Multiproduct Firms and Refunds

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Abstract

We determine how a multi-product firm optimally sells one of its products to consumers who have to pay an inspect cost to discover their value. Consumers can inspect products before or after they have bought them, while the firm chooses product prices and return policies (refunds). One strategy e-commerce firms have adopted is to induce consumers to order many products at once, inspect their fit at home, and then decide what to return. These policies introduce a trade-off as they may result in consumers acquiring products that better fit their taste, at the expense of the private and social costs associated with product returns. We determine the conditions under which firms find it optimal to induce consumers to inspect products simultaneously. We also analyze the efficiency properties of market outcomes and, surprisingly, find that these policies may actually lead to fewer returns. An important part of the analysis characterizes the optimal alternative pricing policy that induces consumers to sequentially inspect products after ordering and find that partial refunds play an important role to extract surplus from consumers.

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1 Introduction

Product returns play an increasingly important role in retail markets. A recent report of the National Retail Federation estimates that total returns in the retail industry of the USA reached \$890 billion in 2024, which is around 16,9% of total annual sales. In the online segment of the retail market return rates were even 21% higher.[1] Given the importance of product returns, firms have started to treat returns strategically by developing optimal return policies. One of these developments is that firms, like Amazon and Zalando, offer consumers the possibility to order multiple items at the same time, inspect them at home to see whether they like them, and to return all items that are considered not to be a good fit.[2]

In this paper we ask how a firm's product return policy could help to generate profits and what the welfare consequences of such policies are. For the welfare analysis it is also important to ask how frequently products are returned, as product returns are associated with environmental costs that are paid by agents not involved in the transaction, while returned products often also cannot be easily resold in the market.[3]

To study product returns, a consumer search framework is appropriate. To learn their value for a product consumers have to inspect it at a (time) cost. There are roughly speaking two ways consumers can perform their inspections. First, they can pay the inspection (search) cost upfront, i.e., before ordering, and then buy the product if they are satisfied with the product features. Alternatively, consumers can order products straightaway and only inspect them after they have been delivered. As inspecting after ordering can usually be done in a more comfortable environment at a time that suits the consumer best, the inspection after ordering is less costly for the consumer. Whether consumers inspect before or after ordering depends on the difference in inspection costs and on the return policies (refund) firms are offering in case consumers learn after ordering that they do not sufficiently like the product. Offering generous refunds comes at a cost to the firm, however, as the salvage value of products that are returned is smaller than the production cost, i.e., returned products are less valuable.

We focus on two aspects of the optimal selling policy of a multi-product firm: (i) are the optimal prices and refund policy of the firm such that it wants to stimulate the consumer to

[1] See, https://nrf.com/research/2024-consumer-returns-retail-industry.

[2] Amazon now labels this 'Prime Try Before You Buy', which previously was called 'Amazon Prime Wardrobe'. See, https://www.amazon.com/gp/help/customer/display.html?nodeId=GCQDLMG7C2YEXSM4 for more details.

[3] These environmental costs include greenhouse gas emissions, non-recycled packaging and products filling up landfills (see, e.g. Tian and Sarkis (2022)), where some websites estimate that only 54 percent of all packaging gets recycled and 5 billion pounds of returned goods end up in landfills each year.

inspect products before or after ordering?, and (ii) does the firm want to induce consumers to inspect and buy (and return) multiple products simultaneously or sequentially even if consumers are only interested to buy one of the products? In particular, under which conditions may inducing simultaneous inspection (like what firms like Amazon or Zalando offer) be a profit maximizing strategy and what consequences does it have for the number of products that are returned? In answering these questions, it is important to note that the firm can offer different prices and refunds for different products, but also condition these on whether or not a consumer orders multiple products simultaneously. The firm cannot, however, condition prices or refunds on whether or not a consumer inspected a product before ordering, as (certainly in online markets) firms cannot observe this. Thus, if the optimal policy is to induce a consumer to inspect products sequentially, then that policy should take into account that the consumer is free to inspect the products before or after ordering.

We consider two important cases, depending on whether the inspection cost before ordering is large or small, and each of our two main results applies to one of these cases. Before explaining our results in detail, it is useful to introduce our methodological contribution to analyze search problems where the consumer can inspect the product after she ordered it and obtain a refund if she returns it. In particular, we redefine the strategy of the firm as follows. The difference between a product's selling price and the refund is a "price" the consumer always pays if she inspects the product after ordering, no matter whether she eventually buys/keeps the product or not. We call this difference the *inspection fee* the firm chooses: it is the price the consumer pays for the right to inspect the product after ordering. The cost for the firm related to the consumer inspecting, but not buying the product, is the *product degradation*. Once the consumer has inspected the product, the relevant decision is whether she returns the product. The price the firm charges for *not* returning the product is the refund, while the cost for the firm of *not* returning the product is the salvage value of the product when it is returned. Thus, we consider that the firm chooses the refund and the inspection fee for its products.[4]

With this redefinition in mind, we first consider scenarios in which consumers never find it optimal to search before ordering, which is the case if the difference in inspection costs before and after ordering is substantial, i.e., when it is difficult for consumers to learn whether the product fits their needs before they have it at their home. In this case, the *optimal selling policy that induces sequential search* is to set very different prices for its products and set them such that the consumer wants to inspect one particular product first (which we term

[4]Thus, the selling price is implicitly defined as the inspection fee plus the refund, while the firm's product cost equals the product degradation plus the salvage value.

the first product). For ease of exposition, we focus on the firm producing two products. The pricing policy is such that for the second product the refund is set equal to the salvage value and the inspection fee is chosen such that all consumer surplus from inspecting the second product is attracted. For the first product, the refund is set equal to the opportunity cost of selling the second product, while the inspection fee is chosen to extract all surplus from the whole search process.[5] The policy is such that (i) the first product has a higher refund than the second product and (ii) if the consumer returns the first product, she never comes back to buy this product again.

The optimal selling policy inducing sequential search has the flavour of a two-part tariff in the sense that the inspection fee is used to extract surplus, while the refund is chosen as a price reflecting the cost to the firm at different stages of the search process. There is one important difference with textbook two-part tariffs, however, in that the inspection fee for the second product also affects the consumers' decision whether or not to inspect the second product. Actually, from a social efficiency perspective, the optimal selling policy sets this inspection fee for the second product too high and this product is not inspected often enough.

The optimal selling policy inducing simultaneous inspection is such that the firm sets the refund equal to the salvage value and an inspection fee that extracts the expected maximum consumer value (given that it is larger than the salvage value). From an efficiency perspective, the consumers' return decision for both products is optimal, but there is too much search, especially when the product degradation or inspection cost after ordering is relatively large.

Comparing the two candidate optimal policies, we find that it is profitable for the firm to induce simultaneous inspection if product degradation and/or the inspection cost after ordering is sufficiently small. This selling policy of inducing simultaneous inspection leads to more returns and a regulation forbidding such policies would reduce the environmental costs related to returns (while in terms of their private well-being consumers are equally well off as they obtain zero surplus in both solutions).

The results are quite different when considering the case where the inspection cost before ordering is sufficiently small so that it severely constrains the sequential selling strategy of the firm.[6] The smaller the inspection cost before ordering, the more credible the threat of the

[5] If the inspection cost after ordering is positive, then it should be deducted from the price.

[6] In the limit when this inspection cost equals 0, the consumer will always want to search the products sequentially before ordering. More generally, the question is whether the firm wants to induce the consumer to inspect products before or after ordering. This question boils down to under which inspection form social surplus is higher and how much of that surplus the firm is able to extract. It is clear that social surplus is potentially higher under inspection after ordering if the difference in inspection costs is relatively large and the product degradation is relatively small. When consumers search before ordering the firm is generically not

consumer to search before ordering and to induce the consumer to inspect after ordering the firm has to give a larger refund for every given price.

Even though consumers return at least one item for sure if they engage in simultaneous inspection, the expected number of returns may (surprisingly) be lower than if consumers order items sequentially. The reason is as follows. If consumers engage in sequential inspection, firms induce consumers to search the first product after ordering, while the consumers will search the second product before or after ordering, depending on the value they observe for the first product. Consumers continue to search the second product if the value of the first product is below their reservation value, which -if the search cost is small- may actually be quite high. This would imply that consumers may almost surely return one product under sequential search and are more likely to return both products as the refund prices under sequential search are much higher than under simultaneous search (where they are equal to the salvage value). We also show that, under a certain condition, the firm makes more profit when inducing consumers to inspect products simultaneously. Thus, a regulation forbidding such simultaneous inspection pricing policies may backfire, as it may create more (socially wasteful) product returns if the inspection cost before ordering is small.

Related literature. The paper combines two strands of literature. The papers most closely related to ours are Janssen and Williams (2024), Jerath and Ren (2023) and Matthews and Persico (2007) in that they also study product returns in a consumer search setting. However, all these papers study a single product firm sell and consumers searching sequentially (where the former paper studies a competitive setting, while the latter two analyze monopoly behavior). They find that the number of refunds is either inefficiently high or low. None of these papers consider a firm that incentivizes consumers to search simultaneously among its multiple products.[7] Petrikaitė (2018a) studies search with returns in a duopoly setting, but also does not consider multiple products per seller or simultaneous search.

The second strand of literature is on multi-product search (Rhodes (2015), Shelegia (2012) and Zhou (2014)), but the focus of these papers is on consumers searching for multiple prod-

able to extract all surplus as it sets price in such a way that consumers find it beneficial to search. When consumers inspect products after ordering the firm is better able to extract surplus by setting inspection fee and refunds appropriately. Only when the search cost of inspecting products before ordering is relatively small and the threat of inspecting before ordering is therefore more severe, the firm has to offer consumers prices and refunds so that they make positive surplus. Thus, the firm may induce consumers to inspect after ordering even if this is not socially optimal.

[7] Another difference with Janssen and Williams (2024) is that we study a setting in which consumers can learn the prices and refunds the firm sets without any cost. This is a feature the paper has in common with the recent literature on price directed search; see, e.g., Armstrong (2017), Choi, Dai, and Kim (2018).

ucts, creating a joint search effect in that once a consumer is at a store, she has a lower search cost to buy other products at that store. These papers do not study product returns or simultaneous search.

The optimal behaviour of the firm if it wants to induce sequential search after ordering has features that also arise in Petrikaitė (2018b) and Gamp (2022) in that a multi-product firm has an incentive to obfuscate search among its products. These papers study a setting where consumers have to inspect products before purchasing one of them and where (together with prices) the firm chooses consumers' search cost directly. They show that the firm has an incentive to set a positive search cost and asymmetric prices so as to induce consumers to search the products in a particular order. In contrast, we allow consumers to order (or buy) products before inspecting them[8] and have a setting where the firm cannot affect the inspection cost of consumers directly. However, by choosing a refund that is smaller than the price, the firm effectively sets an inspection fee that the consumer pays upfront when deciding to inspect. This inspection fee is part of the firm's profits, which makes for another important difference to the above mentioned papers.

More broadly, our paper contributes to the consumer search literature by extending the options for consumers to inspect products, where in the seminal contributions by, for example, Wolinsky (1986), Anderson and Renault (1999) and Armstrong (2017) consumers have to pay this search cost upfront to learn their match value before ordering/purchase. Morgan and Manning (1985) show that if consumers can choose to search sequentially or simultaneously at the same prices, they find it optimal to search items sequentially.[9] This result also applies to our setting if prices and refunds are identical across inspection modes. By offering different prices and refunds if a consumer orders multiple items at once, the firm may, however, incentivize the consumer to search simultaneously.

