Trading Deficits for Investment: Optimal Deficit Rules for Present-Biased Governments

Tobias Bergmann*& Nikolaj Moretti†

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Abstract

We develop a simple two-period principal-agent model in which a present-biased government, the agent, chooses public investment levels given a deficit rule imposed by the principal. The principal sets a deficit cap to curb current debt-financed consumption. In doing so, it also reduces long-term government investment. We characterize the optimal deficit rule that balances these opposing effects. Our analysis yields three key insights. First, a deficit rule is always a second-best instrument resulting in nonzero deficits and inefficiently low public investment. Second, while identifying the optimal deficit rule is challenging in practice, we demonstrate that under general conditions, shocks to the productivity of public investment entail an increase in the optimal deficit cap. Third, we compare the welfare effects of three fiscal rules: a balanced budget rule, the absence of any deficit rule, and a benchmark deficit rule. The benchmark deficit rule limits the agent's deficit to the level incurred by an agent without present bias. For moderate levels of present bias, the absence of a deficit rule leads to higher welfare than the balanced budget rule. The absence of a rule is consistently welfare-dominated by the benchmark deficit rule. Only in cases of substantial present bias does the balanced budget rule result in higher welfare than the benchmark deficit rule.

Keywords: public debt, fiscal rules, present bias, principal-agent, public investment

JEL codes: C72, D72,D82, E62, H30, H41, H63

*Email: tobias.bergmann@pik-potsdam.de, Potsdam Institute for Climate Impact Research (PIK Potsdam), Telegrafenberg A 31, 14473 Potsdam, Germany, and Technical University Berlin, Germany.

†Corresponding author. Email: nikolaj.moretti@pik-potsdam.de, PIK Potsdam and University of Potsdam, Germany.

1 Introduction

The number of countries adopting fiscal rules to restrict the government's ability to run deficits or countries bound by such rules through a supranational entity, has increased substantially since the 1990's (Eyraud et al., 2018; Yared, 2019). Yet, these rules are controversial. Supporters highlight their positive effects on fiscal rigor. Opponents claim they reduce governments' room for maneuver and ultimately reduce welfare. Historically, fiscal rules came up in the 1990s to harness the increase in the debt to GDP ratio observed in many advanced economies since the mid 1970s (Yared, 2019). In an overview article, Yared (2019) argues that this trend could not be explained by standard normative macroeconomic theories of tax smoothing (Barro, 1979; Lucas and Stokey, 1983), safe asset provision (Woodford, 1990; Aiyagari and McGrattan, 1998), or dynamic inefficiency (Diamond, 1965; Blanchard, 1985). Instead, it should be attributed to changes in political factors. He further explains how the competing explanations brought about by a large body of political economy literature all have in common that governments behave as if they had present-biased preferences (see e.g. Laibson, 1997, for a model of hyperbolic discounting that captures this type of preference). This is a powerful narrative, appealing due to its simplicity and intuitiveness (see e.g. Eyraud et al., 2018; Bachmann, 2024). We perceive it as the dominant narrative to justify the introduction and continuation of fiscal rules. However, it is incomplete. In a little noticed reaction to an influent model of political turnover by Tabellini and Alesina (1990) that illustrates the benefits of fiscal rules, Peletier et al. (1999) show how the introduction of a fiscal rule does not only harness government debt but also hampers government investment. Their analysis hinges on the simple idea that "budgetary institutions matter, not only for deficits, but also for public investment" (Peletier et al., 1999, p.1378). This argument is also put forward by the scientific advisory board of the German Ministry of Economic Affairs and Climate Action (Wissenschaftlicher Beirat beim BMWK, 2023) and the resulting deficit-investment trade-off was discussed between Professors Veronika Grimm and Adam Tooze in an important German newspaper.[1] Yet, surprisingly, this deficit-investment trade-off of deficit rules has gained little attention in the theoretical literature.

In this article, we contribute to this literature by providing a simple, conceptual model that sheds light on the deficit-investment trade-off of deficit rules. Its simplicity allows us to characterize the optimal deficit rule analytically and study its reaction to key variables. Furthermore, we contribute to the policy debate by formalizing a recurring criticism of deficit rules in a transparent and tractable way, thus hoping to

[1] "Die Schuldenbremse hätte nie in die Verfassung geschrieben werden dürfen", in: Frankfurter Allgemeine Zeitung, 24.07.2023

provide a point of departure for in-depth, quantitative analyses.

The structure of this article is as follows: In the next section, we locate our contribution in the fragmented theoretical literature on fiscal rules. In Section 3, we describe the model and the deficit-investment trade-off for present-biased governments. Section 4 characterizes the optimal deficit rule, which balances the positive welfare effect of reducing excessive consumption today with the negative future welfare effect of reduced investment. We show that the optimal deficit rule is a second best instrument. Furthermore, we analyze how the optimal deficit cap reacts to changes in the present bias and in the productivity of public investment. In Section 5, we compare different policy options and propose an approximation of the optimal deficit rule before concluding in Section 6.

2 Literature

The theoretical literature on the costs and benefits of fiscal rules predominantly focuses on the trade-off between commitment and flexibility. While fiscal rules are designed to increase the commitment of a government that acts in a time-inconsistent manner, the same commitment (or rigidity) may be unwelcome in moments of crisis or economic downturns, for instance by preventing counter-cyclical fiscal policy.

Amador et al. (2006) explore the commitment-flexibility trade-off in a general setting where individuals face temptation in a consumption-savings model. Halac and Yared (2014) study the optimal level of discretion in a fiscal policy model with a time-inconsistent present biased government. Their analysis is extended to a setting with international spillovers in Halac and Yared (2018) and limited options to enforce the budget rule in Halac and Yared (2022). Dotti and Janeba (2023) use a two-period model to investigate how an optimal deficit rule should accommodate fiscal shocks in the presence of a present-biased government and propose a rule that encompasses a zero structural deficit. Azzimonti et al. (2016) analyze the commitment-flexibility trade-off in a quantitative model for a balanced budget rule. Their political-economy model features a government with a present bias towards non-productive pork-barrel spending, where the benefits of a balanced budget rule include lower long-term debt servicing costs, while the drawbacks are reduced responsiveness in public good provision and increased tax volatility due to limited flexibility in responding to shocks. Although these studies share certain elements with our setup, they focus on the commitment-flexibility trade-off and exclude public investment or intertemporal public goods from their analyses. In our eyes, the reduction in public investment is another cost that should be taken into account when designing fiscal rules. For instance, the polar case of a balanced budget rule can be associated with inefficiently low levels of public investment and provision of durable public goods (Peletier et al., 1999; Bassetto and Sargent, 2006).

Another strand of literature studies the interplay between public debt, investment, and fiscal rules of so-called *golden rules of public finance*, a type of fiscal rule where admissible deficits are conditioned on the level of observed investment. In a seminal paper Bassetto and Sargent (2006) investigate the effects of the introduction of a golden rule in a fully-fledged dynamic model and show that it significantly increases efficiency if Ricardian equivalence does not hold. Bom (2019) explores a negative wealth effect of public investment in the presence of a balanced budget rule. In his setting, the lower market value of firms caused by the increase in public capital, more than offsets the benefits of the increased investment and decreases the current generation's welfare. Introducing a golden rule increases welfare for all generations. In contrast, we do not analyze the effects of a golden rule in our model, since our contribution lies precisely in analyzing the deficit-investment trade-off for fiscal rules that cannot be conditioned on the level of public investment. In practice, the definition of investment is blurry and governments can use *creative accounting* (see e.g. Milesi-Ferretti, 2004; von Hagen and Wolff, 2006) to circumvent conditional rules, e.g. of the golden rule type. This was one of the reasons why Germany abandoned a golden rule for one, that is more similar to a deficit rule (with additional counter-cyclical provisions, see e.g Feld (2024)).

