Naïve Consumers and Financial Mistakes

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Abstract

Financial contracts are complicated and consumers often do not grasp them in their entirety. This may lead to financial mistakes when borrowers do not fully internalize the costs of credit. We develop a quantitative theory of unsecured credit and equilibrium default, where borrowers can sign debt contracts and trade off interest rates for penalty fees. These fees make financial shocks—such as paying late or borrowing over limit—costly. While sophisticated borrowers fully understand the risk of paying penalty fees, naïve borrowers face higher risk without internalizing this fact. Thus, they make financial mistakes by choosing inefficiently high penalty fees. In equilibrium, naïves' fee payments cross-subsidize interest rates for sophisticates. We use our framework to analyze two unexplored features of the CARD act: transparency requirements and penalty fee limits. More transparency makes financial contracts easier to understand, reducing the financial risk for naïve borrowers. Thus, naïves pay lower penalty fees. Fee limits directly ban high-fee contracts for everyone. Both policies reduce the expected revenue from naïve fee payments and consequently interest rates rise. In both cases, naïves make fewer financial mistakes and enjoy a welfare gain. Sophisticates, in contrast, suffer: Since naïves pay lower fees, sophisticates lose cross-subsidization and experience welfare losses.

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1 Introduction

Financial contracts can be quite complicated. According to the Wall Street Journal, credit card contracts in 2013 are up to 50 times longer than in 1980—20,000 words in 2013 versus 400 words in 1980.[1] Complicated contracts are harder to understand. Indeed, many cardholders fail to understand key aspects of their contracts, including when late payments trigger penalty fees (GAO, 2006).[2]

Despite being only partially internalized, penalty fees constitute a significant part of the cost of banking to consumers. In 2011, nearly 28% of US consumer checking accounts experienced non-sufficient funds or overdraft fees. Nearly one third of those accounts accrued more than 10 penalty items with an average fee of $225 (CFPB, 2014). Also credit card borrowers incur penalty fees for repaying financial obligations too late or for exceeding lines of credit. With late fees and over limit fees constituting the largest items, Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015) document that the average credit card holder paid $58 in fees per year.

While some of these penalty fees are certainly the consequence of a rational optimization of one's financial situation, some of these fees seem to occur by mistake. Missing important aspects of their credit contract,[3] consumers can lose track of financial commitments, misunderstand minimum payments, or forget to pay bills on time. These financial mistakes are quite common. For example, 52% of borrowers that were charged overdraft fees do not recall opting in to overdraft at all (PEW, 2014). In addition to unexpected penalty fees, financial mistakes can cause financial distress. According to Warren, Sullivan, and Jacoby (2000), "credit card debt out of control" and "trouble in managing money" are the third and fourth most common reasons given for a consumer bankruptcy. Only job loss and medical reasons rank higher.

Due to their potentially high cost and severe consequences, financial mistakes are at the center of many pieces of regulation. The 2009 Credit Card Accountability Responsibility and Disclosure (CARD) Act in the U.S. is the most prominent recent example. Policy makers tackle financial mistakes along two dimensions: firstly, transparency requirements and reporting standards make contracts easier to understand, lowering the likelihood of incurring penalty fees by mistake. Secondly, the CARD Act caps penalty fees, reducing the cost of financial mistakes that trigger a penalty fee.

Despite the importance of penalty fees and financial mistakes for consumers and policy makers, state-of-the-art quantitative research cannot inform regulators about their costs and consequences. Standard models of consumer debt simply abstract from penalty fees and have no roll for financial mistakes. Without these important ingredients, transparency

[1]See `https://www.wsj.com/articles/SB10001424127887324000704578386652879032748`.

[2]According to the United States Government Accountability Office, this lack of understanding is a consequence of lenders deliberately complicating their contracts. However, this paper will be agnostic about the motivation to design complicated contracts. We take their existence as given.

[3]See GAO (2006).

requirements or fee limits as mandated by the CARD act seem futile. We address this gap in the literature and answer the following questions: What are the consequences of financial mistakes for credit market outcomes? Can regulation similar to the CARD act achieve better outcomes? How do these policies influence the interaction between consumers that are more and less exposed to paying penalty fees?

In a framework where some borrowers make financial mistakes due to their lack of understanding, we explore the implications on interest rates and welfare when consumers borrow on credit contracts that trade off interest rates and penalty fees. Building on the theory of naïveté,[4] we set up a heterogeneous agents model of unsecured debt and default that is inhabited by two types of agents: sophisticates and naïves. While all borrowers are subject to financial shocks that trigger penalty fees, naïve borrowers are unaware of their increased exposure to these shocks and the ensuing penalty fees. Being naïve, these consumers behave just like sophisticated consumers and choose contracts with overly high penalty fees – what we term financial mistakes. Because borrowers behave identically, lenders cannot distinguish between the two types. This naturally leads to (partial) pooling of borrowers.[5] Lenders maximize profit and offer a menu of loan contracts. Conditional on a requested amount and borrower characteristics, lenders offer a continuum of interest rate and penalty fee combinations for both types of borrowers to choose from.

In equilibrium, lenders only offer contracts that yield equal expected revenue. Hence, debt contracts trade off lower interest rates for lower penalty fees. When choosing from the menu of loan contracts, naïve borrowers do not understand the true expected cost of penalty fees. They consequently choose contracts that carry too high penalty fees (that they are naïve about) because they prefer low interest rates. Naïve borrowers make financial mistakes: on average, they pay more for credit than they would if they knew their true exposure to financial shocks. Sophisticated consumers are less prone to financial shocks and incur fewer penalty fees. Being pooled with naïve agents, they benefit from the same set of contracts where low interest rates are cross-subsidized by high penalty fees. These high penalty fees are mainly borne by naïve consumers. Thus, sophisticated consumers face cheaper credit in the presence of naïve consumers than if they were by themselves.

We use this framework to analyze two important aspects of the 2009 CARD Act: (1) The CARD Act defines how late fees, interest rates, and minimum payments are to be reported and communicated to consumers. With stringent transparency requirements, standardized language, and clearer reporting standards contracts are easier to understand. More understandable contracts reduce the risk of financial mistakes. (2) The CARD Act limits how lenders can reset interest rates in response to missed payments and restricts the amount of penalty fees to be charged. Limiting penalty fees for borrowers reduces the

[4]Cf. Armstrong and Vickers (2012), Gabaix and Laibson (2006), and Heidhues and Kőszegi (2010).

[5]This approach has been established by Exler, Livshits, MacGee, and Tertilt (forthcoming) in the context of over-optimistic consumers.

cost of financial mistakes. We find that both pieces of legislation have a similar impact on credit contracts offered in equilibrium. Revenues from penalty fees shrink, either because (1) consumers make fewer mistakes and thus pay lower fees or because (2) lenders are prevented from offering excessive penalty fees. Consequently, equilibrium interest rates rise and cross-subsidization from naïves to sophisticates falls under both policies.

Naïve consumers benefit from both reforms. Their lack of understanding becomes less consequential either (1) because they make fewer mistakes per se due to simpler contracts or (2) because they are forced to choose lower penalty fees and thereby avoid financial mistakes. Naïves pay too much for their credit prior to the reform and both policies reduce their cost of credit. Because sophisticated agents lose cross-subsidization, they stand to lose from these reforms. While inconsequential to their understanding of credit contracts, sophisticates lose out on cross-subsidization as a consequence of (1) transparency requirements. With naïves committing fewer financial mistakes and paying lower penalty fees, sophisticates face higher interest rates. They consequently suffer from transparency requirements. Similarly, (2) limiting penalty fees also reduces sophisticate welfare. Penalty fee limits force naïve borrowers to choose lower fees and reduce cross-subsidization. Thus, sophisticates experience a welfare losses.

1.1 Related Literature

This paper contributes to three strands of literature. First, it relates to empirical studies evaluating the 2009 CARD act. Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015) use panel data to analyze the effects of the 2009 CARD Act and estimate that regulations lead to a decrease in overall borrowing costs. Nelson (2020) also considers the CARD Act and finds that lower markups and limited ability to raise borrowers' interest rates based on new information lead to a fall in average transacted prices. However, he also reports that prices rose in some parts of the market, thereby unveiling possible cross effects which might mitigate the legislator's intent. We add to this literature by proposing a structural model to evaluate two main components of the reform. Besides identifying the importance of financial mistakes, a structural quantitative model allows to identify key mechanisms, gauge effects through equilibrium pricing, and estimate welfare.

Second, our paper extends the concept of naïve agents in credit markets to setups with imperfect enforcement and equilibrium default. Furthermore, it quantitatively gauges the relevance of Naïveté in the credit market and for credit market regulation. Naïveté has long been the focus of theoretical contributions. Armstrong and Vickers (2012) show that in markets with sophisticated and naïve consumers a pooling equilibrium may exist and that competition can work to subsidize the sophisticated at the expense of the naïve. This mechanism is also present in the work by Heidhues and Kőszegi (2015), who study Naïveté-based discrimination and find that firms lend more than is socially optimal to increase unexpected payments of naïve consumers. Heidhues and Kőszegi (2010) develop a model of loan-repayment in competitive credit markets with consumers who value immediate gratification. They show that non-sophisticated consumers take on credit which is cheap in the short term, but would then go on to overborrow and pay large penalties, thereby suffering considerable welfare losses. In Eliaz and Spiegler (2006), agents have dynamically inconsistent preferences and the principal offers a menu of loan contracts in order to screen for sophistication. Gabaix and Laibson (2006) propose a framework in which firms offer cheap baseline contracts in order to hook naïves and earn profits from shrouded prices for additional payments.

Besides constituting an important theoretical tool to study credit markets and contract design, Naïveté is also empirically relevant. DellaVigna and Malmendier (2004) study how firms respond to partially naïve time-inconsistent consumers. They show that contracts are designed to targets consumers' misperception of future behavior. Agarwal, Chomsisengphet, Liu, and Souleles (2015) analyze an experiment from a large U.S bank that offered two contracts to customers. Borrowers could trade off fees and interest rates: One contract offered a lower interest rate but came with an annual fee. The other eliminated the annual fee but charged a higher interest rate. The authors find that about 40% of consumers chose sub-optimally and ended up with higher total cost of credit. Similarly, Campbell, Grant, and Thorp (2022) analyze data from a controlled field trial studying costly credit card delinquency and show that simply sending a reminder to overdue credit card debtors significantly raises repayment rates as well as the amounts repaid by high credit score delinquents. Those results suggest that financial charges often result from consumers making financial mistakes and not correctly incorporating all available information rather than purely from deliberate choices.[6]

Third, we expand the standard framework of unsecured credit and equilibrium default based on the seminal work by Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007).[7] First, we introduce penalty fees as a new dimension to credit contracts: borrowers can not only choose debt levels (and receive interest rate quotes as a function of their state and the amount borrowed), but they can also sign up for higher penalty fee payments to lower their interest rates. Secondly, we add a fraction of naïve agents to the economy that do not fully internalize their likelihood of paying these penalty fees. Consequently, they make mistakes when taking out loans. These novel features create a natural role for information requirements and penalty fee limits, which could not play a role in standard models of unsecured credit. In the context of the 2009 CARD act, We leverage our new framework to investigate these regulatory features that have previously been un(der)studied.

[6]There is also evidence that the interaction of more and less sophisticated consumers through pricing matters. Using transaction data of retail funds, Gao, Hu, Kelly, Peng, and Zhu (2020) document that naïve investors cross-subsidize sophisticated investors. This subsidization is especially pronounced when trading more complex structured funds.

[7]See Exler and Tertilt (2020) for a recent survey.

