# Signals, Interviews, and Hiring: Statistical Discrimination in Labor Market Equilibrium *

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**Abstract**

We incorporate a multi-stage hiring process into an otherwise standard search model of the labor market. Heterogeneous workers apply to firms, and firms decide how many workers to interview, which workers to interview (based on observable characteristics), and ultimately which worker to hire. We characterize the equilibrium and use it to study various policies that limit the types of information that firms are allowed to solicit before deciding which candidates to interview. In particular, we calibrate the model to study the controversial "ban the box" legislation which forbids firms from asking applicants about their criminal record before the interviewing process begins.

**Keywords:** Labor Markets, Discrimination, Interviews, Ban the Box

**JEL Classification:**

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# 1 Introduction

In an effort to reduce the undesirable effects of discrimination and enhance the labor market opportunities of disadvantaged job seekers, policymakers have introduced various regulations that restrict the type of information that employers can collect from (or ask of) job candidates. A recent, well-known example is the so-called "Ban the box" (BTB) initiative, implemented in several states, which prohibits firms from asking workers about their criminal records at the application stage of the hiring process (though does not prohibit collecting this information at later stages). More generally, a variety of laws restrict the extent to which firms can collection information about applicants—including credit checks and drug tests—when determining who to interview and/or hire.

Advocates of these initiatives argue that they will increase the likelihood of disadvantaged workers advancing through the early stages of the interview process, and thus having the opportunity to showcase their (otherwise hard-to-observe) skills and abilities. Since gainful employment is known to, e.g., decrease recidivism rates and increase credit scores, these goals are understandably relevant for improving outcomes of certain disadvantaged workers. However, a number of empirical studies have documented that these initiatives can have harmful, unintended consequences. In particular, when firms cannot observe certain pieces of information about an applicant, they may use the information they *can* observe to make inferences about the applicant's productivity, trustworthiness, and overall job-readiness. As a stark example, Agan and Starr (2018) found that firms who could not observe criminal record relied more heavily on race to determine whom to interview: in states that implemented BTB, they report that the callback rate of fictitious applicants with distinctly black names fell dramatically relative to applicants with otherwise identical resumes containing a distinctly white name. Complementing this audit study, Doleac and Hansen (2020) exploit variation in the timing and adoption of BTB across locations to estimate the effects of the policy on actual employment outcomes for a subset of the population. In particular, they find that BTB had a net negative effect on the employment outcomes of young, low-skilled black men.

These types of empirical findings raise a number of crucial, yet largely unanswered questions. How does restricting the information available to firms affect their willingness to post vacancies, who they interview, and who they ultimately hire? How do these choices depend on *and affect* the composition of the pool of unemployed workers? What are the subsequent implications for the call-back rates, job-finding rates, unemployment rates, and welfare of workers with different characteristics? Is restricting information more or less effective than other policies designed to improve the labor market outcomes of workers with a criminal record, such as subsidizing hiring of such workers?

The goal of this paper is to develop a framework that is capable of confronting these questions. In particular, we develop an equilibrium search model of the labor market in which workers have multiple dimensions of heterogeneity or "attributes" that may (or may not) be relevant for their productivity at a firm. As in the classic model of Mortensen and Pissarides (1994), firms post vacancies and workers of all types randomly contact or "apply to" firms. Unlike most standard models in this literature, we assume that firms can receive many applications in each period, and thus have several decisions to make.

First, we assume that firms must make an ex ante investment in their *interviewing capacity*, which determines the (expected) number of applicants they can interview. Then, if a firm receives a number of applications in excess of its capacity, it must also decide which applicants to interview. To make this decision, we assume that the firm receives a signal about each applicant's productivity *conditional on the attributes that are observable.* This signal allows the firm to rank applicants based on their expected output and, hence, decide which ones to interview. Lastly, after all interviews are conducted, the firm decides whom to hire. We make several assumptions that render this final decision trivial: we assume that firms must interview a worker before hiring him; that interviews fully reveal a worker's productivity; and that all firms make take-it-or-leave-it offers. As a result, firms always hire the worker with the highest productivity, among the set of workers that they interviewed.

Within the context of this framework, we solve for optimal behavior and derive the corresponding implications for labor market outcomes. At the core of the equilibrium characterization is an intricate fixed point problem. In particular, the composition of unemployed workers determines the number of workers that firms endogenously choose to interview, how these interviews are allocated across workers with different attributes, and ultimately the types of workers that are hired. At the same time, these interview and hiring decisions by the firm determine the composition of unemployed workers. A solution to this fixed point problem thus yields testable implications regarding the information that is available to firms—i.e., which of the workers' attributes they can observe—and observable outcomes such as callback rates, job-finding rates, and unemployment rates across different types of workers, along with aggregate outcomes such as vacancy creation, market tightness, output, and welfare.

We calibrate the model to quantitatively evaluate the effects of the somewhat controversial ban the box initiative. In particular, using target moments from before BTB, we identify a set of structural parameters that are consistent with the labor market experience of black and white workers, with and without a criminal record. Then, using targets from the empirical studies of BTB, we study the effects of restricting firms from observing an applicant's criminal record when deciding who to interview.

Matching the change in callback rates observed in audit studies, our model delivers predictions about the corresponding *equilibrium* implications for job-finding and unemployment rates across workers of different races, with and without a criminal record. In particular, the model predicts that the aggregate unemployment rate of white workers is little changed, while the aggregate unemployment rate of black workers rises by approximately 2.5 percentage points—which is roughly in line with the findings of Doleac and Hansen (2020). However, these top line numbers mask significant changes within the set of black and white workers. In particular, through the lens of our model, BTB causes a modest increase in nonemployment of white workers without a criminal record of approximately 1.5 percentage points, but a significant decline in the nonemployment rate of white workers with a criminal record (approximately 7.5 percentage points). In constrast, black workers without a criminal record experience a large increase in nonemployment (approximately 7.5 percentage points), whereas black workers with a criminal record experience a more modest drop in nonemployment (approximately 2.25 percentage points).

We also use the model to conduct two counterfactual exercises. In the first exercise, we study the effects of decreasing the cost of interviewing capacity by 10%, which could represent improved technology (like AI) that makes it less costly to process applicant's information. We find that the proximate effect of this shock is to encourage firm entry, which reduces the number of applicants at each vacancy, increases the callback rates of *all* workers, and reduces nonemployment rates across the board. Hence, this exercise suggests that the frictions implied by a costly interviewing process have significant effects on labor market outcomes, particularly for those workers who are associated with lower average productivity. Second, we explore the effects of a policy that subsidizes firms that hire workers with a criminal record. We find that such a policy has sizable effects on the callback and employment rates of workers with a criminal record, but only minimal negative effects on all other workers.

The structure of the paper is as follows. After reviewing the related literature below, in Section 2 we introduce a one-shot model in which a pool of heterogeneous unemployed workers randomly send applications to a fixed set of firms, who are each looking to hire one worker. This allows us to focus on the novel ingredients that we've introduced in order to study a multi-stage hiring process: we derive the firm's optimal choice of interviewing capacity; we introduce the signaling technology that determines which workers are interviewed when the number of applicants exceeds the firm's interviewing capacity; and we characterize the attributes and expected productivity of the applicant that is ultimately hired. This allows us, in Section 3, to embed this one-period model into a dynamic setting, where the pool of unemployed workers and the measure of open vacancies are endogenously determined. Importantly, the equilibrium of this dynamic model generates testable relationships between the worker attributes that firms can (or are allowed) to observe and the labor market outcomes of each type of worker—e.g., the call-back rates, job-finding rates, and unemployment rates of workers with different attributes. Finally, in Section 4, we calibrate this model and use it to quantitatively evaluate the effects of BTB.

**Related literature.** To be completed.

# 2 Static Model of Applications, Interviews, and Hiring

Consider an environment with a measure $v$ of homogeneous firms who demand one unit of labor and a measure $u$ of unemployed workers who supply one unit of indivisible labor. We assume that unemployed workers have a common outside option that yields them utility $b$, which can be interpreted as unemployment insurance, home production, utility from leisure, or some combination of the three.

Workers are heterogeneous along $N \in \mathbb{N}$ dimensions or "attributes" which we denote by $\mathbf{x} = (x_1, x_2, ..., x_N) \in \mathcal{X}$. We let $\phi(\mathbf{x})$ denote the measure of unemployed workers with characteristics $\mathbf{x}$, with $\int \phi(\mathbf{x}) d\mathbf{x} = u$. A worker's attributes determine the distribution over their match-specific productivity at a randomly selected firm. In particular, a worker of type $\mathbf{x}$ will generate an output $y \sim F_{\mathbf{x}}(y)$.

## 2.1 The Hiring Process

Unemployed workers search for a job by sending an application to a randomly selected firm. As is common in the literature, we assume that the total number of applicants at each firm, which we denote by $n_A$, will be drawn from the Poisson distribution with mean $\lambda \equiv u/v$. Moreover, given the properties of the Poisson distribution, the number of applicants with attributes $\mathbf{x}$ at each firm will also be drawn from the Poisson distribution with mean $\lambda_{\mathbf{x}} \equiv \phi(\mathbf{x})/v$.

**Recruiting intensity.** We assume that firms must interview a worker before hiring her, which perfectly reveals her productivity $y$. However, interviewing is costly. To formalize this idea, we assume that firms choose a *recruiting intensity* $\rho$ at the beginning of the period (before applications arrive) at cost $\rho c$, for some $c > 0$. This choice determines the maximum number of interviews that a firm can conduct, or what we call the firm's "interviewing capacity," which we denote by $n_R$. For tractability, it's helpful to assume that the relationship between $n_R$ and $\rho$ is not deterministic. Instead, we assume that $n_R$ is a random variable drawn from the geometric distribution with parameter $\frac{\rho}{1+\rho} \in [0, 1]$. Hence, the probability that a firm has the capacity to conduct $n \in \mathbb{N}_+$ interviews is

$$\mathbb{P}[n_R = n] = \left(\frac{\rho}{1+\rho}\right)^{n-1} \frac{1}{1+\rho},$$

with $\mathbb{P}[n_R \geq n] = \left(\frac{\rho}{1+\rho}\right)^{n-1}$ and $\mathbb{E}[n] = 1 + \rho$. Note that, by assumption, each firm is always able to interview at least one worker, so that $\rho$ is the expected number of *additional* workers a firm can potentially interview.

**Applicants, signals, and interviews.** The actual number of interviews that a firm conducts can either be constrained by the number of applicants or by the interviewing capacity of the firm. In particular, let $n_I = \min\{n_A, n_R\}$ denote the number of interviews conducted by a firm with $n_A$ applicants and interviewing capacity $n_R$. Once $n_I$ interviews have been completed, the firm can hire at most one of the applicants that it has interviewed.[1] We assume that firms make take-it-or-leave-it offers to workers, so that all workers earn wage $b$. Hence, it follows immediately that the firm's optimal strategy after conducting $n_I \geq 1$ interviews is to hire worker $i \in \text{argmax } \{y_1, ..., y_{n_I}\}$ conditional on $y_i \geq b$.

Moving backwards in the hiring process, when $n_A > n_R$ and firms are constrained by their interviewing capacity, they have to decide which applicants to interview in the first place.[2] To make this decision, firms have a technology that observes an applicant's attributes $\mathbf{x}$ and assigns them a ranking $r \in \{1, 2, ..., K\}$, which is independent of the number and attributes of other rankings. We call this technology a "human resources" (or HR) department and let $\Gamma_{r|\mathbf{x}}$ denote the probability that a worker with observable attributes $\mathbf{x}$ is assigned a rank $r$.