The literature that analyzes the deficit-investment trade-off in the presence of debt-ceilings or deficit-caps, like a balanced budget rule, is very limited. To our knowledge, Peletier et al. (1999) are the first to highlight the deficit-investment trade-off but confine themselves to noticing that "a balanced-budget rule induces the median voter to invest too little" (Peletier et al., 1999, p.1380). In a quantitative model, Uchida and Ono (2021) study how a debt ceiling influences the distribution of the fiscal burden across generations. In their overlapping generations framework, a present bias is induced through the voting process that is influenced by short-sighted, egoistic elderly voters. Through the public education expenditures made by the government, Uchida and Ono (2021) include an intertemporal public investment decision of the type we consider. However, their sophisticated model makes it difficult to discern the mechanism we describe in this article. In particular, the authors do not study optimal deficit rules.

Three papers come closest to our setup. Beetsma and Debrun (2007) develop a theoretical model assessing the revised EU Stability and Growth Pact's impact on fiscal discipline and economic reforms, and, thereby, they analyze many relevant aspects of the deficit-investment trade-off. We extend their analysis in three ways: (i) we analytically derive the optimal deficit rule and provide comparative statics on key parameters, (ii) our simplified model allows for an explicit characterization of three policy alternatives, complementing their numerical approach with greater generality, and (iii) our model's simplicity highlights the core mechanism at play: the deficit-investment trade-off. Boyer et al. (2024) also present a closely related model, focusing on a political economy framework where parties compete for electoral support by directing resources to voter subgroups. While deficit caps constrain policymakers' ability to redistribute future gains, making reforms less appealing, we extend their work by investigating the optimal balance of the deficit-investment trade-off in this context. In concurrent research, Janeba (2025) explores the mixed empirical evidence on fiscal rules and investment, arguing that it does not contradict the existence of an investment-deficit trade-off. He also finds that a German constitutional court ruling, which tightened the national debt brake, negatively affected both public investment levels and its share in the total government budget.

In another related contribution, Bouton et al. (2020) study the effect of fiscal rules in a model with public debt and entitlements, capturing pensions and social security. By doing this, the authors formally capture the argument that focusing on government debt obfuscates the role that other government obligations like pensions play for matters of intergenerational distribution (see e.g Kotlikoff, 1988; Kotlikoff and Burns, 2012). On a more abstract level, the authors make a similar point as we do: By targeting public debt or deficits, one might overlook substitution relationships with other important variables, e.g. entitlements or, as in our case, public investment. Other important theoretical contributions on the effects of fiscal rules were made by Dovis and Kirpalani (2018), who study fiscal rules in the context of a federal state, and Hatchondo et al. (2022), who are concerned with sovereign defaults.

A growing empirical literature on the effectiveness and consequences of fiscal rules is surveyed in Potrafke (2023). Blesse et al. (2023) provide a survey of the literature focused on the consequences on public investment. Both conclude, that there is at best limited evidence for a negative effect of fiscal rules on public investment. However, the articles surveyed encompass analyses of all kinds of fiscal rules, e.g. with investment clauses. There seems to be empirical evidence for a negative effect of more rigid fiscal rules on public investment (Ardanaz et al., 2021). For the case of Germany, the scientific advisory board of the German Ministry of Economic Affairs and Climate Action concludes that the negative effect on investment of the current German deficit rule justifies a reform of the latter (Wissenschaftlicher Beirat beim BMWK, 2023).

3 The Deficit-Investment Trade-off

We analyze a two-period small open economy in which a government maximizes an intertemporal welfare function through spending and borrowing decisions. The resource constraints for each period are defined as follows:

$$c_1 + i = y_1 + b, \tag{1}$$

$$c_2 = y_2 + F(i, A) - R\, b. \tag{2}$$

Initial endowment in both periods is given by y_1 and y_2. The government allocates funds to consumption c_t and investment i. The latter is transformed into second-period resources via the production function $F(i, A)$. This function is continuous, with first derivative with respect to i $F_i > 0$, second derivative $F_{ii} < 0$, and third derivative $F_{iii} > 0$, and fulfills the Inada conditions. Additionally, the productivity of public investment, A, is modeled as a multiplicative factor in the production function: $F(i, A) = A \cdot \tilde{F}(i)$. The budget in the first period can be extended by issuing bonds, b, at the cost of reducing the second-period budget, with R representing the exogenous interest rate on bonds. As the model concludes after the second period, all government debt must be redeemed.[2] We assume perfect commitment. Note that once the first-period government has decided on its control variables c_1, i, and b, all second-period decisions are fully determined. The government then maximizes intertemporal welfare given by

$$W_a = u(c_1) + \beta\, \delta\, u(c_2). \tag{3}$$

We assume $0 \leq \beta \leq 1$ and interpret it as the government's degree of present bias (see Yared, 2019, for an overview of political economy models leading to a government acting *as if* it had present-biased preferences), which reduces the discount factor $0 \leq \delta \leq 1$ we use to compute our benchmark results. The utility function $u(\cdot)$ is continuous and thrice differentiable, with $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'''(\cdot) > 0$, and fulfills the Inada conditions. The government chooses c_1, i and b.[3] However, it is confronted with an upper limit $\bar{b}$ to the deficit it can incur, the deficit cap, so that it always holds that

$$b \leq \bar{b}. \tag{4}$$

We assume perfect enforcement of the deficit rule. The government maximizes the objective function (3) subject to the budget constraints (1) and (2) and the deficit

[2]The assumption of debt redemption is the analog of a transversality condition in an infinite horizon model. It is not necessary for our results to hold. For the qualitative results of this model to hold, it is sufficient that the bonds given out in the first period impose some (expected) cost in the second period, e.g. by increasing the risk of a sovereign debt crisis or expected inflation.

[3]Beetsma and Debrun (2007) formulate a model that is somewhat more realistic in that it encompasses households and a government that raises taxes to pay for a static public good and to alleviate the cost of structural reforms. We abstract from this to keep the model simple while capturing the essential characteristics of their model.

rule (4), where we denote the Lagrange-multiplier from this last constraint with μ. The first-order conditions for this maximization problem can be rearranged to give the following two equations:[4]

$$u'(c_1) = \beta\,\delta\,R u'(c_2) + \mu, \tag{5}$$

$$F_i = R + \frac{\mu}{u'(c_2)\,\beta\,\delta}. \tag{6}$$

To analyze the welfare effects of imposing a deficit cap on the government versus allowing it to act freely, we require a benchmark scenario. We use the decision of an agent without present bias and without a binding deficit cap as the benchmark. The first-order conditions of this benchmark scenario correspond to Equations (5) and (6) with $\beta = 1$ and $\mu = 0$. A government without present bias and no binding deficit cap chooses consumption according to a standard Euler Equation of consumption and determines investment such that the marginal product of the public capital stock equals the exogenously given interest rate. The following proposition establishes that the deficit is excessive in the absence of a deficit rule and that a binding deficit cap effectively harnesses the deficit and current consumption at the expense of public investment.

Proposition 1. *(i) In the absence of a binding deficit cap ($\mu = 0$) a present-biased agent ($\beta < 1$) incurs higher deficits and prioritizes current consumption at the expense of future consumption. Public investment is set at its efficient level, satisfying $F_i = R$.*

(ii) The introduction of a binding deficit cap ($\mu > 0$) reduces the ratio of first period to second period consumption c_1/c_2. Public investment falls below its efficient level, so that $F_i > R$.

