Forward guidance and fiscal rules in HANK

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First version: 31/01/2025.

Abstract

I show that in a canonical HANK model, under a balanced budget fiscal rule, the effect of a nominal interest rate peg is much larger than in a representative agent (RA) model. By contrast, under a standard fiscal rule where tax revenue responds gradually to deviations of the debt-to-GDP ratio from steady-state and depends on economic activity, the effect of forward guidance is much weaker than in the RA model, and becomes linear in the length of the peg. This result is robust to allowing for countercyclical inequality and income risk, and carries over to a quantitative model with capital.

Keywords: Forward guidance, fiscal rules, HANK; JEL Codes: E52, E37

1 Introduction

The development of Heterogeneous Agent New Keynesian (HANK) models has been accompanied by the investigation the effect of forward guidance policies in such models, and whether HANK models offer a solution to the so called "Forward guidance puzzle" (Carlstrom et al. (2015), Del Negro et al. (2012)), i.e. the finding that an the effects of a nominal interest rate peg on output and inflation may be implausible large and explode in the length of the peg. Early results by McKay et al. (2016) suggested that the presence of uninsurable income risk and the associated probability to hit a borrowing constraint would dampen the effects of forward guidance by effectively limiting the households planning horizon. However, as shown by Werning (2015), Hagedorn et al. (2019) and Bilbiie (2020), their result is due to specific assumptions about the distribution of profits and taxes which render income inequality and income risk procyclical. By contrast, with countercyclical inequality and income risk, the effects of forward guidance are larger in HANK models than in the representative agent model, as the policy raises the income share of households with a high Marginal Propensity to Consume (MPC), thus making the forward guidance puzzle larger rather than resolving it.

This paper shows that the Forward guidance puzzle actually *can* be resolved in a HANK model if the fiscal policy is described by an empirically plausible reaction function displaying a small effect of the level of government debt and an important effect of economic activity on tax revenue. I first consider the simple sticky wage, flexible price one-asset model of Auclert et al. (2021) with zero

[*]The opinions expressed here are those of the author and do not necessarily represent those of the National Bank of Belgium or the European System of Central Banks.

[†]I would like to thank Adrien Auclert, Ludwig Straub and Matthew Rognlie for patiently answering many questions on how to use their Sequence Space Jacobian Python toolbox.

profits and thus virtually acyclical pre-tax income. I find that under a balanced budget rule, which keeps the real level of government debt constant by adjusting total tax revenue to compensate for changes in the real interest burden of the government, the effects of forward guidance on output and inflation are stronger than in the representative agent (RA) model and also more explosive in the length of the nominal interest rate peg. The reason is that the policy implies a redistribution from asset rich - low MPC households to households with a high MPC. The decline in the real interest rate implies a decline in asset-rich households income, as well as lower interest rate costs for the government and thus lower total taxes. The decline in taxes is distributed among households proportionally to their labor income, which is more equally distributed than assets. Once I eliminate this redistribution by assuming that agents instead receive an ex-post transfer compensating them exactly for their loss arising from the decline in their real interest income, which I call "Modified balanced budget rule", the effects of the policy are very close to those in the RA model.

By contrast, if instead the fiscal rule mandates that tax revenue responds only gradually in response to deviations of the government debt-to-GDP ratio from a target and its own lag, and responds to economic activity in an empirically plausible way, the effects of the forward guidance policy become very small, even for peg lengths of up to five years, and become linear in the length of the interest rate peg. This "Gradual adjustment rule" attenuates the effect of forward guidance via two avenues. The lower real interest rate reduces the real interest income of asset holders. Lower demand from these households in turn means lower demand and income for all households, further dampening the demand increase. In partial equilibrium logic, one can think of this mechanism as removing the aforementioned transfer that renders the results under the Modified balanced budget rule close to those in the RA model by compensating households ex-post for the income loss they suffer due to the real interest rate decline. Under the modified balanced budget rule, the size of this transfer rises exponentially in the length of the peg, as it is linked to the increase in inflation. The same is then true for the attenuating effect of removing this transfer as we move from the modified balanced budget rule to the gradual adjustment rule. The impact of allowing tax revenue to respond to economic activity can be understood in a similar fashion, as under the Modified balanced budget rule, economic activity increases exponentially in the length of the peg, and so does the attenuating effect linking households tax burden to it as we move to the gradual adjustment rule.

A fiscal rule with the properties of the gradual adjustment rule is an empirically plausible description of the response of tax revenue to the level of government debt. Rules of this form are assumed routinely in estimated DSGE models with fiscal blocks, and these estimations routinely confirm the finding of a very small and gradual effect of the level of debt on taxes and expenditures, see Leeper et al. (2017); Coenen et al. (2013); Gadatsch et al. (2016); Leeper et al. (2010); Zubairy (2014).

The mechanism driving the attenuation of the effects of forward guidance under this fiscal rule in the HANK model partially resembles the manner in which the "Preferences Over Safe Assets" (POSA) assumption analyzed by Rannenberg (2024) attenuates the effect of forward guidance in a RA model. Part of the attenuation in that environment also arises from the persistent decline in government debt observed under a gradual adjustment type fiscal rule, which dampens the consumption increase caused by the policy by raising the marginal utility of debt relative to the marginal utility of consumption. I show that an independently calibrated POSA model with a gradual adjustment rule approximates the results of its HANK counterpart very closely, though how closely is sensitive to the steady-state level of government debt.

I also consider the nominal debt target rule proposed by Hagedorn et al. (2019). This rule also attenuates the effects of forward guidance compared to the RA model and makes the effect linear in the length of the peg by making the price level determinate, thus anchoring inflation expectations. However, the peg remains much more powerful than under the gradual adjustment rule. Moreover, a nominal debt target is empirically much less plausible than the gradual adjustment rule.

Furthermore, I show that my results are robust to a number of extensions of the simple model, including long-term debt, high liquidity, and countercyclical inequality and income risk. Finally, I also repeat the analysis in a quantitative model with nominal rigidities in both wage and price setting and capital accumulation, with households holding a liquid asset and an asset that is illiquid due to portfolio adjustment costs as in Kaplan et al. (2018).

Kaplan et al. (2018) and Auclert et al. (2024a) discuss that the conduct of fiscal policy shapes the effects of ordinary monetary policy shocks. However, to my knowledge, this is the first paper that analyzes the effect of a nominal interest rate peg in a HANK model with a gradual adjustment type fiscal rule. Analyzes of monetary policy in HANK typically assume that government debt is held constant by fiscal policy or is absent (e.g. McKay et al. (2016), Gerke et al. (2024), Herman and Lozej (2023)). An exception are Auclert et al. (2024a) and Kaplan et al. (2018), who however do not consider an interest rate peg. In a short note, Kaplan et al. (2016) seperately simulate a contemporaneous and a future anticipated innovation to the Taylor rule in different variants of their HANK model, which is quite different from simulating an interest rate peg.

The remainder of the paper is structured as follows. Section 2 develops the simple canonical HANK model. Section 3 discusses the calibration. Section 4 describes the nominal interest rate peg that I simulate throughout the paper. Section 5 discusses the results obtained in the canonical HANK model, while Section 6 develops and analyzes the quantitative model. In Appendix A, I repeat the aforementioned simulation of Kaplan et al. (2016) in my models.

2 The model

2.1 Firms

The representative firm produces the output good from labor using a linear technology

$$Y_t = N_t \tag{1}$$

It operates under perfect competition, implying that the real wage w_t is constant at

$$w_t = 1 \tag{2}$$

and profits are zero, implying that

$$Y_t = w_t N_t \tag{3}$$

2.2 Households

2.2.1 Heterogeneous Agent model

The household side of the model follows Auclert et al. (2024b). The economy is populated by unit mass of households indexed with i facing idiosyncratic income uncertainty in the form of time varying labor productivity $e_{i,t}$. The effective labor supply of household i is given by $e_{i,t}n_{i,t}$, with n_{it} denoting household i's labor effort, implying that aggregate hours are given by $N_t = \int e_{it}n_{it}di$.

The nominal wage per unit of effective labor W_t is sticky, implying that households take n_{it} as given. Labor hours are allocated proportionally across households via the rule

$$n_{it} = N_t \tag{4}$$

which imposes the normalization $\int e_{it}$=1. e_{it} follows a an AR(1) process with persistence ρ_e and standard deviation σ_e.

Following Heathcote et al. (2017), I assume a progressive retention function, implying that post-tax labor income z_{it} is given by $z_{it} = \tau_t \left(w_t e_{it} n_{it}\right)^{1-\xi}$, where τ_t is a time varying intercept and $\xi = [0, 1]$ a progressivity parameter. Using the fact that total taxes T_t are given by $T_t = w_t N_t - \int z_{it}$, as well as (3) and (4), τ_t can be pinned down, and z_{it} is then given by

$$z_{it} = \left(Y_t - T_t\right) \frac{e_{it}^{1-\xi}}{\int e_{it}^{1-\xi} di} \tag{5}$$

The household budget constraint is given by

$$a_{it} + c_{it} = \left(1 + r_t\right) a_{it-1} + z_{it} \tag{6}$$

where a_{it} and r_t denote real assets and the (ex-post) real interest rate, respectively. The households is subject to the borrowing constraint

$$a_{it} \geq 0 \tag{7}$$

Households maximize

$$\sum_{i=0}^{\infty} \beta^i \left(\frac{c_{it}^{1-\sigma}}{1-\sigma} - \frac{\chi_N}{1+\eta} n_{it}^{1+\eta} \right) \tag{8}$$

subject to (6) and (7).

2.2.2 Representative Agent model

I also consider two RA version of the model without idiosyncratic income shocks and borrowing constraints. The first is the standard infinite horizon model with the utility function given by (8), while in the second, households have Preferences Over Safe Assets (POSA) as in Rannenberg (2024). The utility function is given by

$$\sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{\chi_N}{1+\eta} N_{t+i}^{1+\eta} + \frac{\chi_b}{1-\sigma_a} A_{t+i}^{1-\sigma_a} \right]$$

with $\chi_a, \sigma_a \geq 0$, while the budget constraint remains as before. The first order conditions with respect to assets and consumption are given by

$$\Lambda_t = \beta E_t \left\{ (1 + r_{t+1}) \Lambda_{t+1} \right\} + \chi_a A_t^{-\sigma_a} \tag{9}$$

$$\Lambda_t = C_t^{-\sigma} \tag{10}$$

where Λ_t denotes the marginal utility of consumption. As discussed in detail in Rannenberg (2024), with POSA ($\chi_a > 0$), effect of future interest changes on current consumption are attenuated in two ways, which are easiest to grasp by linearizing the Euler equation, which yields

$$\hat{\Lambda}_t = \theta \left[\hat{R}_t - E_t \hat{\Pi}_{t+1} \right] + \theta E_t \hat{\Lambda}_{t+1} - (1 - \theta) \sigma_a \frac{Y}{A} \hat{A}_t \tag{11}$$

where a hat on top of a variable denotes the percentage deviation of that variable from the non-stochastic steady state, with the exception of $\hat{A}_t$, which is expressed as a percentage of steady state GDP. $\theta \equiv \beta \frac{R}{\Pi}$, i.e. the product of the steady-state household discount factor and the real interest rate. θ represents the net weight the household attaches to the $t+1$ marginal utility of consumption. Assuming POSA (i.e. $\chi_b > 0$) implies that $\theta < 1$, and thus a smaller effect of future real interest rate changes on current consumption, the more so the further these changes are located in the future. This type of effect is sometimes referred to as "discounting in the Euler equation". Furthermore, depending on the behavior of fiscal policy, an expansionary forward guidance policy may lower real government debt via a lower real interest rate and higher tax revenues. With both POSA and declining marginal utility from safe assets ($\theta < 1$ and $\sigma_a > 0$), a decline in a household's government bond holdings will tend to lower its consumption, as the household attempts to smooth not just consumption but also its real safe asset holdings. This "wealth effect" also contributes to attenuating the effect of forward guidance if fiscal policy allows $\hat{A}_t$ to decline.