**Assumption 1.** *Let*

$$F_r(y) = \frac{\int_{\boldsymbol{x} \in \mathcal{X}} \Gamma_{r|\boldsymbol{x}} \lambda_{\boldsymbol{x}} F_{\boldsymbol{x}}(y) d\boldsymbol{x}}{\int_{\boldsymbol{x} \in \mathcal{X}} \Gamma_{r|\boldsymbol{x}} \lambda_{\boldsymbol{x}} d\boldsymbol{x}} \tag{1}$$

*denote the distribution of productivity for a worker assigned rank $r$. Then, for any $r, r' \in \{1, ..., K\}$, $r' > r \;\Rightarrow\; F_{r'}(y) \geq F_r(y)$ for all $y$ in the support of $F(y)$.*

Assumption 1 implies that the distribution of an applicant's productivity conditional on ranking $r$ is decreasing in the first order stochastic dominant (FOSD) sense: applicants who receive a rank $r$ are at least as productive as applicants who receive a rank $r' > r$. Hence,

[1] We assume, for simplicity, that firms cannot hire a worker that it has not interviewed. Since firms can always interview one applicant at zero cost, one can make assumptions on the distribution of productivity and the cost $c$ such that this assumption is, in fact, optimal even if firms were allowed to hire a worker without an interview.

[2] Given our assumptions, if $n_A \leq n_R$ then it is optimal for firms to interview all applicants.

firms use rankings to follow a pecking order: workers with $r = 1$ will be interviewed first, workers with $r = 2$ will be interviewed second, and so on. In the analysis below, we restrict attention to the binary case, so that $r \in \{1, 2\}$. This assumption is made for analytical convenience: extending our analysis to the case of $K > 2$ would not change any of the main substantive results below (though the algebra would be more complicated).

At this point, we are (intentionally) agnostic about the precise assumptions that generate the relationship between a worker's observable attributes, the signals that are generated, and the rankings that the HR department assigns. Instead, in the analysis below, we treat this relationship as a primitive and show how to characterize the equilibrium for *any* $\Gamma_{r|\mathbf{x}}$. This allows us, down the road, to study how different restrictions on the information available to the firm change $\Gamma_{r|\mathbf{x}}$, and the implications for the firm's decision of who to interview (and who is eventually hired). Section 2.4, below, provides an explicit example and shows how changing the set of available attributes affects the rankings of different types of workers.

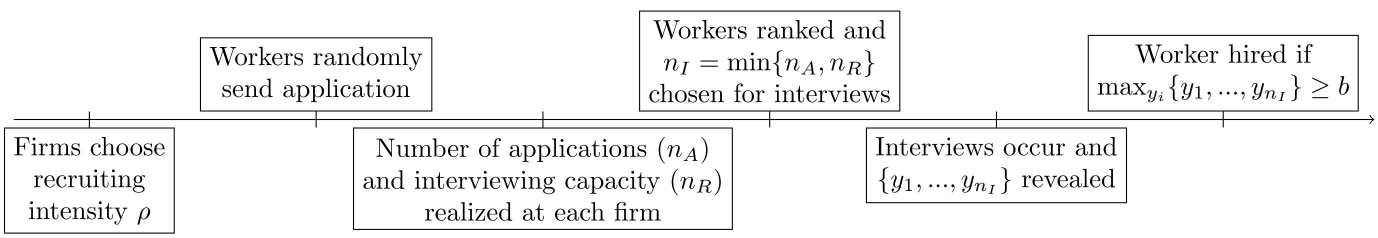


Figure 1: Timeline of Events

## 2.2 Optimal recruiting intensity.

Given a pool of unemployed, summarized by $\{\lambda_{\mathbf{x}}\}_{\mathbf{x}\in\mathcal{X}}$, along with a distribution over rankings assignments, $\Gamma_{r|\mathbf{x}}$, the number of applicants with rank $r \in \{1, 2\}$ is distributed according to the Poisson distribution with mean (or "queue length")

$$\lambda_r = \int_{\mathbf{x}\in\mathcal{X}} \Gamma_{r|\mathbf{x}} \lambda_{\mathbf{x}} d\mathbf{x}.$$

The following lemma characterizes the probability that an applicant is interviewed, given their rank $r$, the measure and rank of other applicants, and the firm's recruiting intensity $\rho$.

**Lemma 1.** *Consider a firm with recruiting intensity $\rho$ and queue lengths $\lambda_r$, for $r \in \{1, 2\}$. The probability for an applicant with $r = 1$ to be interviewed equals*

$$\frac{1 - e^{-\lambda_1/(1+\rho)}}{\lambda_1/(1+\rho)}, \tag{2}$$

*while the probability for an applicant with $r = 2$ to be interviewed equals*

$$e^{-\lambda_1/(1+\rho)} \frac{1 - e^{-\lambda_2/(1+\rho)}}{\lambda_2/(1+\rho)}. \tag{3}$$

*Proof.* See Appendix A.1 □

Given these expressions, we can characterize the probability that a firm hires a worker given recruiting intensity $\rho$, which we denote by $\eta$.

**Lemma 2.** *Consider a firm with recruiting intensity $\rho$ and define $\tau \equiv \rho/(\rho+1)$. The probability that the firm hires a worker with $r = 1$ and productivity exceeding $y$ is given by*

$$\eta_1(y;\rho) = \frac{1 - F_1(y)}{1 - \tau F_1(y)} \left(1 - e^{-\lambda_1(1-\tau F_1(y))}\right) - \int_y^\infty \left(1 - e^{-\lambda_2[1-\tau F_2(y_0)]}\right) \frac{1 - F_2(y_0)}{1 - \tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]}. \tag{4}$$

*The probability that the firm hires any worker with productivity exceeding $y$ is given by*

$$\eta(y;\rho) = \frac{1 - F_1(y)}{1 - \tau F_1(y)} \left(1 - e^{-\lambda_1(1-\tau F_1(y))}\right) + e^{-\lambda_1(1-\tau F_1(y))} \frac{1 - F_2(y)}{1 - \tau F_2(y)} \left(1 - e^{-\lambda_2(1-\tau F_2(y))}\right). \tag{5}$$

*Proof.* See Appendix A.2. □

Given the expression in (5), the objective function of the firm is simply

$$\max_\rho - \int_b^\infty (y - b)\, d\eta(y;\rho) - \rho c. \tag{6}$$

with first-order condition

$$- \int_b^\infty (y - b)\, d\left(\frac{\partial}{\partial \rho} \eta(y;\rho)\right) - c = 0, \tag{7}$$

In the following proposition, we establish that $\eta$ is strictly concave in $\rho$, which ensures a unique optimal recruiting intensity.

**Proposition 1.** *There exists a unique $\rho^\star$ solving (6). In particular, $\rho^\star$ solves (7) if*

$$- \int_b^\infty (y - b)\, d\left(\frac{\partial}{\partial \rho} \eta(y;0)\right) \geq c. \tag{8}$$

*If (8) is violated, $\rho^\star = 0$.*

*Proof.* See Appendix A.3 □

## 2.3 Callback and Hiring Rates

Note that, given an optimal recruiting intensity $\rho^\star$, it is now straightforward to calculate the callback and hiring rates for workers with attributes $\mathbf{x}$. To start, given the expressions in (2) and (3), the callback rate for workers of type $\mathbf{x}$ is immediate:

$$\Gamma_{1|\mathbf{x}}\left[\frac{1-e^{-\lambda_1/(1+\rho^\star)}}{\lambda_1/\left(1+\rho^\star\right)}\right]+\Gamma_{2|\mathbf{x}}\left[e^{-\lambda_1/(1+\rho^\star)}\frac{1-e^{-\lambda_2/(1+\rho^\star)}}{\lambda_2/\left(1+\rho^\star\right)}\right]. \tag{9}$$

Moreover, the probability that a worker with attributes $\mathbf{x}$ gets hired can be written

$$\psi_{\mathbf{x}}\left(\rho^\star\right)=\sum_r \eta_{r\mathbf{x}}\left(b;\rho^\star\right)/\lambda_{\mathbf{x}}, \tag{10}$$

where

$$\eta_{r\mathbf{x}}(b;\rho^\star)=-\int_b^\infty \frac{f_{r\mathbf{x}}(y)}{f_r(y)}\eta_r'(y;\rho^\star)\,dy$$

denotes the probability that a firm hires a worker with rank $r$ and characteristics $\mathbf{x}$, and

$$f_{r\mathbf{x}}(y)=\frac{\Gamma_{r|\mathbf{x}}f_{\mathbf{x}}(y)\lambda_{\mathbf{x}}}{\lambda_r}.$$

.

From the firm's point of view, the probability that a firm hires a worker of rank 1 is given by $\eta_1\left(b;\rho^*\right)$, so that the probability a firm hires a worker of rank 2 is given by $\eta\left(b;\rho^*\right)-\eta_1\left(b;\rho^*\right)$. Since a fraction $\frac{\Gamma_{i|\mathbf{x}}\lambda_{\mathbf{x}}}{\sum_{\mathbf{x}}\Gamma_{i|\mathbf{x}}\lambda_{\mathbf{x}}}$ of workers with rank $i$ have attributes $\mathbf{x}$, the probability that a firm hires a worker with attributes $\mathbf{x}$ is

$$\eta_1\left(b;\rho^*\right)\frac{\Gamma_{1|\mathbf{x}}\lambda_{\mathbf{x}}}{\sum_{\mathbf{x}}\Gamma_{1|\mathbf{x}}\lambda_{\mathbf{x}}}+\left(\eta\left(b;\rho^*\right)-\eta_1\left(b;\rho^*\right)\right)\frac{\Gamma_{2|\mathbf{x}}\lambda_{\mathbf{x}}}{\sum_{\mathbf{x}}\Gamma_{2|\mathbf{x}}\lambda_{\mathbf{x}}}$$

## 2.4 Discussion: Attributes, Signals and Rankings

The formulation described above takes the probability distribution over rankings given an applicant's observable attributes, $\Gamma_{r|\mathbf{x}}$, as a primitive and imposes little structure on it. Before proceeding, we provide two specific examples that generate probability distributions that are consistent with our formulation. These examples are not only useful for fixing ideas, but also useful for understanding how restricting the information available to the firm affects $\Gamma_{r|\mathbf{x}}$. To that end, suppose that a worker's productivity at the firm takes the form

$$y=Z(\mathbf{x})+\varepsilon. \tag{11}$$

We consider two simple technologies or "HR departments" that observe a noisy signal about each worker's productivity and use it go generate a ranking.

**An unconstrained HR department.** Suppose that

$$Z(\mathbf{x}) = \begin{cases} z_H & \text{with probability } \alpha(\mathbf{x}) \\ z_L & \text{with probability } 1-\alpha(\mathbf{x}) \end{cases} \tag{12}$$

for some $z_H > z_L$, and that the HR department perfectly infers the realization of $z \in \{z_L, z_H\}$ from the applicant's observable information, $\mathbf{x}$. However, suppose $\varepsilon$ is an iid draw from some type-independent distribution $G(\varepsilon)$ that can only be observed at the interview stage.

Under this formulation, $\Gamma_{1|\mathbf{x}} = \alpha(\mathbf{x})$ and the rank provided by HR, $r \in \{1, 2\}$, is a sufficient statistic for the distribution of the worker's productivity. That is, before conducting the interview, the firm could learn no more about the distribution over $y$ by knowing the worker's attributes, $\mathbf{x}$.

