Endogenous Incumbency in Repeated Contests*

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February 27, 2025

Abstract

We consider a model of infinitely repeated lottery contests in which the prior contest's winner (incumbent) additionally gains the opportunity to bias the subsequent contest by exerting early effort in an intermediate stage. An effort maximizing contest designer strategically chooses the cost advantage of incumbency. We show that the contest designer prefers to set the cost advantage such that the incumbent only partially discourages the contender, i.e. the contender exerts less, but still positive, effort than in an unbiased contest. In this way, the contest designer achieves higher rent extraction than under independent lottery contests with no intermediate stage, because (i) players compete fiercer to become the incumbent and (ii) the increase in early effort outweighs the decrease in effort in the biased contest. Therefore, we provide some theoretical justification for incumbency advantages, for example in repeated procurement settings.

Keywords: repeated contests, lottery contest, incumbent, discouragement effect
JEL classification: C72, C73, D72

1 Introduction

Fairness is an esteemed value in many competitive situations. Apart from normative concerns, fairness is also often seen as necessary for competition. In unfair competitive situations, both sides may have incentives to shirk from competition: The disadvantaged side is discouraged from competing against a favored opponent, while the advantaged side can free-ride on their advantage. In contrast, when competitions are repeated, rewarding a winner with a future advantage boosts incentives to fight hard today, as players not only fight for today's prize, but also for a future advantage. However, future competitions will then be unfair in the sense that one side has an advantage. This trade-off between fairness and rewards is the center of our analysis.

In this article, we analyze how much an organizer, who wants to maximize overall intensity in competition, should reward performance. Let us illustrate the situation with the following example. Consider a procurement process in which an organizer issues mandates for subcontractors. When a subcontractor wins a mandate, he must implement a task. The organizer can reward a high quality implementation with an advantage in the competition for the next mandate. In this way, the winner can exploit his incumbency status by gaining an advantage in the future. By conditioning the future advantage on the quality of the subcontractor's implementation, the organizer directly incentivizes high

*We thank Lisa Heidelmeier, Florian Herold, Marco Sahm and Stefanie Schmitt for helpful conversations and feedback. We also thank conference and seminar participants from the 10th Contests: Theory and Evidence conference, EARIE 2024, PolEcCon 2024, the 37th BGPE Research Workshop, and the 25th Bavarian Micro Day for helpful comments.

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effort not only in the competition for the mandate, but also in implementation. However, when the organizer rewards quality in implementation, competition for the next mandate is unfair in the sense that the incumbent's competitor is disadvantaged and may react by reducing his effort, or even by completely dropping out of the competition.

We examine the trade-off between fairness and rewards in a potentially infinitely repeated contest model between two ex-ante symmetric players. In particular, we investigate a structure in which today's winner (in the following called *incumbent*) can endogenously bias tomorrow's contest in his favor. Each contest is partitioned into an investment stage and a competition stage. In the investment stage, the incumbent visibly exerts early effort to gain a head start over his opponent (in the following called *contender*). The contest designer may create incentives to invest in early effort by subsidizing early effort. By doing so, the contest designer rewards the incumbent with a lower marginal cost of effort than the contender. In the competition stage, both contestants - the incumbent as well as the contender - observe early effort and exert effort to win the next contest. The game has no fixed time span. Instead, the game continues after each period with the same exogenous probability. This continuation probability can be seen as a measure of the future's relevance compared to the present.[2]

We show that the extent to which the incumbent exerts early effort depends on the degree of compensation for early effort. If compensation is very high such that early effort is cheap, the incumbent fully deters the contender from competition. If compensation is moderate, deterrence is no longer optimal. Instead, the incumbent partially discourages the contender who still participates, but exerts less effort than in a symmetric contest. If compensation is zero (or even negative), the incumbent does not bias the subsequent contest at all. Consequently, the contests remain independent of each other. Anticipating equilibrium behavior, the contest designer maximizes contest intensity by setting marginal compensation for early effort. In this way, the contest designer indirectly determines the value of incumbency.

We find that endogenous incumbency increases the value of winning a contest. The winner does not only get the prize but additionally gains access to potential advantages in the subsequent contest. Conversely, the loser not only loses the competition for the prize, but may also suffer under disadvantages in the subsequent contest. Therefore, the effective prize sum increases in the incumbency advantage. An intensity maximizing contest designer who determines the incumbency advantage moderates two opposing effects: For both the incumbent and the contender, the incumbency advantage intensifies competition as both sides are incentivized to fight hard to become tomorrow's incumbent (*incentive effect*). Conversely, the incumbency advantage partially or fully discourages the contender from competing (*discouragement effect*).

As a measure of contest intensity, we choose rent extraction, defined as effort exerted by the players relative to the total prize sum. We demonstrate that endogenous incumbency can increase rent extraction compared to a game with repeated independent contests if the contest designer sets the incumbency advantage optimally. In this case, the incentive effect dominates the discouragement effect. In particular, we find that the contest designer prefers to induce partial discouragement: he sets the incumbency advantage so that the

[2]This reflects, for example, any exogenous shock to the organizer that leads to termination of the procurement process. If shocks are unlikely, the continuation probability is high. In Section 3, we see that the implementation of this continuation probability resembles discounting of the future. Therefore, we do not incorporate both the continuation probability and discounting into the model as the additional insight would be minimal.

incumbent chooses to bias tomorrow's contest in his favor, but the contender remains active in tomorrow's contest, although with reduced effort compared to an unbiased contest. In addition, the continuation probability affects the contest designer's optimal choice: The higher the continuation probability, the higher the optimal incumbency advantage. This reflects that if the future is important, the incentive effect increases since the incumbency status generates spillover effects from today's contest to all future contests. If future contests exist with higher probability, players will fight harder to become the incumbent.

We also discuss cooperation between players. We show that if the continuation probability is sufficiently high, conditional cooperation equilibria can exist, where players coordinate on playing zero effort, as long as their counterpart does so as well. If the contest designer aims to restrict incentives for players to cooperate, he can achieve this with endogenous incumbency: We show that if the incumbency advantage is very high, cooperation equilibria disappear, as players gain an incentive to deviate from cooperation in order to become the incumbent.

In addition, we cover two extensions of the model. First, we investigate the situation in which the incumbent's early effort is not observable for the contender. In this case, the incumbent loses the commitment opportunity, but still enjoys a cost advantage (if the contest designer subsidizes early effort). We find that if early effort is unobservable, the contest designer prefers to set a higher incumbency advantage than if early effort is observable. Nonetheless, if the contest designer behaves optimally, rent extraction is always higher under observable early effort than under unobservable early effort. Therefore, if early effort is unobservable for the contender, the contest designer should publicly announce the level of early effort to the contender.

Second, we introduce a different objective for the contest designer. In particular, we capture situations in which the contest designer only values winner's effort and not loser's effort. Therefore, we analyze a contest designer that maximizes winner's effort. We find that in this case, the contest designer prefers to induce full discouragement, so that the incumbent fully discourages the contender from competition in every contest.

The paper is structured as follows: Section 2 discusses our contribution to the related literature. In Section 3, we introduce the finite, K-contest game. Section 4 covers the perspective of the contest designer. Section 5 covers the main model, where there is no fixed time span. We compute the equilibrium in this model as the limit of equilibria in the K-contest game from Section 3. Section 6 covers extensions. Section 7 concludes.

2 Related Literature

Incumbent advantages and asymmetric players are a prevalent topic in several branches of the literature. In studies dealing with auctions and procurement processes, incumbents may benefit from existing or emerging market barriers in the form of entry costs or switching costs (e.g. Greenstein (1993); Arozamena et al. (2014); Premik (2023)). Commonly - especially in the contest literature, see e.g. Fu (2006); Epstein et al. (2011); Franke et al. (2014); Clark and Nilssen (2013, 2018); Fu and Wu (2020) only to name a few -, incumbents are modeled to be more efficient, i.e., the incumbent's effort is cheaper or more impactful compared to rivals' efforts, or they are more able in terms of exploiting the prize resulting in a higher valuation[3]. Other models typically use commitment opportunities in sequential

[3]To simplify analysis, authors normally settle on the use of one source of heterogeneity which, under certain conditions, implicitly represents a combination of all three sources under certain conditions (see,

Stackelberg structures to model incumbency (e.g. Dixit (1987); Linster (1993); Morgan (2003); Serena (2017); Gao et al. (2023))[4].

In this paper, we model incumbency as asymmetry in opportunity. In contrast to the contender, the winner of a mandate stays in constant exchange with the contest designer. Therefore, the incumbent gains access to an exclusive investment stage. Investments (early effort) yield a head start for the next competition stage. Consequently, the incumbent can behave as a Stackelberg-leader if he likes and early effort is observable but does not lose the ability for additional investments simultaneously to the rivals' decision. Whether the opportunity to act as a Stackelberg-leader has any value depends on the efficiency of early effort relative to regular effort. Dixit (1987) (Linster (1993), respectively) show that a commitment opportunity in the absence of any other heterogeneity does not have any value in a logit (lottery) contest. However, if early effort is more efficient than regular effort, then the incumbent position contains additional value as it gives the opportunity for manipulating the heterogeneity by investing in an head start to discourage rivals. In contrast to papers about players investing in abilities (see Münster (2007), Fu and Lu (2009), Kwiatkowski (2013), Schaller and Skaperdas (2020)) or sabotaging rivals (Amegashie (2012)) prior to contests, the ability to exert early effort is temporary and bound to the incumbency status in our model. In other words, being the incumbent and utilizing the position arises naturally by winning again and again, but it is also at stake again and again. Thus, we discuss a specific form of endogenous incumbency nested into a repeated contest setting contributing to the literature about previous performance directly affecting abilities in future. For instance, Möller (2012) and Pham (2019) discuss two subsequent contests, but winning the first contest additionally increases the winner's abilities in the subsequent round. In Beviá and Corchón (2013), today's prize shares increases players' strength for subsequent contests. [5]In similar vein, Clark et al. (2020) discuss two lottery contests with the winner additionally gaining a multiplicative bias in her favour. In general, they find that the stronger a first period result increases the heterogeneity, the higher is effort in the first contest and the lower is effort in the second contest as the heterogeneity discourages the weaker player. We contribute to this literature by showing that - additional to the trade-off - endogenous incumbency incorporates an incentive effect which motivates both players in any but the last contest. In that sense, a two-contest model should be seen as a special case because, in the contest where heterogeneity has potentially changed by prior competition, i.e., the second contest, there is no incentive effect at place by design because it is the game's last period.

There is a vast literature on how heterogeneity affects the contest designer's (social planner's or auctioneer's, respectively) objective function and measures he can take to maximize said function. In particular, the contest designer can implement a bias to counteract existing asymmetries, i.e., he uses affirmative action policies to (partially) level the playing field thereby increasing revenue and to induce more players actively competing (e.g. Lazear and Rosen (1981), Dukerich et al. (1990), Schotter and Weigelt (1992), Tsoulouhas et al. (2006), Fu (2006), Fain (2009), Lee (2013), and Kirkegaard (2013)). Lien (1990) and Clark and Riis (2000) discuss public procurement models with heterogeneous firms bribing government officials. On the one hand, firms differ in their costs incurred by the contracted task. On the other hand, government officials may also

e.g., Beviá and Corchón (2024))

[4]Yildirim (2005) provides an analysis of two players who both can visibly commit effort in multiple periods which are then accumulated in an overall contest.

[5]Pham (2019) also discusses cases in which the loser is awarded with reduced effort costs.

treat bribes differently depending on the sender such that some firms are handicapped. Optimal biases in heterogeneous environments are discussed in e.g., Epstein et al. (2011), Li and Yu (2012), Franke et al. (2013), Franke et al. (2014), Barbieri and Serena (2022) with a well-informed contest designer intervening directly by means of biases or head starts.[6] In comparison, our model differs in two ways. First, the contest designer can only indirectly intervene in our model by nudging the contestants to optimally bias upcoming contests by offering a marginal compensation of early effort. Thus, early effort is a decision variable of the incumbent which depends on the decision variable of the contest designer. Second, we find that it is in the contest designer's interest to implement endogenous incumbency and subsidize incumbents which is in contrast to the aforementioned literature that advocates for intervention to reduce or eliminate heterogeneity.

Recently, a few papers have also argued that favoritism may have its merits. For example, Drugov and Ryvkin (2017) discuss a general class of contest success functions with a bias that - depending on the nature of the contest and the bias - may be optimal even when dealing with symmetric players. A procurement entity benefiting from favoritism has been also discussed in Premik (2023) who finds that, in auctions, bid preference programs (e.g. see McAfee and McMillan (1989)) may balance a trade-off between promoting competition by partially leveling a heterogeneous playing field and advantages of lock-in relationships with incumbents. In a dynamic contest setting with incomplete information, Meyer (1991, 1992) also argue for favouring ex-interim leaders. We follow this notion by showing that a contest designer profits from endogenous incumbency in an otherwise symmetric setting with complete information utilizing that, for the contestants, there is more at stake than just one contest's prize. Therefore, an endogenous type of favoritism may be a useful tool for a contest designer to increase contest intensity.[7]

3 Main model

We consider two players $i, j \in I = \{1, 2\}$ who play a potentially infinite number of contests $k \in \mathbb{N}$ in succession. Players are ex-ante symmetric, i.e., their prize valuations and their productiveness coincides, except for one characteristic: one player (henceforth referred to as player i, the incumbent) has won the previous contest whereas the other (henceforth referred to as player j, the contender) has not. In each contest k, players can win a non-divisible prize[8] which is normalized to 1. Each contest is organized as a (potentially biased) lottery contest[9] such that players $i, j \in I$ win contest k with probabilities given by

$$p_{i,k} = \begin{cases} \frac{1}{2} & \text{if } x_{i,k} = x_{j,k} = d_{i,k} = 0, \\ \frac{x_{i,k}+d_{i,k}}{x_{i,k}+x_{j,k}+d_{i,k}} & \text{otherwise,} \end{cases} \tag{1}$$

[6]Usually, an additive advantage is referred to as *head start* whereas a multiplicative advantage is referred to as *handicap* or *bias*.