There have been other approaches to incorporate behavioral consumers into a model of unsecured credit and equilibrium default. Nakajima (2017) introduces hyperbolic discounters but does not allow for any interactions between consumer types. Exler, Livshits, MacGee, and Tertilt (forthcoming) model consumers that are over-optimistic about their future income and interact with rational consumers through the credit market. We follow the authors' assumptions on individual behavior to ensure that only pooling equilibria can exist. Chatterjee, Corbae, Dempsey, and Ríos-Rull (2023) develop a framework where agents hold heterogeneous discount factors. Both, Chatterjee, Corbae, Dempsey, and Ríos-Rull (2023) and Exler, Livshits, MacGee, and Tertilt (forthcoming) focus on how lenders learn about consumer types and do not have a role for information requirements or fee limits, as agents fully understand their credit contracts. Raveendranathan and Stefanidis (forthcoming) analyze the ability to pay check mandated by the 2009 CARD Act, but also have no role for assessing transparency regulations. We see our work as complementing their findings.

The remainder of this paper is structured as follows: Section 2 presents the framework, Section 3 describes the calibration, Section 4 explores the benchmark properties and mechanisms of the model before Section 5 investigates two important components of the 2009 CARD Act and further policies. Section 6 concludes.

2 The Model Framework

We propose a quantitative theory of unsecured debt and equilibrium default that includes naïve agents and offers a penalty fee vs. interest rate trade-off. We study an incomplete-market heterogeneous agent life-cycle model with idiosyncratic uncertainty about earnings, expenses and financial shocks. Interest rates are determined in equilibrium and reflect expected penalty fee revenues as well as write-offs from default.

2.1 Households

We set up a standard overlapping generations model that is populated by a continuum of households, with one model period representing one year. Individuals enter the model at age 21 and die with certainty at age 80. They derive utility from consumption in each period c_t, may experience a disutility from filing for bankruptcy χ, and maximize expected discounted lifetime utility,

$$\mathbb{E}_0 \left\{ \sum_{j=1}^{J} \beta^{j-1} \Big[u(c_j) - \delta_j \chi \Big] \right\},$$

where $\beta < 1$ is the time discount factor and δ_j is an indicator for filing for bankruptcy.

Each agent decides on consumption, whether to save or borrow, and can file for bankruptcy. The model abstracts from secured debt like mortgages and focuses on unsecured credit card debt. Borrowers can default in equilibrium by declaring Chapter 7 bankruptcy. Households face idiosyncratic income shocks, expense shocks, and financial shocks, while the risk-free interest rate is set exogenously.

Labor Productivity. Households are subject to idiosyncratic shocks to their labor productivity p, which represents the wage risk they face. Income y_j is given by $y_j = p_j \cdot e(j)$, where $e(j)$ is an age-dependent efficiency premium. Individual labor productivity is comprised of a persistent AR(1) process z_j and a transitory white noise shock η_j. Hence, for a household of age j, productivity evolves according to

$$\begin{aligned} \log(p_j) &= z_j + \eta_j \\ z_j &= \rho z_{j-1} + \zeta_j, \end{aligned} \tag{1}$$

where $\rho \in [0, 1]$, $\eta \sim \mathbb{N}(0, \sigma_\eta^2)$ and $\zeta \sim \mathbb{N}(0, \sigma_\zeta^2)$.

Expense Shocks. Households are subject to expense shocks. These shocks represent unforeseen expenses that arise from medical bills or family disruptions. Expense shocks are drawn from a discrete finite set $\kappa \in \Omega_\kappa = \{\kappa_0 = 0, \kappa_1, \ldots, \kappa_N\}$ and are modeled as independent and identically distributed with probabilities $p(\kappa_n)$. These expense shocks are unsecured claims and can be dispensed with through bankruptcy.

Financial Shocks. Furthermore, households that borrow on their credit cards are susceptible to financial shocks ε. Financial shocks represent households not meeting their minimal repayment requirements, paying their credit cards late, or over-borrowing on their accounts. If a household incurs such a shock, lenders will charge them a previously contracted upon additional fee $\phi \geq 1$, representing overdraft fees, late payment fees, etc. Thus, borrowers hit by a financial shock ε face additional charges of $\phi \cdot \varepsilon$. Here, ε can be interpreted as a lender's fundamental cost of borrowers mishandling their credit and ϕ is a potential additional fee. These charges are not secured and can be defaulted upon. We model financial shocks as discrete iid shocks, drawn from $\varepsilon \in \Omega_\varepsilon = \{\varepsilon_0 = 0, \varepsilon_1, \ldots, \varepsilon_M\}$ with probabilities $p(\varepsilon_m)$.

Bankruptcy. Households can choose to default and not repay their debts, which includes any expense or financial shocks they incurred. Bankruptcy is modeled according to U.S. Chapter 7. It provides a "Fresh Start:" upon default, all outstanding unsecured debts and claims are forgiven.

However, bankruptcy is costly and requires a good faith effort to repay outstanding debts. This is captured in the model by garnishing a fraction γ of each bankrupt's current income and using it to (partially) repay creditors. Furthermore, bankrupts incur a utility cost χ, which represents other costs of defaulting, such as stigma.

2.2 Naïveté

There are two types of households in the economy: sophisticated consumers (S) and naïve consumers (N). Sophisticated and naïve consumers are identical along all dimensions, except with respect to the financial shock. Both types of agents incur financial shocks, i.e. both types sometimes pay late or borrow over limit. However, naïve borrowers are more prone to these financial shocks, while being naïve about their higher exposure.

In the model, both types' financial shocks share a common support: $\varepsilon^S, \varepsilon^N \in \Omega_\varepsilon$. However, naïve borrowers face higher probabilities that a nonzero financial shock occurs:

$$p(\varepsilon^N = \varepsilon_m) > p(\varepsilon^S = \varepsilon_m) \quad \forall \quad m > 0. \tag{2}$$

The relative difference in probabilities faced by naïves and sophisticated is characterized by a probability spread, ψ, which is constant for all non-zero shocks:

$$\psi = \frac{p(\varepsilon^N = \varepsilon_1)}{p(\varepsilon^S = \varepsilon_1)} = \ldots = \frac{p(\varepsilon^N = \varepsilon_M)}{p(\varepsilon^S = \varepsilon_M)} \tag{3}$$

We define ψ as the extent of naïveté in the economy. If $\psi = 1$, there is no naïveté and all consumers face the identical shock process. If $\psi > 1$, naïve borrowers experience non-zero financial shocks more frequently..

Sophisticated borrowers know the distribution of their shock process and take the expected shocks into account when taking out debt. In contrast, naïve consumers do not understand their higher exposure but expect the same distribution that sophisticates face. Consequently, they underestimate their average financial shock:

$$\mathbb{E}^N(\varepsilon^N) = \mathbb{E}^S(\varepsilon^S) = \mathbb{E}(\varepsilon^S) < \mathbb{E}(\varepsilon^N), \tag{4}$$

where $\mathbb{E}^{N,S}(\cdot)$ denotes the type-specific expectation of consumers and $\mathbb{E}(\cdot)$ the expectation formed with the true underlying risks. In the case of a naïve agent, their expectation differs from a fully information rational expectation.

With identical expectations of financial shocks, naïve agents behave exactly like sophisticated consumers, conditional on all other states. Hence, lenders cannot distinguish between the two types nor separate them by offering distinct contracts. This assumption ensures a unique, tractable pooling equilibrium.[8] Furthermore, we abstract from learning for lenders and borrowers. Even if a consumer keeps experiencing financial shocks, none of the agents update their belief about the borrower's type but just deem this consumer

[8] This setup is similar in spirit to Exler, Livshits, MacGee, and Tertilt (forthcoming), where over-optimistic borrowers face worse income risk but are not aware of it.

very unlucky.[9]

To summarize, naïve borrowers are more exposed to financial shocks, biased in their expectation about future financial shocks, and pooled with sophisticated borrowers. In our benchmark and when conducting policy experiments, we decompose the effects of Naïveté into the effects of higher financial shock risk and the effects of expectation bias. When presenting our policy experiments, we also compare the results from our benchmark pooling equilibrium to the outcomes in a market where lenders perfectly observe borrower types and naïve and sophisticated borrowers are separated.

2.3 Credit Market

Loan Contracts. Households can borrow in one period loans and they have limited commitment to repay. A loan contract consists of the face value of the debt, $d' \in (0, \infty)$, the loan prize, $q \in [0, 1]$ which translates into an interest rate $1/q - 1$, and penalty fees for financial shocks, $\phi \in [1, \infty)$. Borrowers can choose contracts with low penalty fees (e.g. $\phi = 1$) which reduce their exposure to a potential financial shock. Alternatively, they accept higher penalty fees ($\phi > 1$) in exchange for lower interest rates or higher debts.

When an individual takes out a loan, they choose from a menu of interest rates $(1/q - 1)$ and associated penalty fees, ϕ, for each potential debt level d'. There is no asymmetric information and lenders perfectly observe the current state of each borrower and the requested loan size. Whether a given borrower is naïve or sophisticated is unknown to all agents. However, lenders know that debt levels contain information about the fraction of naïves $\lambda(d')$ seeking that loan.

Lenders. Lenders operate in a perfectly competitive market with free entry. Conditional on observables, they earn zero profits in equilibrium. In other words, loan prices q perfectly reflect expected revenue for each borrower state, amount of debt d', and penalty fee ϕ. If a borrower repays their debts, revenue consists of two components: the repaid face value d' of the loan plus the expected revenue from penalty fee payments ϕ. If a borrower is hit by a financial shock ε and repays their debt, the lender earns additional revenue $\phi \cdot \varepsilon$. From that revenue, lenders pay the fundamental cost ε to meet the additional liquidity needs of a borrower that repays late or that over-borrows. Additionally, we assume a proportional transaction cost $(1 - \iota)$ for managing borrowers that do not abide by the loan terms. Consequently, the lender earns a net revenue of

$$\iota \cdot (\phi - 1) \cdot \varepsilon \tag{5}$$

[9]However, lenders can observe the debt level which contains information on the history of shocks. In Section 2.4 we discuss how lenders form expectations on the share of naïves for different debt levels in equilibrium.

from borrowers that are subject to the financial shock and repay.[10]

However, not all households repay their debts. If borrowers declare bankruptcy, a fraction γ of the household's current income y is garnished to repay parts of the outstanding debt. The recovery rate of defaulted loans can thus be written as

$$\rho(d', s) = \gamma y(s) \cdot \frac{d'}{d' + \varepsilon + \kappa}, \tag{6}$$

where $s = (j, z, \eta, \varepsilon, \kappa)$ summarizes the household's (exogenous) state.

Let $\theta(d, \phi, s)$ denote a household's decision to default depending on the contracted repayment d and fee ϕ. Then, a bank's expected profit is given by:

$$\begin{aligned} \Pi(d', \phi, s) = -q(d', \phi, s)d' + \frac{1}{1 + r + \tau} \int \theta(d', \phi, s')\rho(d', s') \\ + \Big(1 - \theta(d', \phi, s')\Big)\Big(d' + \iota(\phi - 1)\varepsilon'\Big)\, \mathrm{d}\mu(s') \end{aligned} \tag{7}$$

where $\mu(s')$ is the probability measure over next period's exogenous states s', r denotes the exogenous refinance interest rate, and τ is a proportional transaction cost of loan creation.

Perfect competition and free entry lead to zero profits conditional on observables d' and s. Consequently,

$$\begin{aligned} q(d', \phi, s) = \frac{1}{1 + r + \tau} \int \theta(d', \phi, s') \frac{\rho(d', s')}{d'} \\ + \Big(1 - \theta(d', \phi, s')\Big)\left(1 + \frac{\iota(\phi - 1)\varepsilon'}{d'}\right) \mathrm{d}\mu(s'). \end{aligned} \tag{8}$$

Equation 8 is an implicit functional relationship $q = q(\phi, \cdot)$ between $q(\cdot)$ and ϕ for any debt level d' and exogenous household state s and has to hold in equilibrium. See Figure A.1 for an illustration of the relationship between $q, \phi,$ and d'.