**A constrained HR department.** Now suppose that $Z(\mathbf{x}) = z$ is drawn from a distribution $F_{\mathbf{x}}(z)$ with mean $\mathbb{E}_{\mathbf{x}} z$, which can only be learned at the interview stage, whereas $\varepsilon$ is drawn from some distribution $G_{\mathbf{x}}(\varepsilon)$, which is perfectly observed by the HR department. Intuitively, one could imagine that $\varepsilon$ is an idiosyncratic feature of the candidate that the HR department can infer from other aspects of the worker's application, such as previous job experience, proficiency in a particular language, or even characteristics of the applicant's cover letter.

In this setting, the firm would ideally like the HR department to use *both* the realization of $\varepsilon$ and any additional information contained in the worker's attributes $\mathbf{x}$ to develop a complete ordering (or ranking) over all applicants. However, we assume that they can not. Instead, perhaps more realistically, we assume that the HR department is "constrained": instead of being able to perfectly rank each candidate, we assume that limitations on time and information processing imply that the HR department can only determine whether or not a candidate is "above the bar."

To formalize this idea, we assume that HR can generate only two rankings: for some threshold $\widetilde{y}$, rank $r = 1$ is assigned to all applicants with $\mathbb{E}_{\mathbf{x}} y = \mathbb{E}_{\mathbf{x}} z + \varepsilon \geq \widetilde{y}$, and rank $r = 2$ is assigned to all other applicants. Since $y$ is clearly strictly increasing in $\varepsilon$, there exists $\widetilde{\varepsilon}(\mathbf{x})$ such that a worker with attributes $\mathbf{x}$ receives rank $r = 1$ only if $\varepsilon \geq \widetilde{\varepsilon}(\mathbf{x})$. Hence, the probability that an applicant with characteristics $\mathbf{x}$ receives rank $r = 1$ is

$$\Gamma_{1|\mathbf{x}} = \mathbb{P}\left[\varepsilon \geq \widetilde{\varepsilon}\left(\mathbf{x}\right)|\mathbf{x}\right] = \frac{\int_{\widetilde{\varepsilon}(\mathbf{x})}^{\infty} \phi\left(\mathbf{x}\right) dG_{\mathbf{x}}(\varepsilon)}{\int_{-\infty}^{\infty} \phi\left(\mathbf{x}\right)\, dG_{\mathbf{x}}(\varepsilon)}.$$

Finally, note that this formulation makes it straightforward to analyze what happens when firms cannot observe certain attributes. For example, suppose we restrict firms from observing $x_1$, but allow them to observe the remaining attributes $\mathbf{x}_{-1}$. Then there is a new threshold function $\widetilde{\varepsilon}\left(\mathbf{x}_{-1}\right)$ such that

$$\mathbb{E}_{\mathbf{x}_{-1}} y = \frac{\int \phi(x_1, \mathbf{x}_{-1}) \mathbb{E}_{\mathbf{x}} z dx_1}{\int \phi(x_1, \mathbf{x}_{-1}) dx_1} + \varepsilon \geq y \;\Leftrightarrow\; \varepsilon \geq \widetilde{\varepsilon}\left(\mathbf{x}_{-1}\right),$$

and the probability that an applicant with observable attributes $\mathbf{x}_{-1}$ is assigned rank $r = 1$ is

$$\Gamma_{1|\mathbf{x}_{-1}} = \frac{\int_{\widetilde{\varepsilon}(\mathbf{x}_{-1})}^{\infty} \int \phi\left(x_1, \mathbf{x}_{-1}\right) dx_1 dG_{\mathbf{x}}(\varepsilon)}{\int_{-\infty}^{\infty} \int \phi\left(x_1, \mathbf{x}_{-1}\right) dx_1 dG_{\mathbf{x}}(\varepsilon)}. \tag{13}$$

Intuitively, consider an applicant of type $\mathbf{x} = (x_1, \mathbf{x}_{-1})$ with average productivity $\mathbb{E}_{\mathbf{x}} y$ who shares attributes $\mathbf{x}_{-1}$ with a group of workers of type $\mathbf{x}' = (x_1', \mathbf{x}_{-1})$ who have lower expected productivity, i.e., $\mathbb{E}_{\mathbf{x}'} y < \mathbb{E}_{\mathbf{x}} y$. Then preventing the firm from observing $x_1$ will hurt this worker: the threshold value $\widetilde{\varepsilon}$ increases, so being ranked first requires a greater realization of $\varepsilon$, and thus the probability of a signal that results in $r = 1$ declines.

# 3 Dynamic Model

We now extend the one-period model from Section 2 to a discrete-time, infinite horizon setting where all agents discount between periods at rate $\beta$. We assume that there is a measure of workers, normalized to one, and a large measure of firms who can post a vacancy by paying a flow cost $k$. As in the previous section, we assume that workers are heterogeneous with respect to their underlying characteristics, and we denote the (exogenous) measure of workers with characteristics $\mathbf{x}$ by $\pi\left(\mathbf{x}\right)$. The measure of unemployed workers of type $\mathbf{x}$—which is now endogenous—is again denoted by $\phi\left(\mathbf{x}\right)$.

Each period consists of 4 different phases. The first phase features job destruction. Among those workers who enter the period employed, we assume that a fraction $\delta$ receive a separation shock and become unemployed. As is often the case, we assume that a worker who loses her job in period $t$ cannot search for a new job until period $t+1$. The second phase features free entry of vacancies, so that the value of vacancy, $V$, is equal to zero in equilibrium. The third phase features the matching process described in Section 2, and depicted in detail in Figure 1.

The final phase consists of production. In a slight change of notation, let $y$ denote the value of the match surplus in excess of worker's outside option, $b$. In particular, for a worker with productivity $z$ and separation rate $\delta$,

$$y(z,\delta) = \frac{z-b}{1-\beta\,(1-\delta)}.$$

Note that we do not take a stance on whether the heterogeneity in $y$ stems from heterogeneity in productivity, separation rates or both. Hence, any joint distribution of $z$ and $\delta$ can be summarized by the implied distribution of $y$.

### 3.1 Equilibrium in the Dynamic Setting

Given this timing, firms post vacancies and choose their optimal recruiting intensity $\rho$, taking as given the measure of unemployed workers of each type $\mathbf{x} \in \mathcal{X}$, the measure of vacancies posted $v$, and the signal structure $\Gamma_{r|\mathbf{x}}$ which maps a worker's underlying characteristics into a ranking $r$. In particular, given the endogenously determined ratios $\lambda_{\mathbf{x}} = \phi\left(\mathbf{x}\right)/v$, the firm's optimal recruiting intensity is determined by the distribution over the number of applications it will receive from workers with rank $r \in \{1,2\}$, summarized by $\lambda_r$, and the distribution of match values, which is $F_r(y)$ for a worker with rank $r$.[3]

As in the one-period model, after firms choose their recruiting intensities, each unemployed worker sends an application to a randomly selected vacancy and the hiring process ensues: the number of applicants and the interviewing capacity are realized at each firm, firms conduct interviews, and a worker is hired if one of the interviewed workers is productive. Again, the probability that the firm hires a worker with productivity exceeding $y$ is denoted by $\eta(y)$. The probability that a worker with rank $r$ is hired is denoted by $\psi_r$ and the probability that a worker with characteristics $\mathbf{x}$ is hired is denoted by $\psi_{\mathbf{x}}$.

By the same arguments as before, the value of an unfilled vacancy satisfies

$$V = \max_{\rho}\ -\rho c - k - \int_b^\infty y\, d\eta(y;\rho), \tag{14}$$

[3]Recall that $\lambda_r = \sum_{\mathbf{x}\in\mathcal{X}} \Gamma_{r|\mathbf{x}}\lambda_{\mathbf{x}}$ and $F_r(y) = \frac{\sum_{\mathbf{x}\in\mathcal{X}} \Gamma_{r|\mathbf{x}} F_{\mathbf{x}}(y)\lambda_{\mathbf{x}}}{\lambda_r}$.

with first-order condition

$$-\int_b^\infty y\, d\left(\frac{\partial}{\partial \rho}\eta\left(y;\rho\right)\right) - c = 0. \tag{15}$$

This problem is again concave, so a unique optimal $\rho^\star$ exists.

Given an optimal recruiting intensity, the probability that a worker with attributes $\mathbf{x}$ is hired, $\psi_{\mathbf{x}}(\rho^\star)$, is given by (10). In a steady-state equilibrium, the measure of unemployed workers of type $\mathbf{x}$ is constant—the flow of such workers into unemployment must equal the outflow. Hence, $\phi\left(\mathbf{x}\right)$ must satisfy

$$\dot{\phi}\left(\mathbf{x}\right) = \left[\pi\left(\mathbf{x}\right) - \phi\left(\mathbf{x}\right)\right]\delta_{\mathbf{x}} - \phi\left(\mathbf{x}\right)\psi_{\mathbf{x}}\left(\rho^\star\right) = 0.$$

Solving reveals that, in any steady-state equilibrium, it must be that

$$\phi\left(\mathbf{x}\right) = \frac{\delta_{\mathbf{x}}\pi\left(\mathbf{x}\right)}{\delta_{\mathbf{x}} + \psi_{\mathbf{x}}\left(\rho^\star\right)}, \tag{16}$$

for all $\mathbf{x} \in \mathcal{X}$. Finally, the steady-state measure of vacancies $v$ must satisfy the free entry condition

$$-\rho^\star c - k - \beta\left[\int_b^\infty y\, d\eta(y;\rho^\star)\right] = 0. \tag{17}$$

**Definition 1.** *Given a distribution of workers $\pi\left(\mathbf{x}\right)$, a production technology $F_{\mathbf{x}}(y)$, and a signal structure $\Gamma_{r|\mathbf{x}}$, an equilibrium is characterized by a tuple $(\rho^\star, \phi(\mathbf{x}), v)$ such that:*

1. *The recruiting intensity $\rho^\star$ satisfies the first-order condition (15).*
2. *The distribution of unemployed workers $\phi(\mathbf{x})$ satisfies (16).*
3. *The measure of vacancies $v$ satisfies the free entry condition (17).*

# 4 Quantitative Exercise

In this section, we use the dynamic equilibrium model to study the quantitative implications of ban the box. In particular, we calibrate the model described above to match a variety of targets from the data. These targets include aggregate moments, such as labor market flows, as well as micro targets that derive from audit studies. We use the calibrated model to back out unobserved parameter values, explore the aggregate implications of imposing BTB more universally, and compare the effects of BTB to other policies that have been proposed to improve the labor market outcomes of workers with a criminal record.

## 4.1 Parameters and Targets

**Preliminaries.** To study ban the box, we assume that $\mathbf{x} = (x_1, x_2)$ is two-dimensional. The first attribute, $x_1 \in \{N, C\}$, represents an applicant's criminal record: we let $N$ denote an applicant with no criminal record and $C$ denote an applicant with a criminal record. The second attribute, $x_2 \in \{B, W\}$, represents an applicant's race, which we restrict to either black ($B$) or white ($W$) due to the focus of BTB. Given these choices we need to parameterize $\pi(\mathbf{x})$, for $\mathbf{x} \in \mathcal{X} = \{N, C\} \times \{B, W\}$, which represents the (exogenous) joint distribution of criminal record and race among the population of interest.