[7]Notice that these effects crucially depend on the contest success function not being perfectly discriminatory. It is straightforward to show that total effort is weakly decreasing when considering repeated all-pay auctions because symmetric all-pay auctions already yield full rent extraction.

[8]A divisible prize fundamentally alters the interpretation of the contest success function, e.g., see Beviá and Corchón (2013).

[9]The use of a biased (unbiased) lottery contest can be justified by axioms presented in Clark and Riis (1998)(Skaperdas (1996), respectively).

and

$$p_{j,k} = \begin{cases} \frac{1}{2} & \text{if } x_{i,k} = x_{j,k} = d_{i,k} = 0, \\ \frac{x_{j,k}}{x_{i,k}+x_{j,k}+d_{i,k}} & \text{otherwise.} \end{cases} \quad (2)$$

where $d_{i,k} \geq 0$ is the head start in favor of player i (henceforth called *early effort*), and $x_{i,k}, x_{j,k} \geq 0$ are players' efforts. Players take into account that player i's effective input into the contest success function is given by $x_{i,k} + d_{i,k}$.

The exact duration of the game is uncertain. After every contest, the game continues with the next contest with probability $\delta \in (0, 1)$ and ends with probability $1 - \delta$. The continuation probability δ is constant over time, so every period k is, conditional on existence, identical. Only after contest K, the game ends with probability 1. The interpretation of δ is any exogenous shock to the contest structure that causes the contest to terminate.[10]

A contest is divided into an investment stage and a subsequent competition stage. In the investment stage I_k of contest k, player i, i.e. the incumbent, can exert early effort $d_{i,k}$ at marginal net cost of early effort $b \geq 0$ to maximize his expected payoff

$$\underset{d_{i,k}}{max} \quad \pi^I_{i,k} = \pi^C_{i,k}(d_{i,k}) - b \cdot d_{i,k} \quad (3)$$

where $\pi^C_{i,k}$ is the expected payoff of the subsequent competition stage, which depends on early effort. A different interpretation is that the incumbent still faces the same marginal cost of effort but is compensated for early effort with $(1 - b)d$ by the contest designer.

In the competition stage, players observe the incumbent's head start $d_{i,k}$ and simultaneously choose effort $x_{i,k}$ ($x_{j,k}$, respectively) to maximize expected payoff, i.e.,

$$\pi^C_{i,k} = p_{i,k} \cdot (1 + \delta\pi^I_{i,k+1}) + (1 - p_{i,k}) \cdot \delta\pi^C_{j,k+1} - x_{i,k}, \quad (4)$$

and

$$\pi^C_{j,k} = p_{j,k} \cdot (1 + \delta\pi^I_{i,k+1}) + (1 - p_{j,k}) \cdot \delta\pi^C_{j,k+1} - x_{j,k}, \quad (5)$$

where $\pi^I_{i,k+1}$ denotes the winner's continuation payoff [11], $\pi^C_{j,k+1}$ denotes the loser's continuation payoff and δ is the continuation probability. By design, $\pi^I_{j,k+1} = \pi^C_{j,k+1}$, because j has no action in the investment stage. In that sense, each contest can be considered a Stackelberg-variant where player i is both a Stackelberg-leader (by exerting early effort in the investment stage) and a Cournot-player (by exerting regular effort in the competition stage).[12] The only exception is the first contest: Since the players are ex-ante symmetric, there is no head start in the first contest, so $d_{i,1} = 0$.

The structure of the contest as well as the marginal costs of (early) effort are common knowledge. The timing of the game is illustrated in Figure 1. [13]

[10] An alternative interpretation of δ is that players discount future periods. Since one can easily show that δ-discounting is mathematically equivalent to the continuation probability, we do not additionally account for discounting in the model.

[11] Notice that depending on the outcome of the contest, the identity of player i may change.

[12] In principle, player i can use both channels of effort. However, we show that player i will solely exert early (regular, respectively) effort if the net cost of early effort is lower (higher) than the cost of regular effort. If the costs of early effort and regular effort are identical, player i will be indifferent between any distribution of efforts across the two stages under the condition that $d_{i,k} + x_{i,k} = \frac{1}{4}$.

[13] I_1 is empty by assumption.

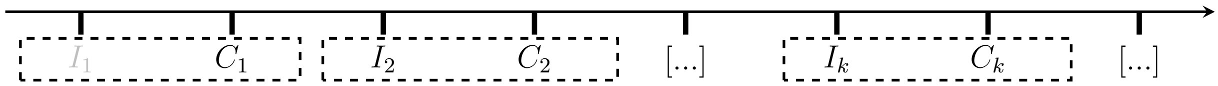

Figure 1: Timing of repeated contests (each rectangle depicts a period).

3.1 Equilibrium behavior in competition stages

In the competition stage of contest k, players observe player i's early effort $d_{i,k}$ and maximize their expected payoffs by simultaneously choosing $x_{i,k}$ and $x_{j,k}$. In every contest, players directly fight for a prize of value 1. However, players take into account that depending on the outcome of the contest they will either be the incumbent or the contender, which changes their continuation payoffs in the subsequent period (which exists with probability δ). Consequently, they compete for a effective prize sum denoted by $\psi_k = 1 + \delta\pi^I_{i,k+1} - \delta\pi^C_{j,k+1}$. In particular, if the difference between continuation payoffs is large (becoming the incumbent is valuable), players fight for a higher effective prize in contest k. Accounting for border cases, optimal efforts and corresponding payoffs are summarized in Lemma 1.

Lemma 1. *In competition stage C_k with observable early effort $d_{i,k}$ by both players, equilibrium efforts and payoffs are given by*

$$x_{i,k} = \begin{cases} \frac{1}{4}\psi_k - d_{i,k} & \text{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ 0 & \text{if } d_{i,k} > \frac{1}{4}\psi_k \end{cases}$$

$$x_{j,k} = \begin{cases} \frac{1}{4}\psi_k & \text{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ \sqrt{d_{i,k}\psi_k} - d_{i,k} & \text{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k \\ 0 & \text{if } d_{i,k} \geq \psi_k \end{cases}$$

$$\pi^C_{i,k} = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} + d_{i,k} & \text{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ \sqrt{d_{i,k}\psi_k} + \delta\pi^C_{j,k+1} & \text{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k \\ \psi_k + \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \geq \psi_k \end{cases}$$

$$\pi^C_{j,k} = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \leq \frac{1}{4}\psi_k \\ \psi_k + \delta\pi^C_{j,k+1} + d_{i,k} - 2\sqrt{d_{i,k}\psi_k} & \text{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k \\ \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \geq \psi_k \end{cases}$$

The additive head start in favor of the incumbent does not change the incumbent's incentive to produce an effective input of $\frac{1}{4}\psi_k$. If the contest is unbiased ($d_{i,k} = 0$), both players are symmetric such that the textbook equilibrium will arise with $x_{i,k} = x_{j,k} = \frac{1}{4}\psi_k$ and $\pi^C_{i,k} = \pi^C_{j,k} = \frac{1}{4}\psi_k$. If $0 < d_{i,k} \leq \frac{1}{4}\psi_k$, the incumbent will exert effort such that his input still equals $\frac{1}{4}\psi_k$, and the contender will react by playing $x_{j,k} = \frac{1}{4}\psi_k$. If $\frac{1}{4}\psi_k < d_{i,k} \leq \psi_k$, then the incumbent will not exert additional effort ($x_{i,k} = 0$), since his effective input is already higher than $\frac{1}{4}\psi_k$ and, thus, functions as a strategic substitute which partially discourages the contender from competing, i.e., $\frac{1}{4}\psi_k > x_{j,k} > 0$. If $d_{i,k} \geq \psi_k$, any positive effort by the contender will imply a negative expected payoff such that $x_{j,k} = 0$ will be dominant. In this case, the competition stage will degenerate and the incumbent will win with certainty due to his early effort investment deterring the contender.

3.2 Equilibrium behavior in investment stages

In the investment stage of contest k, player i who won the previous contest (the incumbent) maximizes expected payoff by choosing optimal early effort $d^*_{i,k}$. Equilibrium early effort and corresponding payoffs are summarized in Lemma 2. The proof is provided in the Appendix A.2.

Lemma 2.

$$d^*_{i,k} = \begin{cases} \psi_k & \text{if } b \leq \frac{1}{2} \\ \frac{\psi_k}{4b^2} & \text{if } \frac{1}{2} < b < 1 \\ 0 & \text{if } b \geq 1 \end{cases}$$

$$\pi^I_{i,k} = \begin{cases} (1-b)\psi_k & \text{if } b \leq \frac{1}{2} \\ \frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} & \text{if } b \geq 1 \end{cases}$$

$$\pi^I_{j,k} = \pi^C_{j,k}$$

Notice that ψ_k, i.e., the value of winning contest k, is not constant but a function depending on the value of incumbency which decreases in the cost of early effort b.

Decreasing b moderates two opposing effects. One the one hand, any $b < 1$ makes the incumbent strong and the contender weak who suffers under a *discouragement effect*. On the other hand, any $b < 1$ implies an increased ψ_k because the incumbent position is valuable which motivates not only the incumbent but also the contender (*incentive effect*). If early effort is cheap ($b \leq \frac{1}{2}$), the incumbent will invest in early effort to fully deter the contender from competing. Consequently, the first contest's winner will also win all other contests with certainty whereas the first contest's loser will not exert any effort in any subsequent contest anymore. However, the incumbent does not win all contests for free (unless $b = 0$)[14]. A low b motivates the contender as well due to the incentive effect. Therefore, a certain level of early effort is necessary to deter the contender. If early effort is moderately cheap ($\frac{1}{2} < b < 1$), the incumbent will commit early effort and partially deter the contender who exerts $x_{j,k} < \frac{1}{4}\psi_k$.[15] Then, the contest's outcome is random, but the incumbent is the favorite because his input outweighs the contender's effort. If early effort is costly ($b \geq 1$), then the incumbent will not invest early effort at all because - in line with Dixit (1987) and Linster (1993) - the commitment opportunity itself without a cost advantage (or even with a cost disadvantage) does not entail any value.

3.3 The K-contest game

Inserting Lemma 2 into Lemma 1 characterizes equilibrium behavior in any contest k, depending on b and the effective prize ψ_k. In order to solve the infinite game, we use a result from Fudenberg and Levine (1983). Theorem 3.3 from Fudenberg and Levine (1983) applied to our game states that under certain conditions, a sequence of subgame perfect

[14]For $b = 0$, any $d_{i,k} > \psi_k$ is optimal. For continuity and without loss of generality, we assume that the incumbent chooses $d_{i,k} = \psi_k$ here.

[15]Notice that it is still possible that the contender's effort $x_{j,k} \geq \frac{1}{4}$, i.e., the contender's effort under endogenous incumbency will be higher than in an unbiased lottery contest, if the incentive effect weakly dominates the discouragement effect. Which effect dominates dominates depends on the specific values of b, k and K.

Nash equilibria in the K-contest game converges for $K \to \infty$ to a subgame perfect Nash equilibrium in the infinite game.[16] Therefore, in this section we solve the finite, K-contest game via backward induction. Figure 2 depicts the timeline of the K-contest game.

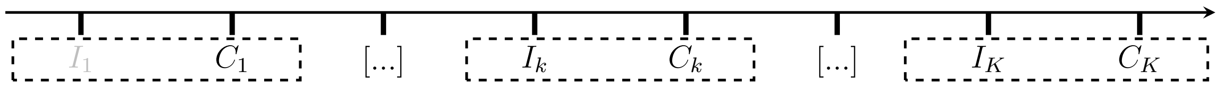

Figure 2: Timing of repeated contests (each rectangle depicts a period).

For any $b \geq 1$, Lemma 3 characterizes the equilibrium of contest k.

Lemma 3. *If $b \geq 1$, then, for any arbitrary contest $k \in \{2, ..., K\}$ where player i is the incumbent, player j is the contender and early effort $d_{i,k}$ is observable, the effective prize sum and equilibrium behavior are given by*

$$\begin{aligned} \psi_k &= 1, \\ d^*_{i,k} &= 0, \\ x_{i,k} &= x_{j,k} = \frac{1}{4}. \end{aligned}$$

As discussed earlier, any $b \geq 1$ implies that the incumbent position is not valuable because it cannot be utilized in a beneficial way. Contest k and, hence, all K contests remain independent such that each period's contests resembles a textbook lottery contest with two symmetric players and an effective prize sum equal to 1.

For any $b \in (0, \frac{1}{2}]$, Lemma 4 characterizes the equilibrium of contest k.[17].

