Coordinating Dividend Taxes and Capital Regulation*

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Abstract

We study the impact of state-contingent dividend taxes (and bans) and capital regulation on a firm's optimal strategy and value. In the model, the firm generates stochastic income under time-varying macroeconomic conditions. Its manager distributes dividends and issues costly equity to maximize shareholder value. We solve the manager's stochastic control problem and derive the firm's reserve distribution in closed form. Imposing dividend taxes (or bans) during crises generates a trade-off, as it encourages reserve accumulation in bad states but promotes payouts in good ones. Also, the policy undermines financial stability by reducing the firm's value and its recapitalization incentives across states. Coordinating dividend taxes with counter-cyclical capital regulation can mitigate value losses and ameliorate the trade-off, but it also creates additional recapitalization disincentives. (JEL: G32; G35; G38)

Keywords: Capital requirements; dividend bans; payout taxation; policy coordination; stochastic control.

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1 Introduction

At the height of the COVID-19 crisis, driven by the observation that banks did not adjust their dividends during the Great Financial Crisis (Acharya et al., 2011; Cziraki et al., 2024; Belloni et al., 2024), banking regulators worldwide recommended, and in some cases enforced, dividend restrictions.[1] These unprecedented measures aimed to preserve banks' credit capacity by ensuring adequate capital buffers and, more critically, preventing systemic defaults.

Empirical evidence suggests that the short-term outcome of these policies has been two-sided. Some studies argue that restrictions were indeed effective in improving banks' balance sheet conditions and, ultimately, in avoiding a credit crunch (Li et al., 2020; Hardy, 2021). Others provide evidence that dividend restrictions and their announcements have negatively impacted banks' equity valuations, because shareholders demanded higher returns for lower and delayed future proceeds (Andreeva et al., 2023; Sanders et al., 2024).

From a theoretical perspective, the problem of evaluating state-contingent dividend regulation, such as bans or, more broadly, dividend taxation, has only recently received attention (see Vadasz, 2022 and Kroen, 2022) and remains poorly understood. Analyzing these policies is complex, especially when considering their endogenous interaction with banks' optimal decisions and other regulations. This paper develops a theory to evaluate how state-contingent dividend taxes (or bans) influence firms' optimal recapitalization and dividend strategies in the short and long term and how they interact with capital requirements.

We model the optimal control problem of a regulated financial firm holding a fixed amount of loans and deposits.[2] Loans generate stochastic cash flows, whose expected returns and volatility depend on the aggregate state of the economy, as in Hackbarth et al. (2006).[3] The government imposes state-contingent dividend taxes and capital requirements. Similarly to Décamps et al. (2011) and Moreno-Bromberg and Rochet (2014), the firm's manager

[1] The ECB advised suspensions to start in March 2020; regular payments resumed in the fourth quarter of 2021. The FED imposed restrictions in June 2020, partially easing them in December 2020. The restrictions ended between June and July 2021, allowing banks to revert to pre-pandemic dividend policies.

[2] In principle, the firm can represent either a financial or a non-financial company. However, our model is particularly suited to represent financial firms, which are usually subject to capital requirements.

[3] Empirical evidence that cash flow uncertainty is a key determinant of firms' payout decisions can be found in Chay and Suh (2009).

decides whether to default or issue equity (and how much) at a cost when capital requirements become binding. She retains cash flows as reserves or pays dividends to avoid costly recapitalization, aiming to maximize shareholder value.

The first part of the paper solves the firm's stochastic control problem in closed form. First, we demonstrate that optimal payouts follow a threshold strategy. In particular, dividends get paid only when their marginal value –proportional to the state-contingent dividend tax rate – exceeds that of accumulating reserves. Otherwise, the firm takes no action. Adopting state-contingent dividend taxes is equivalent to imposing a dividend ban when the tax rate is high enough. Second, we show that recapitalizing the firm is optimal if its costs are sufficiently small ("incentive-compatible"). If so, when the capital constraint binds and there are no dividend taxes, the firm injects equity until reserves reach the dividend payout threshold. In the presence of dividend taxes, the optimal recapitalization target falls below the payout threshold. Third, we analytically derive the stationary distribution of the firm's reserves under the optimal strategy. We interpret this distribution as a measure of the firm's credit capacity ex-ante and use it to evaluate different policies in the long run.

The second part of the paper uses numerical analysis to investigate how dividend taxes and capital regulation affect the firm's optimal strategy and derive policy implications. Motivated by the COVID-19 policy case, we examine a scenario where dividend taxes are higher during a bad economic state, characterized by high cash flow volatility and low expected returns. To isolate the effects of dividend taxes, we consider a-cyclical capital requirements.

Consistent with its scope, the policy encourages reserve accumulation in the targeted state by raising the firm's optimal payout threshold. This happens because higher taxes lower the marginal value of dividends. At the same time, however, it reduces the firm's value not only in the bad state ("ex-post") but also in the good state ("ex-ante") as shareholders internalize the prospect of lower future returns. Accordingly, the firm finds it optimal to increase dividend payouts (i.e., reduce capital buffers) in the good state to compensate for these losses partially. These predictions warn that regulatory uncertainty regarding dividend restrictions may backfire by generating lower capital buffers (on this point, see Attig et al., 2021) in the long run and encourage counter-cyclical equity issuance strategies, as suggested by Baron (2020).

Related to these observations, the second finding of the analysis is that imposing higher dividend taxes (or, equivalently, dividend restrictions) during a bad macroeconomic state reduces the firm's optimal recapitalization targets across all states. Additionally, shareholder value losses lower the incentive-compatible cost threshold beyond which shareholders become unwilling to inject equity when capital constraints are binding. This result suggests that state-contingent tax policies could increase default risk, potentially amplifying stability concerns for the financial system.

Finally, we explore whether coordinating dividend taxes with cyclical capital buffers helps mitigate the adverse effects of the first policy. This analysis is particularly relevant because, during the COVID-19 crisis, regulators combined the recommendation to suspend dividends with more relaxed capital buffers, which Dursun-de Neef et al. (2023) argue was key to sustaining lending. According to our simulations, coordinating counter-cyclical capital regulation with dividend taxes can mitigate the adverse effects of the latter policy by redistributing value losses between states and reducing the dispersion of the firm's distribution in the long run. However, these benefits come at the cost of generating further disincentives for recapitalization.

The paper proceeds as follows. Section 2 situates the paper within the existing literature. Section 3 introduces the model, and Section 4 provides its analytical solution. Section 5 analyzes the model numerically and discusses its policy implications. Section 6 concludes.

2 Literature

From an empirical standpoint, several recent papers examined the (primarily short-term) effects of dividend restrictions during the COVID-19 crisis. For example, Andreeva et al. (2023) and Sanders et al. (2024) find that banks subject to the dividend suspension policy experienced a temporary drop in equity valuation but were able to increase their lending to the economy. Hardy (2021) shows that banks' CDS spreads declined after this measure, suggesting it improved their safety. A notable exception is Mücke (2023), showing that mutual funds permanently reduced their ownership in banks under payout restrictions. We complement this literature by developing a theory studying the joint effect of dividends and capital regulation on a firm's endogenous decisions in the short and long run.

To our knowledge, only a few papers have theoretically examined the effect of firms' dividend regulation. Goodhart et al. (2010) study the impact of dividend restrictions on an interbank market, showing that it may reduce defaults and improve welfare. Lindensjö and Lindskog (2020) solves the optimal control problem of a financial company facing dividend restrictions, finding that they may increase default risk. Unlike our work, these papers abstract from macroeconomic uncertainty.

Other related contributions include Vadasz (2022), which explores the ex-post intervention problem between a regulator and a bank in a two-period game, and Ampudia et al. (2023), which investigates dividend bans in a quantitative DSGE model with banks. Similar to our work, these papers highlight the trade-off between the benefits of increased lending and the losses in bank valuation due to the policy intervention. We differentiate substantially by studying the joint effect of dividends and capital regulation on the firm's optimal recapitalization decisions in a more standard corporate finance setting. In this respect, we connect with extensive literature studying optimal cash management of the firm, such as Décamps et al. (2011) and Gryglewicz (2011). We depart from these studies by considering macroeconomic uncertainty as in Hackbarth et al. (2006) (which does not consider dividends) and state-contingent capital regulation.

Methodologically, we build on the continuous-time stochastic control literature on dividends pioneered by Jeanblanc-Picqué and Shiryaev (1995). In particular, we tackle a problem featuring Markovian regime switching, as in Jiang and Pistorius (2012) and Ferrari et al. (2022). We consider endogenous equity issuance, as in Løkka and Zervos (2008). Our solution adopts a guess-and-verify approach, as in Sotomayor and Cadenillas (2011).

We set apart from these studies in three dimensions. First, we consider macroeconomic uncertainty in both expected cash flows (as in Reppen et al., 2020) and volatility. Second, we model both dividend taxes (bans) and capital regulation. Third, we derive (analytically) the stationary distribution of the reserve process under the firm's optimal strategy.

3 Model set up

Time is continuous and indexed by $t \in [0, \infty)$. As in Guo et al. (2005) and Hackbarth et al. (2006), we consider a firm subject to aggregate uncertainty on the state of the economy, modelled via a bi-variate Markov chain $I_t \in \{1, 2\}$, with transition intensity λ_{I_t}. A risk-neutral manager runs the firm in the best interest of its shareholders.

The firm holds a fixed amount of insured liabilities D (deposits) and assets A_t. Assets include a constant stock of illiquid loans L and time-varying liquid reserves X_t. The book value of the firm's equity at time t satisfies then the balance sheet identity

$$E_t + D = X_t + L. \tag{1}$$

Deposits yield the risk-free interest rate $\rho \geq 0$, while reserves are not remunerated, for simplicity. Loans generate operating cash flows according to the following stochastic differential equation:

$$\bar{\mu}_{I_t} dt + \sigma_{I_t} dW_t, \tag{2}$$

where W_t is a standard one-dimensional Brownian motion defined on some filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} := (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$. The drift and diffusion terms of (2) are contingent on the state of the economy I_t. In particular, we assume that $\bar{\mu}_1 \geq \bar{\mu}_2 > 0$ and $\sigma_2 \geq \sigma_1 > 0$ so that States 1 and 2 represent expansions (higher expected returns and lower volatility) and recessions (lower expected returns and higher volatility), respectively. We will thus refer to $I_t = 1$ as the "good" and to $I_t = 2$ as the "bad" state throughout the paper.

The cash flows can be retained as reserves or paid out as dividends. dZ_t denotes the time-t dividend payment and is the manager's first choice variable. The government taxes dividends depending on I_t. We denote as $\beta_{I_t} \in [0, 1]$ the after-tax value of a 1\$ dividend in State I_t. Consistent with the paper's motivation, we focus on counter-cyclical dividend tax schedules and set $\beta_1 = 1$ and $\beta_1 \geq \beta_2$.

As in Décamps et al. (2011), the firm is required by the regulator to hold sufficient equity to repay debtors fully in the case loans are liquidated at the fire-sale price $\alpha \in [0, 1]$. Provided that $D > \alpha L$, equity must be such that:

$$E_{R,I_t} \geq L(1-\alpha) + \Gamma_{I_t}. \tag{3}$$

The parameter Γ_{I_t} captures additional capital requirements (e.g., systemic capital buffers) when $\Gamma_{I_t} > 0$, or subsidies (e.g., state guarantees to cover default losses partially) when $\Gamma_{I_t} < 0$. By substituting (3) into (1) and rearranging, the capital requirement can be expressed as the following minimum reserves level:

$$X_t \geq D - \alpha L + \Gamma_{I_t} := x_{R,I_t}. \tag{4}$$

Since X_t is stochastic, (4) becomes occasionally binding at the random times $(\tau_n)_{n\geq 1}$. When that happens, the manager can either liquidate or re-capitalize the firm by issuing equity. The outcome is captured by the auxiliary variable b_n, taking values 0 (liquidation) or 1 (re-capitalization). This is the manager's second choice variable.

In the case of a liquidation, shareholders incur no cost but forgo all future dividends. The additional buffer $\Gamma^+_{I_t}$ is rebated to the shareholders. In the case of a recapitalization, shareholders provide the firm with new liquidity G_n and pay a fixed cost $\kappa \geq 0$.

For a given equity issuance schedule, liquid reserves up to a (possibly infinite) liquidation time $\tau_\ell := \inf\{\tau_n \geq 0 : \; b_n = 0\}$ evolve as follows:

$$\begin{cases} X_{\tau_n} = X_{\tau_n^-} + G_n, \qquad n \geq 0, \\ \\ dX_t = \underbrace{(\bar{\mu}_{I_t} - \rho D)}_{:=\mu_{I_t}} dt + \sigma_{I_t} dW_t - dZ_t, \qquad t \in [\tau_n, \tau_{n+1}), \end{cases} \tag{5}$$

with initial values $I_0 = i \in \{1, 2\}$ and $X_0 = x \geq \min\{x_{R,1}, x_{R,2}\}$.

