Contract Shifting *vs.* Contract Splitting in Public Procurement*

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October 31, 2024

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Abstract

This paper investigates how public officials manipulate the procurement process by reducing the contract value to avoid crossing regulatory thresholds, and how this manipulation affects procurement outcomes and welfare. We exploit the unique design of the procurement law in Colombia to document three empirical findings. First, there is substantial manipulation of contracts around the threshold. Second, manipulation takes two distinct forms: *contract shifting* and *contract splitting*. Contract shifting involves reducing the value of the contract while contract splitting refers to dividing a contract into multiple smaller parts. We show that *contract shifting* is the main form of manipulation in this context. Third, we find that the manipulation has no negative effect on almost all procurement outcomes. Specifically, it reduces the final value paid and does not affect the number of bidders, among other outcomes. We propose a model of public procurement that analyzes when a public official chooses each type of manipulation and outlines the associated welfare implications. We show that public officials opt for *contract shifting* when they face small purchase thresholds and for *contract splitting* when they face large ones. Conversely, under reasonable assumptions, the model shows that *contract splitting* enhances welfare for small thresholds but reduces it for large ones while *Contract shifting* is always welfare-enhancing. The model contributes to the policy debate by proposing a design that involves a menu of auction types and threshold values.

*We are fully indebted to Antoine Ferey, Jeanne Hagenbach, Emeric Henry and Stefan Pollinger. We also thank Eric Brousseau, Joaquín Coleff, Walter Cont, Francesco Decarolis, Dolores De La Mata, Ricardo Estada, Philippe Gagnepain, Daniel Herrera, Vladimir Mares, Franz Ostrizek, Leonard Le Roux, Carine Strapoli, Nikiforos Zampetakis and Christian Zehnder as well as the participants in the IOEA 2023, 17th RGS conference, CAF-Development Bank of Latin America, Sciences Po Economics, Governance and Regulation Chair and FCE-UNLP seminars for their useful comments.

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1 Introduction

Public procurement represents, on average, over 13% of the GDP and around 29% of government expenditures in OECD countries (OECD, 2016).[1] Governments face a trade-off when they make purchases. On the one hand, they want to maximize the value of the expenditure that is, getting the best possible quality, fostering equality among bidders, and avoiding corruption. On the other hand, they seek to simplify the procurement process to avoid administrative burdens for public officials, from now on referred to as *buyers*. In practice, governments in most countries resolve this trade-off between maximizing the value of an expenditure and minimizing the complexity of the process as follows. They set regulatory thresholds to determine which procurement method applies: purchases with values below these thresholds are handled through discretion, which means directly selecting the supplier, while those ones with values above the thresholds are handled through formal rules, such as auctions.

This paper investigates buyers' manipulation of the estimated contract value of a purchase to avoid crossing regulatory thresholds when *discretion* is prohibited. Colombia provides an ideal setting to study this issue due to two main characteristics of its procurement law: i) it forbids the use of *discretion* and requires the use of auctions to purchase and, ii) it implements regulatory thresholds that impose different auction formats for small and large purchases. Importantly, it is the buyers themselves who estimate a contract value of a purchase, which is then compared to the regulatory threshold to determine the applicable auction format. Our research answers two key questions. First, in a context where discretion is not allowed, to what extent do buyers manipulate the estimated contract value of a purchase to avoid crossing a regulatory threshold? Second, what are the welfare implications of such manipulation?

To empirically address these questions we exploit the two sources of exogenous variation provided by law. First, the two arbitrary regulatory thresholds that divide the procurement process into three different auction types: i) a first price reverse auction, ii) a restricted scoring rule auction and, iii) an open scoring auction. By crossing each regulatory threshold, the buyer faces a more complex type of auction. Second, buyers are grouped into different categories based on their annual budget. Buyers grouped in different categories face different purchase thresholds.[2] For similar buyers, this exogenous variation in the regulatory regime provides an ideal natural

[1]In Colombia, public procurement accounts for 12.5% of the GDP and 35% of government expenditures (OECD, 2016).

[2]For instance, in 2019 a buyer of category one faced a lower threshold at 7,112 USD, while a buyer of category two faced this threshold at 11,000 USD.

experiment to study the impact of facing these arbitrary thresholds on manipulation of the estimated contract value.[3] We use administrative data from Colombia that covers every contract at the national level of the electronic procurement system between 2018 and March, 2024.

We document significant bunching in the contracting activity around both purchase thresholds. The main implication is an excess mass, to the left, just below each threshold value and a gap to the right of each purchase threshold. Conceptually, the regulation makes the procedure discontinuously more costly at the right of each threshold. Therefore, we can interpret this policy as a regulatory *notch*. Since similar buyers fall into different categories and face different purchase thresholds, by comparing the amount of contracts offered by each group of buyers below and above a purchase threshold, we can detect the causal effect of facing these purchase thresholds on the manipulation of the estimated contract value. Specifically, we can decompose this manipulation to avoid crossing regulatory thresholds in two: either *contract shifting* or *contract splitting*. Contract shifting refers to an arbitrary reduction in the estimated contract value, while contract splitting involves dividing a contract into multiple smaller parts.

Contract shifting and contract splitting have different implications on the distribution of contracts and on the procurement outcomes. Conceptually, contract shifting implies that the number of contracts remains unchanged, but the overall estimated value of the contracts decrease. In contrast, contract splitting implies an increase in the number of contracts while the overall estimated value of these contracts may either increase or decrease. Additionally, contract shifting is a local behavior, whereas contract splitting may occur well beyond the purchase threshold. Using these conceptual arguments we identify that the main form of manipulation in this setting is contract shifting for the lower threshold while contract splitting seems to be the main form of manipulation for the second regulatory threshold. Additionally, we not only infer from the distribution but directly identify contract splitting as labelling as identical those contracts offered by the same buyer within windows of similar characteristics, such as, contract type, date, estimated value and duration.

We test the impact of buyers' manipulation on procurement outcomes and find almost non negative effects around the lower threshold. Specifically, there is no impact on repeated winners, number of bids, difference between estimated and final value of the contracts and duration.[4] Similarly, we do not observe a clear pattern in

[3]Similar buyers in this context refers to buyers that are arbitrarily close to each other but they fall in different buyers categories and face different regulatory thresholds.

[4]Since there is no difference between the final and the estimated value paid in the manipulated contracts, by manipulating a contract the buyer decreases the final value paid.

process quality, as reflected by an increase in the number of contracts modified and additional days required to execute the contracts. These results are consistent with other studies which document non-negative effects of buyer manipulation, such as Carril (2021); Bosio et al. (2022); Coviello et al. (2022).

Motivated by these findings, we propose a stylized theoretical model of public procurement which describes the buyer optimal choice of each manipulation form and provides welfare implications for each. In the model, there is a representative buyer who makes a purchase, and several bidders with heterogeneous marginal costs uniformly distributed. The buyer can either specify additional requirements for the purchase or not. There is an optimal value of the purchase from society's perspective, which the buyer knows, depending on whether additional requirements are attached. The buyer chooses to proceed with either a first price reverse auction without additional requirements or a scoring rule auction with additional requirements. While the buyer values more a purchase when additional requirements are included, doing so also incurs higher transaction costs. The bidders behave optimally, determining the optimal price for the first price reverse auction and optimal price and score for the scoring rule auction.

We study the buyer's optimal decision in two different situations. First, we analyze the buyer's optimal behavior when he freely chooses between the two auctions. Second, we introduce a procurement law that establishes a threshold value for purchases. Above this threshold, procurement must be conducted through a scoring rule auction, while below this threshold, a first price reverse auction is used. In this setting, we derive the buyer's optimal behavior when he can, i) *shift a contract* ii) *split a contract*, iii) do both. We show that the value of the threshold matters for the decision to either shift or split. For small purchase thresholds the buyer opts for contract shifting, while for large purchase thresholds, he prefers contract splitting. Since the purchase threshold limits the maximum value he can get through a first price reverse auction, he prefers not to repeatedly incur the auction's transaction costs when the surplus is insufficient.

After determining the buyer's optimal behavior, we examine the welfare implication of this behavior. We analyze the consumer surplus generated in two scenarios: one in which the agent freely chooses between auctions, referred to as the *no policy intervention* scenario, and another where a policy dictates the choice of the auction. We show that contract shifting is always welfare enhancing in comparison to the no policy intervention scenario. In contrast, contract splitting represents a more intricate form of manipulation. On the one hand, it is more welfare enhancing than contract shifting for small purchase thresholds. On the other hand, it reduces welfare compared to the no policy intervention scenario for large purchases.[5] When both forms of manipulation are available for the buyer, there is a misalignment between the manipulation form that maximizes welfare and the one the buyer wants to implement. On the one hand, for a small purchase threshold, the buyer prefers to implement contract shifting, but the manipulation form that maximizes welfare is contract splitting. On the other hand, for a large purchase threshold, contract shifting is more welfare enhancing but the buyer prefers to implement contract splitting.

We derive policy implications from these insights. First, the threshold value is crucial for both the buyer's choice of a manipulation form and welfare analysis. When the law raises this threshold value, it increases the use of contract splitting. Second, the transaction costs of the least complex auction influence buyer decision to either shift of split. When a law increases these transactions costs, it encourages the buyer to use contract shifting. Therefore, an optimal policy design must consider a menu of transaction costs for the least complex auction and a threshold value that will maximize the welfare, constrained by the buyer's choice of manipulation. For large purchase thresholds, an optimal law would aim to increase the transaction costs of the least complex auction, thereby promoting contract shifting. For small purchase thresholds, the transaction costs should be minimized to encourage contract splitting. In the case of a extremely small purchase threhsold, and assuming no corruption concerns, the law should encourage purchases through discretion.

Related literature

Our paper studies whether and how public buyers manipulate the estimated value of contracts so as not to cross regulatory thresholds and how this manipulation affects procurement outcomes and welfare. This paper contributes broadly to the literature in *procurement and regulation* and the one that uses *bunching designs* to analyze agents' behaviors.

Theoretical analysis of procurement and regulation has received considerable attention since Laffont and Tirole (1993). However, until recently, empirical analysis of these problems remained scarce. This paper contributes to this growing strand of literature that is studying the consequences of institutional designs on buyers' behavior and procurement outcomes.[6] Our paper is in line with other papers like

[5]We show that welfare decreases when both manipulations are available, compared to when only contract splitting is available.

[6]There are many buyers' responses to the institutional design. Some of them analyze the effects of information disclosure on procurement outcomes (Coviello and Mariniello, 2014; Carril et al., 2022), the use of centralized methods to purchase (Bandiera et al., 2009), the use of electronic platforms (Lewis-Faupel et al., 2016; De Michele and Pierri, 2020), analysis over different auction types (Decarolis, 2014, 2018; Baránek et al., 2021; Lisa Chever and Yvrande-Billon, 2017), dynamic

Palguta and Pertold (2017) in the Czech Republic, Carril (2021) in the US, Szucs (2023) in Hungary, Caires et al. (2023) in Portugal and Coviello et al. (2022) in Italy, showing that when arbitrary thresholds change the purchase method, they can induce buyer responses that result in *bunching* in the contracting activity.[7]

Unlike these papers, our paper is the only one measuring this response in a context where buyer discretion is not allowed and every purchase is awarded through an auction. As Coviello et al. (2018) we consider the procurement outcomes of buyers that choose between different auction methods but, differently, in our setting the buyer cannot directly invite any bidder under a restricted or an open scoring rule auction. While in their case, they state that this is an indirect way of having discretion to select the supplier, in the Colombian setting this is impossible. Finally, this paper is complementary to Caires et al. (2023) who tries to identify the main differences between the two ways of manipulation: contract shifting and contract splitting. In contrast to them, we develop an algorithm to detect and measure contract splitting, and we find that the main way of manipulation for the small purchase threshold is contract shifting. Additionally, we provide a graphical rationale for the distribution of both ways of manipulation and a theoretical model that describes when and how these two ways of manipulation take place.

The aforementioned papers find different results of buyer manipulation on procurement outcomes. While some of them show that manipulation of the estimated contract value to buy through discretion can lead to positive outcomes (i.e.: higher quality, better prices, etc.) at the expense of a small increase in corruption, others show that this manipulation can lead to negative outcomes (i.e.: lower quality, higher prices, favoritism, political ties, etc.).[8] The main discrepancy seems to be in line with the results by Bosio et al. (2022) who assert that the differences in procurement outcomes correspond to the quality of institutions between different countries. In this paper, we contribute by showing that this difference could come from the way of manipulating the procurement process even in contexts with few risks of corruption.[9] Ganuza et al. (2024) show theoretically that manipulation to purchase through discretion can be welfare increasing or decreasing depending on the honesty of the bureaucrat. We contribute to this discussion, by identifying when

decisions and the effect of expiring the budget (Liebman and Mahoney, 2017; Cappelletti et al., 2023), bureaucrats' characteristics (Coviello and Gagliarducci, 2017; Decarolis et al., 2021; Best et al., 2023; Tukiainen et al., 2023) workload (Warren, 2014), external audits (Gerardino et al., 2024) and corruption or political connections (Schoenherr, 2019; Brogaard et al., 2021; Ryan, 2020).

[7]From a different perspective, Kang and Miller (2022), study the effect of discretion on the number of bids obtained by estimating a principal-agent model.

[8]For positive outcomes check Bandiera et al. (2021); Carril (2021); Coviello et al. (2022); Decarolis et al. (2020), for negative outcomes check Palguta and Pertold (2017); Szucs (2023).

[9]The model is related to the literature of authority delegation (Aghion and Tirole, 1997; Baker et al., 1999).

and how manipulation can be welfare increasing for honest bureaucrats that do not implement discretion.

Finally, this paper is related to the growing literature in bunching designs used to estimate agent responses to regulations (Saez, 2010; Chetty et al., 2011; Kleven and Waseem, 2013). This paper contributes to later works aimed at identifying intensive and extensive margin responses to the regulation (Carril, 2021; Pollinger, 2023). We propose a novel empirical strategy that relies on the exogenous variation of the law to identify these effects. This paper is also related to (Coles et al., 2022; Hungerman and Ottoni-Wilhelm, 2021) who use a non-parametric counterfactual to measure bunching effects.[10]

The remainder of the paper is organized as follows. Section 2 introduces Colombia's institutional setting, provides the detail of the procurement regulation and describes the data. Section 3 provides the empirical evidence, describes the empirical strategy, the estimation and their main results. Section 4 introduces the model and provides its main results. Finally, Section 5 concludes.

2 Institutional Setting and Data

The law provides two main characteristics that will be useful for the empirical analysis. First, the law establishes how is the procedure to make public purchases, what we call as *procurement formats*. Second, the law allocates buyers into different categories according to their annual budget size. Buyers allocated to different categories face different purchase thresholds.

Procurement formats

According to the Colombian regulation, the open scoring auctions are the default rule to award public contracts, the format is called *Public Auction* in the Colombian law. In order to implement a *Public auction* buyers contracting design a sealed-bid scoring-rule auction applying the following criteria: Most Economically Advantageous Tender (MEAT).[11] However, the law allows to implement two simplified formats when the estimated value of the contract is below two predetermined thresholds: called *Minor* and *Minimum Amount*.[12] Let's denote them k_1 and k_2 respectively with $k_2 > k_1$.

[10]See Bertanha et al. (2023) for an exhaustive revision.

[11]MEAT implies cost-efficiency for the tender in multidimensional terms such that it includes quality criteria (Che, 1993).

[12]*Menor* and *Mínima Cuantía* in Spanish.

Figure 1: Procurement Formats

The first simplified format is an open reverse first price auction, from now on *Reverse Auction*, and allows any bidder to participate and selects the bid with the lowest price. The second simplified format is a restricted sealed bid scoring rule auction, from now on referred to as *Restricted Scoring Rule*, and it is a two step auction. In the first step, they need to manifest interest.[13] If the buyer receives more than ten manifestations of interest he randomly selects ten of them and allow them to participate. These bidders have the opportunity to make an offer and the winner is selected identically in the same way as in a *Public Auction* explained above.

Buyer estimate by themselves the value of a contract for the need that will be acquired.[14] This estimated value of the contract is compared to both regulatory thresholds and then dictates the auction format to use. First, when the estimated contract value is lower than the first purchase threshold, k_1, a buyer is allowed to implement the *Reverse Auction.* Second, when the estimated contract value is between k_1 and k_2 a buyer is allowed to implement a *Restricted Scoring Rule.* Finally, when the estimated contract value is higher than k_2 the buyer needs to use the default auction, the *Public Auction.* The procedure is depicted in figure 1.

In a nutshell, Colombian regulation is designed to simplify the process while reducing buyers discretion by keeping rules for contracting. Simplification then means either simpler rules or less bids to revise.

Buyers' categories

Buyers are grouped in five different categories according to their annual budget. The purchase thresholds and the budget of each buyer is measured in term of the Active Monthly Minimum Wage, which is updated in an annual basis by presidential decree. Buyers' categories and purchase thresholds are presented in table 1. Importantly, similar buyers in terms of buyers size can face very different purchase thresholds given that they fall in different buyers' categories.

For example, the Ministry of Transport had an annual budget equal to 170,700

[13] Suppliers interested in bidding must express their intention through the electronic system (SECOP II) during the three days after the publication of the call for tender.

[14] As mentioned in the introduction, the public entity in charge of buying goods, services or public work, is referred to as *the buyer* along the paper.

minimum wages[15] in 2019, then its corresponding *Minor Amount* and *Minimum Amount* thresholds were equal to 450 and 45 minimum wages accordingly.

Table 1: Thresholds for simplified procurement formats in Colombia

Category	Buyer annual budget	Minor Amount k_2	Minimum Amount $k_1 = 10\% k_2$
5	$> 1{,}200{,}000$	1,000	100
4	(850,000; 1,200,000]	850	85
3	(400,000; 850,000]	650	65
2	(120,000; 400,000]	450	45
1	$< 120{,}000$	280	28

Notes: Values are present in Active Monthly Minimum Wage (AMMW). The AMMW is updated every year by presidential decree. For instance, the AMMW for 2019 was $828,116 Colombian pesos. See Art. 2.2.b. Law 1150 of 2007 for further details.

Data and descriptive statistics

We collected every contract conducted through the electronic system of public procurement (SECOP II) in Colombia from 2018 to 2024 (March).[16] From these contracts we keep only the contracts at the federal level. Thus, the observations are entities from the federal government including delegations and decentralized institutions. Local authorities were not considered because not all of them have fully adopted e-procurement. Moreover, considering the presence of outliers in the estimated contract value, we winsorized the data at 1% in both extremes. We are left with around 120 thousands of contracts. Note that the database does not contain information about neither the purchase thresholds, *Minor* and *Minimum Amount*, nor about buyers' categories. Thus, we collected the annual budgets of each buyer for the same period from the Ministry for Finance to reconstruct such values.

The analysis samples contains more than 120 thousand contracts. The majority, 81%, conducted using the reverse auction format, 14% under the restricted auction, and only 5% followed the open auction format. The data set contains a good number of detailed characteristics for the contracts such as: the estimated contract value, date of publication of the call for tender, the awarding date, duration, type (goods, works, or services). Additionally, it contains information about buyers' and sellers' characteristics: identifiers for both, characteristics of buyers such as power branch, geographical location, sector and for the bidders their size, whether they were joint ventures or SMEs. We also observe indicators of the procurement process such

[15] The legal term is Active Monthly Minimum Wage (AMMW).

