases: $C(Q(p);W^*(p)) > C(Q(p)/n;W^*(p))$. The price $\underline{p}^N = C(Q(\underline{p}^N)/n; W^*(\underline{p}^N))$, at

[26]This is the case for any candidate NE price below the monopoly price given wage offers $W^*(p)$ (considering static NEs with higher prices would be uninteresting).

which firms make zero profits, is an equilibrium price because a deviating firm undercutting $\underline{p}^N$ would end up selling $Q(\underline{p}^N)$ below the corresponding average production cost — i.e., $\pi(\underline{p}^N) = 0 > \pi^D(\underline{p}^N)$. Higher prices, such that $p > C(Q(p)/n; W^*(p))$ and so firms make positive profits, can also be sustained as static NE.[27]

Implications for the analysis of collusion. The results of Proposition 1 have the following implications:

Corollary 1 (static NE). *The static-game equilibrium characterization has the following implications for the supergame:*

- *Perpetual reversion to the zero-profit static NE* $(\underline{p}^N, W^*(\underline{p}^N)) \equiv (\underline{p}^N, \overline{w}^N)$ *following any deviation constitutes an optimal punishment. Therefore, restricting attention to these SPNE in grim-trigger strategies is without loss of generality;*[28]
- *For* $\delta = 0$*, the most profitable SPNE is such that firms play the static NE* $(\overline{p}^N, W^*(\overline{p}^N)) \equiv (\overline{p}^N, \underline{w}^N)$ *for all* t *and so make positive profits.*

3.2 Cartel and collusive outcomes

This section characterizes first the cartel outcome and then the most profitable SPNE of the supergame for any value of δ.

Cartel outcome. The following proposition characterizes the cartel outcome — i.e., the prices and wage offers that maximize firms' joint profits, without imposing stationarity or symmetry assumptions — and provides conditions under which it is sustainable as a SPNE of the supergame:

Proposition 2 (cartel outcome). *The cartel outcome is stationary and symmetric: Maximizing industry profits requires all firms to charge the same price* p^M *to consumers and offer the same wage* w^M *to all workers over time, with*

$$p^M > \overline{p}^N \quad \text{and} \quad w^M < \underline{w}^N.$$

This outcome can be sustained as a SPNE of the supergame if and only if $\delta \geq \delta^M$*, with*

$$\delta^M < \delta_0^M.$$

Maximization of industry profits requires equally splitting production among all firms. The reason is that, because of employer differentiation from workers' viewpoint, the wage level to attract any overall labor force in the industry is minimized when all firms are active and offer

[27]The equilibrium characterization is as in a Bertrand game with increasing marginal costs (Dastidar, 1995).

[28]That is, for any discounted profit that some SPNE can achieve, there always exists a SPNE using these grim-trigger strategies that yields the same discounted profit: The most profitable SPNE is in grim-trigger strategies. The no-rationing assumption is crucial for the existence of this zero-profit static NE, which greatly simplifies the analysis. Assumption (A) ensures that the possibility of rationing does not change qualitatively any of the results of the paper provided that there exists a punishment scheme yielding a discounted profit of zero to a deviating firm in the continuation game following any deviation.

the same wage to all workers: First, by the arguments in Lemma 1, wage discrimination by any firm would not be cost-efficient; second, any asymmetry in wage offers across firms would imply some misallocation of workers — i.e., that some workers do not work for their preferred firm — which necessarily increases the average wages to recruit any overall labor force.

A cartel internalizes the negative cross-firm consumer-demand externalities from setting low prices (for a given labor force) and labor-supply externalities from offering high wages (for a given output). These two anticompetitive effects reinforce each other: The internalization of price-externalities calls for a larger price, and the correspondingly lower production reduces the needed labor force, which depresses the wage offers; through the same mechanism, the lower wage offers because of the internalization of wage-externalities, reducing the hired labor force, make it optimal to reduce production, hence to raise prices. As a result, the cartel price is higher, and the cartel wage is lower than in the most profitable static NE.

The main result of Proposition 2 is that labor market power facilitates collusion — i.e., it lowers the critical discount factor beyond which the cartel outcome is sustainable as a SPNE of the supergame relative to the benchmark scenarios analyzed in Section 2.2. This is because, while punishment profits are still equal to zero (recall Corollary 1), in the presence of employer differentiation from workers' standpoint a firm deviating from the cartel outcome cannot reap the whole industry profit.

This is for two reasons. First, a firm i's deviation profit is strictly below the profit it would make if it were the only firm active in the labor market. Indeed, even though i can eliminate rivals' product market competition by slightly undercutting p^M, it still faces their competition in the labor market, as they do not expect the deviation and so offer w^M to all workers. This competition strictly reduces i's deviation profit: As, for all w_i, $L(w_i, w^M) < L(w_i, -\infty)$, it is more expensive for i to hire workers relative to the case where its rivals are out of the market. Second, even this hypothetical single-firm profit is strictly below the industry profit in the cartel outcome.[29] As argued above, employer differentiation implies that, for all w, $L(w, -\infty) < nL(w, w)$: Even in the absence of its rivals, a firm would need to raise its offers to get the same labor force of a n-firm industry, which implies facing larger costs to produce the cartel output $Q(p^M)$.

Collusion outcomes. Can some collusion manifest itself also when the cartel outcome is not sustainable (i.e., for $\delta < \delta^M$)? The most profitable SPNE, for any given $\delta < \delta^M$, is the pair (p, w) obtained by solving the following problem:

$$\max_{p,w} \pi(p, w) \quad \text{s.t. } \delta \geq 1 - \frac{\pi(p, w)}{\pi^D(p, w)}, \tag{P}$$

where the per-firm profit in the candidate SPNE outcome (p, w) is

$$\pi(p, w) \equiv p\frac{Q(p)}{n} - \left[wL(w, w) + r\,K\left(L(w, w), \frac{Q(p)}{n}\right)\right], \tag{8}$$

[29]This single-firm profit would be obtained by a deviating firm if the stage game was a sequential, production-to-order, game. The main results are robust with respect to the timing of the stage game: see Appendix B.2.

whereas the highest profit that a firm can make when deviating is given by

$$\pi^D(p,w) \equiv pQ(p) - C(Q(p);w), \tag{9}$$

in the period of deviation, and zero afterward, given that (by Corollary 1) firms revert to the zero-profit static NE after any deviation.[30] Then, the incentive-compatibility constraint in Problem (P) follows from the definition of grim-trigger strategies. The following proposition describes the solution to this problem:

Proposition 3 (**multimarket collusion**). *For all $\delta \in [0,\delta^M]$, the most profitable SPNE $(P^M(\delta), W^M(\delta))$ is obtained from the binding incentive-compatibility constraint and the optimality condition*

$$\frac{\partial\pi(\cdot)/\partial p}{\partial\pi^D(\cdot)/\partial p} = \frac{\partial\pi(\cdot)/\partial w}{\partial\pi^D(\cdot)/\partial w}, \tag{10}$$

and is such that, as δ increases from 0 to δ^M, $P^M(\delta)$ continuously increases from $\overline{p}^N$ to p^M and $W^M(\delta)$ continuously decreases from $\underline{w}^N$ to w^M.

For $\delta = 0$, firms are myopic and so cannot sustain any outcome more collusive than the most profitable static NE (Corollary 1); for all $\delta \geq \delta^M$, firms can sustain the cartel outcome, which features both a higher price and a lower wage (Proposition 2). Proposition 3 shows that firms optimally exploit any increase in the discount factor in the range $(0,\delta^M)$ to set both a more collusive (higher) price and (lower) wage. In particular, for any such level of the discount factor, the incentive-compatibility constraint binds, and the optimal collusive scheme equalizes the ratio of the marginal profit from collusion to the marginal profit from deviation across labor and product markets (i.e., with respect to w and p).

The pattern of collusive prices in the presence of labor market power resembles the one arising in oligopoly (with constant marginal costs) when firms sell differentiated products (e.g., Ross, 1992). Indeed, similar to labor market power, product differentiation implies that a deviating firm, undercutting a candidate SPNE price p, can never appropriate the whole industry profit at p,[31] and its incentives to deviate continuously increase with p. The same collusive price patterns thus arise when firms exploit differentiation *vis-à-vis* workers in the labor market, rather than *vis-à-vis* consumers in the product market.[32]

Summing up. The presence of (individual-firm) labor market power implies that collusive behavior has completely different features:

[30] As slightly undercutting p to capture all consumers' demand (and accordingly increasing the wage offer above w to minimize the production cost of $Q(p)$) is the most profitable deviation if $w = W^*(p)$, it is *a fortiori* so starting from a candidate collusive SPNE where $w < W^*(p)$ (colluding firms would never choose $w > W^*(p)$). The reason is that a deviating firm maximizes the gains from its rivals' more accommodating behavior in the labor market by maximally expanding its production, i.e. by serving the whole demand $Q(p)$.

[31] Even if firms sell differentiated products, in the presence of labor market power a deviating firm's average cost would increase when it lowers its price and expands its production, which *ceteris paribus* facilitates collusion.

[32] As shown in Section 4.1 below, this holds irrespective of whether firms collude only on prices or also on wages. Yet, product differentiation also implies that the static NE features positive profits, which limits the severity of punishments, at least in a SPNE in grim-trigger strategies (as considered in Ross, 1992), thereby destabilizing collusion. This effect is absent in this model: When firm differentiation, which facilitates collusion by dampening defection profits, comes from the labor market, severe punishments are ensured in the presence of product homogeneity.

1. *"Collusive" static equilibria*: Besides a zero-profit equilibrium, firms can sustain positive-profits equilibria, with inefficiently low production and employment levels, even absent repeated interactions (i.e., if $\delta = 0$).

2. *"Smooth" collusion*: The ability to collude — i.e., to raise prices and reduce wages — increases continuously with the discount factor (before firms reach the cartel outcome). Moreover, the critical discount factor to sustain the cartel outcome is lower than in the absence of individual-firm labor market power.

3. *Multimarket collusion*: The most profitable SPNE cannot be implemented by only colluding on a price level p, leaving each firm i free to choose its static-profit-maximizing wage offer to produce $q_i = Q(p)/n$: Firms find it optimal to also coordinate their behavior in the labor market — i.e., to suppress their wage offers below $W^*(p)$ in order to reduce the industry production costs (formally, $W^M(\delta) < W^*(P^M(\delta))$ for all $\delta > 0$: see Section 4.1).

4 Policy implications

Section 3 has shown that colluding firms are able to set supra-competitive prices and sub-competitive wage levels, harming consumers and workers alike. To prevent this possibility, antitrust authorities must monitor collusive behavior. Their monitoring activities may target preventing collusion in the labor market, the product market, or both, as examined in Section 4.1. When collusive behavior is hard to monitor — e.g., firms are able to tacitly collude, without leaving hard evidence to the competition watchdog — competition authorities or regulators can nonetheless employ several policy measures, in labor and/or product markets, to make collusion harder to sustain, as examined in Section 4.2.

4.1 Monitoring of collusion

In practice, monitoring of collusive behavior takes place through the detection of price- and wage-fixing agreements. Suppose that collusion in any market leaves hard evidence, and the antitrust authority can commit to a monitoring policy and levy hefty fines on firms caught colluding (as in Motta and Polo, 2003, and Choi and Gerlach, 2013, among many others). Then, if the antitrust authority monitors collusion in both labor and product markets, the best firms can do is play the most profitable static NE $(\overline{p}^N, \underline{w}^N)$ over time (the results of this section are unchanged if firms play any other static NE). If, on the contrary, collusive behavior is left unmonitored, firms reach the *multimarket collusion* outcome characterized in Proposition 3. However, the antitrust authority, when constrained by limited budget, can monitor collusion only in either the labor market or the product market. This section first characterizes the most profitable SPNE in these scenarios and then derives implications of the antitrust authority's monitoring policies.

Preliminaries. Consider a candidate SPNE outcome (p, w). In the analysis of Section 3.2, any deviation from (p, w) triggers the reversion to the zero-profit static NE. This section characterizes the most profitable SPNE if only deviations from p (resp., from w) trigger Nash-reversion

— i.e., (p, w) is played at any t if and only if all firms have set p (resp., w) for all $t' < t$, no matter their choices of w (resp., of p); else, $(\underline{p}^N, \overline{w}^N)$ is played. In these equilibria, the variable whose choice does not trigger the punishment is set by each firm to maximize its static profit, resulting in static Nash behavior taking as given the value of the other variable.

These scenarios arise if collusion in each market requires communication among specialized middle managers — e.g., firms' HR directors need to communicate in order to coordinate on collusive wage levels, given the price set by firms' pricing managers (i.e., to set $w < W^*(p)$) — and such communication can be prevented, in either market, by the competition watchdog. Alternatively, the antitrust authority can only infer whether the prevailing price (resp., wage) is collusive given the observed labor force (resp., consumer demand) — e.g., because it cannot estimate the labor supply function, it is not able to ascertain whether $w < W^*(p)$; in this case, firms can be sued for collusion only if the observed price is supra-competitive given the observed firms' wage and labor force ($p > P^*(w)$ in the notation below). Both these microfoundations are detailed in Appendix B.1.

Monitoring of wage collusion. If wage collusion is monitored, firms collude only on the product price: As described above, given a candidate SPNE price p, along the equilibrium path each firm chooses the wage offers in order to minimize its production cost of $Q(p)/n$ anticipating that rivals do the same. This competitive behavior in the labor market introduces the *competitive wage constraint* $w = W^*(p)$ in the collusion Problem (P).

Denoting by $p^P \equiv \arg\max_p \pi(p, W^*(p))$ the *price cartel outcome*, which would emerge if firms could write down p in a legally binding contract, but could not collude on w, the following results hold:

Proposition 4 (**price collusion**). *Under price collusion, there exists a threshold $\delta^P \in (0, \delta_0^M)$ such that the most profitable SPNE is $(P^P(\delta), W^P(\delta))$, with $W^P(\delta) = W^*(P^P(\delta))$, and $P^P(\delta)$ being increasing in δ and such that $P^P(0) = \overline{p}^N$ and $P^P(\delta) = p^P$ for all $\delta \geq \delta^P$. Moreover,*

$$P^P(\delta) > P^M(\delta) \quad \text{and} \quad W^P(\delta) > W^M(\delta),$$

for all $\delta > 0$.

This proposition shows two main results. First, even though wage collusion is not in place (i.e., firms compete in wages, given the price chosen in a collusive fashion), price collusion also harms workers. Price coordination allows firms to raise their prices, which implies that they face a lower demand, and so find it individually optimal to reduce their wage offers and hire fewer workers. Thus, as the discount factor grows, firms can sustain SPNE featuring both higher prices and lower wages, even if they can coordinate only on their pricing behavior.

Second, the monitoring of wage collusion only is beneficial to workers ($W^P(\delta) > W^M(\delta)$) but harmful to consumers ($P^P(\delta) > P^M(\delta)$). Intuitively, the optimality condition (10) entails that, under multimarket collusion, firms exploit their ability to collude (for a given value of δ) to induce a collusive allocation in both labor and product markets: In particular, $W^M(\delta) < W^*(P^M(\delta))$ for all $\delta > 0$. The higher wage levels prevailing because of competitive behavior in the labor market make it efficient to produce less and more expensive for a firm contemplating a deviation to recruit more workers to expand its production, rendering higher prices incentive-compatible.

Monitoring of price collusion. If price collusion is monitored, firms collude only on their wage offers: As explained above, given a candidate SPNE wage w, along the equilibrium path all firms have labor force $L(w,w)$ and compete in prices. The most profitable SPNE in this class solves Problem (P) subject to the *competitive price constraint* $p \leq P^*(w)$, where $P^*(w)$ is the highest price that prevents a firm from undercutting holding fixed the wage level w, obtained by equating the revenue from capturing rivals' demand and the cost of the extra endowment of capital needed to satisfy the additional demand given that the labor force is fixed at $L(w,w)$:

$$\frac{n-1}{n} pQ(p) = r\left[K(L(w,w),Q(p)) - K\left(L(w,w),\frac{Q(p)}{n}\right)\right]. \tag{11}$$

Denoting by $w^W \equiv \arg\max_w \pi(P^*(w),w)$ the *wage cartel outcome*, which would emerge if firms could write down w in a legally binding contract but could not collude on p, the following results hold:

Proposition 5 (**wage collusion**). *Under wage collusion, there exists a threshold $\delta^W \in (0,\delta_0^M)$ such that the most profitable SPNE is $(P^W(\delta),W^W(\delta))$, with W^W and $P^W \leq P^*(W^W)$ being decreasing and increasing in δ, respectively, and such that $(P^W(0)=\overline{p}^N < P^*(\underline{w}^N), W^W(0) = \underline{w}^N)$ and $(P^W(\delta) = P^*(w^W), W^W(\delta) = w^W)$ for all $\delta \geq \delta^W$. Moreover, for all $\delta > 0$,*

$$P^W(\delta) \leq P^M(\delta) \quad \textit{and} \quad W^W(\delta) \leq W^M(\delta),$$

with strict inequalities whenever the competitive price constraint binds ($p = P^*(w)$).

By colluding in the labor market, firms can sustain higher prices in the supergame, relative to the highest static NE price $\overline{p}^N$, even absent price collusion. By coordinating to lower their wage offers, they end up hiring fewer workers; a lower labor force, in turn, makes individually rational to produce less — i.e., to set higher prices (indeed, $P^*(w)$ is decreasing in w). Wage collusion is thus harmful not only to workers, but to consumers as well.