Lemma 4. *If $b \leq \frac{1}{2}$, then, for any arbitrary contest $k \in \{2, ..., K\}$ where player i is the incumbent, player j is the contender and early effort $d_{i,k}$ is observable, sum of effective prize sums and equilibrium behavior are given by*

$$\begin{aligned} \psi_k =& \frac{1 - [\delta(1-b)]^{K-k+1}}{1 - \delta(1-b)}, \\ d^*_{i,k} =& \frac{1 - [\delta(1-b)]^{K-k+1}}{1 - \delta(1-b)}, \\ x_{i,k} &= x_{j,k} = 0. \end{aligned}$$

where ψ_k and $d^*_{i,k}$ are decreasing in b, increasing in δ and increasing in the number of remaining contests $K - k$.

Lemma 4 is characterized by full discouragement. The incumbent exerts a level of early effort equal to the effective prize sum. In this way, the contender is fully discouraged from competing and prefers to stay out of the contest. The result of the contest is therefore not probabilistic anymore, since the incumbent wins with certainty.

For any $\frac{1}{2} < b < 1$, Lemma 5 characterizes the equilibrium of contest k.

[16] By the term 'K-contest game' we mean that the game has a *maximum* of K contests. The exact duration of the game remains uncertain due to the continuation probability δ. A different formulation is that δ is time-dependent with $\delta_k = \delta$ for $k \in \{1, ..., K-1\}$ and $\delta_K = 0$.

[17] For $b = 0$, the game does not have a unique equilibrium as there exists a continuum of optimal $d^*_{i,k}$ since early effort is costless. We omit this case and later also show that the contest designer has no incentive to ever set $b = 0$.

Lemma 5. *If $\frac{1}{2} < b < 1$, then, for any arbitrary contest $k \in \{2, ..., K\}$ where player i is the incumbent and player j is the contender, the effective prize sum and equilibrium behavior are given by*

$$\psi_k = \frac{1-(\delta A)^{K-k+1}}{1-\delta A},$$
$$d^*_{i,k} = \frac{1-(\delta A)^{K-k+1}}{(1-\delta A)\,4b^2},$$
$$x_{i,k} = 0,$$
$$x_{j,k} = \frac{\left(1-\delta A^{K-k+1}\right)(2b-1)}{(1-\delta A)\,4b^2}.$$

where $\delta A = \delta\left(\frac{5b-1}{4b^2}-1\right)$. The effective prize and early effort are decreasing in b and increasing in δ, and increasing in the number of contests $K-k$.

Lemma 5 is characterized by partial discouragement. The incumbent fully relies on early effort. The contender exerts less effort than he would if the contest was unbiased, but still exerts positive effort. Therefore, the outcome is random, but the incumbent is the favorite. Both players are influenced by the discouragement effect as well as the incentive effect. Decreasing b (increasing, respectively) within the case borders gives the incumbent a higher (lower) cost advantage in this period, but simultaneously increases (decreases) the effective prize sum at stake for both players.

Figure 3 shows the relationship of the effective prize ψ_k with δ and b. Clearly, a larger continuation probability δ means that the future is more valuable, and therefore the value of becoming the incumbent is larger. Similarly, a lower b means that the value of becoming the incumbent is large, since players anticipate that if they win, they can exploit their incumbency position in the next contest at low cost.

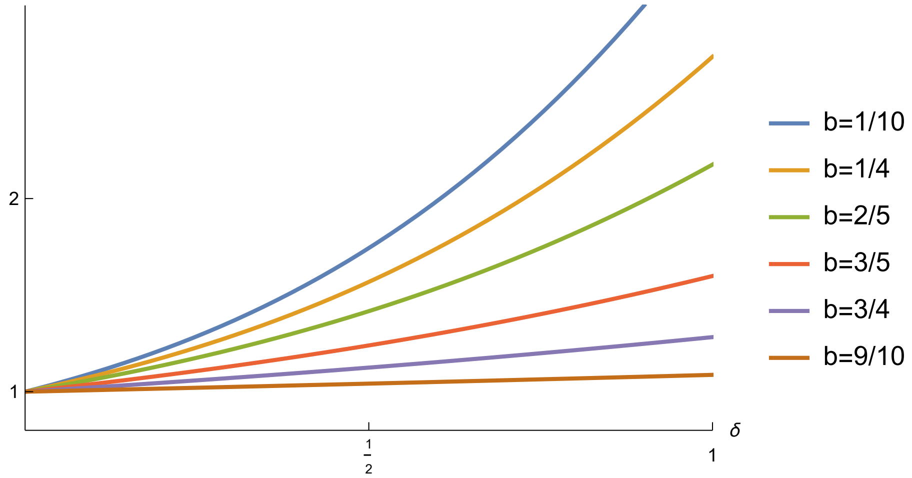

Figure 3: Effective prize sum for $b \in (0,1)$, $K = 6$ and $k = 3$

Figure 4 illustrates the effect of the number of remaining contests on the effective prize. The lower k, the more contests are remaining in the game and the higher the effective prize is. The reason for this are spillover effects: Becoming the incumbent yields an advantage in contest $k+1$, but also indirectly in contests $k+2, ..., K$, as contest $k+1$, in which the incumbent has an advantage, determines who gets an advantage in contest $k+2$ (and so on). Winning an early contest means that the incumbent will enjoy the incumbency advantage for a longer time, and therefore the effective prize of winning an early contest is higher than the effective prize of winning a late contest. In the case of full discouragement ($b < 1/2$), the incumbent will in equilibrium always exert so much early effort that the contender drops out of the contest. Therefore, in equilibrium, the incumbency position is never lost. The incumbency advantage is high in this case, and the effective prize of winning contest k increases steeply with the number of remaining contests.

In the case of partial discouragement ($1/2 < b < 1$), these spillover effects are also present, since winning contest k yields a positive incumbency advantage, such that the incumbent is more likely to also win contest $k + 1$ and enjoy the incumbency advantage also in this contest. The size of the spillover effect is however lower, since the contender still always has some chance of winning. Therefore, the effective prize of winning contest k increases only moderately in the number of remaining contests in the case of partial discouragement.

Another observation is that the effective prize only depends on the number of remaining contests $K - k$ and not on K directly. In particular, behavior in contest $k = 2$ of $K = 3$ will be exactly the same behavior as in contest $k = 99$ of $K = 100$. This is intuitively clear since the continuation payoffs are only forward-looking.

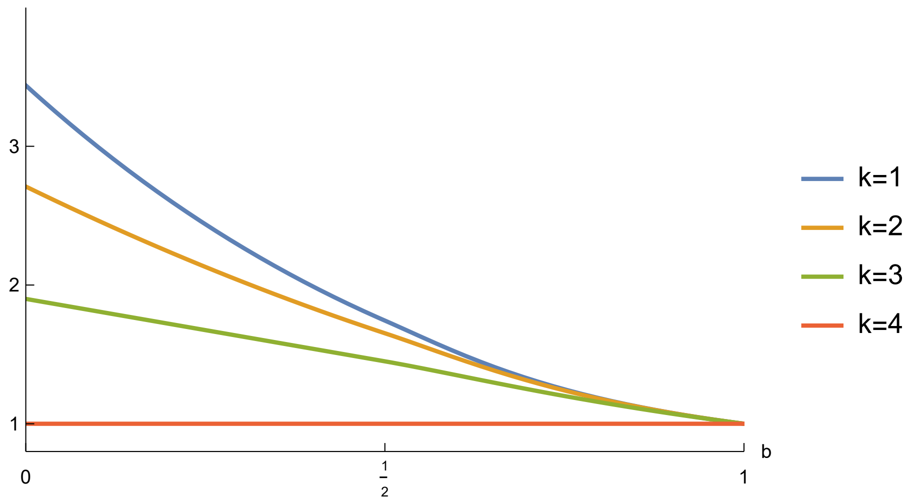

Figure 4: Effective prize sum for $b \in (0, 1)$, $K = 4$ and $\delta = 9/10$

A similar logic applies to the relationship between equilibrium early effort $d^*_{i,k}$ and the parameters b, δ and $K - k$, which is illustrated in Figure 5 and Figure 6. A higher δ implies future contests exist with a higher probability, so the value of winning today is higher. This increases early effort for two reasons: first, the incumbent values a win in the next contest more and therefore has a higher incentive to bias the next contest in his favor. Second, the contender values a win in the next contest more, so the incumbent must exert more early effort to discourage the contender. Early effort is therefore increasing in δ. Since a higher b means a lower incumbency advantage (and therefore, as established, a lower effective prize), early effort is decreasing in b.

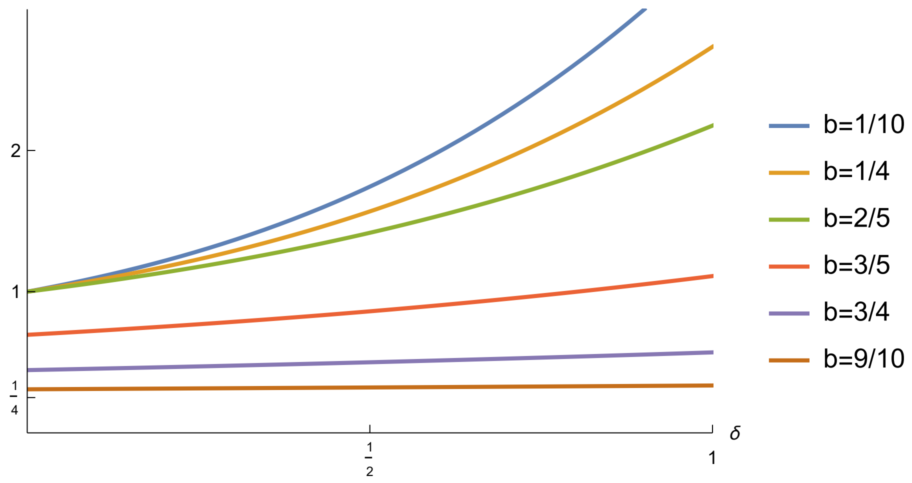

Figure 5: Early effort for $b \in (0,1)$, $K = 6$ and $k = 3$

Figure 6 illustrates the relationship between the number of remaining contests $K - k$ and equilibrium early effort. Since a higher number of remaining contests means a higher effective prize, it is intuitive that the incumbent will exert more early effort.

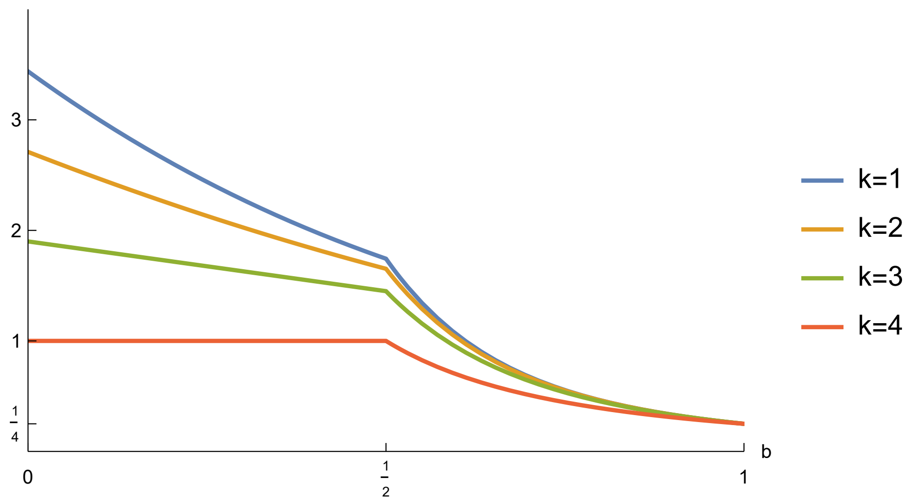

Figure 6: Early effort for $b \in (0,1)$, $K = 4$ and $\delta = 9/10$

Figure 7 illustrates the effort exerted by the contender $x_{j,k}$. The contender is both influenced by the discouragement effect and the incentive effect. If $b \leq 1/2$, the contender is fully discouraged and exerts no effort. If $b \in (1/2, 1)$, the discouragement effect is weakened enough for the incentive effect to be relevant. In part a) of figure 7 we show the interplay of δ with these two effects. The contender reacts in two ways to the cost of early effort b: first, he is discouraged in the present period due to the incumbency advantage working against him. This discouragement effect is higher for a low b. Conversely, for a low b, the incentive effect encourages him to fight hard since this means that conditional on winning, he will enjoy a higher incumbency advantage in the future. The incentive effect depends on future payoffs, and hence, the contender reacts differently to an observed b depending on the value of δ. For a low δ, the future is not very relevant and the discouragement effect dominates. Therefore the contender fights harder with a high b, which mitigates the discouragement effect. For a high δ however, the contender highly values the future. In this case the incentive effect can dominate, so that the contender fights harder with a lower b.

In part b) of figure 7 we show that the incentive effect can boost the contender's effort above the equilibrium level of an unbiased one-shot contest, which is at $x = 1/4$. The incentive effect dominates the discouragement effect for high b and the contender is encouraged to exert effort above the baseline level. As b tends to unity, both effects vanish and the contender's effort converges to the baseline.

