Hope, Noise, and the Efficiency of Perfect Meritocracy

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Abstract

This paper explores the economic effects of imperfect meritocracy in recruitment and career advancement. We compare two career promotion mechanisms: a fully meritocratic system and a "noisy" one, that allows less productive workers to advance. Our model shows that imperfect meritocracy in promotions can boost worker effort through the "hope effect," potentially leading to higher aggregate output and total welfare compared to a strictly meritocratic system. Less skilled workers benefit most under this scenario, while the high skilled are worse off. We conclude that when perfect meritocracy in recruitment is unattainable, it may not be optimal to enforce it in career advancement, offering insights for economic policy.

Keywords: meritocracy; efficiency; recruitment; career advancement.

J.E.L. Classification: A13; D61; D63; J20; M51.

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1 Introduction

The belief that meritocracy has beneficial impacts on society has deep, historical foundations, with roots in the ideas of Confucius, Plato, and Aristotle[1]. Additionally, the notion that meritocracy serves as a means to assign the appropriate roles in the economy to the right individuals or firms has been endorsed by prominent classical thinkers, ranging from Adam Smith to Ricardo, Marshall, Schumpeter, and Hayek. Indeed meritocracy is a multi-facet concept and there is no unanimous consensus on a definition. As Sen (2000) has put it, "the idea of meritocracy may have many virtues, but clarity is not one of them". Such ambiguous nature of merit justifies the interest of both philosophers and economists for a debate on justice and the outcomes of a competitive, meritocratic, system. In fact, meritocracy is a theory of distributive justice, resting on the principle of equal opportunity and of merit-based distributions (Sandel, 2020). Economists also contributed to such a debate on justice by tackling the problem of disentangling success and luck (Frank, 2016). In short, this debate deals with the true nature of merit and the ethical implication of meritocracy but it does not directly address the tenet that it delivers the highest economic efficiency.

A radical attack on the very idea that meritocracy always enhances efficiency and provides a set of virtuous incentives to agents has been taken by Morgan et al. (2022) who show that "increasing meritocracy cannot universally raise the performance of sufficiently diverse contestants, because it discourages the weak and makes the strong complacent (p. 2)". However, it is generally accepted that the overall level of meritocracy of a society is intrinsically associated with the presence of social mobility and thus depends on the meritocratic nature of a plurality of processes. Such processes are often placed in a temporal sequence. For example, exposure to formative stimuli in early childhood impacts intellectual development that is then strengthened in the different, subsequent,

[1]Confucius (551–479 BC) gave an argument for meritocratic politics which shaped Chinese governance for millennia: Meritorious rule is one of the most central ideas in Confucian political thought. The idea, simply put, is that those who occupy positions of power should possess the appropriate virtue and ability. There should be a certain fit between position and virtue. (see Chan (2013). According to Aristotle, justice is done when we give these scarce goods to the people who deserve them. For a complete examination of Aristotle's view of distributive justice, see Keyt (1991).

levels of school education. The level of skill of a person is therefore the result of a complex process in which human capital is accumulated by applying inborn talent to education and learning by doing. Considering the labor market, meritocracy concerns the processes of recruitment, of attributing salary levels, of career advancement,... up to the methods of determining pension benefits. These dimensions also have, obviously, an interdependence over time: for example, the loss of meritocratic access to higher education jeopardizes the subsequent possibility of efficient placement of workers in different job positions.

In this paper we do not address the issue on the overall optimality of meritocracy in terms of economic efficiency. Rather, we consider the implications of non perfect meritocracy in one process on the desirability of perfect meritocracy in another, connected, process. In particular, we focus on two parts of the sequence of a working life path: 1) the recruitment of workers and 2) the subsequent career advancement on a hierarchical ladder. We show that, if part 1) of the sequence is not perfectly meritocratic , then it may be not optimal from the point of view of total output and utilitarian welfare maximization that part 2) of the sequence is perfectly meritocratic. On the contrary, it may be even the case that more meritocracy in the career advancement process worsens the problem.

We assume that the recruitment process of workers produces a slightly imperfect fit between skill and the job position occupied. We then consider two alternative career advancements mechanism: 1) a fully meritocratic one, in which only workers that proved to be the most productive ones in a given position may have a chance to get a promotion, and 2) a "noisy" scenario, where even less productive workers have an opportunity for an advancement. In this second setting, two opposite effects are at work. On the one hand, the more meritocratic the career advancement process is, the better the fit between talent and job positions occupied ("efficiency effect"); on the other hand, if the career advancement mechanism is slightly imperfect then a larger group of people can hope for an advancement and therefore put in greater effort at work ("hope effect").