Proof. (i) Follows directly from Equations (5) and (6) and the concavity of $u(\cdot)$ and $F(\cdot)$ for $\beta < 1$. (ii) Follows directly from Equations (5) and (6) and $\mu > 0$. □

Proposition 1 captures the deficit-investment trade-off for present-biased governments. First, in absence of a deficit rule, a present-biased agent would increase deficits to enable higher present consumption at the expense of future consumption, while investing efficiently into the public capital stock in order to maximize intertemporal resources. This aligns with the explanation by Yared (2019) for the rise in debt-to-GDP ratios between the mid-1970s and 2000s, as well as the narrative that debt rules are essential for harnessing debt-to-GDP ratios and safeguarding the interests of future generations against the preferences of the present. Second, a deficit rule can counteract the present bias distortion by limiting the skew toward first-period consumption.

[4] The strict concavity of the agent's objective function (3) ensures that the conditions are sufficient and that the maximum is unique.

However, this correction comes at a cost. A binding deficit cap introduces a wedge between the marginal product of investment F_i and the interest rate R, leading to a reduction in public investment below its efficient level. The interest of future generations is thus compromised by a reduction in public investment that might affect infrastructure, education, or climate mitigation. We refer to these two effects as the deficit-investment trade-off.

4 The Optimal Deficit Rule

In the previous section, we described the deficit-investment trade-off, demonstrating that imposing a binding deficit cap on a present-biased agent reduces excessive present consumption at the cost of distorting the investment decision. In this section, we examine how to optimally balance these conflicting effects. Following Amador et al. (2006), Halac and Yared (2014), Halac and Yared (2018), and Halac and Yared (2022), we formulate a principal-agent problem where the principal (e.g. an incumbent government with a constitutional majority) sets a binding debt ceiling to influence the decisions of the present-biased agent (e.g. a future government with a simple majority only) to maximize welfare. Hence, we model the introduction of a deficit rule as a game between two governments with different objective functions and control variables. First, we show that the principal always imposes a binding deficit-cap on the agent. Second, we characterize the optimal deficit-cap as a second best instrument. Third, we perform comparative statics and analytically show how the optimal deficit cap reacts to a change in the present bias β and a shock to the productivity of public investment A.

4.1 The Principal-Agent Problem

We analyze the case where the agent optimally chooses investment given an exogenous and binding deficit cap while the principal optimally chooses the deficit cap, anticipating its effect on the agent's investment decision. The case of a binding cap is not only the more interesting one but also the only relevant one since the principal always chooses a binding deficit cap in our model. This is the content of the following proposition:

Proposition 2. *If the agent is present biased (i.e. $\beta < 1$), the principal can increase welfare W_p by imposing a binding deficit cap.*

Proof. See Appendix A.2 □

We now formulate the principal-agent problem, assuming that the principal imposes a deficit cap $\bar{b}$ on the agent that is binding. Formally, we solve two maximization problems. The maximization problem of the principal is given by

$$\begin{aligned}\max_{\{\bar{b}\}}\; W_p =& u(c_1^*) + \delta\, u(c_2^*) \\ \text{s.t.}\quad c_1^* =& y_1 + \bar{b} - i^*(\bar{b}), \\ c_2^* =& y_2 - R\,\bar{b} + F(i^*(\bar{b}), A),\end{aligned} \tag{7}$$

where i^* is the solution to the agent's maximization problem given by

$$\begin{aligned}\max_{\{i\}}\; W_a =& u(c_1) + \beta\,\delta\, u(c_2) \\ \text{s.t.}\quad c_1 =& y_1 + \bar{b} - i, \\ c_2 =& y_2 - R\,\bar{b} + F(i, A),\end{aligned} \tag{8}$$

who takes $\bar{b}$ as exogenously given. Note that the principal acts as leader and the agent as follower in our setting. The only difference between the two objective functions is the present bias β and the different choice variables. Solving both maximization problems gives us two equations pinning down our two variables of interest: investment, i^*, and the deficit cap, $\bar{b}^*$. The optimality condition of the agent, which implicitly pins down a unique[5] i^* is

$$u'(c_1) = F_i u'(c_2)\beta\delta, \tag{9}$$

where both c_1 and c_2 are functions of i, too. The optimality condition of the principal, implicitly pinning down the optimal deficit cap $\bar{b}^*$, is given by:

$$(1 - i_b^*)u'(c_1^*) =\delta\, u'(c_2^*)\left(R - F_i(i^*, A)i_b^*\right), \tag{10}$$

where i_b^*, c_1^*, c_2^*, and i^* are functions of $\bar{b}$. Equation (10) determines a unique solution of the principal's maximization problem if $\partial^2 i^*/\partial\bar{b}^2 < 0$ (i.e. if i^* is a concave function of $\bar{b}$), which we will assume for the rest of the analysis.[6]

The optimal deficit cap maximizes the principal's welfare, taking into account its influence on the present biased agent's investment decision. Since we know that investment is inefficiently low for any binding deficit-cap from Equation (6), we immediately

[5] Existence and uniqueness follow from the concavity of (3).

[6] The second derivative of the principal's objective function W_p with respect to $\bar{b}$ is given by

$$\begin{aligned}\frac{\partial^2 W_p}{\partial \bar{b}^2} =& (1 - i_b^*)^2 u''(c_1^*) + \delta\left(F_i(i^*, A) - R\right)^2 u''(c_2) \\ &+ \delta F_{ii}(i^*, A)(i_b^*)^2 u'(c_2^*) + \delta(1-\beta)F_i(i^*, A)u'(c_2^*)\frac{\partial^2 i^*}{\partial \bar{b}^2}\end{aligned}$$

which is negative for all $\bar{b}$, if $\partial^2 i^*/\partial\bar{b}^2 < 0$. Note that this condition is sufficient but not necessary.

obtain the following corollary:

Corollary 1. *The optimal deficit cap $\bar{b}^*$ is a second best instrument. It implies that investment is inefficiently low, i.e. $F_i(i^*(\bar{b}^*), A) > R$.*

Corollary 1 highlights two key points. First, the optimal deficit rule is only a second-best instrument and cannot achieve the welfare level of the benchmark scenario (as evaluated by the principal). This limitation arises because the principal must balance two opposing forces using a single instrument, making the optimal solution inherently second best. Second, in this simple principal-agent framework, the corollary implies the following: observing a social rate of return F_i higher than the interest rate R is a characteristic of the second-best solution. It does not necessarily mean that the principal would achieve higher welfare by lifting the restrictions imposed on the agent.

4.2 Comparative Statics

We now examine how the optimal deficit cap $\bar{b}^*$ responds to changes in key parameters. More specifically, we analyze its comparative statics with respect to the present bias β and the productivity of public investment A. The response to the present bias β is particularly relevant, as the historical argument for introducing fiscal rules is based on the premise that, starting in the mid-1970s, governments in advanced economies exhibited an increasing degree of present bias. The impact of shocks to the productivity of public investment A is interesting, as it affects the benchmark deficit. Examples of such shocks include the emergence of new productivity-enhancing technologies dependent on public infrastructure (e.g., the internet), sudden shifts in the (international) security landscape, or new scientific evidence on the severity of future climate damages.