The key new object in our model, relative to the standard model of Mortensen and Pissarides (1994), is the function $\Gamma_{r|\mathbf{x}}$, which describes the mapping between a worker's attributes and the probability distribution over the ranking that they are assigned, $r \in \{1, 2\}$. In order to understand how this function changes after imposing restrictions on the information available to firms at the application stage of the hiring process, we have to take a stand on how firms assign rankings. To do so, we follow the so-called "constrained HR" formulation presented in Section 2.4. According to this formulation, the HR department observes the realization of an idiosyncratic shock $\varepsilon \sim G_{\mathbf{x}}(\varepsilon)$ for each applicant, and assigns rank $r = 1$ ($r = 2$) if the match surplus

$$\mathbb{E}_{\mathbf{x}} y = \underbrace{\mathbb{E}_{\mathbf{x}} \left[ \frac{z - b}{1 - \beta\,(1 - \delta_{\mathbf{x}})} \right]}_{Z(\mathbf{x})} + \varepsilon$$

is greater (less) than some threshold $\widetilde{y}$, where $\delta_{\mathbf{x}}$ is an attribute-specific job destruction rate and $z$ is the worker's idiosyncratic productivity that the firm only learns at the interview stage. We assume that $G_{\mathbf{x}}(\varepsilon)$ is independent of $\mathbf{x}$ and takes the form of a Frechet distribution with shape parameter $\nu_\varepsilon$ and scale parameter $\zeta_\varepsilon$, and assume that the distribution of $Z(\mathbf{x})$ takes the form of a Laplace distribution with mean $\mu_{\mathbf{x}}$ and standard deviation $\sigma_{\mathbf{x}}$.

To summarize, calibrating the model requires assigning values to the distribution $\pi(\mathbf{x})$, the job separation rates $\delta_{\mathbf{x}}$, the cost of posting a vacancy $k$, the recruiting intensity cost $c$, and the productivity distribution parameters $(\nu_\varepsilon, \zeta_\varepsilon)$ and $(\mu_{\mathbf{x}}, \sigma_{\mathbf{x}})$.

**Parameters determined outside the model.** We choose values for job separation rates $\delta_{\mathbf{x}}$ and the joint distribution $\pi(\mathbf{x})$ using direct observations from the data. Estimates of the aggregate separation rates by race are taken directly from the Current Population Survey (CPS). Our estimates of the population shares of each race-criminal record pair are built up from three data sources and require a little more discussion. The overall white share of workers is taken from Census tabulations for men ages 18-34. To compute the conviction share within race (again, among men), we start with estimates of felony convictions in Shannon et al. (2017), who draw on administrative data on prisoner releases. We augment these results with estimates of misdemeanor convictions based on the NLSY97. We do not rely exclusively on the NLSY because it appears to understate by half the incidence of felony convictions among black men relative Shannon et al. However, the NLSY is the only source of data on misdemeanors. In the absence of any other guidance, we assume the NLSY also understates the incidence of misdemeanor convictions by half. Therefore, we double the NLSY-implied figure to arrive at our final estimate of 48.2 % for the conviction share among black men aged 18-34. By contrast, the conviction share among white men is taken directly from the NLSY, which largely agrees with the evidence in Shannon et al.

**Parameters calibrated internally.** Turning to the distribution over workers' match-specific productivity, we normalize $\mu_{NW} = \sigma_{NW} = 1$, and assume that $\frac{\sigma_{NW}}{\sigma_{CW}} = \frac{\sigma_{NB}}{\sigma_{CB}}$. The scale parameter $\zeta_\varepsilon$ is determined outside of the calibration; we discuss this further below. This leaves eight parameters to calibrate internally: $\{c, k, \mu_{NB}, \sigma_{NB}, \mu_{CW}, \sigma_{CW}, \mu_{CB}, \nu_\varepsilon\}$. Table 1 summarizes the calibrated parameter values, the target moments in the data, and the corresponding moments in the model. While each parameter potentially affects all moments in the model, it's helpful to discuss (at least informally) the target moments most closely associated with each parameter.

To start, the firm's marginal cost of increasing its recruiting capacity, $c$, is a key determinant of $\rho^\star$, and hence the average number of interviews conducted for each vacancy, which is reported by Barron, Berger, and Black (1997). The entry cost $k$ is a crucial determinant of the aggregate market tightness, and hence the average unemployment rate in the model. To derive the appropriate counterpart in the data, we calculate the average non-employment rate among black and white men between age 18-34 in the CPS, excluding those who are not working because they are in school. We choose the non-employment rate, as opposed to the unemployment rate, to capture discouraged workers; recall that, among men 18-34 years old, very few are not working because of disability or retirement.

Next, the expected match surplus conditional on a worker's observable attributes, $\mu_{\mathbf{x}}$, is a primary determinant of that worker's probability of being ranked $r = 1$ or $r = 2$, which determines his or her likelihood of being interviewed (the so-called "callback rate"). Hence, after normalizing $\mu_{NW} = 1$, the remaining mean values $\{\mu_{NB}, \mu_{CB}, \mu_{CW}\}$ are set to match the callback rate of each group relative to white applicants without a criminal record, which is derived from Agan and Starr (2018). Conditional on each group of applicants' callback rate, the dispersion in the realization of their match-specific productivity—which is learned at the interview stage—is an important determinant of the likelihood that they are hired. Since we have normalized $\sigma_{NW} = 1$ and assumed that $\frac{\sigma_{NW}}{\sigma_{CW}} = \frac{\sigma_{NB}}{\sigma_{CB}}$, it turns out that $\sigma_{NB}$ has a large impact on the probability that a black worker without a criminal record is hired. Hence, taking as given that we have matched the *average* nonemployment rate across these types, $\sigma_{NB}$ helps pin down the *difference* in the non-employment rate between black and white workers, which we calculate using the CPS. Given a value of $\sigma_{NB}$, the remaining parameters that govern the distribution of the match surplus, $\sigma_{CB} = \sigma_{NB}\sigma_{CW}$, determine the difference in non-employment rates between applicants with and without a record, or what we call the "employment penalty from a criminal record."

Finally, note that the discussion of internally calibrated parameters above takes the distribution over $\varepsilon$ as given: for an arbitrary $G(\varepsilon)$, the remaining parameters can adjust to generate the desired callback and employment rates. However, once these parameters have been set, the model delivers precise predictions about the effects of restricting information on the cutoff value $\widetilde{\varepsilon}$. In particular, when firms observe both race and criminal record,

$$\widetilde{\varepsilon}(x_1, x_2) = \mathbb{E}_{(x_1,x_2)} Z(x_1, x_2) - \widetilde{y}, \tag{18}$$

whereas restricting firms from observing $x_1$ yields

$$\widetilde{\varepsilon}(x_2) = \frac{\phi(C, x_2)\mathbb{E}_{x_2} Z(C, x_2) + \phi(N, x_2)\mathbb{E}_{x_2} Z(N, x_2)}{\phi(C, x_2) + \phi(N, x_2)} - \widetilde{y}. \tag{19}$$

But how do these changes in the threshold $\widetilde{\varepsilon}$ affect callback rates? Since $\Gamma_{1|\mathbf{x}} = 1 - G(\widetilde{\varepsilon})$, one can see immediately that the *shape* of $G(\varepsilon)$—but not the scale factor—determines the response of callback rates to changes in the information available to the firm. Hence, to determine the final parameter $\nu_\varepsilon$, we match the change in the ratio of callback rates of black to white applicants, before and after the implementation of BTB. Formally, we target the difference between the log ratio before BTB,

$$\log\left[\frac{\{\phi(C,B)\left[1 - G\left(\widetilde{\varepsilon}(C,B)\right)\right] + \phi(N,B)\left[1 - G\left(\widetilde{\varepsilon}(N,B)\right)\right]\} / \left[\phi(C,B) + \phi(N,B)\right]}{\{\phi(C,W)\left[1 - G\left(\widetilde{\varepsilon}(C,W)\right)\right] + \phi(N,W)\left[1 - G\left(\widetilde{\varepsilon}(N,W)\right)\right]\} / \left[\phi(C,W) + \phi(N,W)\right]}\right], \tag{20}$$

and the log ratio of callback rates after BTB,

$$\log\left[\frac{1 - G\left(\widetilde{\varepsilon}(B)\right)}{1 - G\left(\widetilde{\varepsilon}(W)\right)}\right]. \tag{21}$$

Table 1: Internally Calibrated Parameters and Targets

| Parameters | | | Targets | | |
|---|---|---|---|---|---|
| Description | Variable | Value | Description | Model | Data |
| Recruiting Cost | $c$ | 0.093 | Avg Interviews per Position | 4.86 | 4.86 |
| Entry Cost | $k$ | 1.451 | Average Nonemployment Rate | 0.149 | 0.149 |
| Avg Productivity (CW) | $\mu_{CW}$ | 0.893 | Callback Rate (CW) | 0.593 | 0.593 |
| Avg Productivity (CB) | $\mu_{CB}$ | 0.900 | Callback Rate (CB) | 0.614 | 0.614 |
| Avg Productivity (NB) | $\mu_{NB}$ | 0.963 | Callback Rate (NB) | 0.936 | 0.936 |
| Std Dev Productivity (CW) | $\sigma_{CW}$ | 1.004 | Employment Penalty from Record | 0.116 | 0.116 |
| Std Dev Productivity (NB) | $\sigma_{NB}$ | 1.260 | Black-White Employment Gap | 0.129 | 0.129 |
| Shape Parameter ($\varepsilon$) | $\nu_{\varepsilon}$ | 1.353 | $\Delta$ Relative Callback Rate (BTB) | 0.265 | 0.277 |

## 4.2 Results

In this section, we present two sets of results. First, we illustrate the effects of preventing firms from observing criminal record—formally, comparing equilibrium outcomes when $\mathbf{x} = (x_1, x_2)$ and $\mathbf{x} = x_2$—which is our model analog to ban the box. Then, we explore the effects of two counterfactual scenarios that could potentially improve the labor market prospects of workers with criminal records: reducing the cost of interviewing capacity and subsidizing firms that hire workers with criminal records.

**Ban the Box through the lens of our model.** Comparing the second and third columns of Table 2 reveals the effects of restricting firms from observing an applicant's criminal record. One can see that the primary effect of this policy on model parameters generates a large change in the probability that black workers are assigned a rank $r = 1$: the likelihood of a black worker without a criminal record being assigned $r = 1$ falls substantially, while the likelihood of a black worker with a criminal record being assigned $r = 1$ rises substantially. With restrictions on the ex ante information available, firms respond by increasing the number of interviews they conduct. As a result of the changes in rankings and interviewing capacity, the callback rate of white workers rises by approximately 5 percentage points, while the callback rate of black workers falls by approximately the same amount.

In equilibrium, the total nonemployment rate of white workers is little changed, though this top line number masks significant changes within the set of white workers: those without a criminal record experience an increase in nonemployment (approximately 1.5 percentage points), whereas white workers with a criminal record benefit from a significant drop in nonemployment (approximately 7.5 percentage points). For black workers, our model predicts that the nonemployment rate should rise by approximately 2.5 percentage points after the implementation of BTB—a prediction roughly in line with the evidence presented by Doleac and Hansen (2020). Again, this headline number masks considerable heterogeneity across black workers: those without a criminal record experience a large increase in nonemployment (approximately 7.5 percentage points), whereas black workers with a criminal record experience a more modest drop in nonemployment (approximately 2.25 percentage points).

