Corporate Debt Composition, Access to Finance and Monetary Policy^{*}

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Abstract

In both the U.S. and the euro area, the share of debt securities in aggregate corporate credit has grown over time. To study the implications of the corporate debt structure for the conduct of monetary policy, we develop a New Keynesian dynamic stochastic general equilibrium model where firms have access to different forms of external finance. Embedding heterogeneity in firm productivity levels and financial frictions à la Holmström and Tirole (1997) into our model leads to an equilibrium in which the most productive (and thus profitable) firms are financed through the bond market, whereas less productive firms have to rely on more costly bank lending. The least productive firms have no access to the credit market. Our setup makes both the corporate debt structure and firms' access to credit endogenous. We find that following a monetary policy contraction, aggregate corporate debt contracts and access to credit tightens. At the same time, however, there is substitution from bank loans toward bond finance, as loan supply contracts and loans become more costly relative to bonds due to a squeeze in bank equity. We also show that the model is able to qualitatively replicate empirical impulse responses to monetary policy shocks in the euro area. Our results lend support to the relevance of the bank lending channel of monetary policy transmission. They also shed light on how the corporate debt structure and credit conditions affect this transmission.

JEL classifications: E32, E44, E52, G32

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1 Introduction

The last two decades have witnessed significant transformation in the structure of corporate credit markets across the world. External financing sources alternative to traditional banking have grown substantially. This includes the rising issuance of corporate bonds and other marketable forms of debt, and the growing role of investment funds and other non-banks in corporate lending. This broadly means a transition from intermediated and monitored forms of lending towards direct and unmonitored sources of external finance.

In the U.S., bond markets have provided the most important source of external finance for nonfinancial corporations in the past few decades, while in the euro area, corporate debt has mainly been bank-based. Consequently, the ratio of corporate bonds to loans outstanding is an order of magnitude higher in the U.S. than in the euro area (Figure 1). Nonetheless, both economies have experienced a growing trend towards a greater share of bonds in firms' external financing over the past 20 to 30 years.

The observed shifts in corporate debt composition have been both cyclical and structural in nature. The corporate bond-loan ratio has tended to be counter-cyclical both in the U.S. and in the euro area, and has shown at times very large cyclical shifts (Figure 2). For example, a strong substitution from bank to bond financing occurred in advanced economies in the Global Financial Crisis of 2008, following a massive financial shock that squeezed bank equity and thus deteriorated banks' ability to lend (De Fiore and Uhlig, 2015). However, switches between the two modes of external finance are also prevalent at a smaller scale, at individual firm level, and exhibit regular patterns over the business cycle (Becker and Ivashina, 2014). They have also been a result of longer-term structural and regulatory changes. A process of disintermediation was witnessed in the U.S. in the late 1980's following the abandonment of Regulation Q (Boivin, Kiley and Mishkin, 2010), and similar developments have also been observed in emerging economies (Shin, 2013).

The structure and the cyclicality of external corporate finance is of particular interest to monetary policy transmission. If the bank lending channel of transmission is prevalent, contractionary monetary policy shocks will transmit through the banking sector by squeezing bank equity and reducing the supply of credit, akin to an adverse credit supply shock. But if other forms of external financing are available, firms may substitute bonds for loans, which offers a margin of adjustment and potentially affects the strength and the channels of monetary policy transmission.

We first present empirical evidence that supports the existence of such a bank lending channel using aggregate time series data from the euro area. In a structural vector autoregressive model, we show that a contractionary monetary policy shock leads to an increase in the price of loans relative to bond finance and an increase in the bond share in corporate debt at the aggregate level, while total corporate debt contracts.

As our main contribution, we then develop a New Keynesian dynamic stochastic general equi-



Figure 1: The aggregate corporate bond-to-loan ratio in the euro area (left) and the U.S. (right) *Notes:* The corporate bond-to-loan ratio is calculated as the ratio of stock of debt security liabilities to the stock of loan liabilities outstanding in the non-financial corporate sector. Sources: Eurostat, Euro Area Quarterly Financial Accounts, Non-Financial Corporations; The Board of Governors of the Federal Reserve System, U.S. Financial Accounts, Table Z.1, Nonfinancial Corporate Business; and authors' calculations.



Figure 2: Cyclicality of the aggregate corporate bond-to-loan ratio in the euro area (left) and the U.S. (right)

Notes: The corporate bond-to-loan ratio is calculated as the ratio of stock of debt security liabilities to the stock of loan liabilities outstanding in the non-financial corporate sector. Right panel: shaded areas indicate NBER dated recessions. Sources: Eurostat, Euro Area Quarterly National Accounts and Euro Area Quarterly Financial Accounts; The Board of Governors of the Federal Reserve System, U.S. Financial Accounts, Table Z.1, Nonfinancial Corporate Business; U.S. Bureau of Economic Analysis (BEA); U.S. National Bureau of Economic Research (NBER); authors' calculations.

librium model in which the corporate debt structure (direct versus intermediated finance) as well as firms' access to external financing are determined endogenously. The model embeds an intertemporal version of the static partial equilibrium financial contract of Holmström and Tirole (1997) (henceforth HT) in an otherwise standard New Keynesian setting.

Our model successfully replicates the empirical dynamics of the corporate debt structure following monetary policy shocks. With the model, we are able to disentangle supply effects from demand effects and show that the empirical impulse responses are consistent with an operational bank lending channel, which works through changes in credit supply.

In the model, the production of intermediate goods requires external financing. Due to moral hazard problems and heterogeneous idiosyncratic productivity levels, only the most productive firms can finance their working capital directly by issuing bonds. Less productive ones must rely on bank loans, which are more costly than bonds because bank-based finance requires monitoring. Bank monitoring, however, allows to alleviate the moral hazard and expand the set of firms that can access external finance. The least productive firms are not able to obtain any form of external financing and cannot pursue production.

Furthermore, we assume that the idiosyncratic productivity of each firm is only imperfectly observed when the financing contract is made. The expected productivity of each firm determines the pecking order of the financing mode that they choose. However, this idiosyncratic uncertainty also introduces default risk, as firms which appeared productive (and thus creditworthy) *ex ante* may turn out to be unproductive (and thus insolvent) *ex post* when debt claims are settled. Default losses are absorbed by bank equity, which makes banks' balance sheets vulnerable to aggregate shocks.

These frictions naturally give rise to an environment in which there is a clear distinction both between lenders and financial intermediaries on the one hand, and intermediaries and borrowers on the other. This setup makes the corporate debt structure as well as the extensive margin of credit access endogenous. The total quantities of bank loans and corporate bonds, as well as the external margin of obtaining any type of external funding, are endogenously determined in the general equilibrium. In particular, they depend on the aggregate equity of banks and borrower firms, which in turn depend on the aggregate economic conditions. Our model thus allows to study of the choice between modes of external finance (the internal margin of corporate debt), access to credit (the external margin), the strength of monetary policy transmission and the bank lending channel, and their impact on macroeconomic outcomes.

Our simulations show that, following a contractionary monetary policy shock, firms switch from bank loans to bonds at the internal margin, as bond finance becomes relatively cheaper and more abundant. This happens because bank equity is squeezed by the monetary policy contraction, which hampers their ability to collect deposits and supply credit to firms. Household savings are then increasingly channeled to borrowers through the bond market. This mechanism is line with the large literature stressing the relevance of the bank lending channel. Nevertheless, at the external margin, a larger share of firms are excluded from obtaining any credit as credit conditions tighten in the banking sector, and the total amount of corporate credit contracts as a consequence.

The model also draws a clear distinction between financing choices at the firm level and aggregate credit dynamics. The marginal firm-level choice between bond and bank credit is determined by the terms of the financial contract, the cost of credit and the expected revenue from production. Aggregate amounts are affected not only by per-firm borrowing, but also by the overall fraction of firms choosing a particular mode of finance as well as the share of firms excluded from the credit market.

1.1 Related literature

Our paper speaks to several strands of literature. First, we contribute to the large literature studying monetary policy transmission through the financial markets. The early literature on the credit view of monetary policy has well recognized that the relative size of the banking sector may matter for monetary policy transmission. In fact, the very existence of the bank lending channel relies on the assumption that bank credit is not a perfect substitute for other forms of debt (Bernanke and Blinder, 1988; Kashyap, Stein and Wilcox, 1993). Indeed, more recent studies have confirmed that aggregate bank credit and the ratio of corporate bonds to loans are countercyclical (e.g. Grjebine, Szczerbowicz and Tripier 2018) and that the bond-to-loan ratio tends to increase following contractionary monetary policy shocks both in the U.S. and in the euro area (Lhuissier and Szczerbowicz, 2018; Holm-Hadulla and Thürwächter, 2020).

The evidence supporting the bank lending channel was initially criticized on the grounds that the aggregate bond-to-loan ratio may simply exhibit composition effects and market segmentation between large versus small firms, rather than genuine substitution (Gertler and Gilchrist, 1994; Oliner and Rudebusch, 1996). Instead, these early contributions highlighted the role of frictions in the non-financial corporate sector, stressing the role of demand for credit instead of credit supply. However, strong counter-cyclical substitution effects between bonds and loans at the firm level have been documented more recently in firm-level data (Becker and Ivashina 2014; Crouzet 2021), although these studies provide conflicting results regarding the reaction of the corporate debt composition following exogenous monetary policy contractions.

Our paper also speaks to the theoretical literature on corporate debt structure. Apart from the classic contributions by Diamond (1991) and Holmström and Tirole (1997), theoretical microfoundations for the corporate debt composition have also been developed by, among others, Crouzet (2018) in a dynamic setting, as well as Adrian, Colla and Shin (2013) who instead take the perspective of banks operating on a Value-at-Risk constraint. Repullo and Suarez (2000) and Bolton and Freixas (2006) were the first attempts to model the interactions between the corporate debt

structure and monetary policy, albeit in static settings.

Nevertheless, there have been very few attempts to develop a structural framework in which bonds and loans coexist and in which the corporate debt structure is endogenous. The existing models are either static or do not explore the interactions of the debt composition with monetary policy. Instead, the state of the art New Keynesian monetary policy models rely on a single source of external finance. Our paper aims to fill this gap and proposes a new framework to study these issues.

Chang, Fernández and Gulan (2017) and De Fiore and Uhlig (2011, 2015) develop DSGE frameworks with endogenous corporate debt structures, but abstract from nominal frictions and monetary policy. The former paper models a small open economy to explain the rise in corporate debt issuance in emerging markets by employing the HT framework, similarly to us, while the latter rationalizes the corporate borrowing dynamics during the Global Financial Crisis as a consequence of adverse credit supply shocks. Verona, Martins and Drumond (2013) and Zivanovic (2019) are examples of papers in the New Keynesian framework that distinguish different forms of debt. However, they assume perfect market segmentation and therefore abstract from optimal choice between different forms of external financing and substitution mechanisms at the firm level. To our knowledge, our paper is the first to study the implications of an endogenous corporate debt structure for monetary policy transmission in a New Keynesian DSGE setting.

Finally, the HT double moral hazard framework has also been embedded into a DSGE model by Chen (2001), Aikman and Paustian (2006), Meh and Moran (2010), Christensen, Meh and Moran (2011), Faia (2018), Silvo (2019) and Haavio, Ripatti and Takalo (2022). These contributions, however, study a variant of the HT model where in equilibrium all firms are financed by banks. Contrary to our work, these papers thus abstract away from the endogenous choice between bonds and bank loans and the access to finance, which the flexible HT framework nonetheless allows us to analyze in general equilibrium.

2 Aggregate bond-loan substitution in the euro area

We start by prodiving evidence on aggregate bond-loan substitution following monetary policy shocks using time series data from the euro area. Our data sample covers the period 2001M1–2023M10. To control for the COVID-19 period, we simply omit the observations from 2020M1–2021M12 from our estimation sample.¹

We specify a structural vector autoregressive (SVAR) model and estimate it with Bayesian methods. To identify exogenous variation in monetary policy, we follow the approach in Jarociński and Karadi (2020). It combines a high-frequency identification strategy with sign restrictions to

¹This essentially corresponds to assigning zero weight to these observations in the approach proposed by Lenza and Primiceri (2022). Our results are robust to estimating the SVAR on the pre-COVID sample up to 2019M12.

identify exogenous variation in monetary policy, while controlling for central bank information effects.