2.3 Wage setting

The labor input N_t employed by firms is a CES basket of labor varieties. Hence the demand for labor variety j given by

$$N_t(j) = N_t \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_W}$$

Following Hagedorn et al. (2019), nominal wages paid for individual varieties $W_t(j)$ are set by a union which maximizes the utility of an "average" household subject to a quadratic wage adjustment costs $\frac{\xi_W^J}{2} \left(\frac{\frac{W_t^J(j)}{W_{t-1}^J(j)}}{\Pi} - 1 \right)^2 N_t w_t$. This approach has the advantage that the resulting wage setting equation remains the same regardless of the assumptions regarding household heterogeneity. The unions objective is thus given by

$$E_t \sum_{i=0}^{\infty} \frac{1}{\left(\frac{R_{t+i}}{\Pi_{t+1+i}} \right)^i} \left[-\chi_N \frac{\left(N_{t+i} \left(\frac{W_{t+i}(j)}{W_{t+i}^J} \right)^{-\epsilon_W} \right)^{1+\eta}}{(1+\eta) C_t^{-\sigma}} + \left(\frac{(W_{t+i}(j))^{1-e_W}}{P_{t+i}^J} N_{t+i} \left(\frac{1}{W_{t+i}} \right)^{-\epsilon_W} - \frac{\xi_W^J}{2} \left(\frac{\frac{W_{t+i}(j)}{W_{t-1}(j)}}{\Pi} - 1 \right)^2 N_{t+i} w_{t+i} \right) \right]$$

The first order condition and the fact that all unions set the same wage imply the following non-linear wage Phillips Curve:

$$\left(\frac{\Pi_{W,t}}{\Pi}-1\right)\frac{\Pi_{W,t}}{\Pi}=\kappa_w\left(\frac{\epsilon_w}{\epsilon_w-1}\frac{\chi_N\left(N_t\right)^{\eta}}{w_tC_t^{-\sigma}}-1\right)+E_t\left\{\frac{1}{\left(\frac{R_t}{\Pi_{t+1}}\right)}\frac{N_{t+1}}{N_t}\left(\frac{\Pi_{W,t+1}}{\Pi}-1\right)\frac{\left(\Pi_{W,t+1}\right)^2}{\Pi_{t+1}\Pi}\right\} \tag{12}$$

The linearized version of this equation is given by

$$\hat{\Pi}_{W,t}=\kappa\hat{Y}_t+\frac{1}{1+r}E_t\hat{\Pi}_{W,t+1} \tag{13}$$

with $\kappa\equiv\kappa_w\left(\eta+\sigma\frac{Y}{C}\right)$, where a hat above a variable denotes the percentage deviation from its steady-state, and I have taken into account (1), (2) and the fact that government expenditure is constant. Below I will calibrate η, σ and κ directly and set κ_w to support the chosen combination of values.

Following Hagedorn et al. (2019), I assume that wage and price adjustment costs are "as-if", i.e. they are not actually resource costs though wage setters behave as if they are. The assumption avoids counterfactual "price-adjustment booms" that would be caused by the potentially large price changes caused by the interest peg I simulate.

2.4 Government

The government budget constraint in nominal terms is given

$$B_t=R_{t-1}B_{t-1}+P_t\left(G-T_t\right) \tag{14}$$

where B_t, R_t, P_t and G denote total government bonds, the short-term gross nominal interest rate, the price level and government expenditure, respectively.

Under the first rule I consider, the total tax burden adjusts in order to fix total real debt at a target level, i.e. $\frac{B_t}{P_t}=\frac{B_{t-1}}{P_{t-1}}=b^T$. It is given by

$$T_t=G+b^T\left(\frac{R_{t-1}}{\Pi_t}-1\right) \tag{15}$$

I refer to this policy as "Balanced budget rule".

The second fiscal policy I consider is a "Modified balanced budget rule". In this rule, the government compensates each bondholder for changes in the real interest rate ex-post. So if the real interest rate decline, households receive a transfer at the beginning of the period such that any income change resulting from deviations from the real interest rate is compensated as a lump sum:

$$a_{it}+c_{it}=\frac{R_{t-1}}{\Pi_t}a_{it-1}+z_{it}+t^r_{i,t} \tag{16}$$

with $t^r_{i,t}=\left(\frac{R}{\Pi}-\frac{R_{t-1}}{\Pi_t}\right)a_{it-1}$. However, households do *not* take the dependence of $t^r_{i,t+1}$ on a_{it} into account when making their consumption and saving decision. The government budget constraints is accordingly given by

$$T = G + b^T \left(\frac{R}{\Pi} - 1 \right) \tag{17}$$

and thus total taxes are constant.

The third policy is a "gradual adjustment rule". Total taxes adjust gradually in response to deviations of the debt-to-GDP ratio from a target, a common assumption in the fiscal policy literature and especially in contributions estimating models with fiscal blocks (see Leeper et al. (2017); Coenen et al. (2013); Gadatsch et al. (2016); Leeper et al. (2010). Furthermore, I assume that the level of economic activity also influences tax revenues positively. Total taxes are thus determined as:

$$T_t = T_t^d + \varepsilon_{TY} \hat{Y}_t + \varepsilon_{T,t} \tag{18}$$

$$T_t^d = T + \phi_T \left(\frac{b_{t-1} - b^T}{Y} \right) + \rho_T \left(T_{t-1}^d - T \right) \tag{19}$$

where b^T denotes the debt target. $\varepsilon_{T,t}$ denotes a tax shock that will be used below for the calibration of the POSA model. Government debt evolves as

$$b_t = \frac{R_{t-1}}{\Pi_t} b_{t-1} + G - T_t \tag{20}$$

The fourth policy is the "Modified gradual adjustment rule", which eliminates the effect of economic activity on tax revenue by setting $\varepsilon_{TY} = 0$.

The fifth policy fixes is the "Nominal debt rule", as in Hagedorn et al. (2019). It fixes the *nominal* debt level at a target value B^T:

$$T_t = G + \frac{B^T}{P_t} \left(R_{t-1} - 1 \right)$$

Monetary policy is described by the following interest feedback rule:

$$\hat{R}_t = (1 - d_{p,t}) \left(\phi_\pi \hat{\Pi}_t + \frac{\phi_y}{4} \hat{Y}_t \right) + d_{p,t} \hat{R}_{p,t} \tag{21}$$

where $\hat{Y}_t^p$ denotes flexible price output, and $d_{p,t}$ denotes a dummy variable which takes a value of one if the central bank decides to switch off the interest feedback rule and instead peg the interest rate to $\hat{R}_{p,t}$, and equals 0 otherwise. The net real interest rate earned by households is then determined as

$$r_t = \frac{R_{t-1}}{\Pi_t} - 1 \tag{22}$$

2.5 Equilibrium

The market for the liquid asset clears:

$$A_t = \int a_{it} di \tag{23}$$

$$b_t = A_t \tag{24}$$

The goods market clears:

$$Y_t = C_t + G \tag{25}$$
$$C_t = \int c_{it} di$$

3 Calibration

One period in the model corresponds to one quarter in the data. The calibration is reported in Table 1. I calibrate a number of parameters directly. The elasticity of intertemporal substitution $\frac{1}{\sigma}$ and the Frisch elasticity of labor supply $\frac{1}{\eta}$ are set to one. The calibration of the parameters of the idiosyncratic income process σ_e and ρ_e and the retention function curvature ξ follow Auclert et al. (2024b), with the values modified to be consistent with a quarterly frequency. The share of government expenditure in output equals 20%. For the gradual adjustment rule, I assume $\rho_T = 0.95$ and set ϕ_T to a small value sufficient to guarantee debt stationarity. The literature estimating DSGE models with fiscal blocks typically finds that taxes and government expenditures respond only extremely gradually to changes in government debt, see Rannenberg (2021) Leeper et al. (2017), Gadatsch et al. (2016), Zubairy (2014) and Coenen et al. (2013). Furthermore, in the gradual adjustment rule I set the feedback from economic activity to tax revenue $\varepsilon_{TY} = 0.5$, which equals the semi-elasticity of the budget balance with respect to the output gap as estimated by Price et al. (2015) for the US economy.

Given these choices, I calibrate a number of parameters in order to set the steady-state values of important model variables to (empirical) target values. In all models, I set the wage Phillips Curve slope κ_w such that κ equals the output coefficient of the estimated New Keynesian Phillips curve of Coibion et al. (2018),[1] the household discount factor β to match a real interest rate of 2% annually and the labor disutility weight χ_N to normalize output to one. In the heterogeneous agent model, following Auclert et al. (2024b), I set government debt target in the fiscal rule b^T in order to set the first year Marginal Propensity to Consume (MPC) out of an income windfall to 0.51, as estimated by Fagereng et al. (2021).

[1] See their "Appendix Table 1", first column.

Table 1: Calibration

Directly calibrated parameters		
Parameter	Definition	Value
σ	Consumption curvature	1.0
η	Labor curvature	2.00
σ_e	Idiosyncratic income shock s.d.	0.46
ρ_e	Idiosyncratic income shock persistence	0.98
θ	Retention function curvature	0.18
G	Government expenditure-to-GDP ratio	0.2
ϵ_{YT}	Budget semi-elasticity	0.5
ϕ_T	Fiscal rule debt response	0.03
ρ_T	Fiscal rule lagged tax response	0.95
ϕ_π	Taylor rule inflation	1.5
ϕ_y	Taylor rule output	0.5
μ_w	Wage markup	1.5

Parameters calibrated to match Target values		
Parameter	Definition	Value
β	Household discount factor	0.987
b^T	Debt target fiscal rule	0.72
χ_N	Labor disutility weight	0.83
κ_w	Wage markup coefficient	0.011
POSA: β	Household discount factor	0.95
POSA: χ_a	Safe asset utility weight	0.06
POSA: σ_a	Safe asset utility curvature	0.06

Target values		
Target	Definition	Value
r	Real interest rate, APR	2.0
MPC	First year MPC	0.51
Y	Output	1.0
κ	Phillips curve output coefficient	0.036
POSA: θ	Inverse discounting wedge	0.95
POSA: Debt supply on interest rate	See note below	0.05

Note: The empirical target value for the first year MPC is taken from from Auclert et al. (2024b). Effect of increase of the government debt-to-GDP ratio on the interest rate model counterpart: I simulate a fully anticipated one percentage point increase in the government debt-to-GDP ratio 5 years ahead, implemented as a one-off negative value of the shock $\varepsilon_{T,t}$ during quarter 17 (see equation (18)).

In the POSA model, I keep the government debt level from the heterogeneous agent model and add two additional targets. Following Rannenberg (2024) and Rannenberg (2021), I set the inverse discounting wedge $\theta = 0.95$, in line with the evidence, and calibrate the safe asset curvature parameter σ_a such that the effect of a one percentage point increase of the five-year-ahead government debt-to-annual-GDP ratio on the five-year-ahead ten-year-forward Treasury rate is in the empirical range of 0.03 to 0.06 percentage points found by Gale and Orszag (2004), Engen and Hubbard (2005) and Laubach (2009), which yields $\sigma_a = 0.056$ (see the note below Table 1 for details).[2]

I discretize the AR(1) process for e on an 11 point grid using the Rouwenhorst method. I discretize the asset space using a double-exponentially-spaced grid with 500 grid points.