Since reserves are not remunerated, the firm accumulates them (i.e., pays dividends) only to avoid costly recapitalization or liquidation. Formally, its strategy is a triple of $\mathbb{F}_t$-measurable stochastic processes $A := \big((Z_t)_{t\geq 0}, (b_n, G_n)_{n\geq 1}\big)$. For this strategy to be admissible, dividends cannot be negative or leave the firm with reserves below the capital requirement, and each re-capitalization must be strictly positive.[4]

Formally, the firm's gain functional is

$$J(x, i; A) = \mathbb{E}\left[\int_0^{\tau_\ell} e^{-\delta t}\beta_{I_t} dZ_t - \sum_{n=1}^{\ell-1} e^{-\delta\tau_n}(G_n + \kappa)\right], \forall A \in \mathcal{A}, \tag{6}$$

where δ is a discount rate and $\mathcal{A}$ denotes the set of admissible strategies, with the convention $\sum_{n=1}^{0} = 0$. The firm's value function (i.e., shareholder value) is given by

$$V(x, i) := \sup_{A\in\mathcal{A}} J(x, i; A) \tag{7}$$

$$s.t. \quad (5) \tag{8}$$

4 Model solution

The first part of this section derives the solution to the firm's optimal control problem analytically. For this purpose, we focus on the "baseline" case in which capital requirements are a-cyclical, i.e. not contingent on the state of the economy ($x_{R,1} = x_{R,2} = x_R$). We will consider cyclical capital requirements in Section 5.

The second part of the section derives and analyses the (stationary) distribution of the firm's reserves under the optimal strategy.

4.1 Shareholder value and optimal strategy

To tackle Problem (7), we follow a *guess-and-verify* approach.[5] More specifically, we formulate a set of optimality conditions for a candidate value function v based on heuristic considerations. Then, we use verification arguments to prove that $v = V$.

We expect the firm's value function to solve (in a suitable sense) the following system of

[4] A more formal definition of the decision policy and admissible strategies appears in the online appendix.

[5] This approach is the most used in the classical corporate finance literature starting from Leland (1994). For a *direct* approach to a similar problem, building on the theory of viscosity solutions and stating the optimality conditions as *necessary*, we refer to Akyildirim et al. (2014).

Hamilton-Jacobi-Bellman Variational Inequalities (HJBVI):

$$\max\left\{\mathcal{L}_i v(x,i) - \lambda_i\left[v(x,i) - v(x,3-i)\right], \beta_i - v'(x,i)\right\} = 0, \quad i = 1,2, \quad x > x_R, \tag{9}$$

where $v : [x_R, \infty) \times \{1,2\} \to \mathbb{R}$, and $\mathcal{L}_i$ are differential operators acting on functions $\phi \in C^2(x_R, \infty)$ as follows:

$$\mathcal{L}_i\phi = \frac{1}{2}\sigma_i^2\phi'' + \mu_i\phi' - \delta\phi, \quad i = 1,2.$$

Associated with a smooth enough solution to (9), there are the following *continuation* and *intervention* regions for $i = 1,2$:

$$\mathcal{C}_i := \{x > x_R : \quad v'(x,i) > \beta_i\}, \tag{10}$$

$$\mathcal{S}_i := \{x > x_R : \quad v'(x,i) = \beta_i\}. \tag{11}$$

Equipped with these objects, we conjecture that the optimal control has a threshold structure, as in Sotomayor and Cadenillas (2011), meaning that for each $i = 1,2$, there is a reserve level $\tilde{x}_i$ below which the firm does not pay dividends. Indeed, we expect that it is optimal to accumulate reserves as long as their marginal value ($v'(\cdot,i)$) exceeds that of dividends (β_i).

Formally, we guess that the above regions are $\mathcal{C}_i = (x_R, \tilde{x}_i)$ and $\mathcal{S}_i = [\tilde{x}_i, \infty)$ for $i = 1,2$, implying that

$$\tilde{x}_i = \inf\{x \geq x_R : v'(x,i) \leq \beta_i\}. \tag{12}$$

Next, we make some conjectures to construct a sufficiently smooth solution to (9). First, we assume that $v(\cdot,i) \in C([x_R,\infty)) \cap C^2((x_R,\infty))$. Second, we postulate that $x_R < \tilde{x}_1 < \tilde{x}_2$, based on the intuition that it is optimal to pay more dividends when assets yield higher returns and carry less uncertainty. Third, we impose appropriate boundary conditions at the regulatory threshold x_R. The boundary conditions determine whether recapitalization is optimal at x_R and, if so, the amount of equity to issue.

If the manager liquidates the firm, shareholders receive the maximum between the capital buffer and zero (Γ_i^+). Otherwise, the optimal recapitalization policy ($\hat{G}$) must be feasible and "incentive-compatible", i.e. its benefits must be larger than or equal to its costs. Therefore, we require that $\hat{G} = \operatorname{argmax}\{v(x_R + G; i) - G - \kappa\}$, with $v(x_R + \hat{G}, i) \geq \hat{G} + \kappa$ and $\hat{G} \in \mathcal{G} := [0, \tilde{x}_i - x_R]$. As either liquidation or recapitalization must be optimal, we impose that

$$v(x_R, i) = \max\left\{\max_{G\in\mathcal{G}}\{v(x_R + G, i) - G - \kappa\}, \Gamma_i^+\right\} \qquad \text{for } i = 1, 2. \tag{13}$$

We now construct a function that meets all these conditions.

In the intervals $[\tilde{x}_i, \infty)$, the function must satisfy $v'(\cdot, i) = \beta_i$. In the interval $(\tilde{x}_1, \tilde{x}_2)$, we define $v'(\cdot, 2)$ as the unique solution to

$$\frac{1}{2}\sigma_2^2 v'''(x, 2) + \mu_2 v''(x, 2) - (\delta + \lambda_2)\, v'(x, 2) + \lambda_2 = 0, \tag{14}$$

with boundary conditions $v'(\tilde{x}_2, 2) = \beta_2 > 0$ (optimality condition), and $v''(\tilde{x}_2, 2) = 0$ (super contact condition, see Dumas, 1991). In the interval $(x_R, \tilde{x}_1)$, we find the functions $v'(\cdot, i)$ as unique solutions to the following system:

$$\begin{cases} \frac{1}{2}\sigma_1^2 v'''(x, 1) + \mu_1 v''(x, 1) - (\delta + \lambda_1)\, v'(x, 1) + \lambda_1 v'(x, 2) = 0, \\ \\ \frac{1}{2}\sigma_2^2 v'''(x, 2) + \mu_2 v''(x, 2) - (\delta + \lambda_2)\, v'(x, 2) + \lambda_2 v'(x, 1) = 0. \end{cases} \tag{15}$$

with boundary conditions $v'(\tilde{x}_1, 1) = 1$ (optimality condition), $v''(\tilde{x}_1, 1) = 0$ (super-contact condition), $v'(\tilde{x}_1^-, 2) = v'(\tilde{x}_1^+, 2)$, and $v''(\tilde{x}_1^-, 2) = v''(\tilde{x}_1^+, 2)$ (continuity conditions).

Solving (14) and (15) for $v'(\cdot, i)$ and integrating over the corresponding intervals yields the following.

Proposition 1 (Solution to the HJBVI system). *Recall that $\delta > 0$, $\lambda_i > 0$, $\mu_1 \geq \mu_2$, $\sigma_2 \leq \sigma_1$ and assume that $\beta_2 > \lambda_2/(\lambda_2 + \delta)$. Then, we have the following claims.*

1. *Fix $\tilde{x}_1, \tilde{x}_2$ such that $x_R < \tilde{x}_1 < \tilde{x}_2$ and define*

$$v(x, 1) = K_1 + \begin{cases} (x - \tilde{x}_1), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^{4} A_j \left(e^{\alpha_j (x - \tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1), \end{cases} \tag{16}$$

$$v(x,2) = K_2 + \begin{cases} \beta_2(x-\tilde{x}_2), & x \in [\tilde{x}_2, \infty), \\ \frac{\lambda_2(x-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{j=1}^2 \tilde{A}_j \left(e^{\tilde{\alpha}_j(x-\tilde{x}_1)} - e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}\right), & x \in [\tilde{x}_1, \tilde{x}_2), \\ \frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{j=1}^2 \tilde{A}_j(1-e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}) + \sum_{j=1}^4 B_j \left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1), \end{cases} \tag{17}$$

where $\alpha_j < \alpha_2 < 0 < \alpha_3 < \alpha_4$ *and* $\tilde{\alpha}_1 < 0 < \tilde{\alpha}_2$ *are the real roots of*

$$\underbrace{\left(\delta + \lambda_1 - \alpha\mu_1 - \frac{\sigma_1^2}{2}\alpha^2\right)}_{:=G_1(\alpha)} \underbrace{\left(\delta + \lambda_2 - \alpha\mu_2 - \frac{\sigma_2^2}{2}\alpha^2\right)}_{:=G_2(\alpha)} = \lambda_1\lambda_2, \tag{18}$$

$$\frac{1}{2}\sigma_2^2\tilde{\alpha}^2 + \mu_2\tilde{\alpha} = \delta + \lambda_2, \tag{19}$$

and $(A_1, A_2, A_3, A_4, \tilde{A}_1, \tilde{A}_2) \in \mathbb{R}^6$ *and* $(K_1, K_2) \in \mathbb{R}_+^2$ *solve the following linear system:*[6]

$$\begin{cases} \sum_{j=1}^4 A_j\alpha_j - 1 = 0, \\ \sum_{j=1}^4 A_j\alpha_j^2 = 0, \\ \sum_{j=1}^4 B_j\alpha_j - \frac{\lambda_2}{\delta+\lambda_2} - \sum_{h=1}^2 \tilde{A}_h\tilde{\alpha}_h = 0, \\ \sum_{j=1}^4 B_j\alpha_j^2 - \sum_{h=1}^2 \tilde{A}_h\tilde{\alpha}_h^2 = 0, \\ \sum_{h=1}^2 \tilde{\alpha}_h\tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} - \beta_2 + \frac{\lambda_2}{\delta+\lambda_2} = 0 \\ \sum_{h=1}^2 \tilde{\alpha}_h^2\tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} = 0. \\ \mu_2\beta_2 - (\delta+\lambda_2)K_2 + \lambda_2(K_1 + \tilde{x}_2 - \tilde{x}_1) = 0, \\ \mu_1 - (\delta+\lambda_1)K_1 + \lambda_1\left(K_2 + \frac{\lambda_2}{\delta+\lambda_2}(\tilde{x}_1 - \tilde{x}_2)\right) = 0, \end{cases} \tag{20}$$

with $B_j = A_j\lambda_1^{-1}G_1(\alpha_j)$. *Moreover, assume that*

$$\begin{cases} \sum_{j=1}^4 A_j\alpha_j^3 > 0, \\ \sum_{j=1}^4 B_j\alpha_j^3 > 0, \\ \sum_{j=1}^4 A_j\alpha_j^3 e^{\alpha_j(x_R-\tilde{x}_1)} > 0, \\ \sum_{j=1}^4 B_j\alpha_j^3 e^{\alpha_j(x_R-\tilde{x}_1)} > 0. \end{cases} \tag{21}$$

[6] Assuming that a solution to this system exists.

Then, $v''(\cdot, i) < 0$ in $(x_R, \tilde{x}_i)$ and (16) and (17) solve (9) in a classical sense.

2. *Consider the framework in Point 1 and assume that*

$$\sum_{j=1}^{4} B_j \alpha_j e^{\alpha_j (x_R - \tilde{x}_1)} - 1 > 0. \tag{22}$$

Then, $x_2^* \in (x_R, \tilde{x}_2]$ is the unique solution of

$$\mathbf{1}_{[x_R,\tilde{x}_1]}(x_2^*) \sum_{j=1}^{4} B_j \alpha_j e^{\alpha_j (x_2^* - \tilde{x}_1)} + \mathbf{1}_{(\tilde{x}_1,\tilde{x}_2]}(x_2^*) \left(\frac{\lambda_2}{\delta + \lambda_2} + \sum_{h=1}^{2} \tilde{\alpha}_h \tilde{A}_h e^{\tilde{\alpha}_h (x_2^* - \tilde{x}_1)} \right) - 1 = 0. \tag{23}$$

3. *Consider the framework in Points 1 and 2 and set $\Gamma_i^+ = 0$. Moreover, assume that $\tilde{x}_1, \tilde{x}_2$ solve the following algebraic system:*

$$\begin{cases} \sum_{j=1}^{4} A_j \left(e^{\alpha_j (x_R - \tilde{x}_1)} - 1 \right) + (\tilde{x}_1 - x_R) + \kappa = 0, \\ \frac{\lambda_2 \tilde{x}_1}{\delta + \lambda_2} - \sum_{j=1}^{2} \tilde{A}_j (1 - e^{\tilde{\alpha}_j (\tilde{x}_2 - \tilde{x}_1)}) - \sum_{j=1}^{4} B_j \left(e^{\alpha_j (x_R - \tilde{x}_1)} - 1 \right) + \\ - \left(\frac{\lambda_2 \tilde{x}_1}{\delta + \lambda_2} + \sum_{j=1}^{2} \tilde{A}_j (1 - e^{\tilde{\alpha}_j (\tilde{x}_2 - \tilde{x}_1)}) + \sum_{j=1}^{4} B_j \left(e^{\alpha_j (x_2^* - \tilde{x}_1)} - 1 \right) \right) \mathbf{1}_{(x_R, \tilde{x}_1)} + \\ - \left(\frac{\lambda_2 x_2^*}{\delta + \lambda_2} + \sum_{j=1}^{2} \tilde{A}_j \left(e^{\tilde{\alpha}_j (x_2^* - \tilde{x}_1)} - e^{\tilde{\alpha}_j (\tilde{x}_2 - \tilde{x}_1)} \right) \right) \mathbf{1}_{(\tilde{x}_1, \tilde{x}_2)} - (x_2^* - x_R) + \kappa = 0. \end{cases} \tag{24}$$

Then, (16) and (17) satisfy the boundary conditions (13).

Proof. See Appendix A.1. □

Suppose all the above assumptions hold, and v equals the value function. In that case, we can use this proposition to determine the firm's optimal strategy by solving a system of algebraic equations. The procedure is the following.