Equation 8 can be thought of as a weighted average between what lenders recover in the case of bankruptcy $\rho(\cdot)$, and full repayment plus potential fees in the case of no default, weighted with the appropriate default risk $\theta(\cdot)$. If agents choose $\phi = 1$, they minimize their exposure to financial shocks and pay no additional fee. This comes at the cost of higher interest rates. A choice of $\phi > 1$ implies positive penalty fees that lenders charge in the case of financial shocks but might lead to lower interest rates. Lastly, note that there exists an endogenous limit on penalty fees, since too high fees always cause default: $\exists\ \bar{\phi} \in \mathbb{R}^+ \colon \forall\ \phi > \bar{\phi} : \mathbb{E}[\theta(\cdot)] = 1$. Thus, $q(\phi > \bar{\phi}, \cdot) = 0$.

[10] One can also think of ι more broadly capturing the fact that additional profits derived from fee payments are not fully passed on as reductions in interest rates. However, in a perfectly competitive framework, we prefer the transaction cost interpretation.

Total Cost of Credit Bias and Financial Mistakes. If a borrower seeks out a loan with debt level d', loan prize q and punishment fees ϕ, the total cost of the credit, ToC, are all the payments resulting from interest and fees, that is

$$ToC = d' - qd' + (\phi - 1)\varepsilon \tag{9}$$

where $d' - qd'$ are interest payments and $(\phi - 1)\varepsilon$ are fee payments. Note that $\phi = 1$ means that no extra fees have to be paid. Hence, the cost of credit only exceeds interest payments if $\phi > 1$.

Due to underestimating their proneness to financial shocks, naïve borrowers also underestimate their fee payments, $\mathbb{E}^N(\phi \cdot \varepsilon^N) < \mathbb{E}(\phi \cdot \varepsilon^N)$. Consequently, for any given contract (d', q, ϕ) naïves misjudge the expected total cost associated with this contract. We define the *ToC Bias* as the percentage difference between the true expected total cost of credit and the naïve's biased expectation, expressed as:

$$ToC\ Bias = \frac{\mathbb{E}(ToC) - \mathbb{E}^N(ToC)}{\mathbb{E}^N(ToC)}, \tag{10}$$

where $\mathbb{E}^N(ToC)$ represents the naïve's biased expectation formed under the misperception of their true susceptibility to shocks, and $\mathbb{E}(ToC)$ represents the expectation formed using the true underlying process.

Because of this bias, naïve borrowers might fail to choose an optimal debt contract and opt for contracts with lower interest rates but higher penalty fees. As a result, their true expected total cost of credit will exceed the costs associated with an optimal contract that a social planner would choose for them. Let ToC^* denote the total cost of a credit contract that yields the same debt level d' as chosen by the naïve consumer but with a combination of loan prize and fees (q^*, ϕ^*) that are optimal given the true underlying risks. The extent to which the actual payment of the naïve borrower exceeds the cost of this hypothetical optimal contract is called a *financial mistake*. Hence, in this framework a financial mistake is defined as

$$Mistake = \frac{\mathbb{E}(ToC) - \mathbb{E}(ToC^*)}{\mathbb{E}(ToC^*)}, \tag{11}$$

where the expectation $\mathbb{E}(\cdot)$ is formed using the true risks.

We illustrate these two measures in Figure 3 below when discussing the model mechanisms in the benchmark economy.

2.4 Distribution of naïve consumers

Let λ denote the economy wide average of naïve consumers. When creating the bundle of contracts as defined in Equation 8, the lender takes as given the debt level d' without knowing whether the borrower is naïve or sophisticated. However, because naïve agents are more prone to financial shocks, the lender might find it more likely that they are dealing with a naïve borrower at very high debt levels. At lower debt levels, in contrast, the lender might find it more likely to deal with a sophisticated borrower. Hence, the lender may assign a different probability $\hat{\lambda}(d') \neq \lambda$ to the likelihood of dealing with a naïve agent for each loan size.

When calculating expected repayment and default probabilities for the offered loan prize schedule in Equation 8, lenders form beliefs about the share of naïves over debt levels, $\hat{\lambda}_t(d')$. As long as their belief $\hat{\lambda}_t(\cdot)$ differs from the ex-post realized distribution $\hat{\lambda}_{t+1}(\cdot)$ resulting from the offered loan contracts, the credit market is not in equilibrium. In this case, lenders observe a deviation from expected and realized profits and update their belief accordingly. In equilibrium, lenders' beliefs and the realized distribution coincide and we have $\hat{\lambda}_{t+1}(\cdot) = \hat{\lambda}_t(\cdot)$. Lenders use all available information entailed in the borrowers' chosen debt levels d'.[11]

2.5 Dynamic Programming Problem

A household's decision variables are consumption c, debt level d (where $d < 0$ are positive asset holdings) and a financial contract when taking on a loan. A financial contract is a combination of loan prizes q and fees for financial shocks ϕ. Lenders will offer different loan prizes for different fees and the borrowers choose a contract by deciding on a fee level. Note, however, that this fee choice will only be important in the next period, when the borrower might be faced with a financial shock ε and then has to pay $\phi \cdot \varepsilon$. Also, since credit mishandling is only costly for those in debt, ε does not impact savers.

Let $V(d, \phi; s)$ denote the value function of a consumer in state $s = (j, z, \eta, \varepsilon, \kappa)$, who is holding debts d and chose a fee ϕ last period. If a consumer declares bankruptcy, all debt charges are dismissed ($d = 0$ and $\varepsilon = 0$), but a fraction γ of income is garnished and the household experiences a disutility χ. Also, no saving is allowed for next period, the household simply consumes all income that is not garnished.

With $q(d', \phi'; s)$ being the loan prize for a loan of size d' with fees ϕ' for an individual in state s, the consumer's decision problem reads

$$\begin{aligned} V(d, \phi; s) &= \max_{c,d',\phi'} \left[u(c) + \beta \mathbb{E}^S \max \left\{ V(d', \phi'; s'), B(s') \right\} \right] \\ &\text{s.t. } c + d + \kappa + \phi \cdot \varepsilon \cdot \mathbb{1}_{d>0} \leqslant y(s) + q(d', \phi'; s) d' \end{aligned} \tag{12}$$

where $\mathbb{E}^S(\cdot)$ denotes the fact that all consumers think of themselves as sophisticated when forming expectations about future shocks and B is the value of filing for bankruptcy:

[11] Figure A.2 shows an example of an assumed distribution over asset holdings.

$$B(s) = u(c) - \chi + \beta \mathbb{E}^S \Big\{ V(0, \phi'; s') \Big\}$$
$$\text{s.t. } c = (1-\gamma) y(s) \tag{13}$$

Thus, even though we have two types of agents in the economy, the decision problem is the same for both sophisticated and naïve consumers since the latter do not realize their higher proneness to financial shocks. Consequently, the decisions of sophisticates and naïves will be the same based on their observable state.

An equilibrium is a set of value functions and optimal decision rules for consumption $c(\cdot)$, debt level $d'(\cdot)$, fee choice $\phi'(\cdot)$, and default decisions $\theta(\cdot)$ as well as bond prices $q(\cdot)$ and a stationary belief of the share of naïves for each debt level $\hat{\lambda}(d)$ that solve the consumer's decision problem (Equations 12 and 13) and the competitive loan prize schedule (Equation 8).

3 Calibration

This section presents the parameter values used to numerically solve the benchmark economy. Some parameters are set externally, while others are estimated using a Simulated Method of Moments approach to match important credit market data. We use data of the consumer credit card market from the Consumer Financial Protection Bureau (CFPB, 2014) and credit card account data from the Office of the Comptroller of the Currency in the US (OCC) as reported in Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015).

3.1 Externally Determined Parameters

Households live for 60 periods and one period represents one year. Individuals enter the model at age 21 and die with certainty at age 80. There is no formal retirement, but households' age-dependent experience premium drops after the age of 65. Per-period utility takes the form of a CRRA utility function

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where the risk-aversion parameter, σ, is set to 2, which is standard in the literature. For the age-dependent efficiency premium, $e(j)$, we use the life-cycle component from Livshits, MacGee, and Tertilt (2007), using linear interpolation to create annual values from the original three-year periods. The risk-free rate is set exogenously to 1%.

Persistent labor productivity follows a simple AR(1) process where the autocorrelation parameter is set to $\rho = 0.99$ and the variance of the iid shock chosen to be $\sigma^2_\zeta = 0.007$. The variance of the transitory shock is $\sigma^2_\eta = 0.043$. All these values are within the range

Table 1: Externally Determined Parameters

Economic Parameters		Value
CRRA consumption	σ	2
Persistent Wage Autocorrelation	ϱ	0.99
Persistent Wage Var	σ_ζ^2	0.007
Transitory Wage Var	σ_η^2	0.043
Risk Free Rate	r	1%
Expense Shocks		
Size of expense shocks	κ	$(0, 0.267, 0.8218)$
Probabilities	$[\pi_0, \pi_1, \pi_2]$	$(0.9748, 0.0237, 0.0015)$
Financial Shocks		
Size of mistakes	ε	$(0, 0.0007, 0.0019, 0.0090)$
Unconditional probabilities	$Pr(\varepsilon)$	$(0.698, 0.125, 0.094, 0.083)$
Share of naïves	λ	0.3
Naïve probability spread	ψ	6

of standard literature. In particular, these are the same values employed by Livshits, MacGee, and Tertilt (2007).

For the parametrization of expense shocks to U.S data we follow the estimates of medical expenses, divorces, and unplanned parenthood from Livshits, MacGee, and Tertilt (2007) and Exler, Livshits, MacGee, and Tertilt (forthcoming). Expense shocks can take on three values: $\kappa \in \{0, \kappa_1, \kappa_2\}$. The smaller shock corresponds to 26.4% and the larger one to 82.18% of average yearly income. Annualized probabilities $[\pi_1, \pi_2]$ of these shocks realizing are 2.37% and 0.15%, respectively.

Financial Shocks. Financial shocks are modeled to take on four different values: $\varepsilon \in \{0, \varepsilon_1, \varepsilon_2, \varepsilon_3\}$. These values are meant to represent no occurrence of a financial shock within a year, only one such occurrence, up to three occurrences, and more repeated occurrences. We use data on overdraft from the CFPB (2014) to estimate that a fraction of 12.5% of borrowers experiences only one item per year, while 9.4% of borrowers experience up to three items and 8.3% experience repeated items. Hence, we set $p(\varepsilon_1) = 0.125$, $p(\varepsilon_2) = 0.094$ and $p(\varepsilon_3) = 0.083$. These values represent the economy wide average and we construct different processes for naïve and sophisticated consumers below.

To pin down the size of these shocks, we express the average payments reported for one, up to three, and more occurrences in CFPB (2014) as a fraction of median income in the US. This yields shock sizes of $\varepsilon_1 = 0.0007$, $\varepsilon_2 = 0.0019$ and $\varepsilon_3 = 0.0090$ in terms of median income.

Table 2: Jointly-Targeted Moments

	Data (OCC)	Model
Avg. Interest / Debt (AID)	14.30%	12.97%
Avg. Fees / Debt (AFD)	6.70%	6.62%
Fraction Borrowers	25%	28.8%
Bankruptcy Rate	0.45 %	0.45%
Avg. Debt-to-Income Ratio	6%	5.2%
	Parameter	Value
Discount factor	β	0.9314
Transaction cost	τ	0.1344
Fee efficiency	ι	0.4025
Cost of default	χ	0.0193
Garnishment rate	γ	0.5479

Naïveté. First, we need to proxy the fraction of naïves in the economy. Obviously, Naïveté is not directly observable, so we proxy it by those consumers that pay exorbitantly more penalty fees than the rest of the distribution. This is consistent within the model, as Naïveté implies both, a higher risk of financial shocks and a bias about this risk. In the OCC data, borrowers with FICO scores of 660 and less pay significantly more fees than borrowers above this threshold, see Figure A.3. Those borrowers comprise a share of 30% of the population. Consequently, we set the share of naïves, λ, equal to 30%.