4 Simulation set-up

The simulation setup follows Rannenberg (2024). The economy is initially at its steady state. In quarter one, the central bank announces that it will be "switching off" its interest feedback rule (21) for a total of $D_L + D_p$ quarters, by setting $d_{p,t} = 1$ over those quarters, and to peg the interest rate at values $\hat{R}_{p,t}$. Furthermore, I assume for quarters 1 to D_L, $\hat{R}_{p,t} = 0$. By contrast, for quarters $D_L + 1$ until $D_L + D_p$, the announced trajectory equals $-\Delta$ % . Finally, the central bank promises to return to its standard interest feedback rule (i.e. $d_{p,t} = 0$) in quarter $D_L + D_p + 1$. To summarize, I assume

$$d_{p,t} = \begin{cases} 1 & \text{for } t = 1, 2...., D_L + D_p \\ 0 & \text{for } t > D_L + D_p \end{cases} \tag{26}$$

$$\hat{R}_{p,t} = \begin{cases} 0 & \text{for } t = 1, 2...., D_L \\ -\Delta & \text{for } t = D_L + 1, D_L + 2, ...D_L + D_p \end{cases}$$

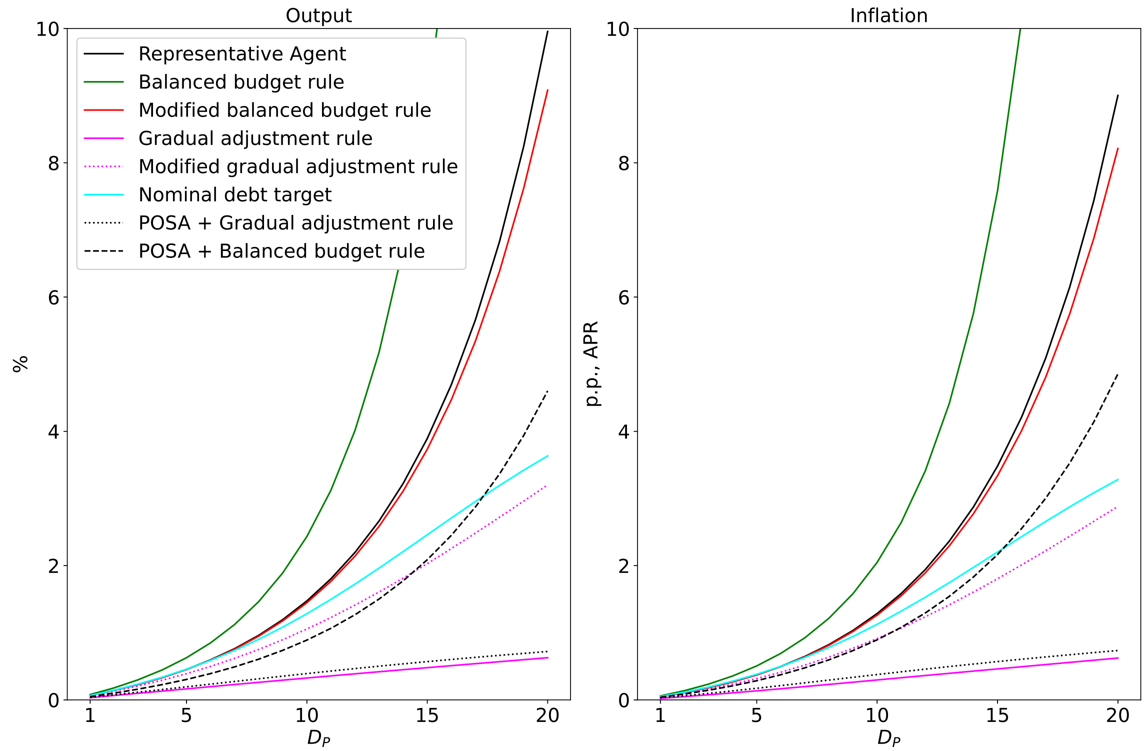
I set $D_L = 6$, following Carlstrom et al. (2015). Furthermore, prior to the forward guidance announcements of the US Federal Reserve in September 2011, January 2012 and September 2013, financial markets expected the federal funds rate to remain at the ZLB for approximately six quarters, according to the evidence reported in Del Negro et al. (2023). I set Δ to an annualized value of 0.2% (i.e. $\Delta = \frac{0.2}{4}$), which is in line with the effect of the announcement on private sector forecasts of three-month treasury bills estimated by Del Negro et al. (2023).[3] Based on these assumptions, below I will investigate the macroeconomic effect of varying D_p, i.e. the length of the period during which the interest rate is pegged Δ percent below its steady-state value.

I perform nonlinear perfect foresight simulations, using the non-linear variant of the Sequence-Space Jacobian method of Auclert et al. (2021) as implemented in their Python toolbox, which allows to easily simulate the interest rate peg just described.

[2]The value of σ_a obtained in this manner depends inter alia on the steady-state level of safe assets, and thus differs from Rannenberg (2024).

[3]See their Figures 1 and 3, respectively.

Figure 1: Forward guidance - Impact effect

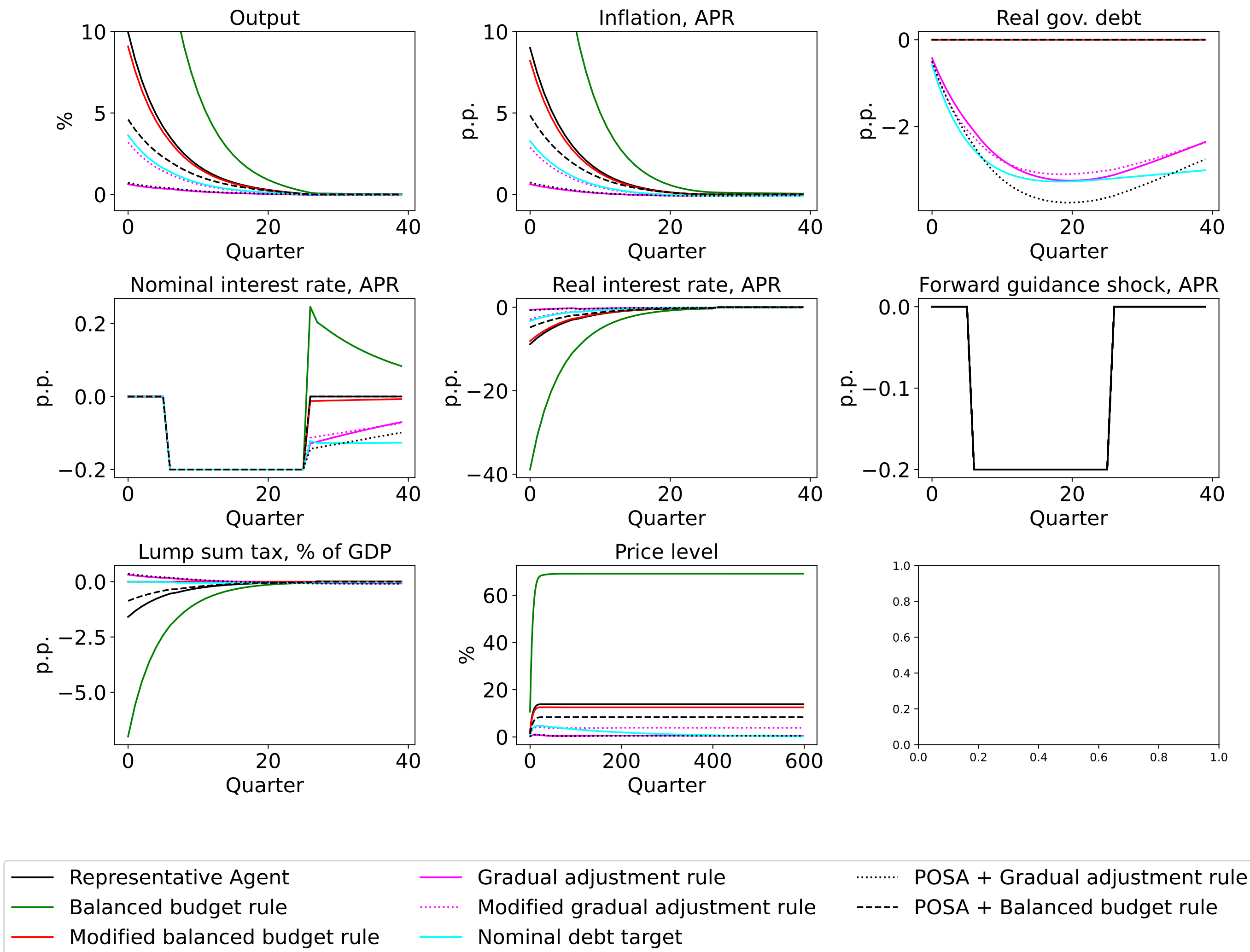


5 Results in the canonical HANK model

5.1 Baseline results

Figure 1 displays the results for the RA model and the six HANK models for different peg lengths. The impact of the policy is largest in the HANK model with the balanced budget rule (green line). Results for the model with the modified balanced budget rule (red line) are very close to the RA model (black line), while results are smallest with the gradual adjustment rule (magenta line). The reason why the effect of the policy is much stronger under the balanced budget rule than in the RA model is that with this setup, the policy implies a redistribution from asset rich - low MPC households to households with a high MPC. The decline in the real interest rate implies a decline in rich households asset income, as well as lower interest rate costs for the government and thus lower total taxes T_t, as can be obtained from Figure 2 (third row). The decline in tax revenue is distributed among households proportionally to their labor income (see equation 5). As labor income is more equally distributed than assets, the policy redistributes income towards households with lower labor income. Figure 3 confirms that the income share of households in the top 30% of the labor income distribution in total income (i.e. including asset income) drops on impact, i.e. post-tax income inequality is countercyclical. Table 2 displays the on-impact MPCs of each labor income decile, confirming that the MPCs in bottom 70% are indeed much higher than in the top 30%. In the context of an ordinary on impact increase of the real interest rate, Auclert et al. (2024a) document the feedback from a lower real interest rate to lower taxes and higher consumption implied by the balanced budget rule.

Figure 2: Forward guidance - dynamic effect



In the model with the modified balanced budget rule, this redistribution is removed by a transfer payment to bond holders which keeps their real interest income and the total lump sum tax constant (see equations 16 and 17), implying acyclical post-tax income inequality (see Figure 3). In line with the literature, the effect of forward guidance then becomes close to the representative agent model (see Bilbiie (2020), Werning (2015)).

However, under the gradual adjustment rule, the effect of the interest rate peg is much lower, and is linear in the length of the peg. One can distinguish two drivers of this attenuation compared to the modified balanced budget rule case. Firstly, the lower real interest rate dampens the income increase of asset holders, the more so the more assets they hold, and drives down their asset holdings very persistently, as can be obtained from Figure 2 for the case of a 20 quarter peg. The decline in their wealth motivates them to persistently save more. A lower demand increase from these households means lower demand and income for all households, further dampening the demand increase. The importance of this element can be gauged by comparing results for the modified gradual adjustment rule (magenta dotted line), which turns off the feedback from economic activity

Table 2: Model MPC according to labor income deciles

Decile	MPC
1	0.98
2	0.95
3	0.91
4	0.86
5	0.71
6	0.71
7	0.22
8	0.07
9	0.04
10	0.03

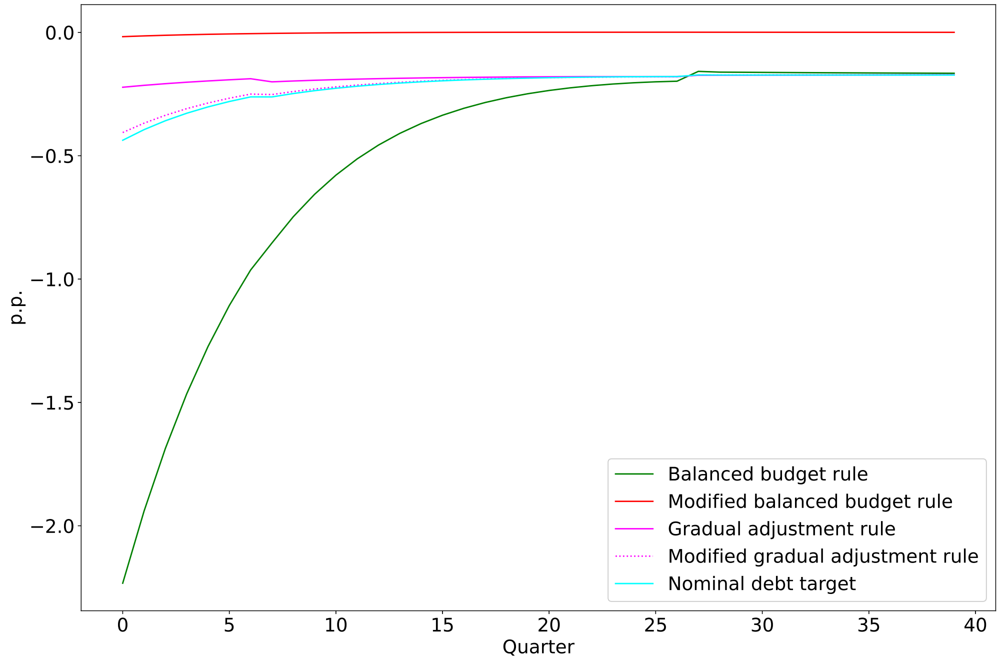
to tax revenue by setting $\varepsilon_{TY} = 0$, to results for the modified balanced budget rule. For peg lengths of about 8 quarters, the effect of the policy on output and inflation are already about 30% weaker under the modified gradual adjustment rule than under with the modified balanced budget rule. For a peg length of 12 quarters, the output effect is 36% lower, and becomes approximately linear in the length of the peg. In partial equilibrium logic, one can think of this mechanism as removing the aforementioned transfer that renders the results under the modified balanced budget rule close to those in the RA model by compensating asset holders ex-post for the income loss they suffer due to the real interest rate decline. As under the modified balanced budget rule, the size of this transfer rises exponentially in the length of the peg, so does the attenuating effect of removing it as we move from the modified balanced budget rule to the modified gradual adjustment rule.