As high-frequency series, we employ the intra-day changes in euro area Overnight Indexed Swap (OIS) rates and the STOXX50 stock market index within narrow windows around the European Central Bank's (ECB) policy announcements, provided in the Euro Area Monetary Policy Event-Study Database (EA-MPD) constructed by Altavilla, Brugnolini, Gürkaynak, Motto and Ragusa (2019). The monetary policy shock is identified through two assumptions: exogeneity of the high-frequency surprises within the narrow window, and a negative co-movement between the high-frequency surprises in the interest rates and stock market index on impact achieved by imposing sign restrictions, following Jarociński and Karadi (2020). The responses of all other variables are left unconstrained.

Our SVAR model specification includes the high-frequency interest rate and stock price surprises, the 2-year euro area OIS rate, the stock of bank loans to non-financial corporations, the stock of debt securities issued by non-financial corporations, an intermediation wedge, the real GDP (interpolated to monthly frequency), and the Harmonised Index of Consumer Prices.

We define the intermediation wedge as the difference between the loan spread and the bond spread. It measures the cost of bank credit relative to market credit. The loan spread is defined as the spread between the average interest rate on new loans to non-financial corporations and the 3-month Euribor, as Euribor rates are commonly used as reference rates for corporate loans in the euro area. The bond spread is measured by the option-adjusted spread on debt securities issued by non-financial corporations relative to maturity-matched OIS swap rates.²

All variables enter the model in log-levels, except the high-frequency surprises and the intermediation wedge, which are measured in percentage points and enter in levels. The model is estimated on monthly data using standard Minnesota priors (Litterman, 1986), with 12 lags to control for seasonal variation. The high-frequency series are aggregated to monthly frequency by summing within-month intra-day surprises. The quarterly volume of GDP is interpolated to monthly frequency with the Chow-Lin method, using monthly euro area industrial production as the highfrequency indicator series. The data are sourced from Eurostat, the ECB, and Bloomberg. The data are described in more detail in Appendix A.

The structural impulse responses to a contractionary monetary policy shock, estimated with the VAR on aggregate euro area data, are shown in Figure 3. Following an exogenous monetary policy contraction, the real GDP and the price level (HICP) contract. In addition, the total stock of corporate debt contracts. The contraction comes at a long lag, however, as the debt stock is slow to adjust. There is also substitution from loans towards bonds: the stock of corporate loans contracts, which drives the contraction in total corporate debt, while the stock of corporate bonds

 $^{^{2}}$ Our results are robust to including the loan spread and the bond spread in the model separately, but the additional variable makes the impulse responses slightly more imprecisely estimated.



Figure 3: Impulse responses to a contractionary monetary policy shock in a Bayesian SVAR model for the euro area.

Notes: The solid lines indicate median impulse responses. Dark shaded areas indicate percentiles 16–84 and the light shaded areas indicate percentiles 5–95 of the identified responses. The impulse responses are scaled to a 25 basis point change in the 2-year OIS rate on impact.

mildly expands.

These responses could be driven by decreases in either credit demand or supply. However, the intermediation wedge (the difference between the loan spread and the bond spread) increases, implying that bank loans become more expensive relative to bonds. The increase in the wedge is driven by an increase in the loan spread (not shown), while the bond spread does not significantly react. That is, financial conditions tighten in bank credit, but not substantially in the bond market. This increase in the relative price of loans together with the observed bond-loan substitution and the contraction in the stock of bank loans is consistent with a bank credit supply-driven mechanism.

Our empirical results lend support to the existence and quantitative relevance of the bank lending channel in the euro area. However, the empirical model cannot inform us about the exact channels of monetary policy transmission, and it does not allow us to directly disentangle credit demand-driven effects from credit supply-driven ones. In the next Section, we turn to formulating a general equilibrium model of corporate debt that can rationalize these empirical findings and that allows us to study the transmission mechanism in detail.

3 The model

In this Section, we present the model economy. We start by describing the general model environment and the timing of events. We then proceed by characterizing in detail the financial contract, which is an adaptation of the Holmström and Tirole (1997) framework to a dynamic setting with aggregate uncertainty. Finally, we briefly present the remaining parts of the model economy, which are standard to the New Keynesian framework.

3.1 A bird's eye view on the economy

The structure of the model economy and the interactions between various agents are summarized in Figure 4. The model economy consists of households, three types of producing firms (intermediate good producers, re-packagers, and final good producers), banks, mutual bond funds, as well as a fiscal authority (government) and a central bank. In addition, individual intermediate good producers are owned by holding companies.

The economy is inhabited by a continuum of representative households of mass one. Households consume final goods, supply labor and rent capital to intermediate good producing firms. The factor markets are perfectly competitive. Moreover, households own the stock of physical capital in the economy and make investment and saving decisions.

The financial sector consists of banks and mutual bond funds. Households can deposit their savings in banks or invest them into shares in mutual funds. In addition, households can invest into riskless government bonds. Mutual funds invest into corporate bonds issued by intermediate good firms. Banks consist of a continuum of bank branches of unit mass. Each branch collects deposits from households and extend loans to intermediate good firms, and engage in costly monitoring of the firms they lend to. The parent bank pools the profits of its branches and redistributes equity to them.

The production sector is organized as follows. The economy is inhabited by a continuum of representative holding companies of mass one ("holdings", for short). Every holding owns a continuum of intermediate good producing firms of mass one. Individual intermediate good producers are *ex ante* identical, but they are subject to idiosyncratic *i.i.d.* productivity shocks that affect the level of their output. Intermediate good producers act in perfectly competitive markets, and produce by hiring labor and capital from households.

However, they are bound by a cash-in-advance constraint on the financing of their working capital. In order to be able to pay their wage and capital rental bills and produce in a given period, they must raise external funding either from banks or from bond investors in the previous period. Each intermediate good firm receives a noisy public signal about its idiosyncratic productivity in next-period production. The signal determines the type of external funding (if any) that the firm is able to secure to finance its next-period production. The exact form of the financial constraint



Figure 4: The structure of the model economy

is described in Section 3.3 below.

The fact that the signal is imperfect means that some individual firms (those with very low realized productivity) become insolvent and default on their debt obligations in each period, when the true idiosyncratic productivity is revealed, even though they look solvent in expectation. This default risk will, as discussed later, give rise to risk for banks. Losses from firm default experienced by mutual bond funds are covered by the government.

Holdings pool together the profits of all solvent firms in their ownership and redistribute equity accumulated through these retained earnings to individual firms. The *i.i.d.* property of the idiosyncratic shock implies that the output and equity of every intermediate good producing firm are independent of its own shock history.³

Monopolistically competitive re-packagers diversify the homogeneous intermediate goods into varieties, subject to a nominal pricing friction as in Calvo (1983). Final goods producers buy these varieties and merge them into a homogeneous final composite good, which is then consumed or used otherwise in the rest of the economy.

The government collects lump-sum taxes from households and use these funds to cover for losses of the mutual bond funds. It also issues riskless nominal bonds, which are in zero net supply in equilibrium, as the government is assumed to run a balanced budget in every period. Finally, the central bank controls the nominal safe interest rate.

 $^{^{3}}$ One can also think of this setup as a perfect risk sharing scheme across all individual intermediate good producers.

3.2 Timing of events

The timing of events is as follows. Each period can be divided into two parts: the production phase and the financing phase.

It is convenient to start the sequence of events in the latter part of period t, the financing phase. The information set contains realizations of all aggregate shocks in t. Intermediate good firms are endowed with identical amounts of equity. Each individual firm obtains a noisy public signal about its idiosyncratic productivity in next-period production. Based on the signal, firms choose whether to be active (produce) in the next period, and if so, how to finance their working capital.

Firms that choose to be active raise funding for production by taking bank loans or by issuing bonds. They combine these external funds with their own equity. These funds will be used in the next period to hire labor and rent capital for production. Equity of inactive firms is invested in the domestic financial market at the nominally riskless rate. Banks pay the costs that accrue from monitoring firms to which they have lent funds. Then period t ends.

Period t + 1 begins with the production phase. At the beginning of the period, all aggregate shocks are realized. Households make their consumption, savings and labor supply choices, as they are not subject to any intra-period uncertainty.

Active intermediate good producers hire labor and rent capital from households. They pay the wage and capital rental bills using the working capital they obtained in the previous period. Then production starts. Now the true value of the idiosyncratic productivity shock of each individual firm is revealed. Some intermediate good firms' productivity turns out to be low and they become insolvent, unable to pay back their creditors. In case a firm declares insolvency, all its output from production is lost.

Successful intermediate good producers then sell their goods to re-packagers, who costlessly differentiate them and sell them onward to final good producers. Re-packagers transfer their monopolistic profits to households. Solvent firms settle their debts using revenue from production, transfer their remaining profits back to the holdings, and end their production activity. Holdings pool together these profits, set aside dividends for consumption, and re-distribute the retained earnings back to all intermediate good firms in their ownership, providing each firm an equal amount of equity for the next period. Similarly, banks pool profits and losses from their individual branches and redistribute equity across them.

Households withdraw their deposits from the banks and collect revenues from their assets. Creditors (banks and bond funds) cover for credit losses from insolvent firms. Banks set aside dividends for consumption. Final good producers sell the final goods to consumers, holdings and banks.

Households combine all sources of income and allocate their resources to consumption and savings according to their choices. They combine the undepreciated capital stock with new investment to obtain capital used in production in the next period. Finally, households, holdings and banks consume. Then, the cycle closes.

3.3 The financing contract

In this section, we describe in detail the equilibrium financing contract and the partial equilibrium in the financial market. The financial sector is subject to two distinct frictions that affect this equilibrium.

First, the external funding of intermediate good producers (henceforth, "firms") is subject to a double moral hazard problem following Holmström and Tirole (1997). Firms are subject to a "cash-in-advance" constraint and must borrow funds to finance working capital for next-period production. To do so, they can either issue bonds or borrow from bank branches. In both cases, the external funding takes the form of a one-period debt contract.

However, firms can neglect production and extract private benefits by shirking, which is not observable to other parties in the financing contract at the time the contract is made. Shirking reduces the firm's revenue from production and thus the income pledgeable to a firms' creditors. Banks can mitigate shirking by monitoring the firm, but this is associated with a non-verifiable pecuniary cost to the bank; thus, the bank may want to forgo proper monitoring. Bond investors are assumed to be "outside", or uninformed, investors who are not able to monitor firms.

Second, the firms are subject to idiosyncratic productivity shocks, which also affect revenue from engaging in production. However, the firm and its creditors only observe an imperfect signal of the productivity *ex ante*, in the period where the financing contract is made. External funding must be secured before the revelation of each firm's true productivity, which introduces individual firm default risk. Firms that receive a good signal and look solvent *ex ante* may turn out to experience unfavorable productivity shocks once production starts, and become insolvent when financial claims are settled.

We start by describing the problems of an individual intermediate good firm, bond investor, bank branch and depositor, and then derive individual equilibrium conditions in the financing contract. Finally, we characterize the partial equilibrium in the financial market with two equilibrium cut-offs that define the external and internal margins of firms' external funding, i.e. its access to external finance and the optimal mode of finance.

3.3.1 The financing problem of an individual intermediate good firm

Let us start by describing an individual firm's financing problem. Firms are risk neutral. In the latter part of period t, each firm owned by a given holding is endowed with an equal amount of equity (or "cash-on-hand"), denoted by K_t^f in nominal terms. The holding is assumed to task every firm with raising an amount of funding of fixed nominal size I, common to all firms in the holding, which will be used to finance the firm's working capital in the next period.

The firms' business is producing intermediate goods. Production starting in period t+1 involves hiring labor h_{t+1}^i and renting physical capital k_t^i from households using the available funds I, which have been raised at the end of period t. We assume that the wage bill and the capital rental costs must be paid before the firm earns revenue from selling its goods. In nominal terms, then, the working capital constraint of an individual firm i takes the form:

$$I \ge W_{t+1}h_{t+1}^i + R_{t+1}^K k_t^i, \tag{1}$$

where $W_{t+1}h_{t+1}^i$ denotes the nominal wage bill and $R_{1+t}^K k_t^i$ the nominal capital rental cost in period t+1 production.