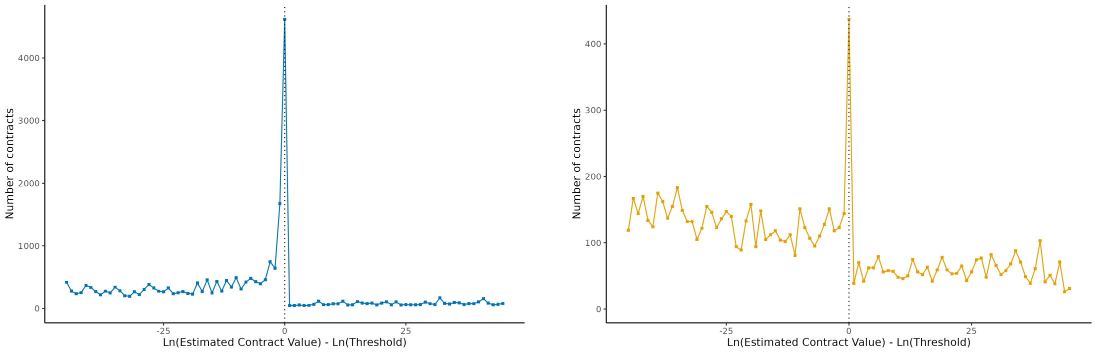
[16] The data sets are publicly available and can be downloaded from `www.colombiacompra.gov.co`. It is important to remark that in Colombia since 2017 the electronic system for procurement is compulsory.

as whether the contract was allocated and afterwards modified or cancelled, the number of bids received, final value paid and the identifier of the winner. Descriptive statistics for buyers, contracts and contracts by procurement format are available in the appendix in tables 2, 3 and 4 respectively.

To shed light on buyers responses towards the regulation, we start by analyzing the distribution of contracts around the regulatory thresholds. Figures 2a and 2b reports the distribution in the number of contracts around both thresholds, the *Minimum Amount* and the *Minor Amount*. Every contract is grouped in bins of equal size in terms of the logarithm of the estimated contract value and re-centered with respect to the corresponding purchase threshold value, k_1 and k_2 respectively. Therefore, figure 2a shows that on the left of the dotted line and at zero the contracts are allocated through a *Reverse Auction* and on the right through a *Restricted Scoring Rule*. In the same way, figure 2b shows on the left and at zero contracts allocated through a *Restricted Scoring Rule* and on the right through a *Public Auction*.

Figure 2: *Estimated contract value manipulation*

(a) Reverse auction *vs* Restricted Scoring R. (b) Restricted Scoring R. *vs* Public Auction



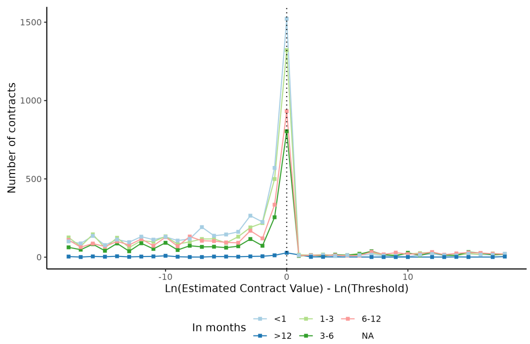
Notes: Both figures report the number of contracts in each bin. Bins group contracts in terms of difference between the natural logarithm of estimated contract value and the threshold value. Contracts with a value equal to the value of the purchase threshold are included in the bin equal to zero. In Figure (a) contracts awarded through a *Reverse Auction* format are grouped in non-positive bins while in positive bins are those contracts awarded through a *Restricted Scoring Rule*. In figure (b) contracts contracts awarded through a *Restricted Scoring Rule* are grouped in non-positive bins while in positive bins are those contracts awarded through a *Public Auction*.

Both graphs show a pronounced spike in the distribution exactly at the threshold followed by a pronounced fall right after. According to Kleven (2016), the pattern is consistent with a systematic sorting due to the presence of a *notch* in agents' constraints introduced by the regulation. In the case of public procurement, the figures are evidence of manipulation of the estimated contract value (Palguta and Pertold, 2017; Liebman and Mahoney, 2017; Coviello et al., 2018; Carril, 2021; Szucs, 2023; Decarolis et al., 2020; Caires et al., 2023; Ganuza et al., 2024). We refer to the term *buyers' manipulation of the procurement process* or simply *manipulation* as the buyer decision to modify the estimated contract value such that he chooses a particular auction format. This manipulation takes the form of an excess and missing mass around the threshold in this descriptive evidence.

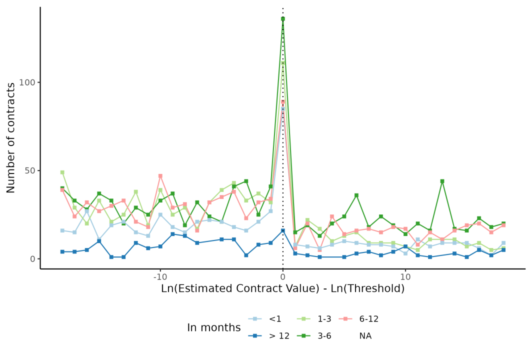
Figure 3: *Bunching Characterization*

Contract Duration

(a) Reverse auction

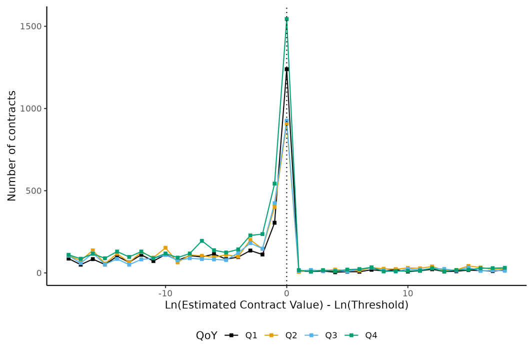


(b) Restricted auction

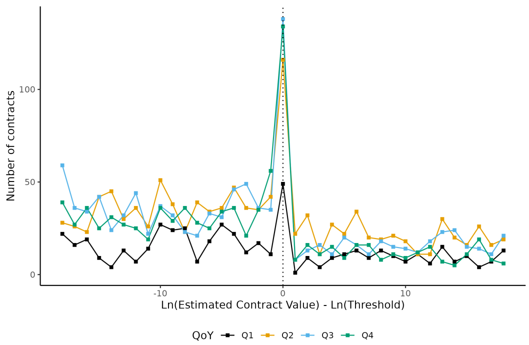


Call for Tender Date in Quarters

(c) Reverse auction



(d) Restricted auction



Notes: Figures on the left side depict contracts around the *Minimum Amount* threshold, while figures on the right side depict contracts around the *Minor Amount* threshold. Every figure plots the number of contracts and the difference between the natural logarithm of the estimated contract value and the natural logarithm of the threshold measured in bins. Panels (a) and (b) depict number of contracts in each bin by duration of the contracts. Panels (c) and (d) depict number of contracts in each bin by the quarter of the year (QoY) when the call for tender was published. Bins group contracts in terms of the benchmark contract value so that any bin includes contracts from both sides of the threshold.

We exploit other observable variables to descriptively characterize the *bunching in the contracting activity*, i.e., the manipulated contracts, and its relationship with the complexity of the auction format.[17] In figure 3 we decompose the bunching by

[17]Other characterizations are offered in the appendix as well. In particular, we find that supplies

contract duration and by post date in the call for tender. From panels 3a and 3b we can see that the main bunching arises in contracts with less duration for both thresholds. While in panel 3a the periods are considerably shorter, less than one month or between one and three months, in panel 3a the periods are a bit longer, between one and six months of duration. The main difference could be explained by the length of the administrative process itself and the extra requirements among the auctions.

Finally, since manipulation might be related with the moment of the civil year, as point out by Liebman and Mahoney (2017), we decompose the *bunching* according to the time of publication of the call for tender within the same fiscal year. Figure 3c shows that for the case of the *Minimum Amount* threshold, manipulation is almost twice more in the last quarter of the year than in quarter two and three, and around one and half more than in the first quarter. Figure 3d depicts the case of the *Minor Amount* threshold, it shows that there is more bunching for contracts offered in the third quarter of the year, slightly followed by the last quarter. Both figures suggest that buyers manipulate to spend their whole budget of the year.[18] As more complex is the process more times it takes to complete the procurement process then buyers initiate the acquisition in advance, which results in a larger manipulation one quarter before the end of the year.[19]

3 Empirical Analysis

In previous section we provided descriptive evidence of buyers' manipulation of the procurement process. In this section we provide: (1) the graphical implications in the contracting activity of buyer manipulation, overall and decomposed in contract shifting and contract splitting, (2) we design an empirical strategy to quantify the manipulation and separate its forms; finally, (3) we test its implications on outcomes of the procurement process.

3.1 Buyer manipulation of the estimated contract value

As previously mentioned, the estimated contract value determines the procurement format, check figure 1. Thus, if the buyer identifies that one procurement format

are the most manipulated contracts around the *Minimum Amount* threshold, and services around the *Minor Amount* one. Additionally, we see manipulation in processes with different amount of bidders (see figure 15).

[18] The final award date of the contract is available in figure 15.

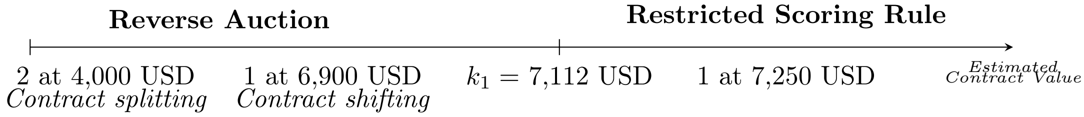
[19] Average contract duration from in a reverse auction is around two weeks, in a restricted scoring rule auction around four weeks, and for a public auction around nine weeks.

is administratively more costly than other one, buyers might manipulate the procurement format to reduce his administrative costs. Note that these costs could be diverse such as monetary, time (see figures (3c) and (3d)). We refer to these overall costs as *buyers' transaction costs.*

The rationale behind manipulation relies on these differences in transactions costs across procurement formats. The simpler the auction, the lower the administrative burden associated with the management of the procurement process. The law raises incentives to manipulate the estimated contract value in favor of simplified procurement formats. This in turn reduces discontinuously the transaction costs of the acquisition.[20] An important point to remark is that the estimated contract value does not imply a maximum value. In other words, buyers can purchase with a final value that is greater than the estimated one.[21]

Buyers' manipulation can take two forms, either *contract shifting* or *contract splitting*. *Contract shifting* refers to the decrease in estimated value of a purchase in order to fall below the purchase threshold k while *contract splitting* implies dividing one contract into several parts such that all these parts fall below the threshold k.[22] See figure 4 for an example, consider a category 1 buyer in 2019, its *Minimum Amount* threshold, k_1, was 7.112 thousand dollars. Assume that this buyer needs to make an acquisition, he estimates the contract value and the true estimation determines a cost of 7.25 thousand dollars. In that way, the buyer can either modify the estimated value and set it up at 6.9 thousand dollars, i.e.: *contract shifting*, or he can split this acquisition into two acquisition of less value of 7.112 thousand dollars each of them for instance two contracts of 4 thousand dollars, i.e.: *contract splitting.* We can anticipate that both ways of manipulation have different implication in the distribution of contracts. In particular, contract shifting is a local effect while contract splitting could be implemented not only on the neighborhood of the threshold value. We discuss that with detail at next.[23]

Figure 4: Example of *contract shifting* and *contract splitting*



[20]The regulation introduces a *notch* in buyer's transaction costs. See Kleven (2016); Bertanha et al. (2023) for compelling reviews.

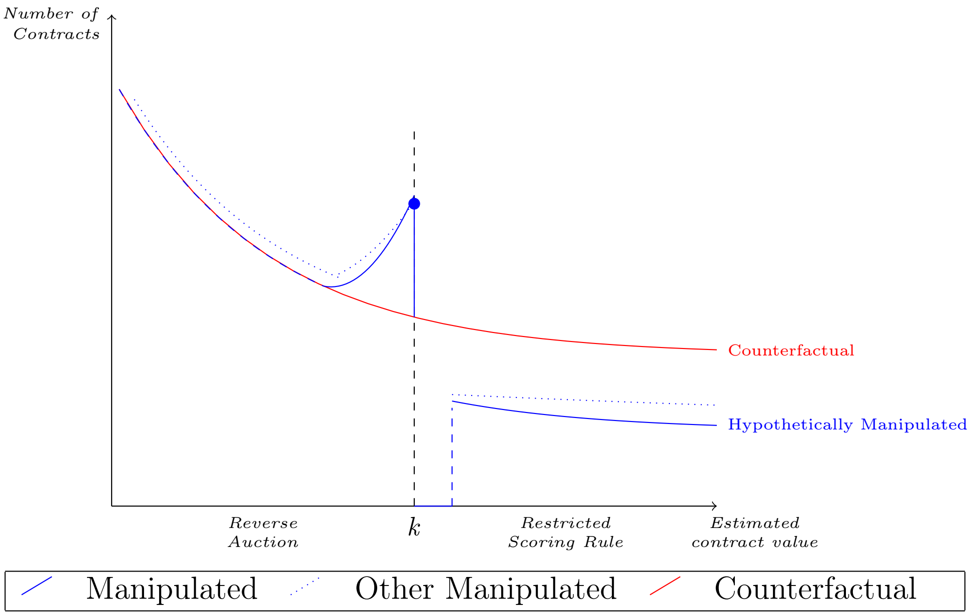
[21]In fact, in our sample, 13% of contracts awarded under the reverse auction, 30% under the restricted auction and 22% under open auctions have higher actual values than the benchmark.

[22]There is no clear consensus on the strict name of these two effects, some papers refer to *contract shifting* as *contract shrinking* and to *contract splitting* as *contract slicing* or *artifical allotment.*

[23]Additionally, it is important to highlight that *contract splitting* is an illegal way of manipulating the procurement process while *contract shifting* is not.

Figure 5 depicts a conceptual illustration of the effect of the overall manipulation on the distribution of contracts. The red line plots a counterfactual distribution of contracts that is not affected by threshold k. Both blue lines, dotted and thick lines, show examples of distributions of contracts that face a threshold around the value k. Being exposed to a regulatory threshold has two main implications. First, there exists a *notch* at the right neighborhood of k. This *notch* implies a missing mass of contracts at the right that shifted to the left neighborhood of k. This shift is a local behavior which often times is called *intensive margin* effect. Second, there is another effect affecting the distribution away from the neighborhood of k. We can see that the dotted line has a shifted support at the left of k, while the thick line does not, both of them are potential distributions. However, for both of them at the right of the threshold the distribution of contracts falls below the counterfactual. These non-local effects are called *extensive margin* in the *bunching* literature.

Figure 5: Manipulated *vs* Counterfactual Distributions

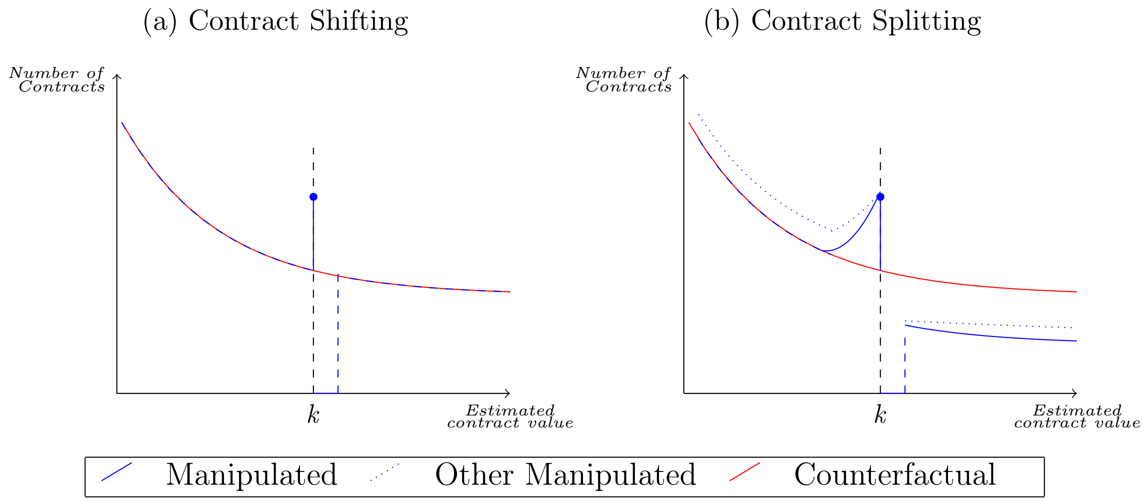


Contract shifting and contract splitting have different implications for the distribution of contracts. Thus, we decompose analyze these effects in figures 6a and 6b. Let's consider figure 6a first. In this figure the buyer can only manipulate by shifting a contract. In that case, we can see that the shifting mass, which is the number of contracts with an estimated contract value exactly at the value of the threshold, which is identical to the missing mass at the right of k, the notch. At the right of the notch the distribution is identical to the counterfactual one.[24] Note

[24]The idea that the shifted contracts move exactly to the value of the threshold is a conceptual

that this is only true if there are no frictions in the shifting, otherwise contracts that are not shifted could disappear which would drop the hypothetically distribution on the right of the *notch*. Conversely, in figure 6b we consider the case in which the buyer can only split contracts. In this case, the splitting generates a *notch* as well, but also affects the distribution beyond the neighborhood. At the left of k, there are two potential effects, either an excess mass focused on the neighbourhood k or a total shift of the distribution. At the right of the neighborhood of k a distribution may have either a permanent shift of the distribution with the same slope or a shift with a slow convergence to the counterfactual distribution. The slow convergence is explained on the difficulty of splitting big contracts. The bigger the contract, the larger the number of small contracts to be awarded and larger the transaction costs.

Figure 6: Forms of Manipulation



To sum up, when the main driver of manipulation is *contract shifting* we should expect an artificial increase on the number of contracts at the left of the threshold which equates the decrease on the number of contracts at the right of the thresholds. In that way, the effect should be mainly local, and the total number of awarded contracts should remain the same as without manipulation. Additionally, if the estimated value and the final value of the contract do not differ considerably, under contract shifting the overall expenditures around the threshold should decrease since there is the same number of contracts and less value paid for them. On the other hand, when *contract splitting* is the main driver we should expect an increase in the overall number of contracts. This increase is explained by a bigger number of contracts at the left of the threshold than those that are split at the right. Additionally,

analysis that reflects *sharp bunching*. There are several arguments to believe that empirically this bunching is not sharp, such as frictions, indivisibilities in the contracts, misunderstanding of the law, etc. See Anagol et al. (2022) for the estimation of diffuse bunching.

the splitting might affect differently the intensive and extensive margin. The split can either shift the whole distribution at the left of the purchase threshold or only appear at the neighborhood of the threshold. This will impact the overall value of expenditures although the direction of this change is not clear.