The results of this section require assumptions about the relative curvature of the utility function $u(\cdot)$ and production function $F(\cdot)$. For the sake of exposition, we simplify the expressions by assuming logarithmic utility, i.e. $u(\cdot) = log(\cdot)$ in this section. In this case, the assumption we must make (necessary for the results with respect to β, sufficient with respect to A) is

$$(1+\delta)F_i F_{iii} > (1+2\delta)\left(F_{ii}\right)^2. \tag{A}$$

Examples of functional forms that satisfy this assumption are the Cobb-Douglas function and the natural logarithm. We are now able to derive the following comparative statics results:

Proposition 3. *(i) Assume logarithmic utility, i.e. $u(\cdot) = log(\cdot)$ and (A) with respect* *to the curvature of $F(\cdot)$. Then:*

$$\frac{\partial \bar{b}^*}{\partial \beta} > 0$$

(ii) Additionally assume that the agent invests more after an increase in the productivity of public investment, i.e. $\frac{\partial i^*}{\partial A} > 0$. *Then:*

$$\frac{\partial \bar{b}^*}{\partial A} > 0$$

Note that Assumption (A) *is sufficient for both results but only necessary for* (i).

Proof. See Appendix A.3 □

Proposition 3 tells us two things. First, the higher the present bias (the lower β), the stricter is the optimal deficit cap the principal chooses. This result is intuitive: the stronger the agent's preference for present consumption due to the present bias, the more the principal will act to counterbalance this tendency by imposing a tighter cap. If the present bias of governments increased over the past decades, this could justify both the introduction and the tightening of fiscal rules during this period. Second, the higher the productivity of public investment, the higher the optimal deficit cap. An increase in investment productivity raises the returns to investment. If the agent reacts to this with an increase in investment, the principal finds it advantageous to relax the binding deficit cap. This result might be an interesting point of departure for discussions about fiscal rule reforms in times of improving scientific evidence about the severeness of future climate damages and increasing geopolitical tensions.

5 Policy

The optimal deficit rule introduced in the previous section might be difficult to implement in practice, since the exact level of the present bias of a specific government is not only unknown but can also vary over the course of time, e.g. when approaching the end of a legislative period. In this section, we thus compare the welfare effects of three specific policy instruments with low informational requirements:

1. A balanced budget rule.
2. The absence of a deficit rule.
3. A benchmark deficit rule, meaning that the agent is prevented from incurring a deficit higher than an agent without present bias would.

This comparison is motivated by the graphical illustration of the principal's objective function for different values of the agent's present bias β in Figure 1.[7] It illustrates

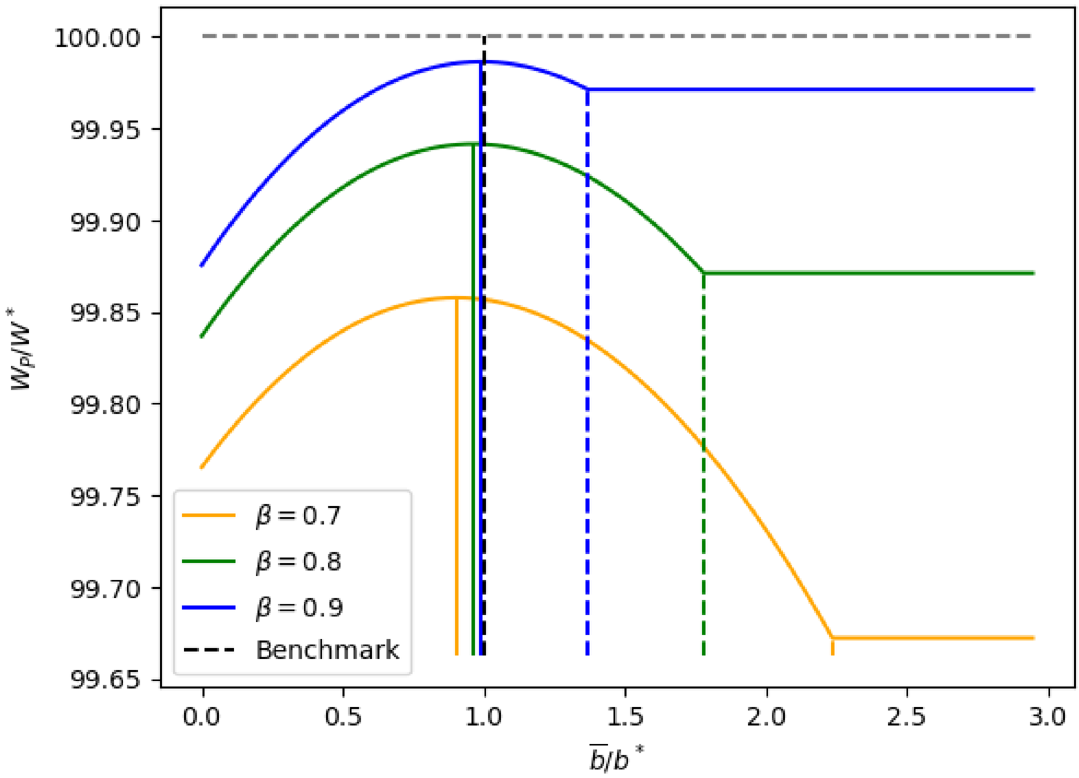

Figure 1: Welfare evaluated by the principal as a function of the imposed debt-ceiling $\bar{b}$ for different levels of present bias β. Solid vertical lines indicate optimal deficit-cap imposed by the principal, dashed vertical lines indicate the deficit level chosen by the agent in absence of a (binding) deficit-cap. Welfare on the y-axis is normalized by benchmark welfare. The deficit-cap on the x-axis is normalized by the benchmark deficit (vertical black dashed line).

the analytical results from the previous sections: First, in absence of a binding deficit-cap, the agent chooses higher deficit levels than in the benchmark scenario (dashed vertical lines). Second, the optimal deficit-cap (solid vertical lines) is a second-best instrument since welfare is always below its benchmark level (horizontal grey dashed line). Third, the optimal deficit cap decreases in the level of present bias (increases in β). Moreover, Figure 1 suggests that for a low present bias (e.g. $\beta = 0.9$), the principal is better-off by not constraining the agent (vertical dashed line) compared to imposing a balanced budget rule (zero-deficit). The opposite is true for higher present bias (e.g. $\beta = 0.7$). Furthermore, the graphical illustration suggests that the benchmark deficit is a promising approximation of the optimal deficit cap for moderate levels of present bias β. This observation is particularly interesting because, as discussed previously,

[7]For the graphical illustration, we use log-utility and Cobb-Douglas production and calibrate the simple 2-period model roughly to the US economy and a 5-year time period. The calibration is described in Appendix B.

the optimal deficit cap is a function of the present bias β of the agent which plausibly changes over time and is difficult to assess. Hence, ignoring the present bias and assuming that $\beta = 1$ might be a good rule of thumb to design a deficit rule.

Figure 1 suggests that there is a threshold value of present bias β', for which the principal prefers a balanced budget rule over the absence of a deficit rule and another, lower threshold value of β'', for which the principal prefers the balanced budget rule over the benchmark deficit rule. The graphical comparison suggests that β'' must be much smaller than β'. While these observations hinge on our calibration and choice of functional forms, the following proposition establishes some general results:

Proposition 4. *(i) The principal always prefers the benchmark rule to the absence of a rule*

(ii) There exists a level of present bias β' for which the principal is indifferent between the balanced budget rule and the absence of a rule. If β' is unique, the principal prefers the absence of a rule for all $\beta > \beta'$ and the balanced budget rule for all $\beta < \beta'$.