The second driver of the attenuation obtained with the gradual adjustment rule is the feedback from economic activity to tax revenues with $\varepsilon_{TY} > 0$ via which the forward guidance policy raises the taxes of all households, thus further dampening the increase in consumption. As can be obtained from Figure 1 (compare the magenta solid and the magenta dotted line), this additional dampening mechanism is extremely important as well. Even for a 20 quarter peg, the impact effect on output equals merely 0.6% under the gradual adjustment rule.

Furthermore, in the RA model with POSA and the gradual adjustment rule, the effect of forward guidance is also much weaker than in the standard RA model, in line with the findings of Rannenberg (2024) (compare the black dotted and the black solid line). Moreover, the results for the POSA model are remarkably close to those obtained for the HANK model with that fiscal rule (compare the blue and the black dotted line). Part of the attenuation compared to the RA model is due to the pure "discounting" effect, i.e. $\theta < 1$, as can be obtained from comparing the results for the RA model to the POSA model with the balanced budget rule (compare black dashed to the black solid line), while the remainder is driven by the wealth effect. Hence forward guidance is another example where a POSA model may proxy the behavior of a HANK reasonably well. Auclert et al. (2024b) make a similar argument in the context of debt financed government expenditure shocks.

Finally, similar to Hagedorn et al. (2019), under the nominal debt targeting rule, are smaller than in the representative agent model for peg lengths exceeding 8 quarters. As discussed in Hagedorn et al. (2019), the reason for the attenuation compared to the representative agent model is that the nominal debt target implies that, unlike in the other models, the price level is determinate. Hence it eventually returns to the value it had before the shock. This fact puts a break on how far the

Figure 3: Share of the top three labor income deciles in total income with an interest rate peg of 20 quarters.



Note: Total income of an individual household equals the sum of interest and post-tax labor income: $\left(\frac{R_{t-1}}{\Pi_t} - 1\right) a_{it-1} + z_{it}$.

long-term real interest rate can decline in response to the forward guidance policy. However, they are still a bit higher than under the modified gradual adjustment rule and much higher than for the (unmodified) gradual adjustment rule with feedback from economic activity to tax revenue.

5.2 Long term bonds

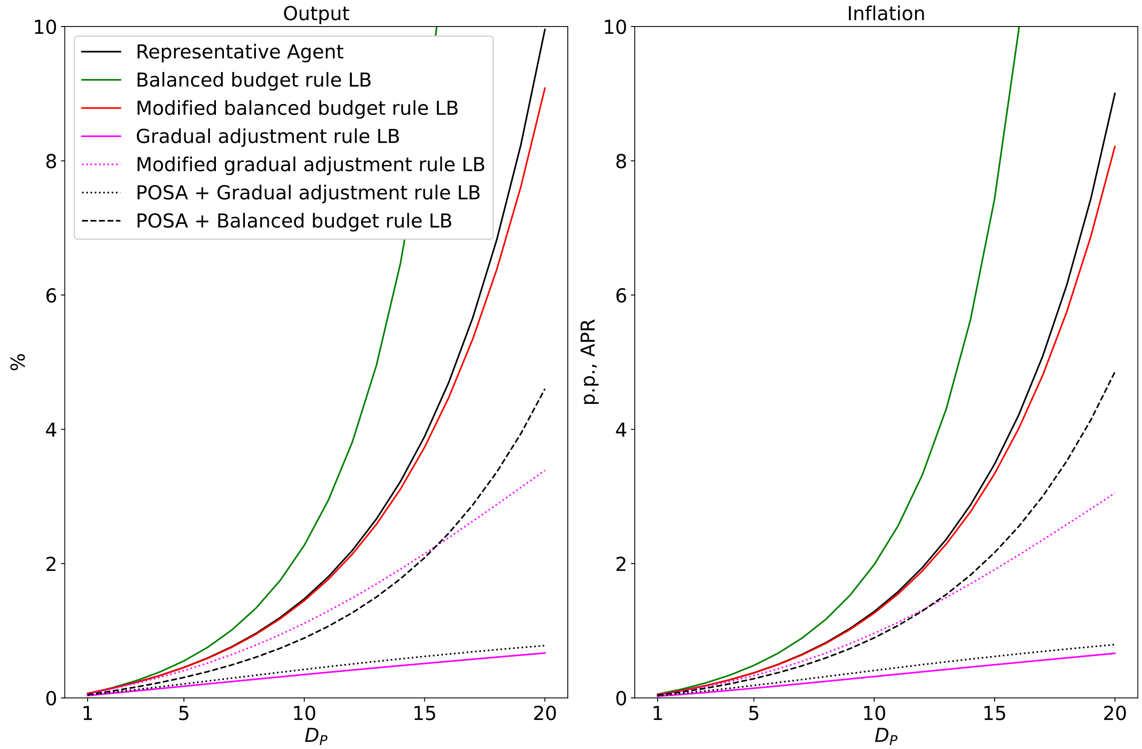
I next assume that government bonds are long-term. This assumption could potentially weaken the attenuation delivered by the gradual adjustment rule because the forward guidance policy will likely cause a jump in the bond price and thus the value of assets, thus counteracting the effect of higher inflation and tax revenues. Following Woodford (2001), I assume that all government debt consists of perpetuities with coupons that decay exponentially. Specifically, I assume that a bond issued of one dollar in period t pays ρ^j dollars $j+1$ periods later. I continue to assume that the bond is nominal. The real return on the bond earned by households is thus given by

$$r_t = \frac{1 + \rho P_{B,t}}{P_{B,t-1}} \frac{1}{\Pi_t} - 1 \tag{27}$$

where which replaces (22). The bond price $P_{B,t}$ is determined as

$$P_{B,t} = \frac{1 + \delta P_{B,t+1}}{R_t} \tag{28}$$

Figure 4: Effect of forward guidance with long-term-debt, impact



while the government budget constraint in real terms can still be written as

$$b_t = (1 + r_t)\, b_{t-1} + (G - T_t)$$

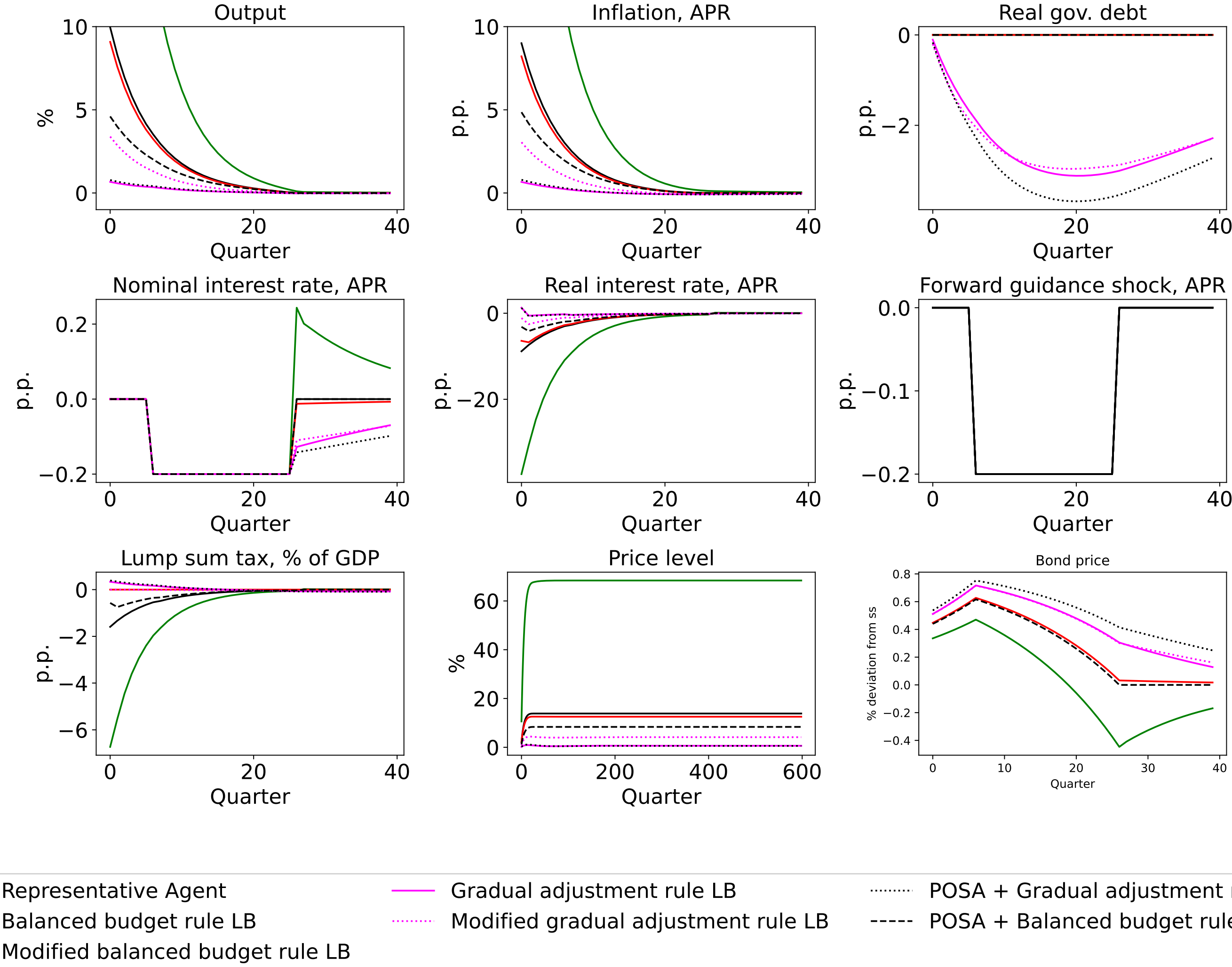
where b_t now denotes the real market value of government bonds, $\frac{B_t P_{B,t}}{P_t}$. Following Auclert et al. (2024a), I set $\rho = 0.95$.

As can be obtained from comparing Figure 4 and 1, the results are quite similar qualitatively and for the most part also in quantitative terms. Effects are smaller under the balanced budget rule because the ex-post real return on bonds declines less on impact (compare Figures 2 and 5, second row, green line), and thus the associated tax cut needed to keep real government debt constant is lower. The jump in the bond price $P_{B,t}$ in the model with long term bonds (see Figure 5, final row) offsets partly the effect of the inflation increase on the ex-post real return (see 27). This effect is also documented by Auclert et al. (2024a). Similarly, effects under the gradual adjustment rule are marginally larger both in the HANK model and with POSA, because with this fiscal rule, the smaller decline in the ex-post real return on bonds implies a smaller decline in total real outstanding bonds.