However, as the competitive price constraint is violated at the multimarket cartel outcome — i.e., $p^M > P^*(w^M)$ — for sufficiently large values of δ, the monitoring of price collusion prevents firms from achieving the profits they make under multimarket collusion.[33] For these values of δ, when they cannot collude on prices, firms inefficiently exploit all their ability to collude to suppress wages, implying that the monitoring of price collusion only is harmful to workers ($W^W(\delta) < W^M(\delta)$) but beneficial to consumers ($P^W(\delta) < P^M(\delta)$). In particular, competitive price behavior depresses the marginal revenue product of labor and entails a more

[33]Wage collusion instead suffices to implement the multimarket collusion outcome for relatively small values of δ. As in the static game firms would be indifferent between undercutting or not the price $P^*(w)$ when holding fixed their wage offers at w, they have strict *static* incentives to undercut $P^*(w)$ given that they can indeed also increase their offers — i.e., $\pi^D(P^*(w),w) > \pi(P^*(w),w)$ for all w. By a continuity argument, so long as δ is sufficiently small, for any given w the price $P^*(w)$ is too large to be sustainable as a SPNE outcome under multimarket collusion. Then, the competitive price constraint does not bind, so (unlike price collusion) wage collusion can achieve the multimarket collusion outcome. This result is, however, specific to perfect Bertrand competition in the product market.

significant increase in demand for a deviating firm undercutting the candidate equilibrium price exacerbating the diseconomies of scale effect, thereby making lower wages both more efficient and incentive-compatible.

Remark (Multimarket-contact effect). To conclude the analysis, it is interesting to notice that, unlike in the multimarket contact model with independent markets (Bernheim and Whinston, 1990), the multimarket cartel outcome (p^M, w^M) might be sustainable at a lower critical discount factor relative to *both* cartel outcomes in each single market under competitive behavior in the other market — i.e., it might be that[34]

$$\delta^M < \min\{\delta^P, \delta^W\}.$$

The reason is that competitive behavior in one market does not eliminate the gains from a deviation in that market once a firm deviates in the other one: $w = W^*(p)$ is a competitive wage level for fixed p (i.e., to produce $q_i = Q(p)/n$), but a deviating firm who undercuts p has also incentives to raise its wage; similarly as for the price level $p = P^*(w)$. For the reasons discussed above, a price (resp., wage) cartel inefficiently pushes the price up (resp., the wage down) relative to a multimarket cartel, which may enhance the profitability of such a joint deviation.[35]

Implications. Suppose that the antitrust authority adopts a consumer-surplus standard, in line with its current *narrow mandate* in many countries. Then, the foregoing analysis has the following immediate policy implications on the monitoring of collusion:

Corollary 2 (monitoring of collusion). *For all $\delta > 0$, $P^P(\delta) > P^M(\delta) \geq P^W(\delta) > \overline{p}^N$, implying that consumers:*

- *benefit from the monitoring of both price and wage collusion;*
- *benefit from the monitoring of wage collusion if and only if price collusion is also monitored;*
- *always benefit from the monitoring of price collusion, and more so when also wage collusion is monitored.*

Labor market power makes collusion easier to sustain, which increases the need to monitor collusive behavior. Moreover, if only price collusion is monitored, firms have the incentives and ability to collude to depress wages, which also causes consumer harm by making higher prices individually rational. Consumer protection thus requires antitrust authorities to monitor also

[34]This is a theoretical possibility in light of the results of Proposition 4 and 5. The possibility result can be established by simulating the model using a Cobb-Douglas production function, an isoelastic product demand, and a logit labor supply function (Matlab code is available upon request). In the case of independent product markets considered by Bernheim and Whinston (1990), instead, in order for the cartel outcome to be sustainable in both markets under multimarket contact, it needs to be sustainable in at least one of the two markets (no matter the behavior in the other market) absent multimarket contact.

[35]Similar to the previous literature on multimarket contact (Matsushima, 2001), labor market power can have further procollusive effects by allowing firms to detect better deviations in settings with demand shocks and imperfect monitoring *à la* Green and Porter (1984). See Appendix B.2 for a discussion.

wage collusion actively.[36] Yet, shifting all their attention and budget towards monitoring wage collusion would not align with their statutory objective: Under pure price collusion, consumers are even worse off than under no monitoring at all of collusive behavior. For these reasons, the welfare stakes for consumer-surplus-oriented authorities deciding whether to monitor price collusion are higher when wage collusion is monitored.

Overall, these results point to a complementarity in the monitoring of collusive behavior in labor and product markets. Granting antitrust authorities a *broad mandate* — specifically, including worker protection in their objectives — is unnecessary to drive their efforts against wage collusion,[37] as long as they are sufficiently resourced to address collusion in both labor and product markets.

4.2 Restraining collusion

Suppose now that antitrust authorities are unable to monitor collusive behavior. Firms' ability to set supra-competitive prices and sub-competitive wages can be constrained by competition or regulatory measures in labor and product markets.

Minimum wage regulation. One of the most important policies, especially in low-skilled labor markets, is minimum wage regulation, which constrains colluding firms' possibility of suppressing wages.

Proposition 6 (minimum wage regulation). *For every $\delta \in (0, \delta^M)$ there exists $\varepsilon > 0$ such that a minimum wage $\underline{w} \in (W^M(\delta), W^M(\delta) + \varepsilon)$ imposed by regulation raises both employment and price levels.*

The effects of minimum wage regulation crucially depend not only on whether labor markets are perfectly competitive or oligopsonistic, as recognized by a long-standing literature (see, e.g., Belman and Wolfson, 2014, for a comprehensive survey) but also on firms' competitive conduct. If labor markets are perfectly competitive, any minimum wage regulation raising the wage above the competitive level w_0 implies that firms find it optimal to hire fewer workers, which, no matter whether they compete or collude in the product market, leads to higher prices. Conversely, in the presence of labor market power, firms would ration workers only if the minimum wage is much larger than the wage level prevailing in equilibrium. A *locally binding* minimum wage instead raises employment levels: Firms no longer have incentives to hire fewer workers in order to be able to pay lower wages when these are set by regulation. This employment-enhancing effect, in turn, makes it optimal to expand production, resulting in lower consumer prices. Once again, this is true in equilibrium under competitive behavior (e.g., Tong and Ornaghi, 2021) and in the cartel outcome (or, under collusion, for $\delta \geq \delta^M$).

The novel implication of this model is that, if firms engage in collusive behavior, but their ability to collude is constrained by incentive-compatibility (i.e., $\delta \in (0, \delta^M)$), the introduction of

[36] Similar arguments justify the *per se illegality* antitrust status of wage cartels even based on a pure consumer surplus standard. Indeed, in the presence of a wage cartel, firms colluding on prices would be able to sustain equilibria with higher prices and lower wages relative to the multimarket collusion outcomes characterized in Proposition 3 for all $\delta < \delta^M$ (see Appendix B.2).

[37] A broad mandate may indeed have unintended consequences, as discussed in Tirole (2023).

a (locally) binding minimum wage, or a (local) increase thereof, implies that firms, being unable to depress wages as much as they would like to, exploit their ability to collude in the product market. That is, firms offer to all workers the minimum wage to comply with regulation (as above, rationing of workers is not optimal if the minimum wage is only slightly higher than the equilibrium wage in the absence of regulation), but they are able to sustain higher prices.

Minimum wage regulation can thus have the same effects as monitoring wage collusion only: it can backfire on consumer surplus if antitrust authorities are not able to monitor collusive behavior in product markets. Labor market power and collusive behavior thus jointly rationalize the evidence in several studies since Card and Krueger (1994) that increases in the minimum wage can raise both employment and prices. Such firms' reactions are thus suggestive of collusive conduct.

Regulation of unions and collective bargaining. In the considered model, labor market power derives from workers' heterogeneous preferences for working at different firms. In reality, however, the extent of firms' labor market power — to be intended as their ability to mark down the competitive wage w_0 even without engaging in collusion — also depends on workers' bargaining power: In many countries, the prevailing wage levels are determined through collective bargaining between employers and unions.

Appendix B.2 develops a model where firms $i \in \mathcal{N}$ choose employment levels ℓ_i and wages are determined through collective bargaining. Suppose that the distribution Ξ is such that the market clearing wage, which would prevail if unions have no bargaining power, would always be below the competitive wage w_0, which would prevail if unions have full bargaining power. The wage prevailing under collective bargaining is the weighted average of the market clearing wage and the competitive wage, where weights reflect unions' bargaining power.[38]

Weaker unions imply a lower cost to hire any labor force for the firms, resulting into lower consumer prices. However, as labor market power facilitates collusion, weakening unions' bargaining power increases firms' ability to set collusive prices, which results into relatively small pass-throughs of the wage mark-downs to consumer prices. As a result, when collusion is a concern, the trade-off between consumer and worker surplus calls for the protection of unions' bargaining power: Increasing firms' labor market power acts as a facilitating device, implying that firms retain much of the corresponding gains, and so workers lose much more than consumers gain.

Open product markets. Policy measures that open up product markets (e.g., free trade agreements), by exposing firms within each local market to competition by firms producing in different geographic markets, can make it impossible for them to sustain collusion, ensuring consumer access to competitively priced products. This section argues that this result does not necessarily hold if, in line with empirical evidence (e.g., Marinescu and Rathelot, 2018), labor markets remain local.

Consider H distinct geographic markets $h \in \{1, ..., H\}$, each composed of a labor and a

[38]Formally, abstracting away from employer differentiation, the market clearing wage as function of industry-wide labor demand $L \equiv \sum_i \ell_i$, $w^*(L) < w_0$, is obtained from $J[1 - \Xi(w_0 - w^*(L))] = L$; the prevailing wage is then $W(L) \equiv \alpha w^*(L) + (1 - \alpha)w_0$, where $\alpha \in [0, 1]$ is an inverse measure of trade unions' bargaining power.

product market as described so far. Such geographic markets are independent and, for simplicity but without loss of insights, identical. Within each market, therefore, the cartel outcome (p^M, w^M) is as in Proposition 2 for all H. Suppose that firms in each local market h can sell their products in any of the H markets: the larger H, the more globalized the product market;[39] still, firms can only recruit workers from their local labor market h. The no-rationing assumption still holds within each market, but a firm charging a price below the one prevailing in some other markets is free to choose how many of these markets it wants to serve.

The following proposition describes the impact of the available number of markets H on the critical discount factor, denoted by $\delta^M(H)$, to sustain the cartel outcome (p^M, w^M) within any geographic market.[40]

Proposition 7 (product market globalization). *There exists a (finite) number of markets H^* such that $\delta^M(H)$ is increasing in H if and only if $H < H^*$ and, for all $H \geq H^*$, $\delta^M(H) = \delta^M(H^*) < (nH^* - 1)/(nH^*)$.*

For $\delta = 0$, at $p = \overline{p}^N$ firms are indifferent between undercutting or not when, if they do so, they have to serve only consumers in their local market. By the diseconomies of scale effect, the opportunity for a deviating firm to serve also consumers in other markets is then valueless: As a result, for all $H \geq 1$, the most profitable SPNE features $(\overline{p}^N, \underline{w}^N)$ in each market. When δ grows larger, however, an increase in H can limit the extent of collusion: In the SPNE characterized in Proposition 3 (played in all markets), outputs and wages are relatively low, implying that a deviating firm may gain strictly more by also serving consumers in some other markets; accordingly, colluding firms in each market have to set lower prices and higher wages to preserve incentive-compatibility. As this is especially true at the cartel outcome, it follows that the critical discount factor to implement (p^M, w^M) may well be larger when $H > 1$. Nevertheless, serving more and more markets requires a deviating firm to recruit more and more workers, which entails paying increasingly high wages (this is true even when the dimension J of the local labor market is sufficiently large — i.e., there are more available workers than a deviating firm would want to hire at w_0 to serve consumers in all H markets). This implies that a deviating firm would not serve consumers in more than H^* markets.

Therefore, for product market globalization to guarantee a competitive outcome, it needs to be accompanied by the globalization of labor markets and the absence of labor market power.As soon as one of these conditions fails, the monitoring of collusion is needed to protect consumers' and workers' interests. Indeed, the procollusive effects of increased labor market power (Yeh et al., 2022) may outweigh the procompetitive impact of the increasing globalization of product markets, thereby leading to higher mark-ups, consistent with the empirical evidence in De Loecker and Eeckhout (2018).

Merger policy. Another traditional policy tool to preserve competition and avoid facilitating collusion consists in preventing concentration, by adopting a strict merger policy that prohibits

[39]Indeed, one can equivalently consider an exogenous number $\overline{H}$ of geographic markets divided into $\overline{H}/H$ disjoint *free trade areas*, so that each firm can sell its products in up to H markets: $H = 1$ corresponds to the case of *local product markets* analyzed so far, $H = \overline{H}$ corresponds instead to a *single market.*

[40]There is no need of *cross-market collusion* to sustain (p^M, w^M) within each geographic market — i.e., firms within a market h only need to collude among each other to sustain (p^M, w^M) (see Appendix A).

horizontal mergers (absent substantial efficiencies). A merged entity, in fact, has weaker incentives to deviate from a collusive arrangement because of the internalization of business-stealing externalities across the merged units. This section argues that, in the presence of labor market power, mergers that increase labor market concentration can produce similar anticompetitive effects even if the merging parties sell demand-independent products. These mergers can therefore be labeled as *conglomerate mergers.*

Indeed, while the foregoing analysis has considered (strategic) firms in only one industry, often workers (especially in low-skilled labor markets) can be employed by firms selling different products to consumers within a geographic market. Suppose that the n firms considered throughout (with n being even for the sake of the exercise) still recruit from the same labor market, but now firms $i \in \{1, ..., n/2\}$ sell product A and the other firms $i \in \{n/2+1, ..., n\}$ sell product B; product markets $z \in \{A, B\}$ are identical and independent, with consumer demand $q_z \equiv Q(p_z)/2$ in each market, where p_z denotes the (minimum available) price for product z (this normalization ensures that the cartel outcome remains as in Proposition 2). This section will contrast the scenario in which the n firms are independent (*single-product firms*) with the one in which the same *multiproduct firms* are active in both product markets — i.e., firm i, selling product A, merges with firm $i + n/2$, selling product B, for all $i \in \{1, ..., n/2\}$. In either case, workers' preferences depend on the features (e.g., the location) of each production plant, irrespective of its ownership: in both scenarios, workers' labor supply is as above. I again restrict attention to the most profitable stationary symmetric SPNE, where all firms set price $p_z \equiv p$ and offer wage $w_z \equiv w$ for $z = A, B$.[41]

As higher wage offers by firms selling product z entail a negative externality on the labor supply of firms selling the other product, firms selling in independent product markets have incentives to collude together to internalize these cross-market wage-externalities.[42] Conglomerate mergers allow to relax incentive-compatibility constraints in this *cross-market collusion* problem. This is because, while along any equilibrium path a multiproduct firm earns the sum of its units' profits, deviation profits are *subadditive.*

The intuition is as follows. A multiproduct firm anticipates that any deviation triggers punishments yielding zero profits in both $z = A, B$, so it optimally deviates by undercutting p in both product markets. Yet, increasing the wage offer to recruit more workers for its production unit in market A imposes a negative externality on its subsidiary in market B, as it makes it increasingly costly to expand the labor force hired for production in market B. As a result, multiproduct firms cannot obtain the same per-market deviation profit of a single-product firm, and so have weaker incentives to deviate from any candidate SPNE (p, w), and in particular from the cartel outcome:

Proposition 8 (conglomerate mergers). *Compared to the scenario with distinct single-*

[41] Once again, such equilibrium is in grim-trigger strategies, as the results of Corollary 1 hold in each product market z, for any w_{-z} offered by firms in the other market — i.e., perfect within-market Bertrand competition ensures the existence of a zero-profit continuation equilibrium following a deviation in either market.

[42] Under such cross-market collusion, firms may be bound, for incentive-compatibility reasons, to reduce consumer prices, relative to the case of within-market collusion, where firms selling each product z only collude among each other correctly anticipating the wage offer by firms in the other product market (see Appendix B.3 for the details). By contrast, the internalization of wage-externalities implies that cross-market collusion unambiguously results in lower wages.

product firms in each product market $z = A, B$, the critical discount factor to sustain the cartel outcome is strictly lower when the same firms operate in both markets.

The result that conglomerate mergers, by inducing multimarket contact, can lead to more collusive outcomes dates back to Bernheim and Whinston (1990). However, in their setting with perfectly competitive input markets, pooling incentive constraints across independent product markets *(i)* has an effect only if markets are asymmetric along some specific dimensions, and *(ii)* cannot yield a strictly higher price in all markets. In contrast, the anticompetitive multimarket-contact effects of conglomerate mergers are much more robust in the presence of labor market power, given that, even absent asymmetries, they simultaneously lead to higher prices in all markets and, on top of this, also entail lower wages.[43] Both effects have been found in the data: Ciliberto and Williams (2014) have shown that, in the airline industry, carriers with a significant amount of multimarket contact can sustain near-perfect cooperation in setting fares; Arnold (2019) has found that mergers that result in significant increases in local labor market concentration produce a decline in wages and these effects are not driven by changes in product market power (see also Prager and Schmitt, 2021, and Berger et al., 2023).

5 Employment contracts and facilitating devices

The foregoing analysis has considered, for simplicity, a spot labor market. In reality, however, workers typically sign long-term employment contracts, which can be subject to horizontal and vertical restraints, such as no-poaching and non-compete agreements, and a body of regulations constraining firms' wage-setting behavior, including pay-transparency and pay-equity provisions. This section explores how these labor-market-specific features affect the sustainability of collusion. The results on labor market collusion do not rely on the assumption that oligopsonists also interact in the same product market (see Appendix B.4). However, the analysis is conducted within the oligopoly-oligopsony setting to also provide insights into price collusion applicable in this scenario.