(iii) There exists a level of present bias β'' for which the principal is indifferent between the balanced budget rule and the benchmark rule. If β'' is unique the principal prefers the benchmark rule for all $\beta > \beta''$ and the balanced budget rule for all $\beta < \beta''$. If both β' and β'' are unique, then $\beta'' > \beta'$.

Proof. See Appendix A.4 □

Proposition 4 gives some conceptual guidance for situations in which we are not able to implement the optimal deficit-cap. With respect to a balanced budget rule, the most prominent version of a deficit-cap, it states that for low levels of present bias, the welfare loss caused by a reduction in investment outweighs the welfare gain from harnessing consumption and leads to net welfare losses. However, there is a present bias threshold for which it becomes preferable to introduce a balanced budget rule over letting the agent incur deficits at will. Furthermore, for a certain range of values of the present bias β, the benchmark deficit seems to be a good approximation of the optimal deficit-cap. Indeed, Proposition 4 establishes that the principal prefers the benchmark deficit over a balanced budget rule even for higher levels of present bias. How much higher? While we are not able to answer this question analytically, a rough calibration of our conceptual model (see Appendix B) suggests that the second threshold for β is indeed much lower than the first, i.e. the balanced budget rule is welfare superior to the benchmark deficit rule only for high degrees of present bias. Note that, since the benchmark deficit is equal to the optimal deficit cap for $\beta = 1$, it increases in the productivity of public investment as stated in Proposition 3.

6 Conclusion

In this article, we propose a simple two-period model of a present-biased government that captures an important deficit-investment trade-off of deficit rules, a type of fiscal rule that solely targets a government's deficit and does not provide exceptions for investment or other expenditure categories. We use a principal-agent formulation of the model to characterize the optimal deficit rule, i.e. the deficit rule that optimally balances the deficit-investment trade-off. This optimal deficit rule is a second best instrument and implies that, on the one hand, deficits are generally non-zero and, on the other hand, public investment is inefficiently low. We show analytically that the optimal deficit cap, i.e. the maximum admissible deficit under the deficit rule, negatively depends on the degree of present bias of the agent and positively depends on the productivity of public investment for plausible functional forms. Computing the optimal deficit cap in practice is difficult, since it depends on a government's present bias, an abstraction that captures a variety of distortions stemming from the political system that can lead to governments attributing more weight to the present than to the future. Therefore, we compare the welfare effects of three simple policies with low informational requirements: a balanced budget rule, a rule that imposes the benchmark level of deficit, and the absence of a deficit rule. We show that for low levels of present bias, a balanced budget rule is welfare-dominated by the absence of a deficit rule. The latter is always dominated by the benchmark deficit rule we propose. The benchmark deficit rule, in turn, seems to be welfare-dominated by the balanced budget rule only for substantial levels of present bias. With our model, we hope to make a valuable contribution not only to the theoretical literature on fiscal rules but also to the policy debate by transparently formalizing a recurring criticism on fiscal rules. Future research could investigate how the deficit-investment trade-off and the commitment-flexibility trade-off interact, how the benchmark deficit rule can be operationalized in a more realistic setting, and how it performs with respect to welfare in a richer, quantitative model.

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A Proofs

A.1 Proof Lemma 1

For the remainder of this section it will be useful to define some implicit derivatives in a Lemma.

Lemma 1. *The partial derivatives of the investment function of the agent, $i^*(\beta, \bar{b}, A)$ with respect to $\bar{b}$ and β are positive and given by*

$$i^*_{\bar{b}} = \frac{\partial i^*}{\partial \bar{b}} = \frac{u''(c_1) + F_i u''(c_2) R\beta\delta}{u''(c_1) + F_i^2 u''(c_2)\beta\delta + F_{ii} u'(c_2)\beta\delta} \in (0,1), \tag{11}$$

$$i^*_{\beta} = \frac{\partial i^*}{\partial \beta} = -\frac{F_i u'(c_2)\delta}{u''(c_1) + F_i^2 u''(c_2)\beta\delta + F_{ii} u'(c_2)\beta\delta} > 0. \tag{12}$$

The derivative with respect to A is given by

$$i^*_A = \frac{\partial i^*}{\partial A} = -\frac{\big(F_A F_i u''(c_2) + F_{iA} u'(c_2)\big)\beta\delta}{u''(c_1) + \big(F_i^2 u''(c_2) + F_{ii} u'(c_2)\big)\beta\delta}, \tag{13}$$

which is positive if $F_A F_i u''(c_2) + F_{iA} u'(c_2) > 0$. Inserting the optimality condition for the principal and the agent as well as log-utility, this condition can be rewritten only in terms of the production function and is given by

$$F_{ii} F\big(R - F_i\beta\big) + F_i^2\big(F_i - R\big)\beta\big(1+\delta\big) > 0. \tag{14}$$

The second-order derivatives are given by

$$\begin{aligned} i^*_{\beta\beta} = \frac{\partial^2 i^*}{\partial\beta\partial\beta} = \frac{F_i u'(c_2)\delta^2}{D^3}\Big(& F_i^4 \beta\delta\big(2\big(u''(c_2)\big)^2 - u'''(c_2)u'(c_2)\big) \\ &+ 2F_{ii} u'(c_2)\big(u''(c_1) + F_{ii} u'(c_2)\beta\delta\big) + F_i^2 u''(c_2)\big(2u''(c_1) + F_{ii} u'(c_2)\beta\delta\big) \\ &+ F_i u'(c_2)\big(u'''(c_1) - F_{iii} u'(c_2)\beta\delta\big)\Big), \end{aligned} \tag{15}$$