For a dense set of parameter values, the "hope effect" can be prevalent over the "efficiency effect" and aggregate output and total welfare measured in a utilitarian way can be greater compared to what obtained with perfect meritocracy in career advancements.

Our model also delivers a “Rawlsian” result: the least talented workers in the economy are always better off under the "noisy" scenario, while the most talented ones are worse off, since the hope effect is maximum for those at the lowest level of the social ladder and equal to 0 for those at the top. Such a result is consistent with Young’s (1958) idea that a meritocratic society can hurt the welfare of those at the bottom of the society since they perceive their subordinate status as personal failure .

The literature on meritocracy is extensive and diverse, spanning the entire range of human and social sciences. Conducting a comprehensive review lies beyond the scope of this paper. Instead, we aim to highlight recent studies relevant to our work, as they challenge some of the virtues often associated with the idea of meritocracy.

The work most closely connected to ours is the already cited paper of Morgan et al. (2022). Using an all-pay auction setting to model the contest between individuals to get a reward for their effort, they find that a perfectly meritocratic society can be “too much of a good thing”. In a framework with homogeneous agents, too much meritocracy makes competition excessively harsh, so that contestants start dropping out of the race and overall output is lower. In a setting with heterogeneous agents, meritocracy discourages the least skilled workers and makes the most talented complacent, two elements that again may decrease output. The idea that too much meritocracy may deprive the least qualified individuals of hope is also an essential feature of our model. However, we use it in a rather different setting, that aims to replicate a key aspect of a meritocratic society: the presence of multiple positions within a hierarchical production process and the opportunity to move between different levels. In fact, the ability to shift one’s position on the social ladder is a crucial element in how individuals perceive the meritocratic nature of a society. In such a context we show that if meritocracy is unattainable in one step then it could not be optimal to pursue perfect meritocracy in the next one.

Fang and Noe (2022) take a different stand, by disputing the common claim that competition and meritocracy go hand in hand. They show that selection contests may become more efficient by policies that reduce competition, such as lowering selection thresholds. The authors introduce a model of contest design where contestants’ performance is influenced by their ability and risk-taking behaviors. In this setting less competitive strategies can lead to more meritocratic outcomes by reducing the role of strategic risk-taking in skewing performance rankings. Even our paper shows laxer standards in the recruitment process may be more efficient. However, in our model less competition may entail superior results because of the "hope" effect and not for its negative impact on the risk taking behaviour of the low skilled.

Carvalho (2022) claims that both the growing dissatisfaction with meritocracy and the widening gap in social outcomes between the college-educated and uneducated stem from a side effect of the meritocratic system. This system sorts individuals based on traits conducive to "merit," separates them into groups, and generates "sorting-separation-externalities" within those groups. Over time, this externality mechanism leads to polarization in social behaviors (e.g., healthier habits in one group, unhealthy ones in another) and reduces overall economic mobility as traits like present bias are transmitted across generations. The system risks mirroring an old caste-like structure and exacerbating stigma and political divides, potentially fueling populism.

In a society based on merit the influence of external factors on individuals' performance should be properly assessed. However, through a series of experiments, Andre (2024) finds that people base reward decisions solely on observed effort, even when they are aware of exogenous circumstances at play. The study highlights differing fairness views among individuals and suggests that merit judgments are often "shallow," holding disadvantaged individuals accountable for choices shaped by factors beyond their control. This "shallow meritocracy" can contribute to perpetuating inequality, raising important questions about fairness in meritocratic systems and policies.

On this last point it is worthwhile to mention the contribution of Mijs (2019), that examines the paradox of rising income inequality in Western societies paired with a lack of public concern. The paper finds that citizens in unequal societies increasingly attribute success to meritocratic factors, reducing their concern about inequality. These beliefs are reinforced by social segregation, limiting interactions across socioeconomic lines and obscuring structural inequality[2]..

The paper is organized as follows. Section 2 presents the basic assumptions of the model. Sections 3 and 4 respectively discuss the perfect meritocratic and the "noisy" scenario. Section 5 illustrates the output and welfare results. Section 6 concludes.