Set-up. Consider an overlapping generation model with cohorts of *myopic* T-period lived workers.[44] Each generation is composed of a measure J of workers j as defined in Section 2 with $\xi_{i,j} \sim^{\text{i.i.d.}} \Xi(\cdot)$ being time-invariant (*persistent type*). Consistent with real-world practices, firms offer long-term stationary contracts, specifying a per-period salary with the possibility of Pareto-improving renegotiation and workers' option to quit at any future period (*one-sided commitment*).[45] To simplify the analysis, suppose that if a worker is not employed in the considered industry at *age* $\tau = 1, ..., T$, it cannot be hired in future periods, i.e., at any age

[43] The subadditivity of deviation profits, which drives the results, generalizes to an arbitrary number of product markets and applies also to asymmetric SPNE outcomes $(p_z, w_z)_{z=A,B}$, which shall be considered if product markets are asymmetric.

[44] The results are qualitatively unchanged if workers are farsighted and discount future payoffs at the common rate δ (see Appendix A for the details). To keep the model stationary, I assume that the first T generations of workers are simultaneously available at the initial time $t = 0$.

[45] Permanent employment contracts specify the current salary, which normally can only be increased in the future, as *downward nominal wage rigidity* is prevalent (Lebow et al., 2003). Nevertheless, employees can decide to leave the firm at any time. Whether firms can fire workers hired in period t in future periods $(t + 1, ...)$ is immaterial to the results.

$\tau' > \tau$ — e.g., it leaves the considered industry or local labor market and enjoys a per-period outside option w_0.[46]

Therefore, at each period t, workers who were aged $\tau = T$ in $t-1$ *retire*, and firms, simultaneously, *(i)* make offers to newcomers ($\tau = 1$); *(ii)* possibly renegotiate wages with their incumbent employees (i.e., previously hired workers of age $\tau = 2, ..., T$) and make offers to rivals' incumbent employees (if allowed: see below); and *(iii)* set their products' price. Then, all workers in the market observe their available offers and choose which firm to work in (if any), and consumers make their purchase decisions. A firm i then needs to employ variable capital $k_i = K(\sum_\tau \ell_{\tau,i}, q_i)$ to satisfy consumers' realized demand, q_i in Eqn. (1), given the measure $\ell_{\tau,i}$ of workers of each age at its disposal, which depends on contract offers.

In particular, newcomers' labor supply, if i's rivals all offer them the same wage w_1, is still given by $\ell_{1,i} = L(w_{1,i}, w_1)$, the function $L(\cdot)$ being defined in Lemma 1. The allocation of the available workers $\sum_i \ell_{1,i}$ of age $\tau = 2, ..., T$ across firms again depends on currently available wage offers in a static fashion — i.e., worker j chooses at each age τ to work for the firm i providing the highest overall utility $w_{i,j} + \xi_{i,j}$ in that period.

5.1 No-poaching agreements

A no-poaching agreement (hereafter, NPA) among any subset of firms prevents the signatories from hiring each others' incumbent workers: If firm i has signed a NPA in period t, it cannot propose any offer to workers who have worked for the other signatories in period $t-1$.[47] If allowed by competition authorities, these agreements are legally binding: Even if a firm deviates from a collusive outcome path, it cannot violate the NPA (e.g., the agreement would be enforced in court, or hefty fines are levied on the violating firm). At each period of the game, each firm decides whether to join the NPA (that rules for one period) before labor- and product-market decisions are made as described above, with the no-poaching constraint for the signatories.

Analysis. Along any stationary equilibrium path, where all employed workers obtain the same wage w and firms set the same price p over time, even in the absence of NPAs, firms do not make poaching offers and only replace *retirees* with newcomers hired at the same wage w. Indeed, so long as a firm wants to hold the same labor force over time, poaching rivals' incumbent workers would not be optimal, as it would require paying them a higher wage than the equilibrium level w at which it can recruit newcomers to replace all its retirees (given that, by revealed preference, rivals' incumbent workers prefer working for their current employer at the equilibrium wage). Thus, the fact that firms do not poach each others' workers *per se* tells little about whether or not their behavior is collusive. However, this does not mean that a ban on NPAs is inconsequential, because the presence of binding NPAs affects the defection profit that can be captured by a firm deviating from a candidate stationary equilibrium path, as argued below.

[46] Together with long-term contracts, this assumption rules out *ratchet effects* deriving from employers learning workers' types $\xi_{i,j}$ over time through their contract acceptance decisions.

[47] In order for NPAs to have a bite, there must be some incumbent workers in the market, so that the following analysis, formally speaking, applies for $t \geq 1$.

Suppose, first, that NPAs are not in place (e.g., they are banned by competition authorities). Then, starting from any candidate most profitable SPNE (p, w), the highest one-shot deviation profit for a firm i is obtained by slightly undercutting the candidate equilibrium price p and making wage offers that minimize the production cost of $Q(p)$: formally,

$$\min_{\{w_{\tau,i}\}} \sum_{\tau=2,\dots,T} \Big[wL(w,w) + w_{\tau,i}\tilde{\ell}_{\tau,i}(w_{\tau,i},w) \Big] + w_{1,i}L(w_{1,i},w) + r\,K(\cdot), \tag{12}$$

as the labor force of a deviating firm i consists of *(i)* its incumbent workers $L(w, w)$, whom it keeps at the candidate equilibrium wage w, *(ii)* poached rivals' workers $\tilde{\ell}_{\tau,i}$, i.e. their incumbent workers that i can hire by offering $w_{\tau,i} > w$,[48] and *(iii)* newcomers, whose labor supply is $L(w_{1,i}, w)$. Then, for any collusive level of wages (i.e., $w < W^*(p)$, this function being defined following the same steps as in Lemma 3), the deviating firm finds it optimal to set $w_{\tau,i} > w$ for all $\tau = 1, ..., T$. The option to make poaching offers to rivals' incumbent workers is thus valuable from a deviating firm's standpoint. The reason is that the wage offer needed to expand its labor force to any given level is lower when it can be offered to a larger pool of workers, which is the case when a deviating firm has the chance of poaching rivals' workers.

Following any deviation, firms revert to a continuation equilibrium where they set prices competitively and the deviating firm makes zero profits; this constitutes an optimal punishment and implies that the best deviation is indeed the one, characterized above, that maximizes profits in the period of defection.

If, instead, NPAs are allowed, a stationary equilibrium where on-path all firms sign these agreements at any period always exists. If one or more firms deviate and do not sign the NPA, this deviation is detected before firms make wage offers and set their prices, so they immediately revert to a zero-profit continuation equilibrium.[49] Therefore, a deviation can be profitable only in the wage- and price-setting stage after signing the NPA. Then, by the arguments above, the impossibility of poaching rivals' workers strictly reduces the profits that a deviating firm can obtain in the deviation period; following any deviation, a continuation equilibrium where no NPAs are signed anymore and the deviating firm makes zero profits is played.[50]

Results and implications. By the above analysis, the possibility of signing a binding NPA weakens defection incentives, allowing firms to sustain more collusive arrangements — i.e., the most profitable SPNE features firms signing binding NPAs, and setting lower wages and higher prices relative to the scenario where these agreements are banned by competition authorities:

[48]Formally, $\tilde{\ell}_{\tau,i}(w_{\tau,i}, w) \equiv (n-1)L(w,w)\Pr\left[w_{\tau,i} + \xi_i \geq w + \max_{i' \in \mathcal{N}\setminus\{i\}} \xi_{i'} \middle| \xi_i < \max_{i' \in \mathcal{N}\setminus\{i\}} \xi_{i'}\right]$. As these workers are identical from i's viewpoint, by the same arguments as in Lemma 1, offering them a uniform wage is optimal. In this model, as a deviating firm deprives its rivals of all consumers and workers are contestable in future periods, these firms would have no incentives to make their workers a counteroffer to the poaching offer.

[49]A deviating firm would derive no advantage from the possibility of making poaching offers if rivals can renegotiate their incumbent employees' wages, given that these workers always prefer remaining where they are if their current employer matches the poaching offer; moreover, the signatories could try to poach the deviating firm's workers, inducing it to increase the remuneration to its incumbent workers. Firms thus remain symmetric in the continuation game, so a zero-profit continuation equilibrium always exists.

[50]Starting from a candidate equilibrium where each firm expects others not to sign the NPA, unilaterally signing it is inconsequential; then, all workers hired by the deviating firm can be poached away by its rivals in the future, exactly as in the game without NPAs, so that a zero-profit continuation equilibrium still exists.

Proposition 9 (no-poaching agreements). *Banning NPAs increases the critical discount factor to sustain the cartel outcome and leads to strictly higher wages and lower prices for all lower values of* δ*.*

This model provides an anticompetitive rationale for the use of NPAs in low-skilled labor markets, where relation-specific investments are not a concern and workers are easily replaceable: These agreements can be employed as facilitating practices — i.e., are instrumental to (wage and, eventually, price) collusion. A ban on NPAs, by tightening incentive-compatibility constraints, thus leads to higher wages and, if employers also interact in the same product market, lower prices. Therefore, the *per se illegality* status of naked NPAs in the US legislation, and their similar consideration as *by object restrictions of competition* in the EU, is consistent with both worker and consumer protection.

The above analysis also rationalizes the recent empirical evidence on franchise NPAs and a ban thereof. Commentators have observed that a series of vertical NPAs between a franchisor and multiple franchisees eliminates competition among the latter as effectively as horizontal agreements among themselves orchestrated by the franchisor.[51] Even if competing firms in the industry are franchisees of different franchisors, all of them imposing franchise NPAs would facilitate intra- and inter-brand collusion (though to a lesser extent than an industrywide NPA) by reducing the pool of available workers that any deviating franchisee can attract. Krueger and Ashenfelter (2022) have documented that, until recently, over half of all franchise agreements in the US, at companies including fast-food restaurants and consumer staples (actually, more than in industries with higher average wages and education levels), included provisions barring franchisees from hiring one another's workers. Then, because of the legal cases and proposed legislation, many chains have removed such clauses from their contracts, which, as shown by Lafontaine et al. (2023) using data from the chain restaurant industry, has led to higher wages.

5.2 Related facilitating devices

Non-compete agreements. A non-compete agreement (hereafter, NCA) between a firm and any of its employees prohibits the latter from working for rival firms in the following period. Formally, when negotiating with a newcomer ($\tau = 1$), a firm i can include a one-period NCA in its offer at an additional remuneration $\omega_{1,i}$, on top of the per-period wage $w_{1,i}$; in this case, if the worker accepts the offer, it commits not to work for a rival firm $i' \in \mathcal{N} \setminus i$ at age $\tau = 2$. In the following period, a firm can renegotiate the wage (i.e., eventually offer $w_{2,i} > w_{1,i}$) and eventually propose a remuneration $\omega_{2,i}$ to sign another one-period NCA, and so on.

As, along any stationary equilibrium path, workers never expect to receive any attractive poaching offer from other firms, NCAs are signed for free (i.e., $\omega_{\tau,i} = 0$ for all τ and i) even by farsighted workers. In an equilibrium where all workers sign NCAs, a firm contemplating a deviation is *de facto* unable to make offers to all rivals' incumbent workers, precisely as in the presence of an industrywide NPA. However, NCAs may not be a perfect substitute for NPAs. The reason is that a deviating firm offering NCAs obtains a larger pool of non-contestable workers, which may make it harder for rivals to punish its deviation. Unless this effect dominates, banning NCAs results in less collusive outcomes.

[51] See, e.g., `https://www.concurrences.com/en/bulletin/special-issues/no-poach-agreements/`.

Corollary 3 (non-compete agreements). *Banning NCAs may increase the critical discount factor to sustain the cartel outcome and lead to higher wages and lower prices for all lower values of δ.*

This result is a possible theoretical explanation for empirical evidence that, similar to NPAs, NCAs cover a significant fraction of less-educated and low-wage workers (Starr et al., 2021), and their ban for these workers resulted in an increase in wages (Lipsitz and Starr, 2022).

Pay-transparency and pay-equity regulations. Pay-equity rules, such as the US Equal Pay Act of 1963, are based on the principle "equal pay for equal work"; pay-transparency regulations, making workers aware of eventual discrimination, are instrumental to their enforcement.

In this model, all employed workers receive equal compensation along any stationary equilibrium path; yet, when a firm deviates, it optimally offers a higher wage to newcomers (and, absent NPAs or NCAs, to rivals' incumbent workers) without correspondingly increasing the wage of its incumbent workers (as these receive no poaching offers in the deviation period). Pay-equity regulations would instead force the deviating firm to do so, reducing its deviation incentives:[52]

Corollary 4 (pay-equity regulations). *Pay-equity regulations forbidding firms from wage-discriminate among their employees reduce the critical discount factor to sustain the cartel outcome and lead to strictly lower wages and higher prices for all lower values of δ.*

The facilitation of employer collusion, coming as an unintended consequence of pay-transparency and pay-equity regulations, may be an explanation for the empirical evidence showing that, on top of narrowing coworker wage gaps, these regulations have often also led to lower average wages (see Cullen, 2024, and references therein).

6 Conclusion

Employer collusion is not a recent phenomenon. The idea that employers have the incentives and the ability to collude to depress wages dates back to Adam Smith; recently, Delabastita and Rubens (2022) have provided historical evidence of wage cartels among Belgian coal firms in the 19th century. The fact that "until recently economists assumed that labor markets are fairly competitive" (Krueger and Posner, 2018) explains why collusive behavior has only been investigated, both theoretically and empirically, in product markets. However, the recent evidence that firms enjoy significant labor market power and a rising number of antitrust cases involving employers' coordinated behavior make it paramount to understand the economics of collusion in labor markets, guiding antitrust and regulatory interventions.

Prima facie, one might think that *turning upside-down* models of collusion in oligopoly would suffice for this purpose: The behavior of firms with labor market power colluding to

[52] Unlike the possibility of signing NCAs, pay-equity regulations cannot help a deviating firm to obtain a positive profit in the punishment phase: Together with downward nominal wage rigidity, pay-equity regulations imply that the deviating firm needs to pay an even (weakly) higher wage in the subsequent periods, which disadvantages it *vis-à-vis* its rivals. Thus, similar to NPAs, pay-equity regulations only add a binding constraint to the (one-shot) deviation profit-maximization problem.

depress wages may simply mirror the well-studied one of firms with product market power colluding to raise prices. Adopting this simplistic view would be misleading, as it overlooks two key aspects. First, firms often interact in the same labor and product markets, and have market power in both. Therefore, it would be erroneous to view labor market and product market collusion as separate phenomena that do not interact: Understanding labor market collusion requires building a theory of multimarket contact in vertically related markets. Second, the labor market has distinctive features absent in product markets. In particular, complex long-term employment contracts, whose terms are partly subject to public regulation, are prevalent, as opposed to simple one-shot transactions at firms' posted prices occurring in most product markets. A theory of employer collusion must therefore unveil what contractual clauses can constitute labor-market-specific facilitating practices and how regulation of employment contracts can affect the scope for collusion.

This paper has developed a theory of collusion in the presence of labor market power that encompasses both these aspects. It has shown that the impact of oligopsony power on collusive behavior is twofold. First, labor market power increases the scope for collusion, in several respects: *(i)* Firms can sustain the cartel outcome at a lower critical discount factor than in the absence of individual-firm labor market power; *(ii)* when firms operating in independent product markets hire from the same labor market, oligopsony power fosters broader cross-market collusion, and conglomerate mergers can produce anticompetitive multimarket-contact effects; *(iii)* firms can employ labor-market-specific horizontal or vertical restraints, such as no-poaching and non-compete agreements, as facilitating practices. Second, labor market power makes collusion a multimarket phenomenon: Firms have incentives to cooperate in both labor and product markets; when colluding in one market is unfeasible (e.g., because of monitoring of collusive behavior by antitrust authorities), cooperation in the other market strengthens.

Since collusion harms workers and consumers alike, antitrust authorities' efforts to monitor also labor market collusion align with consumer protection — i.e., with a *narrow* statutory objective, as opposed to a *broad mandate* including worker protection. Effective monitoring of collusive behavior in labor and product markets is indispensable in safeguarding consumers' and workers' interests because, in the presence of labor market power, standard policy measures to promote product market competitiveness (e.g., free trade agreements) have limited effects on firms' ability to collude, and labor market regulations (e.g., pay-equity provisions) can have unintended procollusive implications.

Finally, some implications of this theory can help infer the presence of collusive behavior from observable wage and price data, which can guide antitrust authorities' monitoring actions. Specifically, *(i)* a simultaneous increase in employment and price levels following a rise in the minimum wage, *(ii)* higher average wages and prices following a ban on no-poaching or non-compete agreements, and *(iii)* a low pass-through to consumer prices of wage mark-downs due to an increase in firms' labor market power (determined by, e.g., the weakening of trade unions' bargaining power), are all indicative of collusive behavior. Existing evidence often aligns with these predictions; this paper will hopefully encourage further investigations.

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Appendix

A Proofs

Technical assumptions. To guarantee firm viability, assume firms sell a positive quantity when pricing at the optimized average cost. This assumption always holds if $Q(c_0) > 0$, with c_0 defined in Eqn. (3), and $\underline{\xi} \geq 0$.

Moreover, I assume that all profit functions considered in the maximization problems are globally concave and admit an interior maximum point, which is therefore obtained from the first-order condition(s). Sufficient conditions for these properties are given by $Q(p) \to 0$ as $p \to \infty$, $Q''(p) \leq 0$, and $\partial^2 L(\cdot)/\partial w_i^2$, $\partial^2 L(\cdot)/(\partial w_i \partial w)$, $\partial^2 L(\cdot)/\partial w^2 \leq 0$.