$$
\begin{aligned}
i^*_{\bar b \bar b} = \frac{\partial^2 i^*}{\partial \bar b \partial \bar b} = \frac{\beta\delta}{D^3}\Big(\\
&- \big(u''(c_1)\big)^2\big(F_{iii}u'(c_2) + F_{ii}u''(c_2)\big(3F_i - 2R\big) + F_i u'''(c_2)\big(F_i - R\big)^2\big) \\
&+ \Big(u'''(c_1)\big(F_i^2 u''(c_2) + F_{ii}u'(c_2)\big)^2 \\
&- 2R\Big(F_i^2\big(u''(c_2)\big)^2\big(2F_{ii}u''(c_1) + F_i u'''(c_1)\big) \\
&+ u'(c_2)\Big(F_{iii}F_i u''(c_1)u''(c_2) - F_{ii}^2 u''(c_1)u''(c_2) \\
&+ F_{ii}F_i\big(u''(c_2)u'''(c_1) - F_i u''(c_1)u'''(c_2)\big)\Big)\Big) \\
&+ F_i\beta\delta\Big(F_i u''(c_2)^2 u'''(c_1) + 2F_{ii}u''(c_1)\big(u''(c_2)^2 - u'''(c_2)u'(c_2)\big)\Big)R^2\Big) \\
&- F_i R^2\beta^2\delta^2\Big(F_{ii}F_i^2 u''(c_2)^3 + F_{iii}F_i u''(c_2)^2 u'(c_2) \\
&+ F_{ii}^2 u'(c_2)\big(u'''(c_2)u'(c_2) - 2u''(c_2)^2\big)\Big)\Big),
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
i^*_{\bar b A} = \frac{\partial^2 i^*}{\partial \bar b \partial A} = \frac{\beta\delta}{D^2}\Bigg(- \frac{1}{u''(c_1) + F_i u''(c_2)R\beta\delta}\Big(\\
&F_{ii}F_A u''(c_1)u''(c_2) + 2F_i F_{iA}u''(c_1)u''(c_2) \\
&+ F_A F_i u''(c_2)u'''(c_1) + F_A F_i^2 u''(c_1)u'''(c_2) \\
&+ F_{iiA}u''(c_1)u'(c_2) + F_{iA}u'''(c_1)u'(c_2) \\
&+ \beta\delta\Big(2F_i^2\big(-F_{ii}F_A + F_i F_{iA}\big)\big(u''(c_2)\big)^2 \\
&+ u''(c_2)u'(c_2)\big(F_{ii}^2 F_A + F_i\big(-F_{iii}F_A + F_{iiA}F_i\big) - F_{ii}F_i F_{iA}\big) \\
&+ F_i^2 u'''(c_2)u'(c_2)\big(F_{ii}F_A - F_i F_{iA}\big) \\
&+ \big(F_{ii}F_{iiA} - F_{iii}F_{iA}\big)\big(u'(c_2)\big)^2\Big)\Big) \\
&+ F_{iA}\Big(u'''(c_1)u'(c_2) + u''(c_1)u''(c_2)R + F_i^2\big(\big(u''(c_2)\big)^2 - u'''(c_2)u'(c_2)\big)R\beta\delta\Big) \\
&+ F_A F_i\Big(u''(c_2)u'''(c_1) + u''(c_1)u'''(c_2)R \\
&+ F_{ii}\big(-\big(u''(c_2)\big)^2 + u'''(c_2)u'(c_2)\big)R\beta\delta\Big)\Bigg),
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
i^*_{\bar b\beta} = \frac{\partial^2 i^*}{\partial \bar b \partial \beta} = \frac{\delta}{D^3}\Bigg(& F_i^4 u''(c_1)\beta\delta\big(-\big(u''(c_2)\big)^2 + u'''(c_2)u'(c_2)\big) \\
& - F_{ii}u''(c_1)u'(c_2)\big(u''(c_1) + F_{ii}u'(c_2)\beta\delta\big) \\
& + F_i^3\beta\delta\Big(u''(c_2)u'''(c_1)u'(c_2) - u'''(c_2)u'(c_2)R\big(u''(c_1) + F_{ii}u'(c_2)\beta\delta\big) \\
& + \big(u''(c_2)\big)^2 R\big(u''(c_1) + 2F_{ii}u'(c_2)\beta\delta\big)\Big) \\
& + F_i^2 u''(c_2)\Big(-\big(u''(c_1)\big)^2 + F_{ii}u''(c_1)u'(c_2)\beta\delta \\
& + u'(c_2)R\beta\delta\big(-u'''(c_1) + F_{iii}u'(c_2)\beta\delta\big)\Big) \\
& + F_i\Big(\big(u''(c_1)\big)^2 u''(c_2)R + F_{iii}u''(c_1)\big(u'(c_2)\big)^2\beta\delta \\
& + F_{ii}\big(u'(c_2)\big)^2\beta\delta\big(u'''(c_1) - F_{ii}u''(c_2)R\beta\delta\big)\Big)\Bigg),
\end{aligned} \tag{18}
$$

where $D = u''(c_1) + \big(F_i^2 u''(c_2) + F_{ii}u'(c_2)\big)\beta\delta$*.*

Proof. First, define the function $H(i, \beta, \bar b, A)$ by rearranging the optimality condition of the agent Equation (9)

$$
H(i, \beta, \bar b, A) := -u'(y_1 + \bar b - i) + F_i(i, A)u'(y_2 + F(i, A) - R\,\bar b)\beta\delta. \tag{19}
$$

Note that since $u(\cdot)$ and $F(\cdot)$ are thrice continuously differentiable, H is continuous in $i, \beta, \bar b$, and A (and twice continuously differentiable in these variables). Furthermore, for all $\beta_0, \bar b_0$, and A_0, there is an $i_0 \in (0, y_1 + \bar b_0)$ such that $H(i_0, \beta_0, \bar b_0, A_0) = 0$. This must hold, since for all $\beta_0, \bar b_0$, and A_0, $W_a(i)$ has a maximum at a value i_0, where necessarily $H(i_0, \beta_0, \bar b_0, A_0) = 0$. This follows from $u(c_1) \to -\infty$ for $i \to y_1 + \bar b_0$ and $F(i) \to \infty$ for $i \to 0$ and the strict concavity of the agent's objective function W_a in i. The derivative of H with respect to i is given by

$$
H_i(i, \beta, \bar b, A) = u''(c_1) + \beta\delta\left(F_{ii}(i, A)u'(c_2) + F_i(i, A)^2 u''(c_2)\right) < 0. \tag{20}
$$

It is strictly negative and in particular non-zero, which guarantees existence and uniqueness of the implicit function $i^*(\beta, \bar b, A)$ defined by Equation (9). The partial derivatives then follow from applying the implicit function theorem. From Equation (11) we also conclude that $i^*_{\bar b} < 1$, since $1 < R < F_i$. □

A.2 Proof Proposition 2

If the agent is present biased (i.e. $\beta < 1$*), the principal can increase welfare* W_p *by imposing a binding deficit rule.*

Proof. Without loss of generality, assume that the principal chooses the debt level b and not only a deficit cap $\bar{b}$ and that the agent chooses i and c_1 in reaction to that. (We will be able to generalize the results since they coincide with the choice of a debt cap for downwards adjustments of the deficit level). We prove the proposition by showing how a tightening of debt increases the principal's welfare W_p evaluated at the level of consumption, investment and debt chosen by a non-constrained agent, $c^*_{1,nc}, i_{nc}$ and b^*_{nc}. The proposition then follows once we remember that the principal is only able to decrease the deficit chosen by the agent but not increase it. It holds that:

$$\begin{aligned}\frac{\partial W_p}{\partial b}(c^*_{1,nc}, i^*_{nc}, b^*_{nc}) =& u'(c^*_{1,nc})(1 - i^*_b) + \delta u'(c^*_{2,nc})(-R + F_i\, i^*_b)\\ =& u'(c^*_{1,nc})(1 - i^*_b) - \delta\, R\, u'(c^*_{2,nc})(1 - i^*_b)\\ =& u'(c^*_{1,nc})(1 - i^*_b)(1 - 1/\beta) < 0\end{aligned}$$

where i^*_b is the derivative of the optimal investment decision of the agent i^* w.r.t to the debt level chosen by the principal. The second line follows from the first-order condition (6) in absence of a binding cap (i.e. $\mu = 0$) and the third line from (5) in absence of a binding cap. The last expression is negative since $\beta < 1$ and $0 < i^*_b < 1$ (see Lemma 1).

□

A.3 Proof Proposition 3

Proof. First, we define the function $G(\bar{b}^*, \beta, A)$ by rearranging the optimality condition of the principal (10) to receive

$$G(\bar{b}^*, \beta, A) = (1 - i^*_b)u'(c^*_1) - \delta\, u'(c^*_2)\,(R - F_i(i^*, A)i^*_b) = 0. \tag{21}$$

(i): Using the implicit function theorem we receive the partial derivative of $\bar{b}^*$ with respect to β given by

$$\frac{\partial \bar{b}^*}{\partial \beta} = -\frac{\frac{\partial G(\bar{b}^*,\beta,A)}{\partial \beta}}{\frac{\partial G(\bar{b}^*,\beta,A)}{\partial \bar{b}}}. \tag{22}$$