The firm only pays dividends in State i when its reserves reach $\tilde{x}_i$. For a given $x_2^* \in (x_R, \tilde{x}_2]$, the payout thresholds solve the system in (24). When reserves reach x_R, liquidation is never optimal ($\hat{\tau}_l = \infty$ or, equivalently, $\hat{b}_n = 1$), provided that κ is sufficiently small to ensure that (13) is positive. If that is the case, the optimal recapitalization in State i injects equity until the firm's reserves reach the "target" level x_i^*. Therefore, $\hat{G}(i) = x_i^* - x_R$.

Since $v(\cdot,i)$ is differentiable, we can use (13) and the boundary condition $v(\tilde{x}_1,1)=1$ to obtain that $x_1^*=\tilde{x}_1$ and $x_2^*\in(x_R,\tilde{x}_2]$ as the unique solution of (23). In other words, the firm finds it optimal to recapitalize exactly to its dividend payout threshold in the good State and *below* the payout threshold in the bad one, ($x_2^*\le\tilde{x}_2$) because the marginal value of dividends (β_2) is smaller than one. Since v is concave, $v(\tilde{x}_2,2)=\beta_2\le v(x_2^*,2)=1$, and each state has a unique payout threshold and recapitalization target.

For a given $\hat{G}(i)$, the following condition defines the maximum cost level that ensures that recapitalization is incentive-compatible:

$$\bar{\kappa}=x_R+\min\left\{v(x_1^*,1)-x_1^*,v(x_2^*,2)-x_2^*\right\}. \tag{25}$$

The next theorem verifies that v is indeed the value function and formally expresses the optimal strategy we described above.

Theorem 1 (Verification). *Let all the assumptions of Proposition 1 hold and let $v(\cdot,i)$ be the functions constructed therein. Moreover, let $(x,i)\in(x_R,\infty)\times\{1,2\}$. Then, $v(x,i)=V(x,i)$ and the control $\hat{A}=(\hat{Z},(\hat{b},\hat{G}))\in\mathcal{A}$ such that*

$$\begin{cases}\hat{b}_n=1,\\ \hat{G}_n=x^*_{I_{\hat{\tau}_n^-}}-x_R,\\ \hat{Z}_t=\hat{Z}_{\tau_n}+\sup_{s\in[\hat{\tau}_n,t)}\left[x^*_{I_{\hat{\tau}_n^-}}+\int_{\hat{\tau}_n}^s(\mu_{I_r}dr+\sigma_{I_r}dW_r)-\tilde{x}_{I_{\hat{\tau}_n^-}}\right]^+, \qquad t\in[\hat{\tau}_n,\hat{\tau}_{n+1}),\end{cases} \tag{26}$$

where $\hat{\tau}_0:=0$ and $\hat{\tau}_n$ is defined recursively as $\hat{\tau}_{n+1}=\inf\{t\ge\hat{\tau}_n:\ \hat{X}_{t^-}=x_R\}$ being $\hat{X}_t$ the associated state process, is optimal.

Proof. See Appendix A.2. □

Remark 1. ***(Dividend tax threshold)*** *The solution structure described in Proposition 1 and Theorem 1 holds under the parametric restriction that dividend taxes are not too high, i.e., $\beta_2>\lambda_2/(\lambda_2+\delta)$. In contrast, when $\beta_2\le\lambda_2/(\lambda_2+\delta)$, an inspection of the conditions set shows that the solution structure used to obtain them breaks down. The same conditions suggest we guess an alternative structure with $\tilde{x}_2=\infty$. The "right" structure has*

$\mathcal{C}_2 = (x_R, \infty)$*, irrespective of the value of* $\beta_2 \in [0, \lambda_2/(\lambda_2 + \delta))$*. For a rigorous treatment of this case, we refer to Appendix A.4.*

Remark 1 clarifies that when taxes are excessively high in the bad state, the firm finds it optimal to withhold dividend payments and wait until the good state occurs to distribute them. Therefore, setting $\beta_2 \geq \lambda_2/(\delta + \lambda_2)$ is equivalent to imposing a dividend ban in the bad state. The level of β_2 that triggers the "ban-like" behaviour increases with the probability of transitioning from the bad to the good state (λ_2) and decreases with the manager's impatience (δ).

4.2 Reserves distribution

This section derives the probability density function of the firm's reserves process in state $i = 1, 2$, denoted as $\pi(x, i)$. We will use this object to evaluate the effects of dividend taxes and capital regulation on the firm's capital buffers (or "credit capacity") in Section 5.

Since the dynamics of reserves obey the controlled process (26), standard arguments can be applied to show that, in the interval $(x_R, x_2^*) \cup (x_2^*, \tilde{x}_1)$, $\pi(x, i)$ satisfies the following system of Kolmogorov Forward Equations (KFE):

$$\begin{cases} \frac{\sigma_1^2}{2}\pi''(x,1) - \mu_1\pi'(x,1) + \lambda_1\left(\pi(x,2) - \pi(x,1)\right) = 0, \\ \frac{\sigma_2^2}{2}\pi''(x,2) - \mu_2\pi'(x,2) + \lambda_2\left(\pi(x,1) - \pi(x,2)\right) = 0, \end{cases} \tag{27}$$

with boundary conditions $\pi(x_2^{*-}, i) = \pi(x_2^{*+}, i)$ (value matching), $\pi'(x_2^{*-}, 1) = \pi'(x_2^{*+}, 1)$ (smooth pasting). By the same logic, in the interval $(\tilde{x}_1, \tilde{x}_2)$ the density function in State 2 satisfies

$$\frac{\sigma_2^2}{2}\pi''(x,2) - \mu_2\pi'(x,2) - \lambda_2\pi(x,2) = 0, \tag{28}$$

with boundary conditions $\pi(\tilde{x}_1^-, 2) = \pi(\tilde{x}_1^+, 2)$ and $\pi'(\tilde{x}_1^-, 2) = \pi'(\tilde{x}_1^+, 2)$. These conditions imply that $\pi(\cdot, i)$ is C^1 in all interior regions for $i = 1, 2$, except for $\pi(\cdot, 2)$ at x_2^*, which is the mass point where reserves accumulate after each recapitalization when $i = 2$.

To characterize the pdf at $\tilde{x}_1$ and $\tilde{x}_2$, we impose the following reflecting barriers:[7]

$$\frac{\sigma_i^2}{2}\pi'(\tilde{x}_i, i) - \mu_i \pi(\tilde{x}_i, i) = 0, \text{ for } i = 1, 2. \tag{29}$$

Coherently, we set $\pi(\cdot, i) = 0$ in the interval $(\tilde{x}_i, \infty)$ for $i = 1, 2$.

To characterize the pdf at the regulatory threshold, we impose that the reserves process is "absorbed" at x_R, in the sense that it is immediately and irreversibly transported to the interior states x_i^* (see Yaegashi et al., 2019, for a discussion of similar conditions in a uni-variate setting). Thus, we set $\pi(x_R, i) = 0$ for $i = 1, 2$. Finally, we impose that

$$\sum_{h=1}^{2} \frac{\lambda_{3-h}}{\lambda_1 + \lambda_2} \int_{x_R}^{\infty} \pi(x, h) dx = 1, \tag{30}$$

where $\lambda_h/(\lambda_1 + \lambda_2) = 1 - \mathbb{P}\{i = h\}$, because $\pi(x, i)$ is a pdf. Solving (27) and (28) under these conditions yields the following.

Proposition 2. ***(Reserves probability density function)*** *Fix $\tilde{x}_1, \tilde{x}_2$ such that $x_R < \tilde{x}_1 < \tilde{x}_2$. Then, the pdf of the firm's reserves in state $i = 1, 2$ satisfies*

$$\pi(x,1) = \begin{cases} P_1 e^{r_1 x} + P_2 e^{r_2 x} + P_3 e^{r_3 x} + P_4 e^{r_4 x}, & x \in (x_R, x^*), \\ \tilde{P}_1 e^{r_1 x} + \tilde{P}_2 e^{r_2 x} + \tilde{P}_3 e^{r_3 x} + \tilde{P}_4 e^{r_4 x}, & x \in (x^*, \tilde{x}_1), \\ 0, & x \in (\tilde{x}_1, \infty), \end{cases}$$

$$\pi(x,2) = \begin{cases} Q_1 e^{r_1 x} + Q_2 e^{r_2 x} + Q_3 e^{r_3 x} + Q_4 e^{r_4 x}, & x \in (x_R, x^*), \\ \tilde{Q}_1 e^{r_1 x} + \tilde{Q}_2 e^{r_2 x} + \tilde{Q}_3 e^{r_3 x} + \tilde{Q}_4 e^{r_4 x}, & x \in (x^*, \tilde{x}_1), \\ H_1 e^{s_1 x} + H_2 e^{s_2 x}, & x \in (\tilde{x}_1, \tilde{x}_2), \\ 0, & x \in (\tilde{x}_2, \infty), \end{cases}$$

[7] For a formal derivation of reflecting barriers for controlled diffusion process, we refer to D.R. and Miller (1965), Chapter 5.

in which $r_1 < r_2 < 0 < r_3 < r_4$ *and* $s_1 < 0 < s_2$ *are the real roots of*

$$\underbrace{\left(\lambda_1 + r\mu_1 - \frac{\sigma_1^2}{2}r^2\right)}_{:=F_1(r)}\underbrace{\left(\lambda_2 + r\mu_2 - \frac{\sigma_2^2}{2}r^2\right)}_{:=F_2(r)} = \lambda_1\lambda_2, \tag{31}$$

$$\frac{\sigma_2^2}{2}s^2 - \mu_2 s = \lambda_2, \tag{32}$$

and the constants $(P_1, P_2, P_3, P_4, \tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \tilde{P}_4, H_1, H_2) \in \mathbb{R}^{10}$ *solve the linear system*

$$\begin{cases} \sum_{j=1}^4 \tilde{P}_j \left(\frac{\sigma_1^2}{2}r_j - \mu_1\right) e^{r_j \tilde{x}_1} = 0, \\ \sum_{j=1}^4 \tilde{P}_j \left(\frac{\sigma_1^2}{2}r_j - \mu_1\right) e^{r_j \tilde{x}_1} = 0, \\ \sum_{j=1}^4 P_j e^{r_j x_R} = 0, \\ \sum_{j=1}^4 Q_j e^{r_j x_R} = 0, \\ \sum_{j=1}^4 e^{r_j x_2^*} \left(P_j - \tilde{P}_j\right) = 0, \\ \sum_{j=1}^4 e^{r_j x_2^*} r_j \left(P_j - \tilde{P}_j\right) = 0, \\ \sum_{j=1}^4 e^{r_j x_2^*} \left(Q_j - \tilde{Q}_j\right) = 0, \\ \sum_{h=1}^2 H_h e^{s_h \tilde{x}_1} - \sum_{j=1}^4 \tilde{Q}_j e^{r_j \tilde{x}_1} = 0, \\ \sum_{h=1}^2 H_h s_h e^{s_h \tilde{x}_1} - \sum_{j=1}^4 \tilde{Q}_j r_j e^{r_j \tilde{x}_1} = 0, \\ \sum_{h=1}^2 \lambda_{3-h} \int_{x_R}^{\tilde{x}_i} \pi(x,i)dx - \lambda_1 - \lambda_2 = 0, \end{cases} \tag{33}$$

where $Q_j = \lambda_1^{-1}F(r_j)P_j$ *and* $\tilde{Q}_j = \lambda_1^{-1}F(r_j)\tilde{P}_j$.

Proof. See Appendix A.3. □

5 Policy implications

In this section, we parameterize the model and numerically analyze its policy implications by examining the effects of dividend taxes on the firm's optimal strategy, value, and reserve distribution.

Parameter	Meaning	Value
μ_1	CF drift, good state	0.05
μ_2	CF drift, bad state	0.02
σ_1	CF vol, good state	0.25
σ_2	CF vol, bad state	0.3
κ	Recapitalization cost	0.087
δ	Discount rate	0.03
x_R	Capital requirement	0.0769
ρ	Return on deposits	0.0043
α	Haircut	0.6
β_2	Dividend regulation	0.87
$\frac{L}{D}$	Loan-to-deposit ratio	1.5385
$1/\lambda_1$	Avg duration, good state	10
$1/\lambda_2$	Avg duration, bad state	6.7
Γ_1, Γ_2	Capital buffers	0

Table 1: Baseline parameters

5.1 Parameters

We normalize the stock of deposits $D = 1$ and choose L to match the US bank deposit-to-loan ratio in Q4 2022 (about 1.5385), according to S&P Global. According to FRED, we set the rate of return on deposits to $\rho = 0.0043$. Consistently, we obtain $x_R = 1 - 0.6 \times 1.5385 = 0.0769$. We calibrate the fixed re-capitalization cost $\kappa = 0.087$ to yield a price-to-book ratio at the dividend payout threshold in the good state $(v(\tilde{x}_1, 1)/(L+\tilde{x}_1-D))$ of about 1.04. This is the value observed across US commercial banks, according to the NYU Stern database. The drift and the diffusion parameters, $\bar{\mu}_i$ and σ_i, and the discount rate δ are similar to Guo et al. (2005). The regime shifting intensities λ_i and the hair cut α come from Hackbarth et al. (2006). Last, we set $\beta_2 = 0.87$, corresponding to an increment of 10 p.p. in the dividend tax rate from 0.3 in the good regime to 0.4 in the bad one.[8]

[8] After normalizing the value of $\beta_1 = 1$, the parameter β_2 can be obtained by solving: $1 - 0.3 = \frac{1-0.4}{\beta_2}$.