2 The Model

2.1 Preferences and Technology

Let us consider a very basic economy, in which just one homogeneous good is produced via a production that function depends on individuals' skills, effort, and a parameter capturing total factor productivity.

The economy is populated by a measure L of individuals. Each individual is born with a certain skill $s_i \in [s_1, s_2, ..., s_I]$ according to a probability mass function $f(s)$. The subscript I is a natural number denoting the highest level of skill. Similarly, there is discrete distribution of job positions in the economy that can be ranked in terms of their total factor productivity $a_j \in [a_1, a_2, ..., a_J]$, with natural number J standing for the highest possible level. For the sake of simplicity we assume that $I = J$.

Any individual enters the labor market in a position that (imperfectly) mirrors her skill s_i, with $i \in [1, 2, ..., I]$. More precisely, we assume that workers with generic skill s_i may find themselves in a job with productivity a_j, with j taking values i, $i + 1$, and $i - 1$[3]. So there are some workers that are employed in a job that perfectly suits their skill (i.e they have skill s_i and are in jobs with productivity a_j, with $i = j$) and other individuals that, for some errors in the recruiting mechanism not modeled in the paper, end up working in a position that does not exactly reflect their abilities[4]. In other terms, there is asymmetric information between workers (that are perfectly aware of her level of

[2]On the role of merit in shaping people' stance towards inequality see also Konow (2000), Fong (2001), Cappelen et al. (2007), and Cappelen et al. (2022).

[3]Of course, workers with skill I (resp. 1) may only find a job with productivity I and $I - 1$ (resp. 1 and 2).

[4]In the paper, we do not explicitly model the behaviour of the employers. We could assume that a fraction of total output in the economy should be used to finance the recruitment process. Without loss of generality we abstract form these considerations. Our recruiter has the same magic aura of the Walrasian auctioneer even though she does not share the same infallibility.

skill s_i) and recruiters that cannot perfectly distinguish workers with slightly different s_i. This mismatch is not large: anyone might be allocated not more than one step next (or before) the right one in the productivity ladder. Specifically, L_j stands for the number of workers employed in the job position with productivity a_j. Then $\gamma_{i=j-1} \cdot L_j$ and $\gamma_{i=j+1} \cdot L_j$ are, respectively, the fraction of them slightly under-qualified (i.e. with skill $s_{i=j-1}$) and slightly over-qualified (i.e. with skill s_{j+1}) for that job. Parameters $\gamma_{i=j-1}$ and $\gamma_{i=j+1}$ capture the extent of "noise" generated by the imperfect recruiting system. We assume that $\gamma_{i=j-1} \approx \gamma_{i=j+1}$. The share of under-qualified workers is not identical to the share of over-qualified ones and, to remain as general as possible, we do not want to impose any specific order of magnitude between them. Still, we assume that the difference is not substantial, so that the recruiting system is not heavily biased towards either kind of misallocation error. More importantly, we impose that the "noise", however important it can be, is not prevalent in the economy: the number of workers in the right job position is greater than the number of those in the wrong place, that is $\gamma_{i=j-1} + \gamma_{i=j+1} < 0.5$.

The model is developed in steady state. At any point in time, a worker with skill s_i (with $i \in [1, 2, ..., I]$) and employed in a job position with productivity a_j (with $j \in [1, 2, ..., J]$) produces an amount $a_j \cdot s_i$ of the unique consumption good in the economy (that is also the *numeraire*). Therefore, in any job position j one can find $\gamma_{i=j+1} \cdot L_j$ workers producing $a_j \cdot s_{i=j+1}$, a larger group of employees, $\gamma_{i=j} L_j$, whose individual production function is $a_j \cdot s_{i=j}$ and the remaining ones, $\gamma_{i=j-1} \cdot L_j$, that produce $a_j \cdot s_{i=j-1}$. Of course, the under-qualified produce less that the rightly qualified ones, that in turn produce less than the over qualified. For technological reason all skills are not perfectly substitute, so workers with skill $s_{i=j+2}$ and $s_{i=j-2}$ in a position j would produce no output.