Finally, using the notation in Section 3, Assumption (A) can be written as

$$\frac{Q(p^M)}{|Q'(p^M)|} \geq r \int_{L(w^M,w^M)}^{nL(w^M,w^M)} \left| \frac{\partial^2 K(\tilde{\ell}, Q(p^M))}{\partial \ell \partial q} \right| \mathrm{d}\tilde{\ell}.$$

Proof of Proposition 0. The results easily follow from the arguments given in the text. $\square$

Proof of Lemma 1. For any rivals' offers $\{w_{i',j}\}_{i' \in \mathcal{N}\setminus\{i\}, j \in [0,J]}$, fix any ℓ_i that firm i may want to hire (on- or off-path). Suppose firm i offers the same wage w_i to all workers $j \in [0, J]$. Then, from Eqn. (2) it follows that w_i must be set such that

$$\ell_i = J\,\mathbb{E}[\Pr[w_i + \xi_i \geq \max\{w_0, \max_{i' \in \mathcal{N}\setminus\{i\}} w_{i'} + \xi_{i'}\}]],$$

the expectation being taken with respect to the vector $\{\xi_i\}_{i \in \mathcal{N}}$ and, eventually, as workers are anonymous, the distribution of rivals' wage offers $w_{i',j}$.

To show that offering a uniform wage is the minimal-cost way to hire ℓ_i workers, suppose by contradiction that firm i makes two distinct offers, $\underline{w}_i$ to a mass J' of workers and $\overline{w}_i > \underline{w}_i$ to the other workers. Then, $\underline{w}_i$ and $\overline{w}_i$ must be such that

$$\ell_i = J'\,\mathbb{E}[\Pr[\underline{w}_i + \xi_i \geq \max\{w_0, \max_{i' \in \mathcal{N}\setminus\{i\}} w_{i'} + \xi_{i'}\}]] + (J - J')\mathbb{E}[\Pr[\overline{w}_i + \xi_i \geq \max\{w_0, \max_{i' \in \mathcal{N}\setminus\{i\}} w_{i'} + \xi_{i'}\}]],$$

where, workers being anonymous, the distribution of rivals' offers is the same across the two groups of workers. As, for all ξ_i and $w_{i'}$,

$$\Pr[\underline{w}_i + \xi_i \geq \max\{w_0, \max_{i' \in \mathcal{N}\setminus\{i\}} w_{i'} + \xi_{i'}\}] < \Pr[\overline{w}_i + \xi_i \geq \max\{w_0, \max_{i' \in \mathcal{N}\setminus\{i\}} w_{i'} + \xi_{i'}\}],$$

the deviation increases the average wage paid to the recruited workers. This argument generalizes to an arbitrary number of different offers. As choosing $w_{i,j} = -\infty$ is equivalent not to approach worker j, this suffices to show that offering uniform wages is a dominant strategy in the static game.

Then, each firm's labor supply function is obtained from Eqn. (2) for $w_{i,j} \equiv w_i$ for all i. As all $\xi_{i,j}$ are drawn from the same distribution Ξ, which is assumed smooth, firms' labor supply is symmetric and almost everywhere differentiable. Then, inspection of Eqn. (2) immediately implies that, in a situation where all i's rivals offer the same wage w to all workers, i's labor supply, denoted by $L(w_i, w)$, is increasing in w_i and decreasing in w. Finally, for $w_i = w$, aggregate labor supply writes as

$$\sum_{i \in \mathcal{N}} L(w, w) = J \Pr[w + \max_{i \in \mathcal{N}} \xi_i \geq w_0],$$

which is clearly increasing in w. □

Proof of Lemma 2. Given the wage w offered (to all workers) by all its rivals, firm i's cost-minimization problem for any given quantity q to produce can be written as

$$\min_{\ell_i, k_i} W(\ell_i, w)\ell_i + r\, k_i$$
$$\text{s.t. } q = F(\ell_i, k_i),$$

where

$$W(\ell_i, w) \equiv L^{-1, w_i}(w_i, w),$$

is firm i's residual (inverse) labor supply, with $L^{-1,w_i}(\cdot)$ being the inverse of the labor supply function $L(\cdot)$ with respect to w_i. As $L(\cdot)$ is strictly increasing in w_i, this inverse function is well defined and is increasing in ℓ_i. Then, by the steps in Gelles and Mitchell (1996), given that $F(\cdot)$ is CRS, at the optimum, the elasticity of the average cost $C(q; w)$, defined in Eqn. (5), with respect to q equals

$$\frac{1}{qC(\cdot)} \ell_i^2 \frac{\partial W_i(\cdot)}{\partial \ell_i} > 0,$$

which implies that the optimized average cost is an increasing function of q. □

Proof of Lemma 3. For any candidate symmetric equilibrium price p and wage w offered by rivals, each firm i expects to sell $q_i = Q(p)/n$ and chooses w_i solving

$$\min_{w_i} w_i L(w_i, w) + r\, K\left(L(w_i, w), \frac{Q(p)}{n}\right).$$

Taking the first-order condition yields

$$L(w_i, w) + \left[w_i + r \frac{\partial K(L(w_i, w), Q(p)/n)}{\partial \ell}\right] \frac{\partial L(w_i, w)}{\partial w_i} = 0. \tag{13}$$

Any symmetric equilibrium is then obtained from Eqn. (13) imposing symmetry — i.e., $w_i = w$. At $w \to -\infty$, its left-hand side is negative, as $L(-\infty, -\infty) = 0$ and the term in square brackets is strictly negative and is multiplied by $\partial L(\cdot)/\partial w_i > 0$. Conversely, for $w \to +\infty$, this left-hand side is clearly positive.

Therefore, the wage-offer game (given p) admits a unique symmetric equilibrium $w_i = W^*(p)$ for all $i \in \mathcal{N}$ if the derivative of the left-hand side of Eqn. (13) for $w_i = w$ with respect to w is positive. This boils down to the following condition:

$$\frac{\partial L(\cdot)}{\partial w_i} + \left[1 + r\frac{\partial L(\cdot)}{\partial w_i}\frac{\partial^2 K(\cdot)}{\partial \ell^2}\right]\left(\frac{\partial L(\cdot)}{\partial w_i} + \frac{\partial L(\cdot)}{\partial w}\right) - \frac{L(\cdot)}{\frac{\partial L(\cdot)}{\partial w_i}}\left(\frac{\partial L^2(\cdot)}{\partial w_i^2} + \frac{\partial L^2(\cdot)}{\partial w_i \partial w}\right) > 0,$$

which, as $[\partial L(\cdot)/\partial w_i + \partial L(\cdot)/\partial w]|_{w_i=w} > 0$, always holds for $\partial L^2(\cdot)/\partial w_i^2 + \partial L^2(\cdot)/(\partial w_i \partial w) \leq 0$.

Finally, the derivative of the left-hand side of Eqn. (13) with respect to p equals

$$r \underbrace{\frac{\partial L(\cdot)}{\partial w_i}}_{+} \underbrace{\frac{\partial^2 K(\cdot)}{\partial \ell \partial q}}_{-} \underbrace{\frac{Q'(\cdot)}{n}}_{-} > 0,$$

which, by the Implicit Function Theorem and the second-order condition of firms' cost-minimization problem, shows that $W^*(p)$ is decreasing in p. □

Proof of Proposition 1. As $\pi(0) < 0 < \pi(p^P)$,[53] and, by concavity, $\pi(p)$ is strictly increasing in p for all $p < p^P$, there exists a price level $\underline{p}^N \in (0, p^P)$ such that $\pi(\underline{p}^N) = 0$. As firms can always overcut any candidate equilibrium price and make zero profits, any candidate equilibrium price is such that $p \geq \underline{p}^N$. Then, for all $p \leq p^P$, the best deviation for a firm always consists in slightly undercutting the price, yielding the deviation profit $\pi^D(p)$ in Eqn. (7).

The results of Lemma 2, which apply here as firms are best-responding to rivals' wages both on- and off-path, imply that holding fixed rivals' wage offers at $W^*(p)$, each firm faces a convex cost function. Then, by the same steps as in Dastidar (1995), it follows that the difference $\pi^D(p) - \pi(p)$ is negative at $p = \underline{p}^N$ and is increasing in p, and so that there is a price $\overline{p}^N \in (\underline{p}^N, p^P]$ such that $\pi(p) \geq \pi^D(p)$ for all $p \leq \overline{p}^N$. Hence, all prices $p \in [\underline{p}^N, \overline{p}^N]$, with corresponding wage levels $w = W^*(p)$, constitute NEs of the static game. As $\pi(p)$ is strictly increasing in p for all $p < p^P$, equilibria with higher prices are associated with a larger profit. □

Proof of Corollary 1. Since optimal punishments profits cannot be negative because firms always have the option to shut down, and the simple perfect equilibrium consisting of the repeated static NE profile $(\underline{p}^N, W^*(\underline{p}^N))$ yields discounted profits of zero to all firms, by the results in Abreu (1988), restricting attention to SPNE in grim-trigger strategies specifying reversion to $(\underline{p}^N, W^*(\underline{p}^N))$ is without loss of generality.

For $\delta = 0$, in any SPNE firms must play a static NE at each period. Hence, the most profitable SPNE is the repetition of $(\overline{p}^N, W^*(\overline{p}^N))$. □

Proof of Proposition 2. As seen in Lemma 1, uniform wage offers ($w_{i,j} \equiv w_i$ for all $j \in [0, J]$) minimize production costs. Then, industry cost minimization requires each worker to be hired by the firm providing the highest non-wage benefit, which is achieved by all firms offering the same wage level w. As a result, industry profits' maximization requires that all firms are active

[53]Note that *(i)* $\pi(p) \to 0$ as $p \to \infty$ (as $Q(p) \to 0$ and so firms are *de facto* inactive, i.e. also $W^*(p) \to 0$), and *(ii)* as firms sell a positive quantity when pricing at the optimized average cost, they can obtain positive profits by raising their price. Hence, p^P is finite and is such that $\pi(p^P) > 0$.

and equally split the production of the monopoly output — i.e., charge the same price p to consumers and offer the same wage w to workers.

Maximizing industry profits is then equivalent to maximize $\pi(p,w)$ in Eqn. (8) with respect to p and w. This yields the first-order conditions

$$Q(p) + \left[p - r\frac{\partial K(L(w,w), Q(p)/n)}{\partial q}\right] Q'(p) = 0, \tag{14}$$

and

$$L(w,w) + \left[w + r\frac{\partial K(L(w,w), Q(p)/n)}{\partial \ell}\right] \left(\frac{\partial L(w_i,w)}{\partial w_i} + \frac{\partial L(w_i,w)}{\partial w}\right)\bigg|_{w_i=w} = 0, \tag{15}$$

respectively.

Defining $\widehat{P}(w)$ as the solution to Eqn. (14) for fixed w, by the Implicit Function Theorem,

$$\frac{\partial \widehat{P}(w)}{\partial w} = -\frac{-rQ'(\cdot)\frac{\partial^2 K(\cdot)}{\partial \ell \partial q}\left(\frac{\partial L(\cdot)}{\partial w_i} + \frac{\partial L(\cdot)}{\partial w}\right)}{\frac{\partial^2 \pi(\cdot)}{\partial p^2}} < 0,$$

as $\partial^2\pi(\cdot)/\partial p^2 < 0$ by the second-order condition of the cartel's problem. Similarly, defining $\widehat{W}(p)$ as the solution to Eqn. (15) for fixed p, the Implicit Function Theorem implies that

$$\frac{\partial \widehat{W}(p)}{\partial p} = -\frac{\frac{r}{n}\frac{\partial^2 K(\cdot)}{\partial \ell \partial q}\left(\frac{\partial L(\cdot)}{\partial w_i} + \frac{\partial L(\cdot)}{\partial w}\right)}{\frac{\partial^2 \pi(\cdot)}{\partial w^2}} < 0,$$

as $\partial^2\pi(\cdot)/\partial w^2 < 0$ by the second-order condition of the cartel's problem.

Comparing Eqn. (15) with Eqn. (13) at a symmetric solution implies that holding fixed the price p, $\widehat{W}(p) < W^*(p)$: This result follows from the second-order conditions because the left-hand side of Eqn. (15) contains a positive extra-term, as $w + r\partial K(\cdot)/\partial \ell < 0$ for the two equations to hold, and $\partial L(\cdot)/\partial w < 0$.

If $p^M > \overline{p}^N$, as $\widehat{W}(p)$ is decreasing in p, this would imply that $w^M < W^*(\overline{p}^N) \equiv \underline{w}^N$. Therefore, given that $\widehat{P}(w)$ is decreasing in w, and it is an upper bound on the cartel price given w (even if $\widehat{P}(w) < P^*(w)$, with $P^*(\cdot)$ defined by Eqn. (11), the cartel, coordinating on the most profitable equilibrium, would still set $\widehat{P}(w)$), one can conclude that indeed $p^M > \overline{p}^N$ and $w^M < \underline{w}^N$.

In order for (p^M, w^M) to be sustainable as a SPNE, it must be that

$$\delta \geq \delta^M \equiv 1 - \frac{\pi(p^M, w^M)}{\pi^D(p^M, w^M)},$$

where $\pi(\cdot)$ and $\pi^D(\cdot)$ are defined in Eqn. (8) and (9), respectively (the result that the profit in Eqn. (9) is the highest one attainable in the deviation period is established in the Proof of Proposition 3 below, and firms' profits are zero afterwards by Corollary 1).

Therefore, $\delta^M < \delta_0^M$ if and only if $\pi^D(p^M, w^M) < n\pi(p^M, w^M)$. Let $\widehat{\pi}^M$ denote the highest attainable profit in the hypothetical scenario where only one firm is active. As firm i's rivals being inactive is equivalent to $w_{i'} = -\infty$ for all $i' \in \mathcal{N} \setminus \{i\}$,

$$\widehat{\pi}^M = \max_{p,w} pQ(p) - [wL(w, -\infty) + r\, K\,(L(w, -\infty), Q(p))]\,.$$

Then, as $L(w, -\infty) > L(w, w^M)$ (because $\partial L(\cdot)/\partial w < 0$), it immediately follows that

$$\widehat{\pi}^M > \max_{p,w} pQ(p) - \left[wL(w, w^M) + r\, K\left(L(w, w^M), Q(p)\right)\right] = \pi^D(p^M, w^M).$$

Next, as, by definition, $\pi(p^M, w^M) = \max_{p,w} \pi(p, w)$, it holds that

$$n\pi(p^M, w^M) = \max_{p,w} pQ(p) - [nwL(w, w) + r\, K\,(nL(w, w), Q(p))] > \widehat{\pi}^M,$$

because employers' differentiation implies that

$$nL(w, w) = J \Pr[w + \max_{i\in\mathcal{N}} \xi_i \geq w_0] > J \Pr[w + \xi_i \geq w_0] = L(w, -\infty).$$

Summing up, $\pi^D(p^M, w^M) < \widehat{\pi}^M < n\pi(p^M, w^M)$, and so $\delta^M < \delta_0^M$. □

Proof of Proposition 3. First, I prove that, given a candidate most profitable SPNE (p, w) and the corresponding per-period profit given in Eqn. (8), the highest deviation profit is the one given in Eqn. (9). To this end, I first establish that, supposing that a deviating firm wants to deviate in price, its best deviation always consists in slightly undercutting p. As $\pi(p, w) > 0$ (this holds at the most profitable static NE, hence *a fortiori* it shall be true at the most profitable SPNE for all $\delta > 0$), and a firm i would make no sales for any $p_i > p$, one can restrict attention to $p_i \leq p$, with the convention that all consumers' demand $Q(p)$ is served by the deviating firm i even if $p_i = p$ (i.e., $p_i = p$ corresponds to a slight undercutting of p). The deviating firm solves

$$\max_{p_i \leq p, w_i} p_i Q(p_i) - [w_i L(w_i, w) + r\, K\,(L(w_i, w), Q(p_i))]\,,$$

and, hence, $p_i = p$ is the best deviation price if and only if

$$Q(p) + \left[p - r\frac{\partial K(L(w_i, w), Q(p))}{\partial q}\right] Q'(p) \geq 0.$$

As the collusive price p is weakly below the monopoly price given $w_i = w$, denoted by $\widehat{P}(w)$ (any higher price would be suboptimal), i's profit is increasing in p_i and so this inequality always holds at $(L(w, w), Q(p)/n)$, or equivalently (given that $\partial K(\cdot)/\partial q$ is homogeneous of degree zero) at $(nL(w, w), Q(p))$. As $\partial^2 K(\cdot)/\partial \ell \partial q < 0$, it holds *a fortiori* in the deviation if $nL(w, w) > L(w_i, w)$. This inequality always holds: As colluding firms at most fully internalize the negative cross-firm wage externalities, $nL(w, w)$ is weakly larger than the overall labor force needed to produce $Q(p)$ at the minimum cost for the industry; hence, a deviating firm who wants to produce $Q(p)$ alone will optimally hire at most $nL(w, w)$ workers; however, given that it needs to raise the wage to attract workers hired by rivals in equilibrium, it follows that

$nL(w,w) > L(w_i,w)$ at the optimal deviation.

Then, the highest deviation profit is given by

$$\tilde{\pi}^D(p,w) \equiv \max_{q_i \in \{Q(p)/n, Q(p)\}, w_i} pq_i - [w_i L(w_i,w) + r\, K\,(L(w_i,w), q_i)]\,.$$

Fix a price level p. If $w = W^*(p)$, firm i is already offering its static profit maximizing wage level to produce $q_i = Q(p)/n$, and so, for all $p > \overline{p}^N$, only choosing $q_i = Q(p)$ strictly increases the deviating firm's profit above $\pi(p,w)$. Colluding firms can, however, set $w < W^*(p)$, which increases the deviation profit from choosing $q_i = Q(p)/n$ less than it increases the deviation profit from choosing $q_i = Q(p)$. Indeed, by the Envelope Theorem,

$$\frac{\partial \tilde{\pi}^D(\cdot)}{\partial w} = L(w_i,w)\frac{\frac{\partial L(w_i,w)}{\partial w}}{\frac{\partial L(w_i,w)}{\partial w_i}},$$

is higher, in absolute value, the higher the deviating firm's w_i, which in turn is higher when it produces more. As a result, any reduction in w below $W^*(p)$ further increases the relative profitability for a deviating firm of choosing $q_i = Q(p)$ rather than $q_i = Q(p)/n$. Therefore, the highest deviation profit is obtained for $q_i = Q(p)$ and is given by Eqn. (9).