Sub FICO 660 borrowers pay roughly six times more fees in the OCC data compared to higher FICO score borrowers. This lets us directly pin down the extent of naïveté, the probability spread between naïves and sophisticates (see Section 2.2). We set $\psi = 6$. The specific probability processes for naïves and sophisticated are then constructed in such a way that their weighted mean (with the share of naïves, λ, used as weight) results in the economy wide financial shock probabilities determined above. This yields the following probabilities: $p(\varepsilon^S) = (0.879, 0.05, 0.038, 0.033)$ and $p(\varepsilon^N) = (0.275, 0.3, 0.226, 0.199)$.

Externally determined parameters are summarized in Table 1.

3.2 Internally Determined Parameters

We are left with five parameters to determine internally: discount factor β, transaction cost of loan creation τ, efficiency of fee collection ι, the garnishment rate γ, and utility cost of default χ. We calibrate those five parameters to five data targets: Average interest payments to debt (AID), average fee payments to debt (AFD), fraction of borrowers, bankruptcy rate, and the average debt-to-income ratio. More technically, we choose these five parameters to minimize the sum of squared relative residuals between model moments

Table 3: Benchmark Outcomes (in %)

	Sophisticated	Naïve	Average
Avg. Interest / Debt (AID)	12.96	12.99	12.97
Average Fees / Debt (AFD)	2.78	15.59	6.62
Fraction Borrowers	28.25	29.96	28.76
Bankruptcy Rate	0.33	0.71	0.45
Debt-to-income ratio	4.8	6.2	5.2
Cross-Subsidization	0.28%	−0.33%	
ToC Bias		64.6%	
Financial Mistake		64.5%	

and data targets.

While all parameters affect all moments jointly, we discuss parameters together with the moment(s) most affected by them. A discount factor $\beta = 0.93$ delivers a debt-to-income ratio of 5.2% vs. 6% in the data and an AID of 14.3% vs. 13.0% in the data. To further achieve this interest rate, a transaction cost for borrowing, $\tau = 0.13$ is chosen. This also allows us to match the share of borrowers: 29% in the model to 25% in the data.

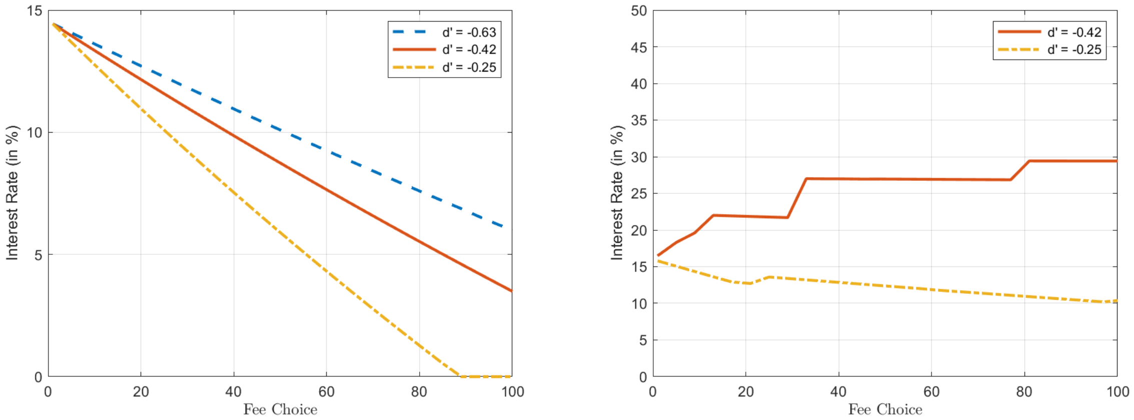
A household who decides to file for bankruptcy experiences stigma costs in the form of a disutility of $\chi = 0.02$ in the period of default and a fraction $\gamma = 0.55$ of current income is garnished. These parameters deliver a fraction of 0.045% bankruptcies per year, exactly as in the data.

Finally, the fee payment efficiency ι, is chosen to match an average ratio of fee payments to debt of 6.6% versus the observed 6.7% in the OCC data.

Overall, the model fits the data very well. Especially achieving very high AFD numbers has proven challenging and lends credibility to the current approach. See Table 2 for a list of all internally determined parameters and the model and data moments.

4 Results

With the previously defined parameters, we solve the model quantitatively by backward iteration on the value function. After briefly discussing the benchmark outcomes in Section 4.1, Section 4.2 illustrates the effects of naïveté on the design of credit contracts and life cycle outcomes. Section 4.3 discusses cross-subsidization and its welfare effects and Section 4.4 highlights the importance of default for the mechanisms.



(a) High income

(b) Low income

Figure 1: Example of Equilibrium Credit Contracts (q,ϕ) for high ($\approx$ 3.2) and low income ($\approx$ 0.53), aged 50. Median income normalized to 1.

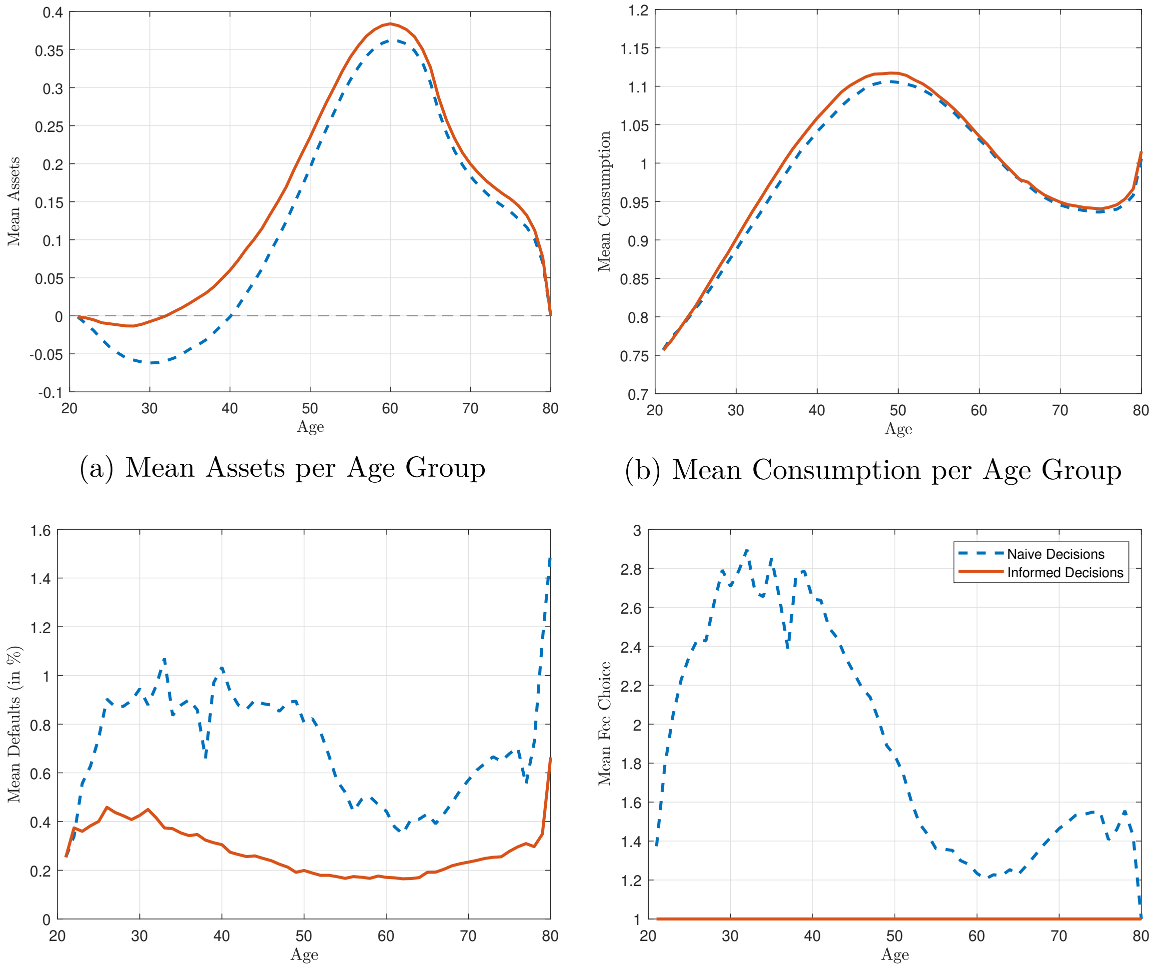
4.1 Benchmark Outcomes

Table 3 presents benchmark outcomes in our calibrated economy. The differences between sophisticated and naïve consumers are informative: Naïveté leads to higher debt (along the intensive and extensive margin) and higher default rates. Still, naïve borrowers pay virtually the same average interest rates. This is explained by substantially higher average penalty fees, with the average fee to debt ratio being nearly 16% compared to the 3% of sophisticated borrowers.

4.2 Illustration of Mechanism

Figure 1 shows an example of possible $(q(\phi, \cdot), \phi)$ schedules as defined in Equation 8 for different loan sizes d' with median income being normalized to 1. For the high income earner in Figure 1a we can see how a choice of higher fees can reduce the real interest rate. However, the $(q(\phi, \cdot), \phi)$ schedule does not have to be monotonic as can be seen in Figure 1b. There, the lender faces a low income earner who might default on future debts. Hence, at some point higher fees lead to an increase in default risk. Thus, the lender expects lower profits and consequently increases the interest rates. Note, however, that all the $(q(\phi, \cdot), \phi)$ schedules to the right of the first jump are strictly inferior to all the points to left. They combine higher fees with higher interest rates and would therefore never be chosen by the consumer. This finding ensures that – conditional on a loan size d' and consumer characteristics – the optimal contract is uniquely defined.

These $(q(\phi, \cdot), \phi)$ schedules become important when analyzing a market in which sophisticated and naïve borrowers are pooled together. Since lenders issue one contract for both types, balancing revenue from interest payments and penalty fees, the $(q(\phi, \cdot), \phi)$ schedules become steeper. This means that interest rates are falling more rapidly with



(a) Mean Assets per Age Group

(b) Mean Consumption per Age Group

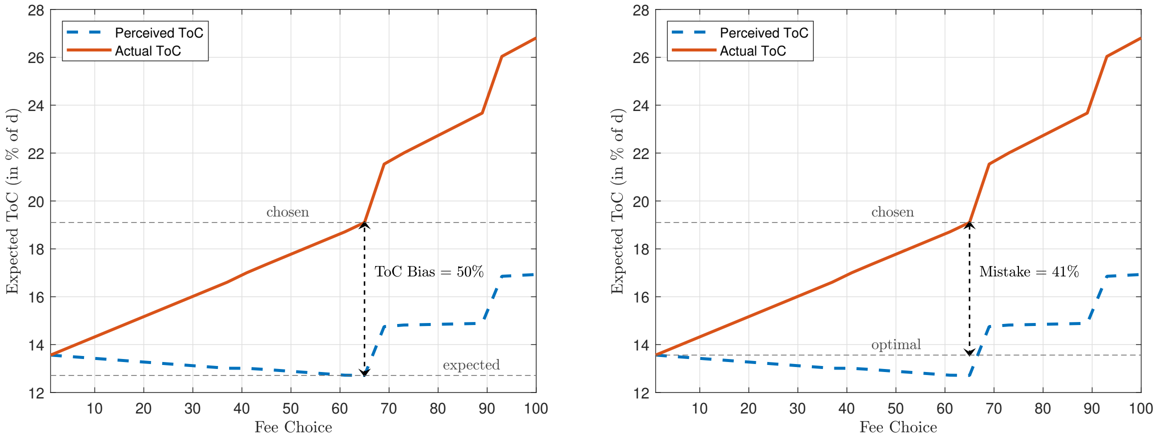
(c) Mean Default per Age Group

(d) Mean Fees per Age Group

Figure 2: Optimal (solid red) and actual decisions (dotted blue) of naïve consumers. Expected cost of Naïveté: 0.83% CEV.

the choice of higher fees, since the foregone interest payments are financed by the higher fee payments resulting from naïveté. Later sections will further elaborate on this cross-subsidization effect and show that sophisticates are indeed choosing different contracts in a pooling equilibrium when compared to a separating equilibrium.