5.3 High liquidity

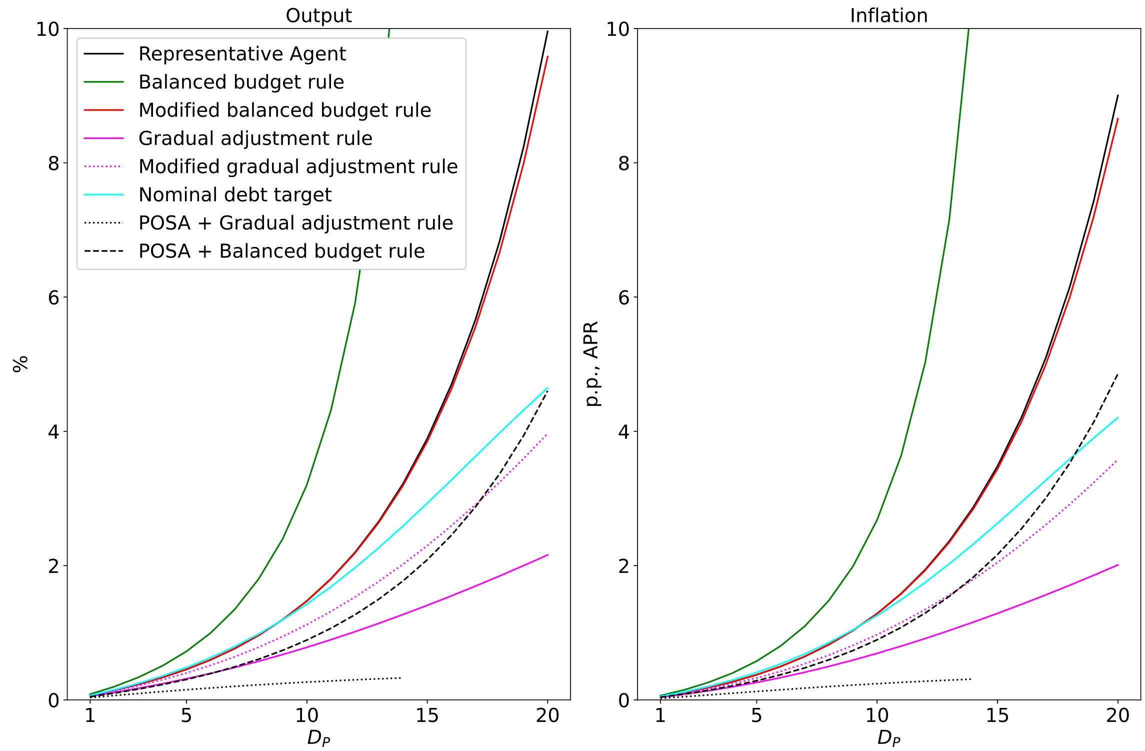
In the baseline, following Auclert et al. (2024b), the supply of liquid assets is calibrated in order to match an empirical target for the first year MPC, implying a low steady-state government debt-to-GDP ratio. I next examine the case of a higher steady-state supply of liquid assets, i.e. I set b^T

Figure 5: Effect of forward guidance with long term debt - dynamic effect



to 4, i.e. 100% of annual GDP, implying that the MPC is now considerably below the target value displayed in Table 1. As can be obtained from Figure 6, the effect of the policy is higher than in the low liquidity scenario in all the HANK model variants. However, the result under the gradual adjustment rule remain much below those for the representative agent model. For instance, for a peg length of 12 quarters, the output effect is less than half of what is observed in the RA model. By contrast, in the POSA model the effects of forward guidance under the gradual adjustment rule are lower than in the baseline. The explanation for this stronger attenuation consists of two elements. Firstly, with a higher steady-state debt-to-GDP ratio, a given decline of the real interest rate causes a stronger decline in the debt-to-GDP ratio, which can be seen by linearizing (20). Secondly, given that my calibration strategy for the wealth curvature parameter σ_a - setting σ_a to match the effect of an expected increase in the government debt-to-GDP ratio on the forward long-term interest rate (see Section 3) keeps $(1-\theta)\sigma_a\frac{Y}{A}$ roughly constant when the steady-state debt-to-GDP ratio $\frac{A}{Y}$ increases, this stronger decline in the debt-to-GDP ratio will lower consumption more strongly via equation (11).

Figure 6: Effect of forward guidance with higher liquidity, impact



5.4 Countercyclical inequality

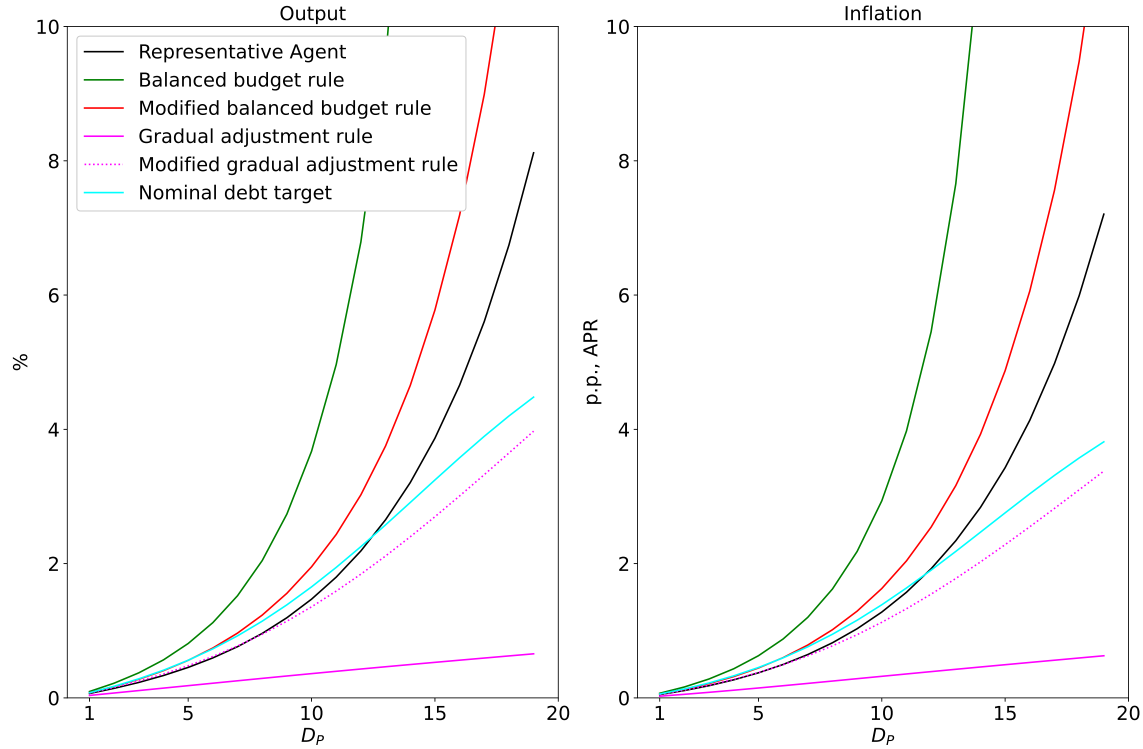
I now examine the effect of introducing countercyclical income inequality and income risk into the model. Bilbiie (2024) argues that HANK models feature a "Catch 22": The -often considered desirable- amplification of aggregate demand shocks compared to the representative agent model requires countercyclical inequality or countercyclical income risk, yet at the same time these feature implies that output responds very strongly to forward guidance policies. Following Auclert et al. (2024b), I introduce countercyclical labor income inequality and risk in a simple way by changing the retention function to $z_{it} = \tau_t \left(w_t e_{it} n_{it}\right)^{(1-\theta)(1+\zeta log(Y_t))}$, implying that (5) is replaced by

$$z_{it} = (Y_t - T_t) \frac{e_{it}^{(1-\theta)(1+\zeta log(Y_t))}}{\int e_{it}^{(1-\theta)(1+\zeta log(Y_t))} di} \tag{29}$$

For $\zeta < 0$, an increase of Y_t above its steady-state value of one lowers the cross sectional variance of income and thus inequality, thus channeling income to high MPC households. It also lowers the risk of a big income loss and thus precautionary saving. I set $\zeta = -0.5$. As can be obtained from Figure 7, as a result, the HANK model with the modified balanced budget now features a markedly stronger effect of forward guidance than the RA model. This amplification follows directly from the two aforementioned amplification mechanisms. Figure 8 reports the effect of a 16 quarter peg on the Top 30% income share. As expected, it now declines under the modified balanced budget rule, whereas before it remained unchanged.

Under the balanced budget rule, countercyclical inequality amplifies the effects of the policy very

Figure 7: Effect of forward guidance with countercyclical income inequality, impact



strongly. By contrast, under the gradual adjustment rule, the effects of the policy on output and inflation increase only marginally. As for this fiscal rule, the output effects of the forward guidance policy are small even in the baseline model with acyclical labor income inequality, the decline in inequality triggered by this comparatively small output increase is much lower than in the other models, which in turn implies little amplification via this new channel.

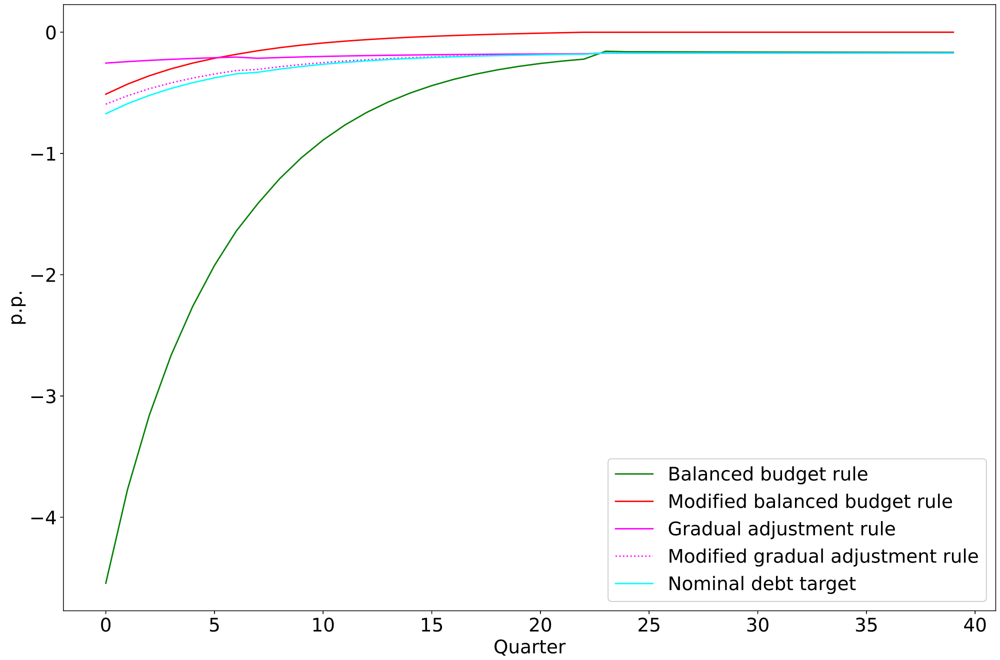
5.5 Real interest rate peg

As some contributions simulate real interest rate pegs (Gerke et al. (2024), McKay et al. (2016)), in this subsection I investigate the robustness of my results to pegging the real instead of the nominal rate. For that purpose, the monetary policy rule 21 is replaced by

$$\hat{R}_t = (1 - d_{p,t})\left(\phi_\pi \hat{\Pi}_t + \frac{\phi_y}{4}\hat{Y}_t\right) + d_{p,t}\left(\hat{R}_{p,t} + E_t\hat{\Pi}_{t+1}\right) \tag{30}$$

The simulation setup remains unchanged. As can be obtained from Figure 9, the effects of a real rate peg are much smaller then the effect of a nominal rate peg of the same size. Pegging the real rate implies neutralizing the effect of inflation on the real interest rate, implying that the decline of the real rate is much smaller than with a nominal rate peg. This neutralization also renders the effects of the peg in the RA model and for the balanced budget rules linear in D_P. The effects become concave in D_P for the gradual adjustment rule, the nominal debt target and the POSA models. The effects of the peg in the HANK and the POSA models under the gradual adjustment rule are still much smaller than in the RA model and the HANK models under the two balanced

Figure 8: Share of the top three labor income deciles in total income with an interest rate peg of 16 quarters, countercyclical inequality



Note: Total income of an individual household equals the sum of interest and post-tax labor income: $\left(\frac{R_{t-1}}{\Pi_t}-1\right)a_{it-1}+z_{it}$.

budget rules. As can be seen from the dynamic effects of a 20 quarter peg (Figure 10), except for the first quarter, results under the two balanced budget rule are now very close.[4]

6 Results in a quantitative model with capital

I now examine the effects of forward guidance in a quantitative model with capital, sticky prices and wages.