Table 2: The Effects of BTB and Alternatives

| **Outcome** | **Benchmark** | **Ban the Box** | **Reduce $c$** | **Subsidize hires** |
|---|---|---|---|---|
| $\Gamma_{1\|N,W}$ | 1.000 | 0.996 | 1.000 | 1.000 |
| $\Gamma_{1\|C,W}$ | 0.443 | 0.996 | 0.443 | 0.704 |
| $\Gamma_{1\|N,B}$ | 0.912 | 0.659 | 0.912 | 0.912 |
| $\Gamma_{1\|C,B}$ | 0.473 | 0.659 | 0.473 | 0.761 |
| Vacancy rate | 0.0129 | 0.0128 | 0.0136 | 0.0132 |
| $\rho^\star$ | 3.860 | 3.927 | 3.745 | 3.722 |
| Callback rate (W) | 0.362 | 0.412 | 0.451 | 0.391 |
| Callback rate (B) | 0.353 | 0.307 | 0.442 | 0.386 |
| Nonemployment rate (W) | 0.126 | 0.124 | 0.094 | 0.117 |
| Nonemployment rate (B) | 0.255 | 0.280 | 0.195 | 0.219 |
| Nonemployment rate (N,W) | 0.108 | 0.122 | 0.082 | 0.110 |
| Nonemployment rate (C,W) | 0.208 | 0.133 | 0.148 | 0.150 |
| Nonemployment rate (N,B) | 0.202 | 0.275 | 0.158 | 0.206 |
| Nonemployment rate (C,B) | 0.311 | 0.286 | 0.235 | 0.233 |

**Two alternatives to BTB.** In this section, we conduct two counterfactual exercises to explore the importance of the recruiting cost friction and study the effects of an alternative policy designed to improve the labor market outcomes of workers with a criminal record.

Consider first the effects of reducing the cost of interviewing capacity ($c$) by 10%, as in Jarosch and Pilossoph (2019). This reduction in cost could be the result of better technology (like AI) that allows the firm to process information and evaluate workers at a lower cost. To isolate the effects of reducing $c$, we hold all other parameters fixed to their benchmark values, which is why $\Gamma_{1|\mathbf{x}}$ is unchanged for all $\mathbf{x} \in \mathcal{X}$. As one can see in column 4 of Table 2, firms respond to this reduction in cost by posting more vacancies. This lowers the ratio of unemployed workers to vacancies, increasing callback rates for *all* workers—despite a small decline in recruiting capacity. As a result, nonemployment rates fall significantly for black and white workers, with and without a criminal record. This result highlights that the estimated costs of interviewing capacity have important ramifications for labor market outcomes.[4]

[4]This result stands in contrast to those of Jarosch and Pilossoph (2019), who find that reducing the cost

Next, consider the effects of a policy that subsidizes firms when they hire workers with a criminal record. In particular, we adjust the model parameters such that the expected surplus that a firm receives when hiring a worker with a criminal record increases by 5%, leaving the payoffs from hiring a worker without a criminal record unchanged from its benchmark value. As one can see in column 5 of Table 2, firms respond to a larger payoff from hiring workers with a criminal record by posting more vacancies and assigning rank $r = 1$ to workers with $x_1 = C$ much more frequently. This generates a large decline in the unemployment rate of workers with a criminal record—particularly for black workers—while leaving workers without a criminal record only slightly worse off.

of screening does not have a large effect in markets where firms attempt to infer worker's productivity from their duration of unemployment. One key difference between the two papers—in addition to the very different contexts—is that we allow for free entry, which ultimately has a large effect in our framework.

# A Proofs

## A.1 Proof of Lemma 1

- To simplify notation, define $\tau = \frac{\rho}{1+\rho}$.
- First, consider applicants with rank 1. Suppose a firm has received $n_1$ applications from such applicants and has a capacity to conduct at most $n_R$ interviews; then the actual number of interviews with rank 1 applicants is $n_I = \min\{n_1, n_R\}$. Hence, the expected number of interviews with applicants with rank 1 is

$$\begin{aligned} I_1 &= \sum_{n_1=0}^{\infty} \sum_{n_R=1}^{\infty} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_R-1} (1-\tau) \min\{n_1, n_R\} \\ &= \sum_{n_1=1}^{\infty} \sum_{n_R=1}^{n_1} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_R-1} (1-\tau) n_R + \sum_{n_1=1}^{\infty} \sum_{n_R=n_1+1}^{\infty} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_R-1} (1-\tau) n_1 \\ &= \frac{1-e^{-\lambda_1(1-\tau)}}{1-\tau}. \end{aligned}$$

- The probability for an applicant with rank 1 to be interviewed therefore equals

$$\frac{I_1}{\lambda_1} = \frac{1-e^{-\lambda_1(1-\tau)}}{\lambda_1(1-\tau)}. \tag{22}$$

- As one would expect, $I_1/\lambda_1 = \left(1-e^{-\lambda_1}\right)/\lambda_1$ if $\tau = 0$, and $I_1/\lambda_1 \to 1$ if $\tau \to 1$.
- Similar logic implies that

$$I_1 + I_2 = \frac{1-e^{-(\lambda_1+\lambda_2)(1-\tau)}}{1-\tau}.$$

- Hence, the expected number of interviews with applicants with rank 2 is

$$I_2 = e^{-\lambda_1(1-\tau)} \frac{1-e^{-\lambda_2(1-\tau)}}{1-\tau}.$$

- The probability for an applicant with rank 2 to be interviewed therefore equals

$$\frac{I_2}{\lambda_2} = e^{-\lambda_1(1-\tau)} \frac{1-e^{-\lambda_2(1-\tau)}}{\lambda_2(1-\tau)}. \tag{23}$$

### A.2 Proof of Lemma 2

We distinguish various cases, based on the composition of the applicant pool. Let $n_i \sim Poi(\lambda_i)$ denote the number of applicants of rank $i$. To simplify notation, define $\tau = \frac{\rho}{1+\rho}$.

**No applicants ($n_1 = n_2 = 0$).** The firm does not match.

**Some rank 1 applicants, but no rank 2 applicants ($n_1 > 0, n_2 = 0$).** The probability that the applicant pool has this composition and that the firm hires a productive applicant with rank 1 is

$$\begin{aligned} A_1(y) &= e^{-\lambda_2} \sum_{n_1=1}^{\infty} \sum_{n_R=1}^{\infty} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_R-1} (1-\tau) \left(1 - F_1(y)^{\min\{n_1,n_R\}}\right) \\ &= e^{-\lambda_2} \frac{1-F_1(y)}{1-\tau F_1(y)} \left(1 - e^{-\lambda_1(1-\tau F_1(y))}\right), \end{aligned}$$

where the second equality follows from Wolthoff (2018).

**No rank 1 applicants, but some rank 2 applicants ($n_1 = 0, n_2 > 0$).** In the same way, the probability that the applicant pool has this composition and that the firm hires an applicant with rank 2 is

$$A_2(y) = e^{-\lambda_1} \frac{1-F_2(y)}{1-\tau F_2(y)} \left(1 - e^{-\lambda_2(1-\tau F_2(y))}\right).$$

**Both rank 1 and rank 2 applicants ($n_1 > 0, n_2 > 0$).** Consider three subcases, based on how the interview constraint binds.

**Only rank 1 applicants are interviewed ($n_R \leq n_1$).** The joint probability of this event and the event that the firm hires a productive applicant with rank 1 is

$$\begin{aligned} B_1(y) &= \left(1 - e^{-\lambda_2}\right) \sum_{n_1=1}^{\infty} \sum_{n_R=1}^{n_1} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_R-1} (1-\tau) \left(1 - F_1(y)^{n_R}\right) \\ &= \left(1 - e^{-\lambda_2}\right) \frac{1-\tau}{\tau} \sum_{n_1=1}^{\infty} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} \sum_{n_R=1}^{n_1} \tau^{n_R} \left(1 - F_1(y)^{n_R}\right). \end{aligned}$$

Note that

$$\sum_{n_1=1}^{\infty} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} \sum_{n_R=1}^{n_1} \tau^{n_R} = \frac{\tau}{1-\tau} \left(1 - e^{-\lambda_1(1-\tau)}\right).$$

Therefore

$$B_1(y) = \left(1 - e^{-\lambda_2}\right)\left[1 - e^{-\lambda_1(1-\tau)} - \frac{(1-\tau)F_1(y)}{1-\tau F_1(y)}\left(1 - e^{-\lambda_1(1-\tau F_1(y))}\right)\right].$$

Naturally, $B_2(y) = 0$.

**All rank 1 and some (but not all) rank 2 applicants are interviewed ($n_1 < n_R < n_1 + n_2$).** A firm interviewing $i \geq 1$ applicants of rank 2 and $j$ applicants of rank 1 will hire an applicant of rank 1 with some particular $y_0$ if and only if all other applicants draw a productivity smaller than $y_0$. The likelihood of this event is $jF_2^i(y_0)\, F_1^{j-1}(y_0)\, f_1(y_0)$. Therefore, the probability that the firm hires an applicant of rank 1 (and that both rank 1 and rank 2 apply and all rank 1 and some (but not all) rank 2 applicants are interviewed) is

$$\begin{aligned} D_1(y) &= (1-\tau)\, e^{-\lambda_1} e^{-\lambda_2} \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{n_R=n_1+1}^{n_1+n_2-1} \frac{\lambda_1^{n_1}}{n_1!} \frac{\lambda_2^{n_2}}{n_2!} \tau^{n_R-1} \int_y^{\infty} F_2(y_0)^{n_R-n_1} dF_1(y_0)^{n_1} \\ &= (1-\tau)\, e^{-\lambda_1} e^{-\lambda_2} \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \frac{\lambda_1^{n_1}}{n_1!} \frac{\lambda_2^{n_2}}{n_2!} \tau^{n_1-1} \int_y^{\infty} \sum_{n=1}^{n_2-1} \tau^n F_2(y_0)^n dF_1(y_0)^{n_1}. \end{aligned}$$

Since

$$\sum_{n=1}^{n_2-1} z^n = \frac{z - z^{n_2}}{1-z},$$

$D_1(y)$ simplifies to

$$D_1(y) = \frac{1-\tau}{\tau} e^{-\lambda_1} e^{-\lambda_2} \int_y^{\infty} \sum_{n_2=1}^{\infty} \frac{\lambda_2^{n_2}}{n_2!} \frac{\tau F_2(y_0) - \tau^{n_2} F_2(y_0)^{n_2}}{1 - \tau F_2(y_0)} d \sum_{n_1=1}^{\infty} \tau^{n_1} \frac{\lambda_1^{n_1}}{n_1!} F_1(y_0)^{n_1}.$$

Further, note that

$$\sum_{n=1}^{\infty} \frac{z^j}{j!} = e^z - 1.$$

As a result,

$$\begin{aligned} e^{-\lambda_1} \sum_{n_1=1}^{\infty} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_1} F_1(y_0)^{n_1} &= e^{-\lambda_1(1-\tau F_1(y_0))} e^{-\lambda_1 \tau F_1(y_0)} \sum_{n_1=1}^{\infty} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_1} F_1(y_0)^{n_1} \\ &= e^{-\lambda_1(1-\tau F_1(y_0))}\left(1 - e^{-\lambda_1 \tau F_1(y_0)}\right) \\ &= e^{-\lambda_1(1-\tau F_1(y_0))} - e^{-\lambda_1}. \end{aligned}$$

Therefore,

$$D_1(y) = \frac{1-\tau}{\tau} \int_y^\infty \frac{\left(1-e^{-\lambda_2}\right)\tau F_2(y_0) - \left(e^{-\lambda_2[1-\tau F_2(y_0)]} - e^{-\lambda_2}\right)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]}.$$

Similarly,

$$\begin{aligned} D_2(y) &= (1-\tau)\, e^{-\lambda_1} e^{-\lambda_2} \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \sum_{n_R=n_1+1}^{n_1+n_2-1} \frac{\lambda_1^{n_1}}{n_1!} \frac{\lambda_2^{n_2}}{n_2!} \tau^{n_R-1} \int_y^\infty F_1(y_0)^{n_1} dF_2(y_0)^{n_R-n_1} \\ &= (1-\tau)\, e^{-\lambda_1} e^{-\lambda_2} \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} \frac{\lambda_1^{n_1}}{n_1!} \frac{\lambda_2^{n_2}}{n_2!} \tau^{n_1-1} \int_y^\infty F_1(y_0)^{n_1} d \sum_{n=1}^{n_2-1} \tau^n F_2(y_0)^n. \end{aligned}$$