We assume individuals have a linear utility function and consume everything they produce, thereby abstracting from consumption/saving decisions. In addition, any worker that is not located at the top of the ladder is aware that can be promoted and pass to a job position with a higher total factor productivity a_{j+1}, thereby consuming a larger amount of output. A necessary (but not sufficient) condition to get a promotion is to exert some extra effort $e_h > 1$ that, however, also implies a certain level of disutility d.[5] The following inequalities present the trade-off in terms of workers' utility:

$$a_{j+1}\, s_i\, e_h - d > a_j\, s_i > a_j\, s_i\, e_h - d \tag{1}$$

for any $a_j \in [a_1, a_2, ..., a_{J-1}]$ and $i \in [j-1,\ j,\ j+1]$. Putting extra effort e_h is valuable if you get a promotion: the gain in terms of higher income that a job with productivity a_{j+1} ensures outweighs the costs in terms of disutility d. However, if the promotion does not arrive, you end up with a lower utility compared to the case in which you do not spend resources for your career advancement.

Climbing the Ladder

How does the society select the workers that succeed at climbing the productivity ladder? Let us denote with P_{j+1} the measure of promotions from j to $j+1$. The exact value of P_{j+1} is obtained via a steady state condition in labor market flows that we will derive in the next subsection. As concerns the number and the "quality" of people eligible for such promotions, we consider two alternative scenarios. In the first one, there is perfect meritocracy in career advancements: only the most productive workers have a chance to get promoted. In the second setting, we add a (relatively) small noise, so that even other less productive workers may have a non zero probability to climb the ladder.

Please note that under a hypothetical fully meritocratic society, where there is perfect and symmetric information about skills at the recruitment process, in our simple model career advancement would not be an issue, since each individual would occupy his/her "right" position in the social ladder with probability one.

We examine the first scenario in section 2 and the second scenario in section 3. Before that, let us present the conditions on labor market flows.

[5] We can also interpret e_h as the sum of basic and extra-time hours worked, with the former normalized to 1.

2.2 Labor Market Flows

At any point in time, an exogenous fraction δ of workers in positions with productivity a_j retire. New workers replace their positions. We assume that a fraction α of such replacements occurs via a job promotion whereas the remaining $1-\alpha$ share happens because new workers occupy that job. For the assumptions presented above, these new workers are in large part endowed with skill $i=j$, but fractions $\gamma_{i=j-1}$ and $\gamma_{i=j+1}$ of them are under-qualified or over-qualified for the position with productivity a_j.

We develop our model in steady state, so that L_j is constant over time for any position $j \in [1, 2, ..., J]$. For the positions at the top, J, and for those at the levels immediately before, $J-1$ and $J-2$ this implies:

$$\delta L_J = \alpha\delta L_J + (1-\alpha)\delta L_J$$

$$\delta L_{J-1} + \alpha\delta L_J = \alpha\delta(L_{J-1} + \alpha L_J) + (1-\alpha)\delta(L_{J-1} + \alpha L_J)$$

$$\delta L_{J-2} + \alpha\delta(L_{J-1} + \alpha L_J) = \alpha\delta\,[\,L_{J-2} + \alpha(L_{J-1} + \alpha L_J)\,] + (1-\alpha)\,[\,L_{J-2} + \alpha(L_{J-1} + \alpha L_J)\,]$$

At level $J-1$ (resp. $J-2$) exits occur both at the exogenous rate δ for retirement and because $\alpha\delta L_J$ (resp. $\alpha\delta(L_{J-1} + \alpha L_J)$) workers get a promotion. Solving recursively for any generic position, we can write the measure of promotions from position j to position $j+1$, P_{j+1}, as follows:

$$P_{j+1} = \alpha\,\delta \sum_{x=1}^{J-j} \alpha^{x-1} \cdot L_{j+x} \tag{2}$$

The term P_{j+1} is therefore a finite number that positively depends on the retirement rate δ and the fraction α of replacements that occur via promotions .

In principle, the value of parameter α depends on the degree of refinement in the selection process for individuals advancing from the previous step of the ladder, relative to that for new workers entering the labor market. One could imagine that employers are more likely to resort to promotions (so choosing a large value for α) if they are confident to find workers better suited for the job (i.e. with a higher level of skill s_i) in the career advancement process. We assume its value is not the identical in the two different scenarios considered in the paper.

As we it will be more clear proceeding in the paper, α cannot be equal to 0, since in both scenarios employers know that workers applying for a job position via the career advancement process are better skilled than the ones newly entering the labor market[6]. In the next sections we elaborate more in depth on this point.

On the other hand, α must be lower than 1, since in steady state there must be a flow of new workers entering the labour market to offset the flow towards retirement.