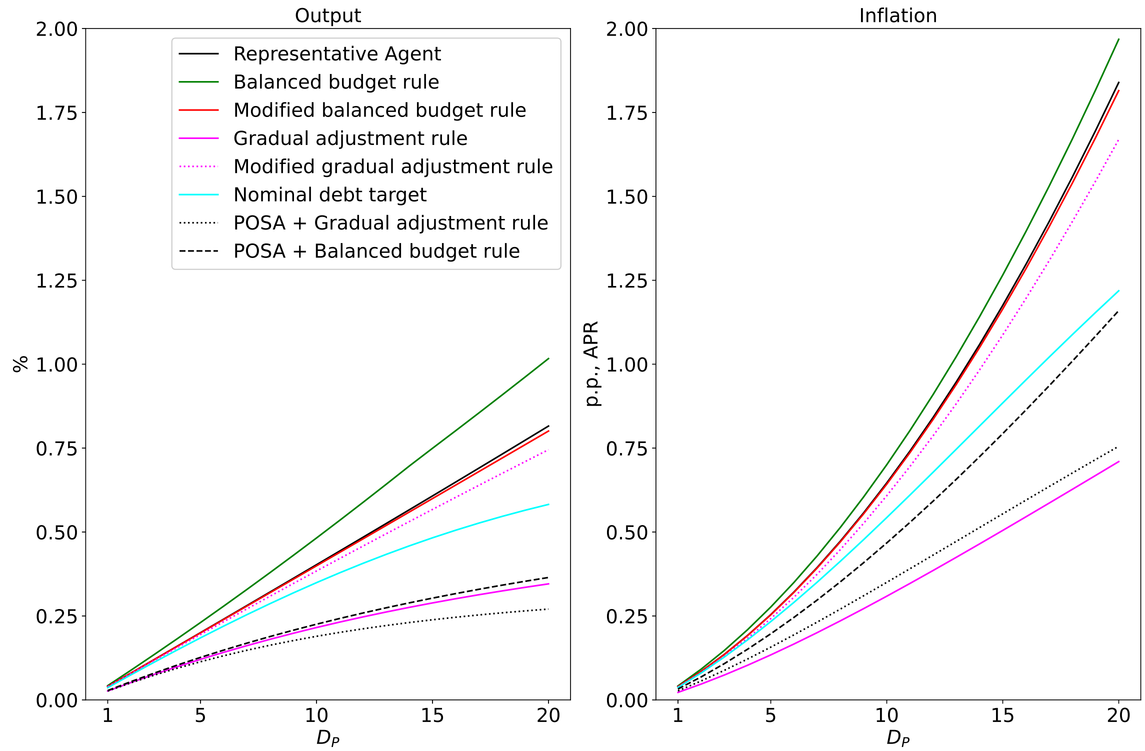
6.1 Households

Households have access to both a liquid account b^h_{it} and an illiquid account a_{it} as in Kaplan et al. (2018), subject to a convex portfolio adjustment cost $\Phi\left(a_{it}, a_{it-1}\right)$. However, I use here the discrete time version of the household block of that model developed by Auclert et al. (2021). The Bellman equation is given by

$$V_t\left(z_{i,t}, b_{it-1}, a_{it-1}\right) = max_{b_{it},a_{it}} u\left(c_{it}\right) + \beta E_t V_{t+1}\left(z_{i,t+1}, b_{it}, a_{it}\right)$$

[4]The upward blip under the balanced budget rule is due to the large transfer households receive due to drop in the real interest rate caused by the quarter one jump in inflation in combination with a $t-1$ steady-state nominal interest rate

Figure 9: Effect of a real rate peg, impact



subject to

$$(1+\tau_C)\left(c_{it}+\Phi\left(a_{it},a_{it-1}\right)\right)+a_{it}+b_{it}^h=z_{it}+\left(1+r_t^a\right)a_{it-1}+\left(1+r_t^b\right)b_{it-1}^h$$

$$a_{it}\geq 0$$

$$b_{it}\geq 0$$

where r_t^a and r_t^b denote the returns on the illiquid and the liquid account, respectively, and z_{it} is the post-tax labor income of households. The adjustment cost function is given by

$$\Phi\left(a_{it},a_{it-1}\right)=\frac{\chi_1}{2}\left(\frac{a_{it}-\left(1+r_t^a\right)a_{it-1}}{\left(1+r_t^a\right)a_{it-1}+\chi_0}\right)^2\left[\left(1+r_t^a\right)a_{it-1}+\chi_0\right]$$

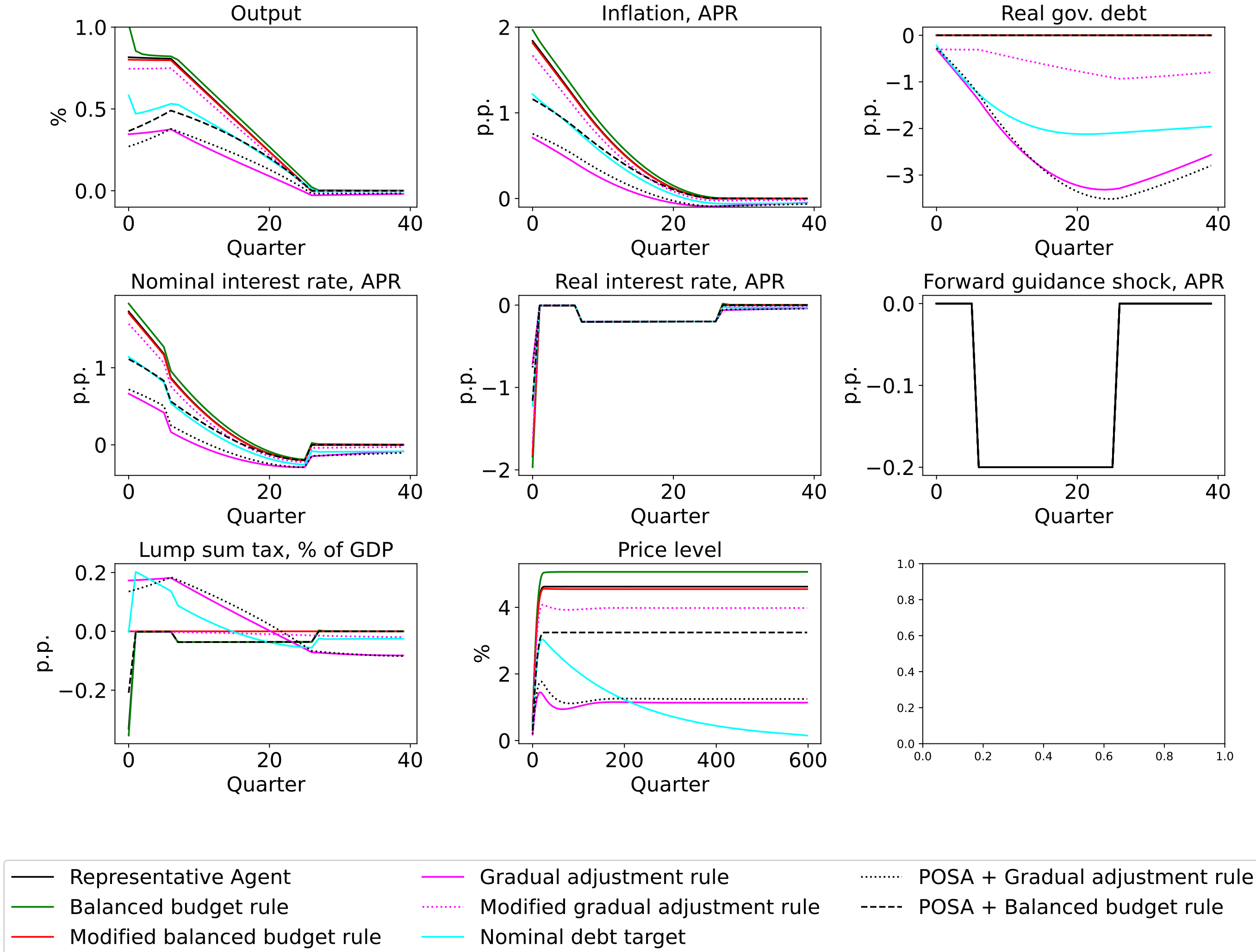
with $\chi_1,\chi_0>0$. In order to ensure maximum comparability of saving and consumption decisions, I assume that a slightly modified version of this portfolio adjustment cost is present also in the RA version of the model, namely

$$\Phi^{RA}\left(a_{it},a_{it-1}\right)=\frac{\chi_1}{2}\left(\frac{a_{it}-\frac{1+r_t^a}{1+r^a}a_{it-1}}{\frac{1+r_t^a}{1+r^a}a_{it-1}+\chi_0}\right)^2\left[\frac{1+r_t^a}{1+r^a}a_{it-1}+\chi_0\right]$$

The modification ensures that in the steady state $\Phi^{RA}\left(a_{it},a_{it-1}\right)$ and its first derivatives equal zero.

The assumptions regarding wage setting and labor demand remain unchanged, implying $z_{it}$is

Figure 10: Real rate peg 20 quarters - dynamic effect



given by

$$z_{it} = \left(\left(1 - \tau_{w,t}\right) w_t N_t - T_t\right) e_{it}$$

where $\tau_{w,t}$ denotes the labor income tax, and the wage setting equation (12) remains unchanged except for the presence of that labor tax:

$$\left(\frac{\Pi_{W,t}}{\Pi} - 1\right)\frac{\Pi_{W,t}}{\Pi} = \kappa_w \left(\frac{\epsilon_w}{\epsilon_w - 1}\frac{\chi_N \left(N_t\right)^\eta}{\left(1 - \tau_w\right) w_t C_t^{-\sigma}} - 1\right) + E_t \left\{\frac{1}{1 + r_t}\frac{N_{t+1}}{N_t}\left(\frac{\Pi_{W,t+1}}{\Pi} - 1\right)\frac{\left(\Pi_{W,t+1}\right)^2}{\Pi_{t+1}\Pi}\right\} \tag{31}$$

Total liquid and illiquid assets, consumption and portfolio adjustment costs are given by

$$b_t^h = \int b_{it}^h di \tag{32}$$

$$A_t = \int a_{it} di \tag{33}$$

$$C_t = \int c_{it} di \tag{34}$$

$$\Phi_t = \int \Phi\left(a_{it}, a_{it-1}\right) di \tag{35}$$

6.2 Capital markets

As in Kaplan et al. (2018), the liquid account of the household consists purely of government bonds, whose nominal return R_t is set by monetary policy. Thus it is safe in nominal terms. Hence the real period t return on the liquid account r_t^b is given by

$$r_t^b = \frac{R_{t-1}}{\Pi_t} - 1$$

The illiquid account is a mutual fund that operates under perfect competition and invests in in physical capital K_t , which earns a rental rate $r_{K,t}$ and shares of the monopolistically competitive firms s_t. The mutual fund pays a fraction ω of its total assets A_t as an administration cost, and earns a return r_t^A. Any fluctuations in r_t^A are passed on to household, implying that the households return on the illiquid account equals

$$r_t^a = r_t^A - \omega \tag{36}$$

The mutual fund can substitute perfectly between shares and physical capital. Hence the share price is given by

$$s_t = E_t \left\{ \frac{s_{t+1} + \left(1 - \tau_K\right) div_{t+1}}{1 + r_{t+1}^A} \right\} \tag{37}$$

where τ_K and div_t denote the tax rate on profits and dividends paid by the monopolistically competitive firms, respectively. Similarly, the expected return on capital has to satisfy

$$E_t \left\{ r_{t+1}^A \right\} = E_t \left\{ \left(1 - \tau_K\right) \left(r_{K,t+1} - \delta\right) \right\}$$

where τ_K and δ denote the capital tax rate and the rate of depreciation, respectively. The ex-post the return of the mutual fund is given by

$$r_t^A = \frac{s_t + \left(1 - \tau_K\right) div_t}{s_{t-1}} \frac{s_{t-1}}{s_{t-1} + K_{t-1}} + \left(1 + r_{K,t}\right) \frac{K_{t-1}}{s_{t-1} + K_{t-1}} - 1 \tag{38}$$

Capital accumulation is given by

$$K_t = \left(1 - \delta\right) K_{t-1} + I_t \tag{39}$$

Due to the earlier assumption of a portfolio adjustment cost I do not assume a capital adjustment cost, but results are robust to adding the assumption of a capital adjustment cost of the form $\frac{\epsilon_I}{2} \left(\frac{K_t}{K_{t-1}} - 1 \right)^2 K_{t-1}$.