Therefore,

$$\begin{aligned} D_2(y) &= \frac{1-\tau}{\tau} e^{-\lambda_1} \int_y^\infty \sum_{n_1=1}^{\infty} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_1} F_1(y_0)^{n_1} d \sum_{n_2=1}^{\infty} \frac{e^{-\lambda_2}\lambda_2^{n_2}}{n_2!} \frac{\tau F_2(y_0) - \tau^{n_2} F_2(y_0)^{n_2}}{1-\tau F_2(y_0)} \\ &= \frac{1-\tau}{\tau} \int_y^\infty \left(e^{-\lambda_1[1-\tau F_1(y_0)]} - e^{-\lambda_1}\right) d \frac{\left(1-e^{-\lambda_2}\right)\tau F_2(y_0) - \left(e^{-\lambda_2[1-\tau F_2(y_0)]} - e^{-\lambda_2}\right)}{1-\tau F_2(y_0)} \\ &= \frac{1-\tau}{\tau} \int_y^\infty \left(e^{-\lambda_1[1-\tau F_1(y_0)]} - e^{-\lambda_1}\right) d \left[\frac{\tau F_2(y_0) - e^{-\lambda_2[1-\tau F_2(y_0)]}}{1-\tau F_2(y_0)} + e^{-\lambda_2}\right] \\ &= \frac{1-\tau}{\tau} \int_y^\infty \left(e^{-\lambda_1[1-\tau F_1(y_0)]} - e^{-\lambda_1}\right) d \left[-1 + \frac{1-e^{-\lambda_2[1-\tau F_2(y_0)]}}{1-\tau F_2(y_0)}\right] \\ &= \frac{1-\tau}{\tau} \int_y^\infty \left(e^{-\lambda_1[1-\tau F_1(y_0)]} - e^{-\lambda_1}\right) d \frac{1-e^{-\lambda_2[1-\tau F_2(y_0)]}}{1-\tau F_2(y_0)}. \end{aligned}$$

Integrating $D_2$ by parts gives

$$\begin{aligned} D_2(y) = &\frac{1-\tau}{\tau} \left[\left(e^{-\lambda_1[1-\tau]} - e^{-\lambda_1}\right) \frac{1-e^{-\lambda_2[1-\tau]}}{1-\tau} - \left(e^{-\lambda_1[1-\tau F_1(y)]} - e^{-\lambda_1}\right) \frac{1-e^{-\lambda_2[1-\tau F_2(y)]}}{1-\tau F_2(y)}\right] \\ &- \frac{1-\tau}{\tau} \int_y^\infty \frac{1-e^{-\lambda_2[1-\tau F_2(y_0)]}}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]}. \end{aligned}$$

Note

$$\int_y^\infty de^{-\lambda_1[1-\tau F_1(y_0)]} + e^{-\lambda_1(1-\tau F_1(y))} = e^{-\lambda_1(1-\tau)} - e^{-\lambda_1(1-\tau F_1(y))} + e^{-\lambda_1(1-\tau F_1(y))} = e^{-\lambda_1(1-\tau)}. \tag{24}$$

**All applicants are interviewed ($n_R \geq n_1 + n_2$).** The joint probability of this event and the event that the firm hires an applicant with rank 1 is

$$
\begin{aligned}
C_1 &= \sum_{n_1=1}^{\infty} \sum_{n_2=1}^{\infty} e^{-\lambda_1} \frac{\lambda_1^{n_1}}{n_1!} e^{-\lambda_2} \frac{\lambda_2^{n_2}}{n_2!} \tau^{n_1+n_2-1} \int_y^{\infty} F_2(y_0)^{n_2} dF_1(y_0)^{n_1} \\
&= \frac{e^{-\lambda_1} e^{-\lambda_2}}{\tau} \int_y^{\infty} \sum_{n_2=1}^{\infty} \frac{\lambda_2^{n_2}}{n_2!} \tau^{n_2} F_2(y_0)^{n_2} d \left[ \sum_{n_1=1}^{\infty} \frac{\lambda_1^{n_1}}{n_1!} \tau^{n_1} F_1(y_0)^{n_1} \right].
\end{aligned}
$$

Note again

$$
\sum_{n=1}^{\infty} \frac{z^j}{j!} = e^z - 1.
$$

Therefore

$$
\begin{aligned}
C_1(y) &= \frac{e^{-\lambda_1} e^{-\lambda_2}}{\tau} \int_y^{\infty} \left( e^{\lambda_2 \tau F_2(y_0)} - 1 \right) de^{\lambda_1 \tau F_1(y_0)} \\
&= \frac{1}{\tau} \int_y^{\infty} \left( e^{-\lambda_2 [1-\tau F_2(y_0)]} - e^{-\lambda_2} \right) de^{-\lambda_1 [1-\tau F_1(y_0)]} \\
&= \frac{1}{\tau} \int_y^{\infty} e^{-\lambda_2 [1-\tau F_2(y_0)]} de^{-\lambda_1 [1-\tau F_1(y_0)]} + \frac{1}{\tau} e^{-\lambda_2} \left( e^{-\lambda_1 (1-\tau F_1(y))} - e^{-\lambda_1 (1-\tau)} \right).
\end{aligned}
$$

Therefore

$$
C_1(y) = \frac{1}{\tau} \int_y^{\infty} \left( e^{-\lambda_2 [1-\tau F_2(y_0)]} - e^{-\lambda_2} \right) de^{-\lambda_1 [1-\tau F_1(y_0)]}.
$$

By a similar argument,

$$
C_2(y) = \frac{1}{\tau} \int_y^{\infty} \left[ e^{-\lambda_1 [1-\tau F_1(y_0)]} - e^{-\lambda_1} \right] de^{-\lambda_2 [1-\tau F_2(y_0)]}.
$$

**Summing the Parts.** Note

$$
\begin{aligned}
A_1(y) + B_1(y) =& e^{-\lambda_2} \frac{1 - F_1(y)}{1 - \tau F_1(y)} \left( 1 - e^{-\lambda_1 (1-\tau F_1(y))} \right) \\
&+ \left( 1 - e^{-\lambda_2} \right) \left[ 1 - e^{-\lambda_1 (1-\tau)} - \frac{(1-\tau) F_1(y)}{1 - \tau F_1(y)} \left( 1 - e^{-\lambda_1 (1-\tau F_1(y))} \right) \right] \\
=& \left[ e^{-\lambda_2} \left[ 1 - \tau F_1(y) \right] - (1-\tau) F_1(y) \right] \frac{1 - e^{-\lambda_1 (1-\tau F_1(y))}}{1 - \tau F_1(y)} + \left( 1 - e^{-\lambda_2} \right) \left( 1 - e^{-\lambda_1 (1-\tau)} \right) \\
=& 1 - e^{-\lambda_2} e^{-\lambda_1 (1-\tau F_1(y))} - \frac{(1-\tau) F_1(y)}{1 - \tau F_1(y)} \left( 1 - e^{-\lambda_1 (1-\tau F_1(y))} \right) - \left( 1 - e^{-\lambda_2} \right) e^{-\lambda_1 (1-\tau)}.
\end{aligned}
$$

Note

$$
\begin{aligned}
C_1(y)+D_1(y) =& \frac{1}{\tau}\int_y^\infty \left(e^{-\lambda_2[1-\tau F_2(y_0)]}-e^{-\lambda_2}\right) de^{-\lambda_1[1-\tau F_1(y_0)]} \\
&+\frac{1}{\tau}\int_y^\infty (1-\tau)\,\frac{\left(1-e^{-\lambda_2}\right)\tau F_2(y_0)-\left(e^{-\lambda_2[1-\tau F_2(y_0)]}-e^{-\lambda_2}\right)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]} \\
=& \int_y^\infty \left[\frac{(1-F_2(y_0))e^{-\lambda_2[1-\tau F_2(y_0)]}+(1-\tau)\,F_2(y_0)}{1-\tau F_2(y_0)}-e^{-\lambda_2}\right] de^{-\lambda_1[1-\tau F_1(y_0)]} \\
=& -\int_y^\infty \left[\frac{(1-F_2(y_0))\left[1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right]}{1-\tau F_2(y_0)}-1+e^{-\lambda_2}\right] de^{-\lambda_1[1-\tau F_1(y_0)]} \\
=& -\int_y^\infty \left(1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right)\frac{1-F_2(y_0)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]} \\
&+\left(1-e^{-\lambda_2}\right)\left[e^{-\lambda_1(1-\tau)}-e^{-\lambda_1(1-\tau F_1(y))}\right].
\end{aligned}
$$

where the last step follows from (24).

Therefore, the probability that the firm hires a worker with rank 1 equals

$$
\begin{aligned}
\eta_1(y) &= A_1(y)+B_1(y)+C_1(y)+D_1(y) \\
&= \frac{1-F_1(y)}{1-\tau F_1(y)}\left(1-e^{-\lambda_1(1-\tau F_1(y))}\right) \\
&\quad -\int_y^\infty \left(1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right)\frac{1-F_2(y_0)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]}
\end{aligned}
$$

Consider now $C_2(y)$ and $D_2(y)$. Note that integration by parts gives

$$
\begin{aligned}
C_2(y) =& \frac{1}{\tau}\int_y^\infty \left[e^{-\lambda_1[1-\tau F_1(y_0)]}-e^{-\lambda_1}\right] de^{-\lambda_2[1-\tau F_2(y_0)]} \\
=& \frac{1}{\tau}\left[e^{-\lambda_1[1-\tau]}-e^{-\lambda_1}\right]e^{-\lambda_2[1-\tau]}-\frac{1}{\tau}\left[e^{-\lambda_1[1-\tau F_1(y)]}-e^{-\lambda_1}\right]e^{-\lambda_2[1-\tau F_2(y)]} \\
&-\frac{1}{\tau}\int_y^\infty e^{-\lambda_2[1-\tau F_2(y_0)]} de^{-\lambda_1[1-\tau F_1(y_0)]}.
\end{aligned}
$$

The sum of the integral parts of $C_2(y)$ and $D_2(y)$ then equals

$$
\begin{aligned}
&-\frac{1}{\tau}\int_y^\infty \left[e^{-\lambda_2[1-\tau F_2(y_0)]}+(1-\tau)\,\frac{1-e^{-\lambda_2[1-\tau F_2(y_0)]}}{1-\tau F_2(y_0)}\right] de^{-\lambda_1[1-\tau F_1(y_0)]} \\
&=-\frac{1}{\tau}\int_y^\infty \left[1-\frac{\tau(1-F_2(y_0))}{1-\tau F_2(y_0)}\left(1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right)\right] de^{-\lambda_1[1-\tau F_1(y_0)]} \\
&=-\frac{1}{\tau}\left[e^{-\lambda_1(1-\tau)}-e^{-\lambda_1(1-\tau F_1(y))}\right]+\int_y^\infty \left(1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right)\frac{1-F_2(y_0)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]}.
\end{aligned}
$$