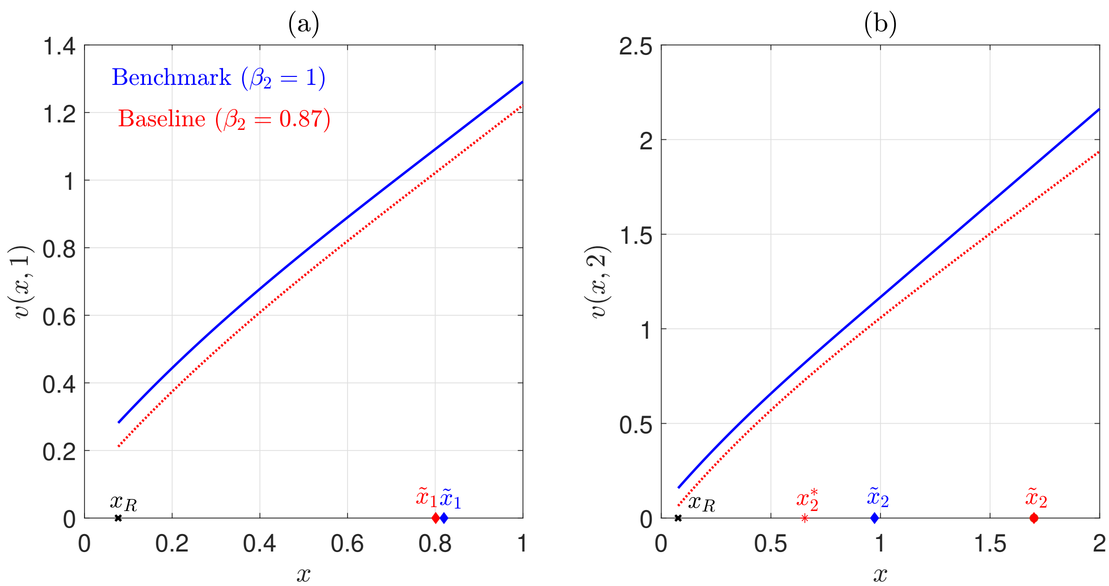

Figure 1: Optimal strategy and shareholder value in the good (Panel (a)) and bad (Panel (b)) states when $\beta_2 = 1$ (solid blue) and $\beta_2 = 0.87$ (dotted red).

5.2 The effects of dividend taxes

5.2.1 Firm's optimal strategy and shareholder value

To evaluate the effect of state-contingent dividend taxes, we compare the firm's optimal strategy and shareholder value (i.e., the value function) when $\beta_2 < \beta_1 = 1$ with the benchmark case where $\beta_2 = \beta_1 = 1$. The red dotted and solid blue lines in Figure 1 display $v(x,i)$ in the good (Panel (a)) and bad (Panel (b)) states as a function of the reserve level in these two cases. The black crosses and the blue (red) diamonds on the x-axis showcase the regulatory threshold (x_R) and the optimal dividend payout ($\tilde{x}_i$) in the good (bad) state. The red star in Panel (b) indicates the optimal recapitalization target. The following four patterns emerge from the comparison.

First, dividend taxes reduce shareholder value for all reserve levels and in each state of the economy. This is expected because the policy introduces an additional distortion beyond the capital requirement, relative to the benchmark. The value losses are more severe in percentage terms for any reserve value in the bad state than in the good state, as the policy affects the latter only indirectly. Additionally, value losses are always more severe when the level of reserves is lower, except in the bad state, where they increase slightly after reaching

β_2	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*
1.000	0.820	0.950	0.950
0.990	0.819	0.941	0.815
0.970	0.814	1.061	0.747
0.950	0.811	1.142	0.724
0.900	0.804	1.421	0.667
0.870	0.804	1.700	0.655
0.850	0.800	2.663	0.649
0.834	0.800	17.024	0.649

Table 2: Optimal dividend threshold and recapitalization target as functions of β_2.

a tipping point around $x = 1$.

Second, we examine the effects of dividend taxes on the firm's optimal strategy. On the one hand, consistent with its intended scope, the policy encourages additional capital accumulation (i.e., fewer dividends) in the bad state, shifting $\tilde{x}_2$ from about 0.95 to 1.7. The higher the tax, the more the firm is willing to wait for a state change before paying dividends. This outcome is apparent in Table 2, which displays the optimal $\tilde{x}_i$ and x_2^* as functions of β_2.[9] On the other hand, the policy "backfires" in the good state, where the firm reduces $\tilde{x}_1$ from 0.82 to 0.80. The reason is that the firm compensates for the bad-state tax by anticipating dividends in the good state. Notice, however, that there is a significant asymmetry in the magnitude of the threshold shifts between the good and bad states. In our parametrization, enforcing dividend restrictions reduces the payout threshold in the good state by only 2.5%, while it increases it by more than 30% in the bad state.

Third, dividend taxes reduce the firm's optimal recapitalization targets (x_i^*) in every state. In the good state, this result directly follows the shift in the payout dividend threshold $\tilde{x}_1$ because it coincides with the recapitalization target ($\tilde{x}_1 = x_1^*$). Conversely, $\tilde{x}_2 > x_2^*$ in the bad state because the marginal cost of paying dividends (β_2) is always lower than that of injecting liquid reserves, and $v''(x,2) < 0$ (see (23)). Notably, x_2^* decreases with β_2 (up to a limit value as $\beta_2 \to \frac{\delta+\lambda_2}{\lambda_2}$) and lies *below* the good-state payout thresholds $\tilde{x}_1$. Hence, dividend taxes reduce the equity shareholders are willing to inject when their reserves reach

[9] Note that, consistent with Remark 1, the last rows of the table showcase that the bad-state payout threshold $\tilde{x}_2$ "explodes" when $\beta_2 \to \lambda_2/(\delta + \lambda_2)$ ($\approx 0.3\bar{3}$ in the baseline parametrization).

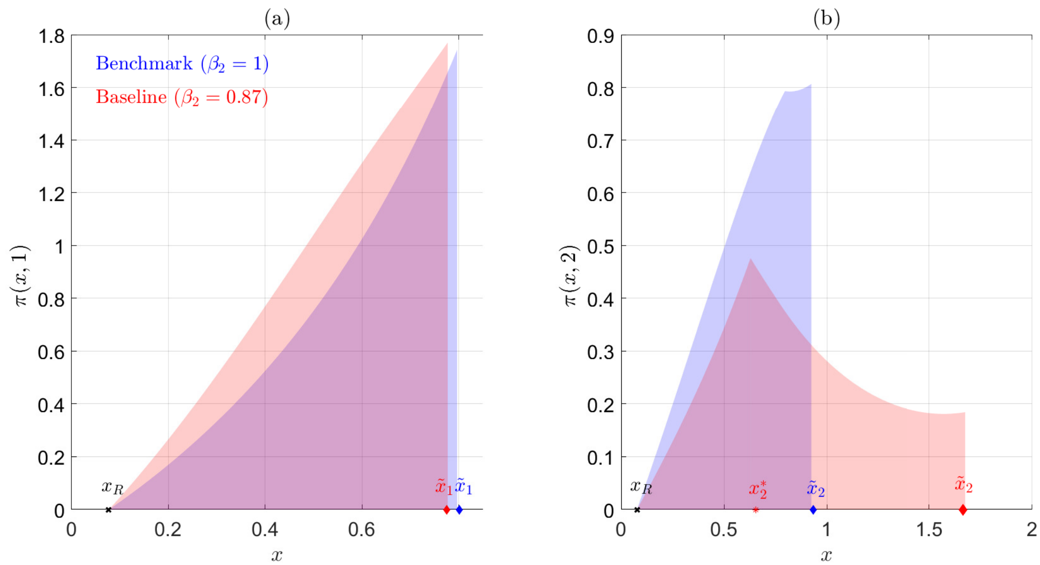

Figure 2: Reserves distribution and the optimal strategy in the good (Panel (a)) and bad (Panel (b)) states when $\beta_2 = 1$ (blue) and $\beta_2 = 0.87$ (red).

the regulatory threshold x_R.

Fourth, the overall influence of the dividend tax policy on the firm's reserve holdings is ambiguous because the ways it affects its optimal strategy are conflicting: while the increase in $\tilde{x}_2$ yields higher capital buffers, the reduction in $\tilde{x}_1$ and x_2^* may curb them. To determine which effect prevails, the next section analyzes the impact of the policy on the firm's stationary reserves distribution.

5.2.2 Credit capacity and recapitalization incentives

This section employs the (stationary) pdf of the firm's reserves ($\pi(x, i)$), characterized in Proposition 2, to compute an ex-ante measure of the firm's credit capacity (i.e., the average level of its reserves) conditional and unconditional on the aggregate state of the economy.

Figure 2 displays $\pi(x, i)$ and the associated optimal strategy when $\beta_2 = \beta_1 = 1$ (blue) and $\beta_2 < \beta_1 = 1$ (red). As a first observation, higher dividend taxes in the bad state do not substantially alter the pdf's shape in the good state (Panel (a)). However, they transfer probability mass towards lower reserve levels because reducing β_2 shifts $\tilde{x}_1$ to the left, but the probability of $i = 1$ remains unchanged. In contrast, the tax policy dramatically affects the shape of the pdf in the bad state (Panel (b)). In particular, the dispersion of $\pi(x, 2)$ increases

β_2	$\mathbb{E}_1^\pi[x]$	$\mathbb{E}_2^\pi[x]$	$\mathbb{E}^\pi[x]$	$\mathbb{V}_1^\pi[x]$	$\mathbb{V}_2^\pi[x]$	$\mathbb{V}^\pi[x]$	$\bar{\kappa}$
1.00	0.6007	0.6525	0.6214	0.0372	0.0387	0.0389	0.1351
0.95	0.5758	0.7473	0.6444	0.0284	0.0609	0.0485	0.1187
0.90	0.5698	0.8289	0.6734	0.0280	0.1015	0.0735	0.1090
0.87	0.5677	0.8913	0.6971	0.0279	0.1496	0.1017	0.1057
0.85	0.5669	0.9448	0.7181	0.0278	0.2118	0.1357	0.1043

Table 3: Firm's credit capacity, its dispersion, and the maximal incentive-compatible recapitalization cost for different dividend tax parameters β_2.

sharply because a lower β_2 shifts $\tilde{x}_2$ to the right, widening the support of the reserves' process. Additionally, dividend taxes generate a steep mass point at the recapitalization target x_2^* because $x_2^* < \tilde{x}_1 < \tilde{x}_2$.[10]

To capture the overall effect of the dividend tax policy, we define the following measure of the firm's "credit capacity" conditional on State i as:

$$\mathbb{E}_i^\pi[x] = \int_{x_R}^{\tilde{x}_i} x\pi(x,i)dx, \tag{34}$$

and its dispersion as $\mathbb{V}_i^\pi[x] = \mathbb{E}_i^\pi[x^2] - \mathbb{E}_i^\pi[x]^2$. Consistently, we define the firm's unconditional credit capacity as

$$\mathbb{E}^\pi[x] = \sum_{h=1}^{2} \mathbb{E}_h^\pi[x] \underbrace{\frac{\lambda_{3-h}}{\lambda_1+\lambda_2}}_{=\mathbb{P}\{i=h\}}, \tag{35}$$

with dispersion $\mathbb{V}^\pi[x] = \mathbb{E}^\pi[x^2] - \mathbb{E}^\pi[x]^2$.[11] Table 3 collects the value of these objects using the parameters in Table 1 and different levels of β_2. The analysis delivers the following implications.

First, state-contingent dividend taxes reduce the firm's credit capacity in the bad state but enhance it in the good state. However, the loss in the former is always more than offset by the gains in the latter (Columns 1 and 2). As a result, imposing dividend taxes increases overall credit capacity (Column 3). Second, higher dividend taxes slightly reduce

[10]Notice that, due to the regime-switching, the pdf of x in State 2 displays a small but interior mass point even when $\beta_2 = 1$, coinciding with the dividend payout threshold $\tilde{x}_1$ in State 1.

[11]While our model features a fixed loan supply, credit capacity captures the ex-ante average resources available to the firm and that may potentially be used to issue new loans.

β_2	$\hat{\tau}(1)$	$\hat{\tau}(2)$	$\hat{\tau}$
1.000	11.89	11.56	11.76
0.970	12.44	11.99	12.26
0.870	13.06	12.66	12.90
0.834	13.12	12.74	12.97

Table 4: Average waiting time (years) between subsequent recapitalization events, in each State i and overall, for different levels of β_2.

the dispersion of reserves in the good state but significantly increase it in the bad state. Hence, the firm's credit capacity dispersion increases overall (Columns 5-7).

The relatively small impact of dividend taxes on the firm's credit capacity in the good state, compared to their more significant effects in the bad state and the overall increase in credit capacity, suggests that the policy's benefits outweigh its costs. Indeed, one can verify that dividend taxes make recapitalization events less frequent in each state i and overall. Table 4 reports a numerical approximation of the average waiting time between subsequent recapitalization events, formally defined as $\hat{\tau}(i) := \inf\{t \geq \tau_n : \hat{X}_t = x_{R,i}, \hat{X}_{\tau_n} = x_i^*\}$ and $\hat{\tau} := \sum_{i=1}^{2} \hat{\tau}(i) \cdot \lambda_{3-i}/(\lambda_1 + \lambda_2)$ for different values of β_2.[12] Note that, despite the tax, the average recapitalization time is higher in the good than in the bad state.