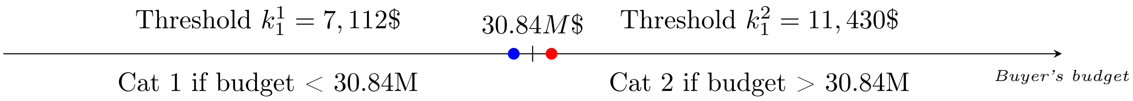
3.2 Empirical Stategy

Buyers grouped into different categories according to their annual budget face different purchase thresholds, that is different values for *Minimum* and *Minor Amounts* (see table 1). Leveraging on this design of the public procurement's law in Colombia we build a counterfactual that provides a causal identification of the effect of a threshold on manipulation.

3.2.1 Identification

The identification relies on the idea that two buyers of similar characteristics, with a small difference in budget size, might fall in different budget categories. From table 1 we see that the buyer in the smaller category face a smaller threshold than the buyer in the subsequent category. Then purchases of buyers allocated to the big category are a good counterfactual of purchases of buyers allocated to the small one around the threshold k_1. Figure (7) summarizes the previous argument for the example two categories of buyers, 1 and 2, in 2019.

Figure 7: Example of identification for two categories



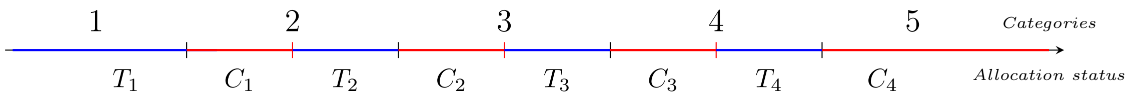
Notes: The figure shows an example for the identification. The blue dot represents a buyer that faces a *Minimum Amount* threshold around the value 7,112 while the red dot a buyer that does not although their buyer size is close. The superscript for k_1^1 and k_1^2 indicates the buyer category, categories 1 and two respectively, the subscript indicates the regulatory threshold, that is the *Minimum Amount*. The annual budget value that divides buyers into categories 1 and 2 is 30.84 million dollars.

The main underlying assumption behind this empirical strategy is that the control group is a good counterfactual for the treatment group. In our case, it means that buyers before and above the threshold are similar in terms of unobservable characteristics. As in the case of other local empirical designs, the assumption is plausible only in the neighborhood of the threshold. Therefore, we have restricted the analysis to buyers located in a certain bandwidth around the boundaries that determine each category.[25] Because there are five categories, restricting the analysis to contracts within a bandwidth also guarantees that no contract belongs simultaneously to the treatment and control group.

3.2.2 Quantifying manipulation

Similar to a *natural experiment* we rely in the exogenous variation provided by the law to allocate a 'Treatment' status to buyers that are in category that faces the lowest purchase threshold and a 'Control' status to those buyers that face the highest purchase threshold in a given neighborhood. In the previous example, category 1 buyers are the treatment group while category 2 are the control group. We use this same argument to classify buyers in the five categories. Note that category 1 is always treatment since its buyers always face the lowest purchase thresholds and category 5 is always control since its buyers face the highest purchase thresholds. For categories 2, 3, and 4, we divide the interval of annual budget size into two. For each half in each category, those buyers that belong to the half interval closer to the smaller category they are labelled as control. In fact, they are control group buyer for those buyers in the smaller category. Buyers in the half closer to the biggest category are labelled as treatment. They are the treatment buyers for the buyeres of the bigger category. The procedure is summarized in figure 8.

Figure 8: Buyers' allocation to treatment or control group



Notes: The figure shows the allocation to either treatment or control group of buyers. Category 1 buyers are always allocated to the treatment group whereas category 5 buyers are always allocated to control group. Categories 2, 3 and 4 are divided in half. Each half has either a 'T' or 'C' and a subscript. Thus, T_2 represents buyers of category two allocated to the treatment group and C_1 are buyers of category two allocated to the control group. In this case, it has a subscript one, because these buyers are the control group for buyers in category one. Colors blue and red refers to treatment and control group respectively. Black ticks represent the thresholds for categories established by law while the red ticks represent the thresholds imposed in the paper to split buyers inside one category into treatment and control.

After each buyer is allocated either to treatment or to control group, we pool the four treatment and control groups into one treatment and control group. We compare the purchases of the treatment and control group in the neighborhood of a purchase threshold k. As above mentioned, the treatment group faces a purchase

[25]In practice, the boundary is half of the segment between categories. We explain that in details in the next section

threshold at this value while the control group does not. Therefore, the treatment group faces a change in procurement format below and above the threshold while the control group performs always the same procurement format. Denote $B = n_b^T - n_b^C$, where B is the difference between the number of contracts n, in the treatment and control group, superscripts T and C, below the threshold k denoted with the subscript b. This difference captures two groups of similar buyers, one group facing a threshold and the other one not facing it, implementing the same procurement format. In other words, if these buyers are similar enough, this difference is capturing the causal effect of facing the threshold at value k. Denote $A = n_a^T - n_a^C$ where n and superscripts T and C are the same as before, but subscript a means after the threshold. This difference captures two groups of similar buyers implementing two different procurement formats. The difference between $B - A$ gets rid between the difference in size of the two groups of buyers and measure the causal effect of implementing different procurement formats for a same group of buyers. Figure (9) depicts the previous argument for the *Minimum Amount* threshold.

Figure 9: Impact of the exposure to the threshold on manipulation

3.2.3 Estimation

We group contracts in bins computed over the difference between the natural logarithm of the estimated contract value and the logarithm of the purchase threshold value. We use the bin size proposed by the McCrary test.[26] We measure this causal effect in two ways. First, with a non parametric estimator that directly captures the differences mentioned in subsection 3.2.2. In other words, we count contracts in treatment and control group below and above the threshold and we compare them. Second, we estimate this same causal effect in a parametric way described at next.

Let T_{ij} be a dummy equal to one if the contract i that belongs to a buyer j who is allocated to the treatment group, and equal to zero if the belongs contract i to a buyer j who is allocated to the control group. A $below_i$ a dummy equal to one if the contract i belongs to a bin below the threshold and equal to zero if the contract i belongs to a bin above the threshold. We fit the number of contracts n_b^T in bin b of group $T \in \{0,\ 1\}$, on a polynomial of degree x of the bin values.

[26]The binsize in the McCrary test is chosen optimally on a data-driven seletion method that minimizes the mean squared error in density estimation. For further discussion over the binsize computation check section 3.2 of McCrary (2008).

$$n_b^T = \sum_{x=1}^{p} \beta_x b^x \times T_{ij} \times below_i + \Lambda X_{ijb} + \mu_{ijb} \tag{1}$$

where X_{ijb} is a set of controls for contracts i and buyers j in bin b. This vector also includes the covariates T_{ij}, $below_i$, as well as the term $T_{ij} \times below_i$. Also, μ_{ijb} is the error term.

By removing the covariate $below_i$ from the equation, the estimated coefficient of term T_{ij} measures the difference in the average number of contracts between the treatment and control group per bin. Second, introducing the term $below_i$ allows to identify the difference between treatment and control groups below and above the thresholds. For instance, by fixing $below_i = 1$, it is possible to estimate the excess mass of contracts in the intensive margin comparing the treatment, i.e., conditional on $T_{ij} = 1$, with its counterfactual ($T_{ij} = 0$). As we mentioned above, this approach provides an intensive margin approximation measured as averages per bin. The complementary approach might be to measure the missing mass above the threshold by fixing $below_i = 0$. The last benefit concerns the estimation of standard errors. The strategy proposed allows statistical inference following standard techniques for hypothesis testing. Hitherto bunching methods require resampling techniques to estimate standard errors.

In addition to the point estimates, we exploit the same strategy to quantify the mass of manipulated contracts in the sample. Similar to Carril (2021), by comparing the estimated counts for each group below the threshold, we estimate the excess mass of contracts ($\hat{m}$) below the threshold as the difference in counts between the treatment and the control group over the number of control group contracts. Symmetrically, by comparing the situation above the threshold, we obtain estimates the missing mass of contracts ($\hat{m}'$) as the difference between control and treatment contracts over the number of control contracts:

$$\hat{m} = \frac{\sum_{b=-k}^{0} (\hat{n_b}^{T=1} - \hat{n_b}^{T=0})}{\sum_{b=-k}^{k} \hat{n_b}^{T=0}}, \quad \hat{m}' = \frac{\sum_{b=0}^{k} (\hat{n_b}^{T=0} - \hat{n_b}^{T=1})}{\sum_{b=-k}^{k} \hat{n_b}^{T=0}} \tag{2}$$

Where $\hat{x} = \hat{m}' - \hat{m}$ is then the corresponding missing mass of contracts in the extensive margin. Both measures are expressed as a share of the number of contracts in the control group in the range k of bins where manipulation is happening. Note that $\hat{n}_b$ is the predicted value in equation (1), therefore, different specifications might be used. To increase the precision of the estimation, we recommend to control with group-specific polynomials of a degree close to the number of bins used in one side of the threshold.

There are three advantages of implementing this parametric estimation. First, it offers a robustness check in terms of the estimation, since we see that the results do not change considerably. Second, it provides standard errors and the possibility to build confidence intervals for the point estimates. Finally, it brings the possibility to test the causal effect of being exposed to a purchase threshold on several procurement outcomes.

3.2.4 Purchases of buyers in category five

Buyers in category five are always allocated to the control group. This means that the previous empirical strategy does not allow to measure manipulation for this category of buyers. This category groups the biggest buyers and consequently those who spend more money in public expenditures. Because of that, we design another empirical strategy that allows to capture the causal effect of the threshold on buyers' manipulation and to study the impact of buyers' manipulation in procurement outcomes.

A first potential solution to this problem could be to measure bunching using standard techniques that relies on parametric assumptions of the counterfactual distribution (Bertanha et al., 2023). However, there is an important extensive margin effect as a consequence of the manipulation. We know that standard techniques of *bunching* with counterfactuals relying on parametrical assumptions have difficulties to measure the extensive margin effects. To account for this problem we propose an identification strategy that relies on the policy variation of the law. We use the pool of the contracts in the counterfactual for the empirical strategy in categories from one to four characterized by figure 8. For this group of contracts we fit different polynomial specifications inside and outside the manipulation window. After that, we re-scale the polynomial with the number of contracts in category five and we repeat the same estimation strategy as in the previous subsection 3.2.2, the details are exposed in the *appendix*.

3.2.5 Impact of manipulation on procurement outcomes

Finally, we apply the especification of equation (1) to test the impact of a contract subjected to be manipulated on procurement outcomes. Among these outcomes we consider several variables related to: i) competition, ii) duration, iii) equality, iv) quality of the process and, v) quality of the products and services. For competition, we use a) share of only one bidder in the process, b) number of bidders and c) difference between estimated and actual value of the purchase. For duration, we use estimated and actual duration of the process. In equality, we consider as dependent variables a) share of joint ventures and SMEs firms. For process of the quality we use: i) share of modified contracts, ii) additional days added to the execution of the contract, iii) contracts signed in the last week of the fiscal year as standard in the literature (Liebman and Mahoney, 2017; Carril, 2021). Finally, for quality of the products and services we **plan to** use methods of Natural Language Processing (NLP) to address differences between the buyer description of their needs and their description of the purchases.[27]

3.3 Results

In this subsection we present: (i) the results for the estimation of equation (1) and the excess and missing mass in equation (2), (ii) we offer an algorithmic characterization of the manipulation into *contract shifting* or *contract splitting*, (iii) we test the impact of manipulation on procurement outcomes; and, (iv) we provide some robustness checks of the experimental validity.

3.3.1 Buyers' Manipulation

Figure 10 shows a visual inspections of the results in the manipulation window. Both panels of the figure plots the number of contracts for the treatment and control group by bin of the logarithm of estimated value re-centered at the threshold value, below and above the respective purchase threshold, the *Minor* and *Minimum Amount*. The blue line contains the number of contracts offered by buyers in the treatment group, those who face the threshold in the bin zero, while the red line contains the number of contracts offered by buyers in the control group, those who do not face the purchase threshold in the bin zero.

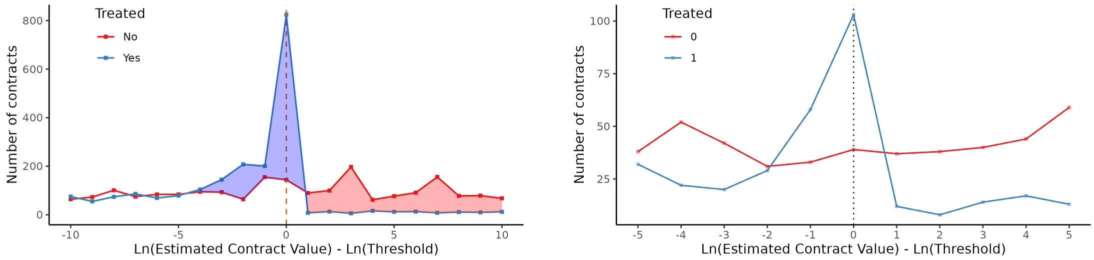
The x-axis of the figure is a re-centered exposition of figure 1 for each threshold k_1 and k_2. At the left of bin zero treatment and control buyers implement the same auction while at the right if bin zero treatment buyers implement a more complex auction and control buyers remain with the same as at the left of the threshold. The blue area that arise at the left of the threshold represents the excess mas while the

[27]We are implementing NLP analysis in two ways. First, through Semantic Textual Similarities in buyers' descriptions of the purchase to determine quality. Once we obtain a score of similarities between the description of what they got and the objective for which they opened the call for tender, we are able to compare whether there are differences in these indices between treatment and control contracts below and above the threshold. Second, an alternative way is to consider the HS codes for goods, forming a dictionary of these products and using its relationship with quality stated in the trade literature (Khandelwal, 2010). Therefore, we could see whether there is an heterogeneity in quality of the manipulated contracts. The main restriction of this last strategy is that we will only be able to handle quality in supplies, but not in services and public work.

red area at the right of the threshold represents the missing mass, measured as in equation (2). We see that both panels show the same behavior, the blue line has an important spike at the bin zero while the red line has a smooth behavior around this neighborhood.

Figure 10: *Overall Manipulation in the manipulation window*

(a) Reverse auction *vs* Restricted Scoring R. (b) Restricted Scoring R. *vs* Public Auction



Notes: Both figures report the number of contracts offered by treatment and control group buyers by bin of estimated contract value. Panel (a) contains contracts with estimated value around the *Minimum Amount* threshold that divides the process into a Reverse Auction and a Restricted Scoring Rule Auction. Panel (b) contains contracts with estimated value around the *Minor Amount* threshold that divides the process into a Restricted Scoring Rule Auction and Public Auction. The estimated contract value is demeaned, thus bin zero is the bin that contains the purchase threshold value. The blue line plots the number of contracts offered by treatment buyers while the red line depicts the number of contracts offered by control buyers. The line is a connection of dots, but the graphs are not continuous. Blue and red areas represent only the difference between treatment and control groups bin by bin but not in a continuous format.

We focus on panel 10a of the figure. As mentioned in subsection 3.1 we can obtain some predictions about the comparison between the manipulated and the counterfactual distribution of contracts, in this case in the manipulation window. First, we can see that at the left of the threshold that the distribution of the treated buyers starts to increase with respect to the counterfactual in the bin -4. Nonetheless, at the left of this bin both distributions have the same slope and levels. The difference between contract buyers offered by treated buyers and those offered by control buyers, depicted as the blue area, is the excess mass. Second, at the the right of the threshold, the distribution of the treated falls close to zero revealing an important notch. The red area, the difference between the contracts offered by the control group and the treatment group of buyers, represents the missing mass.[28]

We report these values in table 7 for different specifications of the excess and missing mass. In the first line, there is the excess mass, 60%, and missing mass, 57%, measured non-parametrically for observed contracts, as in the graph. According to

[28]In the *appendix* we present also some graphs with the estimation specified in equation (1) for several degrees of polynomials and we plot the confidence intervals.

figure 10a the manipulation seems to be local with no shift on the support of the treated distribution, which suggests that if there is contract splitting, those contract split fall in the neighborhood of the threshold. Note that this is consistent with the small net mass effect reported in table 7 of -2.5%.

For figure 10b the results are slightly different. First, we can see that at the left of the threshold the the distribution of the treated buyers starts to increase with respect to the counterfactual in the bin -2. However, different to figure 10a at the left of the threshold both distributions have the same slope but different levels. The other important difference relies on the behavior at the right of the distributions where the number of contracts in the treatment group of buyers do not represent almost zero, representing around 15% of the contracts of bin zero. This evidence is related to some indivisibility characteristics of some contracts, aspect that we resume in subsection 3.3.3. Additionally, we can see that the missing mass, 48%, is considerably more important than the excess mass, 27%. This evidence suggests that independently from the way of manipulating, there is an important *extensive margin* effect of the threshold. That is, when the buyer is around the threshold and he cannot manipulate the procurement process he abandons this public purchase.

Note that importantly, the difference in the slope between the control and the treatment group is a challenge for the identification strategy. The source of this challenge is that we cannot identify with this identification strategy if the difference in numbers between treatment and control group come because these buyers are different or because there are contracts that have been split in the control group. If it is true that we have contracts split in the control group below the threshold k, then we would be underestimating the *excess mass*. We account for that in the robustness check, by implementing other bunching strategies and also by using a different counterfactual with the same identification strategy.

Figure 18 replicates figure 10 extends the window of analysis without restricting to the manipulation window.[29] This figure reinforces the extensive margin effect of the threshold. Finally, we present the results for category five following the empirical strategy described in subsection 3.2.4. In figure 20 and table 14 we present the same result as for the other categories. The results have the same interpretation.

3.3.2 Contract Shifting *vs* Contract Splitting

As mentioned in the previous subsection, manipulation can take different forms: either contract shifting or contract splitting. Before we state that contract shifting is more important than contract splitting for the *Minimum Amount* purchase thresh-

[29] The windows is still restricted, otherwise we would reach a bin where there is the second threshold, *Minor Amount*.

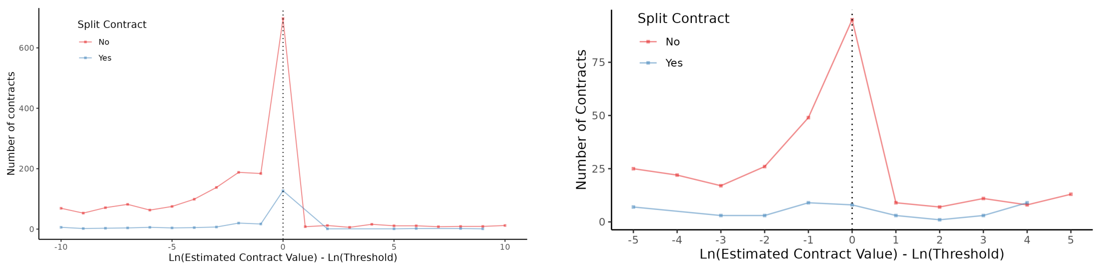
old. We based this statement on how conceptually we predict the distribution of a manipulated distribution with the two ways. In this subsection, we measure contract splitting with two different methods and we compute the excess and missing mass for each type of contracts: shifted and split.