Hence,

$$\begin{aligned}
C_2(y) + D_2(y) &= -\frac{1}{\tau}\left[e^{-\lambda_1(1-\tau)} - e^{-\lambda_1(1-\tau F_1(y))}\right] \\
&+ \left(e^{-\lambda_1[1-\tau]} - e^{-\lambda_1}\right)\frac{1}{\tau} \\
&- \left(e^{-\lambda_1[1-\tau F_1(y)]} - e^{-\lambda_1}\right)\frac{1}{\tau}\frac{1-\tau+\tau(1-F_2(y))e^{-\lambda_2[1-\tau F_2(y)]}}{1-\tau F_2(y)} \\
&+ \int\left(1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right)\frac{1-F_2(y_0)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]} \\
&= \left[e^{-\lambda_1(1-\tau F_1(y))} - e^{-\lambda_1}\right](1-F_2(y))\frac{1-e^{-\lambda_2[1-\tau F_2(y)]}}{1-\tau F_2(y)} \\
&+ \int\left(1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right)\frac{1-F_2(y_0)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]}
\end{aligned}$$

If we now add $A_2$, we get

$$\begin{aligned}
\eta_2(y) = A_2(y) + C_2(y) + D_2(y) &= e^{-\lambda_1(1-\tau F_1(y))}\frac{1-F_2(y)}{1-\tau F_2(y)}\left(1-e^{-\lambda_2(1-\tau F_2(y))}\right) \\
&+ \int\left(1-e^{-\lambda_2[1-\tau F_2(y_0)]}\right)\frac{1-F_2(y_0)}{1-\tau F_2(y_0)} de^{-\lambda_1[1-\tau F_1(y_0)]}
\end{aligned}$$

Hence,

$$\begin{aligned}
\eta(y) &= \eta_1(y) + \eta_2(y) \\
&= \frac{1-F_1(y)}{1-\tau F_1(y)}\left(1-e^{-\lambda_1(1-\tau F_1(y))}\right) + e^{-\lambda_1(1-\tau F_1(y))}\frac{1-F_2(y)}{1-\tau F_2(y)}\left(1-e^{-\lambda_2(1-\tau F_2(y))}\right).
\end{aligned}$$

Applying the definition of the effective queue length then yields the expression in the proposition. □

## A.3 Proof of Proposition 1

To establish the existence of a unique $\rho^\star$, we prove that $\eta(y;\rho)$ is concave in $\rho$. To simplify notation, omit the argument $y$, and let $Z_i = 1-\tau F_i$, $m_i = 1-e^{-\lambda_i Z_i}$ and $H_i = \frac{1-F_i}{1-\tau F_i}$, where again $\tau = \rho/(1+\rho)$. Further, $a_2 = -(1-m_2)(1-m_1)H_2 < 0$ and $a_1 = -(1-m_1)(H_1-H_2)$. Then, we can rewrite (5) as

$$\eta(\rho) = m_1 H_1 + (1-m_1)\, m_2 H_2,$$

such that

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= \frac{\partial^2 m_1}{\partial\rho^2}H_1+2\frac{\partial m_1}{\partial\rho}\frac{\partial H_1}{\partial\rho}+m_1\frac{\partial^2 H_1}{\partial\rho^2}\\ &\quad -\frac{\partial^2 m_1}{\partial\rho^2}m_2H_2-2\frac{\partial m_1}{\partial\rho}\frac{\partial m_2}{\partial\rho}H_2+(1-m_1)\frac{\partial^2 m_2}{\partial\rho^2}H_2\\ &\quad -2\frac{\partial m_1}{\partial\rho}m_2\frac{\partial H_2}{\partial\rho}+2(1-m_1)\frac{\partial m_2}{\partial\rho}\frac{\partial H_2}{\partial\rho}\\ &\quad +(1-m_1)m_2\frac{\partial^2 H_2}{\partial\rho^2}.\end{aligned}$$

Note that

$$\frac{\partial Z_i}{\partial\rho}=-\frac{F_i}{(1+\rho)^2},$$

$$\frac{\partial m_i}{\partial\rho}=-\frac{\lambda_iF_ie^{-\lambda_iZ_i}}{(1+\rho)^2}=-\frac{\lambda_iF_i}{(1+\rho)^2}(1-m_i),$$

$$\begin{aligned}\frac{\partial^2 m_i}{\partial\rho^2} &= -\frac{\lambda_iF_i-2(1+\rho)}{(1+\rho)^4}\lambda_iF_ie^{-\lambda_iZ_i}\\ &= \left[2-\frac{\lambda_iF_i}{1+\rho}\right]\frac{\lambda_iF_i}{(1+\rho)^3}(1-m_i),\end{aligned}$$

$$\frac{\partial H_i}{\partial\rho}=\frac{1}{(1+\rho)^2}\frac{F_i(1-F_i)}{\left(1-\frac{\rho}{1+\rho}F_i\right)^2}=\frac{1}{(1+\rho)^2}\frac{F_i}{Z_i}H_i,$$

and

$$\begin{aligned}\frac{\partial^2 H_i}{\partial\rho^2} &= -2\frac{1}{(1+\rho)^3}\frac{F_i}{Z_i}H_i+\frac{1}{(1+\rho)^2}\frac{F_i}{Z_i}\frac{\partial H_i}{\partial\rho}+\frac{1}{(1+\rho)^4}\frac{F_i^2}{Z_i^2}H_i\\ &= \left[-2\frac{1}{(1+\rho)^3}\frac{F_i}{Z_i}+\frac{1}{(1+\rho)^4}\left(\frac{F_i}{Z_i}\right)^2+\frac{1}{(1+\rho)^4}\frac{F_i^2}{Z_i^2}\right]H_i\\ &= -\frac{2}{(1+\rho)^3}\frac{F_i}{Z_i}\left[1-\frac{1}{(1+\rho)}\frac{F_i}{Z_i}\right]H_i\\ &= -\frac{2}{(1+\rho)^3}\frac{F_i}{Z_i}\left[\frac{(1+\rho)-(1+\rho)F_i}{(1+\rho)Z_i}\right]H_i\\ &= -\frac{2}{(1+\rho)^3}\frac{F_i}{Z_i}H_iH_i\\ &= -\frac{2}{1+\rho}H_i\frac{\partial H_i}{\partial\rho}.\end{aligned}$$

Substituting the expressions of the derivatives of $H_i$ with respect to $\rho$ yields

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= \left[2-\frac{\lambda_1F_1}{1+\rho}\right]\frac{\lambda_1F_1}{(1+\rho)^3}(1-m_1)(H_1-m_2H_2)+\left[-2\frac{\lambda_1F_1}{(1+\rho)^2}(1-m_1)-\frac{2}{1+\rho}H_1m_1\right]\frac{1}{(1+\rho)^2}\frac{F_1}{Z_1}H\\ &\quad -2\frac{\partial m_1}{\partial\rho}\frac{\partial m_2}{\partial\rho}H_2+(1-m_1)\frac{\partial^2 m_2}{\partial\rho^2}H_2\\ &\quad +\left[-2\frac{\partial m_1}{\partial\rho}m_2+2(1-m_1)\frac{\partial m_2}{\partial\rho}-\frac{2}{1+\rho}H_2(1-m_1)m_2\right]\frac{1}{(1+\rho)^2}\frac{F_2}{Z_2}H_2.\end{aligned}$$

Plugging in the derivatives of $m_i$ with respect to $\rho$ then gives

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= \left[2-\frac{\lambda_1F_1}{1+\rho}\right]\frac{\lambda_1F_1}{(1+\rho)^3}(1-m_1)(H_1-m_2H_2)+\left[-2\frac{\lambda_1F_1}{(1+\rho)^2}(1-m_1)-\frac{2}{1+\rho}H_1m_1\right]\frac{1}{(1+\rho)^2}\frac{F_1}{Z_1}H\\ &\quad -2\frac{\lambda_1F_1}{(1+\rho)^2}(1-m_1)\frac{\lambda_2F_2}{(1+\rho)^2}(1-m_2)H_2+(1-m_1)\left[2-\frac{\lambda_2F_2}{1+\rho}\right]\frac{\lambda_2F_2}{(1+\rho)^3}(1-m_2)H_2\\ &\quad +2(1-m_1)\left[\frac{\lambda_1F_1}{1+\rho}m_2-\frac{\lambda_2F_2}{1+\rho}(1-m_2)-H_2m_2\right]\frac{1}{(1+\rho)^3}\frac{F_2}{Z_2}H_2.\end{aligned}$$

Next, we rewrite the above using the definitions of $a_i$ and collect terms with common denominators.

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= \frac{2}{(1+\rho)^3}\left[-(a_1+a_2)\lambda_1F_1-a_2\lambda_2F_2-\frac{m_1F_1H_1^2}{Z_1}-\frac{(1-m_1)m_2F_2H_2^2}{Z_2}\right]\\ &\quad +\frac{1}{(1+\rho)^4}\left[(a_1+a_2)(\lambda_1F_1)^2+a_2(\lambda_2F_2)^2+2a_2\lambda_2F_2\lambda_1F_1\right]\\ &\quad +\frac{1}{(1+\rho)^4}\left[2a_2\lambda_2F_2\frac{F_2}{Z_2}-2\lambda_1F_1(1-m_1)\left[H_1\frac{F_1}{Z_1}-m_2\frac{F_2}{Z_2}H_2\right]\right].\end{aligned}$$

Proceeding with further rewriting, we obtain

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= -\frac{2}{(1+\rho)^3}(\lambda_2F_2+\lambda_1F_1)a_2+\frac{1}{(1+\rho)^4}(\lambda_2F_2+\lambda_1F_1)^2a_2+\frac{2}{(1+\rho)^4}\lambda_2\frac{F_2^2}{Z_2}a_2\\ &\quad -\frac{2}{(1+\rho)^3}\lambda_1F_1a_1+\frac{1}{(1+\rho)^4}(\lambda_1F_1)^2a_1\\ &\quad -\frac{2}{(1+\rho)^3}\left(\frac{m_1F_1H_1^2}{Z_1}+\frac{(1-m_1)m_2F_2H_2^2}{Z_2}\right)-\frac{2}{(1+\rho)^4}\lambda_1F_1(1-m_1)\left[H_1\frac{F_1}{Z_1}-H_2m_2\frac{F_2}{Z_2}\right]\\ &= \frac{2}{(1+\rho)^3}\left[-(\lambda_2F_2+\lambda_1F_1)a_2+\frac{1}{1+\rho}\lambda_2\frac{F_2^2}{Z_2}a_2-\lambda_1F_1a_1-\left(\frac{m_1F_1H_1^2}{Z_1}+\frac{(1-m_1)m_2F_2H_2^2}{Z_2}\right)\right]\\ &\quad -\frac{2}{(1+\rho)^4}\lambda_1F_1(1-m_1)\left[H_1\frac{F_1}{Z_1}-H_2m_2\frac{F_2}{Z_2}\right]+a_2\left(\frac{\lambda_2F_2+\lambda_1F_1}{(1+\rho)^2}\right)^2+a_1\left(\frac{\lambda_1F_1}{(1+\rho)^2}\right)^2.\end{aligned}$$

Note that $1-H_i=\frac{1}{1+\rho}\frac{F_i}{Z_i}$ implies

$$\begin{aligned}-\left(\lambda_2F_2+\lambda_1F_1\right)a_2+\frac{1}{1+\rho}\lambda_2\frac{F_2^2}{Z_2}a_2-\lambda_1F_1a_1 &= -\left(\lambda_2F_2+\lambda_1F_1\right)a_2+\left(1-H_2\right)\lambda_2F_2a_2-\lambda_1F_1a_1\\ &= -H_2\lambda_2F_2a_2-\lambda_1F_1\left(a_2+a_1\right).\end{aligned}$$