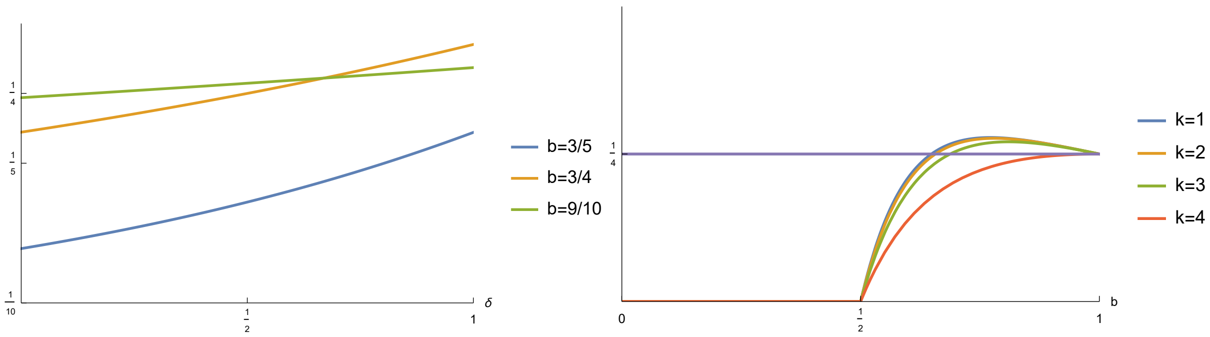

Figure 7: Effort by the contender a) depending on δ and b with $K = 6$ and $k = 3$, and b) depending on b and k with $\delta = 9/10$ and $K = 4$.

Corollary 1 gives the probabilities of winning in equilibrium.

Corollary 1. *For $k \in \{2, ..., K\}$, the probability $p_{i,k}$ with which the incumbent wins contest k is given by*

$$p_{i,k} = \begin{cases} 1 & \text{if } b \leq \frac{1}{2} \\ \frac{1}{2b} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{2} & \text{if } b \geq 1 \end{cases}$$

Two of the three cases are rather obvious. For $b \leq 1/2$, the contender is fully discouraged and exerts no effort, so the incumbent wins with certainty. For $b \geq 1$, the incumbency position holds no advantage and behavior in every contest is the same as in a one-shot contest, so probabilities of winning are $p_{i,k} = 1/2$.

In the nontrivial case $b \in (1/2, 1)$, the probabilities of winning do not depend on δ, K and k. An intuitive explanation for that is that the continuation probability δ affects both players similarly. An increase in δ pushes both players' efforts into the same direction. For the probability of winning, they then cancel out. The same applies to the effect of the number of remaining contests $K - k$ on probabilities of winning. For the cost of early effort b however, the same is not true. A lower b benefits specifically the incumbent and harms the contender in the current contest. Therefore, the probability of winning for the incumbent decreases in b.

Notice that Lemmas 3, 4 and 5 do not specify equilibrium effort in contest 1 , since in contest 1 there is no incumbent. The formulas derived for the effective prize sum ψ_k are nonetheless valid for $k = 1$. Therefore, inserting both ψ_1 and $d_{i,1} = 0$ in Lemma 1 yields Corollary 2.

Corollary 2. *In the subgame perfect equilibrium of the K-contest game, equilibrium efforts* *and effective prize in the first contest is given by*

$$\psi_1 = \begin{cases} 1 & \text{if } b \geq 1, \\ \frac{1-(\delta A)^K}{1-\delta A} & \text{if } \frac{1}{2} < b < 1, \\ \frac{1-[\delta(1-b)]^K}{1-\delta(1-b)} & \text{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

$$x_{1,1} = x_{2,1} = \begin{cases} \frac{1}{4} & \text{if } b \geq 1, \\ \frac{1}{4}\frac{1-(\delta A)^K}{1-\delta A} & \text{if } \frac{1}{2} < b < 1, \\ \frac{1}{4}\frac{1-[\delta(1-b)]^K}{1-\delta(1-b)} & \text{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

where $A = \frac{5b-1}{4b^2} - 1$.

In general, Corollary 2 shows that both equilibrium efforts and payoffs in contest 1 will exceed the values of an independent textbook lottery contest if the incumbent position is valuable, i.e., $b < 1$. In other words, offering compensation for early effort will induce the initial fight for incumbency to be more intense.

4 Contest Designer

The contest designer is concerned with effort revenues. The more effort players put into winning the prize the better for the contest designer. Also, he profits from an incumbent who exerts early effort in any case. As shown, both investments in early effort and regular effort crucially depend on the marginal cost of early effort relative to cost of regular effort. A thoughtful contest designer is aware of this aspect and, consequently, manipulates the players' cost of early effort in his interest by offering a suitable subsidy. In that sense, the contest designer's problem boils down to a specific case of total effort maximization. In particular, we assume that the contest designer maximizes expected rent extraction, defined as the ratio between total effort and the total prize sum.[18]Before the analysis, we can establish the general area of equilibrium rent extraction. Clearly, negative efforts are impossible in our model, so rent extraction is nonnegative. A rent extraction larger than unity would imply that at least one player makes negative expected payoffs. This player could then improve by playing zero effort at all stages. Therefore, rent extraction is bounded in the interval $[0, 1]$.

We assume that, in the absence of any compensation by the contest designer, early effort in the investment stage and effort in the competition stage are equally costly. This can be viewed as a neutrality assumption. In the following we show that the contest designer profits from introducing asymmetry and facilitating an incumbency position even if he partially bears the cost of compensation.

By choosing the cost of early effort b (equivalent to choosing the compensation of early effort $1 - b$), the contest designer maximizes rent extraction:

$$\max_b \quad \rho(b, K) = \frac{1}{\sum_{n=0}^{K-1} \delta^n \cdot 1} \left(\sum_{k=1}^{K} \delta^{k-1}(x_{i,k} + x_{j,k} + b \cdot d_{i,k})\right)$$

[18]We can only express rent extraction in expectation since it is ex-ante uncertain how long the game will last. For shortness of notation, we will in the following simply use the term rent extraction for expected rent extraction.

In order to gain intuition how the choice of b influences total effort, we first analyze an arbitrary contest k with respect to aggregate effort, i.e. the sum of individual efforts:

Corollary 3. *Define aggregate effort in contest k as $E_k = x_{i,k} + x_{j,k} + b \cdot d_{i,k}$. Then, aggregate effort in the equilibrium of contest $k \in \{2, ..., K\}$ is given by*

$$E_k = \begin{cases} \frac{1}{2} & \text{if } b \geq 1, \\ \frac{3b-1}{4b^2}\psi_k & \text{if } \frac{1}{2} < b < 1, \\ b\psi_k & \text{if } 0 \leq b \leq \frac{1}{2}, \end{cases}$$

Manipulating b has a direct and an indirect effect on E_k. Directly, the value of b determines the qualitative behavior of the players in equilibrium, i.e., whether the incumbent does not, partially, or fully deter the contender. Decreasing b boosts player asymmetries and, thus, the discouragement effect. Also, the share of early effort which is compensated by the contest designer increases. Necessarily, this has a negative effect on total effort. Indirectly, the cost of early effort determines whether the incumbent position is valuable ($\psi_k > 1$) or not ($\psi_k = 1$). Decreasing b boosts the incumbent position's value and ψ_k. As a consequence, the incentive effect positively affects both players' efforts in all contests except for contest K, since in contest K continuation payoffs of winning and losing are identical.

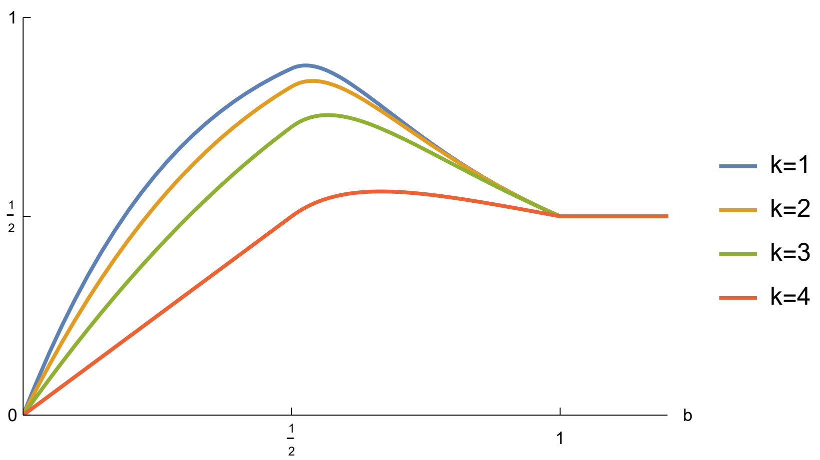

Figure 8: Aggregate effort in contests k depending on b, with $\delta = 9/10$ and $K = 4$.

Figure 8 illustrates the effect of the cost of early effort b and the number of remaining contests $K - k$ on aggregate effort in contest k in equilibrium. If early effort is too costly ($b \geq 1$), the incumbent will not exert early effort and incumbency will not have any value ($\psi_k = 1$). Consequently, all k contests remain independent and unbiased resulting in aggregate effort $E_k = \frac{1}{2}$ per contest. However, if $b < 1$, both the discouragement effect and the incentive effect occur. If the cost of early effort is moderate, the incumbent will exert early effort and will partially discourage the contender such that the contender will exert less than $\frac{1}{4}\psi_k$. The higher revenue generated by inducing early effort will outweigh the negative effect on the contender's effort. It is straightforward to show that $\frac{3b-1}{4b^2} > \frac{1}{2}$ and $\psi_k \geq 1$ for any $\frac{1}{2} < b < 1$ and any fixed $K \geq 2$. Additionally, the first contest ($k = 1$), which is unbiased by assumption, will also generate $E_1 > \frac{1}{2}$ because $\psi_1 > 1$. Consequently, a contest designer will always have an incentive to facilitate an incumbent with an advantage as any $\frac{1}{2} < b < 1$ dominates $b = 1$. If cost of early effort is cheap ($b \geq \frac{1}{2}$), the incumbent will deter the contender from competition while exerting $E_k = b\psi_k$ with $\psi_k > 1$. When choosing b, the contest designer faces the following trade-off here[19]: decreasing b increases ψ_k through the incentive effect, which increases effort and therefore rent extraction. Conversely, decreasing b is equivalent to increasing compensation for early effort, which reduces rent extraction. Corollary 4 shows that the latter effect dominates.

Corollary 4. *Assume the contest designer chooses b to maximize rent extraction. Then, every $b \in (0, 1/2)$ is dominated by $b = 1/2$.*

Figure 8 also shows that aggregate effort increases in the number of remaining contests $K - k$. When a high number of contests are remaining, the total prize at stake is higher and present decisions influence a higher number of contests. This result is also a reflection of Lemma 4 and 5.

With Lemma 3, Lemma 4, Lemma 5, Corollary 3 and Corollary 2 we can now determine rent extraction in the K-contest game.

Proposition 1. *Rent extraction in the subgame perfect equilibrium of the K-contest game is given by*

$$\rho(b,K) = \begin{cases} \frac{1}{2} & \text{if } b \geq 1, \\ \frac{1-\delta}{1-\delta^K}\left(\frac{1}{2}\frac{1-(\delta A)^K}{1-\delta A} + \frac{\delta(3b-1)}{4b^2}\frac{1}{(1-\delta A)}\left[\frac{1-\delta^{K-1}}{1-\delta} - \delta^{K-1}A\frac{1-A^{K-1}}{1-A}\right]\right) & \text{if } \frac{1}{2} < b < 1 \\ \frac{1-\delta}{1-\delta^K}\left(\frac{1}{2}\frac{1-[\delta(1-b)]^K}{1-\delta(1-b)} + \delta b\frac{1}{(1-\delta(1-b))}\left[\frac{1-\delta^{K-1}}{1-\delta} - \delta^{K-1}(1-b)\frac{1-(1-b)^{K-1}}{1-\delta(1-b)}\right]\right) & \text{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

where $A = \frac{5b-1}{4b^2} - 1$.

Figure 9 illustrates the results of Proposition 1. For all $b \in (0, 1)$, rent extraction increases in the length of the game. The maximum is always in the area of partial discouragement. For a higher K, rent extraction increases faster in the area of full discouragement. A lower continuation probability δ however reduces rent extraction.

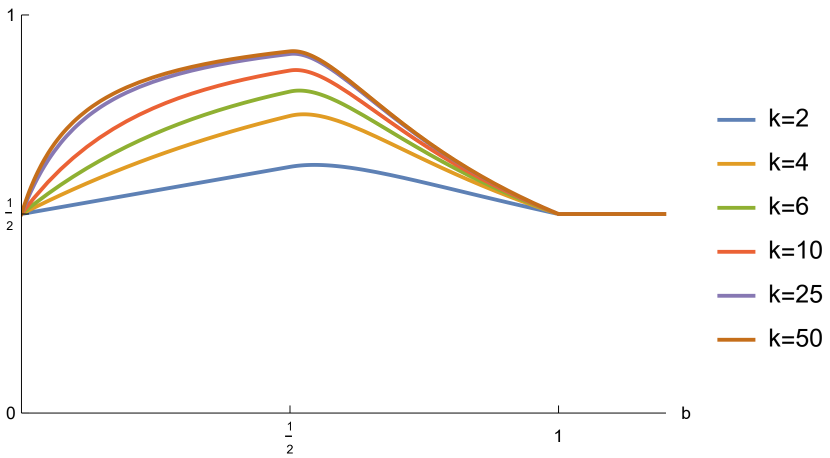

Figure 9: Rent extraction in the K-contest game for $\delta = 9/10$.

[19] If early effort is fully compensated ($b = 0$), no revenue will be generated in any contest except the first one. It is straightforward to show that $TE(b, K) = \frac{K}{2}$ for $b = 0$. Therefore, the optimal b is necessarily positive.