Before deciding on whether to raise funding I and pursuing production, in period t, every firm i obtains a noisy public signal ω_t^i about its idiosyncratic productivity shock in period t+1 production (z_{t+1}^i) . The signal is disturbed by noise ϵ_{t+1}^i , so that:

$$\omega_t^i = z_{t+1}^i \epsilon_{t+1}^i$$

The signal is revealed in t, whereas its components, the true productivity and the noise, only in t + 1. The signal is publicly observable and common knowledge to all agents in the economy.

The components z_{t+1}^i and ϵ_{t+1}^i are independent and both are log-normally distributed, such that $\ln z_{t+1}^i \sim N(\mu_z, \sigma_z^2)$ and $\ln \epsilon_{t+1}^i \sim N(\mu_\epsilon, \sigma_\epsilon^2)$. In consequence, the signal ω_t^i is itself log-normal with parameters $\mu_{\omega} = \mu_z + \mu_{\epsilon}$ and $\sigma_{\omega}^2 = \sigma_z^2 + \sigma_{\epsilon}^2$.

Importantly, the distribution of z_{t+1}^i conditional on observing signal ω_t^i is also log-normal, such that $\ln z_{t+1}^i \mid \omega_t^i \sim N\left[\mu_z + \rho_{z,\omega}\frac{\sigma_z}{\sigma_\omega}\left(\ln\omega^i - \mu_\omega\right), \left(1 - \rho_{z,\omega}^2\right)\sigma_z^2\right]$, where $\rho_{z,\omega} = \frac{\sigma_z^2}{\sigma_z\sigma_\omega}$ is the correlation coefficient between the productivity shock z_{t+1} and the signal ω_t .

In addition, we normalize the expected values of z_{t+1}^i , ϵ_{t+1}^i , and ω_t^i are all equal to 1, so that $\mu_z = -\frac{\sigma_z^2}{2}$, $\mu_\epsilon = -\frac{\sigma_\epsilon^2}{2}$, and $\mu_\omega = -\frac{\sigma_z^2 + \sigma_\epsilon^2}{2}$. This implies that the expected value of (log-)productivity conditional on the signal simplifies to $\frac{\sigma_z^2}{\sigma_\omega^2} \ln \omega_t^i$.

Based on the signal it receives, each firm determines whether or not it will pursue production in the next period. We assume that $K_t^f < I$, so that each firm needs external finance in order to be able to finance its working capital. To cover its external financing needs, each firm can issue bonds, which are purchased by risk-neutral uninformed investors (bond funds), in the amount $I_t^{u,i}$. In this case, the funding constraint of a firm i is:

$$I \le K_t^f + I_t^{u,i} \tag{2}$$

where, given that I and K_t^f are common to all firms in a given holding, implies that $I_t^{u,i} = I_t^u$. The firm promises to pay back a nominal amount $R_t^{u,i}$ to the bond investor in period t + 1.

Alternatively, the firm may take a bank loan from a bank branch. These are also risk-neutral.

It will be useful to distinguish between firms who borrow through bonds and through loans; let us denote by j an individual firm who enters into a contract with a bank branch instead of bond investors. Bank branches combine their own equity $I_t^{b,j}$ and depositors' funds $I_t^{d,j}$ to fund loans. In this case, the funding constraint is:

$$I \le K_t^f + I_t^{b,j} + I_t^{d,j}.$$
 (3)

In this case, the firm agrees to paying back the nominal amounts $R_t^{b,j}$ and $R_t^{d,j}$ to the branch and its depositors, respectively.

When production ends, the firm has to split the proceeds from production between itself and its creditors. As will be explained later, the form of external financing, and whether the firm will be able to borrow at all, will depend on the realization of the signal ω_t^i .

Intermediate goods production is subject to idiosyncratic risk. If the firm decides to become active and its production is financed, its production will yield nominal revenue $z_{t+1}^i R_{t+1}^i$. This total revenue is split between all parties in the financing contract, so that:

$$z_{t+1}^i R_{t+1}^i \ge R_{t+1}^{f,i} + R_t^{u,i} \tag{4}$$

for firms financed with bonds, or

$$z_{t+1}^{j}R_{t+1}^{j} \ge R_{t+1}^{f,j} + R_{t}^{b,j} + R_{t}^{d,j}$$
(5)

for firms financed with bank loans, where $R_{t+1}^{f,i}$, $R_t^{u,i}$, $R_t^{b,j}$ and $R_t^{d,j}$ are the respective nominal payoffs of the firm, uninformed investor, bank branch and depositor. Going forward, we assume these resource constraints always hold with equality, because otherwise some revenue would be wasted.

As will be discussed in depth in Subsection 3.3.2, the part of the revenue denoted by R_{t+1}^k , for $k = \{i, j\}$, is common across all firms that undertake production. Hence we drop the superscript for notational simplicity.

While the productivity signal ω_t^i is known *ex ante*, actual productivity z_{t+1}^i and hence the revenue $z_{t+1}^i R_{t+1}$ is observed only *ex post*. The firm is the residual claimant to the revenue in either contract. It first has to pay back all its creditors the *ex ante* agreed amount, before transferring the remaining revenue to the holding.

Hence, if the realization z_{t+1}^i is low enough, it could yield $R_{t+1}^{f,i} < 0$. However, we assume that the firm is protected by limited liability, such that such strictly negative payoffs to the firm are ruled out. Accordingly, we define the *ex post* default cut-offs as the realized levels of z_{t+1}^k for which

 $R_{t+1}^{f,k} = 0$ for $k = \{i, j\}$. Using (4) and (5), these cut-offs are, respectively:

$$\bar{z}_{t+1}^i = \frac{R_t^{u,i}}{R_{t+1}} \tag{6}$$

in the bond contract, and

$$z_{t+1}^{j} = \frac{R_t^{b,j} + R_t^{d,j}}{R_{t+1}} \tag{7}$$

in the loan contract.

These are the minimum realizations of z_{t+1}^i that allow to cover all claims against the firm in full. If these conditions fail and the firm declares bankruptcy, we assume that its revenue from production is lost in full (zero recovery), so that the firm and all its creditors receive zero payoffs. Importantly, assuming that signals ω_t^k and true productivities z_{t+1}^k are common knowledge rules out strategic default or misreporting.⁴

3.3.2 The moral hazard problem and incentive compatibility

Following the framework in Holmström and Tirole (1997), we assume that every firm has a choice of how much effort to put into production. If the firm decides to behave diligently, production yields full revenue $z_{t+1}^i R_{t+1}$. If it shirks, instead, the revenue will be reduced by a factor of $(1 - \Delta)$ and yield $z_{t+1}^i R_{t+1} (1 - \Delta)$. However, shirking allows the firm to reap a non-observable and nonpecuniary private benefit $b_H I$, proportional to the size of its funding.

Bank branches have the ability to monitor firms, which allows to reduce the private benefit available to the firm from $b_H I$ to $b_L I$ in case the firm shirks, with $0 < b_L < b_H$, and thus increase the income pledgeable to the firm's financiers. However, this is costly to the bank: it incurs a pecuniary monitoring cost cI, with c > 0, that is not verifiable to the firm or the bank's depositors. Banks thus have an incentive to forgo monitoring.

Non-monitored external funding. Consider first the case of a firm i that borrows from bond investors, who have no ability to monitor the firm's activities, i.e. they are uninformed "outside investors". To ensure that a non-monitored firm does not shirk, the following incentive compatibility constraint (ICC) must hold for the firm to obtain external financing:

$$E_t \int_{\bar{z}_{t+1}^i}^{\infty} R_{t+1}^{f,i} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^i|\omega_t^i\right) \ge (1-\Delta) E_t \int_{\bar{z}_{t+1}^i}^{\infty} R_{t+1}^{f,i} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^i|\omega_t^i\right) + b_H I\left(1+i_t\right),$$

 $^{^{4}}$ Differently than in models with asymmetric information, as in *e.g.* Bernanke, Gertler and Gilchrist (1999), we do not have to impose truth-saying incentive compatibility constraints or the revelation principle.

$$E_t \int_{\bar{z}_t^i}^{\infty} R_{t+1}^{f,i} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^i|\omega_t^i\right) \ge \frac{b_H I(1+i_t)}{\Delta} \tag{8}$$

where $1 + i_t$ is the nominal interest rate between periods t and t + 1. In other words, conditional on the signal ω_t^i , firm i must get an expected payoff that is higher if it behaves than if it shirks. The larger b_H is, the more the firm needs to be compensated in order to be incentivized to behave.

In order for bond investors to be willing to lend to a non-monitored firm i, the following participation constraint (PC) must be satisfied:

$$E_t \int_{\bar{z}_{t+1}^i}^{\infty} R_t^{u,i} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^i|\omega_t^i\right) \ge (1+i_t) \, I_t^u \tag{9}$$

The right-hand side of the constraint is the opportunity cost of the uninformed investors' funds, which is just equal to the riskless nominal rate $1 + i_t$.

Given that all firms borrow the same amount, conditional on pursuing production, the righthand side of the PC is common across firms. The left-hand side of the inequality is the expected payoff to bond investors, which is signal-dependent. Perfect competition in the bond market requires that the expected default-adjusted payoffs for uninformed investors are equal, whether they invest in corporate bonds or the riskless government bond. Hence, the inequality has to bind regardless of the signal. The firm-specific agreed nominally fixed payoffs $R_t^{u,i}$ then adjust to compensate for the firm-specific default risk.

Monitored external funding. Next, consider a firm i that borrows from a bank branch. For every such monitored firm j, the ICC is:

$$E_t \int_{\underline{z}_{t+1}^j}^{\infty} R_{t+1}^{f,j} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t^j\right) \ge (1-\Delta) E_t \int_{\underline{z}_{t+1}^j}^{\infty} R_{t+1}^{f,j} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t^j\right) + b_L I\left(1+i_t\right),$$

or, rearranging,

$$E_t \int_{\underline{z}_{t+1}^j}^{\infty} R_{t+1}^{f,j} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t^j\right) \ge \frac{b_L I(1+i_t)}{\Delta} \tag{10}$$

The fact that z_{t+1}^k is firm-specific for k = i, j implies that the expected firm payoff is firm-specific as well. Hence, the inequalities (8) and (10) are not binding for all firms. We impose regularity conditions on the distributions of ω_t^k and z_t^k that guarantee that firms with a higher signal ω_t^k will expect to receive a higher payoff R_{t+1}^{f} .⁵

In order for depositors to be willing to deposit funds in a bank branch that lends to monitored

or

⁵For monitored firms, we also have to assume that the LHS of (10) is less than $\frac{b_H I(1+i_t)}{\Delta}$; otherwise, they will choose to behave diligently even absent monitoring. See Holmström and Tirole (1997), p. 672.

firms, the following participation constraint must be satisfied:

$$R_t^{d,j} \ge (1+i_t) I_t^{d,j} \tag{11}$$

In other words, we assume that the deposit contract involves a nominally fixed payment, so $R_t^{d,j}$ is known *ex ante*.

The lack of adjustment for default losses in (11) reflects the fact that depositors are senior claimants relative to banks. If a debtor firm defaults, the losses of the bank branch will be covered from the parent bank's own equity. We assume that banks are always able to pay back its depositors, and bank deposits are thus nominally riskless for the depositor.⁶

Competition between banks further implies that deposits are paid a common minimum, so that (11) holds with equality. Finally, as argued below, $I^{b,j}$ is common across firms, which also implies a common value for $I_t^{d,j}$ and $R_t^{d,j}$, given equation (3) and the commonality of K_t^f and I. Hence the superscripts j can be dropped for notational simplicity.