The first method consists in identifying contract splitting algorithmically. We define group contracts as a group of split contracts if these contracts are offered by the same buyer and fulfil the following criteria: i) having the same type, ii) being a window of seven days, iii) having less difference that seven days of duration, and iv) having a similar estimated contract value, window of 30% difference. We define this method as *naive contract splitting* since there are several strategies that we cannot detect from a sophisticated bureaucrat, such as: splitting contracts with considerably different values, in different moments of the year, etc. However, we use it as a first approach and the main results are plotted in figure 11 and table 8.

There are two main characteristics to highlight from figure 11. First, the behavior of the buyers around the the *Minimum Amount* (panel 11a) and the *Minor Amount* threshold (panel 11b) are considerably different. In the former, contract shifting and splitting seem to have the same local behavior despite the number of contracts that we detect from each form of manipulation. In the latter, contract shifting reveals a clear response to the threshold while contract splitting contracts seem to be behave quite smooth in the neighborhood of the *Minor Amount*.

Figure 11: *Contract Shifting vs. Contract Splitting in the manipulation window*

(a) Reverse auction *vs* Restricted Scoring R. (b) Restricted Scoring R. *vs* Public Auction



Notes: Both figures report the number of contracts offered by type of manipulation: contract shifting and contract splitting. Panel (a) contains contracts with estimated value around the *Minimum Amount* threshold that splits the process into a Reverse Auction and a Restricted Scoring Rule Auction. Panel (b) contains contracts with estimated value around the *Minor Amount* threshold that splits the process into a Restricted Scoring Rule Auction and Public Auction. The estimated contract value is demeaned, thus bin zero is the bin that contains the purchase threshold value. The blue line plots the number split contracts while the red line depicts the complement, considered as the shifting contracts. The line a connection of dots, but the graphs are not continuous.

Table 8 complements the previous remark. Note that around the *Minimum Amount* threshold, when we divide the two ways of manipulation we get exactly the predictions described in subsection 3.1. That is, the excess and missing mass are exactly identical in contract shifting while the missing mass is importantly higher than the excess mass for contract splitting. However, around the *Minor Amount* threshold two different aspects arise. First, there are contracts with splitting contracts in the control group, and in particular, more than in the treatment group of buyers (negative excess mass). In that way, the control group is biased, since the amount of contracts respond also to contracts that have been manipulated from other categories. In subsection 3.3.4 we perform some robustness checks to ensure that the results for the minimum amount are not challenged by this previous aspect. Second, the missing mass seems to be considerably higher than the excess mass even for contract shifting, which implies that there could be an important amount of contract splitting contracts that we are not detecting from the algorithm previously described. The other approach here proposed will tackle this problem.

3.3.3 Impact of buyer manipulation on procurement outcomes

In this subsection we show how facing the purchase threshold affects the outcomes of the contracts. Tables 9, 10, 11 and 12 in the *appendix* shows the results of equation (1) for the different outcomes proposed in subsection 3.2.5 for the minimum amount threshold.

Tables 9 and 10 shows that manipulation does not affect the amount of bidders, column (1) and (2), in each auction and there is no statistical difference between the estimated contract value and the final value paid by buyers in column (3) and on the number of repeated winners in column (4).[30] Additionally, in table 11 we test the effect on the size of the supplier, the estimated duration and the actual process duration. In every case we do not see any effect. Finally, we check whether being subject to manipulation affects ex-post quality measures of the process. Following Carril (2021) and Liebman and Mahoney (2017) we test whether being affected by the threshold increase: i) the probability to modify a contract in the year, ii) the number of additional days needed to include to finish the contract, and iii) the number of additional days required to finish a contract and iv) the probability of signing a contract in the last week of the civil year.[31] We show in table 12 also that the statistical and economic significance of the coefficients vary considerably with the specification.

[30] In the different specifications proposed in the two tables we can see that there is almost either no statiscal effect or no economic effect. Additional, every effect is extremely senstive to the specification which enforces the idea that there is no effect.

[31] We wanted to use also the number of cancelled contracts, but the variable was not clean enough to bring variation.

3.3.4 Specification and Robustness Checks

Covariate balance. The main assumption behind the empirical strategy is that buyers in the control group are comparable to buyers assigned into the treatment in terms of unobservable characteristics. To provide an empirical evidence on this regard, we examine whether groups are balanced in terms of relevant covariates. The Table 13 reports the means and standar deviation of buyers' observable characteristics. Due to the mechanism of the treatment assignment, buyers in the control group are larger in size than buyers in the treatment group in terms of the annual budget. The evidence presented here suggest that the distribution of covariates is similar across the treatment.

Selection to treatment. Another treat to the specification is the potential self-selection into treatment on the basis of anticipated gains. We consider this treat to be extremely unlikely due to the following reasons. First, treatment selection is based on buyers' annual budget, which is used for the whole operation of the entity and not only for procurement reasons. Second, buyers' annual budget is proposed by the executive and approved by the congress, this process reduces the risk of anticipation. Third, only large entities might have the capacity to bargain over their budget. Nevertheless, it is not clear whether buyers will engage in negotiations only to determine their procurement thresholds.

We provide empirical evidence that confirm our intuition. First, Figure 19 shows a smooth distribution in the number of buyers around the threshold. This suggests that buyers are not systematically self-selecting into treatment. Second, we visually inspect the number of buyers around the multiple thresholds for the various years in the sample and we find a smooth distribution (not shown here). Finally, we **plan to** repeat our estimations for the subs-sample of buyers who remained in the same category.[32]

Alternative control group. Due to the mechanism used for the assignment to treatment and the size of the bandwidth, the treatment effect is estimated using as counterfactual buyers in the close neighborhood of the category cutoff point. For example, to identify manipulation in buyers form category 2, we use the counterfactual distribution of contracts only from buyers in category 3. To show that our results are robust to the specification design, we repeat our analysis using the counterfactual distribution of contracts from other categories (**to be done**).

Placebo tests. Another concern in the empirical analysis is that estimates are

[32]As a descriptive measure 68% of the buyers never changed categories in seven years. Only 15% changed categories more than once, and those buyers are smoothly distributed among categories. It is not the case that buyers from a particular category are regularly moving from one category to another one.

driven by random variations in the number of contracts along the whole distribution. To show that our results are robust to this potential caveat, we **plan to** replicate out empirical analysis using various placebos of the threshold value. In addition, we use as a placebo a sample of contracts not subject to the regulation. Since the visual inspection shown in figure 18 do not suggest manipulation we expect to observe no manipulative behaviors in the analytical tests in this case.

4 The model

In this section we propose a model to rationalize buyers' decision into either contract shifting or contract splitting and study its welfare implications. Every proof is presented in the *appendix*.

4.1 Basics

Players. There is one representative bureaucrat, *the buyer*, who is in charge of the public procurement process and n heterogeneous firms, *bidders*, indexed by f, $f \in \{1, 2, ..., n\}$. We denote $-f$ as any other bidder that is not f.

Institutional Setting. The buyer can either buy through a Reverse First Price Auction, denoted RA, or through a Restricted Scoring Rule Auction, denoted SR.[33] In an RA auction, the buyer purchases from the lowest bid. In a SR, the buyer first establishes observable requirements of the purchase, q, and set up a maximum of participants that can make an offer, $\overline{n} < n$. After that, he designs a score, $s(p, q)$, and allocates the purchase to the bidder with the highest score.

Actions. The buyer decides the purchase amount, y_0, and the observable requirements $q \in [0, \overline{q}]$. Denote $y(q) = (1 + q)y_0$ as the purchase with observable requirements.[34] Bidders decide either the price if they participate in an RA auction or price and score and if they participate in a SR auction.

[33] In the model we consider only one threshold. The extension towards two thresholds does not provide additional insights.

[34] This y_0 is interpreted as the budget used from the government to purchase a particular need and q the extra requirements of this need. Let's consider three examples. First, consider the procurement of a highway, y_0 is the amount transfer to spend on the production of this highway. In this case, q is the extra money that society is up to spend in order to ask for some environmental conditions, often times called *Green Public Procurement* (GPP). An identical example could arise from inclusion requirements, such as q as the extra money spent to buy from a local firm. Second, consider the case of infrastructure procurement, q can be considered as the ex-ante investment required by the government in order to bid for this project. Third, consider the case of health contracts. There is the need to buy a medicine, y_0 is the amount allocated to a particular main component, e.g.: ibroprofen 400mg, q represent the excipients offered by different laboratories and could affect the bioequivalence of the main ingredient.

Payoffs. Buyer's value of a purchase without observable requirements is 1 and with observable requirements is normalized $\frac{1+\phi(q)}{1+q}$ where $\phi(\cdot)$ is increasing, concave and $\phi(0) = \phi'(\overline{q}) = 0$ and $\phi'(0) = +\infty$. Buyer's transaction costs to implement RA are denoted by τ_1 and to implement SR are τ_2. The marginal cost to provide the public purchase for bidder f is c_f.[35]

Information. Buyer's transaction cost are privately known by the buyer. Bidders' marginal cost are privately known by the bidders. However, the buyer knows that marginal costs are independent and uniformly distributed in the unit support, $[0, 1]$. The buyer knows the optimal social amount of a purchase, in terms of social consumer surplus, without and with observable requirements y_0^* and $y_q^* = (1 + q^*)y_0^*$ respectively.[36]

Preferences. Let's denote $\tau := (\tau_1, \tau_2)$ and $\mathbf{c} := (c_1, c_2, ..., c_n)$. The utility for the buyer is:

$$V(y_0, q, \tau, \mathbf{c}) = \begin{cases} y_0 - \mathbb{E}_c[p(c)|RA] - \tau_1 & \text{if RA,} \\ y_0\,(1 + \phi(q)) - \mathbb{E}_c[p(c)|SR] - \tau_2 & \text{if SR.} \end{cases} \tag{3}$$

The utility for bidder f is:

$$U_f(p_f, p_{-f}, c_f) = \begin{cases} (p_f - c_f y_0) & P(p_f \leq p_{-f}, \forall - f) \quad \text{if RA,} \\ (p_f - c_f(1+q)y_0) & P(s_f \geq s_{-f}, \forall - f) \quad \text{if SR.} \end{cases} \tag{4}$$

Where $P(\cdot)$ is the probability function and s represents the score value.

Timing. First, the buyer makes the call for tender under a particular auction. Second, the firms compete in this auction. Third, the public purchase is realized.[37]

Equilibrium Concept. The equilibrium concept is Subgame Perfect Equilibrium. Note that at stage two, the bidders play a Bayesian Nash Equilibrium at the stage game.

Assumptions.

1. $\tau_2 > \rho\tau_1 > \tau_1$. Buyer transaction costs of the most complex auction are higher and the number of split contracts is bounded.

[35]Following Asker and Cantillon (2008) we know that quality plays no role in the selection of the bidder when the scoring function is quasilinear, which is the assumption we impose in the paper.
[36]We refer to the optimal social amount as the value that the society as a whole would be up to spend optimally on this need. This optimal purchase does not consider firms' profits.
[37]Motivated by the empirical fact that the difference between the estimated value and the actual value there is not difference in the manipulation window, we assume that the buyer chooses directly the actual value.

2. $y_0 \in [0, y_0^*]$. The support of the purchase has a maximum at the optimal social value.[38]

3. $s(p,q) = \phi(q) - p$. Following Asker and Cantillon (2008) we assume that the score is a quasilinear function of observable requirements and price.

4. The gross surplus of doing SR with q^* is higher than that gross surplus of RA: $1 + \phi(q^*) - 2\frac{1+q^*}{\overline{n}+1} > \frac{n-1}{n+1}$.[39]

4.2 Analysis

We solve the model by backward induction. Thus, at the second stage we first solve the maximization program of bidder i for each auction RA and SR. The maximization program is presented in the following equation (5).

$$\max_{\left\{p_f, s_{f_{\{\text{if SR}\}}}\right\}} U_f(p_f, p_{-f}, c_f) \tag{5}$$

Lemma 1. *The optimal bid for bidder i and the buyer expected payment are:*

$$p_f = \begin{cases} \frac{1+c_f(n-1)}{n} y_0 & \text{if RA,} \\ \frac{1+c_f(\overline{n}-1)}{\overline{n}}(1+q)y_0 & \text{if SR} \end{cases}, \quad \mathbb{E}_c(p) = \begin{cases} \frac{2}{n+1} y_0 & \text{if RA,} \\ \frac{2}{\overline{n}+1}(1+q)y_0 & \text{if SR} \end{cases} \tag{6}$$

The previous *Lemma* characterizes the buyers' behavior at stage 2 and establishes the expected price that the buyer will pay for this purchase under the two types of auctions. At stage one, the buyer needs to decide either to implement RA or to implement SR. Thus, the buyer chooses y_0 and q such that maximizes his utility:

$$\max_{\left\{y_0, q_{\{\text{if SR}\}}\right\}} V(y_0, q, \tau, \mathbf{c}) \tag{7}$$

The following lemma characterizes his behavior.

Lemma 2. *The optimal observable requirements are $q^* = \overline{q}$. Buyer's optimal purchase is either y_0^* or y_q^* when he implements RA or SR respectively. The buyer implements RA when $y_0^* < \overline{y}$ and SR when $y_0^* \geq \overline{y}$ with cutoff $\overline{y}$ being:*

$$\overline{y} := \frac{\tau_2 - \tau_1}{\phi(q^*) - q^* \frac{2}{\overline{n}+1} + \frac{2(\overline{n}-n)}{(\overline{n}+1)(n+1)}} \tag{8}$$

[38] This assumption is not very restrictive and has two main implications. First, that there is an optimum in the maximization problem we implement afterwards. Second, that as soon as there is no policy intervention in the institutional setting the buyer cannot over purchase.

[39] Abstracting from transaction costs, choosing optimal observable requirements in a SR auction is always better than running the RA auction without them.

There are four points to remark from this lemma. First, the optimal decision over the observable requirements is independent from the amount of the purchase. The decision becomes binary, either to require them or not. Second, buyer's optimal size of the project is always the optimum social size under different auction formats. Finally, buyer's size decision is fully characterized by the cutoff. This cutoff depends on the difference of transactions costs of each auction format, on the observable requirements, q, and the differences in prices paid in each auction format, characterized by the difference in bidders n and $\overline{n}$. Comparative statics are offered in the *appendix*

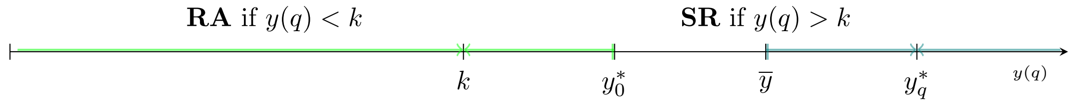
4.2.1 Policy Interventions

Now assume that society decides to implement a procurement law that fixes a purchase threshold, k. This purchase threshold, determines whether the buyer can implement RA, when $y(q) \leq k$ or he needs to implement SR, when $y(q) > k$. Recall that the law is asymmetric, meaning that implementin SR is always allowed. At next we show how buyer's behavior is affected by this intervention. There are some environments where a choice of a purchase threshold is irrelevant, we show this case in the *appendix*. Intuitively, k limits the purchase under RA but not under SR, in that way, the relevant cases happen when $k < y_0^*$, implying that there are certain amount of purchases that the buyer would prefer to buy through RA without policy intervention but will not be able to do with the introduction of procurement law. From the remaining part of the section, we focus on one of these cases.

Assume that $k < y_0^* < \overline{y} < y_q^*$ the threshold affects the buyer's decision as depicted in figure (12).[40] With the implementatoin of the law, we know that the buyer needs to choose either RA with value k given that he cannot purchase at value y_0^* or SR with value y_q^*. We see that optimally, the law does not alter any purchase between $[0, k]$ and SR with y_q^* for any value above $\overline{y}$. However, as soon as the value is in the interval $[k, \overline{y}]$ the buyer cannot react as before given that RA with y_0^* is no longer a feasible option for the buyer then the cutoff $\overline{y}$ is irrelevant. So the new distortion a different buyer's reaction.

[40] Another relevant, but more extreme case, arises when $k < y_0^* < y_q^* < \overline{y}$. This case, makes SR never optimal for the buyer.

Figure 12: Implementation of Purchase Threshold k



Notes: The black line represents the size of the project $y(q)$. We depict four values: 1) purchase threshold k, 2) optimal social value without observable requirements y_0^*, 3) cutoff that characterize buyer behavior without any policy intervention $\overline{y}$ and, 4) optimal social value with observable requirements y_q^*. Both green lines represents buyer's optimal behavior after the implementation of the purchase threshold k. The light green one represents the optimal purchase under RA auction and dark green one the optimal purchase under SR auction.

The buyer has two options in order to face the purchase threshold to handle contracts in the interval $[y_0^*, \overline{y}]$. First, he can either decrease the size and purchase under RA at size k, as he does for the majority of purchases in interval $[k, y_0^*]$, or purchase at size y_q^* under SR, we define this reaction as *contract shifting.* Second, he can divide the size of the contracts into ρ number of pieces with smaller size than k and purchase them under RA or purchase under SR with an optimum size y_q^*, we define this reaction as *contract splitting.* The following *lemma* provides the two cutoff, $\overline{y}_{Shift}$ and $\overline{y}_{Split}$, that make indifferent the buyer to choose either RA or SR under contract shifting or contract splitting. These two cutoffs will be mainly contained in the interval $[y_0^*, \overline{y}]$ and the ordering differs for different purchase threshold values k. However, for some significant large values of k, $\overline{y}_{Split}$ can be higher than $\overline{y}$.

Lemma 3*. Assume that the buyer can either shift or split contracts but not both. Then if $y_0^* < \overline{y}_j$ he implements RA and if $y_0^* \geq \overline{y}_j$ he implements SR, where $j \in \{Shift, Split\}$. Where the cutoffs have the following expression:*

$$\overline{y}_{Shift} := \min\left\{\frac{\tau_2-\tau_1}{1+\phi(q)-(1+q)\frac{2}{\overline{n}+1}} + k\frac{(1-\frac{2}{n+1})}{1+\phi(q)-(1+q)\frac{2}{\overline{n}+1}}, \overline{y}\right\} \tag{9}$$

$$\overline{y}_{Split} := \frac{\tau_2-\rho\tau_1}{1+\phi(q)-(1+q)\frac{2}{\overline{n}+1}} + \rho k\frac{(1-\frac{2}{n+1})}{1+\phi(q)-(1+q)\frac{2}{\overline{n}+1}} \tag{10}$$

Note that both cutoffs are functions of: 1) the purchase threshold imposed, 2) the difference in transaction costs between auctions, and 3) the differences between values and prices for each auction format. Finally, contract splitting cutoff considers, additionally, the number of pieces in which a project is split.