5 The infinite game

In what follows, we consider the number of contests K to be unbounded. In particular, the game has no certain termination date anymore. Nonetheless, the game ends after each period with probability $1-\delta$. Therefore, with probability one the game is finite. The solution concept from Section 4 however does not apply anymore, since for backward induction we need a certain last period. In order to analyze equilibrium behavior by the players and the contest designer, we use a result from Fudenberg and Levine (1983). The theorem loosely states that if the future is not too important, a sequence of equilibria in a K-period game converges for $K \to \infty$ to an equilibrium in the infinite game. We will call this equilibrium the *limit equilibrium.*

Proposition 2. *In the limit equilibrium, rent extraction in the infinite game is given by*

$$\rho(b,\infty) = \begin{cases} \frac{1}{2} & \text{if } b \geq 1, \\ \frac{1}{2} + \frac{\delta b(-8b+11)-3\delta}{8b^2+\delta(8b^2-10b+2)} & \text{if } \frac{1}{2} < b < 1, \\ \frac{1}{2} + \frac{\delta b}{2(1-\delta+\delta b)} & \text{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

In particular, the contest designer optimally sets $b^* = \frac{\delta+\sqrt{9-5\delta}+3}{\delta+11}$.

Figure 10 illustrates the results of Proposition 2.

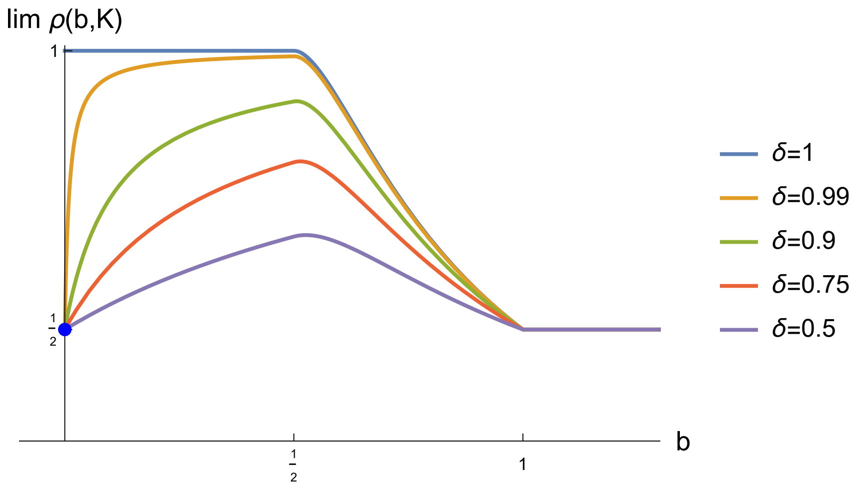

Figure 10: Asymptotic rent extraction for $K \to \infty$.

Proposition 2 shows that for every $\delta \in (0,1)$, the contest designer prefers to set the cost of early effort such that the incumbent always partially discourages the contender. When the continuation probability δ increases, two effects stand out: First, the optimal cost of early effort b^* decreases in the continuation probability. This means that the higher the continuation probability, the more will the incumbent discourage the contender in equilibrium. In the limit, as $\delta \to 1$, the contender wins with probability zero. Second, contest intensity increases with the continuation probability. This is intuitively clear since if δ increases, effort in contest k will with a higher probability also in contests $k+1, k+2, ...$ positively influence payoffs. In the limit, as $\delta \to 1$, the contest designer achieves full rent extraction.

While for every $K \in \mathbb{N}$ the subgame-perfect equilibrium from Section 3 is unique, the limit equilibrium is not the only subgame-perfect equilibrium of the infinite game. Instead, a plethora of subgame-perfect equilibria exist in which the players cooperate to a certain degree. Similar to trigger strategies in the repeated Prisoners Dilemma, players can play strategies in our game in which they cooperate (in the sense of exerting little effort) unless the other player defects. The uncertain end of the game allows these equilibria to exist, if the continuation probability δ is sufficiently high.

In general, any cooperation that yields a payoff improvement for both players can be sustained as a trigger equilibrium, if the continuation probability δ is sufficiently high. We focus our analysis on the most instructive trigger equilibrium, which is the one that yields the highest payoff difference.[20] Both players play effort levels of zero in all competition stages and investment stages, as long as their counterpart also sticks to the strategy. If player $-i$ deviates, player i punishes by playing the limit equilibrium level for the rest of the game. The trigger strategy therefore is an equilibrium if the best deviation, namely exerting an infinitesimal effort ϵ in the first contest and afterwards playing the limit equilibrium in every period, is not profitable.

Proposition 3. *Consider the infinite game. Let $x_{i,k}^L, x_{j,k}^L$ and $d_{i,k}^L$ denote the equilibrium levels of effort and early effort in the limit equilibrium. Consider the trigger strategy $T(b,\delta)$ which is defined by*

$$x_{i,k} = \begin{cases} 0 & \text{if } x_{i,n} = x_{j,n} = 0 \ \forall n \in \{1,...,k-1\} \text{ and } d_{i,n} = 0 \ \forall n \in \{1,...,k\} \\ x_{i,k}^L & \text{otherwise} \end{cases}$$

$$x_{j,k} = \begin{cases} 0 & \text{if } x_{i,n} = x_{j,n} = 0 \ \forall n \in \{1,...,k-1\} \text{ and } d_{i,n} = 0 \ \forall n \in \{1,...,k\} \\ x_{j,k}^L & \text{otherwise} \end{cases}$$

$$d_{i,k} = \begin{cases} 0 & \text{if } x_{i,n} = x_{j,n} = d_{i,n} = 0 \ \forall n \in \{1,...,k-1\} \\ d_{i,k}^L & \text{otherwise.} \end{cases}$$

Then,

(i) For $b \in (0, 1/2)$, the strategy profile in which both players play $T(b,\delta)$ is a subgame perfect Nash equilibrium if $\delta > \frac{1}{1+b}$.

(ii) For $b \in [1/2, 1]$, the strategy profile in which both players play $T(b,\delta)$ is a subgame perfect Nash equilibrium if $\delta > \frac{4b^2}{4b^2+3b-1}$.

(iii) For $b \in (1,\infty)$, the strategy profile in which both players play $T(b,\delta)$ is a subgame perfect Nash equilibrium if $\delta > 2/3$.

Proposition 3 reveals that the strategy profile in which both players play the trigger strategy $T(b,\delta)$ is a subgame perfect Nash equilibrium if the continuation probability δ is sufficiently high, that is, if the future is sufficiently important. The intuition behind this is as follows: the best deviation is to exert a small effort ϵ in the first contest and to play the limit equilibrium in all subsequent contests. This yields a payoff of 1 in the first contest and the limit equilibrium payoffs in all subsequent contests. The deviation therefore improves payoffs in the first contest, but reduces payoffs in all subsequent contests. If the continuation probability is low, it is less likely that the reduced payoffs are actually

[20] In principle, many more Nash equilibria that rely on cooperation and punishment exist, such as a tit for tat equilibrium. We focus on the above mentioned trigger equilibrium as a representative of this class of equilibria, since an exhaustive characterization of all equilibria would be beyond the scope of this paper.

realized. When the contest designer sets b optimally according to 2, numerical methods reveal that the trigger equilibrium exists for $\delta > \delta^* \approx 0.659$. Figure 11 illustrates the existence conditions of the trigger equilibrium.

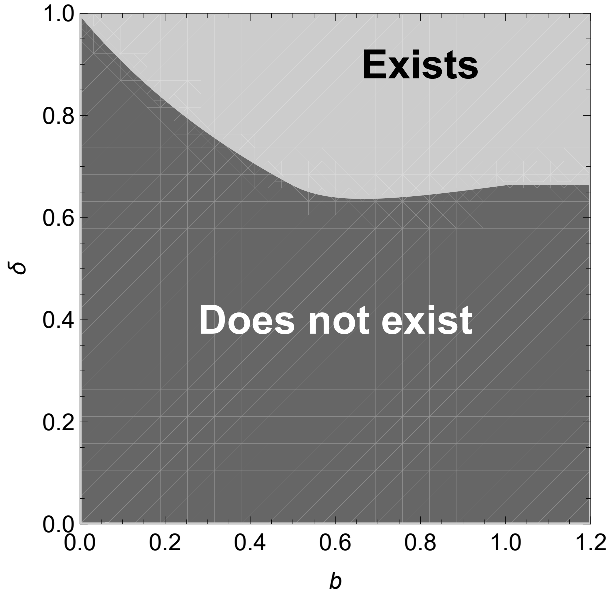

Figure 11: Existence of the trigger Nash equilibrium

Another insight is that the contest designer can use endogenous incumbency to eliminate the possibility of collusion. In the absence of endogenous incumbency, all contests are independent and the trigger equilibrium exists for $\delta > 2/3$. In this case, a contest designer who worries about collusion among players can set a low b since for low values of b, the continuation probability must be very high for the trigger equilibrium to exist. In other words, setting a small b increases the range of continuation probabilities δ for which collusion is not an equilibrium. The reason for this is that for very low b, expected payoffs for the players in the limit equilibrium are rather high. Therefore, after the deviation, the payoff difference between the trigger equilibrium and the limit equilibrium is rather small. The game must in expectation continue for a long time for the sum of this small payoff differences to outweigh the large payoff difference in the opposite direction in the first contest.

6 Extensions

6.1 Unobservable early effort

The Stackelberg structure implicitly assumes that early effort $d_{i,k}$ is observable by the contender before he decides upon $x^C_{j,k}$. If early effort is not observable prior to the contender's decision, the incumbent loses his mover advantage as there is no visible commitment. Therefore, players play as if the decision was simultaneous. The game is then equivalent to a simultaneous move game. For ease of notation, we will analyze the game in the simultaneous version. Nonetheless, the incumbent enjoys a cost advantage and exert positive early effort if $b < 1$. If $b > 1$, the incumbent prefers to exert regular effort instead of early effort. In this case, it is irrelevant whether early effort is observable or unobservable, since the incumbent never exerts early effort. Equilibrium behavior is therefore the same as if early effort was observable, and given by Lemma 3. In this section, we restrict ourselves to $b \leq 1$ to focus on the interesting cases.

Consider an arbitrary contest $k \in \{2, ..., K\}$ where player i is the incumbent and player j is the contender. Then, both players' maximization problems are given by

$$\max_{x_{i,k}} \pi_{i,k} = \frac{x_{i,k}}{x_{i,k} + x_{j,k}}(1 + \delta\pi_{i,k+1}) + (1 - \frac{x_{i,k}}{x_{i,k} + x_{j,k}})\delta\pi_{j,k+1} - bx_{i,1},$$

and

$$\max_{x_{j,k}} \pi_{j,k} = \frac{x_{j,k}}{x_{i,k} + x_{j,k}}(1 + \delta\pi_{i,k+1}) + (1 - \frac{x_{j,k}}{x_{i,k} + x_{j,k}})\delta\pi_{j,k+1} - x_{j,1}.$$

The maximization problems are symmetric except for the marginal cost of effort for the incumbent. Similar to the game with observable early effort, we compute the equilibrium of the infinite game by first computing the equilibrium of the finite, K-contest game by backward induction. Then, we again use Fudenberg and Levine (1983) to get the limit equilibrium. The detailed calculations can be found in the Appendix. We define rent extraction in the game with unobservable effort as

$$\rho(b, K) = \frac{1}{\sum_{k=1}^{K} \delta^{k-1}}(2x_{1,K} + \sum_{k=2}^{K} \delta^{k-1}(b \cdot x^*_{i,k} + x^*_{j,k}))$$

In the infinite game, rent extraction is

Proposition 4.

$$\rho(b, \infty) = \frac{(1-\delta)(1+b)^2 + 4b\delta}{2(1+b)(1+b-\delta+\delta b)}$$

The contest designer who maximizes rent extraction optimally sets

$$b^* = \frac{\delta + 2\sqrt{1-\delta} - 1}{\delta + 3}.$$

Figure 12 illustrates the results from Proposition 4. We find that rent extraction is increasing in the continuation probability δ. This is intuitive as players try harder if the future is valued more. Also if early effort is unobservable, the interplay between the discouragement effect and the incentive effect shapes players' effort choices. The discouragement effect increases efforts in b, while the incentive effect decreases efforts in b. As δ increases, the rent extraction maximizing level of b decreases. This reflects that if the future is important, the incentive effect dominates the discouragement effect. In addition, optimal rent extraction is increasing in δ. In particular, the more important the future is, the harder will the players fight and the more rent the contest designer can extract.

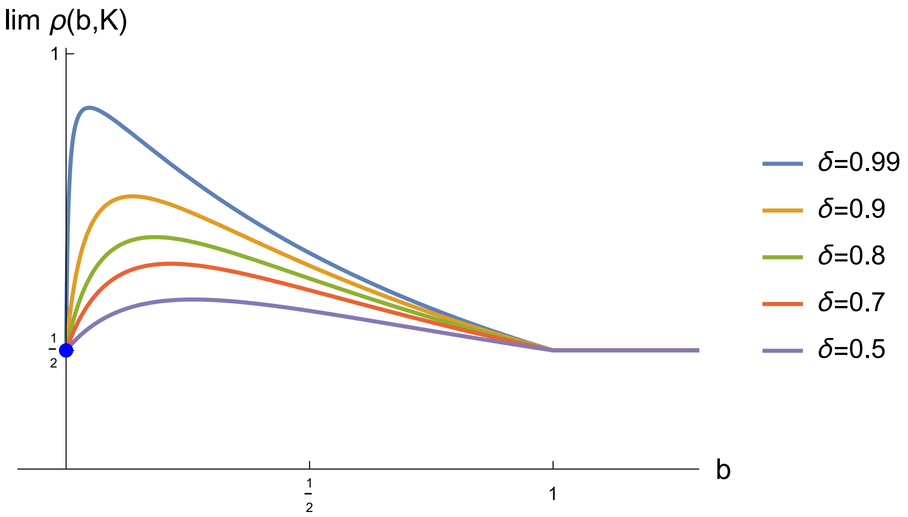

Figure 12: Rent extraction in the infinite game depending on δ

The following corollary relates the game with observable early effort to the game with unobservable early effort:

Corollary 5. *For all $\delta \in (0, 1)$, rent extraction in the infinite game with observable early effort is higher than rent extraction in the infinite game with unobservable early effort, if the contest designer sets the cost of early effort b optimally.*

The proof simply inserts the optimal levels b^* into rent extraction and compares. Corollary 5 highlights that the contest designer prefers early effort to be observable for any δ. In particular, this reveals that if the game reflects a relationship where early effort is unobservable for the contender, the contest designer should always publicly announce the level of the incumbent's total effort. Observable early effort enhances both the discouragement effect and the incentive effect. As the incentive effect dominates if the contest designer sets the cost of early effort optimally, observable early effort leads to a higher level of rent extraction in optimum.