To gauge the importance of misunderstanding the risk of financial shocks and fee payments in this framework, Figure 2 shows mean life-time decisions concerning assets, consumption, fees ϕ and bankruptcy for naïve consumers per age group. While the dotted blue line shows actual decision taken by the naïve consumer, the solid red line shows how the optimal behavior would look like if the consumer was aware of the actual probabilities of experiencing financial shocks but faced the same prices. We call this hypothetical agent *informed naïve*. One can see how the naïve consumers choose significantly higher fees than the informed self, resulting in over-accumulation of debts. These two facts lead to higher default rates and lower consumption on average. Overall, the expected welfare of a naïve



(a) ToC Bias (b) Contract Mistake

Figure 3: Illustration of ToC Bias and Contract Mistake of a naïve consumer at age 45 with high income seeking out a loan of two times the median income.

consumer is 0.83% lower in terms of CEV than that of an informed self facing the same prices.

Financial Mistakes. We measure financial mistakes as the excess cost of credit that results from naïve borrowers not internalizing their true financial shock risk. In other words, the financial mistake is the difference between the total cost of credit associated with the contract that the naïve consumer chose and the optimal contract that an informed consumer would choose for the same debt level d. (see the discussion in Section 2.3 and Equation 11). This is illustrated in Figure 3. The graph shows the true expectation of the total cost of credit (solid red line) and the naïve's expectation (dashed blue line) for different contracts – varying by interest rates and fee choices – for a given loan size d. We can see that given their biased expectation of the resulting costs, the naïve consumer chooses a contract with very high fees (thinking that this choice minimizes their costs). However, as part 3a shows, the actual cost of credit exceeds the naïve's expectation by 50%. This is what we call the *ToC Bias*.

Due to this bias, the choice of the naïve consumer might be different in a situation where they know about their true risks. Figure 3b compares the naïve decision with the decision of a hypothetical informed naïve agent (who forms the correct expectation without affecting prizes). We can see that for the given debt level the optimal contract would entail minimal fees ($\phi = 1$). The difference between the associated costs of this hypothetical optimal contract and the actual chosen one amounts to 41%. This is the percentage that the naïve agent overpays for the given loan size due to underestimating their true risks. We call this a *financial mistake*.

4.3 Cross-subsidization and Welfare

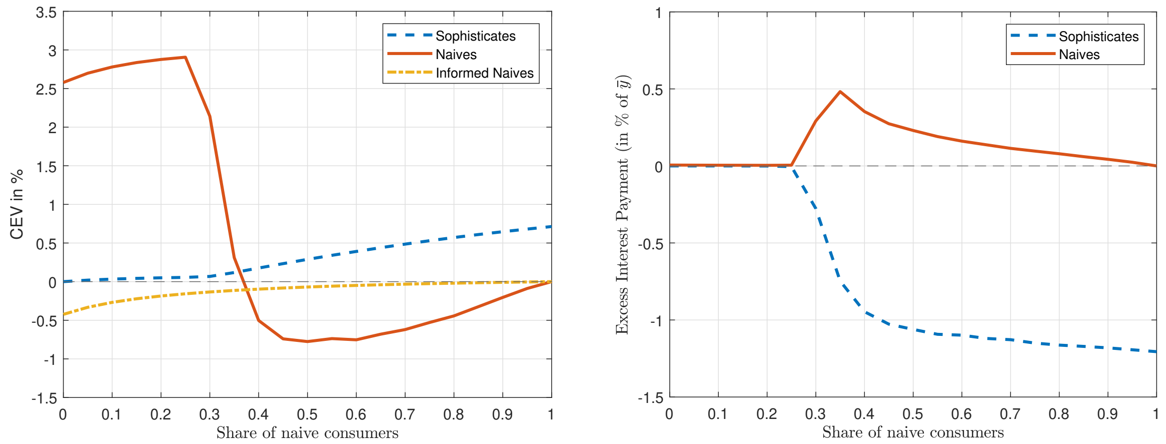
Since lenders cannot separate naïve from sophisticated borrowers, they only issue one set of credit contract for both types. Lenders only offer contracts – i.e. interest rate and penalty fee combinations – that yield the same expected revenue. However, dealing with sophisticates only would lead to contracts with higher interest rates for each level of fees, since sophisticated consumer are less prone to mistakes. Furthermore, in equilibrium, sophisticates understand their exposure to financial shocks and want to insure against this risk, hence they will pick contracts with lower fees.

In contrast, dealing with naïve borrowers would lead to lower interest rates for each level of fees, since they are more likely to experience bad financial shocks and incur these fees. Furthermore, naïves underestimate their true proneness to these shocks and thus, in equilibrium they choose contracts with higher fees and lower interest rates than they should were they aware of their risk.

In an economy where sophisticates and naïves are pooled, lenders have to price credit in such a way as to balance those two mechanisms according to the share of naïves in the market. The welfare effects of both types interacting through pooled credit prices are represented in Figure 4a, which plots welfare of sophisticated and naïve agents depending on the share of naïve consumers in the economy. Welfare changes are measured in consumption equivalence variation in percent and computed relative to the separating equilibrium in which banks can offer different contracts to naïve and sophisticated borrowers. In other words, the point of reference for sophisticated (naïve) borrower is on the left (right). The third line in Figure 4a depicts welfare for an atomistic *informed naïve* agent, i.e. an agent that faces the same prices and the same risk of financial shocks as every naïve but understands their true exposure to these shocks and consequently picks optimal fee levels.

To gauge the importance of cross-subsidization for the observed welfare patterns, Figure 4b plots mean excess interest payments of both types of agents. Excess interest payments are defined as the difference in expected interest ($r(\cdot)d'$) paid by an agent in the pooling equilibrium relative to a separating equilibrium.

Sophisticate welfare level increases as the share of naïve consumers increases. Sophisticates profit from the presence of naïve agents in the economy. They profit from cross-subsidization, since pooled interest rates will fall more rapidly as fees increase because more naïves are around. Figure 4b shows that this effect manifests in decreasing average interest rates (or, equivalently, increasingly negative excess interest payments) compared to the absence of naïves. Average interest paid falls as the share of naïves in the pooled market rises. This happens because lenders shift larger parts of their revenue to fees for the financial mistakes made by naïves, thereby increasing fees and lowering interest rates. Since the decisions made by sophisticates are optimal regarding their risk of financial shocks, they benefit relatively more from lower interest rates than being hurt



(a) Welfare Change

(b) Excess interest paid in pooling economy

Figure 4: The amount of cross-subsidization in dependence of the share of naïve consumers

by higher fees.

In contrast, welfare of naïves is highest in an economy where there are mostly sophisticates and falls sharply as the share of naïves rises, even becoming negative before rising back up again. This might seem counter-intuitive because excess interest payments drop as the fraction of naïves increase and there are fewer sophisticates to enjoy cross-subsidization (cf. Figure 4b).

However, there is a strong opposing effect reducing naïve welfare. Banks shift their focus from interest payments to punishment fees as the probability of dealing with a naïve agent increases. Abstracting from Naïveté and suboptimal fee choices, we first focus on an informed naïve. Her welfare is highest in an economy with only naïve consumers because she can self-insure against her higher risk of mistakes by not choosing any fees while also enjoying contracts with lower interest rates (similar to sophisticates). As the share of naïves in the economy decreases, the set of equilibrium contracts shifts towards higher interest rates. Thus, contracts for the informed naïve get worse and welfare drops.

The naïves, in contrast, do not understand their true proneness to financial shocks and fail to insure themselves by choosing lower fees. Thus, they choose inferior contracts and their welfare drops as lenders focus more on fee payments. However, at higher shares of naïves in the economy, the cross-subsidization effect dominates for high fee contracts, leading to a small increase of naïve welfare above a share of 60%.

The total welfare effect of almost 3% CEV for naïves that live in an economy with mostly sophisticates can be attributed to a better set of contracts being available and forced insurance since there are only contracts with lower penalty fees on offer. Naïves are forced to insure against their risk, making them ex-post better off, although they themselves would prefer lower interest rates. However, as the share of naïves becomes less than 20%, welfare slightly drops again. This is due to the same mechanism driving the welfare of an informed naïve. With the share of naïves being so low, the contracts only focus on interest rates and contracts start becoming worse relative to the optimal point. This change also leads to borrowers shying away from fees altogether as can be seen in Figure 4b, where excess interest payments drop to zero for lower shares of naïves.

Overall, this discussion clearly shows that if the fraction of naïve consumers is high enough, sophisticated borrowers are being cross-subsidized by the mistakes made by naïve individuals. Naïves suffer from insufficient insurance against their higher risk of mistakes.

4.4 Default Matters

When analyzing credit contracts and the interaction between sophisticated and naïve agents, default is important.[12] Default arises because agents have less than perfect commitment to repay. Agents only repay their debts if it is *ex-post* rational to do so. This limits the amount of late fees a lender can charge and consequently limits cross-subsidization. This has important welfare consequences.

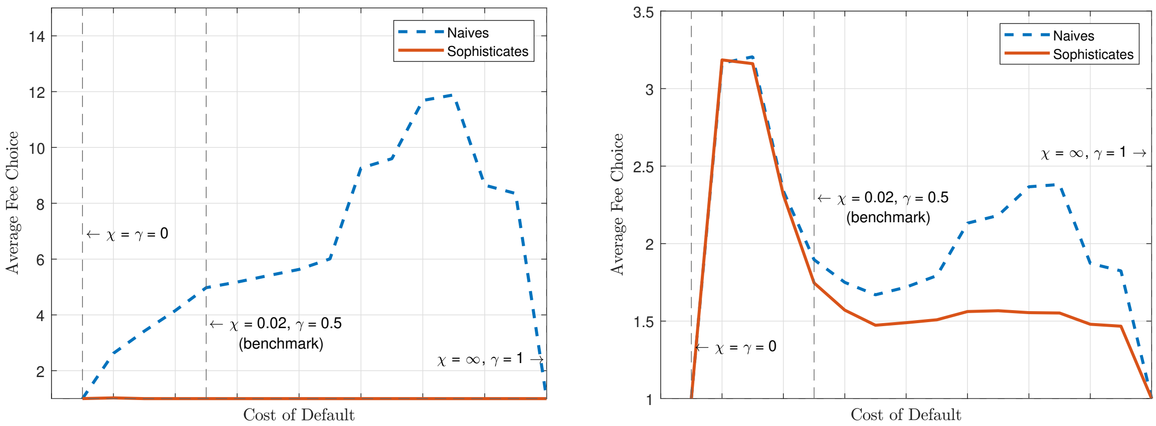
With lower default costs households face less commitment to repay their debts which is taken into account by creditors when pricing the loans. If filing for bankruptcy is completely free, only very low levels of debt can be sustained in equilibrium and lenders will charge high interest rates. In contrast, if bankruptcy is costly enough to ensure nobody ever wants to default, households will take on loans only if they are sure they will be able to repay. This leads to an equilibrium where default is theoretically possible, yet never occurs. Hence, borrowing interest rates are risk-free.