3 Perfect Meritocracy in Career Advancements

The fully meritocratic system works as follows. If all workers in position j exert extra effort e_h, then only the most productive ones may aspire to a job promotion. Since output produced by workers exerting extra effort is equal to $a_j\, s_i\, e_h$, for $i \in [\,j-1,\ j,\ j+1\,]$, this means that only the over-qualified employees (whose skill level is equal to $s_{i=j+1}$) have a chance to climb the ladder. So P_{j+1} workers are randomly chosen only among the pool of the most skilled individuals at level j, $\gamma_{i=j+1} \cdot L_j$. The rationale for this career advancement process can be understood as follows. At the initial stage of the recruitment process asymmetric information is large enough that employers are unable to distinguish between workers with skill $s_{i=j}$, $s_{i=j+1}$, and $s_{i=j-1}$. Conversely, such an ignorance disappears once workers' performance is measured by their output, allowing the most skilled employees to stand out by producing more.

Let us denote with $\mathbb{P}^{j+1}$ the probability of being promoted to position $j+1$. Then, for an individual with skill $s_{i=j+1}$, employed in a position j and that exerts extra effort e_h this probability is equal to:

$$\mathbb{P}^{j+1}_{i=j+1} = \frac{P_{j+1}}{\gamma_{i=j+1} \cdot L_j} \tag{3}$$

For individuals with $s_{i=j}$ and $s_{i=j-1}$, the only chance to pass from a job with productivity a_j to one with productivity a_{j+1} is that they do exert extra effort e_h while employees in the same position but with higher skills do not. In other terms, any worker in position j

[6]Of course, $\alpha = 0$ at $j = 1$, as there are no promotions to the lowest step of the ladder.

with skill $s_{i=j}$ that puts extra effort e_h faces the following probability:

$$\mathbb{P}_{i=j}^{j+1} = \frac{P_{j+1}}{\gamma_{i=j} L_j} \iff \text{workers with } s_{i=j+1} \text{ do not choose } e_h, \tag{4}$$

$$\mathbb{P}_{i=j}^{j+1} = 0 \iff \text{otherwise.} \tag{5}$$

Similarly, for a worker with skill $s_{i=j-1}$ that puts effort e_h we have:

$$\mathbb{P}_{i=j-1}^{j+1} = \frac{P_{j+1}}{\gamma_{i=j-1} L_j} \iff \text{workers with } s_{i=j} \text{ and } s_{i=j+1} \text{ do not choose } e_h, \tag{6}$$

$$\mathbb{P}_{i=j-1}^{j+1} = 0 \iff \text{otherwise.} \tag{7}$$

3.1 Nash Equilibrium with Perfect Meritocracy in Career Advancements

Workers have to decide whether to exert extra effort e_h or not, knowing the inequalities presented in equation (1) and their respective probabilities of being promoted: equations (3), (4), and (6).

The following Proposition summarizes the results of the non-cooperative game between workers with skills $s_{i=j+1}$, $s_{i=j}$, and $s_{i=j-1}$ employed in position j.

Proposition 1 *The game with perfect meritocracy in career advancements admits two alternative Nash equilibria.*

1. *If and only if*

$$d \; > \; s_{i=j+1} \cdot \left[\mathbb{P}^{j+1}_{i=j+1} \cdot e_h(a_{j+1} - a_j) \; + \; a_j(e_h - 1) \right] \tag{8}$$

at the equilibrium no worker exerts any extra effort e_h.

2. *If and only if*

$$d \; < \; s_{i=j+1} \cdot \left[\mathbb{P}^{j+1}_{i=j+1} \cdot e_h(a_{j+1} - a_j) \; + \; a_j(e_h - 1) \right] \tag{9}$$

at the equilibrium workers with skill $s_{i=j+1}$ *exert extra effort* e_h*, while the remaining workers do not put any extra effort.*

Proof. The proof is a direct application of the necessary condition for the over-qualified workers, that is those with skill $s_{i=j+1}$, to apply extra effort e_h:

$$\mathbb{P}^{j+1}_{i=j+1} \cdot (a_{j+1}\, s_{i=j+1}\, e_h \; - \; d) \; + \; \left(1 - \mathbb{P}^{j+1}_{i=j+1}\right) \cdot (a_j\, s_{i=j+1}\, e_h \; - \; d) \; > \; a_j\, s_{i=j+1} \tag{10}$$