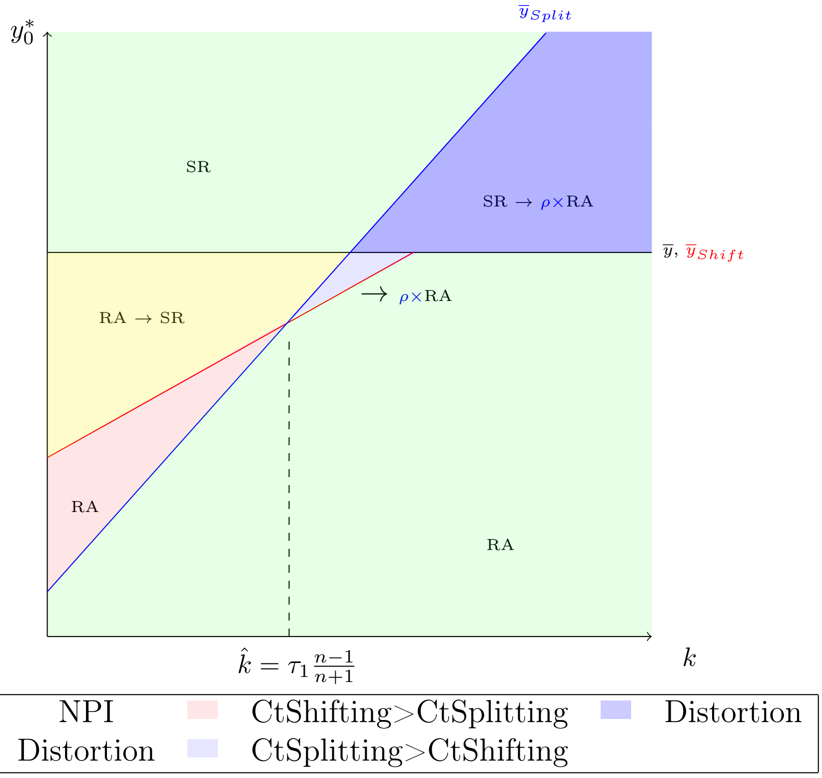
The buyer maximizes his utility either shifting or splitting a contract. In *proposition 1* buyer optimal decision to either split or shift and the overall manipulation.

Proposition 1*. The buyer overall manipulation is characterized by $\overline{y}_{OM} =: \max\{\overline{y}_{Shift}, \overline{y}_{Split}\}$. If $y_0^* < \overline{y}_{OM}$ the buyer implements RA and $y_0^* \geq \overline{y}_{OM}$ the buyer* *implements SR. The buyer is indifferent between shifting and splitting at $\hat{k}$:*

$$\hat{k} := \frac{\tau_1(n+1)}{n-1} \tag{11}$$

Proposition 1 provides three important remarks. First, the overall manipulation has a kink at the value $\hat{k}$. Second, this indifference value is a function of the transaction cost of the less complex auction and the number of bidders that enter in this auction. Third, for $k < \hat{k}$ contract shifting is preferred to contract splitting and the opposite for $k > \hat{k}$. The main results of this proposition are depicted in figure (13).

Figure 13: Results of Proposition 1



Notes: the figure is depicted for $n = 20$, $\overline{n} = 10$ bidders a $\phi(q) = ln(1+q)$ function, $q = 4$ observable requirements and transaction costs values of $\tau_1 = 3$ and $\tau_2 = 7$. *RA* and *SR* represent Reverse Auction and Restricted Scoring Rule respectively. Value k is the purchase threshold value and $\overline{y}, \overline{y}_{Shift}$ and $\overline{y}_{Split}$ are the cutoffs stated in *lemmas* 2 and 3.

The green area represents the coincidence between what the buyer does without policy intervention and his optimal decision with policy intervention. The red area depicts buyer contract shifting manipulation: downsizing the project but implementing the cheapest auction. The light blue area depicts buyer contract splitting manipulation: downsizing the project and paying the cheapest transaction costs ρ times while still doing the action that would be preferred without policy intervention.

Areas yellow and dark blue imply the impact of the overall manipulation: changing completely the auction format. The yellow area implies oversizing the project and covering the highest transaction costs. The dark blue area implies downsizing and paying ρ times the lowest transaction costs, wasting expenditures of $\rho k - y_0^*$ and not profiting from the extra values of the observable requirements q. The impacts of this behavior are summarized in *corollary 1* and described in detail in the next subsection.

4.2.2 Welfare and Policy Implications

We introduce the notion of marginal welfare, MW, as the consumer surplus of a purchase under RA or SR. Welfare in this context will be the integral of this expression, that is the difference between the value of a purchase and the price paid in a given auction type, abstracting from any transaction costs.[41]

$$MW = \begin{cases} \frac{n-1}{n+1} y_0 & \text{if RA} \\ (1 + \phi(q) - (1+q)\frac{2}{\overline{n}+1}) y_0 & \text{if SR} \end{cases} \tag{12}$$

Note that as soon as there is a law implemented, the welfare function is affected by getting a maximum of k under RA, either one or ρ times, depending on the action of the buyer. Then we compare the welfare under five different scenarios. First, the First Best (FB) scenario where the buyer only implements a SR Auction. Second, No Policy Intervention (NPI) scenario where the buyer decides to choose either RA or SR considering that there is no law that establishes how to choose. Third, Contract Shifting (Shift) scenario where there policy intervention and the only manipulation available for the buyer is to shift contracts. Four, Contract splitting (Split) where there is policy intervention and the only manipulation available for the buyer is to split contracts. Five, Overall Manipulation (OM) scenario where there is policy intervention and both forms of manipulation are possible. We denote MW^w as the welfare function for each case $w \in \{FB, NPI, Shift, Split, OM\}$.

Corollary 1. *Let's denote $\overline{k}$ such that $\overline{y}_{Split}(\overline{k}) = \overline{y}$. W^{FB} is always higher than any other scenario for any value of k. Then,*

1. *Contract shifting is always better than no policy intervention for any k,*

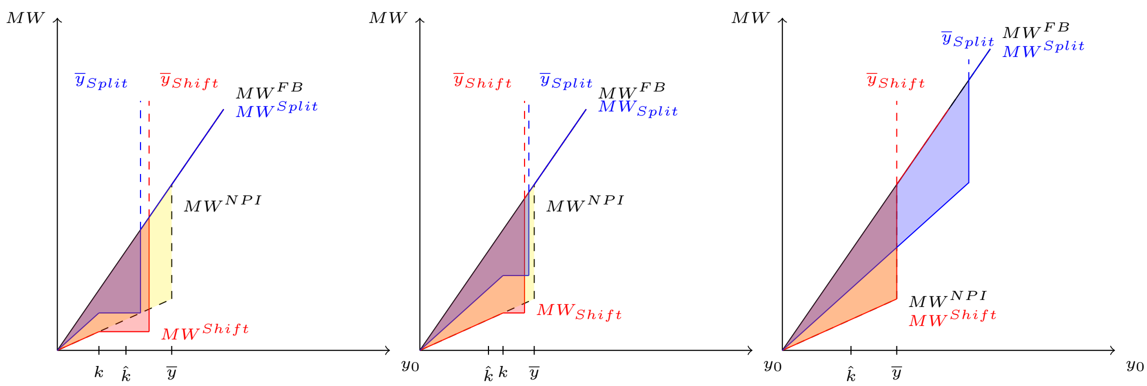
[41] The implicit assumption is that the society and the buyer are affected differently by the transaction costs spent by the buyer. Society in general has more interest on the value of the purchase and they are less affected by the direct transaction costs beared by the buyer. Recall that these transaction costs combine efforts of these buyers, time spent in the purchase, and direct budget allocated to it. However, buyers has an intrinsic interests on these costs, since they affect their utility directly.

2. *It exists a* $\tilde{k} < \overline{k}$ *such that for* $k \in (0, \tilde{k})$ *contract splitting is better for welfare than contract shifting and for* $k > \tilde{k}$ *contract shifting is better for welfare than contract splitting.*

3. *Overall manipulation is worse for welfare than contract splitting alone, and is better for welfare than contract shifting for a considerably larger* k *than* $\tilde{k}$*.*

The previous corollary provides the following messages. First, that contract shifting is welfare enhancing in comparison to a situation where the buyer freely chooses the auctions. Contract shifting implies a quicker switch towards an SR auction, which is a welfare increasing action. Second, contract splitting is welfare increasing when the threshold is sufficiently small. Note that when purchases are small, the added value of an observable requirement is small as well. As a consequence, the buyer can recover some surplus by purchasing ρ times without observable requirements. Finally, overall manipulation implies implementing both manipulations, contract shifting and splitting, in the areas where are welfare decreasing in comparison to the other source of manipulation. The three panels of figure (14) depict these messages.

Figure 14: Welfare Analysis



(a) $k < \hat{k} < \overline{k}$ (b) $\hat{k} < k < \overline{k}$ (c) $\hat{k} < \overline{k} < k$

NPI	Shifting	Splitting
NPI and Shifting	NPI and Splitting	NPI, Shifting and Splitting

Notes: the graph plots the welfare loss for each case, *NPI*, *Contract Shifting*, *Contract Splitting* and *Overall manipulation*, which is simply *Contract Shifting* at the left of $\hat{y}$ and *Contract Splitting* at the right. Recall that $\hat{y} = \overline{y}_{Shift}(\hat{k}) = \overline{y}_{Split}(\hat{k})$ The solid black line represents the First Best marginal welfare, the solid red and blue lines represent the contract shifting and splitting marginal welfare respectively. Finally, the dashed black line represents the NPI marginal welfare. Each colour shade represents the welfare loss of each case labeled in the box. The figure is depicted for $n = 20$, $\overline{n} = 10$ bidders a $\phi(q) = 8\sqrt{q}$ function, $q = 0.25$ observable requirements and transaction costs values of $\tau_1 = 1.5$ and $\tau_2 = 7$.

Figure (14) depicts the welfare loss for each case *First Best, NPI, Contract shift-*

ing, Contract Splitting and Overall Manipulation. Panel (14a) shows the result of the corollary for small values of k. In that case, we can see that contract splitting is welfare increasing with respect to contract shifting, and both forms of manipulation are better than NPI. Panel (14b) shows that for an intermediate value of k contract shifting and contract splitting represent a similar welfare loss, but still both are better than NPI. For this example proposed in the figure, contract splitting is still welfare increasing with respect to contract shifting. Finally, panel (14c) shows how the welfare loss of contract splitting increases considerably when k is very high. In particular, in this case it shows how contract splitting becomes worse for welfare than contract shifting and NPI. Recall that as shown in *proposition* 1 contract shifting is implemented for $k < \hat{k}$ while splitting $k > \hat{k}$. This implementation condition means that contract splitting benefits is not implemented by the buyer for considerable purchase sizes where it is welfare enhancing when both manipulation forms are available. While contract splitting can still be welfare enhancing depending on $\tilde{k}$ it can also be welfare decreasing. Contract shifting reveals to be a strategy for the society that is less welfare-enhancing, but always improving welfare.

Combining *proposition* 1 with *corollary* 1 we learn several relevant policy implications for law design. First, the decision to either shift or split depends on the purchase threshold established by law. Second, contract splitting is better for welfare for small k than contract shifting and conversely for big k. For intermediate values of k, both situations might arise. However, as we know by *proposition* 1, it is not implemented for $k < \hat{k}$. Which means, that the benefits of contract splitting get reduced considerably as soon as the buyer can choose the way to manipulate. Finally, we know that the buyer profitability to either shift or split depend on the transaction cost of the simplest allocation method. Decreasing the transaction cost of the simplest allocation method increase buyer's profitability of RA and decrease the area for which contract shifting is preferred to splitting. According to that, we expect to see 1) less contract splitting for lower thresholds and, 2) which is consistent to observing less contract splitting in the Colombian setting than in any other setting where the simplest allocation method is direct purchase.

These results bring a crucial message for policy design. A procurement law needs to be a menu of transaction cost of the simplest purchase method and a threshold value. In that way, it can minimize welfare losses constrained by the incentive compatibility of buyers choosing their preferred form of manipulation. Given that, there are two main recommendations. First, that for large purchases where contract splitting may reduce welfare and it is implemented by the buyer, the transaction costs of the least complex auction should be high enough to encourage contract shifting under an optimal institutional design. Second, when purchases are small enough contract splitting reveals as more welfare enhancing, which suggests decreasing at maximum the transaction costs of the least complex auctions. In the limit, for considerably small purchases, under no corruption concerns, the law should encourage purchases through discretion.

5 Conclusion

The institutional design of public procurement laws has focused on balancing rules and discretion. The Colombian government proposed an approach to handling the trade-off between ensuring the best value for money and managing process complexity by introducing a procurement law that prohibits discretion but offers different auctions for small and large purchases. We show that this institutional design, although more complex, can offer significant benefits for governments as it directly impacts how public officials manipulate the procurement process.[42] Unlike procurement laws that prioritize either rules or discretion, which tend to favor contract splitting, this framework allows the choice of threshold values and imposes transaction costs of these actions that influence public officials' optimal decisions to either shift or split contracts.

We first provide evidence of buyer manipulation of the estimated contract value in order to avoid crossing regulatory thresholds in contexts without discretion. We show significant bunching in the contracting activity and provide both an indirect and a direct way to characterize these manipulation into contract splitting and distinguish it from contract shifting. We detect that contract shifting is the dominant form of manipulation at the lower regulatory threshold, while contract splitting becomes more prominent at the higher one. We show that manipulation at the lower threshold has minimal negative effects, such as no impact on the number of bidders, repeated winners, difference between estimated and final value paid, duration of the contracts, but a negative effect on the quality of the procurement process.

Second, we study the welfare implications of these two different forms of manipulation. We show that the buyer's decision to engage in either contract shifting or splitting depends on two elements: the value of the regulatory threshold and the transaction cost of the simpler auction format. We show that the use of contract shifting (splitting) increases as the purchase threshold rises (falls) and as the transaction costs of the simpler auction format increases (decreases). However, we show that contract splitting is more beneficial for welfare at low threshold values.

[42] Its main benefits arise even without considering corruption as a main source of distortion in the procurement process. When there are high risks of corruption this system offers even greater benefits due to the absence of discretion.

We compare these two forms of manipulation with a case where a buyer can freely choose their preferred auction format, showing that manipulation can, in some cases, enhance welfare. These results complement other studies in the literature (Carril, 2021; Bosio et al., 2022; Coviello et al., 2022; Ganuza et al., 2024) by showing that the positive welfare effects arise from the impact of the thresholds and transactions costs on the induced form of manipulation.[43]

Finally, this framework offers important extensions to consider. An important one, it is the policy counterfactual of different thresholds and different procurement formats, such as discretion, to perform welfare analysis for Colombia. This extension will help to shed light on the optimal threshold value for the current design and also on the optimal threshold value for each potential design.

References

Aghion, P. and Tirole, J. (1997). Formal and real authority in organizations. *Journal of political economy*, 105(1):1–29.

Anagol, S., Davids, A., Lockwood, B. B., and Ramadorai, T. (2022). Diffuse bunching with frictions: Theory and estimation. *Available at SSRN 4152124*.

Asker, J. and Cantillon, E. (2008). Properties of scoring auctions. *The RAND Journal of Economics*, 39(1):69–85.

Baker, G., Gibbons, R., and Murphy, K. J. (1999). Informal authority in organizations. *Journal of Law, Economics, and organization*, 15(1):56–73.

Bandiera, O., Best, M. C., Khan, A. Q., and Prat, A. (2021). The allocation of authority in organizations: A field experiment with bureaucrats. *The Quarterly Journal of Economics*, 136(4):2195–2242.

Bandiera, O., Prat, A., and Valletti, T. (2009). Active and passive waste in government spending: evidence from a policy experiment. *American Economic Review*, 99(4):1278–1308.

Baránek, B., Titl, V., and Musolff, L. (2021). Detection of collusive networks in e-procurement. *Available at SSRN 3864186*.

[43]In this sense, our findings are closer to Ganuza et al. (2024), which discusses manipulation for pure discretion effects without disentangling its form but offering different characteristics of public officials. However, the argument is different from the one proposed by Bosio et al. (2022) and Coviello et al. (2022) who claim that these effects come from institutional conditions within countries, or from Carril (2021) who argues that manipulation reduces abandoned acquisitions.

Bertanha, M., Caetano, C., Jales, H., and Seegert, N. (2023). Bunching estimation methods. Technical report, Tech. rep., Mimeo.

Best, M. C., Hjort, J., and Szakonyi, D. (2023). Individuals and organizations as sources of state effectiveness. *American Economic Review*, 113(8):2121–2167.

Bosio, E., Djankov, S., Glaeser, E., and Shleifer, A. (2022). Public procurement in law and practice. *American Economic Review*, 112(4):1091–1117.

Brogaard, J., Denes, M., and Duchin, R. (2021). Political influence and the renegotiation of government contracts. *The Review of Financial Studies*, 34(6):3095–3137.

Caires, F. B., Peralta, S., and Mendes, D. (2023). Contract splitting in public procurement. *SSRN Electronic Journal*.

Cappelletti, M., Giuffrida, L. M., Heaton, S., and Siegel, D. S. (2023). Strategic management in public procurement: The role of dynamic capabilities in equity and efficiency. *ZEW-Centre for European Economic Research Discussion Paper*, (23-035).

Carril, R. (2021). *Rules versus discretion in public procurement*. GSE, Graduate School of Economics.

Carril, R., Gonzalez-Lira, A., and Walker, M. S. (2022). Competition under incomplete contracts and the design of procurement policies.

Che, Y.-K. (1993). Design competition through multidimensional auctions. *The RAND Journal of Economics*, pages 668–680.

Chetty, R., Guren, A., Manoli, D., and Weber, A. (2011). Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *American Economic Review*, 101(3):471–75.

Coles, J. L., Patel, E., Seegert, N., and Smith, M. (2022). How do firms respond to corporate taxes? *Journal of Accounting Research*, 60(3):965–1006.

Coviello, D. and Gagliarducci, S. (2017). Tenure in office and public procurement. *American Economic Journal: Economic Policy*, 9(3):59–105.

Coviello, D., Guglielmo, A., Lotti, C., and Spagnolo, G. (2022). Procurement with manipulation.

Coviello, D., Guglielmo, A., and Spagnolo, G. (2018). The effect of discretion on procurement performance. *Management Science*, 64(2):715–738.

Coviello, D. and Mariniello, M. (2014). Publicity requirements in public procurement: Evidence from a regression discontinuity design. *Journal of Public Economics*, 109:76–100.

De Michele, R. and Pierri, G. (2020). Transparency and digital government: The impact of compr. ar in argentina.

Decarolis, F. (2014). Awarding price, contract performance, and bids screening: Evidence from procurement auctions. *American Economic Journal: Applied Economics*, 6(1):108–132.

Decarolis, F. (2018). Comparing public procurement auctions. *International Economic Review*, 59(2):391–419.

Decarolis, F., de Rassenfosse, G., Giuffrida, L. M., Iossa, E., Mollisi, V., Raiteri, E., and Spagnolo, G. (2021). Buyers' role in innovation procurement: Evidence from us military r&d contracts. *Journal of Economics & Management Strategy*, 30(4):697–720.

Decarolis, F., Fisman, R., Pinotti, P., and Vannutelli, S. (2020). Rules, discretion, and corruption in procurement: Evidence from italian government contracting. Working Paper 28209, National Bureau of Economic Research.

Ganuza, J.-J., Llobet, G., and Aspuru, P. (2024). Good and bad discretion in procurement. *mimeo*.