6.3 Firms

There is a continuum of monopolistically competitive firms producing product varieties f. The demand curve for individual varieties is given by

$$Y_t(f) = Y_t \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon_p}$$

Firms face Rotemberg (1982) type price adjustment costs:

$$AC_t(f) = Y_t \frac{\xi_p}{2} \left(\frac{P_t(f)}{P_{t-1}(f)} \frac{1}{\Pi} - 1 \right)^2$$

where $\xi_p > 0$ denotes the adjustment cost curvature and Y_t denotes total output of all firms. Output is produced by combing labor and capital in a Cobb-Douglas production function

$$Y_t(f) = Z K_t^{\alpha}(f) N_t^{1-\alpha}(f) - FC$$

where $FC > 0$ denotes a fixed cost of production. The first order condition with respect to prices results in the price Phillips curve:

$$\begin{aligned} &\kappa_p \left(\left(\frac{\epsilon_p}{\epsilon_p - 1} \right) mc_t - 1 \right) \\ +\beta E_t &\left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1}}{Y_t} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \right\} = \left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} \end{aligned}$$

with $\kappa_p = \frac{\epsilon_p - 1}{\xi_p}$. The FOCs with respect to employment and capital are given by

$$\begin{aligned} w_t &= (1-\alpha) mc_t \frac{Y_t + FC}{N_t} \\ r_{K,t} &= \alpha mc_t \frac{Y_t + FC}{K_{t-1}} \end{aligned}$$

Dividends are given by

$$div_t = (1 - mc_t)(Y_t + FC) - FC \tag{40}$$

6.4 Government

The government budget constraint in real terms is now given by

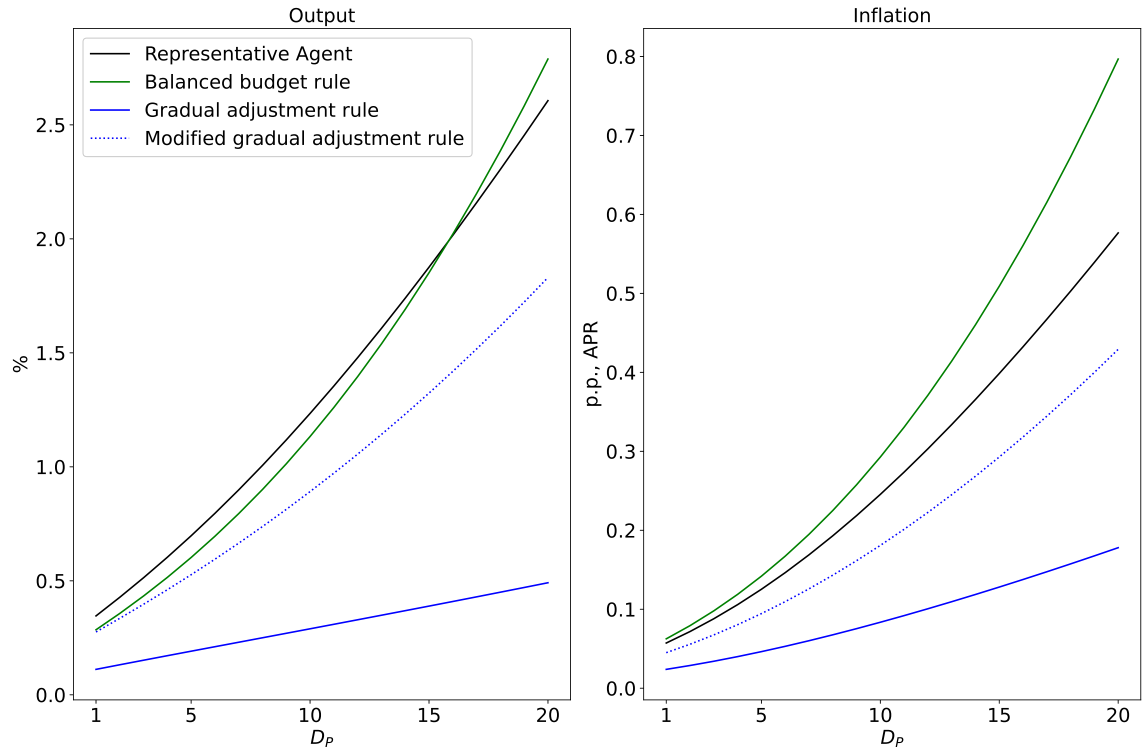
$$b_t = \frac{R_{t-1}}{\Pi_t} b_{t-1} + G - T_t - \tau_w N_t w_t - \tau_C (C_t + \Phi_t) + \tau_K (r_{K,t} - \delta) K_{t-1}$$

The budget rules are the same as those considered in the simple model.

Monetary policy is given by

$$\hat{R}_t = (1 - d_{p,t}) \left((1 - \rho_R) \left(\phi_\pi \hat{\Pi}_t + \frac{\phi_y}{4} \hat{Y}_t \right) + \rho_R \hat{R}_{t-1} \right) + d_{p,t} \hat{R}_{p,t} \tag{41}$$

Figure 11: Effect of forward guidance in quantitative model, impact



6.5 Equilibrium

The market for liquid and illiquid asset clears:

$$b_t^h = b_t \tag{42}$$

$$A_t = Q_t K_t + s_t \tag{43}$$

The goods market clears:

$$Y_t = C_t + G + I_t + \Phi_t + \omega A_{t-1} \tag{44}$$

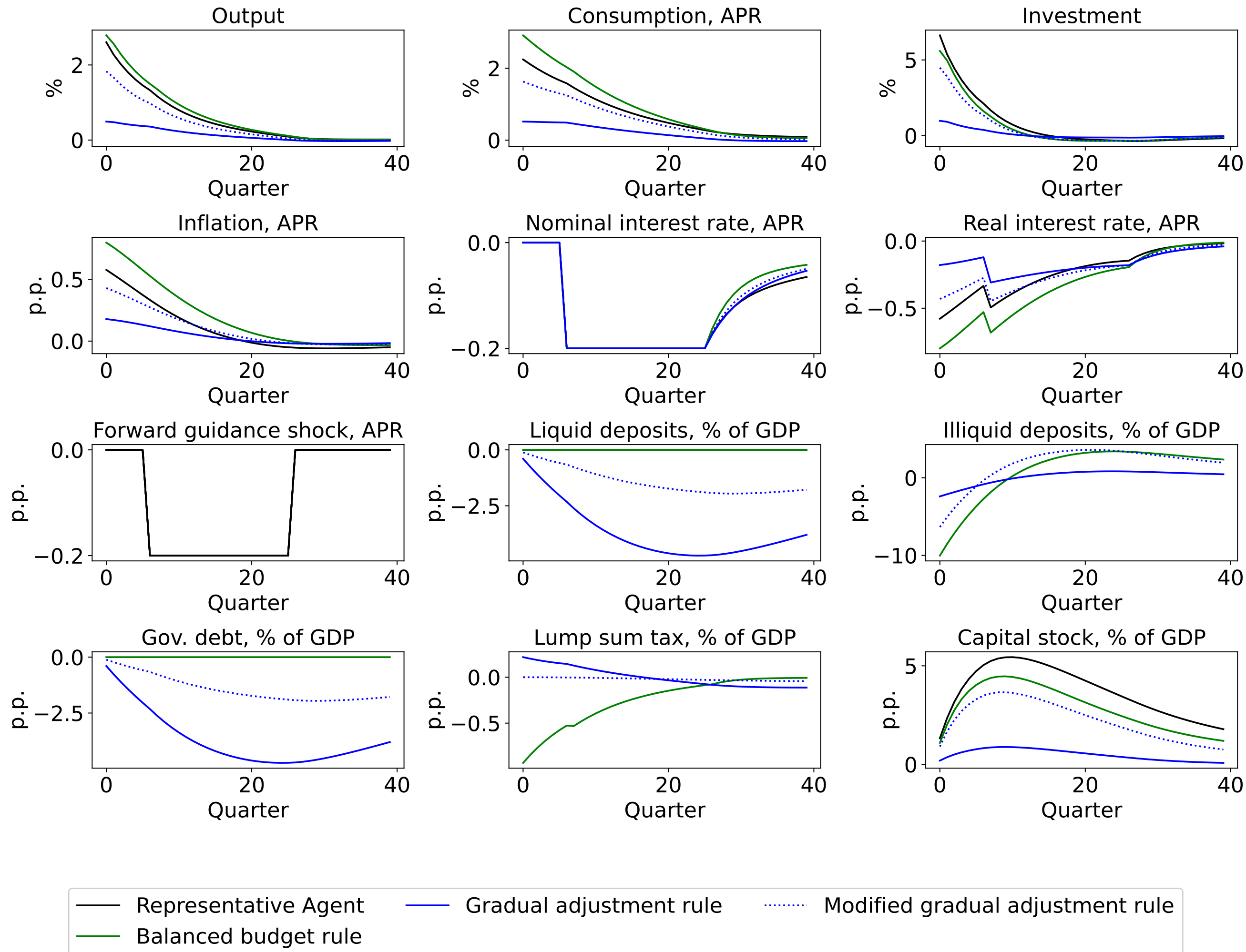
Furthermore, GDP is defined as output without portfolio adjustment and administration costs:

$$GDP_t = C_t + G + I_t \tag{45}$$

6.6 Calibration

I calibrate a number of parameters directly. The elasticity of intertemporal substitution $\frac{1}{\sigma}$, the Frisch elasticity of labor supply $\frac{1}{\eta}$, the wage markup μ_w, the calibration of the idiosyncratic labor income process, the share of government expenditure in GDP and the fiscal rule parameters for the gradual adjustment rule are the same as in the simple model. However, as in Auclert et al. (2021)'s analysis of the two account model, I discretize the AR(1) process for e on a three point grid. The Taylor rule inflation and output coefficients also remain the same, but there is now interest rate

Figure 12: Effect of forward guidance in quantitative model - dynamic effect



smoothing, with $\rho_i = 0.85$. The portfolio adjustment cost pivot parameter χ_0 follows Auclert et al. (2021). I construct the consumption, labor and profit tax rates from BEA NIPA data following Leeper et al. (2010) and average them over the 1987-2019 period. The lump sum tax semi-elasticity is set to the estimate of Price et al. (2015) of the output semi-elasticity of government expenditure in the US. I set the Phillips curve slope parameters in line with recent estimates. The capital depreciation rate δ equals 0.025.

Given these choices, I calibrate 8 parameters in order to set the steady-state values of important model variables to (empirical) target values, namely the household discount factor β, the illiquid asset administration cost ω, the portfolio adjustment cost scale χ_1, the target level of government bonds b in the fiscal rule, the capital elasticity of output α, the steady-state price markup μ_p, fixed costs FC, the level of TFP and the labor disutility weight χ_N. The targets are a real return on the liquid asset of 2% annually, a spread between the return on illiquid and liquid assets $r^A - r$ of 5.1% annualized, a first year average MPC out of an income windfall of 0.44, an investment to GDP ratio of 0.18, a labor share of 0.61, a liquid-asset-to-annual GDP ratio of 0.23 and a ratio of output gross of fixed costs to output $\frac{FC+Y}{Y}$, which measures the degree of returns to scale in the production function, of 1.4. Furthermore, the steady-state levels of employment output are normalized to one. The target for liquid assets equals the value in the model of Kaplan et al. (2018), while the target for the first year average MPC corresponds to the empirical lower bound estimate of Auclert et al. (2024b) based on Italian survey data. The target for $r^A - r$ is computed by first estimating the return on capital as the average of the return on equity and Bonds, and then substracting the three-month treasury bill yield. To compute the return on equity, I combine the estimate of the equity risk premium reported by Damodaran (2022) and the return on 10 year government bonds. For the return on bonds, I use Moody's Seasoned Baa Corporate Bond Yield. The degree of returns to scale in production $\frac{FC+Y}{Y}$ is line with recent estimates for the US economy, e.g. Lindé et al. (2016).

Except for the MPC and the target for liquid assets, the target values are calculated as averages over the 1987 to 2019 period. I also report the implied illiquid asset to GDP ratio, which with 2.4 is somewhat lower than the value of 2.9 estimated by Kaplan et al. (2018). Furthermore, the first year MPC out of illiquid wealth matches closely the stock market wealth MPC estimated by Chodorow-Reich et al. (2021) 0.032.[5]

As in Auclert et al. (2021)'s analysis of the two account model, I discretize the assets spaces using double-exponentially-spaced grids, with 50 and 70 grid points for liquid and illiquid assets, respectively. For the calibration of the RA model, I uses the same values for the directly calibrated parameters and impose the same values on the aforementioned variables, except for the MPC. Correspondingly, I calibrate the value for the portfolio adjustment cost scale directly, and set it to the same value as in the HANK model.

[5] The annual marginal propensity to consume with respect to an increase in illiquid wealth is computed as follows. $\frac{\partial C_t}{\partial r_1^a}$ denotes the partial derivative of total period t household consumption with respect to an increase in the return on illiquid assets in period 1. The period one MPC out of a period one increase in illiquid wealth is accordingly given by $\frac{\partial C_t}{\partial r_1^a}\frac{1}{A}$, and the first year average by $\frac{1}{A}\sum_{i=1}^{4}\frac{\partial C_i}{\partial r_1^a}$.