6.2 Winner's effort

Next, consider a contest designer who maximizes winner's effort. In other words, regular effort is only valuable if the respective effort exerting player also wins the contest. In a repeated contest for mandates, the contest designer prefers to give the mandate to a strong winner, with whom he agrees on a contract. The loser does not get a contract, so the contest designer does not value the loser's effort. Note however, that the contest for a mandate is still a noisy process (depending on b), so for instance with some probability a contender can win with, compared to the incumbent, low effort. In this case, the incumbent's high effort is wasted from the contest designer's perspective. In contrast, we assume that the contest designer always benefits from the incumbent's early effort irrespective of whether the incumbent wins the contest. This reflects the interpretation of early effort as effort in implementation, which the contest designer always values.

The contest designer sets b in order to maximize winner's effort. The players observe b and choose their levels of effort and early effort optimally. The players' behavior is given by Lemma 3, 5 and 4. Then, the contest designer's maximization problem of rent extraction ρ_W is given by

$$\max_{b} \quad \rho_W(b,K) = \frac{1}{\sum_{k=1}^{K}\delta^{k-1}}(\sum_{k=1}^{K}\delta^{k-1}(p_{i,k}x_{i,k} + (1-p_{i,k})x_{j,k} + b \cdot d_{i,k})),$$

The incumbent's probability of winning is given by Corollary 1. Therefore, expected winner's effort in the equilibrium of arbitrary contest $k \in \{2, ..., K\}$ is given by

$$W_k = \begin{cases} \frac{1}{4} & \text{if } b \geq 1, \\ \frac{6b^2-4b+1}{8b^3}\psi_k & \text{if } \frac{1}{2} < b < 1, \\ b\psi_k & \text{if } 0 \leq b \leq \frac{1}{2}. \end{cases} \tag{6}$$

Then, rent extraction ρ_W of the K-contest game with and without discounting can be summarized in Proposition 5.

Proposition 5. *Consider a contest designer that is interested in maximizing winner's effort. Then, rent extraction in the infinite game is*

$$\rho_W(b,\infty) = \begin{cases} \frac{1}{4} & \text{if } b \geq 1, \\ \frac{2b^3-2(b-2)(b-1)b\delta+\delta}{2b(4b^2(\delta+1)-5b\delta+\delta)} & \text{if } \frac{1}{2} < b < 1 \\ \frac{(4b-1)\delta+1}{4(b-1)\delta+4} & \text{if } 0 < b \leq \frac{1}{2} \end{cases}$$

Figure 13 illustrates the results of Proposition 5. When the contest designer maximizes winner's effort instead of total effort, rent extraction changes depending on the value of b. In the area of full discouragement ($b < 1/2$), winner's effort is very close to total effort, since in all contests after the first the incumbent wins with certainty. Only in the first contest both players have a positive chance of winning, and some effort is lost for the contest designer. In the area of partial discouragement ($1/2 < b < 1$), the contender exerts some positive effort in every contest. Therefore, in every contest, the loser exerts positive effort, which is lost from the perspective of the contest designer. However, the incumbent, who is the player that exerts more effort, also wins with higher probability.[21] In the area of no discouragement ($b > 1$), there is no incumbency advantage and the incumbent exerts exactly as much effort as the contender. Therefore, in every contest, the contest designer does not value half of aggregate effort.

[21] Note that there is a discontinuity in rent extraction around $b = 1$. The reason for this is that at this point, the incumbent shifts his effort from the investment stage to the competition stage. In the investment stage, the contest designer always values the incumbent's effort irrespective whether he wins. In the competition stage, the contest designer only values the incumbent's effort if he wins, so in equilibrium half of the time.

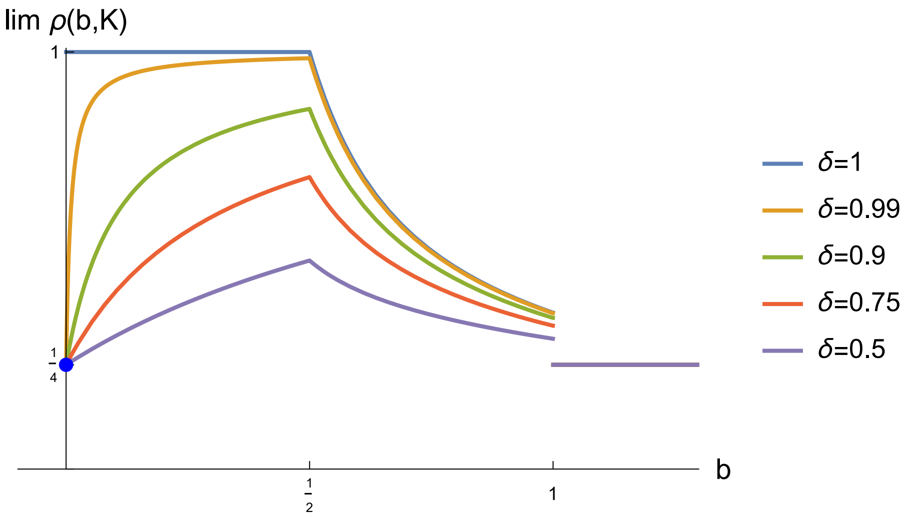

Figure 13: Rent extraction in the infinite game depending on δ, where the contest designer maximizes winner's effort.

Proposition 5 also reveals that a winner's effort maximizing contest designer prefers to always set $b = 1/2$, irrespective of δ. In particular, for this value of b, the incumbent always fully discourages the contender. This means that the contender exerts no effort and the incumbent wins all contests after the first with certainty in equilibrium. This result differs from the optimal cost of early effort when the contest designer maximizes total effort (Proposition 1). When maximizing total effort, the contender always exerts positive effort and has a positive chance of winning each contest. When maximizing winner's effort, such a structure is not optimal from the contest designer's perspective, since in any contest where both players exert some effort, one of the efforts is not valued. As the contest designer does not value loser's effort, he prefers to impose a structure where all effort in the contest comes from the winner.

7 Conclusion

We develop a repeated contest model in which contestants compete for a period-wise prize and the incumbent position in the subsequent contest. The incumbent has the opportunity to gain a head start in the subsequent contest by exerting early effort which can be subsidized by the contest designer. Therefore, the incumbent's advantage is twofold: the incumbent can commit effort earlier than the contender, and early effort may be less costly. In general, we show that a contest designer who maximizes rent extraction always prefers to employ endogenous incumbency over independent contests. The contest designer therefore prefers to introduce a structure which over time generates endogenous asymmetry in an ex-ante symmetric game. The trade-off between fairness and rewards is central to our analysis: We find that it is optimal to sacrifice some fairness in order to generate incentives to fight harder; it is however imperative that the asymmetry results as a reward for performance and not as an idiosyncratic bias.

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A Appendix

A.1 Proof Lemma 1

In competition stage C_k, player i's maximization problem is

$$\pi^C_{i,k} = \frac{x_{i,k} + d_{i,k}}{x_{i,k} + x_{j,k} + d_{i,k}}(1 + \delta\pi^I_{i,k+1}) + \left(1 - \frac{x_{i,k} + d_{i,k}}{x_{i,k} + x_{j,k} + d_{i,k}}\right)\delta\pi^C_{j,k+1} - x_{i,k}$$

and player j's maximization problem is

$$\pi^C_{j,k} = \frac{x_{j,k}}{x_{i,k} + x_{j,k} + d_{i,k}}(1 + \delta\pi^I_{i,k+1}) + \left(1 - \frac{x_{j,k}}{x_{i,k} + x_{j,k} + d_{i,k}}\right)\delta\pi^C_{j,k+1} - x_{j,k}.$$

Solving the two maximization problems and using $\psi_k = 1 + \delta(\pi^I_{k+1} - \pi^C_{k+1})$ yields

$$x_{i,k} = \frac{\psi_k}{4} - d_{i,k}$$
$$x_{j,k} = \frac{\psi_k}{4}$$

Since efforts are nonnegative, the above solution is only valid for $d_{i,k} \leq \psi_k/4$. Otherwise, player i exerts zero effort in optimum. In this case, effort of player j is given by player j's best response to $x_{i,k} = 0$, which is

$$x_{j,k} = \sqrt{d_{i,k}\psi_k} - d_{i,k}.$$

Also player j's effort must be nonnegative, so the above solution is only valid for $d_{i,k} \in [\psi_k/4, \psi_k]$. If instead $d_{i,k} > \psi_k$, player j exerts zero effort in optimum.

Together, the equilibrium levels conditional on the value of $d_{i,k}$ are

$$x^*_{i,k} = \begin{cases} \frac{\psi_k}{4} - d_{i,k} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ 0 & \text{if } d_{i,k} > \frac{\psi_k}{4}, \end{cases}$$
$$x^*_{j,k} = \begin{cases} \frac{\psi_k}{4} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ \sqrt{d_{i,k}\psi_k} - d_{i,k} & \text{if } \frac{\psi_k}{4} < d_{i,k} < \psi_k, \\ 0 & \text{if } d_{i,k} \geq \psi_k. \end{cases}$$

Expected payoffs in competition stage C_k are then given by

$$\pi^C_{i,k}(x^*_{i,k}, x^*_{j,k}) = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} + d_{i,k} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ \sqrt{d_{i,k}\psi_k} + \delta\pi^C_{j,k+1} & \text{if } \frac{\psi_k}{4} < d_{i,k} < \psi_k, \\ \psi_k + \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \geq \psi_k, \end{cases}$$
$$\pi^C_{j,k}(x^*_{i,k}, x^*_{j,k}) = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \leq \frac{\psi_k}{4}, \\ \psi_k + \delta\pi^C_{j,k+1} + d_{i,k} - 2\sqrt{d_{i,k}\psi_k} & \text{if } \frac{\psi_k}{4} < d_{i,k} < \psi_k, \\ \delta\pi^C_{j,k+1} & \text{if } d_{i,k} \geq \psi_k. \end{cases}$$

which proves Lemma 1.

A.2 Proof Lemma 2

Using the expected payoffs given by Lemma 1, player i's maximization problem is given by

$$\max_{d_{i,k}} \quad \pi^I_{i,k} = \begin{cases} \frac{1}{4}\psi_k + \delta\pi^C_{j,k+1} + d_{i,k} - bd_{i,k} & \text{if } d_{i,k} \leq \frac{1}{4}\psi_k, \\ \sqrt{d_{i,k}\psi_k} + \delta\pi^C_{j,k+1} - bd_{i,k} & \text{if } \frac{1}{4}\psi_k < d_{i,k} < \psi_k, \\ \psi_k + \delta\pi^C_{j,k+1} - bd_{i,k} & \text{if } d_{i,k} \geq \psi_k. \end{cases}$$

For each case, we solve the maximization problem separately, Afterwards, we compare the payoffs of the three cases for a given b to determine player i's reaction function $d^*_{i,k}(b)$.

Case 1: $d_{i,k} \leq \frac{1}{4}\psi_k$
Obviously,$\frac{\partial \pi^I_{i,k}}{\partial d_{i,k}} = 1 - b$ is positive for all $b < 1$. Then, $d_{i,k}$ is given by

$$d_{i,k} = \begin{cases} \frac{1}{4}\psi_k & \text{if } b < 1, \\ [0, \frac{1}{4}\psi_k] & \text{if } b = 1, \\ 0 & \text{if } b > 1. \end{cases}$$

and the incumbent's corresponding payoff is

$$\pi^I_{i,k} = \begin{cases} \frac{(2-b)\psi_k}{4} + \delta\pi^C_{j,k+1} & \text{if } b < 1 \\ \frac{\psi_k}{4} + \delta\pi^C_{j,k+1} & \text{if } b \geq 1 \end{cases}$$

Case 2: $\frac{1}{4}\psi_k \leq d_{i,k} \leq \psi_k$
Maximization yields $d_{i,k+1} = \frac{\psi_k}{4b^2}$. However, the solution is only valid for $\frac{1}{4}\psi_k \leq \frac{\psi_k}{4b^2} \leq \psi_k$. If $b > 1$, then $\frac{\psi_k}{4b^2} < \frac{1}{4}\psi_k$, so the lower bound of the interval is optimal. If $b < \frac{1}{2}$, then $\frac{\psi_k}{4b^2} > \psi_k$, so the upper bound of the interval is optimal. Therefore,

$$d_{i,k} = \begin{cases} \psi_k & \text{if } b \leq \frac{1}{2} \\ \frac{\psi_k}{4b^2} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{4}\psi_k & \text{if } b \geq 1 \end{cases}$$

$$\pi_{i,k} = \begin{cases} \psi_k(1-b) + \delta\pi^C_{j,k+1} & \text{if } b \leq \frac{1}{2} \\ \frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} & \text{if } \frac{1}{2} < b < 1 \\ \frac{(2-b)\psi_k}{4} + \delta\pi^C_{j,k+1} & \text{if } b \geq 1 \end{cases}$$

Case 3: $d_{i,k} > \psi_k$
Obviously, $\frac{\partial \pi_{i,k}}{\partial d_{i,k}} = -b$ is negative for all $b > 0$. Therefore,

$$d_{i,k} = \psi_k$$
$$\pi_{i,k} = \psi_k(1-b) + \delta\pi^C_{j,k+1}$$

We compare the case-dependent equilibrium payoffs for all possible values of b and identify three intervals, i.e., $b < \frac{1}{2}$, $\frac{1}{2} < b < 1$ and $b > 1$. Notice that fixing b implies a fixed ψ_k as well in equilibrium.