Gerardino, M. P., Litschig, S., and Pomeranz, D. (2024). Distortion by audit: Evidence from public procurement. *American Economic Journal: Applied Economics*, 16(4):71–108.

Hungerman, D. M. and Ottoni-Wilhelm, M. (2021). Impure impact giving: Theory and evidence. *Journal of Political Economy*, 129(5):1553–1614.

Kang, K. and Miller, R. A. (2022). Winning by default: Why is there so little competition in government procurement? *The Review of Economic Studies*, 89(3):1495–1556.

Khandelwal, A. (2010). The Long and Short (of) Quality Ladders. *The Review of Economic Studies*, 77(4):1450–1476.

Kleven, H. J. (2016). Bunching. *Annual Review of Economics*, 8:435–464.

Kleven, H. J. and Waseem, M. (2013). Using notches to uncover optimization frictions and structural elasticities: Theory and evidence from pakistan. *The Quarterly Journal of Economics*, 128(2):669–723.

Laffont, J.-J. and Tirole, J. (1993). *A theory of incentives in procurement and regulation*. MIT press.

Lewis-Faupel, S., Neggers, Y., Olken, B. A., and Pande, R. (2016). Can electronic procurement improve infrastructure provision? evidence from public works in india and indonesia. *American Economic Journal: Economic Policy*, 8(3):258–283.

Liebman, J. B. and Mahoney, N. (2017). Do expiring budgets lead to wasteful year-end spending? evidence from federal procurement. *American Economic Review*, 107(11):3510–3549.

Lisa Chever, S. S. and Yvrande-Billon, A. (2017). The law of small numbers: investigating the benefits of restricted auctions for public procurement. *Applied Economics*, 49(42):4241–4260.

McCrary, J. (2008). Manipulation of the running variable in the regression discontinuity design: A density test. *Journal of Econometrics*, 142(2):698–714. The regression discontinuity design: Theory and applications.

Menezes, F. M. and Monteiro, P. K. (2004). *An introduction to auction theory*. OUP Oxford.

OECD (2016). *Towards Efficient Public Procurement in Colombia*.

Palguta, J. and Pertold, F. (2017). Manipulation of procurement contracts: Evidence from the introduction of discretionary thresholds. *American Economic Journal: Economic Policy*, 9(2):293–315.

Pollinger, S. (2023). Kinks know more: Policy evaluation beyond bunching with an application to solar subsidies.

Ryan, N. (2020). Contract enforcement and productive efficiency: Evidence from the bidding and renegotiation of power contracts in india. *Econometrica*, 88(2):383–424.

Saez, E. (2010). Do taxpayers bunch at kink points? *American Economic Journal: Economic Policy*, 2(3):180–212.

Schoenherr, D. (2019). Political connections and allocative distortions. *The Journal of Finance*, 74(2):543–586.

Szucs, F. (2023). Discretion and favoritism in public procurement. *Journal of the European Economic Association*.

Tukiainen, J., Blesse, S., Bohne, A., Giuffrida, L. M., Jääskeläinen, J., Luukinen, A., and Sieppi, A. (2023). What are the priorities of bureaucrats? evidence from conjoint experiments with procurement officials.

Warren, P. L. (2014). Contracting officer workload, incomplete contracting, and contractual terms. *The RAND Journal of Economics*, 45(2):395–421.

A Appendix

A.1 Descriptive Statistics

Tables

Table 2: Descriptive statistics for buyers

Statistic	Mean	St. Dev.	Min	Max	N
Budget	1.250	4.355	0.000	37.500	146
Budget growth rate	5.240	13.675	−77.552	32.331	144
Entity power:					
Executive	0.808	0.395	0	1	146
Autonomous Corporation	0.144	0.352	0	1	146
Judicial	0.034	0.182	0	1	146
Legislative	0.014	0.117	0	1	146
Entity sector:					
Defense	0.137	0.345	0	1	146
Sustainable development	0.068	0.253	0	1	146
Education	0.062	0.241	0	1	146
Finance	0.062	0.241	0	1	146
Law and Justice	0.062	0.241	0	1	146
Health and Social Protection	0.055	0.228	0	1	146

Note: The entity sector only includes those with a share larger than 5%. Other sectors include Agriculture, Science and Technology, Sports, National statistics, Mines and energy, Labour, among others. Autonomous corporation refers to as the regional corporations for sustainable development created in Colombia as public entities entrusted by law with the administration of the environment and renewable natural resources. Budget is in Trillions of Colombian pesos.

Table 3: Descriptive statistics for contracts

Variables	Mean	St. Dev.	Min	Max	N
Estimated Contract Value	1,661.336	33,559.170	0.000	3,362,240.000	115,708
Final Contract Value	576.480	9,956.330	0.000	988,369.800	115,708
Procurement format:					
Reverse Auction	0.818	0.386	0	1	115,708
Restricted Scoring Rule	0.140	0.347	0	1	115,708
Public Auction	0.042	0.201	0	1	115,708
Contract type:					
Supplies	0.482	0.500	0	1	115,708
Services	0.451	0.498	0	1	115,708
Public works	0.067	0.250	0	1	115,708
Duration (months)	4.281	5.334	0.000	365.000	115,708
Number of bidders	3.618	5.310	0	189	115,708
Awarded	0.907	0.290	0	1	115,708

Note: This table provides the descriptive statistics for the main contract characteristics.

Table 4: Descriptive Statistics for contracts by auction format

Statistic	Mean	St. Dev.	Min	Max	N
Panel A: Reverse auction:					
Estimated Contract Value	37.826	658.67	0.000	82,810	94,612
Final Contract Value	50.839	1,508.32	0.000	337,664	94,612
Contract type:					
Supplies	0.530	0.499	0	1	94,612
Services	0.439	0.496	0	1	94,612
Public works	0.031	0.172	0	1	94,612
Duration (months)	3.771	4.411	0.000	365	94,612
Number of bidders	3.497	3.736	0	112	94,612
Awarded	0.950	0.217	0	1	94,612
Panel B: Restricted Scoring Rule:					
Estimated Contract Value	2,773.16	31,466.46	0.010	1,063,838	16,193
Final Estimated Contract Value	1,294.561	13,592.31	0.000	907,000	16,193
Contract type:					
Supplies	0.278	0.448	0	1	16,193
Services	0.522	0.500	0	1	16,193
Public works	0.199	0.400	0	1	16,193
Duration (months)	5.263	5.345	0.000	240	16,193
Number of bidders	3.482	6.593	0	144	16,193
Awarded	0.786	0.410	0	1	16,193
Panel C: Public Auction:					
Base price	29,317.79	149,947.8	1.120	3,362,240	4,903
Final value	8,348.06	40,227.1	0.090	988,369.8	4,903
Contract type:					
Supplies	0.234	0.424	0	1	4,903
Services	0.437	0.496	0	1	4,903
Public works	0.329	0.470	0	1	4,903
Duration (months)	10.882	12.245	0.000	123	4,903
Number of bidders	6.411	15.634	0	189	4,903
Awarded	0.469	0.499	0	1	4,903

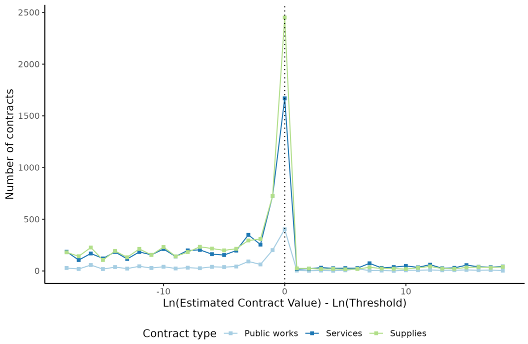
Note: This table provides the descriptive statistics for the main contract characteristics but disaggregated by auction type.

A.1.1 Graphs

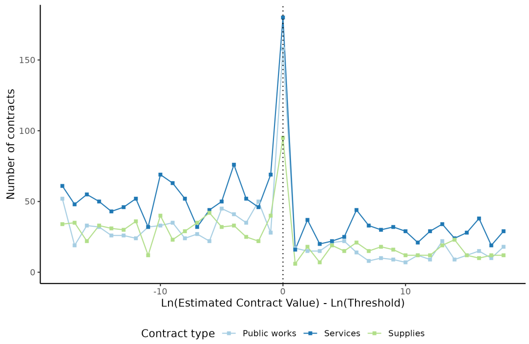
Figure 15: *Bunching Characterization*

Contract Type

(a) Reverse auction *vs* Restricted Scoring R.

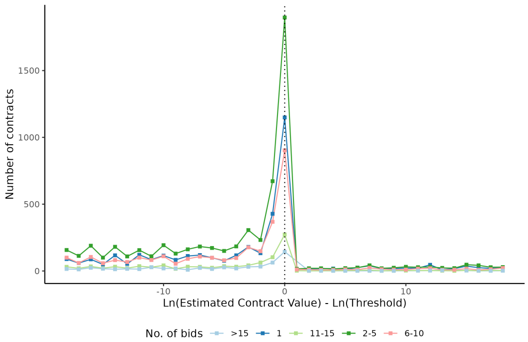


(b) Restricted Scoring R. *vs* Public Auction

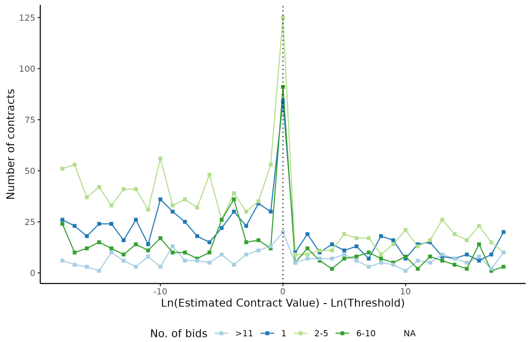


Number of bidders

(c) Reverse auction *vs* Restricted Scoring R.



(d) Restricted Scoring R. *vs* Public Auction



Notes: The Figures report the number of contracts in each bin by the number of bids. Seven groups were created in terms of the number of bids received. Bins group contracts in terms of the logarithm of the estimated contract value so that any bin includes contracts from both sides of the threshold.

A.2 Results

Point estimates

In this specification, excess mass is measured by the sum of coefficients related to the treatment group of buyers, T_{ij}, being below of the purchase thresholds, $below_{ij}$, and the interaction between them, $T_{ij} \times below_{ij}$. Since, the control group does not face any threshold at zero by the design of the law, we expect the coefficient that corresponds to below to be statistically not significant. The missing mass, is the coefficient that corresponds to T_{ij}, which measures the difference between the treatment and the control group while being above the purchase threshold. Finally, the double difference ensures that the interaction, $T_{ij} \times below_{ij}$, provides the net mass. In other words, the net impact of buyers' manipulation at the threshold value. In table 6 we present the same point estimated but for the minor amount threshold. Additionally, in table (7) we present the results of the estimations of equation (2) that represent the blue and red areas in both panels of figure (10). Finally, to account for the confidence intervals of the contracts in the treatment and control group, below and above the thresholds we plot the same figure 9 for the minimum amount under different specifications in figure 16 and for the minor amount in figure 17.

Table 5: Point Estimates for the Minimum Amount Threshold

	Number of contracts			
	(1)	(2)	(3)	(4)
Treatment	−100.710***	−98.015***	−111.594***	−108.892***
	(24.155)	(16.986)	(14.848)	(13.577)
Below	−12.514	534.148***	678.523***	763.803***
	(10.375)	(14.840)	(15.603)	(16.500)
Treatment × Below	427.485***	296.108***	302.157***	291.224***
	(25.543)	(18.079)	(15.793)	(14.444)
Constant	207.285***	−204.845***	−309.948***	−367.359***
	(40.275)	(30.032)	(26.975)	(25.164)
Polynomials	No	1	2	4
Controls	Yes	Yes	Yes	Yes
Observations	4,059	4,059	4,059	4,059
R^2	0.362	0.685	0.760	0.800
Adjusted R^2	0.356	0.682	0.758	0.798
Residual Std. Error	231.944	162.959	142.259	130.032
F Statistic	67.105***	230.374***	318.601***	382.063***

Notes: *p<0.1; **p<0.05; ***p<0.01. The fitted number of contracts, and $\hat{n}_b^T$, were computed estimating equation (1) using polynomials of different degrees x denoted in each row. The interaction terms with the polynomial were included. In the raw defined as *Observed* we compute equation (2) for the the observed values n_b^T.

Table 6: Point Estimates for the Minor Amount Threshold

	Number of contracts			
	(1)	(2)	(3)	(4)
Treatment	-33.071^{***}	-32.013^{***}	-32.035^{***}	-32.330^{***}
	(3.151)	(2.629)	(2.631)	(2.456)
Below	-5.031^{**}	35.868^{***}	35.467^{***}	48.755^{***}
	(1.954)	(2.786)	(2.944)	(3.663)
Treatment × Below	55.697^{***}	47.716^{***}	47.655^{***}	44.661^{***}
	(3.568)	(3.008)	(3.013)	(2.829)
Constant	38.768^{***}	23.557^{***}	24.053^{***}	13.066^{*}
	(7.932)	(6.669)	(6.774)	(6.659)
Polynomial	No	1	2	4
Controls	Yes	Yes	Yes	Yes
Observations	781	781	781	781
R^2	0.366	0.559	0.559	0.617
Adjusted R^2	0.338	0.539	0.539	0.598
Residual Std. Error	20.343	16.967	16.977	15.849
F Statistic	13.046^{***}	27.843^{***}	27.023^{***}	32.349^{***}

Notes: *p<0.1; **p<0.05; ***p<0.01. The fitted number of contracts, and $\hat{n}_b^T$, were computed estimating equation (1) using polynomials of different degrees x denoted in each row. The interaction terms with the polynomial were included. In the raw defined as *Observed* we compute equation (2) for the the observed values n_b^T.

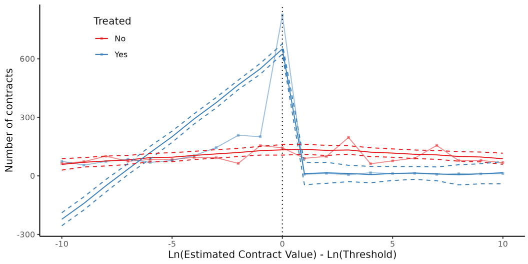
Table 7: Excess and Missing Mass Around Both Thresholds

	Minimum Amount						*Minor Amount*					
	Excess mass		*Missing mass*		*Net mass*		*Excess mass*		*Missing mass*		*Net mass*	
	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
Obs.	931	60.1	889	57.4	-42	-2.7	87	27	154	48	77	21
x=4	903	53.5	995	58.9	92	5.4	87	27	154	48	77	21
x=6	950	58.4	953	58.6	3	0.2	87	27	154	48	77	21
x=8	922	58.9	902	57.5	-21	-1.4	87	27	154	48	77	21

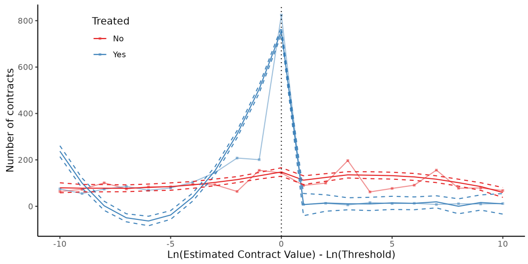
Notes: The table reports the results of estimating equation (2). The fitted number of contracts, and $\hat{n}_b^T$, were computed estimating equation (1) using polynomials of different degrees x denoted in each row. The interaction terms with the polynomial were included. In the raw defined as *Observed* we compute equation (2) for the the observed values n_b^T.

Figure 16: *Fitted Overall Manipulation around the Minimum Amount*

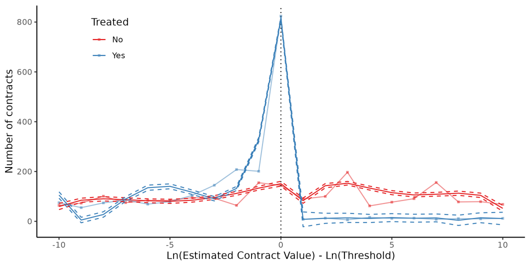
(a) Polynomial of degree 1



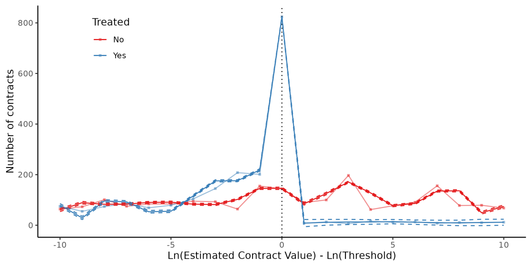
(b) Polynomial of degree 2



(c) Polynomial of degree 4



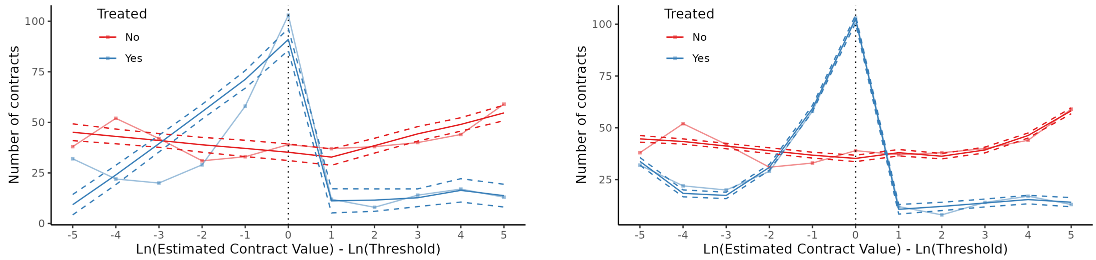
(d) Polynomial of degree 6



Notes: Both figures report the number of contracts offered by treatment and control group buyers by bin of estimated contract value around the *Minimum Amount* threshold that splits the process into a Reverse Auction and a Restricted Scoring Rule Auction. The estimated contract value is demeaned, thus bin zero is the bin that contains the purchase threshold value. The blue line plots the number of contracts offered by treatment buyers while the red line depicts the number of contracts offered by control buyers. The line a connection of dots, but the graphs are not continuous. In every graph there is plot the solid, blue and red, line that represents the actual number of contracts. Additionally, there is the an additional solid, blue and red, line that respresents the fitted number of contracts by different polynomial degrees (1,2,4 and 6). The dashed lines around these lines is the confidence intervals for the polynomial specifications.