The remainder of the paper is organized as follows. The next section introduces the model. Section 3 discusses the case where the inspection cost of inspection before ordering is large, while Section 4 considers the opposite case when this cost is small. Section 5 concludes with a discussion.

[8] Doval (2018) allows consumers to buy blindly, that is without inspecting the product at all. Buying and inspecting after ordering has features that can be considered a generalization of blind buying in the sense that if the refund that the firm gives is zero, the consumer will never inspect the product afterwards and will then also not return the product. However, in our framework the consumer has to pay the inspection fee, a feature that is absent in Doval (2018).

[9] That is if the searcher is patient enough or there is no delay due to sequential search. As we do not want our result that a firm induces consumers to search simultaneously to depend on an exogenously imposed delay because of sequential search, we assume that there is no delay.

2 The Model

A monopoly firm sells two products. Each product has a production cost $c \geq 0$ and a salvage value $\eta \in [0, c]$ to the firm in case the product is bought and then returned. We will define $k = c - \eta$ as the value lost if the product is returned after it is inspected and we will refer to k as the *product degradation.* The firm can set different selling prices and refunds for the different products $i = 1, 2$ and we denote selling price by $p_i \geq 0$ and refund by $\tau_i \in [0, p_i]$, which is the money the firm commits to return to the consumer in case the latter returns the product.[10] Consumers can learn the value of a product by inspecting it before or after ordering, where we interpret "inspecting after ordering" as the act of ordering the product and committing to pay the difference between selling price and refund in case the product is returned after having inspected it, where the difference between p_i and τ_i can not only consist of a firm not giving a full refund, but also of (i) the return cost the consumer has to pay[11] or (ii) a so-called restocking fee a firm charges[12].

The firm cannot condition its prices on whether consumers inspect products before or after ordering (as the firm does not observe this). It can only set prices and refunds such that it incentivizes consumers to inspect products in one way or the other. As a firm can observe whether or not a consumer orders multiple products at once, it can offer different prices and refunds for this situation and we denote them by (p_{sim}, τ_{sim}) with $p_{sim} \geq \tau_{sim}$.[13]

There is a representative consumer with unit demand. The two products are ex-ante identical to the consumer with each product having a valuation that is independently and identically distributed by $v_i \sim F[\underline{v}, \bar{v}]$, with a density $f(v)$ that is positive, continuously differentiable and where f is logconcave.[14] To have an interesting model, we require $\bar{v} > c \geq \eta \geq \underline{v}$. The consumer knows the prices and refunds the firm offers, but has to pay an inspection cost of $s_B > 0$ to learn a product's value before ordering and a cost of s_A if she wants to learn

[10] Note that to prevent arbitrage the firm would never set a refund larger than selling price.

[11] These return costs may consist of the cost of shipping the product back to the firm, which the consumer may have to cover, and further time or "hassle" costs related to the return process. We do not explicitly include such return costs in our model, as they do not qualitatively affect the result. It is clear that from an efficiency perspective such return costs reduce the social salvage value of a returned product. Thus one can show that the model with a certain explicit return cost h is equivalent to our model without such a return cost but with $\eta' := \eta - h$ and $\tau_i' := \tau_i - h$, similar to Janssen and Williams (2024).

[12] See e.g. https://www.zonguru.com/blog/amazon-restocking-fee for a guide that suggests a restocking fee of up to 20 percent of the purchase price.

[13] As the firm will not benefit from setting different prices under simultaneous search, we do not use subscripts for the price and refund of the different products.

[14] It is well-known that this implies that the associated distribution function F and $1 - F$ are then also logconcave; see, e.g., Bagnoli and Bergstrom (2005).

the product's value after ordering, with $s_A \leq s_B$. Thus, if consumers simultaneously order two products they will always inspect them after ordering as this comes at a lower inspection cost. The outside option of the consumer is normalized to 0. For future reference, it will be useful to write $\hat{v}^b$ as the reservation value of inspecting a product before ordering and $\hat{v}_i^a$ as the reservation value of inspecting product i after ordering. They are implicitly defined through the following equations:[15]

$$\int_{\hat{v}^b}^{\infty} (v - \hat{v}^b) f(v) dv = s_B \quad \text{and} \quad \int_{\hat{v}_i^a}^{\infty} (v - \hat{v}_i^a) f(v) dv = s_A + p_i - \tau_i. \tag{1}$$

Note that $\hat{v}_i^a$ is not only a function of exogenous parameters but also of p_i and τ_i, the two strategic variables of the firm for product i. When we write $\hat{v}_i^a$ we implicitly mean the function $\hat{v}_i^a(p_i - \tau_i)$.

Given the firm's choices, the consumer can take one of the following actions:[16] (i) Inspect the products sequentially, (ii) Inspect the products simultaneously after ordering or (iii) Leave and take the outside option with a pay-off of 0. Under (i), the consumer decides in which order to inspect the products and can inspect each product either before ordering or after ordering. Inspecting a product before ordering entails paying the inspection cost of s_B to learn that product's value and then deciding whether to buy it at price p_i or, in case of the first product, continuing to inspect the second product. Inspecting a product after ordering entails paying the inspection cost of s_A to learn that product's value, deciding whether to keep it and pay the price p_i, or, in case of the first product, continuing to inspect the second product, and finally returning and paying $p_i - \tau_i$ for all products inspected after ordering that are not kept.[17] If consumers search sequentially, they have perfect recall. Under (ii), the consumer inspects both products simultaneously after ordering for inspection cost of s_A each and decides whether to buy at most one of the products at the contract (p_{sim}, τ_{sim}) and returns at least one. In the following we will refer to (i) in short as A^{seq} if products are inspected after ordering, while we refer to (ii) as A^{sim}.

[15] In general, we define $\hat{v}(s)$ implicitly through $\int_{\hat{v}}^{\infty}(v - \hat{v}) f(v) dv = s$. Then $\hat{v}^b = \hat{v}(s_B)$ and $\hat{v}_i^a = \hat{v}(s_A + p_i - \tau_i)$.

[16] Note that we have left two possible consumer strategies out of the above list. First, it turns out that it is never optimal for the firm to set prices such that the consumer would choose to buy a product without inspecting it at all (as in Doval (2018)). Second, simultaneous inspection before ordering is also never chosen. In the case of inspection before ordering, at a given price the firm receives the same payoff irrespective of whether the consumer inspects sequentially or simultaneously, while simultaneous search is never optimal for the consumer. Note that, in contrast, the firm's payoffs for simultaneous and sequential search after ordering do differ as firms can make a profit or a loss over their returns.

[17] Note that it does not matter whether p_i is paid at the time of ordering or at the checkout when the final decision is made on which product to keep. By ordering product i, the consumer commits in both cases to pay at least $p_i - \tau_i$ to the firm. See also the next paragraph.

It is important to note that it is possible to redefine inspection after ordering as a structurally simpler problem, which will facilitate the analysis. From the consumer's view inspection after ordering can be re-written as inspection before ordering with certain inspection costs and prices. In particular, at the moment the consumer orders product i to inspect it at inspection cost s_A, she commits to paying at least $p_i - \tau_i$ – which is the part of the price she does not get back if she returns the product. If she instead wants to keep the product she additionally "pays" τ_i, as she forgoes the refund she could have received. Thus, we can redefine inspection after ordering as inspection before ordering with a redefined *inspection cost* of $s_A + p_i - \tau_i$ and a redefined *price* of τ_i. Note that while s_A is lost, $p_i - \tau_i$ is the part of the redefined inspection cost that is paid to the firm. It is thus as if the firm was offering product i for inspection before ordering at an *inspection fee* of $\sigma_i := p_i - \tau_i$ and a *price* for keeping the product $\rho_i := \tau_i$. In line with this redefinition, we can also split the production cost c into two parts that the firm incurs when the consumer respectively *inspects* or *keeps* the product. When the consumer inspects the product, the firm incurs a loss that is equal to the difference between production cost c and salvage value η. We define $k := c - \eta$ and call k the product degradation. When the consumer decides to keep the product, then the firm incurs η as a cost, as it forgoes the product's salvage value. Overall, it is as if the firm chooses for each product an inspection fee σ_i with the associated opportunity cost k and a price (refund) ρ_i with the associated opportunity cost η.[18,19]

3 When Inspection before Ordering is too costly

When the inspection cost before ordering s_B is relatively large, the consumer will not choose this option and when designing the optimal contract conditional on the consumer searching sequentially, the firm's strategy focuses on a consumer that inspects the product sequentially after ordering. In this section, we first construct the optimal contracts for both simultaneous search and sequential search. We then compare profits under both contracts to determine the conditions under which a contract is optimal for the firm, before we compare the number of returns under sequential and simultaneous search.

Consider first the optimal contract under sequential search. The next proposition summarizes the result.

[18] To avoid confusion, we will in the following refer to ρ_i as the "price" and to p_i as the "selling price".

[19] Note that to account for an explicit return cost h as discussed above, the required transformations in this redefined model are $\eta' := \eta - h, \rho_i' := \rho_i - h, k' := k + h$ and $\sigma_i' := \sigma_i + h$, as c and p_i remain unchanged.

Proposition 1 *If s_B is large*[20] *and the firm induces consumers to inspect sequentially after ordering the optimal strategy is as follows:*

$$(\sigma_1^*, \rho_1^*) = (\mathbb{E}[\max(v_1 - ES_2 - \eta, 0)] - s_A, ES_2 + \eta) \text{ and } (\sigma_2^*, \rho_2^*) = (ES_2 + k, \eta)$$

with profits $\pi_{Aseq}^* = \mathbb{E}[\max(v_1 - \eta, ES_2)] - s_A - k$ *and where:*

$$ES_2 = \mathbb{E}[\max(v_2 - \eta, 0)] - s_A - k. \tag{2}$$

The intuition behind the optimality of the strategy seems clear. If the firm incentivizes A^{seq} then Weitzman (1979) implies that the consumer first inspects the product with the higher net reservation value $\hat{v}_1^a - \rho_1 \geq \hat{v}_2^a - \rho_2$ and only inspects product i if it has a non-negative net reservation value $\hat{v}_i^a - \rho_i \geq 0$ (as this is a necessary condition for non-negative utility). Without loss of generality consider that product $i = 1$ is inspected first. Then, as the inspection fee σ_1 for the first inspected product is committed to be paid before inspection starts, the firm can increase it (without distorting consumer decisions) as long as the above inequalities are not violated. This implies that in the optimal contract we should have that $\hat{v}_1^a - \rho_1 = \hat{v}_2^a - \rho_2$, i.e. the net reservation values of the two products will be equal.[21] *If* the firm will choose the contracts for both products such that the net reservation values will be equal to zero $\hat{v}_i^a - \rho_i = 0$, implying that the consumer will buy the first product that has a positive observed net value, $v_i - \rho_i > 0$, then it is clear what is the optimal contract. For the last product in this order, the firm sets the refund (or the price for keeping the product) equal to the opportunity cost, i.e., $\rho_2 = \eta$ and the inspection fee σ_2 such that it extracts ES_2, the efficient surplus from inspection of the second product. Turning to the first product that is inspected, the firm's strategy follows the same principle, but here ρ_1 is priced at the "opportunity cost of selling the first product", which is the sum of the salvage value and the profit that the firm foregoes if the consumer does not inspect the second product. Thus, the firm (realizing it can make a profit of ES_2 and is getting the salvage value if the consumer continues to inspect the second product) will set the refund price such that $\rho_1^* = ES_2 + \eta$ and an inspection fee σ_1^* that extracts all remaining surplus, with $\sigma_2^* \geq \sigma_1^* \geq k$.[22,23]

What is less clear is why it is optimal to set $\hat{v}_i^a - \rho_i = 0$. At one level, this seems obvious as the firm extracts all consumer surplus. However, this is not the efficient surplus as (i)

[20]It is clear that how large s_B should be for it not to impose a constraint on the contract the firm can offer under A^{seq} depends on the other parameters, most notably s_A. If s_A is fairly large itself, then s_B itself should be relatively large for this to be true. If s_B is not large enough, then obviously the profits of the firm under A^{seq} will be lower and it may also be that these profits are smaller than under inspection before ordering.