Sophisticates can only profit from lower interest rates due to the presence of naïves if a positive amount of debt can be sustained in equilibrium. If default is costless, lenders are only willing to provide loans at prohibitively high costs and nobody can take out a loan. Consequently, costless default leads to a situation without debt and without cross-subsidization. Similarly, if default costs are very high households will try to minimize default risks and thus choose debt contracts without fees ($\phi = 1$). Again, this leads to a situation without cross-subsidization since sophisticates can only profit from the presence of naïves if the lender expects additional revenue from fee payments.

Figures 5 and 6 show the effects of different levels of default on average fee choice and excess interest payments. To that end, we interpolate between zero default cost and prohibitively high default cost. Default costs have two components: the fraction of garnished income γ and the utility cost χ. Costless default corresponds to $\gamma = \chi = 0$, while in the benchmark we set $\gamma = 0.5$ and $\chi = 0.02$ which leads to an average default rate of 0.45%. Prohibitively high default costs are $\gamma = 1$ and $\chi = \infty$.

Figure 5a shows the effects of higher defaults costs on the average fee choice in an economy without pooling where there is no interaction between sophisticated and naïve households. We can see that in such a separating environment choices of naïve and sophis-

[12] We extend Heidhues and Kőszegi (2010) to incorporate default and better gauge the magnitude of cross-subsidization and its welfare effects.



(a) No Pooling (b) Pooling

Figure 5: Effects of default costs on average fee choice.

ticated borrowers differ drastically. While sophisticates never choose any fees, the average fee choice of naïves rises with the cost of default. Because of limited commitment to repay debts, lenders' expectations of profits from naïve decisions rises as default becomes more costly and thus less likely. Hence, they offer contracts with low interest rates and high fees. Since naïves underestimate their proneness to incurring these financial costs, they choose high fees to benefit from the low interest rates. As the costs of filing for bankruptcy increase further, however, the average fee choice falls again. Now, borrowers never want to file for bankruptcy and thus, even naïve consumers start to insure themselves against possible default risks. The choices of sophisticates, in contrast, are unaffected by the cost of default. Since there is no cross-subsidization without pooling, sophisticates don not profit from choosing higher fees and remain at $\phi = 1$.

Figure 5b again shows the effects of changing default costs, but here sophisticates and naïves are pooled and face the same contracts. One striking difference to Figure 5b is that now sophisticates always choose contracts with higher fees than in the separating economy, exposing themselves to higher costs associated with financial shocks. Due to pooling, however, sophisticates can profit from these contracts. At the same time, naïve borrowers choose lower fees (a peak of ca. 3 when pooled vs. ca. 12 when separated) when in the presence of sophisticates. This comes from the fact that – through cross subsidization – naïve interest rates drop slower in increasing fees. Pooled naïves thus find high fees less attractive and choose lower average levels. Also, note how higher default costs again lead to a decrease in average fee choice. If default becomes so costly that nobody wants to file for bankruptcy anymore the average chosen fee in the economy even drops to 1, meaning that consumers want to insure against financial shocks. Agents ensure that unexpected financial costs cannot lead to a situation in which they cannot repay and have to default.

The effect of default costs on cross-subsidization is depicted in Figure 6. It plots the costs of default against the excess interest payments compared to an economy without

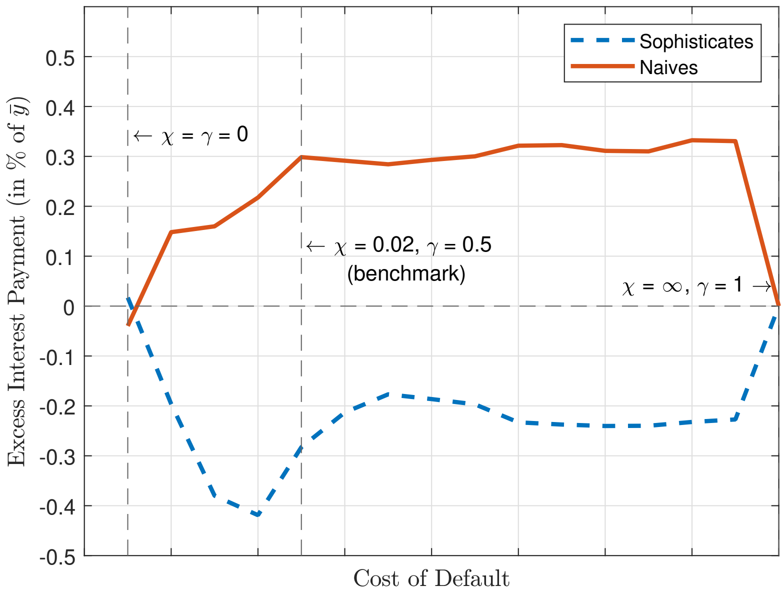


Figure 6: Excess Interest Payments and Default Cost

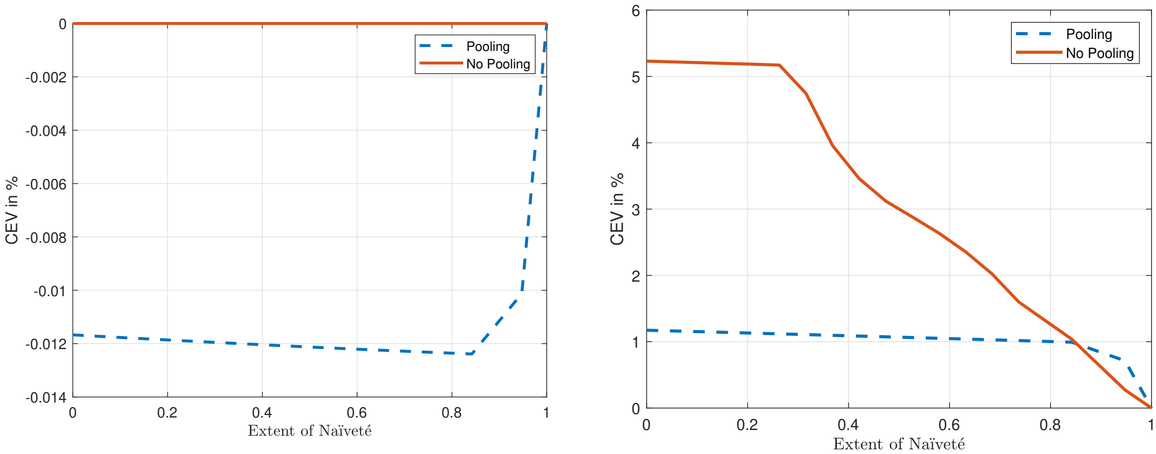
pooling, taking as given the penalty fee choices discussed before. We can see that as long as agents hold debt and there is default in the economy, naïves cross-subsidize sophisticates.[13] This supports the idea that sophisticates choose higher fees when pooled together with naïves only to profit from the lower interest rates. If default becomes too expensive, average penalty fees drop sharply and cross-subsidization all but vanishes. Thus, cross-subsidization crucially hinges on the occurrence of default in equilibrium. In a model with a continuum of credit contracts that trade off interest rates for penalty fees, naïve agents would otherwise self-insure against their perceived risk of mistakes and break the cross-subsidization between them and sophisticates.[14]

5 Policy Experiments

Policy makers often pass legislations to reduce the likelihood that financial mistakes occur (e.g. standardized language, information requirements or transparency rules for contracts). Such policies have not been studied in previous quantitative models, because they have no effect on perfectly informed rational agents. In this model, however, we can easily simulate such policies by scaling the extent of naïveté, i.e. reducing the difference between the likelihood of financial shocks of sophisticated and naïve consumers. Another policy which is easily studied within our framework is imposing a limit on possible fee charges.

[13]In the case of free default, debt is not sustainable in equilibrium. In the case of infinitely costly default, agents will never default. In both of these extreme cases, cross-subsidization breaks down.

[14]We do not see a theoretical argument why prohibitively high bankruptcy cost must always lead to no fees and no cross-subsidization. Rather, in our calibration, financial shocks are large enough that some borrowers sometimes cannot pay them. However, in the opposite case of free bankruptcy and no sustainable debt in equilibrium, there can never be cross-subsidization.



(a) sophisticates

(b) naïves

Figure 7: Welfare Effects of Transparency Requirements.

Note: The horizontal axis is normalized so that 1 corresponds to the benchmark economy ($\psi = 6$), and 0 represents an economy without naïveté ($\psi = 1$)

In the following, we therefore discuss these two policies as well as the introduction of borrowing limits and assess them in terms of their effects on welfare which is calculated as consumption equivalence relative to the benchmark. Also, we always compare our benchmark economy, where naïve and sophisticated borrowers are pooled together, with a hypothetical economy in which markets are separated and lenders can offer different contracts to naïve and sophisticated borrowers. This enables us to disentangle the direct effects these policies have on consumer behavior from effects through cross-subsidization.

Furthermore, our framework enables a detailed analysis of how different policies affect both the ToC Bias and the financial mistakes made by naïves. Hence, we not only compare the welfare effects of different regulatory interventions but also their potential to guide naïves toward more informed financial decisions.

5.1 Transparency Requirements

Credit market regulations and legislations which are aimed at reducing the likelihood of financial shocks by making contracts easier to understand should have an effect on naïve consumers, since they will become better at understanding the true underlying risks they are facing. Sophisticated consumers, in contrast, will not be affected since they are already aware of the true risks. Hence, we model an improvement in the transparency of financial contracts as a reduction in the spread between the probabilities of financial shocks of sophisticates and naïves. With ψ denoting this spread as defined in Equation 3, we vary the extent of naïveté from benchmark ($\psi = 6$) to no naïveté ($\psi = 1$) where naïves face exactly the same shocks as sophisticates. The results of this experiment are shown in Figure 7, where welfare is given as consumption equivalence (CEV) relative to the benchmark in percent.

Figure 7b shows that the effect of improving the transparency of financial contracts is unambiguously positive for naïves, whose welfare in terms of consumption equivalence can be improved up to 5%. This is not surprising, since they receive fewer adverse financial shocks. However, note that these welfare gains rise sharply at the beginning, while becoming flatter as transparency is already high and naïveté is rather weak. Improving transparency by only 20%, for example, already leads to welfare gains between 1% and 1.3% in CEV. Interestingly, comparing the effects of transparency requirements in the pooling equilibrium to an equilibrium with separated contracts reveals that welfare gains are nearly always higher for naïves when they are operating in separated markets. While cross-subsidizing sophisticates, interest rates do not react to higher fees as quickly as in a separating equilibrium. Thus, equilibrium fee levels are lower (see Figure 5) and gains from transparency regulation are smaller in the pooling economy.

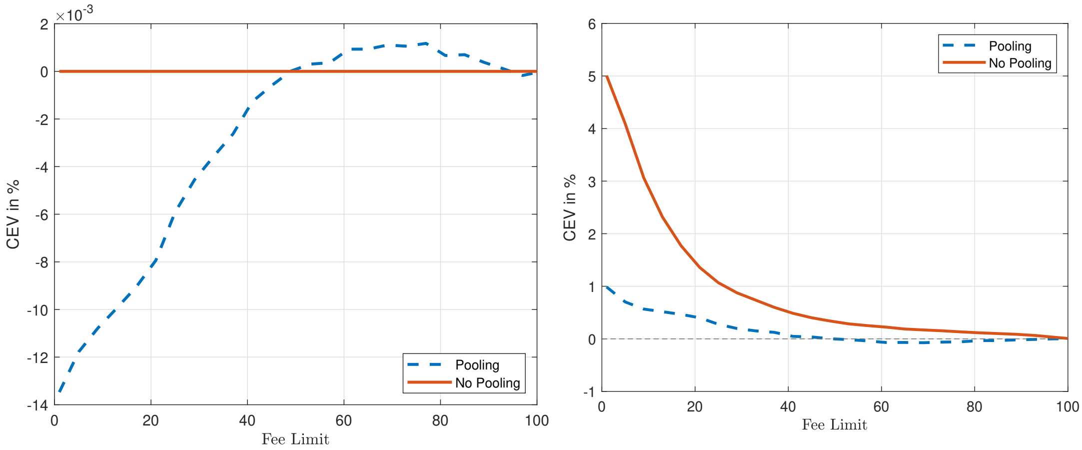
Figure 7a shows that the effects of higher transparency requirements have vastly different effects on sophisticates depending on being in an economy with or without naïves. The policy has no effect in the economy without pooling, since lenders can perfectly distinguish between sophisticated and naïve borrowers. Sophisticated borrowers have already considered their true risks when deciding on a credit contract and thus, their decisions are unaffected by the extent of naïveté, because banks do not change their offers to sophisticates. Looking at the pooling equilibrium, in contrast, reveals that sophisticates are hurt by the new policies when they are in an economy with naïves. Since improved transparency of financial contracts reduces the revenue banks can expect from fee payments from naïve borrowers, sophisticates lose some cross-subsidization on their interest payments. Hence, a legislation aimed at reducing the likelihood of financial mistakes will help naïves while hurting sophisticates, meaning that overall welfare effects are ambiguous and depend on the fraction of naïves in the economy.