So, we get

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= \frac{2}{(1+\rho)^3}\left[-H_2\lambda_2F_2a_2-\lambda_1F_1\left(a_2+a_1\right)\right]\\ &\quad -\frac{2}{(1+\rho)^3}\left[\left(\frac{m_1F_1H_1^2}{Z_1}+\frac{(1-m_1)m_2F_2H_2^2}{Z_2}\right)+\frac{1}{1+\rho}\lambda_1F_1(1-m_1)\left[H_1\frac{F_1}{Z_1}-H_2m_2\frac{F_2}{Z_2}\right]\right]\\ &\quad +a_2\left(\frac{\lambda_2F_2+\lambda_1F_1}{(1+\rho)^2}\right)^2+a_1\left(\frac{\lambda_1F_1}{(1+\rho)^2}\right)^2.\end{aligned}$$

Once again using the definitions of $a_i$, we obtain

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= \frac{2}{(1+\rho)^3}\left[\left(1-m_2\right)\left(1-m_1\right)\lambda_2F_2H_2^2+\left(1-m_1\right)\lambda_1F_1\left(H_1-m_2H_2\right)\right]\\ &\quad -\frac{2}{(1+\rho)^3}\left[\left(\frac{m_1F_1H_1^2}{Z_1}+\frac{(1-m_1)m_2F_2H_2^2}{Z_2}\right)+\frac{1}{1+\rho}\lambda_1F_1(1-m_1)\left[H_1\frac{F_1}{Z_1}-H_2m_2\frac{F_2}{Z_2}\right]\right]\\ &\quad +a_2\left(\frac{\lambda_2F_2+\lambda_1F_1}{(1+\rho)^2}\right)^2+a_1\left(\frac{\lambda_1F_1}{(1+\rho)^2}\right)^2.\end{aligned}$$

Using again $1-H_i=\frac{1}{1+\rho}\frac{F_i}{Z_i}$, we get

$$\left(1-m_1\right)\lambda_1F_1\left[\left(H_1-m_2H_2\right)-\frac{1}{1+\rho}\left[H_1\frac{F_1}{Z_1}-H_2m_2\frac{F_2}{Z_2}\right]\right]=\left(1-m_1\right)\lambda_1F_1\left(H_1^2-m_2H_2^2\right).$$

So, we obtain

$$\begin{aligned}\frac{\partial^2\eta}{\partial\rho^2} &= \frac{2}{(1+\rho)^3}\left[\left(1-m_2\right)\left(1-m_1\right)\lambda_2F_2H_2^2-\left(\frac{m_1F_1H_1^2}{Z_1}+\frac{(1-m_1)m_2F_2H_2^2}{Z_2}\right)\right]\\ &\quad +\frac{2}{(1+\rho)^3}\left(1-m_1\right)\lambda_1F_1\left(H_1^2-m_2H_2^2\right)+a_2\left(\frac{\lambda_2F_2+\lambda_1F_1}{(1+\rho)^2}\right)^2+a_1\left(\frac{\lambda_1F_1}{(1+\rho)^2}\right)^2.\end{aligned}$$

Collecting the terms with $H_2^2$ and $H_1^2$ gives

$$\frac{\partial^2\eta}{\partial\rho^2} = \frac{2}{(1+\rho)^3}\left[(1-m_1)(1-m_2)\left[\lambda_2 F_2 - \frac{m_2}{1-m_2}\frac{F_2}{Z_2} - \frac{m_2}{1-m_2}\lambda_1 F_1\right] H_2^2\right]$$
$$+\frac{2}{(1+\rho)^3}(1-m_1)\left[\lambda_1 F_1 - \frac{m_1}{1-m_1}\frac{F_1}{Z_1}\right] H_1^2 + a_2\left(\frac{\lambda_2 F_2 + \lambda_1 F_1}{(1+\rho)^2}\right)^2 + a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2}\right)^2.$$

Hence,

$$\frac{\partial^2\eta}{\partial\rho^2} = \frac{2}{(1+\rho)^3}\left[(1-m_1)(1-m_2)\left[\lambda_2 F_2 - \frac{m_2}{1-m_2}\frac{F_2}{Z_2}\right] H_2^2 + (1-m_1)\left[\lambda_1 F_1 - \frac{m_1}{1-m_1}\frac{F_1}{Z_1}\right] H_1^2\right]$$
$$-\frac{2}{(1+\rho)^3}(1-m_1)(1-m_2)\left[\frac{m_2}{1-m_2}\lambda_1 F_1\right] H_2^2 + a_2\left(\frac{\lambda_2 F_2 + \lambda_1 F_1}{(1+\rho)^2}\right)^2 + a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2}\right)^2.$$

Note that

$$\begin{aligned}
a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2}\right)^2 &= a_1\left[\left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2 + \left(\frac{\lambda_1 F_1}{(1+\rho)^2}\right)^2 - \left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2\right] \\
&= a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2 + a_1\left[\left(\frac{\lambda_1 F_1}{(1+\rho)^2}\right)^2 - \left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2\right] \\
&= a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2 + a_1\frac{m_2 H_2}{1+\rho}\left(\frac{2\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right).
\end{aligned}$$

We therefore obtain

$$\frac{\partial^2\eta}{\partial\rho^2} = \frac{2}{(1+\rho)^3}\left[(1-m_1)(1-m_2)\left[\lambda_2 F_2 - \frac{m_2}{1-m_2}\frac{F_2}{Z_2}\right] H_2^2 + (1-m_1)\left[\lambda_1 F_1 - \frac{m_1}{1-m_1}\frac{F_1}{Z_1}\right] H_1^2\right]$$
$$+ a_2\left(\frac{\lambda_2 F_2 + \lambda_1 F_1}{(1+\rho)^2}\right)^2 + a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2$$
$$- \frac{2}{(1+\rho)^3}\left[(1-m_1)(1-m_2)\left[\frac{m_2}{1-m_2}\lambda_1 F_1\right] H_2^2\right] + a_1\frac{m_2 H_2}{1+\rho}\left(\frac{2\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right).$$

Note that we can write the third line as

$$
\begin{aligned}
& - \frac{2}{(1+\rho)^3}\left[(1-m_1)\,(1-m_2)\left[\frac{m_2}{1-m_2}\lambda_1 F_1\right] H_2^2\right] + a_1\frac{m_2 H_2}{1+\rho}\left(\frac{2\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right) \\
=& - \frac{2}{(1+\rho)^3}\,(1-m_1)\, m_2\lambda_1 F_1 H_2^2 - (1-m_1)(H_1-H_2)\frac{m_2 H_2}{1+\rho}\left(\frac{2\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right) \\
=& - \frac{2}{(1+\rho)^3}\,(1-m_1)\, m_2\lambda_1 F_1 H_2\,(H_2 + H_1 - H_2) + (1-m_1)(H_1-H_2)\left(\frac{m_2 H_2}{1+\rho}\right)^2 \\
=& \frac{(1-m_1)\, m_2 H_2}{(1+\rho)^2}\left[-\frac{2}{1+\rho}\lambda_1 F_1 H_1 + (H_1-H_2) m_2 H_2\right].
\end{aligned}
$$

The first term in the square brackets is negative. Further,

$$
\begin{aligned}
H_1 - H_2 =& \frac{1-F_1}{Z_1} - \frac{1-F_2}{Z_2} = \frac{\left(1-F_2\frac{\rho}{1+\rho}\right)(1-F_1) - \left(1-F_1\frac{\rho}{1+\rho}\right)(1-F_2)}{Z_2 Z_1} \\
=& \frac{\left(1-F_2\frac{\rho}{1+\rho}\right) - \left(1-F_2\frac{\rho}{1+\rho}\right)F_1 - \left(1-F_1\frac{\rho}{1+\rho}\right) + \left(1-F_1\frac{\rho}{1+\rho}\right)F_2}{Z_2 Z_1} = \frac{1}{1+\rho}\frac{F_2-F_1}{Z_2 Z_1} \\
=& \frac{1}{1+\rho}\frac{\left(1-F_1\frac{\rho}{1+\rho}\right)F_2 - \left(1-F_2\frac{\rho}{1+\rho}\right)F_1}{Z_2 Z_1} = \frac{1}{1+\rho}\frac{Z_1 F_2 - Z_2 F_1}{Z_2 Z_1} = \frac{1}{1+\rho}\left(\frac{F_2}{Z_2} - \frac{F_1}{Z_1}\right).
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
\frac{\partial^2\eta}{\partial\rho^2} \le& \frac{2}{(1+\rho)^3}\left[(1-m_1)\,(1-m_2)\left[\lambda_2 F_2 - \frac{m_2}{1-m_2}\frac{F_2}{Z_2}\right] H_2^2 + (1-m_1)\left[\lambda_1 F_1 - \frac{m_1}{1-m_1}\frac{F_1}{Z_1}\right] H_1^2\right] \\
&+ a_2\left(\frac{\lambda_2 F_2 + \lambda_1 F_1}{(1+\rho)^2}\right)^2 + a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2 \\
&+ \frac{(1-m_1)\, m_2 H_2}{(1+\rho)^3}\left(\frac{F_2}{Z_2} - \frac{F_1}{Z_1}\right) m_2 H_2.
\end{aligned}
$$

Combining the third line with the first term in the first line gives

$$
\begin{aligned}
\frac{\partial^2\eta}{\partial\rho^2} \le& \frac{2}{(1+\rho)^3}\,(1-m_1)\,(1-m_2)\left[\lambda_2 F_2 - \frac{m_2}{1-m_2}\frac{F_2}{Z_2} + \frac{1}{2}\left(\frac{F_2}{Z_2} - \frac{F_1}{Z_1}\right)\frac{m_2^2}{1-m_2}\right] H_2^2 \\
&+ \frac{2}{(1+\rho)^3}\,(1-m_1)\left[\lambda_1 F_1 - \frac{m_1}{1-m_1}\frac{F_1}{Z_1}\right] H_1^2 \\
&+ a_2\left(\frac{\lambda_2 F_2 + \lambda_1 F_1}{(1+\rho)^2}\right)^2 + a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2.
\end{aligned}
$$

Since $F_1/Z_1 > 0$, we get

$$\begin{aligned}\frac{\partial^2 \eta}{\partial \rho^2} \leq & \frac{2}{(1+\rho)^3}(1-m_1)(1-m_2)\left[\lambda_2 F_2 - \frac{m_2}{1-m_2}\frac{F_2}{Z_2} + \frac{1}{2}\frac{F_2}{Z_2}\frac{m_2^2}{1-m_2}\right] H_2^2 \\ & + \frac{2}{(1+\rho)^3}(1-m_1)\left[\lambda_1 F_1 - \frac{m_1}{1-m_1}\frac{F_1}{Z_1}\right] H_1^2 \\ & + a_2\left(\frac{\lambda_2 F_2 + \lambda_1 F_1}{(1+\rho)^2}\right)^2 + a_1\left(\frac{\lambda_1 F_1}{(1+\rho)^2} - \frac{m_2 H_2}{1+\rho}\right)^2.\end{aligned}$$

The last line is negative. Note that The second line is negative as well because $1 - \frac{m_1}{1-m_1}\frac{1}{\lambda_1 Z_1} = -\frac{1-e^{-\lambda_1 Z_1} - \lambda_1 Z_1 e^{-\lambda_1 Z_1}}{\lambda_1 Z_1 e^{-\lambda_1 Z_1}} < 0$. Hence, a sufficient condition for $\frac{\partial^2 \eta}{\partial \rho^2}$ to be negative is that

$$1 - \frac{m_2}{1-m_2}\frac{1}{\lambda_2 Z_2} + \frac{1}{2}\frac{1}{\lambda_2 Z_2}\frac{m_2^2}{1-m_2} < 0.$$

The left-hand side of this expression is a function of one variable which is indeed always negative.

□

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