[21]From (1) it follows that $\partial \hat{v}_i^a / \partial \sigma_i = -1/[1 - F(\hat{v}_i^a)] \leq -1$.

[22]$\sigma_1^* = \mathbb{E}[\max(v - ES_2 - \eta), 0] - s_A = \mathbb{E}[\max(v - \eta, ES_2)] - \mathbb{E}[\max(v - \eta, 0)] + k \geq k$.

[23]It is relatively easy to see how this optimal solution can be generalized to selling one out of n products.

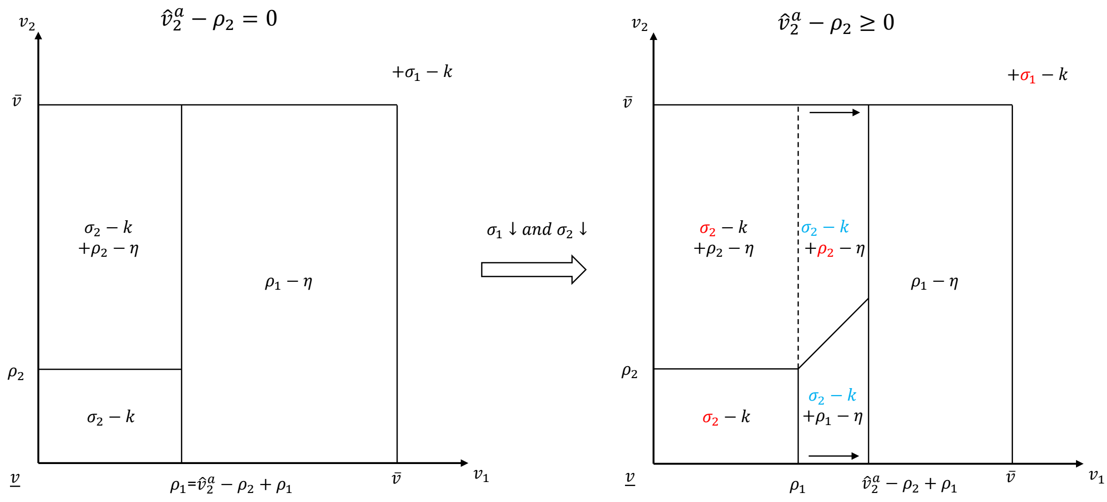

Figure 1: Possible deviation (right) from the optimal A^{seq} strategy (left). Gains from the deviation are in blue while losses are in red.

the inspection fee for the second product causes an inefficiency as the first product may be kept, ending search, even though the second product has a higher (net) value, while (ii) the difference in refunds for the first and second product also creates an inefficiency as it may well happen that the first product is returned, while the second product turns out to have a lower net value.[24]

The issue can also be illustrated by means of Figure 1. In the optimal solution, we have that the whole value area can be divided into three parts as in the left part of the figure: (i) if the consumer has a value $v_1 > \rho_1$ she will buy product 1, (ii) if the consumer has a value $v_1 < \rho_1$ she will continue to search the second product and purchase that product if $v_2 > \rho_2$, and (iii) if the consumer has a value $v_1 < \rho_1$ and $v_2 < \rho_2$, she will buy none of the products. In the right part of the figure, we indicate the different consumer behaviours in case $\hat{v}_i^a - \rho_i > 0$. Here, after inspecting the first product, the consumer may decide not to buy the product immediately even if she discovers that $v_1 > \rho_1$. Inspecting the second product delivers another inspection fee of σ_2 to the firm and the consumer may still decide to buy product 1. The largest part of the proof in the appendix is dedicated to showing that this is not optimal and the firm indeed wants to set $\hat{v}_i^a - \rho_i = 0$ if $f(v)$ is logconcave.

We finalize the discussion of the optimal sequential contract after ordering with a numerical example and a few general remarks.

[24]Note that even if the first product is returned only after the second is inspected, the consumer would still return the first product as it has a higher refund.

Example. The following example illustrates the nature of the optimal solution under A^{seq} and shows why the optimal solution involves an asymmetric contract even if the products are ex ante symmetric. Suppose that $s_A = k = \eta = 0$ and that values are uniformly distributed over $[0,1]$. If the firm would have one product to sell, it is clear that the optimal contract would have $\tau = \rho = 0$ and $p = \sigma = 1/2$. The firm sets the refund efficiently, namely equal to the salvage value, and then extracts all surplus by setting the selling price equal to the expected surplus of searching. This is also the optimal contract for the second product if the firm sells two products. Consider then the first product. The firm knows it can make a profit of $1/2$ and that the consumer gets an expected surplus of zero if the consumer continues to inspect the second product. It is then optimal to set the refund in the first period $\tau_1 = \rho_1 = 1/2$ as this is the opportunity cost of the refund: a higher refund yields some extra consumers returning the product with a refund that is larger than the profit it generates. Given the choice of the refund and a selling price p_1 in the first period consumers start searching if their expected surplus is non-negative, which yields the following constraint: $-\sigma_1 + 1/2 * (3/4 - \rho_1) + 1/2 * 0 \geq 0$. It is optimal for the firm to set the largest selling price given this constraint, yielding $p_1 = \sigma_1 + 1/2 = 5/8$. The total profit is thus equal to $5/8$ as the consumer pays the first inspection fee σ_1 of 1/8 and then pays the additional price τ_1 of 1/2 if the valuation is larger than 1/2 (which happens with probability 1/2) and if the valuation is smaller than 1/2 the consumer continues to search the second product, pays the inspection fee σ_2 of 1/2 and always keeps the product.

Thus, the firm finds it optimal to make inspection costly by creating an inspection fee σ_i, which is the difference between the selling price and the refund. Consumers know that they lose σ_i when they inspect a product. The example shows that even though the actual inspection cost equals 0, this optimal inspection fee can actually be quite large, especially for the second product. Second, it is interesting to see that the resulting profit under A^{seq} equals $\mathbb{E}[\max(v - \eta, ES_2)] - s_A - k$, which is exactly identical to the efficient surplus if there was no recall. In addition, the firm makes this profit independent of whether the consumer eventually purchases product 1, 2 or no product at all, i.e., even if the consumer returns both products the firm makes the same profit as when it sells. Third, as in Petrikaitė (2018b), the profit maximizing strategy of the firm distorts the consumer's optimal search behavior in such a way as to remove their ability to recall any earlier inspected product. However, in our case it is further able to extract all that surplus by setting the inspection fees appropriately. The fact that the inspection fees are another source of revenue create the technical complications alluded to above to show that indeed the firm wants to set $\hat{v}_i^a - \rho_i = 0$.

We now consider the optimal contract and profits when consumers search simultaneously after ordering so that the consumer pays the inspection fee σ_{sim} and the inspection cost s_A for both products upfront as long as their expected utility is non-negative. Recall that the consumer can buy at the terms of contract $(\sigma_{sim}, \rho_{sim})$ only if she chooses the action A^{sim}, i.e. ordering both products simultaneously. The firm does not have to consider therefore a potential deviation of the consumer when incentivizing A^{sim} as it can in principle set very unattractive terms for the consumer to search sequentially. When consumers search simultaneously, they will buy the product with the higher net value $v_i - \rho_{sim}$, as long as either of them is non-negative. So, the profit-maximizing contract is essentially a two-part tariff where the optimal price ρ^*_{sim} is set at marginal cost η and the optimal inspection fee σ^*_{sim} extracts all surplus. In particular, as the expected social surplus is given by

$$\mathbb{E}[\max(v_1 - \eta, v_2 - \eta, 0)] - 2(s_A + k) \tag{3}$$

the profit $\pi^*_{Asim} = 2(\sigma^*_{sim} - k)$ is equal to this expression.[25] From an efficiency standpoint, the number of inspections is too large, but products are returned at an efficient level: the product with the lowest valuation will always be returned and this is efficient as the consumer has no (additional) value for it, while the firm has a salvage value and the product with the highest valuation will be returned if its value is smaller than the firm's salvage value.

Example continued. Keeping the same parameter values, it is clear that under A^{sim}, the firm wants to set $\rho_{sim} = \eta = 0$. The firm then wants to set the selling price for the two products such that it attracts $\mathbb{E}[\max(v_1, v_2)] = 2/3$. Thus, it will set the selling price for each product equal to $1/3$.

Finally, we are able to compare the profits for A^{sim} to those for A^{seq} and evaluate the impact of A^{sim} on the number of products returned. We find the following:

Proposition 2 *If s_B is large, then there exists a function $\underline{S}_A(\eta) > 0$ such that for all (s_A, k, η):*

$$s_A + k \leq \underline{S}_A \Leftrightarrow \pi^*_{Asim} \geq \pi^*_{Aseq}.$$

Moreover, the expected number of returns under A^{sim} is larger than under A^{seq}.

[25] Note that any other contract with asymmetric inspection fees σ^i_{sim} satisfying $\sigma^1_{sim} + \sigma^2_{sim} = 2\sigma^*_{sim}$ would have resulted qualitatively in the same outcome.

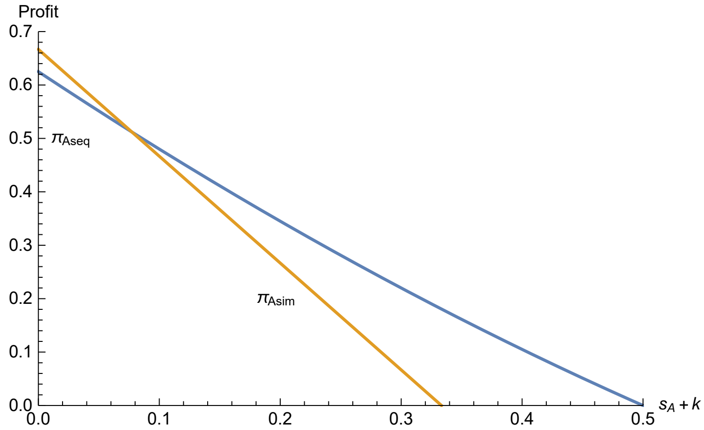

Figure 2: Profits π_{Asim} and π_{Aseq} as functions of the sum of inspection and degradation costs $s_A + k$ for values distributed uniformly on $[0, 1]$ and $\eta = 0$.