If $b \leq \frac{1}{2}$, then

$$\psi_k(1-b) + \delta\pi^C_{j,k+1} > \frac{(2-b)\psi_k}{4} + \delta\pi^C_{j,k+1} \quad \Leftrightarrow \quad b < \frac{2}{3},$$

which is always true if $b < 1/2$. Therefore,

$$d^*_{i,k} = \psi_k \quad \text{if } 0 < b \leq \frac{1}{2}$$

If $\frac{1}{2} < b < 1$, then

$$\frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} > \psi_k + \delta\pi^C_{j,k+1} - b\psi_k \quad \Leftrightarrow \quad (2b-1)^2 > 0$$

and

$$\frac{\psi_k}{4b} + \delta\pi^C_{j,k+1} > \frac{\psi_k(2-b)}{4} + \delta\pi^C_{j,k+1} \quad \Leftrightarrow \quad (b-1)^2 > 0$$

are both always true. Therefore,

$$d^*_{i,k} = \frac{\psi_k}{4b^2} \quad \text{if } \frac{1}{2} < b < 1.$$

If $b \geq 1$, then

$$\frac{\psi_k}{4} + \delta\pi^C_{i,k+1} \geq \frac{\psi_k(2-b)}{4} + \delta\pi^C_{j,k+1} \quad \Leftrightarrow \quad b \geq 1$$

and

$$\frac{\psi_k}{4} + \delta\pi^C_{j,k+1} > \psi_k + \delta\pi^C_{j,k+1} - b\psi_k \quad \Leftrightarrow \quad b \geq \frac{3}{4}$$

are both always true.

Therefore, bringing the three cases together,

$$d^*_{i,k} = \begin{cases} \psi_k & \text{if } b \leq \frac{1}{2}, \\ \frac{\psi_k}{4b^2} & \text{if } \frac{1}{2} < b < 1, \\ 0 & \text{if } b \geq 1 \end{cases}$$

which proves Lemma 2.

A.3 Proof Lemma 3

For any $b \geq 1$, Lemma 2 yields $d^*_{i,k} = 0 \; \forall k \in \{2, ..., K\}$. Therefore, Lemma 1 and Lemma 2 immediately reveal $\pi^I_{i,k} = \pi^C_{j,k}$ for all k, so $\psi_k = 1$ and $x_{i,k} = x_{j,k} = 1/4$.

A.4 Proof Lemma 4

Assume $b < 1/2$. Given that $\psi_k = 1 + \delta(\pi^I_{i,k+1} - \pi^C_{j,k+1})$ and inserting the values of $\pi^I_{i,k+1}$ and $\pi^C_{j,k+1}$ from Lemma 1 and Lemma 2, we have

$$\psi_k = 1 + \delta(1-b)\psi_{k+1}.$$

From that we can immediately see that ψ_k is increasing in δ and decreasing in b. We now show that

$$\psi_k = \frac{1 - (\delta(1-b))^{K-k+1}}{1 - \delta(1-b)} \tag{7}$$

for all $k \in \{1, ..., K\}$ by induction, where the base case covers period K and the induction step is from $k+1$ to k.

In period K, there is no next period, so continuation payoffs of winning and losing are identical, meaning $\psi_K = 1$. Indeed, from (7), we have

$$\psi_K = \frac{1 - (\delta(1-b))^{K-K+1}}{1 - \delta(1-b)} = 1.$$

For the induction step, assume the statement holds for $k+1$. Then,

$$\psi_k = 1 + \delta(1-b)\psi_{k+1} = \delta(1+b)\frac{1 - (\delta(1-b))^{K-k}}{1 - \delta(1-b)} = \frac{1 - \delta(1-b) + \delta(1-b)(1 - (\delta(1-b))^{K-k})}{1 - \delta(1-b)}$$
$$= \frac{1 + \delta(1-b)(1 - (\delta(1-b))^{K-k} - 1)}{1 - \delta(1-b)} = \frac{1 - (\delta(1-b))^{K-k+1}}{1 - \delta(1-b)}.$$

From this we can immediately see that ψ_k is decreasing in k, so increasing in the number of remaining contests, which concludes the proof of (7). From Lemma 1 and Lemma 2 we then immediately get the rest of Lemma 4.

A.5 Proof Lemma 5

Assume $b \in (1/2, 1)$. Given that $\psi_k = 1 + \delta(\pi^I_{i,k+1} - \pi^C_{j,k+1})$, inserting continuation payoffs from Lemma 1 and Lemma 2 yields

$$\psi_k = 1 + \delta(\frac{\psi_{k+1}}{4b} + \delta\pi^C_{j,k+2} - 1 - \delta\pi^I_{i,k+2} - \frac{\psi_{k+1}}{4b^2} + \frac{\psi_{k+1}}{b})$$

which is, after using the definition $\psi_{k+1} = 1 + \delta(\pi^I_{i,k+2} - \pi^C_{j,k+2})$, equivalent to

$$\begin{aligned}\psi_k &= 1 + \delta(-\psi_{k+1} + \frac{\psi_{k+1}}{4b} - \frac{\psi_{k+1}}{4b^2} + \frac{\psi_{k+1}}{b} \\ &= 1 + \delta\psi_{k+1}(\frac{5b-1}{4b^2} - 1)\end{aligned}$$

We now show by induction, similar to the proof of Lemma 4, that

$$\psi_k = \frac{1 - (\delta A)^{K-k+1}}{1 - \delta A} \tag{8}$$

where $A := \frac{5b-1}{4b^2} - 1$.
Base case:

$$\psi_K = \frac{1-(\delta A)^{K-K+1}}{1-\delta A} = 1.$$

Induction step: assume the statement holds for $k+1$. Then,

$$\begin{aligned}\psi_k &= 1 + \delta A \psi_{k+1} = 1 + \delta A \frac{1-(\delta A)^{K-k}}{1-\delta A} = \frac{1-\delta A + \delta A(1-(\delta A)^{K-k})}{1-\delta A} \\ &= \frac{1+\delta A(1-(\delta A)^{K-k} - 1}{1-\delta A} = \frac{1-(\delta A)^{K-k+1}}{1-\delta A}\end{aligned}$$

which proves (8). Inserting this expression for ψ_k into the respective values for $d^*_{i,k}$ and $x_{j,k}$ proves the rest of Lemma 5.

A.6 Proof Proposition 1

Given that the contest designer discounts future payoffs as well and shares the common discount factor δ, the present value in period 1 of total effort can be rewritten as

$$\begin{aligned}TE(b,K) &= \sum_{k=1}^{K} \delta^{k-1}(x_{i,k} + x_{j,k} + b \cdot d_{i,k}) = \\ &= 2x_{1,1} + \sum_{k=2}^{K} \delta^{k-1} E_k.\end{aligned} \tag{9}$$

using the definition of E_k from Corollary 3. We can now, depending on b, insert the results from Corollary 2, Corollary 3 and Lemma 3-5.

For $b \geq 1$ this yields

$$TE(b,K) = \frac{1}{2} + \frac{1}{2}\sum_{k=2}^{K} \delta^{k-1} = \frac{1}{2} + \frac{1}{2} \cdot \frac{\delta(1-\delta^{K-1})}{1-\delta}.$$

For $b \in (1/2, 1)$ this yields

$$\begin{aligned}TE(b,K) &= \frac{1}{2} \cdot \frac{1-(\delta A)^K}{1-\delta A} + \frac{3b-1}{4b^2} \sum_{k=2}^{K} \delta^{k-1} \frac{1-(\delta A)^{K-k+1}}{1-\delta A} \\ &= \frac{1}{2} \cdot \frac{1-(\delta A)^K}{1-\delta A} + \frac{3b-1}{4b^2(1-\delta A)} (\sum_{k=2}^{K} \delta^{k-1} - \sum_{k=2}^{K} \delta^K A^{K-k+1}) \\ &= \frac{1}{2} \cdot \frac{1-(\delta A)^K}{1-\delta A} + \frac{3b-1}{4b^2(1-\delta A)} (\sum_{k=2}^{K} \delta^{k-1} - \delta^K \sum_{k=1}^{K-1} A^k) \\ &= \frac{1}{2} \cdot \frac{1-(\delta A)^K}{1-\delta A} + \frac{3b-1}{4b^2(1-\delta A)} (\frac{\delta(1-\delta^{K-1})}{1-\delta} - \frac{\delta^K A(1-A^{K-1})}{1-A}).\end{aligned}$$

For $b \in (0, 1/2)$ this yields

$$
\begin{aligned}
TE(b,K) &= \frac{1}{2} \cdot \frac{1-(\delta(1-b))^K}{1-\delta(1-b)} + b\sum_{k=2}^{K} \delta^{k-1} \frac{1-(\delta(1-b))^{K-k+1}}{1-\delta(1-b)} \\
&= \frac{1}{2} \cdot \frac{1-(\delta(1-b))^K}{1-\delta(1-b)} + \frac{b}{1-\delta(1-b)} \sum_{k=2}^{K} \delta^{k-1} - \delta^K (1-b)^{K-k+1} \\
&= \frac{1}{2} \cdot \frac{1-(\delta(1-b))^K}{1-\delta(1-b)} + \frac{b}{1-\delta(1-b)} (\sum_{k=2}^{K} \delta^{k-1} - \delta^K \sum_{k=2}^{K} (1-b)^{K-k+1}) \\
&= \frac{1}{2} \cdot \frac{1-(\delta(1-b))^K}{1-\delta(1-b)} + \frac{b}{1-\delta(1-b)} (\sum_{k=2}^{K} \delta^{k-1} - \delta^K \sum_{k=1}^{K-1} (1-b)^{k}) \\
&= \frac{1}{2} \cdot \frac{1-(\delta(1-b))^K}{1-\delta(1-b)} + \frac{b}{1-\delta(1-b)} (\frac{\delta(1-\delta^{K-1})}{1-\delta} - \delta^K \frac{(1-b)(1-(1-b)^{K-1})}{b})
\end{aligned}
$$

which proves Proposition 1.

A.7 Proof Proposition 2

Consider the K-contest game. Then, the subgame perfect Nash equilibrium of the K-contest game, depending on b, is given by Lemma 3, Lemma 5, Lemma 4, Corollary 2 and Proposition **??**. Let g_K denote the subgame perfect Nash equilibrium for $K \in \mathbb{N}$. Then, Theorem 3.3 from Fudenberg and Levine (1983) states that $g^* = \lim_{K\to\infty} g_K$ is a subgame perfect Nash equilibrium of the infinite game (for $T(n) = n$ and $\epsilon_n = 0 \; \forall n$).

Then, rent extraction in the infinite game is the limit of rent extraction in the K-contest game for $K \to \infty$. Letting $K \to \infty$ in Proposition 1 yields rent extraction for the infinite game. One can quickly verify that $\rho(b, \infty)$ is increasing for $b < 1/2$ and has the unique maximum

$$b^* = \frac{\delta + \sqrt{9-5\delta} + 3}{\delta + 11}$$

which always satisfies $b^* \in (1/2, 1)$.

A.8 Proof Proposition 3

Consider the trigger strategy $T(b, \delta)$. Assume both players play according to $T(b, \delta)$. Consider a unilateral deviation by some player p. First, note that an optimal deviation is characterized by some deviation in some stage k, and always playing the limit equilibrium in all stages $k' > k$. This is because player p knows that player $-p$ will react to player p's deviation by pulling the trigger and playing the limit equilibrium in all future periods. Player p's best response to the limit equilibrium is the limit equilibrium.

The optimal deviation therefore occurs in exactly one period. At what point in time should player p optimally deviate? Note that the profitable deviation must increase payoffs in the exact period of the deviation, since afterwards cooperation ends and payoffs decrease relative to the cooperative payoffs. The increase in payoffs in the period of deviation must therefore offset the discounted sum of payoff losses in all future periods. Since future periods are uncertain, it must be optimal for the deviation to occur as early as possible, that is, in the first contest.