At the LHS we have the expected payoff of putting extra effort e_h. At the RHS we have the output obtained in case the workers decide not to invest in a promotion. Rearranging, we get inequality (9). If workers with skill $s_{i=j+1}$ decide to put extra effort e_h, the best response strategy for workers with skill $s_{i=j}$ and $s_{i=j-1}$ is to not choose e_h, as their chances to be promoted are equal to zero and they would only suffer disutility d. As concerns the necessary part of Proposition 1, it is evident that an equilibrium in which workers with skill $s_{i=j+1}$ decide to exert extra effort e_h is possible only if inequality (9) is respected. If, on the contrary, inequality (8) holds, workers with skill $s_{i=j+1}$ do not find it optimal to choose e_h.

We also prove that workers with skill $s_{i=j}$ and those with skill $s_{i=j-1}$ would also not choose e_h if inequality (8) is respected. Notice that workers with skill $s_{i=j}$ would exert e_h

if:

$$\begin{aligned} &\mathbb{P}^{j+1}_{i=j} \cdot (a_{j+1}\, s_{i=j}\, e_h \,-\, d) \,+\, \left(1-\mathbb{P}^{j+1}_{i=j}\right) \cdot (a_j\, s_{i=j}\, e_h \,-\, d) \,>\, a_j\, s_{i=j} \iff \\ &d \,<\, s_{i=j} \left[\, \mathbb{P}^{j+1}_{i=j} \cdot e_h(a_{j+1}-a_j) \,+\, a_j(e_h-1) \,\right] \end{aligned} \tag{11}$$

If inequality (8) is respected, the condition in (11) does not hold, because the term at the RHS of (11) is lower than the term at the RHS of (8), for two reasons. First, we have that $\mathbb{P}^{j+1}_{i=j}$ (defined in equation (4)) is lower than $\mathbb{P}^{j+1}_{i=j+1}$ (defined in equation (3)), as we have assumed that workers in the right position in their job are more numerous (we have imposed $\gamma_{i=j-1} + \gamma_{i=j+1} < 0.5$). Second, notice that the term multiplying the square brackets in the second inequality of Proposition 1, $s_{i=j+1}$, is larger than the term $s_{i=j}$ at the RHS of (11). Because of their higher skill, the capital gain of getting a promotion for the over-skilled workers is larger than the one of the rightly allocated employees.

The same reasoning holds if we consider workers with skill $s_{i=j-1}$. They would exert e_h if:

$$d \,<\, s_{i=j-1} \left[\, \mathbb{P}^{j+1}_{i=j-1} \cdot e_h(a_{j+1}-a_j) \,+\, a_j(e_h-1) \,\right] \tag{12}$$

Again, if inequality (8) is respected, the condition in (12) does not hold, because the term at the RHS of (12) is lower than the term at the RHS of (8). Since we have assumed that $\gamma_{i=j-1} \approx \gamma_{i=j+1}$, from equations (3)and (6) we have that $\mathbb{P}^{j+1}_{i=j} \approx \mathbb{P}^{j+1}_{i=j-1}$. In addition, $s_{i=j-1} < s_{i=j+1}$, so it is not possible that workers with skill $s_{i=j-1}$ find it optimal to exert extra effort e_h if workers with skill $s_{i=j+1}$ do not. The only Nash equilibrium if the inequality in point 1 of Proposition 1 holds implies that all workers do not exert any extra effort e_h.

□

The inequalities in Proposition 1 are easy to interpret: employees with skill $s_{i=j+1}$ are more likely to choose e_h if the disutility d is small, if the probability to move up $\mathbb{P}^{j+1}_{i=j+1}$ is high, or the gap between the productivity in the new potential position and the current one $a_{j+1} - a_j$ is large.

4 Noise in Career Advancements

We introduce a noise in the career advancement process. While in the previous section we had assumed that only the most productive workers could have a chance to get a promotion (provided that they exerted extra effort e_h), in this new setting we add a probability ϵ that employees that have produced less may also have chance to climb the ladder, by putting extra effort e_h.

One could advance several explanations for this noise. Since our paper aims to compare a meritocratic process with an *a*-meritocratic one, we rule out hypotheses that see noise as a tool that systematically favours one specific category of people[7]. Instead, one may imagine that some residual asymmetric information problem between employers and workers persists after hiring, so that the former are not able to perfectly measure the individual amount of output produced by the latter. Noise can also be interpreted as a sort of second chance instrument. Assigning less productive workers a non-zero probability of a promotion means giving another opportunity to workers whose lower performance compared to peers in the same role may stem from some non-insurable negative shocks (poor inborn talents, bad parenting, etc...).