The intuition behind the proposition is clear. Under both search protocols the firm extracts all surplus. However, the surplus is quite different. Under simultaneous search, the consumer inspects both products and chooses the one with the higher net value. The potential loss in surplus is due to inspection costs and product degradation related to the purchase and return of at least one product. Under sequential search, the consumer inspects the first product and keeps it if it has a higher net value than the expected value of the second inspection, including the inspection fee the firm imposes. Compared to simultaneous search, surplus is lower if the consumer decides not to inspect the second product even though it would have had a higher net value if she would have done so, or if the consumer continues to inspect the second product, but then does not keep the product with the highest value due to the difference in refunds. If the loss in surplus under simultaneous search due to unnecessary inspection costs and product degradation is relatively small, simultaneous search leads to higher profits. If, on the other hand, $s_A + k$ is relatively large, then A^{seq} yields more profits as one can find a good fit already with the first product and save on inspection cost and product degradation. Figure 2 presents a numerical example. What is interesting is that for a single product firm both solutions yield the same outcome and that the outcome is efficient. The inefficiencies that are created under both A^{sim} and A^{seq} are due to the multi-product nature of the firm and the associated search.[26]

[26] It can be argued that the consumer may use the contract of A^{sim} in a different way: While she does have to pay σ_{sim} for both products upfront, once she has them "at home" she does not necessarily have to inspect both simultaneously, but can do so sequentially instead. This is indeed optimal if some part of the inspection cost s_A comes from the effort of "testing the product at home". If instead s_A only comes from the effort of selecting a product before ordering it, then nothing changes in our analysis as presented in this section. We now show that the extreme opposite, where that effort is zero and all of s_A instead comes from testing the product at

Thus, the firm induces consumers to search simultaneously if the sum of inspection and degradation costs $s_A + k$ is small and this leads to more product returns than in an alternative contract if $1+F^2(\eta) > F(\rho_1^*)(1+F(\eta))$. The proof of the proposition shows that this condition follows from the logconcavity of $1 - F(v)$. A policy where such simultaneous contracts would be forbidden would therefore reduce the number of returns if s_B is large enough. The next section shows that this is not necessarily the case if s_B is small.

Alternatively, a regulator could choose to impose that consumers get full refunds. In our framework this would imply that $\sigma_i = 0, i = 1, 2$. It is not difficult to see that in that case the firm's profits when setting a price p are equal to

$$(1 - F^2(p))(p - c - k) - 2F^2(p)k = (1 - F^2(p))(p - \eta) - 2k,$$

for the simultaneous search contract, and

$$(1 - F^2(p))(p - c - k) - 2F^2(p)k + F(\hat{v}_a)k = (1 - F^2(p))(p - \eta) - 2k + F(\hat{v}_a)k,$$

for the sequential contract, where $\hat{v}_a$ is defined as the usual reservation price relative to the search cost s_A (and $\sigma_i = 0$). Thus in both cases the firm optimally sets the price such that it maximizes joint monopoly profits given a cost η, and the profit in case of sequential search after ordering is higher as the firm may economize on the cost related to product degradation. Unless, the reservation value $\hat{v}_a < \rho_1^*$, it is clear that mandating full refunds leads to an increase in product returns as it leads to much higher refunds. In the absence of inspection fees, consumers are able, however, to enjoy more surplus.

4 Small Inspection Cost

When the inspection cost before ordering s_B is relatively small, the consumer may choose to inspect a product before instead of after ordering while she is searching sequentially. As we argued before, the firm cannot observe whether the consumer has inspected a product before or after ordering, and therefore cannot set different prices in each of those cases. Note that

home, does not change our result in a substantial way. In that case, before any inspection the consumer pays $2\sigma_{sim}$ to the firm, who anticipates a cost of $2k$. Then the consumer inspects the two products sequentially at their inspection cost s_A and if she decides to keep one product, she pays ρ_{sim} to the firm, who realizes an additional cost of η in that event. From an efficiency view, the maximum surplus is realized if the firm sets $\rho_{sim} = \eta$, and the firm is able to extract all that surplus using σ_{sim}. This surplus - and therefore firm profit - is bigger than what we derived in the above section as sequential search is more efficient than simultaneous. This implies that the threshold of Proposition 2, below which $\pi^*_{Asim} > \pi^*_{Aseq}$ would be "higher" - note, however, since now s_A and k are not both invested at the same time, we would need to adjust Proposition 2 such that $k \leq \widetilde{S}_A(s_A, \eta) \Leftrightarrow \pi^*_{Asim} > \pi^*_{Aseq}$.

if the firm sets the same contract as in the previous section, the consumer would deviate to inspecting before ordering, which is a viable alternative if s_B is relatively small. Therefore the firm has to adjust its contract accordingly to ensure the consumer does not deviate. Naturally, this implies reduced profits under A^{seq} compared to the previous section. However, that loss in profits is not the only implication of a relatively small s_B. In this section, we will show, perhaps surprisingly, that contracts inducing simultaneous inspection can actually lead to a lower number of returns than those inducing sequential inspection after ordering when s_B is relatively small. The presence of the threat of the consumer deviating to inspection before ordering turns out to be important in facilitating this result.

Before characterizing the optimal contract when s_B is a small positive number, we will characterize what type of search behavior the firm will induce in the optimal contract. Here, we have two Lemmas. First, regarding the product the consumer inspects first, the firm sets the contract such that the consumer searches in the way that is socially optimal, i.e., if $s_A + k$, which is the social cost of inspection after ordering, is smaller than s_B, the social cost of inspection before ordering, then the optimal contract induces the consumer to search the first product after ordering.

Lemma 1 *The optimal sequential contract $\{(\rho_i^*, \sigma_i^*)\}_{i=1,2}$ is such that consumers inspect the first product after ordering, if $s_A + k < s_B$.*[27]

An important goal of this section is to identify conditions under which a firm offering a simultaneous contract induces fewer product returns than when these contracts are not offered and consumers instead search sequentially. It is clear, however, that when the first product is inspected before ordering, the number of returns cannot be larger under sequential search than under simultaneous search. Therefore, in the rest of this section we consider that $s_A + k < s_B$.

The second Lemma shows that the inspection mode of the second product depends on the value of the first product the consumer uncovered. In other words, for some values of v_1 the consumer (weakly) prefers to inspect the second product before ordering, and for others after ordering. If $v_1 \geq \rho_1$, searching the second product *after ordering* yields an additional benefit relative to v_1 of

$$\int_{v_1-\rho_1+\rho_2}^{\bar{v}} (1 - F(v_2))dv_2 - s_A - \sigma_2$$

while searching the second product *before ordering* yields an additional benefit relative to v_1

[27]The condition $s_A + k < s_B$ is sufficient, but not necessary. The firm could also profit by setting a contract with a higher σ_1 and a lower ρ_1, where the latter would not affect profits significantly due to the envelope theorem.

of

$$\int_{v_1-\rho_1+\rho_2+\sigma_2}^{\overline{v}} (1 - F(v_2))dv_2 - s_B.$$

When these expressions are equal, the consumer is indifferent between these two options and we will denote this value by $\widetilde{v}_1^a$, which is uniquely defined by:

$$\int_{\widetilde{v}_1^a-\rho_1+\rho_2}^{\widetilde{v}_1^a-\rho_1+\rho_2+\sigma_2} F(v_2)dv_2 = s_B - s_A. \tag{4}$$

Note that $\widetilde{v}_1^a$ is an implicit function of the firm's contract (except σ_1) and the inspection costs of the consumer. Thus, we have the following result.

Lemma 2 *If $s_A + k < s_B$, the optimal contract is such that consumers inspect the second product before ordering if, and only if, $v_1 > \widetilde{v}_1^a$, where $\rho_1^* < \widetilde{v}_1^a < \overline{v}$.*

This result can be intuitively understood as follows. Compared to inspection before ordering, inspection after ordering comes at a lower inspection cost, but implies to pay part of the full price of the product upfront. This option is better in situations where the outside option, i.e., the previously observed v_1, provides a low value, and it is therefore more likely that the second product will ultimately be bought. If instead the observed v_1 is already large, then an improvement on it is unlikely and the consumer will not be willing to make an upfront payment for the second product.

A consequence of this more complex search behavior is that it is difficult to explicitly solve for the optimal contract for general parameters. Instead, we identify the optimal contract for the limit case where $s_B = s_A = k = 0$ and utilize this contract to derive properties of the optimal contract approximate for the case where $s_B < \bar{s}_B$, where $\bar{s}_B$ is a positive number sufficiently close to zero.

Thus, now we derive the optimal contract in the limit case $s_A = s_B = k = 0$ and focus on the case where the first product is inspected after ordering. Given that both inspection before and inspection after have the same inspection cost, the consumer is not willing to pay part of the price of the product upfront and thus it must be that $\sigma_i = 0$ for both products. Then log-concavity of the distribution implies that the optimal prices ρ_i^* will be symmetric (see Petrikaitė (2018b)). The firm thus maximizes $(1 - F^2(\rho))(\rho - \eta)$. We denote the unique price that maximizes this expression as $\rho^{JM}(\eta)$, the joint monopoly price. For the consumer to be willing to start search at this price, the reservation value must be weakly larger than this price. This means there is an upper bound on the inspection cost for which this price maximizes profits. This upper bound is implicitly defined as the solution to $\hat{v}(s) = \rho^{JM}(\eta)$.[28]

[28] $\hat{v}(s)$ is as before the reservation value for search cost s, as implicitly defined by $\int_{\hat{v}}^{\overline{v}}(1 - F(v))dv = s$. In

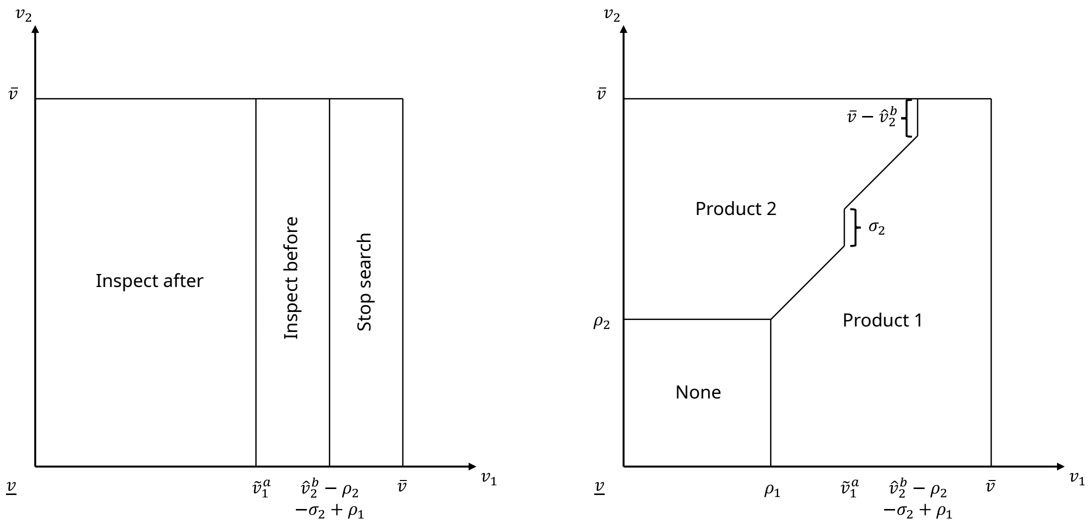

Figure 3: Optimal contract for $s_A + k < s_B \leq \bar{s}_B$ with $\bar{s}_B$ sufficiently small: (Left) shows the consumer's search behavior for the second product given the observed value v_1; (Right) shows the product the consumer eventually buys/keeps, if any. Proportions are exaggerated: For the considered parameters we have $\hat{v}_2^b \to \bar{v}$ and $\sigma_2 \to 0$ such that the consumer almost always inspects the second product and the jumps in the diagonal are negligible (see Fig. 4).