The denominator of Equation (22) is equal to the second-order condition of the principals maximization problem and has to be negative for a maximum, $\frac{\partial G(\bar{b}^*,\beta,A)}{\partial \bar{b}^*} = \frac{\partial^2 W_p}{\partial \bar{b}^2} < 0$. To prove that $\frac{\partial \bar{b}^*}{\partial \beta} > 0$, the numerator of Equation (22) has to be positive. We are thus left to prove that

$$\begin{aligned}\frac{\partial G(\bar{b}^*,\beta,A)}{\partial\beta} &= i^*_\beta\Big(i^*_b\delta\big(F_{ii}u'(c_2)+F_i^2u''(c_2)\big) \\ &\quad -\big(1-i^*_b\big)u''(c_1)-R\delta F_iu''(c_2)\Big) \\ &\quad +\big(\delta F_iu'(c_2)-u'(c_1)\big)i^*_{b\beta}>0.\end{aligned} \tag{23}$$

We can simplify this expression by using Lemma 1 and the optimality conditions of the agent and the principal given by Equations (9) and (10). Furthermore, if we apply log-utility and by this the following relations $u''(c_t)=u'(c_t)^2$ and $u'''(c_t)=2u'(c_t)^3$ we can simplify Equation (23) to

$$\frac{\partial G(\bar{b}^*,\beta,A)}{\partial\beta}=\frac{\delta(R-F_i\beta)^2}{(1-\beta)^2F_i^2(R+F_i\beta\delta)^2}\frac{(R-F_i)}{F_{ii}}\mathcal{A}. \tag{24}$$

Since, we know that $F_i>R$ from Proposition 1 and the sign of the derivatives of the production function, this expression is positive if (and only if) $\mathcal{A}:=(1+\delta)F_iF_{iii}-(1+2\delta)F_{ii}^2>0$ which is Equation (A) from the main text.

(ii): Analogous to (i), we use the implicit function theorem to receive the partial derivative of $\bar{b}^*$ with respect to A given by

$$\frac{\partial\bar{b}^*}{\partial A}=-\frac{\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A}}{\frac{\partial G(\bar{b}^*,\beta,A)}{\partial\bar{b}^*}}. \tag{25}$$

The denominator of Equation (25) is again equal to the second-order condition of the principals maximization problem and negative. To prove that $\frac{\partial\bar{b}^*}{\partial A}>0$, the numerator of Equation (25) has to be positive. We are thus left to prove that

$$\begin{aligned}\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} &= i^*_b\delta\Big(F_AF_iu''(c_2)+F_{iA}u'(c_2)\Big)-F_ARu''(c_2)\delta \\ &\quad +i^*_A\Big(\big(u''(c_1)+F_i^2u''(c_2)\delta+F_{ii}u'(c_2)\delta\big)i^*_b-u''(c_1)-F_iRu''(c_2)\delta\Big) \\ &\quad +i^*_{bA}\big(F_iu'(c_2)\delta-u'(c_1)\big)>0.\end{aligned} \tag{26}$$

We can simplify this expression by using Lemma 1 and by applying log-utility and by this the following relations $u''(c_t)=u'(c_t)^2$ and $u'''(c_t)=2u'(c_t)^3$ as well as linearity of A in $F(i,A)$, so that $F_A=\frac{F(i,A)}{A}$, $F_{iA}=\frac{F_i}{A}$ and $F_{iiA}=\frac{F_{ii}}{A}$. Furthermore, we insert the optimality conditions of the agent and the principal given by Equations (9) and (10).

Thus, we can rewrite Equation (26) to

$$\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} = -\frac{\big(R-F_i\beta\big)^2\delta}{AF_{ii}F_i^4\big(F_i-R\big)\big(\beta-1\big)^2\big(1+\delta\big)\big(R+F_i\beta\delta\big)^2}\Big[\tag{27}$$
$$- F_{ii}F_{iii}F_iF\big(F_i-R\big)\big(F_i\beta-R\big)\big(1+\delta\big)$$
$$+ F_{iii}F_i^3\big(F_i-R\big)^2\beta\big(1+\delta\big)^2$$
$$+ F_{ii}^3F\Big(R^2\big(1+2\delta\big)-2F_iR\big(1+\delta+\beta\delta\big)+F_i^2\beta\big(1+\delta+\beta\delta\big)\Big)$$
$$- F_{ii}^2F_i^2\big(1+\delta\big)\Big(R^2\beta\big(1+2\delta\big)+F_i^2\beta\big(1+\delta+\beta\delta\big)-F_iR\big(1+\beta+4\beta\delta\big)\Big)\Big]$$

Since, we know that the term in front of the parentheses is positive, we have to show that the expression in parentheses is positive as well. Rearranging this we receive

$$\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} = -\frac{\big(R-F_i\beta\big)^2\delta}{AF_{ii}F_i^4\big(F_i-R\big)\big(\beta-1\big)^2\big(1+\delta\big)\big(R+F_i\beta\delta\big)^2}\Big[\tag{28}$$
$$\Big(\big(F_i-R\big)\Big(F_{ii}F\big(F_i\beta-R\big)-F_i^2\big(F_i-R\big)\beta\big(1+\delta\big)\Big)\Big)$$
$$\Big(F_{ii}^2\big(1+2\delta\big)-F_iF_{iii}\big(1+\delta\big)\Big)$$
$$+ F_{ii}^2F_i\big(1-\beta\big)\big(R+F_i\beta\delta\big)\Big(F_i^2\big(1+\delta\big)-F_{ii}F\Big)\Big],$$

where the last line $F_{ii}^2F_i\big(1-\beta\big)\big(R+F_i\beta\delta\big)\Big(F_i^2\big(1+\delta\big)-F_{ii}F\Big) > 0$. The second line is Equation (A), that states $F_iF_{iii}\big(1+\delta\big) > F_{ii}^2\big(1+2\delta\big)$. To show that the first line is negative we know from Proposition 1 that $F_i > R$ and from Equation (14) in Lemma 1 that $F_{ii}F\big(F_i\beta-R\big)-F_i^2\big(F_i-R\big)\beta\big(1+\delta\big) < 0$. Thus, for $\frac{\partial i^*}{\partial A} > 0$ the first line is negative, resulting in an overall positive expression. Note that both conditions (A) and (14) are sufficient and not necessary to prove that $\frac{\partial G(\bar{b}^*,\beta,A)}{\partial A} > 0$. □

A.4 Proof Proposition 4

(i) The principal always prefers the benchmark rule to the absence of a rule

(ii) There exists a level of present bias β' for which the principal is indifferent between the balanced budget rule and the absence of a rule. If β' is unique, the principal prefers the absence of a rule for all $\beta > \beta'$ and the balanced budget rule for all $\beta < \beta'$.

(iii) There exists a level of present bias β'' for which the principal is indifferent between the balanced budget rule and the benchmark rule. If β'' is unique the principal prefers the benchmark rule for all $\beta > \beta''$ and the balanced budget rule for all $\beta < \beta''$. If both β' and β'' are unique, then $\beta'' > \beta'$.

Proof. In this proof, we formalize the graphical intuition that can be derived from Figure 1. Interpret the objective function of the principal (7) as function of both deficit cap $\bar{b}$ and present bias β, i.e. $W_p(\bar{b}, \beta)$.

Furthermore, let $\bar{b}^*(\beta)$ denote the optimal deficit cap for present bias β, $b^*(\beta)$ the choice of an agent with present bias β in absence of a deficit rule, and $b^*(1)$ the benchmark deficit. We prove the three parts of the proposition by using that, facing an agent with present bias β, the principal prefers policy option A associated with deficit level b_A to option B associated with deficit level b_B, if and only if $W_p(b_A, \beta) \geq W_p(b_B, \beta)$.