Bank branches separately face an incentive problem, because monitoring is costly but non-verifiable. For them to diligently monitor firms, the following ICC must hold:

$$E_{t} \int_{\underline{z}_{t+1}^{j}}^{\infty} \left(R_{t}^{b,j} + R_{t}^{d} \right) dF_{z|\omega} \left(z_{t+1}^{j} | \omega_{t}^{j} \right) - cI \left(1 + i_{t} \right) - R_{t}^{d} \ge$$
$$(1 - \Delta) \left[E_{t} \int_{\underline{z}_{t+1}^{j}}^{\infty} \left(R_{t}^{b,j} + R_{t}^{d} \right) dF_{z|\omega} \left(z_{t+1}^{j} | \omega_{t}^{j} \right) - R_{t}^{d} \right]$$

or

$$E_t \int_{\mathbb{Z}_{t+1}^j}^{\infty} R_t^{b,j} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t^j\right) - E_t \int_0^{\mathbb{Z}_{t+1}^j} R_t^d \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t^j\right) \ge \frac{cI\left(1+i_t\right)}{\Delta}.$$
 (12)

The inequality (12) reflects the risk faced by banks. The bank branch will get revenue from its assets only if its borrowers remain solvent, which happens only when $z_{t+1}^j \ge z_{t+1}^j$. But it has to pay back its depositors in every state of the world. The incentive scheme has to take this into account. This implies that for a given loan size, the bank branch will require a signal-specific expected nominal payoff $E_t \int_{z_{t+1}^j}^{\infty} R_t^{b,j} dF_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right)$ to compensate for the corresponding default risk. Given equation (7), default cut-offs of firms borrowing from banks are signal-specific (not common, as in the bond market).

Equation (12) must hold with equality for any signal ω_t^j . To see this, observe that when the inequality binds, the lower the signal (i.e. the higher the firm's default risk *ex ante*), the higher the expected payoff to the bank branch has to be. Hence the firm with the lowest admissible signal must promise the highest expected payoff to the branch.

 $^{^{6}}$ This is facilitated by our assumption of a parent bank that is able to transfer resources across its branches, as discussed in Subsection 3.3.3.

Now assume that all firms j promise this expected payoff to the bank branch. All firms with a strictly higher signal have a strictly lower risk of default ex-ante than the firm with the lowest admissible signal. Then, equation (12) would hold with strict inequality and the bank branch would be making an expected positive profit from lending to these firms. But these firms would then be subject to an unnecessarily strict default criterion (7) *ex post*. They would potentially have to declare bankruptcy, even though they would still be economically solvent if they had promised some lower expected payoff to the bank branch. Hence, such a contract would impose unnecessary loss of resources and be inefficient.

In addition, to participate in lending, bank branches must obtain a minimum acceptable nominal rate of return on their equity $1 + i_t^{b,j}$. This rate has to be strictly higher than the gross nominal interest rate $1+i_t$, because it has to additionally remunerate bank branches for monitoring expenses and borrower default risk. The following participation constraint must then hold for an individual bank branch:

$$E_t \int_{z_{t+1}^j}^{\infty} \left(R_t^{b,j} + R_t^d \right) \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) - R_t^d \ge \left(1 + i_t^{b,j} \right) I_t^{b,j},$$

or, rearranging,

$$E_t \int_{\mathbb{Z}_{t+1}^j}^{\infty} R_t^{b,j} \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t^j\right) - E_t \int_0^{\mathbb{Z}_{t+1}^j} R_t^d \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t^j\right) \ge \left(1 + i_t^{b,j}\right) I_t^{b,j} \tag{13}$$

Although we assume that there is no free access to the banking sector, we assume that banks and their branches compete with each other. Perfect competition drives constraint (13) to hold with equality for any signal ω_t^j . Moreover, competition leads all bank branches to demand the same expected rate of return for any loan, so that $1 + i_t^{b,j} = 1 + i_t^b \forall j$.

Borrowing $I_t^{b,j}$ from banks is more costly than borrowing from depositors, so it is kept at a common minimum: $I_t^{b,j} = I_t^b$, which in turn also implies $I_t^{d,j} = I_t^d$.

Combining equations (12) and (13), and observing that both must hold with equality for any signal ω_t^j yields the following expression for the required amount of bank funding:

$$I_t^{b,j} = I_t^b = \frac{cI}{\Delta} \frac{(1+i_t)}{(1+i_t^b)}$$
(14)

3.3.3 Risk in the financing contract

Contracts between firms and its creditors are intertemporal and subject to two types of risk: aggregate and idiosyncratic. Idiosyncratic risk arises from the fact that the productivity signal ω^i is noisy, so that in practice $z_{t+1}^i \neq \omega_t^i$. Aggregate risk stems from aggregate shocks that occur after the financial contract has been written down, but before the repayment of debts is settled.

Consider first idiosyncratic risk, abstracting away from aggregate risk. A firm can borrow by

issuing a bond that is then purchased by a bond fund, where both parties are risk neutral. All firms that obtain financing are *ex ante* expected to be solvent based on their signal, but some firms nevertheless default *ex post*. Bond funds hold a representative portfolio of bonds. Hence, all firm-specific risk is diversified away. This is similar to the Holmström and Tirole (1997) setup. Households, which are risk averse, lend to firms only indirectly, by investing in a bond fund. The idiosyncratic firm risk is diversified away from the households' point of view.

A firm may alternatively borrow from a risk-neutral bank branch. An individual bank branch lends to a continuum of firms, and following Holmström and Tirole (1997), we assume that bank branches are "large" relative to each firm it lends to. We further assume that there is perfect correlation between all projects that a given bank branch finances. This perfect correlation relates both to the signal ω^i (*i.e.*, all firms funded by a given bank branch have the same signal value) as well as to actual productivities *ex post* (*i.e.*, all firms financed by a single bank branch have ex post the same value of z^i). Hence, the signal and its realization can be thought of as occurring at the bank branch level. More generally, this setup can be interpreted as bank branches specializing in lending to a particular sector of firms.⁷

There is a continuum of bank branches owned by an economy-wide, representative bank. The bank collects profits from its branches and reallocates equity to them. When branches invest in mutually uncorrelated projects, the parent bank can perfectly diversify all branch (and by extension firm-level) default risk.

When there is aggregate risk, the *ex post* state of the economy will in general be different than expected *ex ante*. Importantly, this applies to variables such as aggregate firm revenue R_{t+1} and hence to the bankruptcy cutoffs (6) and (7). Following an aggregate shock, a smaller or larger fraction of firms may go bankrupt than expected. *Ex post* revenues of banks and bond funds will then be different than expected, and they may make unexpected profits or losses.

We assume that unexpected losses of bond funds are subsidized by lump-sum transfers from the government, so that the bond funds always make zero profits *ex post*. The subsidy is financed by lump-sum taxes on households. Similarly, unexpected profits are transferred lump-sum to households.

The unexpected aggregate default risk of firms (and thus bank branches) is fully borne by the bank itself and absorbed by its equity. Households are paid back their nominal deposit amount, with accrued nominal interest, regardless of the aggregate state of the economy. This means that a bank's equity could potentially be fully wiped out by a large enough negative aggregate shock. With no deposit insurance scheme in place, this would make bank deposits risky. We abstract away

⁷The key technical reason that necessitates perfect correlation is that with independent productivity outcomes, the bank branch could cross-pledge, *i.e.* use the proceeds from one loan to cover for losses from another one. As the number of loans in the bank approaches infinity, the amount of a bank's own required equity goes to zero (see Tirole (2006)). Alternatively, one could assume that firms have independent signals and/or productivity realizations, but that cross-pledging is precluded by some friction, such as legal restrictions.

from this possibility by considering only small shocks and studying local dynamics around a steady state with no bank default.

The bond fund subsidy implies that investing in the bond fund is nominally riskless from the household's perspective. Hence, this assumption is necessary to prevent investing in bond funds riskier than investing in deposits. Given that both bond funds and banks diversify away all idiosyncratic risk, from the households' point of view both bank deposits and shares in a mutual bond fund are nominally riskless debt contracts. Households are then indifferent between investing in these two asset types as well as in government bonds.

3.3.4 Contract cut-offs and financial market equilibrium

We are now equipped to characterize the partial equilibrium in the financial market, given the incentive compatibility and participation constraints that define the individual equilibrium contract. Following Holmström and Tirole (1997), the financial market equilibrium can be characterized by two cut-off values for the productivity signal.

Consider first non-monitored financing. Define $\bar{\omega}_t$ as the minimum observed value of the signal ω_t^i necessary to obtain bond funding. To derive this cut-off signal, start by combining (8) with (4) to arrive at:

$$E_t \int_{\bar{z}_{t+1}}^{\infty} z_{t+1}^i R_{t+1} \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^i | \omega_t^i \right) - \frac{b_H I(1+i_t)}{\Delta} \ge E_t \int_{\bar{z}_{t+1}}^{\infty} R_t^u \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^i | \omega_t^i \right)$$

Substituting in equations (9) and (2), taking into account that the expressions hold with equality, and rearranging yields:

$$E_t \int_{\bar{z}_{t+1}}^{\infty} z_{t+1}^i \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^i|\bar{\omega}_t\right) = \frac{I\left(1+i_t\right)\left(1-\frac{K_t^I}{I}+\frac{b_H}{\Delta}\right)}{E_t R_{t+1}}.$$
(15)

This equilibrium condition for $\bar{\omega}_t$ is an implicit function of aggregate variables. It defines the smallest expected productivity conditional on the signal, $E_t[z_{t+1}^i|\omega_{t+1}^i]$, required to obtain funding from uninformed lenders. We later discuss the properties and monotonicity of the LHS of (15) in detail. For now, we assume that the integral is increasing in $\bar{\omega}$, so that a higher RHS implies a higher $\bar{\omega}$.

Next, consider monitored financing. Define $\underline{\omega}_t$ as the minimum observed value of the signal ω_t^i necessary to obtain bank funding. Combining equation (10) with (5) yields:

$$E_t \int_{\mathcal{Z}_{t+1}^j}^{\infty} \left[z_{t+1}^j R_{t+1} - R_t^{b,j} \right] \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) - \frac{b_L I(1+i_t)}{\Delta} \ge E_t \int_{\mathcal{Z}_{t+1}^j}^{\infty} R_t^d \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) + \frac{b_L I(1+i_t)}{\Delta} \ge E_t \int_{\mathcal{Z}_{t+1}^j}^{\infty} R_t^d \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) + \frac{b_L I(1+i_t)}{\Delta} \ge E_t \int_{\mathcal{Z}_{t+1}^j}^{\infty} R_t^d \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j \right) \, \mathrm{d}F_{z|\omega} \left(z_{t+1}^j | \omega_t^j$$

Substituting in equations (11), (12), (3) and (14), taking into account that the expressions hold

with equality, and rearranging yields:

$$E_t \int_{\underline{z}_{t+1}^j}^{\infty} z_{t+1}^j \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t\right) = \frac{I\left(1+i_t\right)\left[1-\frac{K_t^f}{I}-\frac{c}{\Delta}\frac{(1+i_t)}{(1+i_t^b)}+\frac{(c+b_L)}{\Delta}\right]}{E_t R_{t+1}} \tag{16}$$

This allows us to similarly define ω_t as an implicit function of aggregate variables, where \underline{z}_{t+1}^j is the *ex post* default cut-off corresponding to the firm with signal value ω_t .

Following Holmström and Tirole (1997), we conjecture that $\bar{\omega}_t \geq \omega_t$. This and other regularity conditions are verified and further discussed in Subsection ??. Moreover, *ex ante* (and in the steady state), it must be the case that default cut-offs are strictly smaller than the corresponding financing cut-offs, i.e., $\bar{\omega}_t \geq E_t \bar{z}_{t+1}^i$ for any *i* and $\omega_t \geq E_t z_{t+1}^j$ for any *j*. This is because the default cut-offs correspond to a situation in which the firm receives zero revenue. However, zero revenue would violate any firm's incentive compatibility constraint *ex ante*. For a firm to be solvent in expectation, we require $E_t z_{t+1}^k | \omega_t^k > E_t \bar{z}_{t+1}^k$ for any $k = \{i, j\}$. However, these are only necessary conditions, as the revenue of the firm needs to be not only positive, but also sufficiently high to satisfy all incentive and participation constraints, which is described by the signal cut-offs.