This noise is not large. Specifically, we impose that ϵ is the probability that the society is not able to distinguish between the output produced by workers with skill s_i (equal to $a_j\, s_{i=j}\, e_h$) and that produced by workers with skill s_{i+1} (equal to $a_j\, s_{i=j+1}\, e_h$) when both groups exert extra effort e_h. If this is the case, for any job position with productivity a_j, the measure of workers that may aspire to get an upgrade is equal to $(\gamma_{i=j+1} + \gamma_{i=j}) \cdot L_j$, conditional on all putting extra effort e_h. So, if an error with probability ϵ occurs, the probability of a career advancement becomes:

$$\mathbb{P}^{j+1}_{i=j\,\cup\, i=j+1} \;\equiv\; \frac{P_{\epsilon,\, j+1}}{(\gamma_{i=j+1} + \gamma_{i=j}) \cdot L_j} \tag{13}$$

in which

$$P_{\epsilon,\, j+1} \;=\; \alpha_\epsilon\, \delta \sum_{x=1}^{J-j} \alpha_\epsilon^{x-1} \cdot L_{j+x} \tag{14}$$

[7]In other words, we exclude cronyism and other un-meritocratic systems.

with $j \in [1, 2, ..., J-1]$. Notice that in this scenario the effective number of promotions to step $j+1$ is different in this scenario because we are considering a different value for α, denoted with α_ϵ. As we have anticipated at the end of section 1, it is reasonable to assume that the share of workers that occupy a new position via a career advancement process may be lower if employers know there is some noise in that system. This implies $\alpha_\epsilon < \alpha$. So, we have that $\mathbb{P}^{j+1}_{i=j \cup i=j+1} < \mathbb{P}^{j+1}_{i=j+1}$ because the denominator in (3) is lower than denominator in (13) and because $\alpha_\epsilon < \alpha$.

The mistake in the advancement process is present only when slightly similar employees all provide extra effort. So, if workers with skill $s_{i=j}$ do not choose e_h, the probability for a worker with $s_{i=j+1}$ to get promoted is denoted with

$$\mathbb{P}^{j+1}_{\epsilon,\, i=j+1} = \frac{P_{\epsilon,\, j+1}}{\gamma_{i=j+1} \cdot L_j}$$

The formula is the same used in the perfect meritocracy scenario in equation (3). The only change is the different parameter α_ϵ in the steady-state equation for the promotions (14). Similarly, if employees with skill $s_{i=j+1}$ do not exert extra effort, the probability for a worker $s_{i=j}$ to get an advancement is $\mathbb{P}^{j+1}_{\epsilon,\, i=j} = \frac{P_{\epsilon,\, j+1}}{\gamma_{i=j} \cdot L_j}$

We also assume that the noise does not affect workers with skill $s_{i=j-1}$. The society might not be able to differentiate between the output produced by workers with skill $s_{i=j}$ and that produced by $s_{i=j+1}$ workers, but the chance that under-qualified $s_{i=j-1}$ employees get a promotion remain the same presented in equation (6) (apart from substituting α with α_ϵ).

Consider now the best response strategy of workers employed in a position a_j with skill $s_{i=j}$ if workers with skill $s_{i=j+1}$ provide extra effort e_h. Their optimal strategy will be to choose e_h as well if and only if

$$\epsilon \cdot \mathbb{P}^{j+1}_{i=j \cup i=j+1} \cdot (a_{j+1}\, s_{i=j}\, e_h - d) + \left(1 - \epsilon \cdot \mathbb{P}^{j+1}_{i=j \cup i=j+1}\right) \cdot (a_j\, s_{i=j}\, e_h - d) > a_j\, s_{i=j} \quad (15)$$

By the same token, if workers with skill $s_{i=j}$ provide extra effort, e_h is also the best response strategy for workers employed in a position a_j with skill $s_{i=j+1}$ if and only if

$$\begin{aligned}&\left[\epsilon\cdot\mathbb{P}^{j+1}_{i=j\,\cup\,i=j+1}+(1-\epsilon)\cdot\mathbb{P}^{j+1}_{\epsilon,\,i=j+1}\right]\cdot\left(a_{j+1}\,s_{i=j+1}\,e_h\,-\,d\right)\;+\\&+\;\left[1\,-\left(\epsilon\cdot\mathbb{P}^{j+1}_{i=j\,\cup\,i=j+1}+(1-\epsilon)\cdot\mathbb{P}^{j+1}_{\epsilon,\,i=j+1}\right)\right]\cdot\left(a_j\,s_{i=j+1}\,e_h\,-\,d\right)\;>\;a_j\,s_{i=j+1}\end{aligned}\tag{16}$$