The overall evaluation of the dividend tax policy becomes less straightforward when noticing that, due to the firm's value losses, higher dividend taxes reduce the maximal equity issuance cost that is incentive-compatible ($\bar{\kappa}$, see (25)). The last column of Table 3 displays this phenomenon, showing that, for example, a ten p.p. increment in dividend taxes is associated with a 28 per cent reduction in the level of $\bar{\kappa}$. It is relevant to emphasize that all the figures reported in the table exceed the baseline cost level adopted in our parametrization ($\kappa = 0.087$), ensuring that the firm always finds it optimal to recapitalize when $x = x_R$. We nevertheless interpret this tightening of the incentive-compatibility constraint as evidence that dividend restrictions may endogenously increase default risk.

[12]The numbers are obtained by averaging 5,000 Monte Carlo simulations of the firm's reserves process under the optimal strategy (26).

	$\tilde{x}_1$		$\tilde{x}_2$		x_2^*		$\bar{\kappa}$	
β_2	1.00	0.87	1.00	0.87	1.00	0.87	1.00	0.87
Baseline	0.821	0.801	0.954	1.700	0.954	0.655	0.135	0.106
$\sigma_2 = 0.35$	0.827	0.811	1.051	1.938	1.051	0.710	0.118	0.087
$\mu_1 = 0.07$	0.794	0.776	0.952	1.698	0.952	0.653	0.288	0.251
$\mu_2 = 0.03$	0.820	0.798	0.941	1.651	0.941	0.637	0.194	0.157
$\lambda_1 = 0.05$	0.816	0.806	0.954	1.700	0.954	0.655	0.186	0.160
$\lambda_2 = 0.2$	0.821	0.804	0.950	3.324	0.950	0.672	0.164	0.141
$\Gamma_1 = \Gamma_2 = 0.05$	0.871	0.851	1.007	1.750	1.007	0.705	0.133	0.106
$\Gamma_1 = \Gamma_2 = -0.05$	0.769	0.751	0.923	1.650	0.923	0.605	0.133	0.106

Table 5: Comparative statics analysis.

5.2.3 Comparative statics

The previous section qualitatively examined the effects of dividend taxes in our baseline calibration. Table 5 reports a comparative statics analysis to demonstrate that our results hold qualitatively for significant variations in the model's main parameters.

A higher drift in the good or bad states lowers the payout thresholds because it releases the firm's precautionary motive (Rows 3-4). Raising volatilities leads to an opposite effect (see, e.g., Row 2). Decreasing the probability of visiting the bad state (λ_1) fosters dividend payouts without the tax while mitigating the additional payout incentive in the good state induced by the tax. Moreover, it increases the maximum incentive-compatible cost level while leaving the optimal recapitalization target unaffected. The probability of transitioning from the bad to the good state (λ_2) has similar effects on $\tilde{x}_1$, x_2^*, and $\bar{\kappa}$, while also leading to a further postponement of dividends in the bad state. (Row 6).

A particular aspect worth emphasizing concerns the effects of a change in the regulatory buffer/subsidy parameters Γ_i, for $i = 1, 2$. The last two rows of the Table 5 examine the cases where $\Gamma_1 = \Gamma_2 = \Gamma = \pm 0.05$. Figure 3 displays the effect of changing Γ on the firm's value function. The analysis will serve as a benchmark when we discuss cyclical capital regulation (i.e. when $\Gamma_1 \neq \Gamma_2$) in Section 5.3.2. As intuition suggests, tighter (looser) capital requirements are associated with higher (lower) recapitalization thresholds and lower (higher) firm valuations. Additionally, it is worth noting that changes in Γ do not affect the maximum incentive-compatible recapitalization costs. This happens because shareholder

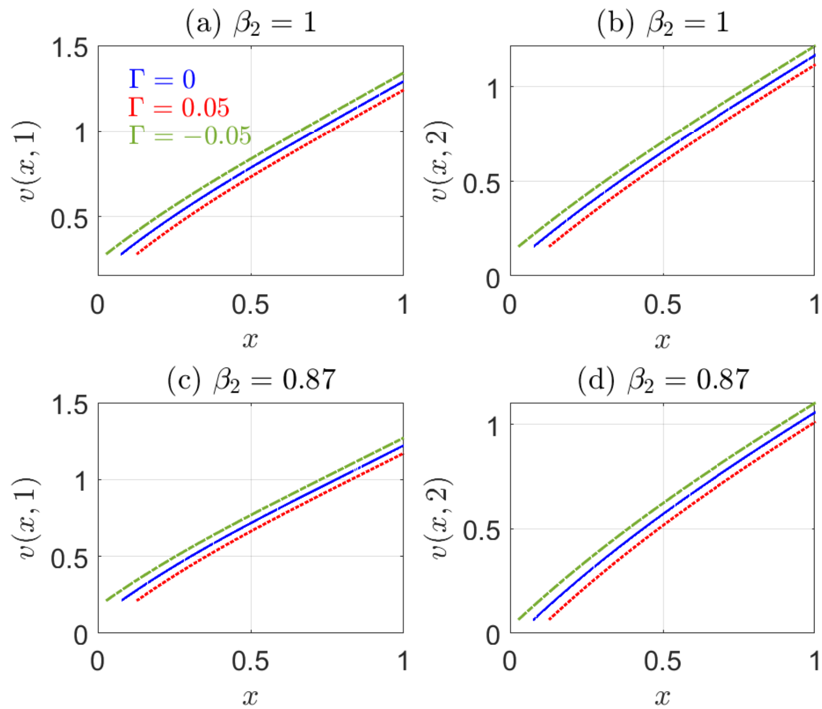

Figure 3: Shareholder value in the good (left panels) and the bad states (right panels) for different combinations of Γ when $\beta_2 = 1$ (top panels) and $\beta_2 = 0.87$ (bottom panels).

value shifts in the opposite direction of the capital requirement change.

Although relatively straightforward, these effects are significant as they suggest regulators can mitigate shareholder value losses–one of the adverse impacts of the dividend tax policy–by adjusting capital requirements simultaneously. We explore this aspect further in the next section.

5.3 Coordinating dividend taxes and capital regulation

This section extends the model to consider counter-cyclical capital requirements and discusses how that modifies the associated solution structure. Then, it examines numerically the policy challenge of coordinating dividend taxes and capital regulation.

Our analysis is primarily motivated by the fact that, while recommending dividend suspensions, the ECB temporarily eased capital requirements for banks during the COVID-19 crisis (see Matyunina and Ongena, 2022). The Basel III regulatory framework justifies our focus on counter-cyclical capital requirements. At the same time, we neglect pro-cyclical

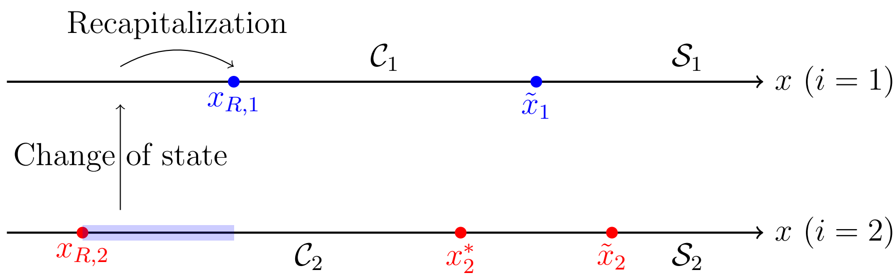

Figure 4: Solution structure with counter-cyclical capital requirements.

capital buffers based on the extensive literature showing that it may destabilize financial institutions and thus exacerbate crises. (On this point, see, for example Repullo and Suarez, 2013; Valencia and Bolanos, 2018, and the references therein).[13]

5.3.1 Counter-cyclical capital requirements

To model counter-cyclical capital requirements, we consider $\Gamma_1 > \Gamma_2$ or, equivalently, $x_{R,1} > x_{R,2}$. This assumption requires adjusting the model's solution structure by adding the region $x \in (x_{R,2}, x_{R,1})$ to the state space, as shown in Figure 4. The regulator requires the firm to immediately recapitalize (or liquidate) in this region should the economy shift from State 2 to State 1. Accordingly, we set

$$v(x,1) = \max\left\{v(\tilde{x}_1, 1) - (\tilde{x}_1 - x) - \kappa, 0\right\}, \tag{36}$$

and find $v(x,2)$ as the unique solution of

$$\frac{1}{2}\sigma_2^2 v''(x,2) + \mu_2 v'(x,2) - (\delta + \lambda_2)\, v(x,2) + \lambda_2 v(x,1) = 0, \tag{37}$$

with boundary conditions $v(x_{R,1}^+, 2) = v(x_{R,1}^-, 2)$ and $v'(x_{R,1}^+, 2) = v'(x_{R,1}^-, 2)$.

Since the firm always finds it optimal to recapitalize when κ is small enough (see (36)), in the remaining regions (i.e., when $x > x_{R,1}$), the value function equals the one described in Section 4.1 after setting $x_{R,1} = x_R$. Hence, we can still characterize the model's solution

[13] A brief discussion on how to adapt the model's solution structure to accommodate pro-cyclical capital requirements can be found in the online appendix.

$x_{R,1}$	$x_{R,2}$	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*	$\mathbb{E}_1^\pi[x]$	$\mathbb{E}_2^\pi[x]$	$\mathbb{E}^\pi[x]$	$\mathbb{V}_1^\pi[x]$	$\mathbb{V}_2^\pi[x]$	$\mathbb{V}^\pi[x]$	$\bar{\kappa}$
0.077	0.077	0.801	1.700	0.655	0.568	0.891	0.697	0.028	0.150	0.102	0.106
0.087	0.062	0.806	1.688	0.643	0.571	0.884	0.696	0.028	0.148	0.099	0.102
0.097	0.047	0.812	1.677	0.631	0.579	0.868	0.694	0.028	0.150	0.097	0.098
0.117	0.017	0.824	1.656	0.609	0.600	0.847	0.693	0.027	0.151	0.092	0.087

Table 6: Firm's optimal strategy, its credit capacity and dispersion, and the maximal incentive-compatible recapitalization cost for different levels of Γ ($x_{R,1}$ and $x_{R,2}$).

analytically by solving a system of algebraic equations. We report the details in the online appendix.

5.3.2 The effects of counter-cyclical capital regulation

We now numerically examine the effects of counter-cyclical capital requirements on the firm's optimal strategy and their interaction with dividend taxes.

To benchmark our results against the a-cyclical case discussed in Section 4, we assume that the regulator sets Γ_i to maintain the *mean* capital requirement x_R across states. Accordingly, we use $\Gamma_1 = \Gamma > 0$ and $\Gamma_2 = -\Gamma_1$, such that

$$x_R = \underbrace{(D - \alpha L + \Gamma)}_{=x_{R,1}} \underbrace{\frac{\lambda_2}{\lambda_1 + \lambda_2}}_{=\mathbb{P}\{i=1\}} + \underbrace{(D - \alpha L - \Gamma)}_{=x_{R,2}} \underbrace{\frac{\lambda_1}{\lambda_1 + \lambda_2}}_{=\mathbb{P}\{i=2\}}. \tag{38}$$

Table 6 reports the firm's optimal policy, credit capacity and dispersion, and the maximal incentive-compatible recapitalization cost for different levels of Γ. Row 1 illustrates the benchmark case where $\Gamma = 0$. We obtain the following predictions.[14]

First, adopting counter-cyclical capital requirements increases the firm's credit capacity in the good state (Columns 3 and 6) while reducing it in the bad states (Columns 4 and 6) and overall (Column 8), compared to the benchmark. Notably, when both dividend taxes *and* capital regulation are used, the firm's overall credit capacity remains higher than when dividend regulation is not applied (i.e., $\beta_2 = \beta_1 = 1$). Moreover, coordinating the two policies

[14] The appendix examines a simpler case where the regulator relaxes capital requirements only in the bad state (i.e., $x_{R,1} = x_R$ and $x_{R,2} = x_{R,1} - \Gamma_2$ with $\Gamma_2 > 0$). As expected, the policy lowers the dividend payout threshold, diminishes recapitalization incentives, and reduces average credit capacity and dispersion across states. Furthermore, it increases the firm's value in both states.

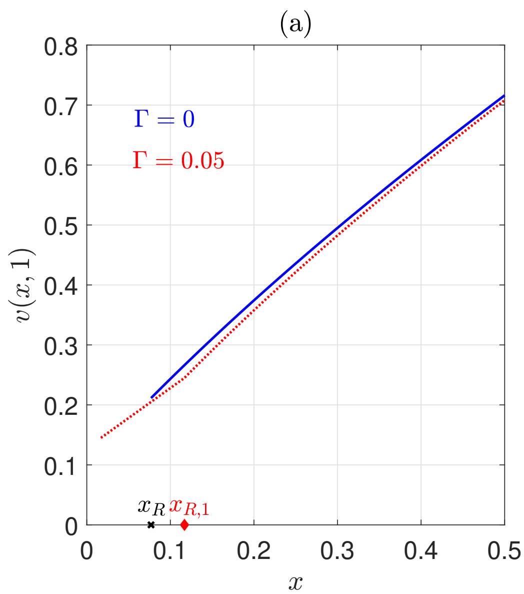

Figure 5: Effect of counter-cyclical capital regulation on shareholder value in the good (Panel (a)) and bad (Panel (b)) states.

allows for the implementation of dividend taxes without curbing capacity in the good state (on these points, see also the results in Table 3).

The policy's second effect is to reduce the dispersion of the reserve distribution, even after accounting for its (lower) expected value. For example, our simulations show that the reserves variation coefficient ($\sqrt{\mathbb{V}^\pi[x]}/\mathbb{E}^\pi[x]$) decreases by approximately 3.2 per cent when $x_{R,1} = 0.117$ and $x_{R,2} = 0.017$, compared to when $x_{R,1} = x_{R,2} = 0.077$.