The financial market equilibrium is fully characterized by the two cut-offs. Upon obtaining the idiosyncratic productivity signal ω_t^i , each firm will end up in one of three possible situations:

- Case 1. $\omega_t^i \in [0, \omega_t)$. The firm is not be able to borrow external funds and pursue production. It remains inactive and invests its equity at the riskless rate in the domestic financial market. Such firms will be labeled "Category 1" firms.
- Case 2. $\omega_t^i \in [\omega_t, \bar{\omega}_t)$. The firm borrows from a bank branch, using both the bank's (I_t^b) and its depositors' funds (I_t^d) to pursue production, and is monitored. Such firms will be labeled "Category 2" firms.
- Case 3. $\omega_t^i \in [\bar{\omega}_t, \infty)$. The firm borrows obtains direct financing from the bond market (I_t^u) . Its expected productivity (and thus revenue) is high enough not to be monitored. Such firms will be labeled "Category 3" firms.

The distribution of firms according to their signal is illustrated in Figure 5. The lower cut-off ω_t defines the extensive margin of external funding, and the upper cut-off $\bar{\omega}_t$ the intensive margin, i.e. the choice between bank and bond funding. Firms with the lowest expected productivity and revenue are not able to raise any external funds. In other words, the equilibrium financing contract involves credit rationing. Firms in the intermediate region rely on bank funding, whereas the most productive firms are able to access direct market finance. As both cut-offs depend on aggregate variables, both the extensive and the intensive margin will be endogenously determined in general equilibrium, which we discuss next.



Figure 5: The productivity signal distribution and firm selection into forms of external finance

3.4 The New Keynesian block

We now describe the production sector, the banking sector, households, and government. With the exception of intermediate good producers and banks, the model follows the standard New Keynesian framework, and we refrain from detailed presentation of these elements. To keep this part of the model simple and adhere to standard notation in the literature, we assume that the aggregate masses of intermediate, re-packaged and final goods in the economy are the same.

3.4.1 Intermediate good producers and holdings

Intermediate good producers that receive signals above the lower cut-off $\underline{\omega}_{t-1}$ secure either bank or bond funding in period t-1 and become active in the next period. Production starting in period t involves hiring labor h_t^i and renting physical capital k_{t-1}^i using the available funds I, such that, in real terms:

$$\frac{I}{P_t} \ge w_t h_t^i + r_t^K k_{t-1}^i,$$
(17)

where w_t is the real wage rate and r_t^K is the real rental rate of capital.

Intermediate goods $y_t^{m,i}$ are homogeneous and produced using a Cobb-Douglas technology with capital and labor as input factors:

$$y_t^{m,i} = z_t^i A_t \left(k_{t-1}^i\right)^\alpha \left(h_t^i\right)^{1-\alpha} \tag{18}$$

where A_t is the aggregate total factor productivity shock.

The goods are sold at a real price p_t^m . Intermediate good producers maximize their expected profit, conditional on their signal ω_{t-1}^i and taking the price as given:

$$\max_{k_{t-1}^{i},h_{t}^{i}} p_{t}^{m} \int_{\bar{z}_{t}^{i}}^{\infty} z_{t}^{i} A_{t} \left(k_{t-1}^{i}\right)^{\alpha} \left(h_{t}^{i}\right)^{1-\alpha} \mathrm{d}F_{z|\omega}\left(z_{t}^{i}|\omega_{t-1}^{i}\right) - \left(w_{t}h_{t}^{i} + r_{t-1}^{K}k_{t-1}^{i}\right)$$
(19)

subject to the working capital constraint (17). We assume that the constraint binds for all active intermediate good producers. This, together with the common Cobb-Douglas production technology, implies that the amounts of capital and labor demanded are common across all producers, i.e. $h_t^i = h_t$ and $k_{t-1}^i = k_{t-1} \forall i$. In equilibrium, the firm's production function thus consists of an idiosyncratic and a common component: $y^{m,i}: t = z_t^i y_t^m$, where we define $y_t^m = A_t k_{t-1}^\alpha h_t^{1-\alpha}$

After the choice of capital and labor inputs, as production starts, the true idiosyncratic productivity z_t^i is revealed. The revenue of the firm is then:

$$z_t^i R_t = z_t^i P_t p_t^m y_t^m$$

The producers whose productivity z_t^i falls below the relevant threshold, defined in equations (6) and (7), become insolvent and declare default. Their output is lost in full. Solvent firms, having obtained their revenue, pay back their debts and transfer the remaining profit to the holding.

Making use of equations (6), (7) and (17), the nominal profit of the representative holding can then be expressed as:

$$\Pi_{t}^{f} = \underbrace{K_{t-1}^{f} (1+i_{t-1}) \int_{0}^{\omega_{t-1}} \int_{0}^{\infty} dF_{z,\omega} \left(z_{t}^{i}, \omega_{t-1}^{i}\right)}_{1} + \underbrace{R_{t} \int_{\omega_{t-1}}^{\omega_{t-1}} \int_{z_{t}^{i}}^{\infty} z_{t}^{j} dF_{z,\omega} \left(z_{t}^{j}, \omega_{t-1}^{j}\right)}_{2}}_{2} + \underbrace{R_{t} \int_{\overline{\omega}_{t-1}}^{\infty} \int_{\overline{z}_{t}^{i}}^{\infty} z_{t}^{i} dF_{z,\omega} \left(z_{t}^{i}, \omega_{t-1}^{i}\right)}_{3} - \underbrace{R_{t} \int_{\omega_{t-1}}^{\omega_{t-1}} \int_{z_{t}^{i}}^{\infty} z_{t}^{j} dF_{z,\omega} \left(z_{t}^{j}, \omega_{t-1}^{j}\right)}_{4}}_{4} - \underbrace{R_{t} \int_{\overline{\omega}_{t-1}}^{\infty} \int_{\overline{z}_{t}^{i}}^{\infty} \overline{z}_{t}^{i} dF_{z,\omega} \left(z_{t}^{i}, \omega_{t-1}^{i}\right)}_{5}$$
(20)

The terms in the profit function are:

- 1. Safe nominal return earned on the equity of Category 1 firms
- 2. Revenue from the production of solvent Category 2 firms
- 3. Revenue from the productoin of solvent Category 3 firms
- 4. Payments to banks by solvent Category 2 firms

5. Payments to bond holders by solvent Category 3 firms

The holding is assumed to retain a fraction $0 < \phi^f < 1$ of the profit and pay out as dividend a fraction $1 - \phi^f$. The retained earnings become equity of the holding for the next period. The holdings' equity thus evolves according to:

$$K_t^f = \phi^f \Pi_t^f \tag{21}$$

The holding consumes the dividends itself, such that holding consumption C_t^f is, in real terms:

$$C_t^f = \left(1 - \phi^f\right) \frac{\Pi_t^f}{P_t} \tag{22}$$

3.4.2 Re-packagers

Monopolistically competitive re-packagers buy homogeneous intermediate goods from solvent intermediate good producers. They costlessly diversify these homogeneous goods into differentiated varieties y_t^k , indexed by k, that are imperfect substitutes, and sell these varieties to final good producers.

The real price p_t^m of the intermediate good that they purchase becomes their real marginal cost, denoted by φ_t . Re-packagers minimize their expenditure on intermediate goods, subject to their production technology that turns one unit of intermediate good y_t^m into one unit of a variety y_t^k :

$$\min_{y_t^m} p_t^m y_t^m + \varphi_t \left\{ y_t^k - y_t^m \right\}$$

Given their marginal costs and the Calvo pricing friction, re-packagers then set the price P_t^k of their variety to maximize their profits:

$$\max_{P_t^k} E_t \sum_{s=0}^{\infty} \left(\xi\beta\right)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\left(\frac{P_t^k}{P_{t+s}}\right)^{1-\theta} - \varphi_{t+s} \left(\frac{P_t^k}{P_{t+s}}\right)^{-\theta} \right] y_{t+s},$$

subject to the demand for variety y_t^k by final good producers. Assuming a symmetric equilibrium, the maximization problem yields the standard optimality condition for a re-packager able to reset its price in period t. We denote the optimal re-set price by P_t^* .

3.4.3 Final good producers

Final goods producers purchase varieties y_t^k from re-packagers. They combine them in bundles with the constant elasticity of substitution (CES) aggregation technology to produce the final good y_t :

$$y_t = \left[\int_0^1 \left\{y_t^k\right\}^{\frac{\theta-1}{\theta}} \mathrm{d}k\right]^{\frac{\theta}{\theta-1}},\tag{23}$$

where θ is the elasticity of substitution between varieties k. The final good producers' problem is then:

$$\max_{y_t} P_t y_t - \int_0^1 P_t^k y_t^k \,\mathrm{d}k$$

subject to (23). The solution yields the previously aforementioned demand for variety k. The aggregate price level can then be derived by substituting this solution into (23). Given that only a share $1 - \xi$ of re-packagers re-optimizes prices in every period, the aggregate price level evolves according to:

$$P_t^{1-\theta} = (1-\xi) \left(P_t^*\right)^{1-\theta} + \xi P_{t-1}^{1-\theta}$$
(24)

3.4.4 Banks

Banks and their branches are risk neutral and maximize their profits from lending to intermediate good producers. The equilibrium conditions for bank branches to engage in lending are given by the incentive and participation constraints (12) and (13).

Ex post, as each intermediate good firm's productivity is revealed, some firms may become insolvent. That risk is borne by banks. Deposits, by contrast, are assumed to be riskless to the depositor and always paid back in full. Further, we assume that banks are "large", i.e. always have enough equity to cover for losses from firm default, so that they cannot default themselves.

Under these assumptions, and using equation (7), the representative bank's nominal profit can be expressed as:

$$\Pi_{t}^{b} = \underbrace{R_{t} \int_{\underline{\omega}_{t-1}}^{\underline{\omega}_{t-1}} \int_{z_{t}^{j}}^{\infty} z_{t}^{j} \, \mathrm{d}F_{z,\omega}\left(z_{t}^{j}, \omega_{t-1}^{j}\right)}_{1} - \underbrace{R_{t-1}^{d}\left[F_{\omega}\left(\bar{\omega}_{t-1}\right) - F_{\omega}\left(\underline{\omega}_{t-1}\right)\right]}_{2} - \underbrace{cI\left[F_{\omega}\left(\bar{\omega}_{t}\right) - F_{\omega}\left(\underline{\omega}_{t}\right)\right]}_{3},\tag{25}$$

where the terms represent:

- 1. Branch revenue from solvent Category 2 firms
- 2. Payments to the bank branches' depositors
- 3. Monitoring costs

Analogously to the holding, the representative bank is assumed to retain a fraction $0 < \phi^b < 1$ of its profit and pay out as dividend a fraction $1 - \phi^b$. Bank equity thus evolves according to:

$$K_t^b = \phi^b \Pi_t^b, \tag{26}$$

and bank consumption C_t^b equals, in real terms:

$$C_t^b = \left(1 - \phi^b\right) \frac{\Pi_t^b}{P_t}.$$
(27)

3.4.5 Households

The representative household chooses consumption, investment, saving and labor supply to maximize its expected lifetime utility:

$$\max_{\{C_t, X_t, S_t, H_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(H_t)^{1+\tau}}{1+\tau} \right]$$
(28)

subject to the budget constraint, expressed in real terms as:

$$C_t + X_t + \frac{S_t}{P_t} + \frac{T_t}{P_t} = w_t H_t + r_{t-1}^K K_{t-1} + \frac{S_{t-1}}{P_{t-1}} \left(1 + r_t\right) + \frac{\Pi_t^m}{P_t}$$
(29)

and the law of motion for the capital stock, K_t :

$$K_{t} = (1 - \delta) K_{t-1} + \left[1 - \frac{\kappa}{2} \left(\frac{X_{t}}{X_{t-1}} - 1\right)^{2}\right] X_{t}$$
(30)

Here C_t is consumption, X_t is investment in new capital, S_t is the nominal amount saving into financial assets and T_t are nominal lump-sum taxes. The real return $1+r_t$ is defined according to the Fisher identity as $\frac{1+i_{t-1}}{\pi_t}$, where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate. The parameter $1 > \beta > 0$ is the household's subjective discount rate, $\sigma > 0$ governs the intertemporal elasticity of substitution, and $\tau > 0$ is the inverse Frisch elasticity of labor supply. The capital stock is subject to a quadratic investment cost, the extent of which is captured by the parameter $\kappa > 0$.