Example continued. If $s_B = 0$, setting a selling price p for each of the products, the firm makes a profit of $(1 - p^2)p$. Maximizing this expression with respect to p yields the FOC $3p^2 = 1$, or $p = \sqrt{1/3}$. Thus, the total profit of the firm is $\frac{2}{3}\sqrt{1/3}$. Note that both the selling price and the profit are larger than for a single product monopolist, but that the profit is considerably smaller than the profit under A^{seq} we derived in the previous section.

The following proposition then characterizes the optimal contract under A^{seq}.

Proposition 3 *There exists an $\bar{s}_B > 0$ such that for all $s_A, s_B, k \geq 0$ with $s_A + k < s_B < \bar{s}_B$ the optimal sequential contract $\{(\rho_i^*, \sigma_i^*)\}_{i=1,2}$ is such that:*
(i) $\rho_i^* \approx \rho^{JM}(\eta)$ and $\sigma_i \approx k$
(ii) $F(\widetilde{v}_1^a) = \frac{1+F^2(\rho^{JM})}{2}$

Figure 3 visualizes the optimal contract for the specified parameter combinations and the resulting consumer search behavior. The visible "jump" in the diagonal is due to the consumer changing from inspecting after ordering to inspecting before, at which point σ_2 is no longer

the following, we assume that the inspection costs are smaller than this upper bound, which given that we are focusing on inspection costs close to zero, only becomes a relevant constraint if η is close to $\bar{v}$.

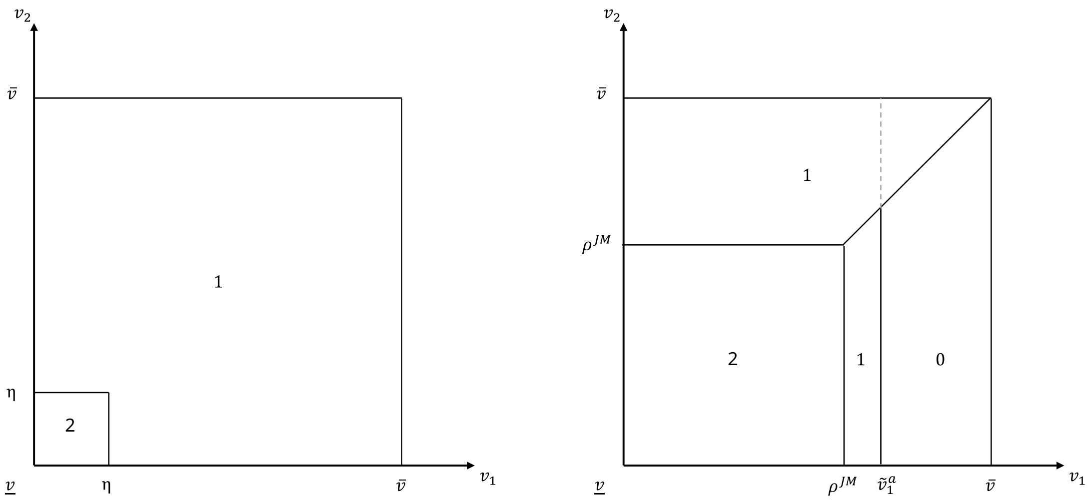

Figure 4: The number of returns under A^{sim} (left) and A^{seq} (right) for different realized values (v_1, v_2) for the uniform distribution and $\eta > \underline{v}$. The relatively bigger lower-left area where both products are returned under A^{seq} is responsible for the overall higher number of expected returns under the sequential contract.

a sunk cost. Under such an optimal sequential contract, the firm makes (approximately) no profit from inspections and only profits from selling the products. As we have shown in the previous section, in the optimal simultaneous contract, the exact opposite is true: The firm sells the products at marginal cost and all its profits come from the inspection fees.

Given the optimal sequential contract, we now show that if the inspection costs are small, a pricing strategy that induces consumers to buy many products, inspect them after ordering and return the products they do not want to keep can lead to fewer products being returned than pricing strategies that lead to consumers buying and inspecting products sequentially. The expected number of returns under A^{sim} and A^{seq} for the specified set of parameters are given by

$$n_{Asim} = 1 + F(\eta)^2 \quad \text{and} \quad n_{Aseq} \approx F(\widetilde{v}_1^a) + F^2(\rho^{JM}) + \frac{1}{2}(1 - F(\widetilde{v}_1^a))^2.$$

Figure 4 illustrates the number of returns under both pricing policies. The number of returns under A^{sim} is easily understood as both products are always inspected and one of them is returned with certainty. Both are returned only if both values are smaller than $\rho^*_{sim} = \eta$, the efficient return price and the lowest price the firm will ever set. The number of returns under A^{seq} is comprised of the following parts. The first two terms result from the consumer inspecting the second product after ordering, which happens if $v_1 \leq \widetilde{v}_1^a$. In that case, she will certainly return one of the products, and she will return both products if they both have a value $v_i < \rho_i \approx \rho^{JM}$. The third term results from the consumer inspecting the second product before ordering, where she will return the first product in case the second has a higher net value, which happens in approximately half of the cases where both products have a value above $\widetilde{v}_1^a$.

Using these expressions for the expected number of returns and point (ii) of Proposition 3, one can easily derive the condition under which A^{sim} or A^{seq} create more returns.

Proposition 4 *There exists an $\bar{s}_B > 0$ such that for all $s_A, s_B, k \geq 0$ with $s_A + k < s_B < \bar{s}_B$ the number of returns is larger under a contract inducing sequential inspection than under a contract inducing simultaneous inspection if, and only if,*

$$F(\rho^{JM}(\eta)) > \sqrt{-5 + \sqrt{28 + 8F^2(\eta)}}. \tag{5}$$

Thus, banning simultaneous contracts may actually lead to more rather than less returns. The intuition behind this result is the following. Under A^{seq}, the low inspection costs and inspection fee σ_2 makes inspection of the second product attractive to the consumer, while the high refund price ρ_1 makes it unlikely that the consumer will consider the first product a good enough fit. Thus, there is a high chance that the second product will be inspected, in which case at least one product will be returned with certainty. Due to the similarly high refund price ρ_2, it is however also likely that the consumer finds neither of the two products a good enough fit, implying that both would be returned. For small s_B this effect is most severe, leading to a higher number of returns under A^{seq} than under A^{sim}.

Condition (5) depends solely on the given distribution F and on the value of η. Inspecting the RHS, we see that it ranges from approximately 0.54 to 1 as η changes from $\underline{v}$ to $\bar{v}$. Thus, any distribution with a large enough joint monopoly value (at a minimum larger than the 54th percentile of the value distribution) will fulfill the condition and results in contracts inducing sequential inspection leading to more returns than contracts inducing simultaneous inspection. This is in general the case for distribution functions that are not particularly skewed to the left. It can be shown that for the uniform and the exponential distribution, the condition holds for any value of η, i.e., there will always be more returns under contracts inducing sequential inspection for small enough inspection and product degradation cost. Figure 5 shows the difference in the expected number of returns for different values of η for the uniform distribution and for the exponential distribution. For the uniform distribution, we observe for values of η below the mean of the value distribution that A^{seq} leads to between around 5% and 13% more returns on average than A^{sim}. For the exponential distribution,

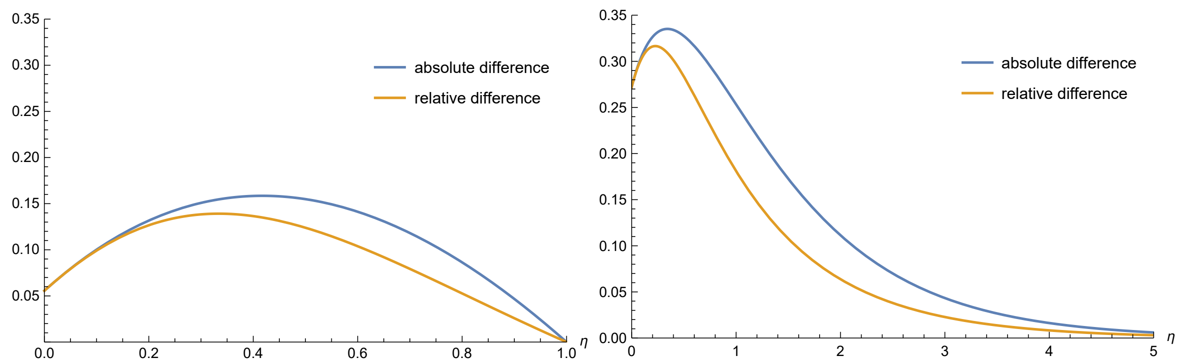

Figure 5: The difference in expected number of returns between A^{seq} and A^{sim} over varying salvage value η for uniform distribution $U[0,1]$ (left) and exponential distribution with $\lambda = 1$ (right).[30] The absolute difference is $n_{Aseq} - n_{Asim}$ and the relative difference is $\frac{n_{Aseq} - n_{Asim}}{n_{Asim}}$.

that number even lies between 18% and 32%. Note that both figures are valid independent of the precise values of s_A, s_B and k, as long as $s_A + k < s_B < \bar{s}_B$ holds.

While we derived Proposition 2 for large values of s_B, the implication for when A^{sim} leads to higher profits than A^{seq} is also sufficient for small s_B. The reason is that the profit from A^{seq} will be strictly smaller for small s_B than what we derived in the previous section for large s_B, while the profit under A^{sim} remains unchanged. Thus, Propositions 2 and 4 together imply the following:

Corollary 1 *There exists an $\underline{S}_A(\eta) > 0$ as defined in Proposition 2 and an $\bar{s}_B > 0$, such that for all $s_A, s_B, k \geq 0$ with $s_B < \bar{s}_B$ and $s_A + k < \max[s_B, \underline{S}_A(\eta)]$ and for all (F, η) that fulfill condition (5), the profit maximizing strategy for the firm is to induce the consumer to inspect products simultaneously, leading to a lower number of returns than when this policy would be banned.*

5 Discussion and Conclusion

This paper shows that multi-product firms may find it profitable to induce consumers to inspect many products simultaneously and get a refund for the products they want to return. Especially in online markets this may be an interesting proposition for consumers as they may

[30]It can be shown that the figure for the uniform distribution is valid for arbitrary lower and upper bound values. Although the figure for the exponential distribution was obtained by maximizing the original profit function, in a linear Taylor approximation of that profit function η and the distribution parameter λ only occur in form of their product $\lambda\eta$. The error of that approximation is at most 0.008 at $\eta = 0$. Therefore the figure is approximately valid for all values of λ by scaling the x-axis accordingly by $\frac{1}{\lambda}$.

inspect products at their own ease at home. Presented with this option, consumers buy the product with the highest valuation and are willing to pay a higher price.

To show that this may be a profitable strategy for firms, we also consider alternative pricing policies where consumers inspect products sequentially. Sequential inspection may be done either before or after ordering. The optimal contract under sequential search may be to induce consumers to inspect after ordering. The way to do so is to set asymmetric contracts where the contract for the first product to be inspected has a lower inspection fee and a higher refund price. These contracts have features in common with optimal obfuscation contracts as in Petrikaitė (2018b), with the main difference that the optimal contracts here have features of a two-part tariff where the firm benefits from creating an inspection fee, which is the difference between price and refund (made possible through a partial refund).