Figure 5 displays the model's third prediction by showing the firm's value function for $\Gamma = 0$ ($x_{R,1} = x_{R,2} = 0.077$) and $\Gamma = 0.05$ ($x_{R,1} = 0.117$, $x_{R,2} = 0.017$).[15] The plot reveals that relaxing the capital requirement in the bad state helps mitigate shareholder value losses, which can be substantial when dividends are taxed, as discussed in the previous sections (Panel (b)). However, the gain comes at the cost of reducing the firm's value in the good state (Panel (a)). These outcomes materialize because tighter (looser) capital requirements curb (foster) the managers' precautionary motive in the good (bad) state, leading them to anticipate (delay) their dividend payments (Columns 3 and 4).

These results suggest that coordinating dividend taxes (or bans) with counter-cyclical capital regulation can mitigate some of the adverse effects of the former policy. This provides

[15] Computing plots for different values of Γ yields the same qualitative results.

$x_{R,1}$	$x_{R,2}$	$\hat{\tau}(1)$	$\hat{\tau}(2)$	$\hat{\tau}$
0.077	0.077	13.06	12.66	12.90
0.087	0.062	13.11	12.97	13.05
0.097	0.047	13.22	12.94	13.10
0.117	0.017	13.38	12.84	13.16

Table 7: Average waiting time (years) between each subsequent recapitalization, in each State i and overall, for different levels of Γ when $\beta_2 = 0.87$.

a theoretical foundation for the regulator's decision to associate dividend restrictions with looser capital requirements during the COVID-19 crisis.

However, the positive outcomes of policy coordination come with a cautionary note. Indeed, the model's fourth policy prediction indicates that adopting counter-cyclical capital requirements lowers the optimal recapitalization target in the bad state (x_2^*) and, correspondingly, the incentive-compatible recapitalization cost limit, $\bar{\kappa}$ (Columns 5 and 12). These effects arise because, while the policy increases the firm's value for each point in the state space above x_R, it reduces the value *at* $x_{R,2} < x_R$, which is a key determinant of its recapitalization decisions (see (13)).

The last step of the analysis assesses the effect of counter-cyclical capital regulation on the average waiting time (years) after each recapitalization. Table 7 presents a numerical approximation of these quantities conditional on each State i and overall for various levels of Γ when $\beta_2 = 0.87$. According to these simulations, coordinating counter-cyclical dividend taxes and capital requirements can effectively reduce the frequency of recapitalization (see Columns 3-5). However, when the policy becomes excessively cyclical, its effectiveness during downturns diminishes (see Column 4). This occurs because when $x_{R,2}$ becomes too low, the positive effect on capital buffers from a higher $\tilde{x}_1$ in the good state is partially offset by the increasingly negative impact of having lower $\tilde{x}_2$ and x_2^* in the bad one (see Table 6).

6 Conclusion

We have modelled and solved the optimal control problem of a firm choosing dividends and re-capitalization strategies under macroeconomic uncertainty, cyclical capital requirements, and dividend taxes (or bans). Our framework provides several testable policy implications, complementing recent empirical literature on the (short-term) effects of bank dividend suspension policies in the EU and the US.

First, the model predicts that state-contingent dividend taxes negatively affect shareholder value not only during crises (ex-post) but also in good times (ex-ante). Second, taxing dividends in bad macroeconomic states incentivizes the firm to pay out more in good times, reducing its corresponding capital buffers (credit capacity). This creates a trade-off between maintaining capital buffers (credit capacity) in good versus bad macroeconomic conditions. Third, dividend taxes may generate dispersion in the firm's capital buffers over the long run. Furthermore, they may undermine financial stability by diminishing the firm's recapitalization incentives.

Policymakers can coordinate dividend restrictions with counter-cyclical capital requirements to reallocate value losses and credit capacity between good and bad states, reducing the firm's reserve distribution dispersion. However, this approach may generate further disincentives for recapitalization.

Similar to other studies in the literature, our tractability assumptions carry a few limitations. For example, our firm's optimization problem does not include investments and assumes fixed loans and deposits. As a result, even though our analysis of credit capacity proxies the potential support the firm may provide to the real economy under different policies, it does not capture how that interacts with its risk-taking incentives. Second, the firm's optimal responses in our model perfectly anticipate the policy the regulator will enact in each possible state. A non-trivial extension of the model could explore the case where, if the economy deteriorates, the regulator may refrain from intervening with a certain probability. Although these extensions lie beyond the scope of the current paper, they open promising avenues for future research.

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A Proofs and derivations

A.1 Proof of Proposition 1

1. We divide the proof into the following steps.

   (i) Integrating $v'(\cdot,i) = \beta_i$ in $[\tilde{x}_i, x]$ yields the expression of $v(\cdot,i)$ in the interval $[\tilde{x}_i,\infty)$ with $K_i := v(\tilde{x}_i,i) > 0$.

   (ii) In the interval $(\tilde{x}_1,\tilde{x}_2)$, our guess is that $v(\cdot,1)$ solves (14). Hence, it has the following structure:

$$v'(x,2) = \frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2} \tilde{\alpha}_h \tilde{A}_h e^{\tilde{\alpha}_h (x-\tilde{x}_1)}. \tag{39}$$

   with $\tilde{\alpha}_i$ as in (19). By imposing $v(\cdot,2) \in C^2$ at $x = \tilde{x}_2$, we obtain that $v'(\tilde{x}_2,2) = \beta_2$ and $v''(\tilde{x}_2,2) = 0$, that reflect in the conditions

$$\sum_{h=1}^{2} \tilde{\alpha}_h \tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} = \beta_2 - \frac{\lambda_2}{\delta+\lambda_2}, \qquad \sum_{h=1}^{2} \tilde{\alpha}_h^2 \tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} = 0. \tag{40}$$

   (iii) To show that $v''(\cdot,2) < 0$ in $(\tilde{x}_1,\tilde{x}_2)$, we set $u(s) = v'(\tilde{x}_2 - s, 2)$ and $w(s) = v''(\tilde{x}_2 - s, 2)$. Then, from (14) and the related boundary conditions, we have that

$$\begin{cases} u'(s) = -w(s), \qquad u(0) = \beta_2, \\ w'(s) = \frac{2}{\sigma_2^2}\left[\mu_2 w(s) - (\delta+\lambda_2)u(s) + \lambda_2\right], \qquad w(0) = 0. \end{cases}$$

   Given that $(\delta+\lambda_2)\beta_2 > \lambda_2$, an analysis of this system shows that $w'(s) < 0$ for $s > 0$. This means that $v'''(\cdot,2) > 0$ in $(\tilde{x}_1,\tilde{x}_2)$. We verify our claims by taking into account that $v''(\tilde{x}_2,2) = 0$.

   (iv) In the interval $(x_R,\tilde{x}_1)$, the functions $v'(\cdot,i)$ satisfy the coupled ODE system (44) with boundary conditions

$$v'(\tilde{x}_1,1)=1,\;\; v''(\tilde{x}_1,1)=0,\;\; v'(\tilde{x}_1,2)=\frac{\lambda_2}{\lambda_2+\delta}+\sum_{h=1}^{2}\tilde{\alpha}_h\tilde{A}_h,\;\; v''(\tilde{x}_1,2)=\sum_{h=1}^{2}\tilde{\alpha}_h^2\tilde{A}_h. \tag{41}$$

By plugging in (41) the guesses

$$v'(x,1)=\sum_{j=1}^{4}A_j\alpha_j e^{\alpha_j(x-\tilde{x}_1)},\quad v'(x,2)=\sum_{j=1}^{4}B_j\alpha_j e^{\alpha_j(x-\tilde{x}_1)}, \tag{42}$$

and matching coefficients, we obtain the characteristic equations

$$A_j\left(\alpha_j\mu_1+\frac{\sigma_1^2}{2}\alpha_j^2-(\delta+\lambda_1)\right)+\lambda_1B_j=0$$

and

$$B_j\left(\alpha_j\mu_2+\frac{\sigma_2^2}{2}\alpha_j^2-(\delta+\lambda_2)\right)+\lambda_2A_j=0,$$

for $j=1,2,3,4$. Solving the first equation yields B_j. Substituting B_j in the latter equation and rearranging yields (18). To verify that (18) has four real roots, let us define

$$f(\theta):=\underbrace{\left(\delta+\lambda_1-\theta\mu_1-\frac{\sigma_1^2}{2}\theta^2\right)}_{:=G_1(\theta)}\underbrace{\left(\delta+\lambda_2-\theta\mu_2-\frac{\sigma_2^2}{2}\theta^2\right)}_{:=G_2(\theta)}-\lambda_1\lambda_2,$$

and let θ_j^i be the roots of $G_i(\theta_j)$. It is straightforward to verify that $f(0)>0$, $f(\infty)>0$, $f(-\infty)>0$, and $f(\theta_j^i)=-\lambda_1\lambda_2<0$ for $i=1,2$ and $j=1,2,3,4$. Then, by continuity and using that $\theta_1^i\theta_2^i=-2\left(\delta+\lambda_i\right)/\sigma_i^2<0$, (18) has four different four roots, two positive and two negative. Then, (41) reads as

$$\begin{cases}\sum_{j=1}^4 A_j\alpha_j=1,\\ \sum_{j=1}^4 A_j\alpha_j^2=0,\\ \sum_{j=1}^4 B_j\alpha_j=\frac{\lambda_2}{\delta+\lambda_2}+\tilde{A}_1\tilde{\alpha}_1+\tilde{\alpha}_2\tilde{A}_2,\\ \sum_{j=1}^4 B_j\alpha_j^2=\tilde{A}_1\tilde{\alpha}_1^2+\tilde{\alpha}_2^2\tilde{A}_2.\end{cases} \tag{43}$$

We obtain then the expressions of $v(\cdot,i)$ by integrating (41) in $[x,\tilde{x}_1]$ and using that, by value matching,

$$v(\tilde{x}_1,2)=K_2+\frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2}+\sum_{j=1}^{2}\tilde{A}_j\left(1-e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}\right)$$

and having also set $K_1:=v(\tilde{x}_1,1)>0$.

(v) To show that $v''(\cdot,i)<0$ in $(x_R,\tilde{x}_1)$ under (21) we use that, in this interval, the functions $u(\cdot,i):=v'''(\cdot;i)$ solve the following ODE system:

$$\begin{cases}\frac{1}{2}\sigma_1^2u''(x,1)+\mu_1u'(x,1)-(\delta+\lambda_1)\,u(x,1)+\lambda_1u(x,2)=0,\\ \\ \frac{1}{2}\sigma_2^2u''(x,2)+\mu_2u'(x,2)-(\delta+\lambda_2)\,u(x,2)+\lambda_2u(x,1)=0.\end{cases}\tag{44}$$

The Feynman-Kac representation of u provides

$$u(x,i)=\mathbb{E}\left[e^{-\delta\tau}u(X_\tau^{x,i,\circ},I_\tau^i)\right],\tag{45}$$

where $\tau=\inf\{t\geq 0:\ X_t^{x,i,\circ}\notin(x_R,\tilde{x}_1)\}$, being $X_t^{x,i,\circ}$ the solution to

$$dX_t=\mu_{I_t}dt+\sigma_{I_t}dW_t,\qquad X_0^{x,i,\circ}=0.$$

Condition (21) entails $u(x_R,i)>0$ and $u(\tilde{x}_1,i)>0$ for all $i=1,2$. Thus, from (45) we get $u(\cdot,i)>0$. Hence, $v''(\cdot,i)$ is strictly increasing on $(x_R,\tilde{x}_1)$ for $i=1,2$. Since $v''(\tilde{x}_1^-,1)=0$ and $v''(\tilde{x}_1^-,2)<0$, we get that v'' is negative on $(x_R,\tilde{x}_1)$ and the claim follows.

(vi) Here we show that

$$v'(\tilde{x}_1,2)<\frac{\lambda_1+\delta}{\lambda_1}.\tag{46}$$

In the interval $(x_R,\tilde{x}_1)$, the function $v'(\cdot;1)$ solve

$$\frac{1}{2}\sigma_1^2v'''(x,1)+\mu_1v''(x,1)-(\delta+\lambda_1)\,v'(x,1)+\lambda_1v'(x,2)=0.\tag{47}$$

By (21), we have $v'''(\tilde{x}_1^-, 1) > 0$. Recalling also that $v''(\tilde{x}_1, 1) = 0$ and $v'(\tilde{x}_1, 1) = 1$, plugging all these information into (47), and passing to the limit as $x \to \tilde{x}_1^-$, we get $\lambda_1 v'(\tilde{x}_1, 2) > (\lambda_1 + \delta)$, which verifies the claim.

(vii) To identify the free parameters K_1 and K_2, we impose the following conditions:

$$\mathcal{L}_i v(\tilde{x}_i, i) - \lambda_i \left[v(\tilde{x}_i, i) - v(\tilde{x}_i, 3-i)\right] = 0, \quad i = 1, 2,$$

which are obtained by passing the equality $\mathcal{L}_i v(x, i) - \lambda_i \left[v(x, i) - v(x, 3-i)\right] = 0$ to the limit as $x \to \tilde{x}_i^-$. Since $v'(\tilde{x}_i, i) = \beta_i$ and $v''(\tilde{x}_i, i) = 0$, they rewrite as

$$\mu_2\beta_2 - (\delta + \lambda_2)K_2 + \lambda_2(K_1 + \tilde{x}_2 - \tilde{x}_1) = 0, \tag{48}$$

and

$$\mu_1 - (\delta + \lambda_1)K_1 + \lambda_1 \left(K_2 + \frac{\lambda_2}{\delta + \lambda_2}(\tilde{x}_1 - \tilde{x}_2)\right) = 0, \tag{49}$$

which completes the system (20).