Next, to obtain the most profitable SPNE for any $\delta \in [0, \delta^M]$, I first show that the ratio $\pi(p,w)/\pi^D(p,w)$ is decreasing in p and increasing in w at any candidate most profitable SPNE. As, for all $p \leq \widehat{P}(w)$, $\partial\pi(\cdot)/\partial p \geq 0$ and, by the arguments above,

$$\frac{\partial \pi^D(\cdot)}{\partial p} = Q(p) + \left[p - r\frac{\partial K(L(w_i,w), Q(p))}{\partial q}\right] Q'(p) > 0,$$

it holds that

$$\frac{\partial}{\partial p}\left(\frac{\pi(\cdot)}{\pi^D(\cdot)}\right) < 0 \iff \frac{\pi(\cdot)}{\pi^D(\cdot)} > \frac{\partial \pi(\cdot)/\partial p}{\partial \pi^D(\cdot)/\partial p},$$

which, using the fact that $K(\cdot)$ is homogeneous of degree one, and so $K(\ell,q) = \ell\,\partial K(\cdot)/\partial \ell + q\,\partial K(\cdot)/\partial q$, can be written (omitting arguments) as

$$\frac{\frac{1}{n}\left[\left(p - r\frac{\partial K}{\partial q}\right)Q - \left(w + r\frac{\partial K}{\partial \ell}\right)L\right]}{\left(p - r\frac{\partial K^D}{\partial q}\right)Q - \left(w^D + r\frac{\partial K^D}{\partial \ell}\right)L^D} > \frac{\frac{1}{n}\left[Q + \left(p - r\frac{\partial K}{\partial q}\right)Q'\right]}{Q + \left(p - r\frac{\partial K^D}{\partial q}\right)Q'},$$

where w^D is the best deviation wage, $L^D \equiv L(w^D, w)$, and $K^D \equiv K(L^D, Q(p))$, whilst L and K denote the corresponding values in the candidate SPNE outcome. The two sides of this inequality are equal if $w^D = w$ and $L^D = nL$. Yet, as argued above, because $w^D > w$, the deviating firm sets $L^D < nL$. By a revealed preference argument, $\pi^D(\cdot)$ is then smaller, hence the left-hand side is larger; conversely, the denominator of the right-hand side is larger (as its derivative with respect to L^D equals $-rQ' \cdot \partial^2 K^D/(\partial\ell\partial q) < 0$), so that the right-hand side is smaller. As a result, the inequality is always satisfied.

Similarly, as, for all $w \geq \widehat{W}(p)$, $\partial\pi(\cdot)/\partial w \leq 0$ and, as seen above,

$\partial \pi^D(\cdot)/\partial w = L^D(\partial L^D/\partial w)/(\partial L^D/\partial w_i) < 0$, it holds that

$$\frac{\partial}{\partial w}\left(\frac{\pi(\cdot)}{\pi^D(\cdot)}\right) > 0 \iff \frac{\pi(\cdot)}{\pi^D(\cdot)} > \frac{\partial \pi(\cdot)/\partial w}{\partial \pi^D(\cdot)/\partial w}.$$

As colluding firm always set $w \leq W^*(p)$,

$$\frac{\partial \pi(\cdot)}{\partial w} > L\frac{\partial L/\partial w}{\partial L/\partial w_i},$$

and so a sufficient condition for the above inequality to hold is $\pi(\cdot)/\pi^D(\cdot) > L/L^D$, or, equivalently,

$$\frac{\pi^D(\cdot)}{L^D} < \frac{\pi(\cdot)}{L}.$$

This inequality always holds because a deviation, entailing an increase in the wage to be paid to attract more workers, unambiguously reduces the profit per worker (i.e., the deviating firm is less efficient than the whole industry in producing the same output $Q(p)$).

Therefore, the colluding firms would always like to raise p (holding fixed w) or reduce w (holding fixed p), but cannot do so when $\delta < 1 - \pi(\cdot)/\pi^D(\cdot)$ and so the incentive-compatibility constraint is violated. It follows that the incentive-compatibility constraint is binding for all $\delta < \delta^M$, and colluding firms optimally exploit any increase in δ to both raise the price and reduce the wage offers. By the Implicit Function Theorem, the results that $\pi(\cdot)/\pi^D(\cdot)$ is decreasing in p and increasing in w implies that the incentive-compatibility constraint defines an upward-sloping function $P^{\mathrm{IC}}(w)$ (resp., $W^{\mathrm{IC}}(p)$) in the (w,p)-plane (resp., (p,w)-plane), and the colluding firms choose the point on this function that maximizes $\pi(p,w)$. In particular, the Lagrangian of Problem (P) writes as

$$\mathcal{L}(\cdot) = \pi(p,w) - \lambda\left[\delta - 1 + \frac{\pi(p,w)}{\pi^D(p,w)}\right],$$

with λ denoting the Lagrange multiplier associated to the incentive-compatibility constraint. Taking the first-order condition with respect to p and w yields, respectively,[54]

$$\frac{\partial \pi(\cdot)}{\partial p} = \frac{\lambda}{[\pi^D(\cdot)]^2}\left[\frac{\partial \pi(\cdot)}{\partial p}\pi^D(\cdot) - \frac{\partial \pi^D(\cdot)}{\partial p}\pi(\cdot)\right],$$

and

$$\frac{\partial \pi(\cdot)}{\partial w} = \frac{\lambda}{[\pi^D(\cdot)]^2}\left[\frac{\partial \pi(\cdot)}{\partial w}\pi^D(\cdot) - \frac{\partial \pi^D(\cdot)}{\partial p}\pi(\cdot)\right].$$

Diving these equations and rearranging yields the optimality condition (10). □

Proof of Proposition 4. Maximizing $\pi(p, W^*(p))$ w.r.t. p yields the first-order condition

$$Q(\cdot) + Q'(\cdot)\left[p - r\frac{\partial K(\cdot)}{\partial q}\right] + n\frac{\partial L(\cdot)/\partial w}{\partial L(\cdot)/\partial w_i}L(\cdot)\frac{\partial W^*(\cdot)}{\partial p} = 0,$$

[54]The functions $\pi(p,w)$ and $\pi^D(p,w)$ defined in Eqn. (8) and (9) are continuously differentiable. Hence, the following expressions are always well-defined.

which, as $\partial W^*(p)/\partial p < 0$, contains a positive extra-term relative to the first-order condition (14) of the multimarket cartel: hence, the price cartel has incentives to set a higher price holding fixed w. Together with $W^*(p) > \widehat{W}(p)$, this implies that $p^P > p^M$ and $W^*(p^P) > w^M$.

Under Assumption (A), $p^P > \overline{p}^N$ is such that $\pi(p^P, W^*(p^P)) < \pi^D(p^P, W^*(p^P))$, implying that the critical discount factor to implement the price cartel outcome is

$$\delta^P = 1 - \frac{\pi(p^P, W^*(p^P))}{\pi^D(p^P, W^*(p^P))} > 0.$$

Indeed, as $w^M < W^*(p^M)$, also $L(w^M, w^M) < L(W^*(p^M), W^*(p^M))$, and so

$$p^M \geq r\frac{\partial K(L(w^M, w^M), Q(p^M))}{\partial q} > r\frac{\partial K(L(W^*(p^M), W^*(p^M)), Q(p^M))}{\partial q}.$$

Therefore, Assumption (A), which is equivalent to the first inequality above, implies that a firm is willing to undercut p^M even though it must stick to $w = W^*(p^M)$, implying that *a fortiori* $\pi(p^M, W^*(p^M)) < \pi^D(p^M, W^*(p^M))$. Then, as the difference $\pi^D(p, W^*(p)) - \pi(p, W^*(p))$ is increasing in p (see the Proof of Proposition 1), and, as seen above, $p^P > p^M$, one can conclude that $\pi(p^P, W^*(p^P)) < \pi^D(p^P, W^*(p^P))$.[55]

Next, the result $\delta^P < \delta_0^M$ follows from Lemma 2. Indeed, offering $w_i = W^*(p)$ when rivals do the same implies that firm i produces at the minimized average cost function $C(Q(p)/n; W^*(p))$. Then, as rivals still offer $w = W^*(p)$ when i deviates, the minimized average cost to produce $Q(p)$ is larger: $C(Q(p); W^*(p)) > C(Q(p)/n, W^*(p))$, from which one can conclude that, at $p = p^P$,

$$\pi^D(p^P, W^*(p^P)) = [p^P - C(Q(p^P); W^*(p^P))]Q(p^P) < [p^P - C(Q(p^P)/n; W^*(p^P))]Q(p^P) = n\pi(p^P, W^*(p^P)),$$

which implies that $\delta^P < (n-1)/n = \delta_0^M$.

Next, consider values of $\delta > 0$ such that both $P^P(\delta) < p^P$ and $P^M(\delta) < p^M$ hold — i.e., the incentive-compatibility constraint binds under both price collusion and multimarket collusion. For all such values of δ, the multimarket collusion outcome $(P^M(\delta), W^M(\delta))$ violates the wage constraint $w = W^*(p)$. Indeed, at $w = W^*(p)$, the inequality

$$\frac{\partial \pi(p, W^*(p))/\partial p}{\partial \pi^D(p, W^*(p))/\partial p} < \frac{\partial \pi(p, W^*(p))/\partial w}{\partial \pi^D(p, W^*(p))/\partial w}$$

simplifies, using the notation introduced in the proof of Proposition 3 above, as

$$\frac{\frac{1}{n}\left[Q + \left(p - r\frac{\partial K}{\partial q}\right)Q'\right]}{Q + \left(p - r\frac{\partial K^D}{\partial q}\right)Q'} < \frac{-L\frac{\partial L(\cdot)}{\partial w}\frac{\partial L(\cdot)}{\partial w_i}}{-L^D\frac{\partial L^D}{\partial w}\frac{\partial L^D}{\partial w_i}}.$$

The two sides of this inequality are equal when $w^D = w$ and $L^D = nL$; then, $w^D > w_i$ implies that $L^D < nL$ (see the Proof of Proposition 2), which reduces the denominator of the left-hand side and increases the denominator of the right-hand side, implying that this inequality always

[55] This implies that $\overline{p}^N < p^P$ is such that $\pi(\overline{p}^N, W^*(\overline{p}^N)) = \pi^D(\overline{p}^N, W^*(\overline{p}^N))$, and so $(\overline{p}^N, W^*(\overline{p}^N))$ is the most profitable SPNE under price collusion for $\delta = 0$.

holds.

Therefore, as the optimality condition (10) is violated under the wage constraint $w = W^*(p)$, it follows that, under price collusion, this constraint must bind for all $\delta > 0$. Then, as under multimarket collusion firms would never set $w > W^*(p)$ and the binding incentive-compatibility constraint defines an upward-sloping function in the (p, w)-plane, it follows that, for any given δ, the price collusion outcome is such that $P^P(\delta) > P^M(\delta)$ and $W^P(\delta) = W^*(P^P(\delta)) > W^M(\delta)$. Still, as any increase in δ entails an outward shift of the locus defined by the incentive-compatibility constraint and firms' profits increase in p for all $p \leq \widehat{P}(w)$, $P^P(\delta)$ is increasing in δ, and so $W^P(\delta) = W^*(P^P(\delta))$ is decreasing in δ.

Therefore, if the value $P^P(\delta^M)$, obtained by the intersection between the upward-sloping function $W^{\mathrm{IC}}(p)$ defined by the incentive-compatibility constraint at δ^M and the downward-sloping function $W^*(p)$, is smaller than p^P, then $\delta^P > \delta^M$ (in this case, $P^P(\delta)$ keeps increasing, and $W^P(\delta) = W^{\mathrm{IC}}(P^P(\delta))$ keeps decreasing for all $\delta \in [\delta^M, \delta^P]$, where the multimarket cartel outcome can be sustained); else, $\delta^P < \delta^M$ (in this case, $P^M(\delta)$ keeps increasing, and $W^M(\delta)$ keeps decreasing for all $\delta \in [\delta^P, \delta^M]$, where the price cartel outcome can be sustained). $\square$

Proof of Proposition 5. The constraint $p \leq P^*(w)$ is obtained by imposing that a firm does not gain by undercutting p holding fixed its wage offer at w — i.e.,

$$\frac{n-1}{n} pQ(p) - r\left[K(L(w,w), Q(p)) - K(L(w,w), Q(p)/n)\right] \leq 0. \tag{16}$$

To see that this inequality defines an upper bound on p, note that its left-hand side is negative at $p = 0$ and is increasing in p: its derivative with respect to p equals

$$\frac{n-1}{n}\left[Q(p) + pQ'(p)\right] - r\left[\frac{\partial K(L(w,w), Q(p))}{\partial q} - \frac{1}{n}\frac{\partial K(L(w,w), Q(p)/n)}{\partial q}\right],$$

this value being positive as $\partial K(L(w,w), Q(p))/\partial q > \partial K(L(w,w), Q(p)/n)/\partial q$ and $Q(p) + pQ'(p) - r\,\partial K(L(w,w), Q(p)/n)/\partial q \geq 0$ for all $p \leq \widehat{P}(w)$. Therefore, $P^*(w)$ is the value that satisfies (16) with equality. Moreover, $P^*(w)$ is a decreasing function of w. This result follows from the Implicit Function Theorem and the fact that the left-hand side of inequality (16) is increasing in p, as seen above, and in w: Indeed, its derivative with respect to w equals

$$-r\left[\frac{\partial K(L(w,w), Q(p))}{\partial \ell} - \frac{\partial K(L(w,w), Q(p)/n)}{\partial \ell}\right]\left(\frac{\partial L(w,w)}{\partial w_i} + \frac{\partial L(w,w)}{\partial w}\right) > 0.$$

Next, the most profitable static NE $(\overline{p}^N, \underline{w}^N)$ is the most collusive SPNE outcome under wage collusion only for $\delta = 0$: Under Assumption (A), it is such that $\pi(\cdot) = \pi^D(\cdot)$, so any different wage level would imply that firms have incentives to deviate in wage offers holding fixed their price. As argued in the text, $\overline{p}^N < P^*(\underline{w}^N)$, because a firm i is indifferent between undercutting or not $p = \overline{p}^N$ when it can optimally expand its labor force, which implies that it is strictly better off by not undercutting when it cannot choose $w_i > \underline{w}^N$. By a continuity argument, the price constraint $p \leq P^*(w)$ does not bind for sufficiently low values of δ, so that wage collusion can replicate the multimarket collusion outcome.

Conversely, Assumption (A) implies that, at the multimarket cartel outcome, any firm would have strict incentives to undercut $p = p^M$ even though it cannot raise its wage, or, equivalently, that $p^M = \widehat{P}(w^M) > P^*(w^M)$. Therefore, for all $\delta \geq \delta^M$, the price constraint binds, implying that under wage collusion, firms have to move down along the upward-sloping function in the (p, w)-plane defined by the incentive-compatibility constraint for $\delta = \delta^M$ by choosing lower price and wage levels such that the binding price constraint is satisfied. Therefore, by a continuity argument, $W^W(\delta) < W^M(\delta)$ and $P^W(\delta) = P^*(W^W(\delta)) < P^M(\delta)$ for all δ sufficiently large.

This argument also implies that the price constraint must bind at the wage cartel optimum. Maximizing $\pi(P^*(w), w)$ yields the first-order condition

$$\left[L(\cdot) + \left(w + r\frac{\partial K(\cdot)}{\partial \ell}\right)\left(\frac{\partial L(\cdot)}{\partial w_i} + \frac{\partial L(\cdot)}{\partial w}\right)\right] - \frac{1}{n}\left[Q(\cdot) + \left(p - r\frac{\partial K(\cdot)}{\partial q}\right)Q'(\cdot)\right]\frac{\partial P^*(\cdot)}{\partial w} = 0,$$

which, as $\partial P^*(w)/\partial w < 0$, contains a positive extra-term relative to the first-order condition (15) of the multimarket cartel. Together with the fact that, under Assumption (A), $\widehat{P}(w) > P^*(w)$ at the optimum, by the corresponding second-order condition this implies $w^W < w^M$ and $p^W < w^M$.

Therefore, $\delta^W > \delta^M$ if the wage level $W^W(\delta)$ pinned down, as explained above, by the intersection (in the (p, w)-plane) between the upward-sloping locus of incentive-compatible points given $\delta = \delta^M$ and the price constraint $p = P^*(w)$, is such that $W^W(\delta) > w^W$; else, $\delta^W < \delta^M$.