We show that despite the appearance of creating unnecessary refunds, inducing consumers to inspect many products simultaneously at home may actually lead to fewer (rather than more) products being returned. This has interesting implications for regulatory policies aiming to reduce the environmental impact of product returns. Our paper suggests that it is important to investigate in more detail in what type of markets abandoning simultaneous inspection options are more likely to lead to more or less returns. The leading issue in this regard is to focus on markets where inspection before ordering is costly for consumers.

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A Appendix

A.1 Proof of Proposition 1

For clarity, we denote $\hat{v}_i^a$ as $\hat{v}_i$ in this proof. We further define $\overline{\rho} = \eta + \frac{1-F(\overline{\rho})}{f(\overline{\rho})}$.

The proof is in several steps. First, note that as long as the consumer continues to inspect the first product first we can always increase σ_1 to increase profits. Thus, we should have $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2 \geq 0$. It is easy to show that *if* $\widehat{v}_2 - \rho_2 = 0$, the optimal contract is as specified in the Proposition. If $\widehat{v}_2 - \rho_2 = 0$, the firm's profit equals

$$\begin{aligned} & \sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)(1 - F(\rho_2))(\sigma_2 + \rho_2 - c) + F(\rho_2)(F(\rho_1))(\sigma_2 - k) \\ = \ & \sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)\left(\int_{\rho_2}(1 - F(v))dv - s_A + \rho_2 - c - F(\rho_2)(\rho_2 - \eta)\right). \end{aligned}$$

The derivative wrt ρ_2 equals $-f(\rho_2)(\rho_2 - \eta)$. Thus, we should have $\rho_2 = \eta$ and it then follows from $\widehat{v}_2 - \rho_2 = 0$ that $\sigma_2 = \int_\eta (1 - F(v))dv - s_A$. Thus, the profit on the second product equals $\int_\eta (1 - F(v))dv - s_A - k$ and overall profit is then equal to

$$\int_{\rho_1}(1 - F(v))dv - s_A - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)\left(\int_\eta (1 - F(v))dv - s_A - k\right).$$

The derivative wrt ρ_1 yields

$$-f(\rho_1)(\rho_1 - \eta) + f(\rho_1)\left(\int_\eta (1 - F(v))dv - s_A - k\right),$$

which implies that the optimal ρ_1 is

$$\rho_1 = \eta + \int_\eta (1 - F(v))dv - s_A - k.$$

The rest of the proof shows that it cannot be the case that $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2 > 0$. This part of the proof is by contradiction. If $\widehat{v}_2 - \rho_2 > 0$ the firm can increase either σ_2 and ρ_1 or ρ_2 and ρ_1 or σ_2 and σ_1 such that $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2 > 0$. By analyzing these joint increases in turn, we successively rule out different subcases that together imply that it cannot be that $\widehat{v}_2 - \rho_2 > 0$.

First consider that we jointly increase σ_2 and ρ_1 such that $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2$. We can do that by changing them such that $(1 - F(\widehat{v}_2))d\rho_1 = d\sigma_2$. The profit function is equal to

$$\begin{aligned} & \sigma_1 - k + F(\widehat{v}_2 + \rho_1 - \rho_2)(\sigma_2 - k) + \\ & \left[\int_{\rho_1}^{\widehat{v}_2 + \rho_1 - \rho_2} F(v_1 - \rho_1 + \rho_2)f(v_1)dv_1 + 1 - F(\widehat{v}_2 + \rho_1 - \rho_2)\right](\rho_1 - \eta) + \\ & \left[\int_{\rho_2}^{\widehat{v}_2} F(v_2 + \rho_1 - \rho_2)f(v_2)dv_2 + F(\widehat{v}_2 + \rho_1 - \rho_2)\left[1 - F(\widehat{v}_2)\right]\right](\rho_2 - \eta). \end{aligned}$$

The increase in profits equals

$$
\begin{aligned}
F(\widehat{v}_1)\left(1-F(\widehat{v}_2)\right)+ \quad & \int_{\rho_1}^{\widehat{v}_1} F(v_1-\rho_1+\rho_2)f(v_1)dv_1+1-F(\widehat{v}_1)- \\
& \left[\int_{\rho_1}^{\widehat{v}_1} f(v_1-\rho_1+\rho_2)f(v_1)dv_1+F(\rho_2)f(\rho_1)\right](\rho_1-\eta)+ \\
& \left[\int_{\rho_2}^{\widehat{v}_2} f(v_2+\rho_1-\rho_2)f(v_2)dv_2\right](\rho_2-\eta),
\end{aligned}
$$

which can be rewritten as

$$
\begin{aligned}
& \int_{\rho_1}^{\widehat{v}_1} F(v_1-\rho_1+\rho_2)f(v_1)dv_1+1-F(\widehat{v}_1)F(\widehat{v}_2)-F(\rho_2)f(\rho_1)(\rho_1-\eta)+ \\
- \quad & \left[\int_{\rho_1}^{\widehat{v}_1} f(v_1-\rho_1+\rho_2)f(v_1)dv_1\right](\rho_1-\rho_2).
\end{aligned}
$$

This is equal to

$$
\begin{aligned}
& \int_{\rho_1}^{\widehat{v}_1}\left[F(v_1-\rho_1+\rho_2)-F(\rho_2)\right]f(v_1)dv_1+1-F(\widehat{v}_2)F(\widehat{v}_1)-F(\rho_2)(1-F(\widehat{v}_1)) \\
& +F(\rho_2)\left[(1-F(\rho_1)-f(\rho_1)(\rho_1-\eta)\right]-\int_{\rho_1}^{\widehat{v}_1} f(v_2+\rho_1-\rho_2)(\rho_1-\rho_2)f(v_2)dv_2,
\end{aligned}
$$

which, as $1-F$ is logconcave and $1-F(\widehat{v}_2)F(\widehat{v}_1)-F(\rho_2)(1-F(\widehat{v}_1))=(1-F(\rho_2))(1-F(\widehat{v}_1))+F(\widehat{v}_1)(1-F(\widehat{v}_2))>0$, is strictly larger than 0 if $\rho_1\leq\min\{\rho_2,\overline{\rho}\}$. Thus, if $\widehat{v}_2-\rho_2>0$ we should have $\rho_1>\min\{\rho_2,\overline{\rho}\}$.

Next, we argue that raising both ρ_1 and ρ_2 to the same extent (keeping $\widehat{v}_1$ and $\widehat{v}_2$ constant) increases in profits if $\rho_i\leq\overline{\rho}, i=1,2$. The increase in profits in this case is equal to

$$
\begin{aligned}
& (1-F(\rho_1+\widehat{v}_2-\rho_2))+F(\rho_1+\widehat{v}_2-\rho_2)(1-F(\widehat{v}_2))+ \\
& \int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)F(v+\rho_2-\rho_1)dv-(\rho_1-\eta)f(\rho_1)F(\rho_2)+ \\
& \int_{\rho_2}^{\widehat{v}_2} f(v)F(v-\rho_2+\rho_1)dv-(\rho_2-\eta)f(\rho_2)F(\rho_1) \\
= \quad & 1-F(\rho_1)F(\rho_2)-(\rho_1-\eta)f(\rho_1)F(\rho_2)-(\rho_2-\eta)f(\rho_2)F(\rho_1) \\
= \quad & F(\rho_2)(1-F(\rho_1)-f(\rho_1)(\rho_1-\eta))+F(\rho_1)(1-F(\rho_2)-f(\rho_2)(\rho_2-\eta)) \\
& +\left(1-F(\rho_1)\right)\left(1-F(\rho_2)\right),
\end{aligned}
\tag{6}
$$

which by logconcavity of $1-F$ is clearly positive if $\rho_i\leq\overline{\rho}$, $i=1,2$.[31] Moreover if $\rho_2=\eta$ this is positive if $F(\rho_2)(1-F(\rho_1)-f(\rho_1)(\rho_1-\eta))+1-F(\rho_2)>0$, which is the case if $1-F(\rho_2)F(\rho_1)-F(\rho_2)f(\rho_1)(\rho_1-\eta)>0$.

[31] Note by the way that at $\rho_1=\rho_2$ this equals 0 if $\rho_1=\rho_2$ is equal to the joint monopoly price that solves $\rho=\eta+\frac{1-F^2(\rho)}{2f(\rho)F(\rho)}$.

We next argue that $\rho_2 \geq \eta$. If not, then a decrease in σ_2 and an increase in ρ_2 such that $\widehat{v}_2 - \rho_2$ is constant (so that $d\sigma_2 = -(1 - F(\widehat{v}_2))d\rho_2$ increases profits. Profits can be written as

$$\sigma_1 - k + F(\rho_1)\left[\sigma_2 - k + (1 - F(\rho_2)(\rho_2 - \eta)\right] + \left(F(\rho_1 + \widehat{v}_2 - \rho_2) - F(\rho_1)\right)(\sigma_2 - k) +$$
$$(\rho_1 - \eta)\int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)F(v + \rho_2 - \rho_1)dv + (\rho_2 - \eta)\int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)(1 - F(v + \rho_2 - \rho_1))dv$$
$$+(1 - F(\rho_1 + \widehat{v}_2 - \rho_2))(\rho_1 - \eta)$$

so that the increase in profits is equal to

$$\begin{aligned}
& F(\rho_1)\left[-(1 - F(\widehat{v}_2)) + (1 - F(\rho_2)) - f(\rho_2)(\rho_2 - \eta)\right] \\
& -(F(\rho_1 + \widehat{v}_2 - \rho_2) - F(\rho_1))(1 - F(\widehat{v}_2)) \\
& + \int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)(1 - F(v + \rho_2 - \rho_1))dv \\
& +(\rho_1 - \rho_2)\int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)f(v + \rho_2 - \rho_1)dv \\
= \; & F(\rho_1)\left[F(\widehat{v}_2) - F(\rho_2) - f(\rho_2)(\rho_2 - \eta)\right] \\
& + \int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)(F(\widehat{v}_2) - F(v + \rho_2 - \rho_1))dv \\
& +(\rho_1 - \rho_2)\int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)f(v + \rho_2 - \rho_1)dv
\end{aligned}$$

which is clearly positive if $\rho_2 - \eta \leq 0$ and $\rho_1 \geq \rho_2$. Thus, the optimal solution can only involve $\rho_2 \leq \eta$ and $\widehat{v}_2 - \rho_2 > 0$ if $\rho_1 < \rho_2 \leq \eta$, which cannot be the case as $\rho_1 > \min\{\rho_2, \overline{\rho}\}$.