(viii) Next, we show that the solution constructed in Points (i)-(vii) solves (9). Most of the work has already been done. Indeed, looking at the previous steps, we see that we have constructed v such that all the following are met: (a) $v(\cdot, i) \in C^2((x_R, \infty); \mathbb{R})$; (b) $v'(x, i) = \beta_i$ in $[\tilde{x}_i, \infty)$; (c) $\mathcal{L}_i v(\cdot, i) - \lambda_i(v(\cdot, i) - v(\cdot, 3-i)) = 0$ in $(x_R, \tilde{x}_i)$; (d) $v(\cdot, i)$ are concave, which entails $v'(\cdot, i) \geq \beta_i$ for $i = 1, 2$. So, it remains to show that

$$H(x, i) := \mathcal{L}_i v(x, i) - \lambda_i(v(x, i) - v(x, 3-i)) \leq 0, \qquad \forall x \in [\tilde{x}_i, \infty), \quad i = 1, 2.$$

First, we prove that $H(x, 2) \leq 0$ in $[\tilde{x}_2, \infty)$. Indeed, by (48), we have $H(\tilde{x}_2, 2) = 0$. Recalling that $v'(x, i) = \beta_i$ for $x \geq \tilde{x}_2$ and using that $\beta_2 > \lambda_2/(\lambda_2 + \delta)$, we have

$$H'(x, 2) = -(\lambda_2 + \delta)\beta_2 + \lambda_2 < 0, \quad \forall x \geq \tilde{x}_2.$$

Next, we prove that $H(x, 1) \leq 0$ in $[\tilde{x}_1, \infty)$. Indeed, by (49), we have $H(\tilde{x}_1, 1) = 0$.

Recalling that $v'(x,1) = 1$ for $x \in [\tilde{x}_1, \infty)$ and using concavity of $v(\cdot, 2)$ and (46), we get

$$H'(x,1) = -(\lambda_1 + \delta) + \lambda_1 v'(x,2) < 0, \quad \forall x \geq \tilde{x}_1.$$

2. The fact that (23) admits a unique solution in $(x_R, \tilde{x}_1]$ is due to the structure of $v(\cdot, 1)$ defined in Point 1 (notably the strict concavity of $v(\cdot, 1)$ in that interval) and the fact that $v'(\tilde{x}_2) = \beta_2 \leq 1$ together with (22) entailing $v'(\tilde{x}_R^+, 2) > 1$.

3. This is immediate as (24) is nothing but the rewriting of (13) given the structure determined in the previous points.

A.2 Proof of Theorem 1

We only sketch the proof, as a rigorous argument would be highly technical. We refer to two papers dealing with similar problems and providing complete proofs: Løkka and Zervos (2008), in the case of no regime switching and no recapitalization; Ferrari et al. (2022), in the case of regime-switching but with no recapitalization or dividend taxes.

As a first step, we prove that $v(x,i) \geq V(x,i)$. Let $A = (Z, (b, G)) \in \mathcal{A}$ be an arbitrary control and define $\tau_0 := 0$ and, recursively on $n \geq 0$, $\tau_{n+1} = \inf\{t \geq \tau_n : \ X_{t^-} = x_R\}$, being X_t the associated state process. Then, we have, by verification arguments in the interval $[0, \tau_1)$ (see Ferrari et al., 2022)

$$v(x,i) \geq \mathbb{E}\left[\int_0^{\tau_1^-} e^{-\delta t} \beta_{I_t} dZ_t + e^{-\delta \tau_1} v(x_R, I_{\tau_1})\right]. \tag{50}$$

By using (50) and (13) we get

$$v(x,i) \geq \mathbb{E}\left[\int_0^{\tau_1^-} e^{-\delta t} \beta_{I_t} dZ_t + e^{-\delta \tau_1} \left(v(x_R + G_1; I_{\tau_1}) - G_1 - \kappa\right)\right].$$

Iterating the argument yields

$$v(x,i) \geq \mathbb{E}\left[\int_0^{\tau_n^-} e^{-\delta t} \beta_{I_t} dZ_t - \sum_{k=1}^{n} e^{-\delta \tau_k} \left(G_k + \kappa\right) + e^{-\delta \tau_n} v(x_R, I_{\tau_n})\right].$$

Letting $n \to \infty$ and observing that $\tau_n \to \infty$, we get $v(x,i) \geq J(x,i;A)$. By arbitrariness of A, we conclude this part of the proof.

As a second step, we take $A = \hat{A}$ into Step 1. Then, by construction, all previous inequalities become equalities, which allows us to conclude that $v(x,i) = J(x,i;\hat{A})$. Together with Step 1, this last condition entails $J(x,i;\hat{A}) = v(x,i) = V(x,i)$, which verifies our first claim.

A.3 Proof of Proposition 2

To obtain $\pi(\cdot,i)$ over the intervals $(x_R, \tilde{x}_i)$ for $i = 1,2$, we plug in (27) the following guesses:

$$\pi(x,1) = \sum_{j=1}^{4} P_j e^{r_j x}, \; \pi(x,2) = \sum_{j=1}^{4} Q_j e^{r_j x}.$$

Matching coefficients and solving for P_j and Q_j yields the characteristic equation (31) and the relationship $Q_j = \lambda_1^{-1} F(r_j) P_j$. Similarly, we obtain $\pi(\cdot,2)$ over the interval $(\tilde{x}_1, \tilde{x}_2)$ and (32) by plugging in (28) the guess

$$\pi(x,2) = \sum_{j=1}^{2} H_j e^{s_j x}$$

and matching coefficients. Utilizing these equations to impose the boundary and mass preservation conditions as they appear in the main text yields (33).

A.4 Solution structure with dividend bans

First, we show that when $\beta_2 \leq \lambda_2/(\lambda_2 + \delta)$, the solution structure in Proposition 1 does not hold. For this purpose, we solve the linear system (40) to obtain

$$\begin{bmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{bmatrix} = \left(\beta_2 - \frac{\lambda_2}{\delta + \lambda_2}\right) \begin{bmatrix} \frac{e^{\tilde{\alpha}_1(\tilde{x}_1 - \tilde{x}_2)}}{\tilde{\alpha}_1(\tilde{\alpha}_2 - \tilde{\alpha}_1)} \\ -\frac{e^{\tilde{\alpha}_2(\tilde{x}_1 - \tilde{x}_2)}}{\tilde{\alpha}_2(\tilde{\alpha}_2 - \tilde{\alpha}_1)} \end{bmatrix}.$$

Plugging these coefficients in (39) and rearranging, we get

$$v'(x,2) = \frac{\lambda_2}{\delta+\lambda_2} + \underbrace{\frac{e^{-\tilde{\alpha}_1(\tilde{x}_2-x)} - e^{-\tilde{\alpha}_2(\tilde{x}_2-x)}}{\tilde{\alpha}_2-\tilde{\alpha}_1}}_{>0} \underbrace{\left(\beta_2 - \frac{\lambda_2}{\delta+\lambda_2}\right)}_{\leq 0}.$$

which violates the optimality condition that $v'(x,2) > \beta_2$ in $(\tilde{x}_1, \tilde{x}_2)$.

Second, we build an alternative solution of (9) in which $\tilde{x}_2 = \infty$, and show that it is consistent with the parametric restriction $\beta_2 \leq \lambda_2/(\lambda_2+\delta)$. Following the same steps of Section 4.1, we look for a function v such that, for $i = 1,2$, the following hold:

a) $v(\cdot,i) \in C([x_R,\infty)) \cap C^1((x_R,\infty))$;

b) The associated continuation and intervention regions have the following structure:

$$\mathcal{C}_2 = (x_R,\infty), \qquad \mathcal{C}_1 = (x_R,\tilde{x}_1); \quad \mathcal{S}_1 = [\tilde{x}_1,\infty);$$

c) $v(\cdot,1) \in C^2(\mathcal{C}_1)$ and $v(\cdot,2) \in C^2(\mathcal{C}_2 \setminus \{\tilde{x}_1\})$.

Accordingly, we have $\tilde{x}_1 = \inf\{x > x_R : v'(x,1) \leq 1\}$ and $\tilde{x}_2 = \infty$. The boundary conditions at x_R are specified as in the main text.

Following the same approach of Appendic A.1, we now construct a function that fulfils all these guesses and translates them into a list of algebraic requirements. In the interval $[\tilde{x}_1,\infty)$, we set $v'(\cdot,i) = 1$ and

$$v'(x,2) = \frac{\lambda_2}{\delta+\lambda_2} + \tilde{\alpha}_1 \tilde{A}_1 e^{\tilde{\alpha}_1(x-\tilde{x}_1)},$$

where $\tilde{\alpha}_1 < 0$ is the negative root of (19) and $\tilde{A}_1 < 0$. In the interval $(x_R,\tilde{x}_1)$, we set the functions $v'(\cdot,i)$ as unique solutions to the system (15). This entails the same structure as in (42), whose coefficients solve the following linear system:

$$\begin{cases} \sum_{j=1}^4 A_j\alpha_j = 1, \\ \sum_{j=1}^4 A_j\alpha_j^2 = 0, \\ \sum_{j=1}^4 B_j\alpha_j = \frac{\lambda_2}{\delta+\lambda_2} + \tilde{A}_1\tilde{\alpha}_1, \\ \sum_{j=1}^4 B_j\alpha_j^2 = \tilde{\alpha}_1^2\tilde{A}_1. \end{cases}$$

By integrating $v'(\cdot, i)$ we get

$$v(x,1) = K_1 + \begin{cases} (x-\tilde{x}_1), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^4 A_j \left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1), \end{cases}$$

$$v(x,2) = K_2 + \begin{cases} \frac{\lambda_2}{\delta+\lambda_2}(x-\tilde{x}_1) + \tilde{A}_1\left(e^{\tilde{\alpha}_1(x-\tilde{x}_1)} - 1\right), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^4 B_j \left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1). \end{cases}$$

Under similar assumptions as Proposition 1 and using that $v(\cdot, i)$ is differentiable, one gets that $G(1)^* = \tilde{x}_1 - x_R$ and $G(2)^* = x_2^* - x_R$, where $x_2^* \in (x_R, \infty)$ is the unique solution of

$$\mathbf{1}_{[x_R,\tilde{x}_1]} \sum_{j=1}^4 B_j\alpha_j e^{\alpha_j(x_2^*-\tilde{x}_1)} + \mathbf{1}_{(\tilde{x}_1,\infty)} \left(\frac{\lambda_2}{\delta+\lambda_2} + \tilde{\alpha}_1\tilde{A}_1 e^{\tilde{\alpha}_1\left(x_2^*-\tilde{x}_1\right)}\right) - 1 = 0.$$

To pin down the remaining coefficients ($K_1 > 0$, $K_2 > 0$, and $\tilde{A}_1 < 0$) and the dividend threshold ($\tilde{x}_1 > x_R$) we enforce that $\mathcal{L}_1 v(\tilde{x}_1, 1) - \lambda_1 [v(\tilde{x}_1, 1) - v(\tilde{x}_1, 2)] = 0$ and $\mathcal{L}_2 v(\tilde{x}_2, 2) - \lambda_2 [v(\tilde{x}_1, 2) - v(\tilde{x}_1, 1)] = 0$ as $x \to \tilde{x}_1^-$, which rewrites as

$$\begin{cases} \mu_2 \sum_{j=1}^4 B_j\alpha_j + \frac{\sigma_2^2}{2}\sum_{j=1}^4 B_j\alpha_j^2 - (\delta+\lambda_2) K_2 + \lambda_2 K_1 = 0, \\ \mu_1 - (\delta+\lambda_1)K_1 + \lambda_1 K_2 = 0, \end{cases}$$

and impose the two boundary conditions in (13).

B Supplementary material (online appendix)

B.1 Formal definition of the admissible control

The firm's control strategy is a triple of measurable stochastic processes $A \coloneqq (Z, b, G) = \big((Z_t)_{t\geq 0}, (b_n, G_n)_{n\geq 1}\big)$ such that:

(i) The cumulative dividend $(Z_t)_{t\geq 0}$ is a right-continuous, non-decreasing, $\mathbb{F}$-adapted process such that, setting $Z_{0^-} = 0$, each increment $\Delta Z_t \coloneqq Z_t - Z_{t^-} < X_t - x_{R,i}$, $\forall t \geq 0$ and $i = 1, 2$. This condition ensures that the firm can never issue dividends and equity simultaneously.

(ii) The auxiliary function $(b_n)_{n\geq 1}$ is a $\mathcal{F}_{\tau_n}$-measurable process taking values $b_n = 0$ or $b_n = 1$ if a liquidation or recapitalization takes place at time τ_n, respectively.

(iii) The equity issuance $(G_n)_{n\geq 1}$ is a strictly positive $\mathcal{F}_{\tau_n}$-measurable process representing the new equity issued at time τ_n when $b_n = 1$.

B.2 Solution structure with counter-cyclical capital requirements

Let us fix $\tilde{x}_1, \tilde{x}_2$ such that $x_{R,2} < x_{R,1} < \tilde{x}_1 < \tilde{x}_2$, Then, the model's solution structure encompasses the following four regions: (i) $x \in (x_{R,2}, x_{R;1})$, (ii) $x \in (x_{R,1}\tilde{x}_1)$, (iii) $x \in (\tilde{x}_1, \tilde{x}_2)$, and (iv) $x \in (\tilde{x}_2, \infty)$.