In either case, $\delta^M < \delta_0^M$. This result follows from the same argument given in the Proof of Proposition 2: At $w = w^W$, the industry is producing $Q(p^W)$ at the minimal cost; a deviating firm that wants to produce the same quantity alone will always face higher costs, because *(i)* of employer differentiation $(nL(w^W, w^W) > L(w^W, -\infty))$, and *(ii)* of labor market competition by rivals $(L(w_i, -\infty) > L(w_i, w^W)\ \forall w_i)$; as a result, $\pi^D(P^*(w^W), w^W) < n\pi(P^*(w^W), w^W)$, and so $\delta^W < (n-1)/n = \delta_0^M$. □

Proof of Corollary 2. The results in Proposition 3, 4, and 5 yield the inequalities in the statement. Then, the results immediately follow from consumer surplus being a decreasing function of the equilibrium price. □

Proof of Proposition 6. Take $\hat{\delta} \in (0, \delta^M)$ and consider the multimarket collusion outcome $(P^M(\hat{\delta}), W^M(\hat{\delta}))$, which is the point on the upward-sloping function $P^{\text{IC}}(w)$ defined by the binding incentive-compatibility constraint that maximizes the equilibrium profit, i.e. satisfies condition (10). Suppose that the regulator introduces a minimum wage $\underline{w} = W^M(\hat{\delta}) + \varepsilon$, with $\varepsilon > 0$ small enough, and denote by $(\underline{p}^M, \underline{w}^M)$ the most profitable SPNE under the wage constraint $w \geq \underline{w}$. Then, this constraint must bind: $\underline{w}^M(\delta) = \underline{w}$; moreover, as the regulatory minimum wage is still lower than the wage at which firms would like to ration workers, $L(\underline{w}, \underline{w}) > L(W^M(\hat{\delta}), W^M(\hat{\delta}))$. As the incentive-compatibility constraint is still binding, $\underline{p}^M = P^{\text{IC}}(\underline{w}) > P^{\text{IC}}(W^M(\hat{\delta})) = P^M(\hat{\delta})$. □

Proof of Proposition 7. Irrespective of H, the cartel outcome in each product market is $(p^M, w^M) \equiv \arg\max_{p,w} \pi(p, w)$, with $\pi(p, w)$ given in Eqn. (8) as in the baseline model because each firm only sells to consumers in its geographic market. The critical discount factor to sustain

(p^M, w^M) as a SPNE of the supergame in each product market is

$$\delta^M(H) = 1 - \frac{\pi(p^M, w^M)}{\pi^D(p^M, w^M)},$$

where

$$\pi^D(p^M, w^M) \equiv \max_{h \in \{1,\dots,H\}} \left\{ hp^M Q(p^M) - \min_{w_i} \left[w_i L(w_i, w^M) + r\, K(L(w_i, w^M), hQ(p^M)) \right] \right\}.$$

This is because, as seen in the baseline model, a deviating firm always finds it optimal to undercut the cartel price and serve all consumers within its local market. Yet, it may be profitable to serve consumers in $h > 1$ markets, if possible (i.e., if $H > 1$). However, if $H \to \infty$, the maximand H^* in the above problem is always finite. The reason is that a deviating firm faces a globally increasing average cost function (this is true by the arguments of the baseline analysis whenever the dimension J of the local labor market, whose workers can be all attracted by paying w_0, is not binding, and even more so for higher production levels, where the deviating firm can expand production only by increasing its endowment of capital) and hence has a finite optimal level of production.

Therefore, $\delta^M(H)$ is strictly increasing in H for $H < H^*$, and constant at $\delta^M(H^*)$ for all $H \geq H^*$. Finally, $\delta^M(H^*) < (nH^* - 1)/(nH^*)$ because a deviating firm serving consumers in h markets always obtains $\pi^D(p^M, w^M) < hn\pi(p^M, w^M)$: This holds for $h = 1$ by Proposition 2, and *a fortiori* holds for $h > 1$ as the deviating firm serves consumers also in geographic markets from which it cannot attract any worker.

Finally, note that there is no need for *cross-market collusion* to sustain (p^M, w^M) within each geographic market. The reason is as follows. Suppose firms in market h expect this outcome to prevail in all other markets. Then, a single firm's best deviation is always more profitable than a joint deviation of all firms in market h from (p^M, w^M), given that a joint deviation, implying higher wage offers by all firms in local labor market h, makes it costlier to expand production for any individual firm. Hence, for all $\delta \geq \delta^M(H)$, this critical discount factor being pinned down by the individual-firm incentive-compatibility constraint defined above, all firms in market h optimally choose (p^M, w^M) even if they only collude among each other. □

Proof of Proposition 8. In what follows, $\ell_{i,z} = L(w_{i,z}, w_z; w_{i',-z}, w_{-z})$ denotes the labor force hired by firm i in market z as function of the own wage offer, the wage offer of firm i' in the other product market $-z$ with which it can merge (as detailed above), and the symmetric offers made by any other firm in each of the two markets.

If all firms selling product z set the same price p_z and offer the same wage w_z to all workers, each firm in product market $z \in \{A, B\}$ makes a per-period profit

$$\pi(p_z, w_z; w_{-z}, w_{-z}) \equiv p_z \frac{Q(p_z)}{n} - \left[w_z L(w_z, w_z; w_{-z}, w_{-z}) + r\, K\left(L(w_z, w_z; w_{-z}, w_{-z}), \frac{Q(p_z)}{n} \right) \right].$$

With single-product firms, the highest deviation profit equals

$$\pi^D(p_z, w_z; w_{-z}, w_{-z}) \equiv p_z Q(p_z)/2 - \min_{w_i} \left[w_i L(w_i, w_z; w_{-z}, w_{-z}) + r\, K\left(L(w_i, w_z; w_{-z}, w_{-z}), Q(p_z)/2 \right) \right].$$

Comparing this deviation profit of a single-product firm with that of a multiproduct firm, denoted here by $\pi_m^D(p_z, w_z; w_{-z})$, at a symmetric SPNE candidate ($p_z \equiv p$, $w_z = w_{-z} \equiv w$, and also $w_{i',-z} = w$ with single-product firms), one has that $\pi_m^D(p, w; w) < 2\pi^D(p, w; w, w)$. To see this, note that, at the optimal deviation wage w_z^D for a single-product firm operating in market z,

$$\frac{\partial \pi^D(p, w; w_{i',-z}, w)}{\partial w_{i',-z}} = -\left(w_z^D + r\frac{\partial K(L(w_z^D, w; w_{i',-z}, w), Q(p)/n)}{\partial \ell}\right)\frac{\partial L(w_z^D, w; w_{i',-z}, w)}{\partial w_{i',-z}} =$$

$$= -L(w_z^D, w; w_{i',-z}, w)\frac{\frac{\partial L(w_z^D, w; w_{i',-z}, w)}{\partial w_{i',-z}}}{\frac{\partial L(w_z^D, w; w_{i',-z}, w)}{\partial w_i}} < 0.$$

Therefore, the deviation profit in market z is lower when the deviating multiproduct firm raises its wage in the other market $-z$. As a result, a deviating multimarket firm, raising its wage offers to expand production in both markets, cannot obtain twice as much the deviation profit of a single-product firm.

As the cartel outcome (p^M, w^M) — which, by construction, is as in the baseline model — does not depend on whether firms are single- or multiproduct, the results $\pi_m^D(p, w; w) < 2\pi^D(p, w; w, w)$ and, using the same notation for on-path profits, $\pi_m(p, w; w) = 2\pi(p, w; w, w)$ for all (p, w) immediately imply that

$$1 - \frac{\pi_m(p^M, w^M; w^M)}{\pi_m^D(p^M, w^M; w^M)} < 1 - \frac{\pi(p^M, w^M; w^M, w^M)}{\pi^D(p^M, w^M; w^M, w^M)},$$

which concludes the proof. □

Proof of Proposition 9. Take a stationary equilibrium (p, w) and any wages $w_{\tau,i}^D \geq w$ offered by the deviating firm in the period of deviation (as undercutting the equilibrium wage is never profitable). Then, there always exists a continuation equilibrium where the deviating firm makes zero profit, which constitutes an optimal punishment by the results in Abreu (1988).

To establish this claim, as firms' products are perfect substitutes, it is sufficient to prove that the average cost of the deviating firm is weakly larger than the average cost of the other firms. To this end, consider a continuation equilibrium where no NPAs are signed anymore. Such a continuation equilibrium always exists because when each firm expects rivals not to sign the NPA, unilaterally signing it would be inconsequential.

Consider the first period of the punishment phase. The aggregate labor force in the industry of workers of age $\tau \geq 3$ is $nL(w, w)$ (irrespective of whether NPAs were in place along the equilibrium path), as they were first hired when all firms offered the equilibrium wage: This is a symmetric pool of workers, in the sense that no firm can hire more workers from this pool than its rivals unless it offers a higher wage. This is true also for newcomers ($\tau = 1$), given that all workers are available for hire at the initial age. Finally, the industry labor force of workers of age $\tau = 2$ is

$$nL(w, w) + J\Pr[w_{1,i}^D + \xi_i \geq \max\{w_0, w + \max_{i' \in \mathcal{N}\setminus\{i\}} \xi_{i'}\} | w + \max_{\tilde{i} \in \mathcal{N}} \xi_{\tilde{i}} < w_0],$$

with $w^D_{1,i}$ denoting the offer they received from the deviating firm in the deviation period. That is, this pool of workers consists of the on-path industry labor force and the additional workers attracted to the deviating firm i because $w^D_{\tau,i} \geq w$. As $\xi_{i,j}$ are i.i.d. draws from the same distribution Ξ, also this is a symmetric pool of workers in the sense described above.

Therefore, irrespective of its wage offers in the deviation period, the deviating firm has never a cost-advantage *vis-à-vis* its rivals — i.e., it can never recruit a given labor force paying a lower average wage than its rivals.[56] These arguments also hold in subsequent periods, implying that the deviating firm makes zero profits forever after the deviation period.

Given that, for any profile of deviation wages, the deviating firm is guaranteed a payoff of zero in the continuation game, its highest deviation profit is obtained by minimizing the production cost of $Q(p)$ in the period of deviation. This is because the best static deviation, by the same arguments as in the baseline model, consists in slightly undercutting the candidate equilibrium price p — i.e., starting from any candidate SPNE (p, w), one can define the deviation profit as

$$\pi^D(p, w) \equiv pQ(p) - \min_{\{w_{\tau,i}\}} \sum_{\tau=2,...,T} \left[wL(w, w) + w_{\tau,i}\tilde{\ell}_{\tau,i}(w_{\tau,i}, w)\right] + w_{1,i}L(w_{1,i}, w) + r\,K(\cdot).$$

In the presence of a binding NPA in the period of deviation, this minimization problem is subject to the constraint $w_{\tau,i} = w$ for all $\tau = 2, ..., T$, so that $\tilde{\ell}_{\tau,i}(w, w) = 0$, given that offering rivals' workers any $w_i \leq w$ is equivalent not to make a poaching offer.

With a ban on NPAs, the first-order condition of the deviating firm's problem with respect to $w_{\tau,i}$, for all $\tau = 2, ..., T$, is given by

$$\tilde{\ell}_{\tau,i}(\cdot) + \left(w_{\tau,i} + r\frac{\partial K(\cdot)}{\partial \ell}\right)\frac{\partial \tilde{\ell}_{\tau,i}(\cdot)}{\partial w_{\tau,i}} = 0.$$

For $w_{\tau,i} = w \,\forall \tau \geq 2$, from the definition of $\tilde{\ell}_{\tau,i}(\cdot)$ it immediately follows that $\tilde{\ell}_{\tau,i}(\cdot) = 0 < \partial\tilde{\ell}_{\tau,i}(\cdot)/\partial w_{\tau,i}$, and so the left-hand side of the above equation equals

$$\left(w + r\frac{\partial K(\cdot)}{\partial \ell}\right)\frac{\partial \tilde{\ell}_{\tau,i}(\cdot)}{\partial w_{\tau,i}} < -\frac{L(w, w)}{\frac{\partial L(\cdot)}{\partial w_{\tau,i}}}\frac{\partial \tilde{\ell}_{\tau,i}(\cdot)}{\partial w_{\tau,i}} < 0,$$

where the first inequality follows from *(i)* $W^M(\delta) < W^*(P^M(\delta))$ and *(ii)* as $\sum_\tau L(w_{\tau,i}, w) < n\sum_\tau L(w, w)$ by the diseconomies of scale effect (*a fortiori* so if $w_{\tau,i} = w$ for all $\tau = 2, ..., T$), $\partial K(\sum_\tau L(w_{\tau,i}, w), Q(p))/\partial\ell < \partial K(\sum_\tau L(w, w), Q(p)/n)/\partial\ell$. By the second-order condition of the cost-minimization problem, this inequality implies that $w_{\tau,i} > w$ at the optimum for all τ. Therefore, the constraint $w_{\tau,i} = w$ for all $\tau = 2, ..., T$ (so that $\tilde{\ell}_{\tau,i}(\cdot) = 0$) imposed by the presence of NPAs binds, strictly lowers the maximum deviation profit.

By the same arguments of the baseline analysis, wage discrimination, even across workers of

[56] The latter indeed collectively have a strict cost advantage because *(i)* they know they can keep all their incumbent workers $\tau \geq 2$ even offering a wage lower by an amount $w^D_{\tau-1,i} - w$ relative to the one offered by the deviating firm (this only holds for $\tau = 2$ aged workers in the presence of a NPA on-path), and *(ii)* downward nominal wage rigidity is less stringent for them than for the deviating firm who has offered higher wages in the period of deviation (this gives them an advantage if the wage level in the punishment phase is lower than the deviation wage).

different ages, is always inefficient, implying that the cartel outcome is stationary and symmetric and so is defined as

$$(p^M, w^M) \equiv \arg\max_{p,w} \pi(p,w) \equiv p\frac{Q(p)}{n} - \left[wTL(w,w) + r\,K\left(TL(w,w), \frac{Q(p)}{n}\right)\right],$$

where maximization with respect to p and w gives the same first-order conditions (14)-(15) as in the baseline model. As punishment profits are zero, the cartel outcome is sustainable as a SPNE of the dynamic game if and only if

$$\delta \geq \delta^M \equiv 1 - \frac{\pi(p^M, w^M)}{\pi^D(p^M, w^M)}.$$

For lower values of the discount factor, by the same steps as in Proposition 3, the optimal collusive outcome (within the class of stationary and symmetric allocations) is $(P^M(\delta), W^M(\delta))$ obtained from the binding incentive-compatibility constraint and the optimality condition (10), with $P^M(\delta)$ and $W^M(\delta)$ being increasing and decreasing in δ, respectively, and such that, for all $\delta \in (0, \delta^M)$, $W^M(\delta) < W^*(P^M(\delta))$, with the static equilibrium offer $W^*(p)$ defined by Eqn. (13).

Then, as the constraint $w_{\tau,i} = w$ for all $\tau \geq 2$ imposed by NPAs in the deviating firm's problem strictly reduces the deviation profits from any candidate stationary equilibrium (p, w), in the presence of NPAs along the equilibrium path: *(i)* the critical discount factor δ^M is lower, and *(ii)* whenever the incentive-compatibility constraint binds absent NPAs, firms will optimally use the resulting slackness in the incentive constraint in the presence of NPAs to sustain an equilibrium with both higher price and lower wage levels.

The equilibrium where all firms sign a NPA at all periods is always sustained by the threat of reverting to a zero-profit equilibrium immediately if a firm does not sign the NPA. Such a zero-profit continuation equilibrium exists because the deviating firm can poach the signatories' incumbent workers and *vice versa*, and firms compete *à la* Bertrand. This drives the deviating firm's profit to zero, making a deviation not to sign the NPA always unprofitable. Therefore, as the most profitable equilibrium with NPAs features higher profits than the most profitable equilibrium without NPAs, the only way to prevent firms from signing NPAs is through a ban on these agreements, which, by the arguments above, has the effects stated in Proposition 9.

Finally, the same results hold if workers are *farsighted* and discount payoffs at the common rate δ. Along any stationary equilibrium path, newcomers anticipate that they will not get higher wages in future periods, so their acceptance decisions are unchanged. Conversely, when a firm deviates by offering newcomers higher wages, farsighted workers anticipate getting higher offers in the future because firms are reverting to a competitive equilibrium, which changes their labor supply decision.[57] This effect, however, is identical no matter whether NPAs are banned or not as, in either case, firms will not sign NPAs in the competitive continuation equilibrium following the deviation. □

[57]This is true if newcomers can observe all offers or have *symmetric beliefs* — i.e., each believes that the deviating firm is offering the same contract to all other newcomers. In the case of private contracting and *passive beliefs*, as each worker still believes that the deviating firm is offering the equilibrium contract to any other worker and so does not expect a wage war, the labor supply is identical to the case of myopic workers.

Proof of Corollary 3. If NCAs are banned, the most collusive stationary SPNE is as in the Proof of Proposition 9 above in the scenario of a ban on NPAs. Otherwise, firms always find it optimal to sign NCAs, for two reasons. First, workers are willing to sign these clauses at no extra remuneration ($\omega_{\tau,i} = 0$ for all τ and i), as they do not expect to receive poaching offers along the equilibrium path, so signing these clauses is costless for firms. Second, NCAs signed by other firms with their workers reduce a deviating firm's profit in the best static deviation, as they play the exact same role as binding NPAs, which can only help sustaining more profitable equilibria. Then, if the possibility by the deviating firm of signing NCAs in the deviation period is valuable — i.e., if it increases its profits in the punishment phase — this firm will do so anyway. Otherwise, its punishment profits are zero, implying that its best deviation is indeed the optimal static deviation, and hence, the presence of NCAs hinders its deviation incentives. In this case, a ban on NCAs produces the same effects as those of a ban on NPAs characterized in Proposition 9. □

Proof of Corollary 4. Also in the presence of pay-equity regulations, following any deviation firms can play a continuation equilibrium in which the deviating firm makes zero profits, which constitutes an optimal punishment. This result follows from the same arguments in the Proof of Proposition 9 above and from the fact that the regulatory constraint further penalizes the deviating firm, which, because of downward nominal wage rigidity and the deviation wage w^D being higher than w, faces the tighter constraint $w_i \geq w^D$ in the punishment phase if it wants to hire newcomers. Therefore, the optimal deviation from any stationary symmetric SPNE (p, w) is the best static deviation — i.e., slightly undercutting p and choosing wage offers to minimize the production cost of $Q(p)$:

$$\min_{w_i} \sum_{\tau=2,\ldots,T} \left[w_i L(w, w) + w_i \tilde{\ell}_{\tau,i}(w_i, w) \right] + w_i L(w_i, w) + r\, K(\cdot),$$

whose first-order condition immediately yields that $w_i > w$. Then, the deviation profit is strictly lower than absent such regulations because keeping the wage at $w_i = w$ to own incumbent workers is optimal absent pay-equity rules. Therefore, introducing pay-equity regulations dampens deviation incentives from any candidate SPNE outcome (p, w). As a result, it reduces the critical discount factor to sustain the cartel outcome (which, being symmetric, does not depend on whether these rules are in place), and allows firms to sustain SPNE with strictly higher prices and lower wages for all lower values of δ. □

B Additional material

B.1 Microfoundations

Price or wage collusion and antitrust monitoring. Suppose that firms need pre-play communication to coordinate on collusive price and wage levels and that such communication takes place among specialized middle managers of each firm, say pricing managers for prices and HR directors for wage offers (each acting in its employing firm's overall best interests), who then report to the top management that takes final decisions.