Figure 17: *Fitted Overall Manipulation around the Minor Amount*

(a) Polynomial of degree 1 (b) Polynomial of degree 2

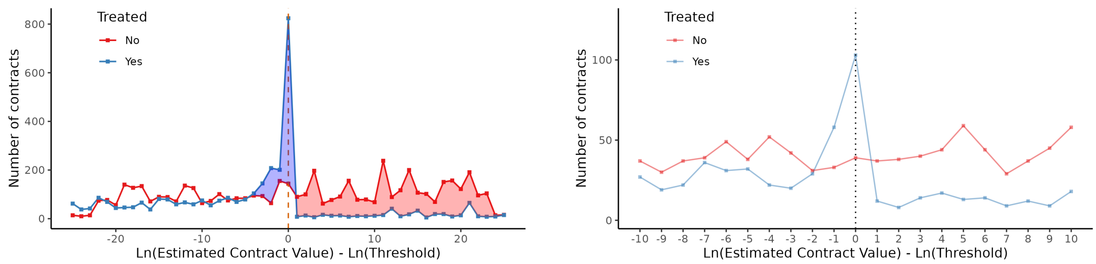


Notes: Both figures report the number of contracts offered by treatment and control group buyers by bin of estimated contract value around the *Minor Amount* threshold that splits the process into a Restricted Scoring Rule and a Public Auction. The estimated contract value is demeaned, thus bin zero is the bin that contains the purchase threshold value. The blue line plots the number of contracts offered by treatment buyers while the red line depicts the number of contracts offered by control buyers. The line a connection of dots, but the graphs are not continuous. In every graph there is plot the solid, blue and red, line that represents the actual number of contracts. Additionally, there is the an additional solid, blue and red, line that respresents the fitted number of contracts by different polynomial degrees (1 and 2). The dashed lines around these lines is the confidence intervals for the polynomial specifications.

Overall Manipulation

Figure 18: *Overall Manipulation*

(a) Reverse auction *vs* Restricted Scoring R. (b) Restricted Scoring R. *vs* Public Auction



Notes: Both figures report the number of contracts offered by treatment and control group buyers by bin of estimated contract value. Panel (a) contains contracts with estimated value around the *Minimum Amount* threshold that splits the process into a Reverse Auction and a Restricted Scoring Rule Auction. Panel (b) contains contracts with estimated value around the *Minor Amount* threshold that splits the process into a Restricted Scoring Rule Auction and Public Auction. The estimated contract value is demeaned, thus bin zero is the bin that contains the purchase threshold value. The blue line plots the number of contracts offered by treatment buyers while the red line depicts the number of contracts offered by control buyers. The line a connection of dots, but the graphs are not continuous. Blue and red areas represent only the difference between treatment and control groups bin by bin but not in a continuous format.

Contract shifting *vs* contract splitting

Table 8: Contract shifting *vs* Contract Splitting

	Excess mass		*Missing mass*		*Excess mass*		*Missing mass*	
Man. Form	No.	%	No.	%	No.	%	No.	%
Ct. Shifting	807	**52.1**	807	**52.1**	48	10.8	114	25.2
Ct. Splitting	124	**8**	82	**5.3**	-19	-4.2	40	8.8

Note: The table reports the excess, missing and net mass of observed contracts for split and non-split manipulated contracts. The fitted contracts are not reported since splitting contracts have not enough observations to have a correct prediction.

Impact of manipulation on procurement outcomes

Table 9: Manipulation Impact on Outcomes Around the Minimimum Amount (1)

	1 Bidder	Bidders	ΔValue	Repeated Winner
	(1)	(2)	(3)	(4)
Treatment	0.100*	−1.038*	−0.106*	0.158***
	(0.051)	(0.570)	(0.060)	(0.053)
Below	−0.021	0.384*	−0.003	0.039*
	(0.019)	(0.213)	(0.023)	(0.020)
Treatment x Below	−0.054	0.530	0.012	−0.017
	(0.047)	(0.525)	(0.056)	(0.049)
Constant	0.252***	4.657***	0.334***	0.023
	(0.085)	(0.948)	(0.100)	(0.089)
Polynomial	No	No	No	No
Controls	Yes	Yes	Yes	Yes
Observations	4,059	4,059	4,059	4,059
R^2	0.047	0.076	0.037	0.075
Adjusted R^2	0.038	0.067	0.028	0.067
Residual Std. Error	0.423	4.736	0.502	0.443
F Statistic	5.236***	8.707***	4.051***	8.614***

Notes: *p<0.1; **p<0.05; ***p<0.01. The table reports the results of estimating equation (1) with and without polynomials of different degrees and the interaction terms with the polynomial were included. The outcome variable that we use are: (1) the share of one bidder, (2) the number of bidders, (3) difference between final value paid and estimated value of a contract, (4) share of repeated winners.

Table 10: Manipulation Impact on Outcomes Around the Minimimum Amount (2)

	1 Bidder	Bidders	ΔValue	Repeated Winner	1 Bidder	Bidders	ΔPrices	Repeated Winner
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Treatment	0.180*	−2.637**	0.051	0.014	0.011	−3.507	1.142***	−0.516
	(0.108)	(1.206)	(0.128)	(0.113)	(0.359)	(4.018)	(0.425)	(0.377)
Below	−0.030	0.541	0.025	0.038	−0.067	1.930	0.301**	−0.039
	(0.038)	(0.425)	(0.045)	(0.040)	(0.124)	(1.381)	(0.146)	(0.130)
Treatment x Below	−0.141	2.127*	−0.087	0.074	0.130	0.680	−1.206***	0.584
	(0.110)	(1.229)	(0.130)	(0.115)	(0.363)	(4.060)	(0.430)	(0.381)
Constant	0.262***	4.899***	0.217**	0.116	0.239*	5.158***	−0.102	0.244*
	(0.080)	(0.894)	(0.095)	(0.084)	(0.136)	(1.521)	(0.161)	(0.143)
Polynomials	1	1	1	1	2	2	2	2
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4,059	4,059	4,059	4,059	4,059	4,059	4,059	4,059
R^2	0.045	0.075	0.037	0.073	0.046	0.079	0.042	0.075
Adjusted R^2	0.036	0.066	0.028	0.065	0.036	0.069	0.032	0.065
Residual Std. Error	0.423	4.740	0.502	0.444	0.423	4.732	0.501	0.444
F Statistic	4.931***	8.522***	4.097***	8.385***	4.581***	8.153***	4.143***	7.732***

Note: *p<0.1; **p<0.05; ***p<0.01. The table reports the results of estimating equation (1) with and without polynomials of different degrees and the interaction terms with the polynomial were included. The outcome variable that we use are: (1) the share of one bidder, (2) the number of bidders, (3) difference between final value paid and estimated value of a contract, (4) share of repeated winners.

Table 11: Manipulation Impact on Outcomes Around the Minimimum Amount (3)

	Small Firm	Estimated Duration	Process Duration	Small Firm	Estimated Duration	Process Duration	Small Firm	Estimated Duration	Process Duration
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8) (9)	
Treatment	−0.001	−0.538	−1.296	−0.102	0.355	−0.420	0.057	−3.998	7.804
	(0.051)	(0.426)	(2.639)	(0.109)	(0.879)	(5.674)	(0.363)	(2.933)	(19.234)
Below	0.004	0.059	0.969	0.019	0.122	4.433**	−0.119	−0.953	9.929
	(0.019)	(0.159)	(0.985)	(0.038)	(0.310)	(1.983)	(0.125)	(1.008)	(6.377)
Treatment x Below	−0.047	−0.266	−1.738	0.077	−1.898**	−2.378	−0.101	2.983	−16.191
	(0.047)	(0.392)	(2.434)	(0.111)	(0.896)	(5.777)	(0.367)	(2.963)	(19.425)
Constant	0.872***	1.293**	15.741***	0.791***	3.487***	13.539***	0.896***	4.304***	11.739*
	(0.085)	(0.615)	(3.817)	(0.081)	(0.651)	(4.175)	(0.137)	(1.110)	(7.054)
Polynomials	No	No	No	1	1	1	2	2	2
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4,059	4,059	3,688	4,059	4,059	3,688	4,059	4,059	3,688
R^2	0.054	0.137	0.032	0.052	0.179	0.032	0.053	0.180	0.034
Adjusted R^2	0.045	0.129	0.022	0.043	0.171	0.022	0.043	0.171	0.023
Residual Std. Error	0.427	3.541	20.994	0.427	3.455	20.994	0.427	3.454	20.991
F Statistic	6.017***	18.236***	3.413***	5.759***	23.069***	3.221***	5.386***	20.991***	3.035***

Notes: *p<0.1; **p<0.05; ***p<0.01. The table reports the results of estimating equation (1) with and without polynomials of different degrees and the interaction terms with the polynomial were included. The outcome variable that we use are: in (1), (4) and (7) the share of small firms, in (2), (5) and (8) the estimated duraction of a contract, and in (3), (6) and (9) the actual duration of a contract.

Table 12: Manipulation Impact on Outcomes Around the Minimimum Amount (4)

	%Modified Contracts (1)	Nb Modified Contracts (2)	Additional Days (3)	Last Week (4)	%Modified Contracts (5)	Nb Modified Contracts (6)	Additional Days (7)	Last Week (8)
Treatment	0.328***	0.290**	0.037	−0.003	−0.118	−0.102	0.002	0.014
	(0.117)	(0.116)	(0.105)	(0.023)	(0.102)	(0.101)	(0.091)	(0.020)
Below	0.066	0.006	0.121***	0.007	0.179***	0.079	0.196***	0.028**
	(0.044)	(0.043)	(0.039)	(0.009)	(0.066)	(0.065)	(0.059)	(0.013)
Treatment x Below	0.076	−0.008	−0.133	0.010	0.024	−0.048	−0.150	0.007
	(0.108)	(0.106)	(0.097)	(0.021)	(0.108)	(0.107)	(0.097)	(0.021)
Constant	−0.649***	−0.546***	−0.146	0.044	−0.046	−0.012	−0.095	0.006
	(0.194)	(0.192)	(0.175)	(0.038)	(0.173)	(0.171)	(0.155)	(0.034)
Polynomials	No	No	No	No	1	1	1	1
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4,063	4,063	4,063	3,689	4,063	4,063	4,063	3,689
R^2	0.066	0.040	0.039	0.031	0.066	0.040	0.039	0.031
Adjusted R^2	0.057	0.031	0.030	0.021	0.057	0.031	0.030	0.021
Residual Std. Error	0.971	0.961	0.875	0.182	0.971	0.961	0.875	0.182
F Statistic	7.520***	4.394***	4.281***	3.047***	7.520***	4.394***	4.281***	3.047***

Notes: *p<0.1; **p<0.05; ***p<0.01. The table reports the results of estimating equation (1) with and without polynomials of different degrees and the interaction terms with the polynomial were included. The outcome variable that we use are: in (1) and (5) the share of modified contracts, in (2) and (6) number of contracts, in (3) and (7) additional days and (4) and (8) contracts signed in the last week of the civil year.

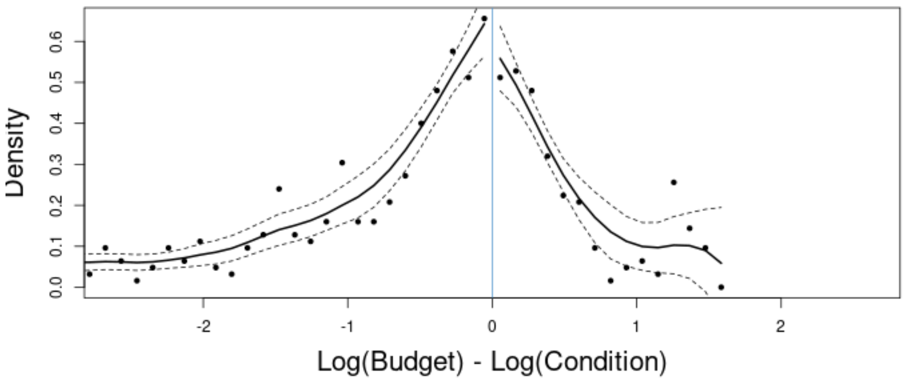
Robustness checks

Table 13: Buyers Covariates

Variables	Control	Treated
log(Budget)	28.192	24.922
	(0.8886)	(1.9491)
Executive	0.865	0.663
	(0.3418)	(0.4728)
Judicial	0.102	0.069
	(0.3033)	(0.2544)
Sector: Defense	0.286	0.204
	(0.4521)	(0.4027)
Sector: Environment	0.023	0.242
	(0.1504)	(0.4284)
Sector: Justice	0.17	0.069
	(0.3761)	(0.2544)
Sector: Labour	0.186	0.01
	(0.3894)	(0.0988)
Sector: Transport	0.168	0.006
	(0.3744)	(0.0798)

Notes: The table reports means and standard deviation for the variables used for buyers in treatment and control groups.

Figure 19: Non-Discontinuity in Buyers Budget



Notes: The figure show buyers continuity in terms of distance between the logaritm of buyers' annual budget and the condition that allocates them in different categories.

A.3 Purchases of buyers in category five

Description of the estimation

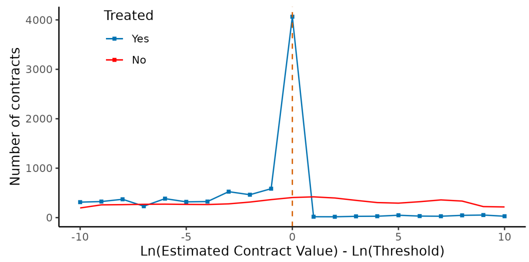
As mentioned, bunching estimations have difficulties to estimate extensive margin effects. Therefore, we rely on the design of the law to obtain counterfactual of category five. The identification assumption relies on the idea that the behavior below and above the threshold is not category specific. In other words, the reaction to the threshold needs to be similar.

We estimate several polynomials on the contracts offered by the counterfactual buyers below and above the thresholds on the manipulation window and three times the manipulation window.[44] We do not extend more the distribution, because the polynomials start to assume extreme values. Once we obtain the estimated values of these polynomials, we re-scale these estimations by a coefficient that captures the ratio between the amount of contracts offered by category five of buyers and the amount of contracts offered by the pool of categories two to five (buyers in the control group).

Once obtained this parametrical counterfactual, we repeat the estimation in 3.2.2 where the control group contracts are those countefactual contracts obtained by the polynomial and re-scaled. The results are just below in the next subsection. We present the main results of this estimation only for the *Minimum Amount* threshold, where there is few contract splitting that could challenge the main result. Since, we have already stated that contract splitting seems more prevalent for the *Minor Amount* this estimation strategy becomes less relevant.

Results

Figure 20: *Category Five*



Notes: The figure reports the number of contracts offered by treatment and control group buyers by bin of estimated contract value. It contains contracts with estimated value around the *Minimum Amount* threshold that splits the process into a Reverse Auction and a Restricted Scoring Rule Auction. The estimated contract value is demeaned, thus bin zero is the bin that contains the purchase threshold value. The blue line plots the number of contracts offered by treatment buyers while the red line depicts the number of contracts offered by control buyers. The line a connection of dots, but the graphs are not continuous. The control group is build according to the specifications in subsection 3.2.4.

[44] The manipulation window is the one obtained through the McCrary test and represents ten bins of difference between the logarithm of the estimated value and the logarithm of the threshold. Three times the manipulation window represents thirty bins.

Table 14: Excess and Missing Mass for buyers of category five

	Excess mass $\hat{m}$		*Missing mass* $\hat{m}'$	
	No.	%	No.	%
x = 3	4758	74.6	2900	45.5
x = 6	4752	74.5	2894	45.4
x = 9	4764	74.7	2905	45.6

The table reports the results of estimating equations (2). The fitted number of contracts were computed estimating equation (1) using polynomials of different degrees x denoted in each row. The control group is build according to the specifications in subsection 3.2.4.

A.4 Model

Proof of *Lemma 1*.

Consider the Reverse First Price Auction. We start the search for a symmetric Bayesian Nash equilibrium by adopting the point of view of one bidder, f. We denote $-f$ as every other bidder that is not f. and $c_m = \min\{c_{-f}, \forall - f\}$. This bidder has marginal cost c_f and he chooses the price p in order to maximize his utility, (3), under the auction auction RA. Let's assume that $p(\cdot)$ is symmetric, strictly increasing on c and differentiable. Note that bidding either 0 or 1 is strictly dominated. Then it must exist a value $x \in (0, 1)$ such that $p(x) = p_f$. In other words, the problem stated in equation (5) for RA is now equivalent to:

$$\max_x (p(x) - c_f y_0) \quad P(p(x) \leq p(c_m)). \tag{13}$$

Since the strategy $p(\cdot)$ is symmetric, differentiable and monotonically increasing $P(p(x) \leq p(c_m)) = P(x \leq c_m)$.[45] Before maximizing we can write explicitly the probability of the minimum using the statistic of order for every other $n-1$ bidders,

[45]We can show that every property stated for $p(x)$ is true after computing the optimal price. The strategy is symmetric since every bidder faces the same maximization program. The proof for differentiable and monotonically increasing are omitted because they are standard. To show that $p(x)$ is increasing we need to differentiate (24) with respect to c_f. For differentiability would be enough to show that $p(c_f)$ is continuous at $c_f = 1$. For a detailed exposition of independent private values (IPV) auction models check Menezes and Monteiro (2004).

$P(x \leq c_m) = (1 - F(x))^{(n-1)}$. Where $F(x)$ is the cumulative uniform distribution. Taking the first derivative we obtain:

$$\frac{\partial U_f}{\partial x} = -(p(x) - c_f y_0)(n-1)f(x)(1-F(x))^{n-2} + p'(x)(1-F(x))^{n-1}. \tag{14}$$

Note that in a symmetric equilibrium, the expression is maximized at $x = c_f$.[46] Thus, we can rewrite the first order condition as:

$$\frac{\partial U_f}{\partial x} = 0 \Longleftrightarrow \left[p(c_f)(1-F(c_f))^{n-1}\right]' = -c_f(n-1)f(c_f)y_0(1-F(c_f))^{n-2}. \tag{15}$$

Using the Fundamental Theorem of Calculus we get:

$$p(c_f)(1-F(c_f))^{n-1} = -\int_{c_f}^{1} y_0 c_f (n-1) f(c_f)(1-F(c_f))^{n-2} dc_f + C. \tag{16}$$

Where C is a constant of integration.[47] This expression, for the uniform distribution with support in the interval $[0,1]$ yields the following expression:

$$p^*(c_f) = \begin{cases} \frac{1+c_f(n-1)}{n} y_0 & \text{if } c_f < 1, \\ 0 & \text{if } c_f = 1. \end{cases} \tag{17}$$

Which yields the expression in equation (6). In order to simplify notation in the main body of the article, we do not present the price strategy when $c_f = 1$. The buyer expected price will be then the lowest price:

$$\mathbb{E}_c(p) = y_0 \int_0^1 \frac{1 + x(n-1)}{n} \, n(1-F(x))^{n-1} f(x) dx. \tag{18}$$

Which yields the expression in equation (6).

Consider the Restricted Scoring Rule Auction. We start the search for a symmetric Bayesian Nash equilibrium by adopting the point of view of one bidder, f. Let's denote $\overline{s}$ as the score that makes one buyer win the auction. Note that we can write the utility of bidder f under a SR auction as follows:

$$\max_{\{p,q\}} (p - c_f(1+q))y_0, \quad \text{s.t } s(p,q) = \overline{s}. \tag{19}$$

[46] This is a direct revelation argument. The detail can be revised in capter 6 of Menezes and Monteiro (2004).

[47] The constant is equal to zero since the left hand side is bounded to zero when $c_f \to 1$.