5.2 Limiting Fees

Another possible intervention by policy makers concerns restrictions on the maximum amount of fees which can be charged for mishandling once credit (i.e. when incurring a financial shock). Again, such policies can easily be simulated within our model by imposing an upper bound $\bar{\phi}$ on possible fees ϕ.

The results of such policies are shown in Figure 8 which plots the effects on welfare of a variety of different fee limits, $\bar{\phi}$, on possible credit contracts. Again, the effects are calculated for an economy with and without pooling and measured as consumption equivalence relative to the benchmark in percent.

Figure 8a shows that a limit on fees has no effects on sophisticated borrowers in a separated economy. Since sophisticates already insure themselves against the risk of financial shocks, they are not affected by restrictions. However, if sophisticates are pooled together with naïves, there is first a very small positive and for tighter limits a negative



(a) sophisticates

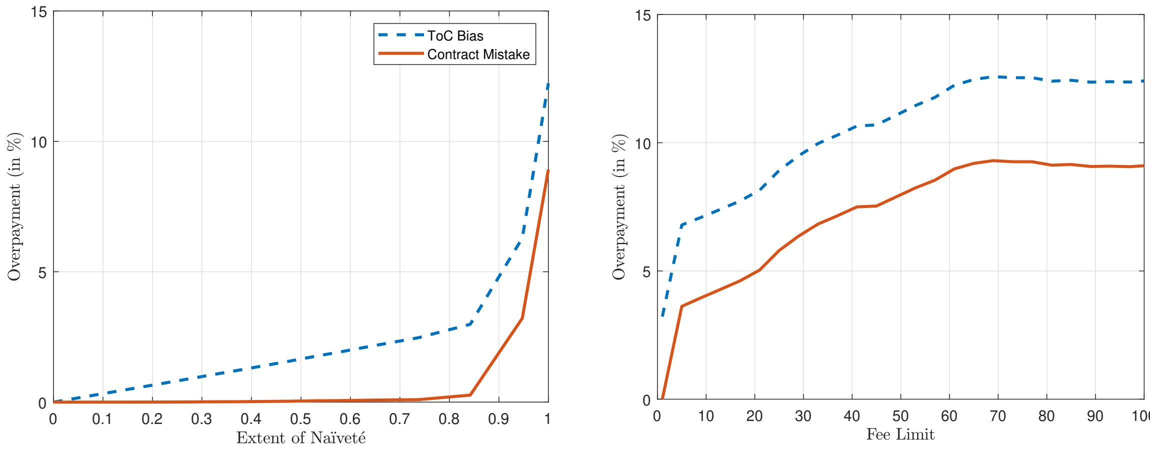
(b) naïves

Figure 8: Welfare effects of fee limits. Benchmark: $\overline{\phi} = 101$.

welfare effect. Laxer fee constraints force naïves to choose lower fees. This makes pooling for sophisticates less costly as they do not have to choose extremely high fee levels to enjoy cross-subsidization. However, for tighter limits, expected fee revenue drops enough for lenders to offer higher interest rates when compared to the benchmark economy in order to keep the same expected revenue as before. Hence, sophisticates lose cross-subsidization and end up paying more for their credit then in the benchmark economy.

Turning to Figure 8b we see that while the effects of a fee-limit are unambiguously positive for naïves, they profit relatively more if they face separated contracts than when they are pooled together with sophisticates. This is because in the separated economy contracts are geared more towards fee payments, since lenders do not have to account for the presence of sophisticates. Hence, limiting possible fee payments has a stronger effect on contracts in the separating equilibrium than in the pooling equilibrium. The mechanism, however, is the same in both cases: naïves are profiting from the fact that upper bounds on fees reduce the size of their potential mistakes. Stricter fee limits force naïve borrowers to avoid mistakes and choose credit contracts which are closer to the optimal contracts they would choose if they knew their true risks.

Lastly, comparing Figure 8a and Figure 8b shows that welfare effects are evolving at different rates for sophisticated and naïve consumers. While the loss of welfare for sophisticates exhibits a steady fall, welfare gains for naïves rise slowly at first but very sharply at tighter limits. The overall welfare effects of such a policy therefore crucially depend on the specific choice of the fee limit. Note, however, that welfare gains by naïves always seem to outweigh the welfare losses of sophisticates, even when accounting for their different shares in the economy.



(a) Improving transparency requirements

(b) Imposing fee limit

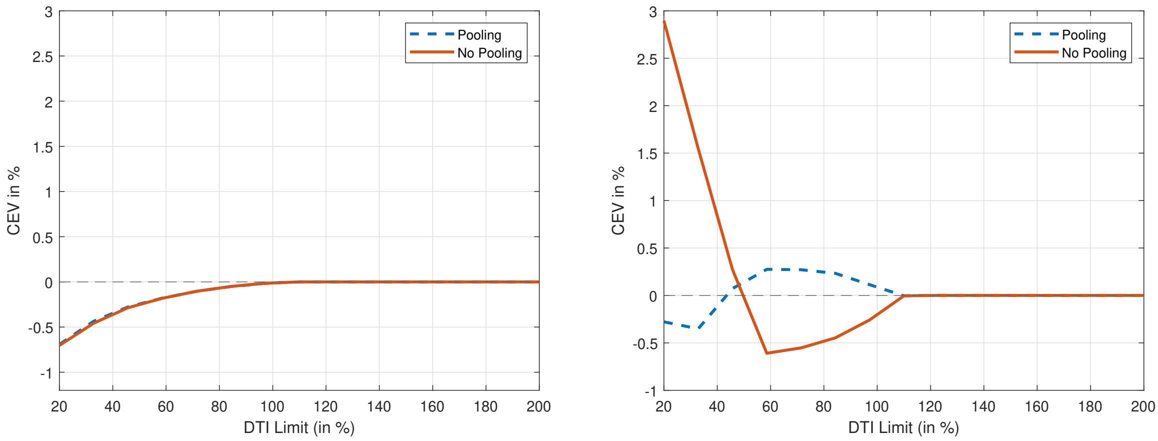
Figure 9: Policy effects on average size of over-payment by naïves.

5.3 Policy Interventions and Costs of Mistakes

We have shown that both policies are successful at making naïve borrowers better off. However, policy makers might not only care about improving overall outcomes but also about helping naïve borrowers avoid mistakes. We therefore analyze the effects of different policies on the average size of mistakes of naïve borrowers. Mistakes are measured as the amount by which the expected total cost of a chosen credit contract exceeds the expected cost of the optimal contract (cf. Equation 11 and Figure 3 in Section 4.2). We also report the Total Cost of Credit (ToC) Bias of the naïve consumer which is defined as percentage difference between the true expected cost of credit and the naïve 's biased expectation (cf. Equation 10 in Section 2.3).

Figure 9 shows the change in average financial mistakes (i.e. the average over-payment in percent) and average ToC Bias of naïves for both policies presented in Section 5, reducing the likelihood of financial shocks or limiting fees. In the extreme case of completely avoiding mistakes – either by directly abolishing naïveté or by prohibiting lenders to charge any fees – the contract mistake is zero.

However, improving transparency of financial contracts and thereby reducing the extent of naïveté in the economy (see Figure 9a) is more effective than limiting fees (see Figure 9b). While the average over-payment falls sharply with the initial reduction of naïveté in the economy, it remains relatively constant with lax fee limits and only decreases if the limit becomes very tight. For example, reducing naïveté in the economy by 40% already reduces the contract mistake to nearly 0%, and also reduces the ToC Bias from more than 10% to less than 2.5%. To achieve the same improvement in the average mistake through fee limits requires nearly a total abolishment of all fees. This suggests that imposing a limit on possible fees is not as effective at reducing mistakes due to Naïveté, as is working to reduce the bias itself.



(a) Sophisticates (b) Naïves

Figure 10: Welfare effects of implementing a limit on the debt-to-income ratio.

5.4 Borrowing Limits

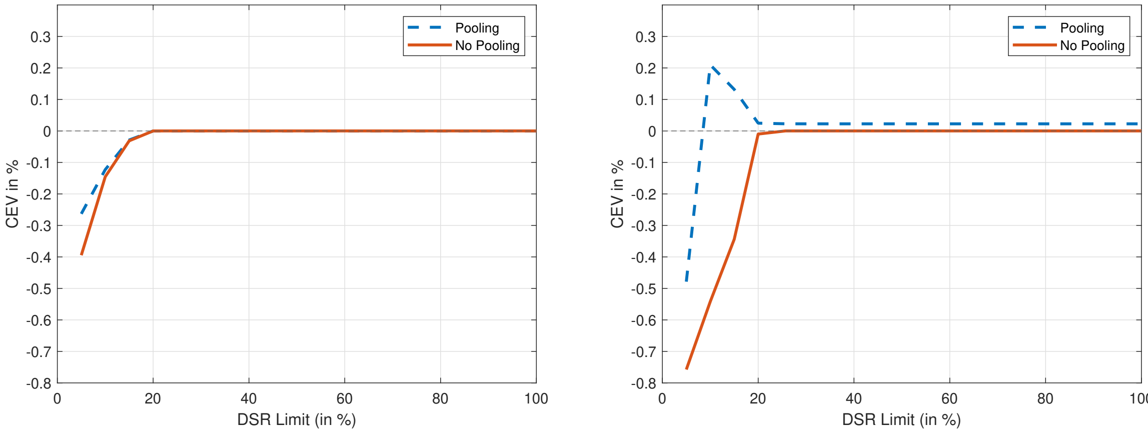
Naïve borrowers' misperception of their susceptibility to financial shocks not only draws them into unfavorable contracts with high potential fee payments but also results in excessive debt accumulation over the life cycle (see Figure 2). This observation motivates the evaluation of policies that focus not on altering contract terms but on limiting borrowing based on consumer characteristics.

In this section, we examine two policies designed to reduce the debt burden borrowers may take on: a *debt-to-income* (DTI) limit and a limit on the *debt service ratio* (DSR). The first introduces a limit, $\bar{D}$, on the amount of borrowing based on the current wage of the borrower:

$$q(d', \phi, s) = \begin{cases} q(d', \phi, s), & \text{if } q(\cdot)d'/y(s) \leq \bar{D} \\ 0, & \text{else.} \end{cases} \tag{14}$$

Figure 10 illustrates the welfare effects of implementing such a DTI limit. Focusing first on Panel 10a, we can see that the policy unambiguously reduces welfare of sophisticated consumers. Since sophisticated borrowers make rational decisions based on a true understanding of their financial risks, any restriction on their choice set can only make them worse off. Furthermore, this negative welfare effect is nearly identical in both a pooling economy and a separated economy.