3.4.6 Monetary policy

The central banks sets the nominal interest rate i_t with the following Taylor rule:

$$1 + i_t = \frac{1}{\beta} \left[\beta \left(1 + i_{t-1} \right) \right]^{\rho_i} \left[\pi_t^{\rho_\pi} \left(\frac{Y_t}{Y_t^p} \right)^{\rho_Y} \right]^{1 - \rho_i} \left(\frac{\pi_t}{\pi_{t-1}} \right)^{\rho_{d\pi}} \left(\frac{Y_t / Y_t^p}{Y_{t-1} / Y_{t-1}^p} \right)^{\rho_{dY}} M_t, \quad (31)$$

where ρ_i is an interest rate smoothing parameter, ρ_{π} is the weight on inflation, and ρ_Y is the weight on the output gap $\frac{Y_t}{Y_t^p}$, where Y_t^p denotes the potential output. The parameters $\rho_{d\pi}$ and ρ_{dY} are the weights on the change in inflation rate and output gap, respectively. M_t is an exogenous monetary policy shock process.

3.4.7 Government

The government collects lump-sum taxes T_t from households and issues one-period bonds B_t^g that yield the nominal riskless rate $1 + i_t$. These funds are used to cover for unexpected differences in the revenue of the bond fund due to aggregate risk. The budget of the government is assumed to be always in balance, so that the bonds are in zero net supply in equilibrium.

The government budget constraint is thus:

$$T_{t} = \int_{\bar{\omega}_{t-1}}^{\infty} \int_{\bar{z}_{t}^{i}}^{\infty} R_{t-1}^{u,i} \, \mathrm{d}F_{z,\omega}\left(z_{t}^{i}, \omega_{t-1}^{i}\right) - E_{t-1} \int_{\bar{\omega}_{t-1}}^{\infty} \int_{\bar{z}_{t}^{i}}^{\infty} R_{t-1}^{u,i} \, \mathrm{d}F_{z,\omega}\left(z_{t}^{i}, \omega_{t-1}^{i}\right)$$

where the right-hand side of the equation is government spending G_t . The first integral term is the *ex post* revenue of the mutual fund from bond purchases made in period t - 1 and remunerated in t. The second integral term is the liability of the fund to its investors. When the realized payoffs to bond funds fall short of its liabilities to its investors, the government covers this difference. In the opposite case, the difference is transferred lump-sum to the households.

3.4.8 Market clearing

To complete the description of the model economy, we collect here the market clearing conditions and characterize the general equilibrium.

Equating the demand and supply for the intermediate good in the production process and aggregating over the distribution of intermediate good firms' production, the aggregate amount of final output in the economy, Y_t , is given by:

$$Y_{t} = y_{t} \left[\int_{\underline{\omega}_{t-1}}^{\bar{\omega}_{t-1}} \int_{\underline{z}_{t}^{j}}^{\infty} z_{t}^{j} \, \mathrm{d}F_{z,\omega}\left(z_{t}^{j}, \omega_{t-1}^{j}\right) + \int_{\bar{\omega}_{t-1}}^{\infty} \int_{\bar{z}_{t}^{i}}^{\infty} z_{t}^{i} \, \mathrm{d}F_{z,\omega}\left(z_{t}^{i}, \omega_{t-1}^{i}\right) \right]$$
(32)

Aggregate profits of the monopolistically competitive re-packaging sector are ultimately paid to households as dividends. The nominal profit of the representative re-packager is given by:

$$\Pi_t^m = P_t \left[y_t - \varphi_t y_t^m \right] \left[\int_{\underline{\omega}_{t-1}}^{\overline{\omega}_{t-1}} \int_{\underline{z}_t^j}^{\infty} z_t^j \, \mathrm{d}F_{z,\omega} \left(z_t^j, \omega_{t-1}^j \right) + \int_{\overline{\omega}_{t-1}}^{\infty} \int_{\overline{z}_t^i}^{\infty} z_t^i \, \mathrm{d}F_{z,\omega} \left(z_t^i, \omega_{t-1}^i \right) \right]$$
(33)

The market clearing condition for the final good implies:

$$Y_t = C_t + C_t^f + C_t^b + X_t + \frac{cI}{P_t} \left[F_\omega \left(\bar{\omega}_t \right) - F_\omega \left(\underline{\omega}_t \right) \right]$$
(34)

The final good is consumed by the households, consumed as dividends by holdings and banks, used by households for investment, and used by banks to cover for monitoring costs. The function $F_{\omega}(x) \equiv \int_{0}^{x} \int_{0}^{\infty} f_{z,\omega} \left(z_{t+1}^{i}, \omega_{t}^{i} \right) dz_{t+1}^{i} d\omega_{t}^{i}$ is the marginal cumulative distribution of ω_{t}^{i} .

Market clearing in the labor market requires that labor supply and demand are equated:

$$H_t = h_t \left[1 - F_\omega \left(\underline{\omega}_{t-1} \right) \right] \tag{35}$$

where H_t denotes the aggregate labor supply and h_t is the "per firm" demand for labor.

In the financial market, aggregate stock of net savings of the household S_t must equal the aggregate demand for funds:

$$S_t = -K_t^f F_\omega\left(\underline{\omega}_t\right) + D_t + B_t + B_t^g \tag{36}$$

where

$$D_t = I_t^d \left[F_\omega \left(\bar{\omega}_t \right) - F_\omega \left(\underline{\omega}_t \right) \right] \tag{37}$$

are aggregate bank deposits, and

$$B_t = I_t^u \left[1 - F_\omega \left(\bar{\omega}_t \right) \right] \tag{38}$$

is the aggregate stock of corporate bonds. The term B_t^g is savings into government bonds. The term $K_t^f F_{\omega}(\omega_t)$ is the aggregate amount of inactive intermediate good producers' idle equity, invested in the financial market. It enters with a negative sign, reflecting the fact that these firms are effectively lending money to the households.

Aggregate bank assets in t are simply all period t loans granted to intermediate good producers, due in t + 1, and expressed in nominal terms as:

$$L_t = \left(I - K_t^f\right) \left[F_\omega\left(\bar{\omega}_t\right) - F_\omega\left(\underline{\omega}_t\right)\right] \tag{39}$$

Market clearing for bank funds in period t requires that:

$$K_t^b = I_t^b \left[F_\omega \left(\bar{\omega}_t \right) - F_\omega \left(\underline{\omega}_t \right) \right] \tag{40}$$

where K_t^b is aggregate bank equity, and the right-hand side is the demand for bank funds.

Finally, market clearing in the government bond market requires $B_t^g = 0$ in all periods t due to the assumption of a balanced budget.

After normalizing the nominal price level to $P_t = 1$, the model consists of 44 endogenous variables in 44 equations. The full model equations are given in Appendix ??. In addition, there are three exogenous variables A_t , M_t and J_t , which we assume to evolve according to the following autoregressive stochastic processes:

$$\ln A_t = \rho_A \ln A_{t-1} + (1 - \rho_A) \ln A + \epsilon_t^A, \ 0 \le \rho_A < 1, \ \epsilon_t^A \stackrel{i.i.d}{\sim} N\left(0, \sigma_A^2\right)$$
(41)

$$\ln J_t = \rho_J \ln J_{t-1} + (1 - \rho_J) \ln J + \epsilon_t^J, \ 0 \le \rho_J < 1, \ \epsilon_t^J \stackrel{i.i.d}{\sim} N\left(0, \sigma_J^2\right)$$
(42)

$$\ln M_t = \rho_M \ln M_{t-1} + (1 - \rho_M) \ln M + \epsilon_t^M, \ 0 \le \rho_M < 1, \ \epsilon_t^M \stackrel{i.i.d}{\sim} N\left(0, \sigma_M^2\right)$$
(43)

4 Parametrization

To parametrize the model, we take the following approach. We parametrize the model to quarterly frequency. We calibrate a subset of the parameters using aggregate euro area data over the period 2001–2022, most importantly those governing the behavior of the financial sector, and borrow conventional parameter values from the New Keynesian literature for the rest. The parametrization is described in detail in Table 1.

In the New Keynesian model block, we use fairly standard parameter values. We either calculate values for these parameters directly from data, or apply commonly used values in the New Keynesian literature. In most cases, we use parameter values estimated in Coenen et al. (2018). We set the elasticity of substitution between varieties of intermediate goods θ to 3.857, corresponding to a gross markup of $\frac{\theta}{\theta-1} = 1.35$, in line with Coenen et al. (2018). The subjective discount factor β is set to match the long-run average of the quarterly overnight EONIA/ESTR rate of 0.0026. The Cobb-Douglas labour share $1 - \alpha$ is set to match the long-run average total labor share of 0.607, implying $\alpha = 0.393$. The Calvo probability is set to $\xi = 0.82$ following Coenen et al. (2018), which corresponds to re-pricing on average every 1.5 periods. Likewise, we employ value for the investment adjustment cost κ reported in Coenen et al. (2018). For the relative risk aversion σ and inverse Frisch elasticity τ , we use the values $\sigma = 2$ and $\tau = 2$, which are within the ranges commonly used in the New Keynesian literature. The monetary policy rule parameters are obtained from Coenen et al. (2018).

Parametrizing the financial sector is more involved. We select parameter values to match the steady state of the model to observed long-run averages in euro area data. The monitoring cost c can be recovered directly from observable data through the following steady-state relationship:

$$BOCBA \equiv \frac{cI \left[F_{\omega} \left(\bar{\omega}\right) - F_{\omega} \left(\bar{\omega}\right)\right]}{\left(I - K^{f}\right) \left[F_{\omega} \left(\bar{\omega}\right) - F_{\omega} \left(\bar{\omega}\right)\right]} = \frac{c\frac{I}{K^{f}}}{\frac{I}{K^{f}} - 1}$$

Object	Description	Baseline value	Source		
Financial market block					
с	Monitoring cost	0.0034	Data		
Δ	Revenue reducing factor	0.0073	Data		
b_H	High private benefit	0.0013	Calibrated		
b_L	Low private benefit	0.0010	Calibrated		
σ_z	Idiosyncratic productivity shock dispersion	0.0614	Calibrated		
σ_{ω}	Idiosyncratic signal dispersion	0.0923	Calibrated		
ϕ^f	Holding retained earnings-to-equity ratio	0.8932	Calibrated		
ϕ^b	Bank retained earnings-to-equity ratio	0.9837	Calibrated		
New Keynesian block					
α	Cobb-Douglas parameter	0.3933	Data		
β	Subjective discount factor	0.9974	Data		
δ	Capital depreciation rate	0.0045	Data		
κ	Investment adjustment cost	10.78	Coenen et al. (2018)		
σ	Relative risk aversion	2.0	Havranek et al. (2015)		
au	Inverse Frisch elasticity	2.0	Coenen et al. (2018)		
θ	Elasticity of substitution between intermediate goods	3.857	Coenen et al. (2018)		
ξ	Calvo price stickiness	0.82	Coenen et al. (2018)		
$ ho_r$	Interest rate smoothing in Taylor rule	0.93	Coenen et al. (2018)		
$ ho_{\Pi}$	Inflation weight in Taylor rule	2.74	Coenen et al. (2018)		
ρ_Y	Output gap weight in Taylor rule	0.03	Coenen et al. (2018)		
$ ho_{d\Pi}$	Inflation smoothing in Taylor rule	0.04	Coenen et al. (2018)		
ρ_{dY}	Output gap smoothing in Taylor rule	0.10	Coenen et al. (2018)		
Exogenous shock processes					
ρ_A	TFP shock persistence	0.92	Coenen et al. (2018)		
$ ho_M$	Monetary policy shock persistence	0			
$ ho_J$	Bank equity shock persistence	0			

Table 1: Baseline calibration of the model

Notes: In the column labelled "Source", 'Data' refers to a parameter calculated directly from observable data, and 'Calibrated' refers to a parameter calibrated to match long-run empirical targers, described in Table 2. The rest of the parameters are obtained from the New Keynesian literature as cited.