Table 3: Calibration two-account model

Directly calibrated parameters		
Parameter	Definition	Value
σ	Consumption curvature	1.0
η	Labor curvature	2.0
χ_0	Portfolio adj. cost pivot	0.25
σ_e	Idiosyncratic income shock s.d.	0.46
ρ_e	Idiosyncratic income shock persistence	0.98
G	Government expenditure-to-GDP ratio	0.2
ϵ_{YT}	Expenditure semi-elasticity	0.46
τ_C	Consumption tax	0.07
τ_w	Labor tax	0.24
τ_K	Profit tax	0.21
ϕ_T	Fiscal rule debt response	0.03
ρ_T	Fiscal rule lagged tax response	0.95
ϕ_π	Taylor rule inflation	1.5
ϕ_y	Taylor rule output	0.5
ρ_i	Taylor rule lagged interest rate	0.85
μ_w	Steady-state wage markup	1.5
κ_w	Wage Phillips curve slope	0.011
κ_p	Price Phillips curve slope	0.011
ϵ_I	Capital adjustment cost curvature	0.00
δ	Depreciation rate	0.025
Parameters calibrated to match target values		
Parameter	Definition	Value
β	Household discount factor	0.989
ω	Asset administration cost	0.009
χ_1	Portfolio adj. cost scale	37
b^T	Government debt target fiscal rule	0.92
α	Capital elasticity of output	0.35
μ_p	Steady-state price markup	1.49
FC	Fixed cost	0.40
Z	TFP	0.71
χ_N	Labor disutility weight	0.59
Target values		
Parameter	Definition	Value
r	Real interest rate (safe assets, APR)	2.0
$r^A - r$	Real return (illiquid account, APR)	5.1
MPC	First-year MPC	0.44
$b/(4Y)$	Liquid asset-to-GDP ratio	0.23
$\frac{I}{Y}$	Investment-to-GDP ratio	0.18
$\frac{wN}{Y}$	Labor share	0.61
$\frac{FC+Y}{Y}$	Output gross of fixed costs-to-output ratio	1.40
Y	Output	1.0
N	Employment	1.0
$\frac{A}{4Y}$	Not targeted: Illiquid asset-to-GDP ratio	2.4
$\frac{1}{A}\sum_{i=1}^{4}\frac{\partial C_i}{\partial r_1^a}$	Not targeted: First-year MPC out of illiquid assets	0.032

Note: The empirical target value for the first-year MPC is taken from Auclert et al. (2024b).

6.7 Results

The simulation setup remains as described in Section 4. In the RA version, the effects of the peg on output and inflation are much smaller than in its simple model counterpart (see Figure 11). With both price and wage stickiness, the overall degree of nominal rigidity is larger and thus a given increase in output causes a much smaller increase in inflation and correspondingly a smaller decline in the real interest rate (see Figure 12). Under the balanced budget rule, the smaller decline of the real interest rate means a much smaller redistribution from asset holders to lower income households than in the simple model via lower lump sum taxes (see Figure 12), implying that the effects of forward guidance are now closer to the representative agent model. With the gradual adjustment rule, the results of the policy continue to be much smaller than under the balanced budget rule or in the RA model. For instance, for a 12 quarter peg, the effect of the policy on GDP equals less than one fourth of the effects in the RA model and in the HANK model with the balanced budget rule. As can be obtained from Figure 12, liquid assets strongly decline under the gradual adjustment rule, thus dampening the increase in consumption. The value of the illiquid account A_t drops on impact driven by a drop in the share price s_t, which declines due to a decline in dividends caused an increase in the real wage. It then recovers due to the increase in the physical capital stock. Assuming a capital adjustment cost would dampen the drop of A_t due to the on-impact increase in the value of the capital stock as a consequence of higher Tobin's Q.

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A Contemporaneous and anticipated monetary policy shocks

In a short note, Kaplan et al. (2016) compare the effects of of contemporaneous and anticipated monetary policy shocks (as opposed to interest rate pegs) in variants of their HANK model. I now perform the same simulation shocks in some of the models considered in the main text. For that purpose, I change the monetary policy rule (21) by adding a contemporaneous and an anticipated monetary policy shock

$$\hat{R}_t = \phi_\pi \hat{\Pi}_t + \frac{\phi_y}{4}\hat{Y}_t + \varepsilon_t^0 + \varepsilon_{t-8}^8 \tag{46}$$

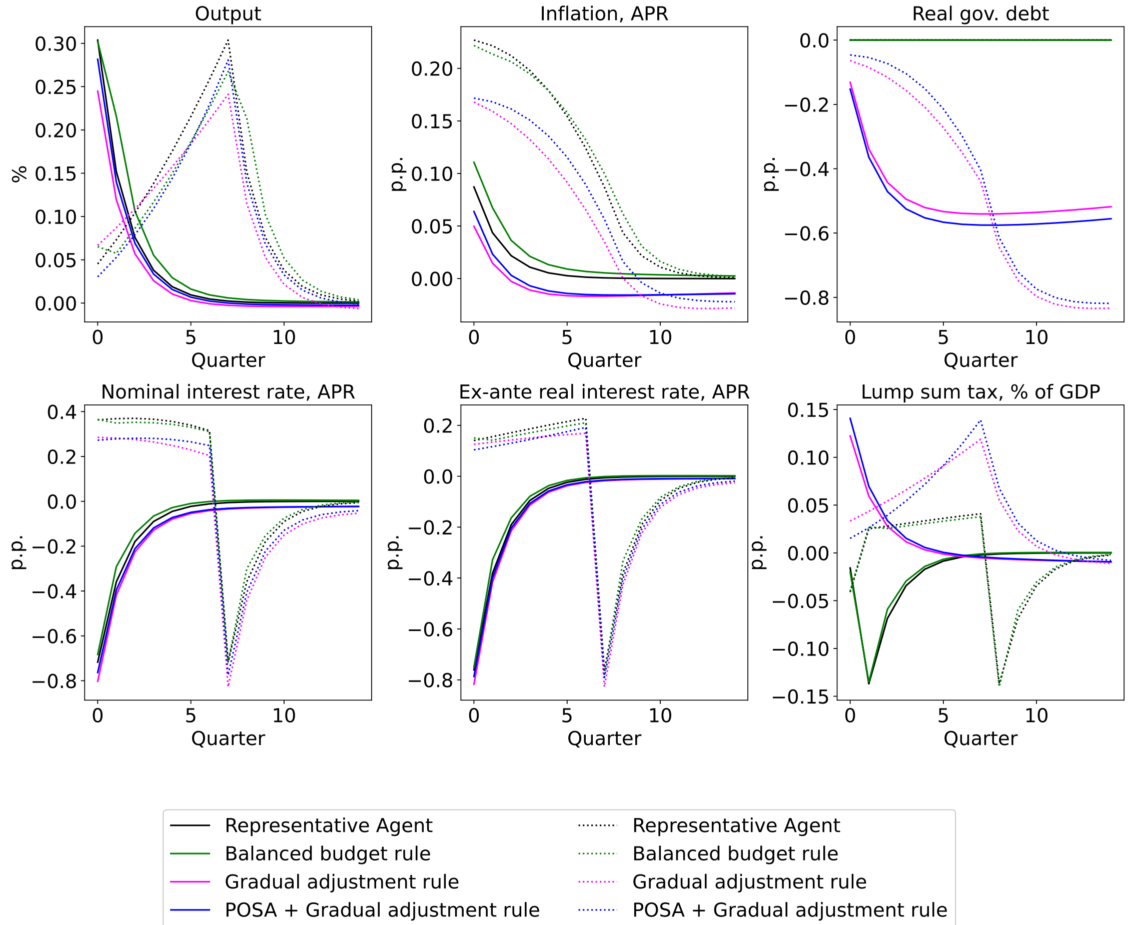
with

$$\begin{aligned}\varepsilon_t^0 &= \rho\varepsilon_{t-1}^0 + \eta_t^0\\ \varepsilon_t^8 &= \rho\varepsilon_{t-1}^8 + \eta_t^8\end{aligned}$$

I set $\rho = 0.5$ and seperately simulate first period increases of η_t^0 and η_t^8 by 0.0025 (1% annually). I do not display the results for the modified balanced budget rule, as they are almost identical to the RA model, both for the contemporaneous and the anticipated shock. As can be Figure 13, for the contemporaneous shock, the impact effect on output is strongest in the RA model (black solid line), and almost identical to the output effect under the balanced budget rule (green solid line). From the second quarter onwards, the output trajectory is larger under the balanced budget rule. This change is driven by the decline in the quarter two decline of the ex-post real interest rate $\hat{R}_1 - \hat{\Pi}_2$, which lowers the governments real interest rate burden and thus under the balanced budget rule taxes. As discussed in Section 5.1, as taxes are more equally distributed than assets, this shift redistributes income from low to high MPC households.

By contrast, under the gradual adjustment rule, the effects are a bit smaller than in the RA model as the policy raises tax revenue via higher output (see equation 18) and lowers government debt, which has a negative effect on consumption in both the HANK and the POSA model. Furthermore, the decline in government debt is highly persistent, which is why starting in quarter 7, output remains slightly below steady-state for a very long period. As up to first order, via the Phillips curve inflation depends positively on current and future output, the ratio of the on impact increase in inflation under the gradual adjustment rule to the inflation increase in the RA model is smaller than the corresponding ratio for output. The finding the output and inflation effects are larger under the balanced budget than under the gradual adjustment rule is consistent with Kaplan et al. (2016).

Figure 13: Effect of contemporaneous and anticipated monetary policy shocks, impact



For the anticipated shock, in the RA model and all other models, the impact effect on output is much smaller than for the contemporaneous monetary policy shock, and then rises gradually until the shock arrives. Thus these output dynamics are not specific to HANK models, unlike what is suggested by Kaplan et al. (2016). In all models, inflation jumps and then declines gradually. The increase in inflation and output causes an increase in the nominal and the ex-ante real interest rate on impact via the interest feedback rule (46), with both remaining above steady-state until the shock arrives. In the RA model (black dotted line), the output profile can then be explained by the fact that via the consumption Euler equation, up to first order, consumption depends negatively on the sum of current and future real inters rate: $\hat{C}_t = -\frac{1}{\sigma} E_t \left\{ \sum_{i=0}^{\infty} \hat{R}_{t+i} - \hat{\Pi}_{t+1+i} \right\}$. As the moment of the arrival of the shock approaches, $\frac{1}{\sigma} E_t \left\{ \sum_{i=0}^{\infty} \hat{R}_{t+i} - \hat{\Pi}_{t+1+i} \right\}$ becomes more negative, thus explaining the increasing output path up until the arrival of the shock, when the real interest rate drops to its minimum value.

In the HANK model under the balanced budget rule, the output trajectory is lower than in the RA starting from the second quarter until after the arrival of the shock, after which it is higher than in the RA model. These dynamics are again explained by the dynamics of the lump sum tax, driven by the dynamics of the ex-post real interest rate.

The HANK model with the gradual adjustment rule displays the stronger output effects than all other models during the first three quarters, and after the fifth quarter a lower output trajectory than the other models, while the effects on inflation are the weaker than in the other models throughout. The finding of a larger on-impact output effect but a smaller peak effect than under the balanced budget rule and the results for inflation are consistent with findings of Kaplan et al. (2016) [6] The reason for this result appears to be the lower real interest rate trajectory than under the balanced budget rule and the RA model. Like the contemporaneous shock, the anticipated shock persistently lowers real government debt, and in a more back loaded fashion. Lower government debt tends to lower consumption, implying that after quarter 12, output remains slightly but very persistently below steady-state, which limits the on-impact inflation increase via the Phillips curve. Via the monetary policy rule, the lower inflation trajectory implies lower nominal and real interest rate trajectories than in the RA model and under the balanced budget rule.

By contrast, in the POSA model, the on-impact output increase is smaller than in all other models. The difference with respect to the HANK model under the gradual adjustment rule maybe due to the fact that the POSA Euler equation features not merely a wealth effect from government bonds like the HANK model, but also a smaller effect of future real interest rate deviations on current consumption, the more so the more distant these deviations are located in the future (see equation (11)).

[6] See their “transfers adjust” and “debt adjusts” scenarios, Figures 2 and 4.