Consider then an increase in σ_1 and σ_2 so that $\widehat{v}_1 - \rho_1 = \widehat{v}_2 - \rho_2 \geq 0$. As $-(1 - F(\widehat{v}_i))\frac{\partial \widehat{v}_i}{\partial \sigma_i} = 1$, this implies that $\frac{(1-F(\widehat{v}_1))}{(1-F(\widehat{v}_2))} = \frac{d\sigma_1}{d\sigma_2}$. We write the firm's profit as

$$\sigma_1 - k + (1 - F(\rho_1 + \widehat{v}_2 - \rho_2))(\rho_1 - \eta) + F(\rho_1 + \widehat{v}_2 - \rho_2)(1 - F(\widehat{v}_2))(\sigma_2 + \rho_2 - c) +$$
$$F(\rho_2)F(\rho_1)(\sigma_2 - k) + (\sigma_2 + \rho_1 - c)\int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)F(v + \rho_2 - \rho_1)dv +$$
$$(\sigma_2 + \rho_2 - c)\int_{\rho_2}^{\widehat{v}_2} f(v)F(v - \rho_2 + \rho_1)dv.$$

So that the increase in profit equals

$$\frac{(1 - F(\widehat{v}_1))}{(1 - F(\widehat{v}_2))} + \frac{f(\rho_1 + \widehat{v}_2 - \rho_2)}{(1 - F(\widehat{v}_2))}(\rho_1 - \eta) + F(\rho_1 + \widehat{v}_2 - \rho_2)(1 - F(\widehat{v}_2)) +$$
$$\frac{F(\rho_1 + \widehat{v}_2 - \rho_2)f(\widehat{v}_2) - f(\rho_1 + \widehat{v}_2 - \rho_2)(1 - F(\widehat{v}_2))}{(1 - F(\widehat{v}_2))}(\sigma_2 + \rho_2 - c) +$$
$$F(\rho_2)F(\rho_1) + \int_{\rho_1}^{\rho_1+\widehat{v}_2-\rho_2} f(v)F(v + \rho_2 - \rho_1)dv + \int_{\rho_2}^{\widehat{v}_2} f(v)F(v - \rho_2 + \rho_1)dv$$
$$-(\sigma_2 + \rho_1 - c)\frac{f(\rho_1 + \widehat{v}_2 - \rho_2)F(\widehat{v}_2)}{(1 - F(\widehat{v}_2))} - (\sigma_2 + \rho_2 - c)\frac{F(\rho_1 + \widehat{v}_2 - \rho_2)f(\widehat{v}_2)}{(1 - F(\widehat{v}_2))}.$$

This can be rewritten as

$$\begin{aligned}
& \frac{(1-F(\widehat{v}_1))}{(1-F(\widehat{v}_2))}+\frac{f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}(\rho_1-\eta)+F(\widehat{v}_1) \\
& -(\rho_1-\rho_2)\frac{f(\widehat{v}_1)F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}-\frac{f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}(\sigma_2+\rho_2-c) \\
= \; & \frac{(1-F(\widehat{v}_1))}{(1-F(\widehat{v}_2))}+\frac{f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}(\rho_1-\eta)+F(\widehat{v}_1) \\
& -(\sigma_2+\rho_1-c)\frac{f(\widehat{v}_1)F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}-\frac{f(\widehat{v}_1)(1-F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}(\sigma_2+\rho_2-c) \\
= \; & \frac{1-F(\widehat{v}_1)}{(1-F(\widehat{v}_2))}-(\sigma_2-k)\frac{f(\widehat{v}_1)F(\widehat{v}_2)}{(1-F(\widehat{v}_2))}+F(\widehat{v}_1)-f(\widehat{v}_1)(\sigma_2-k-\rho_1+\rho_2) \\
= \; & \frac{1-F(\widehat{v}_1)}{(1-F(\widehat{v}_2))}-\frac{(\sigma_2-k)f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}+F(\widehat{v}_1)+f(\widehat{v}_1)(\rho_1-\rho_2),
\end{aligned}$$

which because $\sigma_2=\int_{\widehat{v}_2}(1-F(v))dv-s_A$ is equal to

$$\frac{1-F(\widehat{v}_1)}{(1-F(\widehat{v}_2))}-\frac{(\int_{\widehat{v}_2}(1-F(v))dv-s_A-k)f(\widehat{v}_1)}{(1-F(\widehat{v}_2))}+F(\widehat{v}_1)+f(\widehat{v}_1)(\widehat{v}_1-\widehat{v}_2).$$

This is positive if

$$\frac{1-F(\widehat{v}_1)F(\widehat{v}_2)}{f(\widehat{v}_1)}-\left(\int_{\widehat{v}_2}(1-F(v))dv-s_A-k\right)+(1-F(\widehat{v}_2))(\widehat{v}_1-\widehat{v}_2)>0. \tag{7}$$

That is certainly the case if $\widehat{v}_2=\overline{v}$. The derivative of this expression wrt $\widehat{v}_2$ equals

$$-\frac{F(\widehat{v}_1)f(\widehat{v}_2)}{f(\widehat{v}_1)}+(1-F(\widehat{v}_2))-(1-F(\widehat{v}_2))-f(\widehat{v}_2))(\widehat{v}_1-\widehat{v}_2).$$

This is clearly nonpositive if $-\frac{F(\widehat{v}_1)f(\widehat{v}_2)}{f(\widehat{v}_1)}-f(\widehat{v}_2))(\widehat{v}_1-\widehat{v}_2)\leq 0$, which is the case if $\widehat{v}_1\geq \widehat{v}_2-\frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$. So, if we decrease $\widehat{v}_2$ starting from $\widehat{v}_2=\overline{v}$, then 7 remains positive if $\widehat{v}_1\geq\widehat{v}_2-\frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$. So, the only possibility for an equilibrium with $\widehat{v}_2>\rho_2$ is that $\widehat{v}_1<\widehat{v}_2-\frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$.

To rule out that $\widehat{v}_1<\widehat{v}_2-\frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}$ we finally consider that we increase σ_2 and decrease ρ_2 such that $\widehat{v}_2-\rho_2$ is constant. We can do that by changing them such that $-(1-F(\widehat{v}_2))d\rho_1=d\sigma_2$. The profit function is equal to

$$\begin{aligned}
& \sigma_1-k+F(\widehat{v}_2+\rho_1-\rho_2)(\sigma_2-k)+ \\
& \left[\int_{\rho_1}^{\widehat{v}_2+\rho_1-\rho_2}F(v_1-\rho_1+\rho_2)f(v_1)dv_1+1-F(\widehat{v}_2+\rho_1-\rho_2)\right](\rho_1-\eta)+ \\
& \left[\int_{\rho_2}^{\widehat{v}_2}F(v_2+\rho_1-\rho_2)f(v_2)dv_2+F(\widehat{v}_2+\rho_1-\rho_2)\left[1-F(\widehat{v}_2)\right]\right](\rho_2-\eta).
\end{aligned}$$

The increase in profits equals

$$F(\widehat{v}_1)\,(1-F(\widehat{v}_2)) - \left[\int_{\rho_2}^{\widehat{v}_2} F(v_2+\rho_1-\rho_2)f(v_2)dv_2 + F(\widehat{v}_1)\,[1-F(\widehat{v}_2)]\right]$$
$$-\left[\int_{\rho_1}^{\widehat{v}_1} f(v_1-\rho_1+\rho_2)f(v_1)dv_1\right](\rho_1-\eta) + [-F(\widehat{v}_1)f(\widehat{v}_2)]\,(\rho_2-\eta) +$$
$$\left[\int_{\rho_2}^{\widehat{v}_2} f(v_2+\rho_1-\rho_2)f(v_2)dv_2 + F(\rho_1)f(\rho_2) + F(\widehat{v}_1)f(\widehat{v}_2)\right](\rho_2-\eta),$$

which can be rewritten as

$$\begin{aligned}
&- \int_{\rho_2}^{\widehat{v}_2} F(v_2+\rho_1-\rho_2)f(v_2)dv_2 \\
&+ \left[\int_{\rho_2}^{\widehat{v}_2} f(v_1+\rho_1-\rho_2)f(v_2)dv_2\right](\rho_2-\rho_1) + F(\rho_1)f(\rho_2)(\rho_2-\eta) \\
\geq\ & \int_{\rho_2}^{\widehat{v}_2}\left[\frac{f(v_1+\rho_1-\rho_2)}{F(v_2+\rho_1-\rho_2)}\frac{F(\widehat{v}_1)}{f(\widehat{v}_1)}-1\right]F(v_2+\rho_1-\rho_2)f(v_2)dv_2 + F(\rho_1)f(\rho_2)(\rho_2-\eta).
\end{aligned}$$

As F is logconcave, f/F is decreasing and therefore $\frac{f(v_1+\rho_1-\rho_2)}{F(v_2+\rho_1-\rho_2)} > \frac{f(\widehat{v}_1)}{F(\widehat{v}_1)}$. Thus, the term in square brackets is positive and the whole expression is strictly positive as $\rho_2 > \eta$. $\square$

A.2 Proof of Proposition 2

The profits under the two search modes are

$$\pi^*_{Asim} = \mathbb{E}[\max(v_1-\eta, v_2-\eta, 0)] - 2(s_A+k),$$

and

$$\pi^*_{Aseq} = \mathbb{E}[\max(v_1-\eta, \mathbb{E}[\max(v_2-\eta,0)] - s_A - k)] - s_A - k$$

respectively, where in the second equation it is important to note that the second product is only inspected if inspection of the first product results in a low value. Thus, we have that $\pi^*_{Asim} \geq \pi^*_{Aseq}$, if and only if,

$$\mathbb{E}[\max(v_1-\eta, v_2-\eta, 0)] \geq \mathbb{E}[\max(v_1-\eta+s_A+k, \mathbb{E}[\max(v_2-\eta,0)])]$$

It is immediately evident that for $s_A+k=0$ and for any value of η, A^{sim} leads to strictly higher profits. Thus, by continuity of the RHS in s_A+k, it follows that there exists a threshold $\underline{S}_A(\eta)$ such that A^{sim} yields larger profit if $s_A+k \leq \underline{S}_A(\eta)$. On the other hand, as the RHS of the above inequality is weakly increasing in s_A+k, and strictly increasing in s_A+k if s_A+k is large enough, it also follows that A^{seq} yields larger profit if $s_A+k > \underline{S}_A(\eta)$. This proves the first part of the proposition.

For the second part we need to prove that

$$1 + F^2(\eta) > F(\rho_1^*)(1 + F(\eta)).$$

This can be written as

$$\frac{(1 + F(\eta))^2 - 2F(\eta)}{1 + F(\eta)} = 1 + F(\eta) - \frac{2F(\eta)}{1 + F(\eta)} > F(\rho_1^*). \tag{8}$$

The LHS equals 1 at $\eta \leq \underline{v}$ and at $\eta = \overline{v}$. The RHS equals 1 at $\eta = \overline{v}$ and is smaller than 1 at $\eta \leq \underline{v}$. The derivatives of the LHS and the RHS wrt η are respectively $f(\eta) - \frac{2f(\eta)}{(1+F(\eta))^2} = f(\eta)\left(1 - \frac{2}{(1+F(\eta))^2}\right)$, which is first decreasing and then from $F(\eta) = \sqrt{2} - 1$ it is increasing, and $f(\rho_1^*)F(\eta) > 0$. At $\eta = \overline{v}$ the derivative of the LHS is smaller than that of the RHS.