Substituting the constraint in the maximization program, it becomes:

$$\max_{\{q\}}(\phi(q) - c_f(1+q) - \overline{s})y_0. \tag{20}$$

From equation (20) we see that the observable requirements choice is independent from the score: $q^* := \phi'^{-1}(c_f)$. Following Asker and Cantillon (2008) we can construct a pseudotype, $\theta(c_f)$ for bidder f as:

$$\begin{aligned}\theta(c_f) &:= \phi(q^*) - c_f(1+q^*) \\ &= \phi(\phi'^{-1}(c_f)) - c_f(1+\phi'^{-1}(c_f)).\end{aligned} \tag{21}$$

This pseudotype is decreasing on c_f.[48] This pseudotype is the amount of surplus that a bidder of type c_f can generate while choosing the optimal observable requirements q^*. Maximizing this surplus is the aim of the buyer, so this problem becomes equivalent to choosing the bidder with the highest bid, where this bid b is a function of the pseudotype. Additionally, assuming that the bidding strategy are symmetric for each bidder, this problem is equivalent to finding the bidder with the lowest cost subject to the optimal choice of quality.[49]

Note that:

$$\begin{aligned}P(b_f(\theta(c_f)) \geq b_{-f}(\theta(c_m))) &= P(b(\theta(c_f)) \geq b(\theta(c_m))) \textit{ by symmetry} \\ &= P(\theta(c_f) \geq \theta(c_m)) \textit{ by } b(\cdot) \textit{ increasing} \\ &= P(c_f \leq c_m) \textit{ by } \theta(\cdot) \textit{ decreasing.}\end{aligned} \tag{22}$$

Let's denote $b_M(\theta(c_{-f})) = \max\{b_{-f}(\theta(c_{-f})),\ \forall -f\}$. Thus, the two following programs become equivalent:

$$\max_{b_f(\theta(c_f))}(\theta(c_f) - \overline{s})P(b_f(\theta(c_f)) \geq b_M(\theta(c_{-f}))) \Longleftrightarrow \max_x(p(x) - c_f(1+q^*)y_0)P(x \leq c_{-f}). \tag{23}$$

[48]Computing the derivative we get:

$$\frac{\partial\theta(c_f)}{\partial c_f} = \frac{\partial\phi(q^*)}{\partial q^*}\frac{\partial q^*}{\partial c_f} - (1+q^*) - c_f\frac{\partial q^*}{\partial c_f}.$$

Note that $\frac{\partial\phi(q^*)}{\partial q^*} = \phi'(q^*) = \phi'(\phi'^{-1}(c_f)) = c_f$ and $\frac{\partial q^*}{\partial c_f} = \frac{1}{\phi''(q^*)}$. Replacing these values, we get:

$$\frac{\partial\theta(c_f)}{\partial c_f} = c_f\frac{1}{\phi''(q^*)} - (1+q^*) - c_f\frac{1}{\phi''(q^*)} = -(1+q^*) < 0.$$

Since $q^* = \phi'^{-1}(c_f)$ is always a positive function given that $\phi'(\cdot) > 0$.

[49]An explicit proof of the symmetry in bidders strategy is provided by Asker and Cantillon (2008).

The left hand side (LHS) requires $b(\theta(c_f)) = s(p,q)$. Focusing on the right hand side (RHS), the solution of this problem is almost identical to the one proposed by equation (13) and yields the expression:

$$p^*(c_f) = \begin{cases} \frac{1+c_f(\overline{n}-1)}{\overline{n}}(1+q^*)y_0 & \text{if } c_f < 1, \\ 0 & \text{if } c_f = 1. \end{cases} \tag{24}$$

Where $q^* := \phi'^{-1}(c_f)$ as showed above and yields the expression in equation (6). The buyer expected price will be then the lowest price:

$$\mathbb{E}_c(p) = y_0(1+q^*)\int_0^1 \frac{1+x(\overline{n}-1)}{\overline{n}}\,\overline{n}(1-F(x))^{\overline{n}-1}f(x)dx. \tag{25}$$

Which yields the expression in equation (6).

□

The proof is a direct conclusion from Theorem 1 of Asker and Cantillon (2008), which states that 'every equilibrium in the scoring auction is typewise equivalent to an equilibrium in the scoring auction where suppliers are constrained to bid only on the basis of their pseudotypes'.

Proof of *Lemma 2*.

The first part of the proof is the maximization of the pseudotype $\theta(c_f)$, it does not consider any score $\overline{s}$. The optimal requirements imposed by the buyer are identical to the optimal choice made by firms since buyer's surplus for observable requirements, $(1+\phi(q)-c_m(1+q))\,y_0$ and the winner bidder pseudotype, $(\phi(q)-c_m(1+q)-\overline{s})\,y_0$, are an affine transformation.

The second part of the proof is a direct comparison from the buyer's utility when implementing either RA or SR. That is,

$$V(y_0,q^*,\tau,n,\overline{n}) = \begin{cases} (1-\frac{2}{n+1})y_0 - \tau_1 & \text{if } RA \\ (1+\phi(q^*)-\frac{2}{\overline{n}+1}(1+q^*))y_0 - \tau_2 & \text{if } SR \end{cases} \tag{26}$$

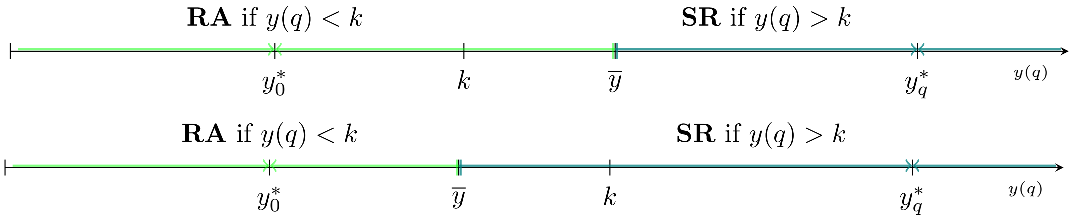
The y_0 that solves this equality is $\overline{y}$ in equation (8).

□

Policy interventions: irrelevant cases.

Note that if either $k > \overline{y} > y_0^*$ or $\overline{y} > k > y_0^*$ the threshold does not affect the decision. In the first case, this relies on the assumption that $y_0 \in [0, y_0^*]$.

Figure 21: Irrelevant Cases



Notes: The black line represents the size of the project $y(q)$. We depict four values: 1) purchase threshold k, 2) optimal social value without observable requirements y_0^*, 3) cutoff that characterize buyer behavior without any policy intervention $\overline{y}$ and, 4) optimal social value with observable requirements y_q^*. The green line represents buyer's optimal behavior after the implementation of the purchase threshold k. The light green represents purchases made with an RA auction while the dark green those with a SR auction.

Proof of *Lemma 3*.

Equation (9) is the direct comparison of buyer utility under the two different auctions when he cannot split contracts:

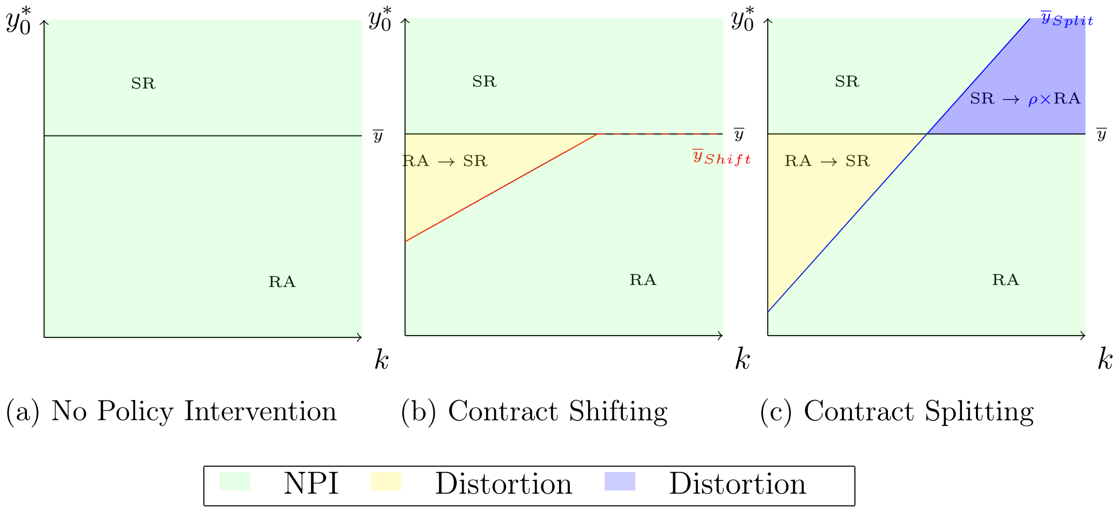
$$V(y_0, q^*, \tau, k, n, \overline{n}) = \begin{cases} (1 - \frac{2}{n+1})k - \tau_1 & \text{if } RA \\ (1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1 + q^*))y_0 - \tau_2 & \text{if } SR \end{cases} \quad (27)$$

Equation (10) is the direct comparison of buyer utility under the two different auctions when he can split a project in ρ contracts of smaller size, that is:

$$V(y_0, q^*, \tau, k, \rho, n, \overline{n}) = \begin{cases} \rho((1 - \frac{2}{n+1})k - \tau_1) & \text{if } RA \\ (1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1 + q^*))y_0 - \tau_2 & \text{if } SR \end{cases} \quad (28)$$

□

Figure 22: Buyer optimal purchase



(a) No Policy Intervention (b) Contract Shifting (c) Contract Splitting

Notes: the figure is depicted for $n = 20$, $\overline{n} = 10$ bidders a $\phi(q) = ln(1 + q)$ function, $q = 4$ observable requirements and transaction costs values of $\tau_1 = 3$ and $\tau_2 = 7$. RA and SR represent Reverse Auction and Restricted Scoring Rule respectively. Value k is the purchase threshold value and $\overline{y}, \overline{y}_{Shift}$ and $\overline{y}_{Split}$ are the cutoffs stated in *lemmas* 2 and 3.

Proof of *Proposition 1*.

Equation (11) is the solution for k of the equality between equations (9) and (10).

□

Proof of *Corollary 1*.

Let's start by computing the welfare in the following fifth cases: first best, no policy intervention, only contract shifting, only contract splitting and overall manipulation. Finally, we compute the differences.

For welfare, we consider the gross surplus that the purchase generate with and without observable requirements. We do not consider the transaction costs of the buyer, since they are private costs afforded by the buyer. Otherwise, the welfare analysis would reduce also to the results of *proposition 1*.

Let's compute each case. The first best welfare considers the case where the acquisition always contains observable requirements, since it generates higher gross surplus and the transaction costs are not directly paid by the society. Also, because by default the in public procurement purchases are implemented through an open sealed bid scoring auction.

$$W^{FB} = \int_0^{y_0^*} \left(1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1+q^*)\right) xdx. \tag{29}$$

As soon as the buyer has the opportunity to choose between two different auctions, the welfare for society becomes the following.

$$W^{NPI} = \int_0^{\overline{y}} \frac{n-1}{n+1} xdx + \int_{\overline{y}}^{y_0^*} \left(1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1+q^*)\right) xdx. \tag{30}$$

The difference between equation (29) and (30) establishes that there is always a loss in welfare by allowing the buyer to implement different auction formats, $W^{FB} - W^{NPI} = \left(1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1+q^*) - \frac{n-1}{n+1}\right) \int_0^{\overline{y}} xdx \geq 0$. This loss in welfare is the consequence of the agency problem of the buyer who is incurring in private transaction costs.

Let's consider the case when there is a policy intervention and the only possible reaction is contract shifting.

$$W^{Shift} = \int_0^{k} \frac{n-1}{n+1} xdx + \int_k^{\overline{y}_{Shift}} \frac{n-1}{n+1} kdx + \int_{\overline{y}_{Shift}}^{y_0^*} \left(1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1+q^*)\right) xdx. \tag{31}$$

Conversely, the welfare when the only potential reaction is to split contracts is:

$$\begin{aligned} W^{Split} = \int_0^{k} \rho \frac{n-1}{n+1} xdx + \int_k^{\overline{y}_{Split}} & \rho \frac{n-1}{n+1} kdx \\ & + \int_{\overline{y}_{Split}}^{y_0^*} \left(1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1+q^*)\right) xdx. \end{aligned} \tag{32}$$

Finally, we consider the case under overall manipulation. By *Proposition 1* we know that $\hat{k}$ makes contract shifting indifferent to contract splitting, we denote by $\hat{y} = \overline{y}_{Shift} = \overline{y}_{Split}$. Then, for each $y \in (0, \hat{y})$ the buyer prefers contract shifting, at $\hat{y}$ is indifferent between shifting and splitting, and when $y > \hat{y}$ he splits contracts. Thus, we can write the welfare of the overall manipulation as follows.

$$
\begin{aligned}
W^{OM} = &\int_0^k \frac{n-1}{n+1} x dx + \int_k^{\min\{\hat{y}, \overline{y}_{Shift}\}} \frac{n-1}{n+1} k dx \\
& + \mathbb{1}\left\{\overline{y}_{Shift} < \hat{y}\right\} \int_{\overline{y}_{Shift}}^{y_q^*} \left(1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1+q^*)\right) x dx \\
+ \left(1 - \mathbb{1}\left\{\overline{y}_{Shift} < \hat{y}\right\}\right) & \left(\int_{\overline{y}_{Shift}}^{\overline{y}_{Split}} \rho k \frac{n-1}{n+1} dx + \int_{\overline{y}_{Split}}^{y_0^*} \left(1 + \phi(q^*) - \frac{2}{\overline{n}+1}(1+q^*)\right) x dx \right).
\end{aligned} \tag{33}
$$

Now we can compare, how these forms of manipulation affect welfare in comparison to the no policy intervention case.

$$
\begin{aligned}
W^{NPI} - W^{Shift} = &\int_k^{\overline{y}_{Shift}} \frac{n-1}{n+1}(x-k) dx + \\
& \int_{\overline{y}_{Shift}}^{\overline{y}} \left(\frac{n-1}{n+1} - 1 - \phi(q^*) + \frac{2}{n+1}(1+q^*) \right) x dx.
\end{aligned} \tag{34}
$$

By *lemma 2* we know that $\left(\frac{n-1}{n+1}\right)\overline{y} - \tau_1 = \left(1 + \phi(q^*) - \frac{2}{n+1}(1+q^*)\right)\overline{y} - \tau_2$. The equality remains, if we integrate the expression on the variable $\overline{y}$ in the range $(0, \overline{y})$. Similarly, by *lemma 3*, $\frac{n-1}{n+1}k - \tau_1 = \left(1 + \phi(q^*) - \frac{2}{n+1}(1+q^*)\right)\overline{y}_{Shift} - \tau_2$. The equality remains, if we integrate the expression on the variable $\overline{y}^{Shift}$ in the range $(0, \overline{y}^{Shift})$. Thus, adding $(\tau_1 - \tau_1 + \tau_2 - \tau_2)\overline{y}$ and $(\tau_1 - \tau_1 + \tau_2 - \tau_2)\overline{y}^{Shift}$ and rearranging conveniently, we get:

$$
W^{NPI} - W^{Shift} = \frac{k^2}{2}\frac{n-1}{n+1} + (\tau_2 - \tau_1)(\overline{y}_{Shift} - \overline{y}) \leq 0. \tag{35}
$$

Note that $\overline{y}_{Shift} < \overline{y}$ and $k < \overline{y}$ since the k that makes $\overline{y}_{Shift} = \overline{y}$ is $k = \overline{y}$ and then contract shifting and NPI becomes the same problem. By replacing $(\overline{y}_{Shift} - \overline{y})$ by its expression, we are left with the following expression:

$$
\frac{k^2}{2}\frac{n-1}{n+1} - \frac{\frac{n-1}{n+1}}{1 + \phi(q^*) - \frac{2}{n+1}(1+q^*)} \left(k - \frac{(\tau_2 - \tau_1)^2}{1 + \phi(q^*) - \frac{2}{n+1}(1+q^*) - \frac{n-1}{n+1}} \right) \leq 0.
$$

Denota $A = 1 + \phi(q^*) - \frac{2}{n+1}(1+q^*) - \frac{n-1}{n+1}$ and $B = \frac{n-1}{n+1}$, we know that $B < 1 < A$ by assumption four. Therefore, $A - B > \lambda > 0$ and sufficiently big to make the SR implementable under NPI. Without loss of generality, for any $\alpha \in (0,1)$ we can write $A = 1 + \lambda\alpha$ and $B = 1 - \alpha\lambda$. The two roots that make this expression equal to zero are either one negative or the other one is higher than $\overline{y}$. Therefore, contract shifting is always welfare enhancing.

For contract splitting there are two cases if $\overline{y} \geq \overline{y}_{Split}$ or if $\overline{y} < \overline{y}_{Split}$. We show one, the reasoning is identical in the other case. Assume $\overline{y} \geq \overline{y}_{Split}$, the difference is:

$$W^{NPI} - W^{Split} = (1-\rho)\int_0^k \left(\frac{n-1}{n+1}x\right) dx + \int_k^{\overline{y}_{Split}} \left(\frac{n-1}{n+1}(x-\rho k)\right) dx + \int_{\overline{y}_{Split}}^{\overline{y}} \left(\frac{n-1}{n+1}x - \left(1+\phi(q^*) - \frac{2}{n+1}(1+q^*)\right) x\right) dx. \quad (36)$$

By *lemma 2* we know that $\left(\frac{n-1}{n+1}\right)\overline{y} - \tau_1 = \left(1+\phi(q^*) - \frac{2}{n+1}(1+q^*)\right)\overline{y} - \tau_2$. Similarly, by *lemma 3*, $\rho\left(\frac{n-1}{n+1}k - \tau_1\right) = \left(1+\phi(q^*) - \frac{2}{n+1}(1+q^*)\right)\overline{y}_{Split} - \tau_2$. Then, we can rewrite this expression as:

$$W^{NPI} - W^{Split} = \frac{\rho k^2}{2}\frac{n-1}{n+1} - (\tau_2 - \tau_1)\overline{y} + (\rho\tau_1 - \tau_2)\overline{y}_{Split}. \quad (37)$$

Replacing $\overline{y}$ and $\overline{y}_{Split}$ we can see that the quadratic equation on k has two roots. Additionally, writing $A = 1 + \alpha\lambda$ and $B = 1 - \alpha\lambda$ for any $\alpha \in (0,1)$ it is simple to see that the only positive root for k is larger than $\hat{k}$.[50] Let's denote this value that makes equation (37) equal to zero as $\tilde{k}$. This value might be larger or smaller than $\overline{k}$ depending on the value of the parameters.

The difference between shifting and splitting, using equations (36) and (37) becomes the following:

$$W^{Shift} - W^{Split} = \frac{(1-\rho)k^2}{2}\frac{n-1}{n+1} - (\tau_2 - \tau_1)\overline{y} + (\rho\tau_1 - \tau_2)\overline{y}_{Split}. \quad (38)$$

Which implies that shifting becomes welfare enhancing with respect to splitting for a value $k \in (\hat{k}, \tilde{k})$. Additionally, by simple inspection of equations (33) and (32) is clear that $W^{Split} - W^{OM} \geq 0$. In the same way, inspections of equations (33) and (31) shows that W^{OM} can only be greater than W^{Shift} for a $k > \tilde{k}$.

□

[50] We do not pay attention to the negative root since it has no economic meaning.