How should the optimal deviation in the first contest look like? This is rather intuitive. Since in the trigger strategy, player $-p$ exerts an effort of zero, player p can exert an arbitrarily small effort to win the first contest with certainty and become the incumbent. In particular, assume player p deviates as follows:

$$x_{p,k} = \begin{cases} \bar{x} & \text{if} \quad k = 1 \\ x^L_{p,k} & \text{if} \quad k > 1 \end{cases} \qquad\qquad d_{p,k} = d^L_{p,k} \quad \forall k$$

In other words, player p exerts an effort of $\bar{x}$ in the first contest and plays the limit equilibrium afterwards. Then, player p receives a total payoff of

$$\pi^D = 1 - \bar{x} + \delta \pi^I_{i,2}$$

The payoff $\pi^I_{i,2}$ can be calculated as the limit of the payoff in the K-contest game, as in Proposition 2. In contrast, if both players play the trigger strategy, both players enjoy a payoff of 1/2 in every period. The present value of this at the start of the game is

$$\pi^T = \sum_{k=1}^{\infty} \frac{1}{2}\delta^{k-1} = \frac{1}{2(1-\delta)}$$

The trigger strategy $T(b,\delta)$ is then a symmetric equilibrium if $\pi^T > \pi^D$. We use the following expression for $\pi^I_{i,2}$ to compare the two payoffs:

Lemma 6. *The payoff $\pi^I_{i,2}$ is given by*

$$\pi^I_{i,2} = \begin{cases} \frac{1}{4(1-\delta)} & \text{if} \quad b \geq 1 \\ \frac{A}{1-\delta A} + \frac{(2b-1)^2}{(1-\delta A)(1-\delta)4b^2} & \text{if} \quad \frac{1}{2} < b < 1 \\ \frac{1-b}{1-\delta(1-b)} & \text{if} \quad b \leq \frac{1}{2} \end{cases}$$

where $A = \frac{5b-1}{4b^2} - 1$.

The proof of this Lemma is below. Using this, we can express π^D as

$$\pi^D = \begin{cases} 1 - \bar{x} + \frac{\delta}{4(1-\delta)} & \text{if} \quad b \geq 1 \\ 1 - \bar{x} + \delta\left(\frac{A}{1-\delta A} + \frac{(2b-1)^2}{(1-\delta A)(1-\delta)4b^2}\right) & \text{if} \quad \frac{1}{2} < b < 1 \\ 1 - \bar{x} + \delta\left(\frac{1-b}{1-\delta(1-b)}\right) & \text{if} \quad b \leq \frac{1}{2} \end{cases}$$

Since any positive $\bar{x}$ ensures this payoff, player p sets $\bar{x}$ arbitrarily close to zero. As the usual convention goes, we set $\bar{x} = 0$. Then, a simple comparison reveals that the deviation is profitable if $\pi^D > \pi^T$, that is

$$\pi^D > \pi^T \Leftrightarrow \begin{cases} \delta < \frac{2}{3} & \text{if} \quad b \geq 1 \\ \delta < \frac{4b^2}{4b^2+3b-1} & \text{if} \quad \frac{1}{2} < b < 1 \\ \delta < \frac{1}{1+b} & \text{if} \quad b \leq \frac{1}{2} \end{cases}$$

which proves Proposition 3.
Proof of Lemma 6:
We prove the three cases separately. Suppose first $b \geq 1$. Then, for all k, $d_{i,k} = 0$ and

$x_{i,k} = x_{j,k} = 1/4$. Therefore $\pi^I_{i,k} = \sum_{k=0}^{\infty} \frac{1}{4}\delta^k$ for all k. In particular, also for $k = 2$ we have

$$\pi^I_{i,2} = \frac{1}{4(1-\delta)}.$$

Suppose now $b \in (1/2, 1)$. We calculate the payoff $\pi^I_{i,2}$ of the infinite game as the limit of payoffs $\pi^I_{i,2}$ of the K-contest game and then make again use of Theorem 3.3 from Fudenberg and Levine (1983). Then, according to Lemma 1 and Lemma 2, we can express $\pi^C_{j,k}$ recursively:

$$\pi^C_{j,k} = \psi_k + \delta\pi^C_{j,k+1} + \frac{\psi_k}{4b^2} - 2\sqrt{\frac{\psi_k^2}{4b^2}} = \frac{(2b-1)^2}{4b^2}\psi_k + \delta\pi^C_{j,k+1}.$$

By induction we show that $\pi^C_{j,k}$ has the representation

$$\pi^C_{j,k} = \frac{(2b-1)^2}{4b^2}\sum_{n=k}^{K}\delta^{n-k}\psi_n.$$

The base case $k = K$ is clear from $\psi_K = 1$ and Lemma 1. Induction step from $k+1$ to k:

$$\begin{aligned}\pi^C_{j,k} &= \frac{(2b-1)^2}{4b^2}\psi_k + \delta\pi^C_{j,k+1} = \frac{(2b-1)^2}{4b^2}\psi_k + \delta\frac{(2b-1)^2}{4b^2}\sum_{n=k+1}^{K}\delta^{n-(k+1)}\psi_n \\ &= \frac{(2b-1)^2}{4b^2}(\psi_k + \sum_{n=k+1}^{K}\delta^{n-k}\psi_n) = \frac{(2b-1)^2}{4b^2}\sum_{n=k}^{K}\delta^{n-k}\psi_n.\end{aligned}$$

Inserting this into $\pi^I_{i,k}$ from Lemma 2, we obtain

$$\begin{aligned}\pi^I_{i,k} &= \frac{1}{4b}\psi_k + \delta\pi^C_{j,k+1} = \frac{1}{4b}\psi_k + \delta(\frac{(2b-1)^2}{4b^2}\sum_{n=k+1}^{K}\delta^{n-(k+1)}\psi_n) \\ &= \frac{1}{4b}\psi_k + \delta^0\frac{(2b-1)^2}{4b^2}\psi_k - \delta^0\frac{(2b-1)^2}{4b^2}\psi_k + \delta\frac{(2b-1)^2}{4b^2}\sum_{n=k+1}^{K}\delta^{n-(k+1)}\psi_n = \\ &= \left(\frac{5b-1}{4b^2} - 1\right)\psi_k + \frac{(2b-1)^2}{4b^2}\sum_{n=k}^{K}\delta^{n-k}\psi_n.\end{aligned}$$

From Lemma 5 we can insert the value of ψ_n into this expression and obtain

$$\begin{aligned}\pi^I_{j,k} &= A\cdot\frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2}\sum_{n=k}^{K}\delta^{n-k}\frac{1-(\delta A)^{K-n+1}}{1-\delta A} \\ &= A\cdot\frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2(1-\delta A)}(\sum_{n=k}^{K}\delta^{n-k} - \sum_{n=k}^{K}\delta^{K-k+1}A^{K-n+1}) \\ &= A\cdot\frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2(1-\delta A)}(\sum_{n=0}^{K-k}\delta^n - \delta^{K-k+1}\sum_{n=1}^{K-k+1}A^n) \\ &= A\cdot\frac{1-(\delta A)^{K-k+1}}{1-\delta A} + \frac{(2b-1)^2}{4b^2(1-\delta A)}(\frac{1-\delta^{K-k+1}}{1-\delta} - \delta^{K-k+1}\frac{A(1-A^{K-k+1})}{1-A})\end{aligned}$$

Setting $k = 2$ and letting $K \to \infty$ in this expression yields $\pi^I_{i,2}$ in the infinite game:

$$\lim_{K\to\infty} \pi^I_{i,2} = \frac{A}{1-\delta A} + \frac{(2b-1)^2}{(1-\delta A)(1-\delta)4b^2}.$$

Suppose now $b \in (0, 1/2)$. Then, from Lemma 4, $\pi^I_{i,k} = (1-b)\psi_k$. Inserting the value of ψ_k from Lemma 4 yields

$$\pi^I_{i,k} = (1-b)\frac{1-(\delta(1-b))^{K-k+1}}{1-\delta(1-b)}.$$

Now, again, for $k = 2$ and $K \to \infty$ we obtain

$$\pi^I_{i,2} = \frac{(1-b)}{1-\delta(1-b)}$$

which finalizes the proof of the Lemma.

A.9 Proof Proposition 4

For the game with unobservable early effort, we show by induction that the effective prize in period k is given by

$$\psi_k = \frac{1-(\delta B)^{K-k+1}}{1-\delta B}, \qquad B = \frac{1-b}{1+b}.$$

The base case $k = K$ is clear since $\psi_K = 1$. Induction step $k + 1 \to K$:

$$\begin{aligned}
\psi_k &= 1 + \delta(\pi_{i,k+1} - \pi_{j,k+1}) = 1 + \delta(\frac{\psi_{k+1}}{(1+b)^2} - \frac{b^2\psi_{k+1}}{(1+b)^2}) \\
&= 1 + \frac{\delta(1-b)}{1+b}\psi_{k+1} = 1 + \frac{\delta(1-b)}{1+b}\frac{1-(\delta B)^{K-k}}{1-\delta B} = 1 + \frac{\delta B(1-(\delta B)^{K-k})}{1-\delta B} \\
&= \frac{1-\delta B + \delta B(1-(\delta B)^{K-k})}{1-\delta B} = \frac{1-(\delta B)^{K-k+1}}{1-\delta B}
\end{aligned}$$

Since $x_{i,k} = \frac{1}{(1+b)^2}\psi_k$ and $x_{j,k} = \frac{b^2}{(1+b)^2}\psi_k$, we directly have

$$\begin{aligned}
x_{i,k} &= \frac{1}{(1+b)^2}\frac{1-(\delta B)^{K-k+1}}{1-\delta B} \\
x_{j,k} &= \frac{b}{(1+b)^2}\frac{1-(\delta B)^{K-k+1}}{1-\delta B}.
\end{aligned}$$

Since the expression of ψ_k is also valid for $k = 1$ and players in the first contest both exert effort of a quarter of the prize, we have

$$x_{i,1} = x_{j,1} = \frac{1-(\delta B)^K}{4(1-\delta B)}.$$

Inserting these values into $\rho(b,K)$, we obtain

$$\begin{aligned}
\rho(b,K) &= \frac{1}{\sum_{k=1}^{K}\delta^{k-1}}(2\frac{1-(\delta B)^K}{4(1-\delta B)}+\sum_{k=2}^{K}\delta^{k-1}\frac{2b}{(1+b)^2}\frac{1-(\delta B)^{K-k+1}}{1-\delta B}) \\
&= \frac{1-\delta}{1-\delta^K}(\frac{1-(\delta B)^K}{2(1-\delta B)}+\frac{2b}{(1+b)^2(1-\delta B)}\sum_{k=2}^{K}\delta^{k-1}(1-(\delta B)^{K-k+1})) \\
&= \frac{1-\delta}{1-\delta^K}(\frac{1-(\delta B)^K}{2(1-\delta B)}+\frac{2b}{(1+b)^2(1-\delta B)}(\sum_{k=2}^{K}\delta^{k-1}-\delta^K\sum_{k=2}^{K}B^{K-k+1})) \\
&= \frac{1-\delta}{1-\delta^K}(\frac{1-(\delta B)^K}{2(1-\delta B)}+\frac{2b}{(1+b)^2(1-\delta B)}(\frac{\delta-\delta^K}{1-\delta}-\frac{\delta^K(B-B^K)}{1-B}))
\end{aligned}$$

In the infinite game, again using Fudenberg and Levine (1983), rent extraction $\rho(b,\infty)$ is given by

$$\rho(b,\infty)=\lim_{K\to\infty}\rho(b,K)=\frac{(1-\delta)(1+b)^2+4b\delta}{2(1+b)(1+b-\delta+\delta b)}.$$

A.10 Proof Proposition 5

Rent extraction when the contest designer maximizes winner's effort is given by

$$\rho_W(b,K)=\frac{1}{\sum_{k=1}^{K}\delta^{k-1}}(\sum_{k=1}^{K}\delta^{k-1}(p_{i,k}x_{i,k}+(1-p_{i,k})x_{j,k}+b\cdot d_{i,k})). \tag{10}$$

We use equation 6 to explicitly calculate $\rho_W(b,K)$. For $b\geq 1$, we can immediately see that $W_k=(E_k)/2$ and therefore $\rho_W(b,K)=\rho(b,K)/2$.
For $b\in(0,1/2)$, we can immediately see that $W_k=E_k$ and therefore

$$\rho_W(b,K)=\rho(b,K)-\frac{1}{\sum_{k=1}^{K}\delta^{k-1}}x_{1,1}.$$

Therefore all that is left to cover is $b\in(1/2,1)$. We have established that in this case W_k is given by

$$W_k=\frac{6b^2-4b+1}{8b^3}\psi_k.$$

Rent extraction in the K contest game is therefore

$$\begin{aligned}
\rho_W(b,K) &= \frac{1-\delta}{1-\delta^K}(\frac{1}{4}\cdot\frac{1-(\delta A)^K}{1-\delta A}+\frac{6b^2-4b+1}{8b^3(1-\delta A)}(\sum_{k=2}^{K}\delta^{k-1}-\delta^K\sum_{k=2}^{K}A^{K-k+1})) \\
&= \frac{1-\delta}{1-\delta^K}(\frac{1}{4}\cdot\frac{1-(\delta A)^K}{1-\delta A}+\frac{6b^2-4b+1}{8b^3(1-\delta A)}(\frac{\delta(1-\delta^{K-1})}{1-\delta}-\delta^K\frac{A(1-A^{K-1})}{1-A})).
\end{aligned}$$

We then again use Theorem 3.3 from Fudenberg and Levine (1983) to get rent extraction in the infinite game, which is

$$\rho_W(b,\infty)=\lim_{K\to\infty}\rho_W(b,K)=\frac{2b^3-2(b-2)(b-1)b\delta+\delta}{2b\,(4b^2(\delta+1)-5b\delta+\delta)}.$$