In Region (i), under the assumption that κ is small enough (i.e., it is always optimal to recapitalize), we set

$$v(x, 1) = v(\tilde{x}_1, 1) - (\tilde{x}_1 - x) - \kappa > 0. \tag{51}$$

Given (51) and (37), we guess and verify that $v(\cdot, 2)$ has the following form:

$$v(x, 2) = \frac{\mu_2 \lambda_2}{(\delta + \lambda_2)^2} + \frac{\lambda_2}{\delta + \lambda_2} \left(v(\tilde{x}_1, 1) - (\tilde{x}_1 - x) - \kappa\right) + C_1 e^{\tilde{\alpha}_1 (x - \tilde{x}_1)} + C_2^{\tilde{\alpha}_2 (x - \tilde{x}_1)},$$

where $(C_1, C_2) \in \mathbb{R}^2$ are two constants coefficients given below and $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are the real roots of (19).We find $v(\cdot, i)$ in Regions (ii)-(iv) by using (16) and (17) with $x_R = x_{R,1}$.

By using these equations to impose the usual boundary (value matching, smooth pasting, and super-contact) conditions, we get the following linear system:

$$
\begin{cases}
\sum_{h=1}^{4} A_h \alpha_h - 1 = 0, \\
\sum_{h=1}^{4} A_h \alpha_h^2 = 0, \\
\sum_{h=1}^{4} B_h \alpha_h - \sum_{h=1}^{2} \tilde{A}_h \tilde{\alpha}_h - \frac{\lambda_2}{\delta+\lambda_2} = 0, \\
\sum_{h=1}^{2} \tilde{A}_h \tilde{\alpha}_h^2 e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} = 0, \\
\sum_{h=1}^{4} B_h \alpha_h^2 - \sum_{h=1}^{2} \tilde{A}_h \tilde{\alpha}_h^2 = 0, \\
\frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2} \tilde{A}_h \tilde{\alpha}_h e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)} - \beta_2 = 0, \\
\frac{\mu_2\lambda_2}{(\delta+\lambda_2)^2} + \sum_{h=1}^{2} \left[C_h e^{\tilde{\alpha}_h\left(x_{R,1}-\tilde{x}_1\right)} - \tilde{A}_h(1-e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)}) \right] - K_1 + \\
+\frac{\lambda_2}{\delta+\lambda_2}\left(K_1 - (\tilde{x}_1 - x_{R,1}) - \kappa\right) - \frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2} - \sum_{h=1}^{4} B_h \left(e^{\alpha_h(x_{R,1}-\tilde{x}_1)} - 1\right) = 0, \\
\frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2} \tilde{\alpha}_h e^{\tilde{\alpha}_h\left(x_{R,1}-\tilde{x}_1\right)} - \sum_{j=1}^{4} B_j \alpha_j e^{\alpha_j(x_{R,1}-\tilde{x}_1)} = 0, \\
\mu_2\beta_2 - (\delta+\lambda_2)K_2 + \lambda_2(K_1 + \tilde{x}_2 - \tilde{x}_1) = 0, \\
\mu_1 - (\delta+\lambda_1)K_1 + \lambda_1\left(K_2 + \frac{\lambda_2}{\delta+\lambda_2}(\tilde{x}_1-\tilde{x}_2)\right),
\end{cases}
$$

whose solution (if it exists) yields $(A_1, A_2, A_3, A_4, \tilde{A}_1, \tilde{A}_2, C_1, C_2) \in \mathbb{R}^8$ and $(K_1, K_2) \in \mathbb{R}_+^2$.

By using these coefficients to compute $v(\cdot, i)$, we can obtain the recapitalization target x_2^* and the payout thresholds $\tilde{x}_1 = x_1^*$ and $\tilde{x}_2$ by imposing the boundary conditions (13) and the optimality condition $v'(x_2^*, 2) = 1$, which requires to solve the following non-linear system:

$$\begin{cases} \mathbf{1}_{[x_{R,2},\tilde{x}_1]}\frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2}\left[\mathbf{1}_{[x_{R,2},x_{R,1})}C_h e^{\tilde{\alpha}_h(x-\tilde{x}_1)} + \mathbf{1}_{(x_{R,1},\tilde{x}_1]}\tilde{A}_h\tilde{\alpha}_h\left(e^{\tilde{\alpha}_h(x_2^*-\tilde{x}_1)}\right)\right] + \\ +\mathbf{1}_{(\tilde{x}_1,\infty)}\sum_{h=1}^{4} B_h\alpha_h e^{\alpha_h(x_2^*-\tilde{x}_1)} - 1 = 0 \\ \sum_{h=1}^{4} A_h\left(e^{\alpha_h(x_{R,1}-\tilde{x}_1)} - 1\right) + \tilde{x}_1 - x + \kappa = 0 \\ \frac{\mu_2\lambda_2}{(\delta+\lambda_2)^2} + \frac{\lambda_2}{\delta+\lambda_2}\left(K_1 + x - \tilde{x}_1 - \kappa\right) + \sum_{h=1}^{2} C_h e^{\tilde{\alpha}_h\left(x_{R,1}-\tilde{x}_1\right)} + \\ +\mathbf{1}_{[x_{R,2},x_{R,1})}\left[\frac{\mu_2\lambda_2}{(\delta+\lambda_2)^2} + \frac{\lambda_2}{\delta+\lambda_2}\left(K_1 - \tilde{x}_1 + x_2^* - \kappa\right) + \sum_{h=1}^{2} C_h e^{\tilde{\alpha}_h\left(x_2^*-\tilde{x}_1\right)}\right] - \\ +\mathbf{1}_{(x_{R,1},\tilde{x}_1]}\left[K_2 + \frac{\lambda_2(x_2^*-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{h=1}^{2}\tilde{A}_h\left(e^{\tilde{\alpha}_h(x_2^*-\tilde{x}_1)} - e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)}\right)\right] - \\ +\mathbf{1}_{(\tilde{x}_1,\infty)}\left[K_1 + \frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{h=1}^{2}\tilde{A}_h(1 - e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)}) + \sum_{h=1}^{4} B_h\left(e^{\alpha_h(x_2^*-\tilde{x}_1)} - 1\right)\right] + \\ + (x_2^* - x) + \kappa = 0. \end{cases}$$

B.3 Solution structure with pro-cyclical capital requirements

Pro-cyclical capital requirements associate with the parametric condition $\Gamma_1 < \Gamma_2$ or, equivalently, $x_{R,1} < x_{R,2}$. Thus, modeling this case requires us to expand the support of the firm's reserves to include the region $x \in (x_{R,1}, x_{R,2})$. The rest of the state space when $x > x_{R,2}$ is that described in Section 4.1 after setting $x_{R,2} = x_R$.

When $x \in (x_{R,1}, x_{R,2})$ and there is a random transition from State 1 to State 2, the regulatory constraint $x > x_{R,2}$ is not satisfied. Thus, we assume the regulator requires the firm to immediately recapitalize or default and, following the logic of Section 5.3.1, set

$$v(x,2) = \max\left\{v(x_2^*, 2) - (x_2^* - x) - \kappa, 0\right\}. \tag{52}$$

Given (52), one can find $v(x, 1)$ as the unique solution of

$$\frac{1}{2}\sigma_1^1 v''(x,1) + \mu_1 v'(x,1) - (\delta+\lambda_1)\, v(x,1) + \lambda_1 v(x,2) = 0,$$

with boundary conditions $v(x_{R,2}^+, 1) = v(x_{R,2}^-, 1)$ and $v'(x_{R,2}^+, 1) = v'(x_{R,2}^-, 1)$.

Unlike the case of counter-cyclical capital requirements, the parametric condition (36) does *not* ensure that the firm is always willing to recapitalize after a change of state. In

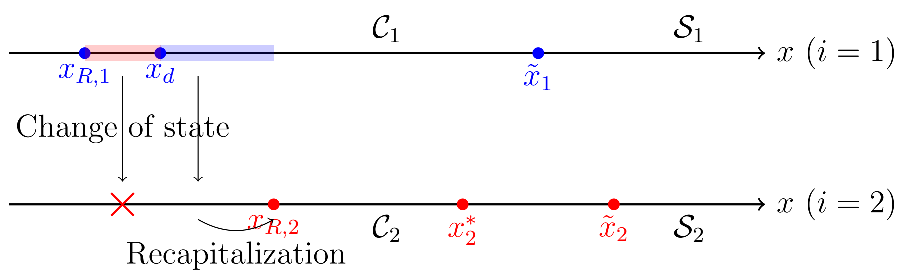

Figure 6: Solution structure with pro-cyclical capital requirements.

other words, there may be some reserves level $x_d \in (x_{R,1}, x_{R,2})$ such that (52) equals zero. Figure 6 visually represents this issue.

To adapt the solution structure, we split the region $x \in (x_{R,1}, x_{R,2})$ into the following two sub-intervals: $x \in (x_{R,1}, x_d)$ and $x \in (x_d, x_{R,2})$. In the former, we find $v(x,1)$ by solving

$$\frac{1}{2}\sigma_1^1 v''(x,1) + \mu_1 v'(x,1) - (\delta + \lambda_1)\, v(x,1) + \lambda_1 \left(v(x_2^*,2) - v(x_2^*,2) - x_2^* + x\right) = 0$$

with boundary conditions $v(x_{R,2}^+,1) = v(x_{R,2}^-,1)$ and $v'(x_{R,2}^+,1) = v'(x_{R,2}^-,1)$. In the latter, we solve

$$\frac{1}{2}\sigma_1^1 v''(x,1) + \mu_1 v'(x,1) - (\delta + \lambda_1)\, v(x,1) = 0$$

with boundary conditions $v(x_d^+,1) = v(x_d^-,1)$ and $v'(x_d^+,1) = v'(x_d^-,1)$. We find the endogenous threshold x_d by finding the reverse level so that (52) equals zero, which yields $x_d = \kappa + x_2^* - v(x_2^*,2)$.

B.4 Counter-cyclical capital requirements: comparative statics

Table 8 reports the firm's optimal strategy, its credit capacity and dispersion, and the maximal incentive-compatible recapitalization cost for different levels of Γ_2 ($x_{R,2}$).

The results of this analysis are broadly consistent with those of the mean-preserving counter-cyclical capital requirements discussed in the main text. However, the policy's effects are more straightforward, as it does not impose tighter capital requirements in the good state. Specifically, relaxing capital requirements in the bad state lowers the dividend payout threshold in both states (Columns 2 and 3). At the same time, it diminishes the firm's

$x_{R,1}$	$x_{R,2}$	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*	$\mathbb{E}_1^\pi[x]$	$\mathbb{E}_2^\pi[x]$	$\mathbb{E}^\pi[x]$	$\mathbb{V}_1^\pi[x]$	$\mathbb{V}_2^\pi[x]$	$\mathbb{V}^\pi[x]$	$\bar{\kappa}$
0.077	0.069	0.799	1.691	0.646	0.565	0.968	0.726	0.028	0.162	0.130	0.104
-	0.057	0.797	1.687	0.637	0.564	0.958	0.722	0.027	0.163	0.119	0.102
-	0.049	0.795	1.674	0.629	0.562	0.950	0.717	0.027	0.163	0.118	0.101
-	0.037	0.794	1.666	0.622	0.561	0.941	0.713	0.027	0.163	0.116	0.099
-	0.027	0.792	1.657	0.612	0.559	0.932	0.709	0.027	0.163	0.115	0.098

Table 8: Firm's optimal strategy, its credit capacity and dispersion, and the maximal incentive-compatible recapitalization cost for different levels of Γ_2 ($x_{R,2}$).

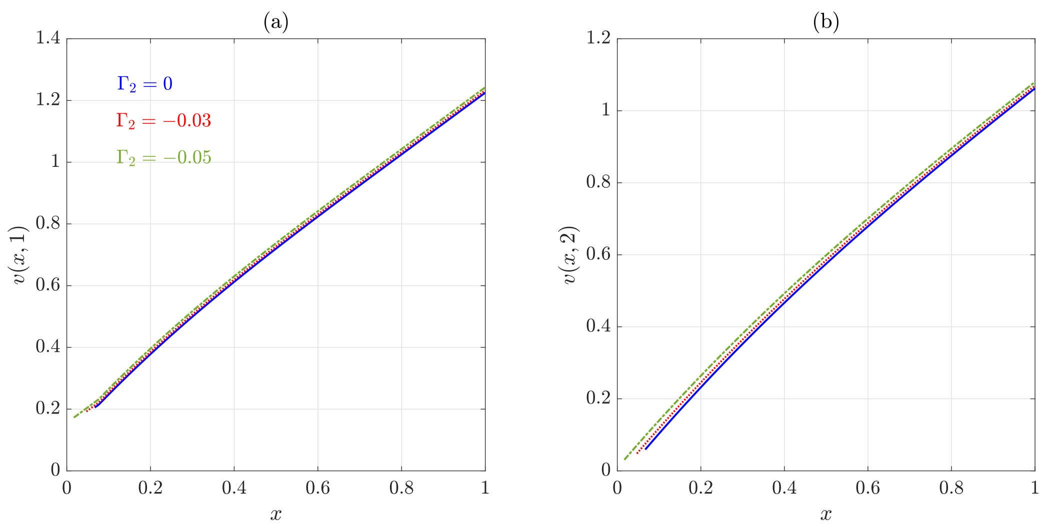

Figure 7: Effect of counter-cyclical capital regulation on shareholder value in the good (Panel (a)) and bad (Panel (b)) states.

recapitalization incentives (see Columns 5 and 12). Consequently, the policy reduces the average credit capacity and its dispersion across states. Figure 7 reports $v(x,i)$ for different values of Γ_2 (corresponding to Row 3 and 5 of Table 8) in State 1 (Panel (a)) and 2 (Panel (b)), showing that relaxing capital requirements increases the firm's value in both states.