If such communication is not prevented by antitrust authorities, it leads firms to implement the multimarket collusive outcome, yielding the results of Section 3.2; if, on the contrary, antitrust authorities are able to prevent all middle managers from communicating, the best firms can do is playing the most profitable static NE $(\overline{p}^N, W^*(\overline{p}^N))$ over time. However, antitrust authorities may be able to prevent communication only *in one market*. If only communication among pricing managers is prevented (*monitoring of price collusion*), firms will agree upon w, and so only a different wage offer made by any firm will be considered a deviation and trigger Nash reversion; accordingly, each firm is free to choose its price in its individual best interest; anticipating this introduces the constraint $p \leq P^*(w)$ in the collusion Problem (P), yielding the wage collusion outcome described in Proposition 5. Conversely, if only communication among HR directors is prevented (*monitoring of wage collusion*), firms will only agree upon p, each firm will then set its individual profit-maximizing wage, which adds the constraint $w = W^*(p)$ to the collusion Problem (P), yielding the price collusion outcome described in Proposition 4.

The same results hold if antitrust authorities can only look for ex-post evidence of communication when they have a clue of collusive behavior. Assume the authorities know the production function and can observe product prices and wage offers, and firms' sales and labor force. Then, suppose they also know $L(\cdot)$, but not $Q(\cdot)$. In that case, they can observe whether the prevailing wage level is too low, i.e., $w < W^*(p)$ (as this equilibrium value only depends on the equilibrium quantity and not on the demand function), and in this case will look for, and eventually find, evidence of collusion. Anticipating this, colluding firms will optimally set $w = W^*(p)$, which implements the price collusion outcome (as authorities, not knowing $Q(\cdot)$, cannot infer whether a price is collusive). *Vice versa*, if authorities know $Q(\cdot)$, but not $L(\cdot)$, firms will optimally implement the wage collusion outcome. Authorities' knowledge of both consumer demand and labor supply functions or of none leads, respectively, to $(\overline{p}^N, W^*(\overline{p}^N))$ or the multimarket collusive outcome.

Search frictions. While, in the baseline model, firms have oligopsony power because of their differentiation *vis-à-vis* workers, similar conclusions hold in the presence of labor market power deriving from search frictions in the labor market.

To make this point in the simplest way, suppose that there is no employer differentiation: $\xi_{i,j} \equiv \xi_j \sim \Xi$ for all $i \in \mathcal{N}$. Then, worker j will join (one of) the firm(s) i from which it has received the highest offer $w_{i,j}$, provided that $w_{i,j} + \xi_j \geq w_0$. Yet, because of search frictions in the labor market, firms cannot reach all workers. For simplicity, say that a fraction λ/n of workers only receive the offer from one firm i, whereas the other workers actively search and consider offers from all firms.[58] Then, denoting $\underline{w} \equiv \min_{i \in \mathcal{N}} w_i$, one has

$$\ell_i = L(w_i, \underline{w}) \equiv J\left(\frac{\lambda}{n} + \mathbb{1}[w_i = \underline{w}]\frac{1-\lambda}{\#\{i : w_i = \underline{w}\}}\right)[1 - \Xi(w_0 - w_i)].$$

Starting from any candidate SPNE where all firms set price $p_i = p$ and try to reach all workers (which is always optimal to maximize the labor force for any given wage) offering $w_i = w$, a

[58]The following argument applies, more generally, whenever each firm i cannot reach a sufficiently large fraction of workers within the deviation period.

deviating firm, by slightly overcutting w, would hire

$$\ell_i = J\left(\frac{\lambda}{n} + 1 - \lambda\right)[1 - \Xi(w_0 - w)] < J[1 - \Xi(w_0 - w)] = nL(w, w)$$

workers. Under the no-rationing assumption, this translates into a diseconomies of scale effect, yielding similar results to those in the baseline analysis.

To see this, consider the cartel outcome (p^M, w^M), which is independent of λ.[59] As $nL(w^M, w^M)$ is the cost-efficient labor force to produce $Q(p^M)$, a deviating firm, who shall serve all this demand alone, would necessarily face a larger average cost (because it cannot recruit the same labor force when offering w^M to the subset of workers it can reach). This implies $\pi^D(p^M, w^M) < n\pi(p^M, w^M)$, and hence $\delta^M < \delta_0^M$ $\forall \lambda > 0$.[60] Moreover, as the measure of workers that a deviating firm can steer away from its rivals by slightly overcutting w^M is decreasing in λ, it immediately follows that

$$\frac{\partial \delta^M}{\partial \lambda} < 0,$$

so that more significant search frictions (captured by a larger value of λ), which translate into labor market power even absent firm differentiation, unambiguously facilitate collusion.

B.2 Discussion and robustness

No-rationing assumption. The analysis has maintained the standard assumption of Bertrand models that firms are always committed to satisfy all consumers' demand at their posted price. How would the results change if firms could choose how much to produce? To answer this question in a standard model, suppose that firms simultaneously choose, on top of p and w, also the amount of the other production factor k to employ. Then, the static game would not admit a pure-strategy NE with zero profits: In the candidate NE where firms are supposed to sell at $p = c(Q(p)/n)$, they would choose (w, k) so that $F(L(w, w), k) = Q(p)/n$ (i.e., no excess capacity in equilibrium); but then, any firm would have incentives to overcut this price, as it would face no competition for the residual demand.

Indeed, even if k could be adjusted ex-post, the deviating firm's rivals would have no incentives to serve the consumers it has left because doing so would entail an increase in the average production cost. In other words, the only role played in the analysis by the possibility of adjusting the endowment of k after demand is realized is that jointly with the no-rationing assumption, it implies that overcutting the competitive price $p = c(Q(p)/n)$ is not profitable. In turn, this entails the existence of a zero-profit static NE, which allows to restrict attention

[59] As in the baseline model, it is straightforward to prove that the cartel outcome is indeed stationary and symmetric, and so is defined by:

$$(p^M, w^M) \equiv \arg\max_{p,w}\, pQ(p) - n\left[wL(w, w) + r\,K\left(L(w, w), \frac{Q(p)}{n}\right)\right] =$$
$$= pQ(p) - \left[w\,J[1 - \Xi(w_0 - w)] + r\,K\left(J[1 - \Xi(w_0 - w)], Q(p)\right)\right],$$

where the equality follows from the definition of $L(\cdot)$ and from $K(\cdot)$ being homogeneous of degree one.

[60] Indeed, a zero-profit static NE to which firms can revert in the punishment phase, characterized as in the baseline analysis, always exists under this specification of the model.

to grim-trigger strategies.

Nevertheless, provided that there exists a punishment yielding a discounted profit of zero to a deviating firm in the continuation game, the qualitative results are unchanged under the simultaneous timing considered here.[61] The reason is that the price-overcutting incentives described above are absent in the most profitable SPNE: Colluding firms' ability to charge high prices is instead constrained by their incentives to undercut the candidate equilibrium price. Such undercutting incentives are stronger than in the baseline analysis whenever, starting from a candidate SPNE (p, w), a deviating firm would find it optimal to sell a quantity $q \in (Q(p)/n, Q(p))$, rather than $q = Q(p)$ as is constrained to do in the base model. In these circumstances, this alternative timing of the game strengthens deviation incentives, resulting in lower prices and higher wages in the most profitable SPNE. However, this is the case only for relatively low values of δ: As Assumption (A) implies that serving all the demand is optimal for a firm deviating from the cartel outcome, by a standard continuity argument it follows that the most profitable SPNE is unchanged for sufficiently large values of δ.

Sequential stage game. The main results are robust under sequential-moves specifications of the stage game. In a *production-to-order* game, in which prices are set (and publicly-observed) in advance of purchasing production factors, a firm undercutting a candidate equilibrium price benefits from its rivals optimally avoiding hiring any worker — i.e., $p_i < p_{i'}$ implies no demand for all firms $i' \in \mathcal{N} \setminus \{i\}$, and so $w_{i'} = k_{i'} = 0$. This feature of the production-to-order game strengthens undercutting incentives, resulting in less collusive equilibria; yet, because of employer differentiation, it does not eliminate the diseconomies of scale effect. Except for this, the analysis is unchanged (in particular, there exists a zero-profit static NE, implying that grim-trigger strategies are without loss of generality).[62]

In a *production-in-advance* game, where firms purchase production factors before setting prices — i.e., they first make wage offers and choose capital endowments, and then, upon observing these choices, set their prices — as known since Kreps and Scheinkman (1983), prices will be non-cooperatively set at the market clearing level — i.e., $p = Q^{-1}\left(\sum_{i \in \mathcal{N}} F(\ell_i, k_i)\right)$ — which maximizes individual and industrywide profits for the chosen *capacities*. Therefore, a production-in-advance game boils down to a model of collusion in production capacities, making monitoring of wage collusion even more crucial to ensure low prices. The incentives to deviate from a collusive outcome (w, k) — or, equivalently, (p, w), with each firm buying $k = K(L(w, w), Q(p)/n)$ — are weaker than in the simultaneous-choice stage game as a deviating firm not only has to pay a higher wage to produce more but will also sell at a lower price. In a SPNE in grim-trigger strategies, this effect would be at least in part outweighed by the fact that reversion to the static NE does not lead to zero profits. As above, however, the result that, through the diseconomies of scale effect, labor market power facilitates collusion always holds provided that firms can employ optimal punishment schemes yielding zero discounted continuation profits to a defector.

[61] As shown by Abreu (1988), grim-trigger strategies are not optimal punishments unless, as in the baseline model, they yield a discounted profit of zero. Otherwise, stick-and-carrot strategies suffice to yield zero discounted profits in the continuation game following any deviation for values of δ *not too small.*

[62] Note that a deviation only in the wage offers is *a fortiori* suboptimal given that a price-deviation is more profitable than in the baseline model.

Production function. Throughout the analysis, firms only employ two variable production factors. As in Yeh et al. (2022), the analysis immediately generalizes to additional variable factors traded in competitive markets. Formally, let $k_1, ..., k_V$ be variable production factors, and r_v be the unit price of each factor k_v, $v = 1, ..., V$. Suppose that the production function is CRS with respect to $\{\ell, k_1, ..., k_V\}$. Then, for any quantity q_i that firm i needs to produce, given its labor force ℓ_i, it chooses $(k_1, ..., k_V)$ solving

$$\min_{k_1,...,k_V} \sum_{v=1,...,V} r_v k_V$$
$$\text{s.t. } q_i = F\left(\ell_i, \sum_{v=1,...,V} k_V\right).$$

Letting $k_v = K_V(\ell_i, q_i)$, for $v = 1, ..., V$, denote the solution of the above cost-minimization problem, the same expression for firms' profit obtains by considering the vector notation $r = (r_1, ..., r_V)$ and $K = (K_1(\ell_i, q_i), ..., K_V(\ell_i, q_i))$; the analysis is unchanged as each $K_v(\cdot)$ is homogeneous of degree one.

Next, suppose that on top of the (at least two) variable production factors considered so far, also fixed factors enter the production function. For conciseness, consider the two variable factors (ℓ, k) as in the base model and only one fixed factor, denoted by z; its level is first set by each firm and becomes common knowledge at the outset of period $t = 0$, then can be publicly revised at the outset of period $t = T$, for some $T \geq 1$, then again at the outset of period $t = 2T$, and so on (except for this, the timing of the game is as in the baseline model). Provided that any deviation in the choice of the fixed factor triggers a zero-profit equilibrium of the continuation game, colluding firms optimally choose the value of z to facilitate price and wage collusion. Formally, they solve the following problem:

$$\max_{p,w,z} \pi(p, w, z) \equiv p\frac{Q(p)}{n} - \left[r_z z + wL(w, w) + r\, K\left(z, L(w, w), \frac{Q(p)}{n}\right)\right]$$
$$\text{s.t. } \delta \geq 1 - \frac{\pi(p, w, z)}{\pi^D(p, w, z)},$$

where r_z is the unit price of z, $K(\cdot) \equiv F^{-1,k}(z, \ell, q)$ is the inverse of the production function $F(z, \ell, k)$ w.r.t. k, and the deviation profit is

$$\pi^D(p, w, z) \equiv \max_{q_i \in \left\{\frac{Q(p)}{n}, Q(p)\right\}, w_i} pq_i - [w_i L(w_i, w) + r\, K\,(L(w_i, w), z, q_i) + r_z z]\,,$$

given that the deviating firm cannot change its choice of z even at the outset of periods $t \in \{0, T, 2T, ...\}$, because doing so would entail triggering a zero-profit continuation equilibrium. The first-order condition w.r.t. z, when the incentive-compatibility constraint binds, gives

$$\frac{\partial \pi(\cdot)}{\partial z} = \lambda \frac{\partial}{\partial z}\left[\frac{\pi(\cdot)}{\pi^D(\cdot)}\right],$$

with λ being the Lagrange multiplier of the above problem, so that firms account for how the choice of z affects deviation incentives. In particular, setting an inefficiently low z from an industrywide profit maximization standpoint (given p and w) can be optimal to weaken price-undercutting incentives.

Importantly, if the production function is CRS with respect to all factors $\{\ell, k, z\}$, the presence of a fixed factor facilitates collusion through a diseconomies of scale effect, given that a deviating firm cannot expand its endowment of z, and so faces decreasing returns w.r.t. $\{\ell, k\}$. This, in turn, further exacerbates the diseconomies of scale effect brought up by the presence of labor market power: The decreasing returns of labor when z is fixed imply that paying higher wages to attract more workers is less appealing than in the baseline analysis.

Imperfect monitoring. Imperfect labor market competition can have further pro-collusive effects by allowing firms to detect better deviations in settings with demand shocks and imperfect monitoring *à la* Green and Porter (1984) — see, e.g., Matsushima (2001). In perfectly competitive labor markets, firms can recruit as many workers as they want at the competitive wage. So, the labor market delivers no information on whether a rival has deviated from the collusive price. Conversely, in the presence of labor market power, even if there is uncertainty on the distribution Ξ at every period, firms ending up with an unlikely low labor force (given their wage offer) can draw inferences on the fact that a rival has increased its wage to recruit more workers.

Hence, a deviating firm undercutting the price either also increases its wage offer, which maximizes its deviation profit but increases the chances of triggering the punishment phase, or it does not deviate in the labor market, which makes price-undercutting less profitable in the first place. In either case, it has weaker deviation incentives.

Wage cartels. Suppose that wage cartels are allowed and firms reach an agreement, enforceable in court, which constrains them to offer a wage w to all workers. As explicit price cartels are still illegal, colluding firms solve Problem (P) with a deviation profit

$$\max_{q \in \{Q(p)/n, Q(p)\}} p\,q - [wL(w,w) + r\,K(L(w,w),q)] < \pi^D(p,w),$$

which softens the incentive-compatibility constraint. That is, by making it impossible to recruit more workers at any period, a legally binding wage cartel reduces the profitability of undercutting any candidate SPNE price. The reason is that a deviating firm can satisfy the increased demand by only increasing its endowment of variable capital, which is not cost-efficient.

Then, for all $\delta \in [0, \delta^M)$ — i.e., whenever the incentive-constraint binds without an explicit wage cartel — firms would find it optimal to write down a legally binding wage-fixing agreement to sustain both lower wages and higher prices than in the most profitable SPNE characterized in Proposition 3. This argument provides a rationale for the *per se illegality* antitrust status of explicit wage-fixing agreements, even based on a pure consumer surplus standard.

Collective bargaining. Consider a model where firms open vacancies, and the wage they end up paying workers in equilibrium depends on industrywide trade unions' bargaining power.

Formally, each firm $i \in \mathcal{N}$ simultaneously chooses how many workers ℓ_i to employ and its product price p_i. The latter choices determine consumers' demand as in the main model.

The former choices determine the prevailing wage level. Suppose for simplicity that firms are homogeneous from workers' viewpoint — i.e., as above, $\xi_{i,j} \equiv \xi_j \sim \Xi$. Absent trade unions' bargaining power, the wage level $w^*(L) < w_0$, where $L \equiv \sum_{i \in \mathcal{N}} \ell_i$, is obtained from the market clearing condition $J[1-\Xi(w_0 - w^*(L))] = L$. If, on the contrary, trade unions have full bargaining power, they are always able to impose the competitive wage: $w_0(L) \equiv w_0 \; \forall L$. More generally, suppose that the expected or average wage paid by each firm equals $W(L) \equiv \alpha w^*(L) + (1-\alpha) w_0$, where the parameter $\alpha \in [0,1]$ is an inverse measure of trade unions' bargaining power — e.g., there is a probability α with which trade unions will manage to ex-post impose competitive wages for all workers or a fraction α of workers is unionized and is ex-post able to extract the competitive wage.