Empirical target	Data	Model	Calibration strategy	Data source
Return on bank equity r^b	0.024	0.024	Calculated from data	Eurostat
Bank operating costs-to-assets ratio (%)	0.340	0.340	Calculated from data	ECB
Bank leverage	2.198	2.198	Calculated from data	ECB
Bank retained earnings-to-equity ratio ϕ^b	0.992	0.984	Implied	Eurostat
NFC retained earnings-to-equity ratio ϕ^f	0.984	0.893	Targeted	Eurostat
NFC leverage	1.941	1.832	Targeted	Eurostat
NFC bond-to-loan ratio	0.115	0.119	Targeted	Eurostat
NFC loan default rate $(\%)$	0.180	0.188	Targeted	ECB
NFC bond default rate $(\%)$	0.008	0.007	Targeted	S&P
NFC debt-to-output ratio $\frac{B+L}{V}$	3.914	0.610	Targeted	Eurostat
NFC loans-to-output ratio $\frac{L}{V}$	3.508	0.545	Implied	Eurostat
NFC bonds-to-output ratio $\frac{B}{V}$	0.406	0.065	Implied	Eurostat
Non-residential capital-to-output ratio $\frac{K}{Y}$	10.81	34.27	Implied	Eurostat
Non-residential investment-to-output ratio $\frac{X}{Y}$	0.154	0.154	Calculated from data	Eurostat

Table 2: Empirical targets for parameter calibration

Notes: Data targets are averages of euro area aggregates, calculated over the common sample period 2001–2022 for which all time series are available. See Appendix B for a detailed description of the data.

This formula allow us to directly calibrate the parameter c, assuming some value for individual firm leverage $\frac{I}{K^{f}}$ and calculating the value for bank operating costs-to-bank assets ratio (*BOCBA*) from the data.

Using this result, we are able to calibrate Δ by using the observed long-run averages for the short-term interest rate *i*, rate of return on bank equity i^b , and bank leverage (*BLEV*):

$$\Delta = BLEV \times BOCBA \times \frac{1+i}{1+i^b}$$

We are then left with six parameters $\{b_H, b_L, \sigma_z, \sigma_\omega, \phi^f, \phi^b\}$. To calibrate them, we match key steady state ratios in the model to corresponding long-run ratios in the data with a general method of moments (GMM) type algorithm. This involves solving a constrained non-linear minimization problem in six unknowns and requires six relevant empirical targets to match. Finally, we obtain a value for the capital depreciation rate δ by using the long-run non-residential investment-to-output ratio observed in the data and the non-residential capital-to-output ratio implied by the calibration match. The empirical targets and the model's match to them are described in Table 2.

The six empirical targets we aim to match are the NFC retained earnings-to-equity ratio, NFC leverage, the NFC bond-to-loan ratio, the default rate on NFC loans, the default rate on NFC bonds, and the NFC debt-to-output ratio.

Data on the outstanding stocks of assets and liabilities of non-financial corporations are obtained from euro area financial accounts, and data on fixed assets, fixed investment and GDP are from euro area national accounts, published by Eurostat. Data on euro area banks' assets and liabilities, as well as banks' operating costs and loan charge-off rates, are obtained from the European Central Bank. The bank leverage calculated as total assets/liabilities over banks' capital and reserves significantly overestimates the exposure of banks to the non-financial corporate sector, as loans to NFCs account for only a fraction of bank assets, the rest being mainly household loans and government debt. For that reason, we adjust the empirically observed bank leverage by a factor corresponding to the average fraction of NFC loans in bank assets to map the empirical target more closely to the model's concept of bank leverage. Finally, data on corporate bond default rates are obtained from S&P Global Ratings Credit Research. The data used in the calibration are described in more detail in Appendix B.

Our calibration succeeds in matching the key steady state financial ratios in the model to the corresponding long-run empirical targets very well (Table 2). NFC leverage and bond-to-loan ratios, the default rates on corporate loans and bonds are very closely matched. The NFC retained earnings-to-equity ratio in the model also comes very close to its empirical counterpart. The only target we are not able to match is the corporate debt-to-output ratio, which we severely undershoot. One reason for this is that our model describes only a specific use of corporate credit, namely the financing working capital. In reality, non-financial firms use credit for various purposes: for example, financing investment, working capital, and trade credit. Our model can capture only a part of this activity.

5 Results

In this section, we first examine monetary policy transmission in the model using the baseline calibration, which targets the euro area economy where the corporate bond-to-loan ratio is relatively low. Then, we perform a counterfactual experiment: we look at how this transmission changes in an economy that is otherwise similar to the baseline model economy, but that has a structurally higher corporate bond-to-loan ratio. That is, we study how the transmission of monetary policy shocks is affected by a deeper bond market.

5.1 Baseline model dynamics: inspecting the mechanism

Figure 6 shows the impulse responses of the model to an *i.i.d.* shock to the monetary policy rule that increases the nominal riskless interest rate. This contractionary monetary policy shock leads to a contraction in output and a decline in the inflation rate, as expected. As the real interest rate rises, households' consumption decreases and saving increases through intertemporal substitution.

In the financial market, the bank lending channel kicks in. A contractionary monetary policy shock increases banks' funding costs relative to the riskless rate, *i.e.*, the bank funding spread $(i_t^b - i_t)$ increases. This is due to two factors.

First, the moral hazard problem in firms' external funding worsens. The hike in the riskless rate



Figure 6: Model impulse responses to a contractionary monetary policy shock *Notes:* The shock is scaled to a 25 bp annualized increase in the nominal interest rate on impact (25/4 bp at the quarterly frequency).

increases the banks' opportunity cost of monitoring, such that they must be compensated more in order to be incentivized to diligently monitor debtor firms. Similarly, the opportunity cost of behaving diligently increases for firms, worsening their incentives as well.

Second, the monetary policy contraction also worsens aggregate economic conditions and makes servicing existing debt more difficult. The default rate on existing bank loans unexpectedly increases due to the shock, manifested in a higher required idiosyncratic productivity requirement \underline{z}_t for debtor firms to remain solvent. These losses are absorbed by bank equity. Consequently, banks also require a higher return on loans to compensate for the higher default risk.

This feeds into higher rates charged on bank loans, and a higher spread between loan rates and the riskless rate. But the increasing default risk and worsening incentives of firms to behave diligently also lead to an endogenous increase in the lower signal cut-off ω_t . In other words, access to bank credit tightens and credit rationing becomes more prevalent.⁸

The increasing default losses and funding costs erode bank profits, as the rates charged on loans do not fully compensate for them. This leads to a squeeze in bank equity. Consequently, depositors are less willing to provide deposits to the banking sector, as incentives of both banks and firms have become worse. But since households' aggregate supply of savings increases through the intertemporal substitution channel, these funds are channelled to the bond market instead. The upper signal cut-off $\bar{\omega}_t$ declines, and at the internal margin, more firms are able to switch from bank loans to direct finance. As a consequence, the aggregate bond-to-loan ratio increases. However, the increase in bond finance is not enough to fully offset the decline in bank lending, and aggregate corporate debt contracts.

The aggregate model dynamics closely replicate the empirical behavior of the corporate debt stocks and the loan spread reported in Section 2 in qualitative terms. It confirms that the impulse responses estimated from the SVAR are consistent with a bank lending channel, which we have explicitly modelled. This is our first main contribution.

Moreover, the model also provides testable implications on distinguishing monetary policy shocks from "pure" credit supply shocks. Monetary policy shocks are conventionally classified as aggregate demand shocks, because they cause aggregate output and the price level to co-move in the same direction. However, they also look like credit supply shocks when the bank lending channel is active, as they lead to a squeeze in bank equity, an increase in the loan spread, and a contraction in aggregate credit supply by banks, as discussed above.

Our model allows us to simulate a credit supply shock (J_t) that causes a contraction in bank equity unrelated to exogenous changes in monetary policy. The model impulse responses to a 10% negative shock to bank equity are reported in Appendix D. Such a shock leads to a contraction in corporate loans, an increase in the loan spread, a tightening access to credit, and a rebalancing from loans towards bonds in the aggregate corporate debt structure, much like the monetary policy shock. However, unlike a contractionary monetary policy shock, such a bank equity shock leads to a contraction in output but a simultaneous increase in inflation and a reduction in the real interest rate. It can thus be classified as a genuine aggregate supply shock.

5.2 Counterfactual experiment: a deeper bond market

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⁸Note that since we assume the funding needs of firms to be fixed at I, the monetary contraction does not reduce the credit demand of individual firms. Our mechanism thus focuses on transmission through a contraction in credit supply.

6 Conclusions

In this paper, we develop a New Keynesian DSGE model where both the aggregate corporate debt structure and firms' access to finance are endogenous, and determined by the aggregate amounts of bank and firm equity. In the model, firms' idiosyncratic productivity levels are imperfectly observed at the time when external financing contracts are made. This introduces corporate default risk and makes banks' balance sheets vulnerable to macroeconomic conditions.

Our model allows us to study the implications of an endogenous corporate debt structure for monetary policy transmission through the financial sector. We show that following a contractionary monetary policy shock, bank equity is squeezed because of an increase in their funding costs and default losses on their existing loans. This hampers banks' ability to collect deposits and supply credit to firms, and makes loans more expensive relative to bond finance. Firms that are able to do so substitute bonds for loans, which cushions some of the impact of the monetary policy contraction. However, overall access to credit tightens and aggregate corporate debt contracts. This mechanism rationalizes the empirically observed substitution from loans towards bonds following contractionary monetary policy shocks that we uncover in aggregate euro area data.

Our model successfully replicates the empirically estimated impulse responses. Our empirical results also confirm findings in previous studies that have observed bond-loan substitution following monetary policy shocks. They lend support to the relevance of the bank lending channel of monetary policy, which our model is able to rationalize.

Our model then allows us to analyze many further topical issues on the implications of the corporate debt composition for monetary policy both in the long run and at the business cycle frequency. For instance, in the long term, what implications would a deepening of the Capital Market Union have for the conduct of monetary policy in the euro area? How should optimal monetary policy be characterized in an environment where firms have access to various forms of external finance? What are the welfare implications of a bank-based versus a market-based corporate credit market in terms of investment and output? We leave these important questions for future research.