The derivatives are equal to each other if $\left(\frac{1}{F(\eta)} - \frac{2}{F(\eta)(1+F(\eta))^2}\right) = f(\rho_1^*)/f(\eta)$. As $1-F$ is logconcave, $\frac{1-F(v)}{f(v)}$ is decreassing in v and thus $\frac{f(\rho_1^*)}{f(\eta)} > \frac{1-F(\rho_1^*)}{1-F(\eta)}$ as $\rho_1^* > \eta$. Thus, the derivatives can only be equal to each other if

$$\left(\frac{1}{F(\eta)} - \frac{2}{F(\eta)\,(1 + F(\eta))^2}\right) > \frac{1 - F(\rho_1^*)}{1 - F(\eta)},$$

which can be rewritten as

$$\left(\frac{1}{F(\eta)} - \frac{2(1 - F(\eta))}{F(\eta)\,(1 + F(\eta))^2}\right) = \frac{-1 + 4F(\eta) + F^2(\eta)}{F(\eta)\,(1 + F(\eta))^2} > 2 - F(\rho_1^*),$$

or

$$-1 + 3F(\eta) - 2F^2(\eta) - F^3(\eta) > (1 - F(\rho_1^*))\,(1 + F(\eta))^2\,F(\eta).$$

As the LHS is negative for any $0 \leq F(\eta) \leq 1$, while the RHS is positive, this inequality can never hold. Thus, the the derivative of the LHS of (8) is always smaller than the derivative of its RHS and therefore (8) holds. $\square$

A.3 Proof of Lemma 1

Suppose to the contrary that the consumer inspects the first product before ordering and the contract for the first product has (ρ_1, σ_1). If the firm would offer an alternative contract for the first product with

$$(\rho_1 + \sigma_1, s_B - s_A)$$

then the two contracts are identical from the consumers' perspective if they search the product afterwards using this alternative contract, i.e. searching before with the initial contract is identical to searching afterwards with the alternative contract. The consumer will never search before using the alternative contract as the conditions are worse. This also implies that the consumer does not want to switch the order of inspecting the two products, since for them the situation is effectively the same as with the initial contract. Now, importantly, the firm's expected profit from the second product is the same for both contracts, as the consumer's search behavior and expected search result are identical. However, the firm's profits from the first product are strictly larger under the alternative contract, as it now receives additional profits from inspection of $s_B - s_A - k$, which is strictly positive due to our initial assumption. Because the firm strictly prefers the alternative contract and the consumer is indifferent, it must be the case that the first product is searched afterwards. $\square$

A.4 Proof of Lemma 2

We prove that in the optimal contract, it must be that $\rho_1^* < \widetilde{v}_1^a < \bar{v}$. In other words, for some values of v_1 the consumer weakly prefers to inspect the second product before ordering and for others after ordering.

On the one hand, it cannot be that the consumer strictly prefers to inspect the second product after ordering for all values of v_1 as in that case the firm could raise σ_2 to increase profits. Raising σ_2 does not affect the order of search as it makes it less attractive to search this product first and a marginal increase will continue making it optimal for the consumer to search the product afterwards.

On the other hand, it cannot be the case that there are no values of v_1 for which the consumer wants to inspect the second product afterwards. To see why, we show that in the optimal contract there must exist a value of v_1 at which the consumer is indifferent between searching the second product before and after ordering, denoted by $\widetilde{v}_1^a$, with $\widetilde{v}_1^a \geq \rho_1$. That value of $\widetilde{v}_1^a$ for a given contract can be obtained from (4), which we restate here for quick reference:

$$\int_{\widetilde{v}_1^a - \rho_1 + \rho_2}^{\widetilde{v}_1^a - \rho_1 + \rho_2 + \sigma_2} F(v_2) dv_2 = s_B - s_A.$$

Then for $v_1 \geq \widetilde{v}_1^a$ the consumer finds it optimal to inspect before ordering and for $v_1 < \widetilde{v}_1^a$ the consumer finds it optimal to inspect afterwards. Assume now, contrary to our assertion, that the contract of the firm is such that the consumer always inspects the second product before ordering, regardless of v_1. Then the profit of the firm is

$$\sigma_1 - k + (1 - F(\rho_1))(\rho_1 - \eta) + F(\rho_1)(1 - F(\rho_2 + \sigma_2))(\rho_2 + \sigma_2 - k - \eta) +$$
$$(\rho_2 + \sigma_2 - \rho_1 - k) \int_{\rho_1}^{\hat{v}^b - (\rho_2 + \sigma_2 - \rho_1)} \int_{\rho_2 + \sigma_2 + v_1 - \rho_1}^{\overline{v}} f(v_1) f(v_2) dv_2 dv_1.$$

Note that this expression depends on the sum $\rho_2 + \sigma_2$, but not on the individual components ρ_2 and σ_2. However, the decision whether to inspect the second product before or after depends on these components ρ_2 and σ_2. In particular, if σ_2 is small enough, consumers want to inspect the second product afterwards certainly if $v_1 \leq \rho_1$. Then the firm can decrease σ_2 and increase ρ_2 in such a way that the sum $\rho_2 + \sigma_2$ stays constant, and it can do so just enough that the consumer becomes indifferent between inspecting the second product before and after ordering in those cases where $v_1 \leq \rho_1$, while still preferring to inspect the second product before for $v_1 > \rho_1$. This means that $\widetilde{v}_1^a = \rho_1$. If the consumer then for $v_1 \leq \rho_1$ inspects the product after ordering, then the third term in the above profit expression becomes $F(\rho_1)[(1 - F(\rho_2)(\rho_2 - \eta) + \sigma_2 - k]$, which is a strict increase in profits if $\sigma_2 - k > 0$. Referring to (4), we see that the LHS is strictly smaller than σ_2 as long as $s_B - s_A$ is positive, which implies that $\sigma_2 > s_B - s_A$. As we require $s_B - s_A > k$, we find that $\sigma_2 > k$ and therefore the above change in contract indeed yields a strict increase in profits if the consumer inspects the second product after ordering for $v_1 \leq \rho_1$. Should the consumer still continue to inspect the second product before ordering for $v_1 \leq \rho_1$, as is permissible given her indifference, then the firm would have an incentive to further lower σ_1 to make the consumer strictly prefer inspecting the second product after ordering for $v_1 \leq \rho_1$. This must be profitable given that the firm strictly prefers the consumer to inspect the second product after ordering and the consumer is indifferent. □

A.5 Proof of Proposition 3

(i) We have already shown that in the limit of $s_B = s_A = 0$ the optimal contract is $\sigma_i^* = k = 0$ and $\rho_i^* = \rho^{JM}(\eta)$. We now show that for $s_A + k < s_B \leq \bar{s}_B$ the optimal contract will be approximately the same. As Lemma 2 shows that (4) must hold in the optimal sequential contract, it is straightforward to see that as the RHS goes towards zero, on the LHS σ_2 has to go towards zero as well. We have also argued in Lemma 2 that it is always possible for the firm to set $\sigma_2 > k$ for positive $s_B - s_A$. Therefore in the limit the firm will set $\sigma_2^* = k = 0$, and for $s_A + k < s_B \leq \bar{s}_B$ the optimal σ_2^* will be $\sigma_2^* \approx k$. The firm's profit is equal to:

$$\begin{aligned}
&\sigma_1 - k + F(\widetilde{v}_1^a)(\sigma_2 - k) + F(\rho_1)(1 - F(\rho_2))(\rho_2 - \eta) + (1 - F(\rho_1))(\rho_1 - \eta) \\
&+ (\rho_2 - \rho_1) \int_{\rho_1}^{\widetilde{v}_1^a} \int_{v_1 - \rho_1 + \rho_2} f(v_1) f(v_2) dv_2 dv_1 \\
&+ (\sigma_2 + \rho_2 - \rho_1 - k) \int_{\widetilde{v}_1^a}^{\hat{v}^b + \rho_1 - \rho_2 - \sigma_2} \int_{v_1 - \rho_1 + \rho_2 + \sigma_2} f(v_1) f(v_2) dv_2 dv_1
\end{aligned}$$

It is immediately clear that the optimal σ_1^* will be as large as possible. Raising σ_1 too much relative to σ_2 would prompt the consumer to start inspection with the second product instead of the first. Therefore σ_1^* must similarly be approximately equal to k. Given that the profit from inspection is approximately zero, the optimal prices ρ_i^* are determined as in the limit, and take on the same value ρ^{JM}.

(ii) It is possible to derive the relation between $\widetilde{v}_1^a$ and $\rho^{JM}(\eta)$ near the limit. We will make use of this relation in the next proposition. The relation follows from the consideration when the consumer prefers to inspect the first product after ordering to inspecting it before ordering. We are focusing on the case where the first product is always inspected after ordering, however, as we have argued before, in the limit of $s_B, s_A \to 0$ the difference between the two inspection modes vanishes as $\sigma_i^* - k \to 0$. Given that $\sigma_i - k$ is so small, the comparison between the cases where the first product is inspected after and before ordering is fully determined by considering when the consumer pays σ_i in both modes. The prices ρ_i are paid in nearly the same instances (whenever a product is paid or kept), and the expected gain through the product values is also nearly the same. The main difference is then that under inspection after ordering, the consumer pays σ_1 upfront in a significant number of cases. Therefore, to determine when the consumer weakly prefers to inspect the first product after ordering, we consider when the expected expenditure in inspection costs is smaller under inspection after ordering.

If the consumer inspects the first product before, then if $v_1 > \widetilde{v}_1^a$ she inspects the second product also before, but she always buys one of the products so she pays either σ_1 or σ_2 as part of the price of the product.[32] If $v_1 < \widetilde{v}_1^a$ she inspects the second product after ordering, so she always pays σ_2, but she also pays σ_1 (in addition) as part of the price of the first product if $v_1 > \max\{v_2, \rho_1\}$. Thus, taking $\sigma_1 = \sigma_2 = \sigma$ the consumer implicitly or explicitly pays approximately $\sigma(1 + \frac{1}{2}\left(F^2(\widetilde{v}_1^a) - F^2(\rho_1)\right)$.

If the consumer inspects the first product afterwards, then if $v_1 > \widetilde{v}_1^a$ she inspects the second product before ordering, and she buys the second product if $v_2 > v_1$. If $v_1 < \widetilde{v}_1^a$ she also inspects the second product after ordering, so she always pays $\sigma_1 + \sigma_2$. Thus, taking $\sigma_1 = \sigma_2 = \sigma$ the consumer implicitly or explicitly pays approximately $\sigma(1 + F(\widetilde{v}_1^a) + \frac{1}{2}\left(1 - F^2(\widetilde{v}_1^a)\right))$.

To weakly prefer inspecting the first product after ordering we should have that

$$s_B + \sigma(1 + \frac{1}{2}\left(F^2(\widetilde{v}_1^a) - F^2(\rho_1)\right) \geq s_A + \sigma(1 + F(\widetilde{v}_1^a) + \frac{1}{2}\left(1 - F(\widetilde{v}_1^a)\right)^2),$$

or

$$\sigma \leq \frac{2(s_B - s_A)}{1 + F^2(\rho_1)}.$$

For σ small (4) becomes approximately $\sigma F(\widetilde{v}_1^a) = s_B - s_A$ which together with the above

[32] Note that if the first product is inspected before, then the relevant threshold value is $\widetilde{v}_1^b$ which solves $\int_{\widetilde{v}_1^a-\rho_1-\sigma_1+\rho_2}^{\widetilde{v}_1^a-\rho_1-\sigma_1+\rho_2+\sigma_2} F(v_2)dv_2 = s_B - s_A$. But as in the limit σ_1 is approximately zero, it holds that $\widetilde{v}_1^a \approx \widetilde{v}_1^b$.

yields

$$F(\widetilde{v}_1^a) = \frac{1 + F^2(\rho_1)}{2},$$

concluding the proof. □