For $\alpha = 0$, this model is identical to the competitive input markets benchmark examined in Section 2.2: The cartel outcome is thus sustainable by colluding firms if and only if $\delta \geq \delta_0^M$. For $\alpha = 1$, the analysis is similar to Section 3. In particular, starting from a symmetric candidate SPNE (ℓ, p), a deviating firm undercutting the price finds it optimal to expand its labor demand. Doing so, however, implies a rise in the market clearing wage. Because of this diseconomies of scale effect, the critical discount factor to sustain the cartel outcome, defined here as $(p^M, L^M) \equiv \arg\max_{p,L} pQ(p) - [W(L)L + r\,K(L, Q(p))]$, is $\delta^M < \delta_0^M$. More generally, for $\alpha \in [0,1]$ one has that

$$\frac{\partial \delta^M}{\partial \alpha} < 0.$$

Therefore, policies weakening trade unions' bargaining power facilitate collusion.

B.3 Local labor markets and cross-market collusion

This section considers the model with independent product markets described in Section 4.2, with $n/2$ different single-product firms in each market.

Suppose that firms operating in market z only collude among each other (*within-market collusion*). Then, for any given δ and wage w_{-z} offered by firms in the other market, their optimal collusive scheme (p_z, w_z) maximizes the profit $\pi(p_z, w_z; w_{-z}, w_{-z})$ subject to the incentive-compatibility constraint $\delta \geq 1 - \pi(\cdot)/\pi^D(\cdot)$, where the highest deviation profit, similar to the baseline analysis, is given by $\pi^D(p_z, w_z; w_{-z}, w_{-z})$ (the functions $\pi(\cdot)$ and $\pi^D(\cdot)$ are defined in the Proof of Proposition 8).

Then, the within-market cartel outcome is defined as the solution to the fixed-point problem: $(p^S, w^S) \equiv \arg\max_{p_z, w_z} \pi(p_z, w_z; w^S, w^S)$, and it is sustainable for all $\delta \geq \delta^S$, where

$$\delta^S \equiv 1 - \frac{\pi(p^S, w^S; w^S, w^S)}{\pi^D(p^S, w^S; w^S, w^S)} < \delta_0^M.$$

For lower values of δ, the most profitable SPNE $(P^S(\delta), W^S(\delta))$ is again pinned down by the binding incentive-compatibility constraint and the optimality condition (10), where derivatives are taken with respect to variables in the own market z, imposing symmetry ($p_z = p$ and $w_z = w$ for $z = A, B$).

As within-market cartels do not internalize the cross-market externalities taking place through the labor market, w^S is too high from an aggregate (cross-industry) profit maximization viewpoint: A cross-market cartel would profitably set $(p^M, w^M) \equiv \arg\max_{p,w} \pi(p, w; w, w)$, with $w^M < w^S$ and, as hiring fewer workers makes it optimal to reduce production, $p^M > p^S$.

Firms selling their products in independent product markets thus have incentives to collude together. Under *cross market collusion*, the most profitable SPNE, denoted by $(P^M(\delta), W^M(\delta))$, solves the following problem:

$$\begin{aligned} &\max_{p,w} \pi(p, w; w, w) \\ &\text{s.t. } \delta \geq 1 - \frac{\pi(p, w; w, w)}{\pi^D(p, w; w, w)}. \end{aligned}$$

As cooperative and defection profits are unchanged, sustaining the more collusive cross-market cartel outcome requires a higher critical discount factor:[63]

$$\delta^M = 1 - \frac{\pi(p^M, w^M; w^M, w^M)}{\pi^D(p^M, w^M; w^M, w^M)} \in (\delta^S, \delta_0^M).$$

For all $\delta \in [0, \delta^M)$, the incentive-compatibility constraint binds, and the cross-market optimal collusive scheme satisfies the optimality condition

$$\frac{\partial\pi(\cdot)/\partial p_z}{\partial\pi^D(\cdot)/\partial p_z} = \frac{\partial\pi(\cdot)/\partial w_z + \partial\pi(\cdot)/\partial w_{-z}}{\partial\pi^D(\cdot)/\partial w_z + \partial\pi^D(\cdot)/\partial w_{-z}}. \tag{17}$$

The internalization of cross-market externalities, as reflected by the extra-terms on the right-hand side of Eqn. (17) relative to Eqn. (10), implies that $W^M(\delta) < W^S(\delta)$ for all $\delta > 0$. Whenever the incentive-compatibility constraint also binds under within-market collusion, however, colluding firms need to lower their price to offset the stronger incentives to deviate deriving from the more collusive wage level. Summing up:[64]

Proposition (cross-market collusion). *The most profitable SPNE $(P^M(\delta), W^M(\delta))$ with multiple (identical and independent) product markets is only achievable through cross-market collusion and is such that $W^M(\delta) < W^S(\delta)$ for all $\delta > 0$, and $P^M(\delta) > P^S(\delta)$ if and only if $\delta > \widetilde{\delta}$, where $\widetilde{\delta} \in (\delta^S, \delta^M)$.*

Labor market power in local labor markets creates the scope for cross-market collusion, strengthening the case for antitrust authorities' monitoring of collusive behavior. Interestingly, this broadening of collusive behavior does not necessarily translate into higher consumer prices: The presence of significant wage mark-downs unambiguously reveals that cross-market collusion is in place.[65]

[63] Still, $\delta^M < \delta_0^M$ as, by the same arguments outlined in Section 3.2, $\pi^D(p^M, w^M; w^M, w^M) < n\pi(p^M, w^M; w^M, w^M)$.

[64] The complete proof is omitted for brevity and available upon request.

[65] How would the above results change if markets are asymmetric? In the presence of significant asymmetries across markets, there may be values of δ such that the within-market cartel outcome is attainable in only some markets. In these circumstances, a cross-market collusive scheme can exploit the slackness of the incentive constraint in these markets to depress wage offers in all markets further. Then, prices can simultaneously rise in these markets where the incentive-compatibility constraint does not bind and drop in markets where the lower wage levels tighten the incentive constraint.

B.4 NPAs/NCAs as facilitating practices in perfectly competitive markets

This section considers the model of Section 5 assuming away product market oligopoly and employer differentiation, to derive the procollusive implications of NPAs and NCAs in the simplest setting.

Formally, suppose that firms can sell any quantity of their products at the exogenous, competitive price p_0 and are perfectly homogeneous from workers' viewpoint — i.e., $\xi_{i,j} \equiv \xi_j$ for all i. In this simple setting, firms' per-period (on-path) profit can be written as

$$\pi_i = \sum_{\tau=1,\ldots,T} (p - w_{\tau,i})\ell_{\tau,i},$$

where $\ell_{\tau,i}(\cdot)$ is firm i's labor force of age-τ workers (which will be characterized below) when each firm i offers the same wage $w_{\tau,i}$ to all workers of age τ (which holds, both on- and off-path, in the stationary equilibria considered below), and

$$p \equiv -r\frac{\partial K(\sum_\tau \ell_{\tau,i}, q^*(\sum_\tau \ell_{\tau,i}))}{\partial \ell},$$

with

$$q^*\left(\sum_\tau \ell_{\tau,i}\right) = \arg\max_q p\, q - \left[\sum_\tau w_\tau \ell_{\tau,i}(\cdot) - r\, K\left(\sum_\tau \ell_{\tau,i}, q\right)\right],$$

being a constant (i.e., p is independent of all $\ell_{\tau,i}$).[66]

No-poaching agreements. To simplify the exposition, suppose for the moment that workers are myopic. The dynamic game admits an equilibrium where, at any period and for any history of the game, no NPAs are signed and all firms offer $w = p$ to all available workers (i.e., all newcomers and all incumbent workers in the industry). Indeed, there are no profitable deviations from this equilibrium: At any t, unilaterally signing NPAs when other firms do not is inconsequential and, by a standard Bertrand-type argument, offering $w_{\tau,j} < p$ to any worker j of age τ implies not hiring it with probability one, while offering $w_{\tau,j} > p$ implies making a negative per-period surplus $p - w_{\tau,j} < 0$ from its hire. In this *competitive equilibrium*, firms make zero profits at each period.

By contrast, in the *monopsony equilibrium*, the wage offered to all available workers at every period would be $w^M \equiv \arg\max_w (p - w)TL(w) < p$, where $L(w) \equiv J\Xi(w_0 - w)$ is the *myopic labor supply* of each generation of workers.[67] As, in this simple model, firms are homogeneous

[66] This result follows from the production function being CRS. Profit maximization with respect to q yields that $q^*(\sum_\tau \ell_{\tau,i})$ must satisfy the first-order condition $p_0 - r\frac{\partial K(\cdot)}{\partial q} = 0$, which, substituted into the profit, and using the fact that $K(\cdot)$ is homogeneous of degree one, gives the above expression for π_i. Finally,

$$\frac{\partial p}{\partial \ell_{\tau,i}} = -r\left[\frac{\partial^2 K(\cdot)}{\partial \ell_{\tau,i}^2} + \frac{\partial^2 K(\cdot)}{\partial \ell_{\tau,i}\partial q}\frac{\partial q^*(\cdot)}{\partial \ell_{\tau,i}}\right] = 0,$$

as, by the Implicit Function Theorem, $\partial q^*(\cdot)/\partial \ell_{\tau,i} = -\frac{\partial^2 K(\cdot)/\partial \ell_{\tau,i}\partial q}{\partial^2 K(\cdot)/\partial q^2}$, and, by the homogeneity of $K(\cdot)$, $(\partial^2 K(\cdot)/\partial \ell_{\tau,i}^2)(\partial^2 K(\cdot)/\partial q^2) = (\partial^2 K(\cdot)/\partial \ell_{\tau,i}\partial q)^2$.

[67] As seen in Section 3.1, wage discrimination would only increase the average wage needed to recruit any overall labor force. In this setting, there is no incentive to offer to available workers of any age τ a wage different from the statically optimal one w^M: Offering $w < w^M$ to a newcomer entails losing forever the profitable possibility of hiring it; and, once a worker is hired at w^M, it is pointless to increase its wage later on, and impossible to

from workers' viewpoint, the cartel outcome would feature the same wage w^M offered by all firms to all available workers over time, yielding per-firm profit $\pi(w^M) = (p - w^M)TL(w^M)/n$ at any period.

Suppose, first, that NPAs are not in place — e.g., they are banned by competition authorities. Then, starting from any stationary equilibrium with wage $w \in [w^M, p)$, the best deviation for any firm consists in slightly overcutting this wage offer to all available workers — i.e., all rivals' incumbent workers and all newcomers, which will then all join the deviating firm. By doing so, it obtains a profit

$$\pi^D(w) = (p - w)TL(w) = n\pi(w)$$

in the period of deviation. Afterward, however, firms will play the equilibrium where they offer $w = p$ to all available workers and so make zero profits, which therefore constitutes an optimal punishment.[68] Hence, absent NPAs, any stationary equilibrium wage $w \in [w^M, p)$ can be sustained as a SPNE of the dynamic game if and only if

$$\delta \geq \delta_0^M.$$

Consider, next, the case where NPAs are allowed, and are signed by all firms at any period along a stationary equilibrium path.[69] If any of the firms deviates and does not sign the NPA at some period t, all firms will immediately revert to a continuation equilibrium where they offer $w = p$ to all available workers starting from the same period t — i.e., all firms offer the competitive wage to newcomers and renegotiate at the competitive level the wage of their incumbent workers, so that the possibility of poaching rivals' incumbent workers cannot increase the deviating firm's profit above zero.[70] A firm contemplating a deviation in period t shall thus sign the NPA and then deviate in the subsequent wage-offer game. In this case, the optimal deviation still consists in slightly overcutting the candidate equilibrium wage w; however, because of the binding NPA, this offer can only be made to newcomers and not to rivals' incumbent workers. As a result, given that firms revert to the continuation equilibrium where NPAs are not signed anymore and they offer $w = p$ to all available workers from period $t + 1$ onwards, the deviation profit is

$$\pi^D(w) = \frac{1}{n}(p - w)(T - 1)L(w) + (p - w)L(w) < n\pi(w).$$

reduce it by the assumption of Pareto-improving renegotiation.

[68] As this equilibrium of the continuation game following the deviation is an optimal punishment no matter the wage the deviating firm has offered in the previous period and no matter how many workers it has employed, the optimal deviation is indeed the statically optimal one described above.

[69] In order for NPAs to have a bite, there must be some incumbent workers in the market, so the following analysis only applies for $t \geq 1$. However, one may consider that, at $t = 0$, only one generation of workers is already in the market, with the second generation coming in at $t = 1$, and so on, so that T generations of workers are available for all $t \geq T - 1$ (this does not affect the monopsony wage level w^M). This makes deviating in the first periods less profitable than in $t \geq T - 1$, implying that, both with and without NPAs, the critical discount factor is pinned down by the binding incentive-compatibility constraint at any $t \geq T - 1$.

[70] This is indeed a continuation equilibrium because, as seen above, offering $w = p$ to any worker is a best response for any firm when rivals do the same; moreover, by yielding a profit of zero to the deviating firm already in the deviation period, playing this continuation equilibrium constitutes an optimal punishment.

Therefore, a stationary equilibrium with NPAs and wage $w \in [w^M, p)$ can be sustained as a SPNE of the dynamic game for all

$$\delta \geq \delta^M \equiv \frac{n-1}{n+T-1}, \quad \text{with} \quad \delta^M < \delta_0^M.$$

The above analysis implies that, for all $\delta \in [\delta^M, \delta_0^M)$, a ban on NPAs makes it impossible for firms to sustain the cartel outcome and so would result in higher equilibrium wages.[71]

Finally, the result that banning NPAs increases the critical discount factor to sustain collusive wages also holds if workers are farsighted and discount future profits at the common rate δ. Indeed, along any stationary equilibrium path, newcomers anticipate not getting higher wages in future periods. So, their decision to accept is the same, whether or not they are farsighted. Conversely, when a firm deviates by overcutting the candidate equilibrium wage, farsighted workers (provided they have *symmetric beliefs* or all offers are publicly observed) anticipate getting higher offers in the future because firms revert to the competitive equilibrium and are more willing to enter the industry (see below). This effect, however, is identical no matter whether NPAs are banned or not as, in either case, the competitive equilibrium played after the period of deviation entails that no NPAs are signed anymore and these workers obtain $w = p$.

Non-compete agreements. A myopic worker is always willing to sign a NCA without asking for compensation, given that this clause could only impact its wage in the future. Therefore, a meaningful analysis of NCAs requires considering farsighted workers (and *symmetric beliefs* or public offers). For simplicity of exposition, let me consider two-period lived workers ($T = 2$), and focus on the critical discount factor to implement the cartel outcome, i.e., the wage level w^M characterized above.

Under a ban on NCAs, for any deviation wage $w > w^M$, newcomers anticipate the wage war in the future period (i.e., that they will earn $w = p$ in the successive period) and so accept the deviating firm's offer if and only if $w + \delta p \geq (1+\delta)(w_0 - \xi_j)$, yielding a labor supply $L^D(w) \equiv 1 - \Xi[w_0 - (w+\delta p)/(1+\delta)] > L(w)$ for all w. As this labor supply is less elastic to wage relative to the on-path labor supply $L(w)$, the unconstrained monopsony wage would be lower, and so a deviating firm optimally slightly overcuts w^M *vis-à-vis* newcomers, besides with rivals' incumbent workers, and gets a deviation profit

$$\pi^D = (p - w^M)(L(w^M) + L^D(w^M)) > 2(p - w^M)L(w^M) = n\pi$$

in the deviation period, and zero afterward (because of wage war in the punishment phase for all available workers, who are all contestable). As a result, the critical discount factor to sustain the cartel outcome is strictly higher than δ_0^M.

Next, suppose NCAs are allowed, and consider a stationary equilibrium where, at all periods, all firms offer w^M, sign NCAs with all their workers, and do not make offers to rivals' incumbent workers. Then, as on-path workers do not expect to receive any attractive offer

[71] Note that, once explicit, legally binding NPAs are banned, the possibility of firms tacitly agreeing not to poach each other's workers is valueless: Poaching rivals' workers is optimal only for a firm who wants to overcut the candidate equilibrium wage level to expand its labor force; the deviating firm would then also renege on the implicit NPA, which therefore cannot help firms to sustain collusive wage levels.

from their incumbent employer's rivals in the future, they are willing to sign these clauses at no extra compensation. Hence, a deviation consisting only of not offering NCAs to any worker is immaterial: The only relevant deviations are in wage offers. Newcomers anticipate that, by accepting NCAs from the deviating firm, they will earn the same wage w^M in the second period rather than the competitive wage p that they would get absent NCAs. Hence, one needs to distinguish two cases. First, if the deviating firm offers NCAs to newcomers, their labor supply will be $L(w)$ as in the myopic case. Then, the deviating firm optimally slightly overcuts w^M *vis-à-vis* newcomers, whom it employs at the same wage in the following period (no other profits are made from that period onwards). This deviation is not profitable if and only if

$$(p - w^M)\frac{L(w^M)}{n} + (p - w^M)L(w^M) + \delta(p - w^M)L(w^M) \leq \frac{1}{1-\delta}2(p - w^M)\frac{L(w^M)}{n},$$

which is equivalent to $\delta \geq \delta_0^M$.

Second, if the deviating firm does not offer NCAs to newcomers, it is *de facto* providing them lifetime utility $w + \delta p$ when making an offer w, because of the wage-war in the punishment phase. In order to attract them, this must be higher than the lifetime utility $(1 + \delta)w^M$ offered by rivals (as they still offer the equilibrium contract, which features NCAs, and so would be able to employ them at w^M also during the punishment phase). As rivals offer the profit-maximizing lifetime utility, a deviating firm would optimally only slightly overcut it: $w = (1 + \delta)w^M - \delta p$; this implies that it again hires $L(w^M)$ and obtains a deviation profit

$$(p - w^M)\frac{L(w^M)}{n} + [p - ((1 + \delta)w^M - \delta p)]L(w^M),$$

which, rearranged, is equal to the deviation profit in the previous case.

As a result, the cartel outcome can be sustained in a SPNE with NCAs for all $\delta \geq \delta_0^M$: Firms' possibility of signing NCAs thus facilitates collusion.