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A Data used in SVAR estimation

Variable	Description	Source/Details					
	Low-Frequency Varia	bles					
Loan rate	Loans to non-financial corporations (new business), annualized agreed rate, Euro Area (changing composition), monthly.	Source: ECB ECB Series Key: MIR.M.U2.B.A2A.A.R.A.2240.EUR.N https://data.ecb.europa.eu/data/datasets/MIR					
Bond stock	Debt securities by non-financial corporations, Index of notional stocks, Euro Area (changing composition), monthly.	Source: ECB ECB Series Key: CSEC.M.N.U2.W0.S11.S1.N.L.LE.F3. L. Z.IX_R_LE.X1.M.V.NT https://data.ecb.europa.eu/data/datasets/CSEC					
Loan stock	Loans vis-a-vis euro area NFCs by MFIs (excl. ESCB), Index of notional stocks, Euro Area (changing composition), monthly. Calculated from the outstanding stock of loans (ECB Series Key: BSI.M.U2.N.A.A20.A.1.U2.2240.Z01.E) and the monthly financial transactions (ECB Series Key: BSI.M.U2.N.A.A20.A.4.U2.2240.Z01.E).	Calculated variable Data source: ECB https://data.ecb.europa.eu/data/datasets/BSI					
Price level	Euro Area, Consumer Price Index (HICP), Overall Index, Calendar Adjusted, SA.	Source: ECB Series Key: ICP.M.U2.Y.000000.3.INX https://data.ecb.europa.eu/data/datasets/ICP					
Real GDP	Quarterly GDP (Euro Area 20, Gross Domestic Product, 2010 reference year, constant prices, chained, EUR) interpolated to monthly frequency using industrial production (Euro Area 20, Production in industry, Total industry (B-D), 2021=100, Calendar Adjusted, SA, Index) as indicator series, Chow-Lin method.	Calculated variable Data source: Eurostat.					
2Y OIS	Euro Area 2-year Overnight Indexed Swap (OIS) rate, monthly average.	Source: Bloomberg.					
3M Euribor	3-month Euribor interbank rate (Act/365), monthly average.	Source: European Money Markets Institute (EMMI)					
Bond spread	Option-Adjusted Spread rate calculated from ICE BofA Non-Financial Index (against Swap Rate)	Source: Bloomberg Bloomberg Ticker: EN00					
Loan spread	Calculated as the difference between variables Loan Rate and 3-month Euribor.	Calculated variable					
Intermediation wedge	Calculated as the difference between variables Loan Spread and Bond Spread.	Calculated variable					
High-Frequency Variables							
Surprise in OIS rate	Change in the first principal component of the OIS rates in relevant monetary policy window in basis points, monthly sum.	Source: Euro Area Monetary Policy Event Study Database (EA-MPD) https://www.ecb.europa.eu/pub/pdf/annex/Dataset_ EA-MPD.xlsx, and Jarociński and Karadi (2020)					
Surprise in STOXX50 index	Euro STOXX50 index change in the relevant monetary policy window in percentage points, monthly sum.	Source: Euro Area Monetary Policy Event Study Database (EA-MPD) https://www.ecb.europa.eu/pub/pdf/annex/Dataset_ EA-MPD.xlsx, and Jarociński and Karadi (2020)					

Table 3: Aggregate euro area data used in the Bayesian SVAR estimation

B Empirical targets and methodology for model calibration

C Full model equations

$$I_t^u = I - K_t^f \tag{2}$$

$$I_t^d = I - K_t^f - I_t^b \tag{3}$$

$$\bar{z}_t = \frac{\bar{R}_{t-1}^u}{R_t} \tag{6}$$

$$\underline{z}_t = \frac{\underline{R}_{t-1}^b + R_{t-1}^d}{R_t} \tag{7}$$

$$E_t \int_{\bar{z}_{t+1}}^{\infty} \bar{R}_t^u \,\mathrm{d}F_{z|\omega}\left(z_{t+1}^i|\bar{\omega}_t\right) = (1+i_t) I_t^u \tag{9}$$

$$R_t^d = (1+i_t) I_t^d$$
 (11)

$$E_t \int_{\underline{z}_{t+1}}^{\infty} \underline{R}_t^b \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t\right) - E_t \int_0^{\underline{z}_{t+1}} R_t^d \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t\right) = \frac{cI\left(1+i_t\right)}{\Delta} \tag{12}$$

$$I_t^b = \frac{cI}{\Delta} \frac{(1+i_t)}{(1+i_t^b)} \tag{14}$$

$$E_t \int_{\bar{z}_{t+1}}^{\infty} z_{t+1}^i \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^i|\bar{\omega}_t\right) = \frac{I\left(1+i_t\right)\left(1-\frac{K_t^f}{I}+\frac{b_H}{\Delta}\right)}{E_t R_{t+1}} \tag{15}$$

$$E_t \int_{\underline{z}_{t+1}}^{\infty} z_{t+1}^j \, \mathrm{d}F_{z|\omega}\left(z_{t+1}^j|\omega_t\right) = \frac{I\left(1+i_t\right)\left[1-\frac{K_t^f}{I}-\frac{c}{\Delta}\frac{(1+i_t)}{(1+i_t^b)}+\frac{(c+b_L)}{\Delta}\right]}{E_t R_{t+1}} \tag{16}$$

$$I = w_t h_t + r_t^K k_{t-1} \tag{17}$$

$$y_t^m = A_t k_{t-1}^\alpha h_t^{1-\alpha} \tag{18}$$

$$h_t = \frac{(1-\alpha)}{w_t} I \tag{??}$$

$$R_t = p_t^m y_t^m \tag{(??)}$$

$$\Pi_{t}^{f} = K_{t-1}^{f} \left(1 + i_{t-1}\right) \int_{0}^{\omega_{t-1}} \int_{0}^{\infty} \mathrm{d}F_{z,\omega} \left(z_{t}^{i}, \omega_{t-1}^{i}\right) \\ + R_{t} \int_{\bar{\omega}_{t-1}}^{\infty} \int_{\bar{z}_{t}^{i}}^{\infty} z_{t}^{i} \mathrm{d}F_{z,\omega} \left(z_{t}^{i}, \omega_{t-1}^{i}\right) + R_{t} \int_{\omega_{t-1}}^{\bar{\omega}_{t-1}} \int_{\bar{z}_{t}^{j}}^{\infty} z_{t}^{j} \mathrm{d}F_{z,\omega} \left(z_{t}^{j}, \omega_{t-1}^{j}\right) \\ - R_{t} \int_{\bar{\omega}_{t-1}}^{\infty} \int_{\bar{z}_{t}^{i}}^{\infty} \bar{z}_{t}^{i} \mathrm{d}F_{z,\omega} \left(z_{t}^{i}, \omega_{t-1}^{i}\right) - R_{t} \int_{\omega_{t-1}}^{\infty} \int_{\bar{z}_{t}^{j}}^{\infty} z_{t}^{j} \mathrm{d}F_{z,\omega} \left(z_{t}^{j}, \omega_{t-1}^{j}\right)$$
(20)

$$K_t^f = \phi^f \Pi_t^f \tag{21}$$

$$C_t^f = \left(1 - \phi^f\right) \Pi_t^f \tag{22}$$

$$L_t = \left(I - K_t^f\right) \left[F_\omega\left(\bar{\omega}_t\right) - F_\omega\left(\underline{\omega}_t\right)\right] \tag{39}$$

$$K_t^b = I_t^b \left[F_\omega \left(\bar{\omega}_t \right) - F_\omega \left(\underline{\omega}_t \right) \right]$$
(40)

$$\Pi_{t}^{b} = R_{t} \int_{\underline{\omega}_{t-1}}^{\overline{\omega}_{t-1}} \int_{\underline{z}_{t}^{j}}^{\infty} \underline{z}_{t}^{j} \,\mathrm{d}F_{z,\omega}\left(z_{t}^{j}, \omega_{t-1}^{j}\right) - R_{t-1}^{d}\left[F_{\omega}\left(\overline{\omega}_{t-1}\right) - F_{\omega}\left(\underline{\omega}_{t-1}\right)\right] - cI\left[F_{\omega}\left(\overline{\omega}_{t}\right) - F_{\omega}\left(\underline{\omega}_{t}\right)\right]$$

$$(25)$$

$$C_t^b = \left(1 - \phi^b\right) \Pi_t^b \tag{27}$$

$$K_t^b = \phi^b \Pi_t^b J_t \tag{26}$$

$$\varphi_t = p_t^m \tag{(??)}$$

$$p_t^* = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \left(\xi\beta\right)^s y_{t+s} \lambda_{t+s} \varphi_{t+s} \pi_{t,t+s}^{\theta}}{E_t \sum_{s=0}^{\infty} \left(\xi\beta\right)^s y_{t+s} \lambda_{t+s} \pi_{t,t+s}^{\theta - 1}}$$
(??)

$$1 = (1 - \xi) \left(p_t^* \right)^{1-\theta} + \xi \pi_t^{\theta - 1}$$
(24)

$$y_t = \frac{y_t^m}{d_t^p} \tag{??}$$

$$d_t^p = \xi \pi_t^\theta d_{t-1}^p + (1 - \xi) (p_t^*)^{-\theta}$$
(??)

$$Y_{t} = y_{t} \left[\int_{\underline{\omega}_{t-1}}^{\overline{\omega}_{t-1}} \int_{\underline{z}_{t}^{j}}^{\infty} z_{t}^{j} \, \mathrm{d}F_{z,\omega} \left(z_{t}^{j}, \omega_{t-1}^{j} \right) + \int_{\overline{\omega}_{t-1}}^{\infty} \int_{\overline{z}_{t}^{i}}^{\infty} z_{t}^{i} \, \mathrm{d}F_{z,\omega} \left(z_{t}^{i}, \omega_{t-1}^{i} \right) \right]$$
(32)

$$\Pi_t^m = (y_t - \varphi_t y_t^m) \left[\int_{\underline{\omega}_{t-1}}^{\overline{\omega}_{t-1}} \int_{\underline{z}_t^j}^{\infty} z_t^j \, \mathrm{d}F_{z,\omega} \left(z_t^j, \omega_{t-1}^j \right) + \int_{\overline{\omega}_{t-1}}^{\infty} \int_{\overline{z}_t^i}^{\infty} z_t^i \, \mathrm{d}F_{z,\omega} \left(z_t^i, \omega_{t-1}^i \right) \right]$$
(33)

$$1 + r_t = \frac{1 + i_{t-1}}{\pi_t} \tag{(??)}$$

$$H_t = h_t \left[1 - F_\omega \left(\underline{\omega}_{t-1} \right) \right] \tag{35}$$

$$S_t = -K_t^f F_\omega \left(\underline{\omega}_t\right) + D_t + B_t \tag{36}$$

$$D_t = I_t^d \left[F_\omega \left(\bar{\omega}_t \right) - F_\omega \left(\underline{\omega}_t \right) \right] \tag{37}$$

$$B_t = I_t^u \left[1 - F_\omega \left(\bar{\omega}_t \right) \right] \tag{38}$$

$$\lambda_t = E_t \beta \left(1 + r_{t+1} \right) \lambda_{t+1} \tag{(??)}$$

$$\lambda_t = C_t^{-\sigma} \tag{(??)}$$

$$w_t = \frac{H_t^{\tau}}{C_t^{-\sigma}} \tag{(??)}$$

$$K_t = k_t \left[1 - F_\omega \left(\underline{\omega}_t \right) \right] \tag{??}$$

$$K_{t} = (1 - \delta) K_{t-1} + \left[1 - \frac{\kappa}{2} \left(\frac{X_{t}}{X_{t-1}} - 1\right)^{2}\right] X_{t}$$
(30)

$$1 = q_t \left\{ \left[1 - \frac{\kappa}{2} \left(\frac{X_t}{X_{t-1}} - 1 \right)^2 \right] - \kappa \left(\frac{X_t}{X_{t-1}} - 1 \right) \frac{X_t}{X_{t-1}} \right\} + E_t \beta \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \kappa \left(\frac{X_{t+1}}{X_t} - 1 \right) \left(\frac{X_{t+1}}{X_t} \right)^2$$
(??)

$$q_{t} = E_{t}\beta \frac{\lambda_{t+1}}{\lambda_{t}} \left[r_{t+1}^{K} + q_{t+1} \left(1 - \delta \right) \right]$$
(??)

$$(1+i_t) = \left[\beta \left(1+i_{t-1}\right)\right]^{\rho_i} \pi_t^{(1-\rho_i)\rho_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{(1-\rho_i)\rho_Y} \left(\frac{\pi_t}{\pi_{t-1}}\right)^{\rho_{d\pi}} \left(\frac{Y_t/Y_t^f}{Y_{t-1}/Y_{t-1}^f}\right)^{\rho_{dY}} \frac{M_t}{\beta}$$
(31)

$$T_{t} = R_{t} \int_{\bar{\omega}_{t-1}}^{\infty} \int_{\bar{z}_{t}^{i}}^{\infty} \bar{z}_{t}^{i} \,\mathrm{d}F_{z,\omega}\left(z_{t}^{i},\omega_{t-1}^{i}\right) - (1+i_{t-1}) I_{t-1}^{u} \left[1 - F_{\omega}\left(\bar{\omega}_{t-1}\right)\right] \tag{??}$$

$$Y_t = C_t + C_t^f + C_t^b + X_t + cI \left[F_\omega \left(\bar{\omega}_t \right) - F_\omega \left(\underline{\omega}_t \right) \right]$$
(34)



D Model impulse responses to a bank equity shock

Figure 7: Model impulse responses to a contractionary bank equity shock J_t Notes: The shock is scaled to 10 percent reduction in bank equity on impact.