Turning to Figure 10b, in contrast, reveals vastly differing effects on naïve borrowers in a pooling economy compared to a separated economy with only naïves. In a pooling economy, a moderate borrowing limit between 50% and 100% can slightly improve naïve welfare. By limiting the amount of borrowing, this policy prevents naïves from accruing more debt, thereby reducing their total cost of credit. This can lead to welfare gains, even if naïves would prefer higher debt levels. When the limit becomes overly restrictive,



(a) Sophisticates (b) Naives

Figure 11: Welfare effects of implementing a limit on the debt service ratio.

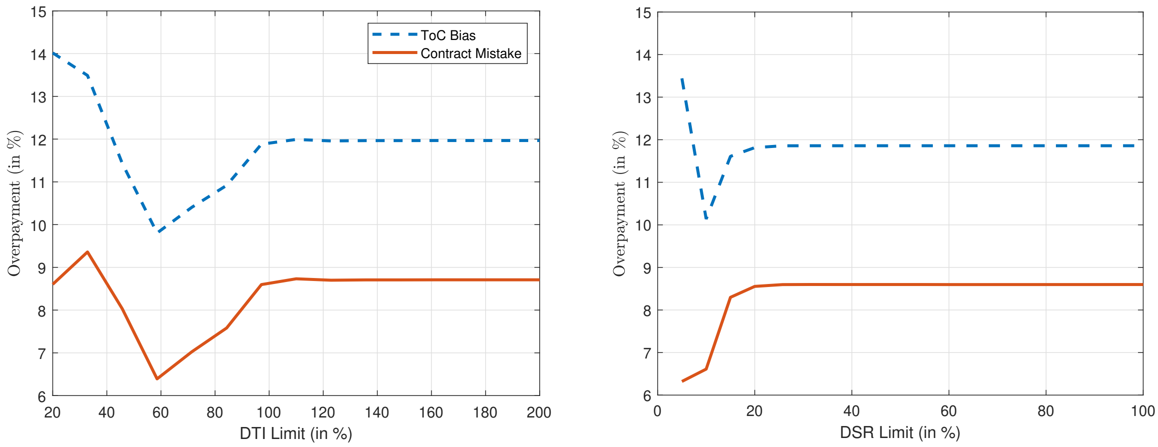
however, naïve borrowers are incentivized to accept even worse contracts than before in order to get below the DTI limit. This, in turn, raises their total cost of credit and reduces their welfare.

In a separated economy, naïve welfare is driven by the opposite effect. Without sophisticated borrowers interest rates fall more rapidly with an increase in chosen fees. Hence, at moderate borrowing limits naïves accept contracts with slightly higher fees to get below the DTI limit with the same debt level as before. This increases their costs once the true shocks realize and thus, reduces welfare. A tighter limit, in contrast, leads to a reduction in chosen fees and consequently, improves welfare. This is because the tighter limit finally reduces the amount of debt and incentivizes naïves to accept contracts which are more beneficial for them.

A borrowing limit, as described here, does not account for the interest rate associated with the credit. Borrowers may select contracts that meet the DTI threshold but result in a heavy repayment burden in the future. Therefore, instead of focusing solely on the amount of borrowing, we could also consider limiting interest payments. The interest paid on debt d' with chosen contract q is $(1-q)d'$. The debt service ratio (DSR) measures these interest payments relative to income. Thus, a DSR limit is given by

$$q(d', \phi, s) = \begin{cases} q(d', \phi, s), & \text{if } (1-q(\cdot))d'/y(s) \leq \bar{D} \\ 0, & \text{else.} \end{cases} \tag{15}$$

Figure 11 shows the welfare effects of introducing such a limit on DSR. Focusing first on sophisticates, Figure 11a reveals a similar picture as the DTI limit above. Again, Limiting the choices of sophisticates always hurts them, since they are already making optimal decisions based on the correct processes. Also, forcing naïves away from more costly contracts reduces the amount of cross-subsidization. Consequently, welfare of sophisticates drops by nearly 0.4%.



(a) DTI Limit

(b) DSR Limit

Figure 12: Policy effects on average size of over-payment by naïves.

Next, looking at Figure 11b shows that a DSR limit has a similar effect on naïves as a DTI limit as long as they are pooled together with sophisticates. As before, a moderate limit between 10% and 20% can slightly improve naïve welfare in the pooling economy. Tightening the limit even further, however, forces naïves to accept higher fees in order to reduce the interest rate and get below the DSR limit. This increases their total cost of credit, thereby reducing their welfare.

The main difference between a DSR limit and a DTI limit lies in the effects on naïves in a separating economy. While tight DTI limits vastly improve naïve welfare, a DSR limit has unanbigusouly negative effects. In the separating economy contracts exhibit high fees and low interest rates, because lenders can focus on naïve borrowers only. Hence, moderate DSR limits have no effects in the separating economy. Only when the limit becomes very tight can naïves no longer afford their preferred contracts. If they want to sustain the same debt level as before, they have to opt for higher fees. This increases their cost of credit and reduces their welfare. In the pooling economy, in contrast, lenders also have to account for sophisticated borrowers and the trade-off between interest rates and fee payments is less steep.

Lastly, to analyze again how these two policies affect overpayment by naïves, Figure 12 plots the ToC Bias and the contract mistake done by naïves with the introduction of either a DTI limit or a DSR limit. First, note that the two policies have similar effects both qualitatively and quantitatively. Moderate limits have no sizable effects, while tighter limits lead to slight improvements, reducing the contract mistake by nearly 30% (from about 9% to about 6%). Very tight limits, however, act to increase the size of the ToC Bias until it even slightly surpasses the benchmark value. This is because these tight limits force naïves to choose contracts with even higher fees than before, but because they underestimate their proneness to financial shocks this increases their bias (and reduces their welfare as seen in Figure 10b and Figure 11b).

Overall, this analysis demonstrates that borrowing limits can improve naïve welfare. However, these policies are poorly suited, if the aim is to help naïve borrowers to make better informed decisions and to reduce their financial bias.

6 Conclusion

We propose a novel quantitative theory of Naïveté in the credit market. We incorporate Naïveté in a standard framework of unsecured credit and equilibrium default. The model gives rise to policy interventions as naïve borrowers misunderstand their contracts and make financial mistakes. In equilibrium, they pay high penalty fees that benefit sophisticated borrowers through cross-subsidized interest rates.

We find that the 2009 CARD Act tackles two important dimensions of financial mistakes. Firstly, it makes contracts easier to understand and thereby reduces financial mistakes. Through the lens of our model, the welfare effects of such a policy are ambiguous as sophisticated consumers are hurt by higher interest rates, while naïves make less mistakes. The overall welfare effect therefore crucially hinges on the fraction of naïve consumers in the economy.

Secondly, the CARD Act limits the amount of penalty fees that lenders can charge. This limit has a clear and considerably positive impact on naïve borrowers. They are forced to accept contracts with lower fees, which also lowers the size of potential mistakes from wrong contract choices. Hence, by banning high-fee contracts that naïves could otherwise have wrongly chosen, the limit insures them against their future risks, while they themselves would not have done so. Sophisticates, in contrast, are again hurt by such a policy. By reducing the possible size of mistakes made by naïves, cross-subsidization of lower interest rates is also reduced and sophisticates end up paying more for their credit than in the benchmark economy. Also, the welfare gains of naïves and losses of sophisticates rise at different rates. Hence, the overall welfare effects of such a policy can be highly ambiguous and not only depend on the fraction of naïves in the economy, but also on the tightness of the limit.

Lastly, comparing the effects of these two policies on the size of the mistakes made by naïve consumers, we find that transparency requirements are more effective as sizable results are more easily achieved. Also, in light of the welfare effects, transparency requirements seem to have a less strong negative impact on sophisticates. Hence, while limiting fees might be the easier policy, improving the language used in financial contracts and raising standards for transparency and information requirements seems to be the better solution.

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A Additional Figures

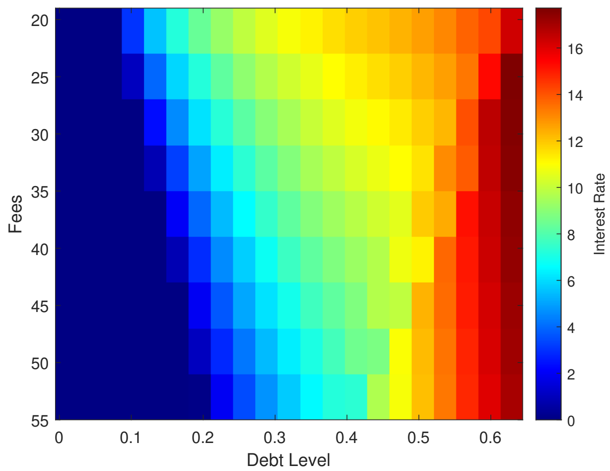

Figure A.1: Example contract space

Note: Choosing higher fees does not necessarily lower interest rates. In this examples, for debts above 0.5, borrowers cannot credibly commit to repay in the event of a financial shock. Thus, higher fees can lead to *higher* interest rates.

Figure A.1 shows an example of the contract space and the relationship between interest rates and fees at different debt levels for a middle-aged borrower with median persistent idiosyncratic productivity.

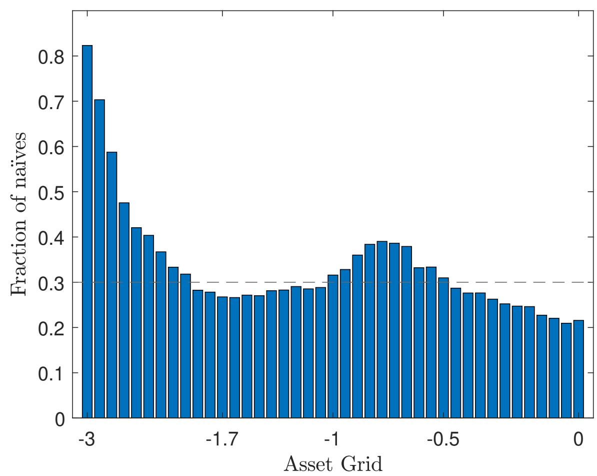

Figure A.2: Example of distribution of naïve consumers over different debt levels (share of naïves in whole economy $\lambda = 0.3$).

Figure A.2 depicts the distribution of naïve borrowers over debt levels. The fraction of naïves typically rises with higher debt levels, while the fraction of naïves at low debt levels is smaller than the economy wide average.

Figure A.3 shows the data on fee payments relative to FICO scores. Above a score of 660, borrowers pay constantly low fees relative to the amount borrowed. Below 660

(B) Fee Income

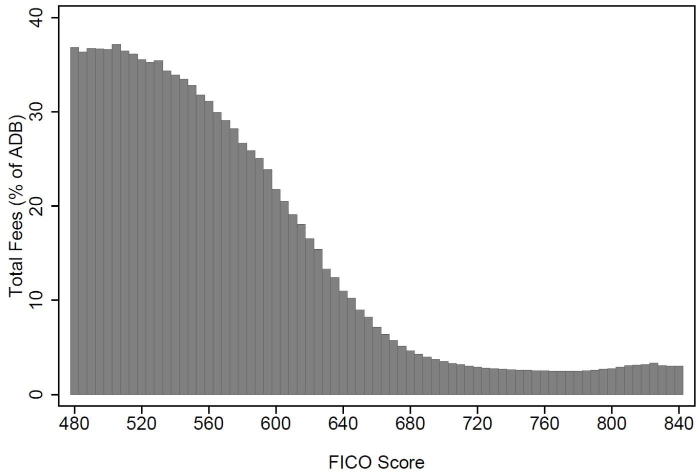

Figure A.3: Fee Payments by FICO Score
Source: Agarwal, Chomsisengphet, Mahoney, and Stroebel (2015, Figure 2 (B))

however, fee payments rise sharply. The share of borrowers with scores below 660 is 30% in the data.