(i): First, from Proposition 3 we know that the optimal deficit cap is increasing in the level of β, i.e. $\partial\bar{b}^*/\partial\beta > 0$. Hence, for all $\beta < 1$, we know that the benchmark deficit (which is the optimal deficit for $\beta = 1$) is bigger than the optimal deficit cap, i.e. $b^*(1) \equiv \bar{b}^*(1) > \bar{b}^*(\beta)$. Second, we know that for a non-constrained agent, the incurred deficit is increasing in the level of present bias (decreasing in β), i.e. $\partial b^*/\partial\beta < 0$.[8] Hence, for all $\beta < 1$, the deficit chosen by the agent in absence of a deficit rule is higher than the benchmark deficit, $b^*(\beta) > b^*(1)$. The result follows since W_p is decreasing for $\bar{b} \,\epsilon\, [\bar{b}^*(\beta), b^*(\beta)]$ given our assumption that i^* is concave in $\bar{b}$.

(ii): We start by proving that there exists a $\beta' \,\epsilon\, [0, 1]$ for which $W_p(0, \beta') = W_p(b^*(\beta), \beta')$. Note that in the absence of present bias (i.e. $\beta = 1$), b^* is the optimal deficit and therefore $W_p(0, 1) \leq W_p(b^*(\beta), 1) = W_p(b^*(1), 1)$ and the inequality is strict if $b^* \neq 0$ because of the strict concavity of W_p in $\bar{b}$. Next, note that for perfect present bias $\beta = 0$ (i.e. the agent not caring about the second period), the agent would always transfer the maximum possible amount from period 2 to period 1 (including potential returns to investment) using debt. In addition, the agent would not invest anything as soon as the principal sets a deficit ceiling that is lower than second period endowment y_2. Thus, if $u(c) \to -\infty$ for $c \to 0$, the principal prefers any allocation of the intertemporal resources $y_1 + y_2/R$ that involves $c_1 > 0$ and $c_2 > 0$ over $c_1 = y_1 + y_2/R$ and $c_2 = 0$. In particular, the principal prefers a balanced budget rule over the absence of a deficit rule, i.e. $W_p(0, 0) > W_p(b^*(\beta), 0)$ for $\beta = 0$.

Define $\Delta W(\beta) := W_p(0, \beta) - W_p(b^*(\beta), \beta)$. It is thus the case that $\Delta W(0) = W_p(0, 0) - W_p(b^*(\beta), 0)$ is positive and for $\Delta W(1)$ is negative (equal to zero if $b^* = 0$). We now show that ΔW is continuous in β. Note that ΔW is continuous in β as soon as $W_p(\bar{b}, \beta)$ is continuous for any value of $\bar{b}$. The only part of $W_p(\bar{b}, \beta)$ that depends on β is the agent's optimal decision i^* that is implicitly defined in Equation (9) and

[8] To see this, consider the first order Equations for a non-constrained agent's decision problem which are given by Equations (5) and (6) for $\mu = 0$. Since investment is pinned down by R and does not change with present bias β, the only possibility to adjust the intertemporal consumption path to the benefit of c_2 with increasing β, as implied by the Euler Equation (5), is by reducing b.

which has been shown to be continuously differentiable in β in the proof of Lemma 1, thus in particular continuous. Since both utility $u(\cdot)$ and production function $F(\cdot)$ are assumed to be continuous, $W_p(\bar{b}, \beta)$ is continuous, too. The existence of a root $\beta' \epsilon [0, 1]$ then follows from Bolzano's theorem (corollary of intermediate value theorem).

If the root β' is unique, the fact that the principal prefers the balanced budget rule over the absence of a rule for $\beta < \beta'$ follows from $\Delta W(0) > 0$. By the same token, the principal prefers the absence of a rule over the balanced budget rule for $\beta > \beta'$ since $\Delta W(1) < 0$.

(iii): The proof of the existence of β'' is analogous to the proof of the existence of β' in *(ii)*. If β'' is unique, the preference relationship follows by the same argument as in *(ii)*, too. To see that indeed $\beta'' < \beta'$ if both β' and β'' are unique, note that from *(i)* we know that $W_p(0, \beta') = W_p(b^*(\beta), \beta') < W_p(\beta^*(1), \beta')$ and hence $\widetilde{\Delta W}(\beta') := W_p(0, \beta') - W_p(\beta^*(1), \beta') < 0$. The only way how this is compatible with $\widetilde{\Delta W}(0) > 0$ and $\widetilde{\Delta W}(1) < 0$ (which can be shown in the same way as for $\Delta W(\beta)$ in part *(ii)*) is that the unique root β'' is smaller than β'.

□

B Calibration

To facilitate the calibration, we assume that public investment contributes to the public capital stock K_t for the numerical analysis. The dynamics of the stock are given by

$$K_2 = (1 - \delta_K)K_1 + i, \tag{29}$$

where δ_K is the depreciation of public capital. We assume that utility is logarithmic and the production function from public capital Cobb-Douglas:

$$F(K_t) = \frac{A}{\alpha} \cdot K_t^{\alpha}. \tag{30}$$

We calibrate our model to match features of the US economy in 2022. Note that this is only a very rough calibration. Our goal is to make qualitative points. We do not interpret the results quantitatively.

We choose to interpret our model time period to represent 5 years, the approximate length of legislative terms in many democracies. We set the yearly real interest rate on 5 year government bonds equal to 1% and thus the period interest rate $R - 1$ to approximately 5%. The discount factor of the principal is set to be equal to the inverse of period interest rate, $\delta = 1/R$. The productivity parameter of the production function (30) is set to $A = T \cdot \tilde{A} = 5$, which reflects a normalization of yearly productivity to $\tilde{A} = 1$. To calibrate Y_1, Y_2, K_1 and α, we proceed as follows. We take current prices GDP for the US from 2022 from the World Bank and set the capital stock $K_1 = 0.63 \cdot GDP_1$, where the value for the share is taken from Ramey (2020) for core public capital. Next, we take output elasticity of core public capital estimates $\varepsilon_{Y,K}$ from Bom and Ligthart (2014) and determine α by defining model GDP to be equal to $\tilde{Y}_1 = Y_1 + F(K_1) + S_1$, where S_1 is an exogenous gross savings term taken from FRED St. Louis, and solving the equation

$$\frac{\partial \tilde{Y}_1}{\partial K_1} \frac{K_1}{\tilde{Y}_1} = \varepsilon_{Y,K} \qquad \Longrightarrow \qquad \frac{T \cdot K_1^{\alpha}}{\tilde{Y}_1} = \varepsilon_{Y,K}. \tag{31}$$

This gives us a value of $\alpha = 0.52$ to match the output elasticity estimate from Bom and Ligthart (2014). The exogenous endowment of the economy is then set equal to $Y_1 = T \cdot GDP_1 - T \cdot S_1 - F(K_1)$ and $Y_2 = (1 + g_y)^T Y_1$, where data on the annual GDP growth rate g_y are taken from the World Bank. Finally, the 5-year depreciation rate of public capital δ_K is computed from the quarterly depreciation rate provided in Ramey (2020). Table 1 summarizes the calibration.

Table 1: Model Calibration

Parameter	Value	Source
R	1.05	5-year model period, 1% yearly interest rate
δ	0.95	$1/R$
β	Range of values	
A	5	$5 \cdot \tilde{A}$, where $\tilde{A}$ is yearly productivity normalized to 1
α	0.52	Match output elasticity from Bom and Ligthart (2014)
K_1/GDP_1	0.63	Ramey (2020)
S_1	5	Billion USD, FRED St. Louis
Y_1	78.75	World Bank, FRED St. Louis
Y_2	86.95	World Bank
δ_K	0.18	Ramey (2020)