For the overqualified worker with skill $s_{i=j+1}$ employed in a position j, receiving a promotion may occur via two alternative channels. The first one depends on the term $\epsilon\cdot\mathbb{P}^{j+1}_{i=j\,\cup\,i=j+1}$, which is the probability that she is selected among other $(\gamma_{i=j+1}+\gamma_{i=j})\cdot L_j$ workers because of the noise in the selection process. The second reason depends on the term $(1-\epsilon)\cdot\mathbb{P}^{j+1}_{\epsilon,\,i=j+1}$, which is the probability of a promotion if the noise does not occur and she is selected among $\gamma_{i=j+1}\cdot L_j$ workers.

4.1 Nash Equilibrium with Noise

The following Proposition summarizes the result.

Proposition 2 *The game with noise in career advancements admits three alternative Nash equilibria.*

1. *If and only if*

$$d \ > \ s_{i=j+1} \cdot \left[\mathbb{P}^{j+1}_{\epsilon,\ i=j+1} \cdot e_h(a_{j+1} - a_j) \ + \ a_j(e_h - 1) \right]$$

at the equilibrium no worker exerts any extra effort e_h

2. *If and only if*

$$s_{i=j} \left[\epsilon \, \mathbb{P}^{j+1}_{i=j \cup i=j+1} \cdot e_h(a_{j+1} - a_j) + a_j(e_h - 1) \right] < d < s_{i=j+1} \left[\mathbb{P}^{j+1}_{\epsilon,\ i=j+1} e_h(a_{j+1} - a_j) + \ a_j(e_h - 1) \right]$$

at the equilibrium only workers with skill $s_{i=j+1}$ *exert extra effort* e_h.

3. *If and only if*

$$d < s_{i=j} \cdot \left[\epsilon \, \mathbb{P}^{j+1}_{i=j \cup i=j+1} \cdot e_h(a_{j+1} - a_j) + a_j(e_h - 1) \right]$$

both workers with skill $s_{i=j+1}$ *and workers with skill* $s_{i=j}$ *exert extra effort* e_h.

Proof. See Appendix 1. □

Let us compare the results of Proposition 2 with those of Proposition 1. If perfect meritocracy prevails in career advancements, two alternative equilibria are possible. A first type of equilibrium occurs if the condition (8) is fulfilled: the disutility d is greater than the expected gain obtained by a promotion for the overqualified, so nobody exerts extra effort. Even in this setting with noise we have a similar condition for an equilibrium in which nobody chooses e_h: the only difference between the inequality in point (1) of Proposition 2 and the condition (8) is that $\mathbb{P}^{j+1}_{\epsilon,\ i=j+1} \neq \mathbb{P}^{j+1}_{i=j+1}$ for $\alpha \neq \alpha_\epsilon$.

We have also seen that if parameter d is not too large, the perfect meritocracy scenario allows for the occurrence of an alternative equilibrium, in which only workers with skill $s_{i=j+1}$ exert e_h. This outcome is also possible in this scenario (point 2 of Proposition 2).

The novelty of this setting is the existence of a third equilibrium, in which both workers with skill $s_{i=j+1}$ and the ones with skill $s_{i=j}$ put extra effort. This is possible if the disutility of extra effort d is sufficiently small, so that workers with skill $s_{i=j}$ find it profitable to exert extra effort e_h even when those with skill $s_{i=j+1}$ do the same. Condition in point 3 of Proposition 2 is the same presented in (15) expressed in terms of d. Of course the equilibrium in which $s_{i=j}$ and $s_{i=j+1}$ workers exert extra effort is more likely to occur if $\epsilon\, \mathbb{P}^{j+1}_{i=j\,\cup\, i=j+1}$ is large. If the probability to get a promotion when the poll of eligible applicants is composed by over-qualified and rightly qualified workers is high, the the latter are more willing to exert extra effort e_h.

A final remark concerns the possibility that adding noise may push workers with skill $s_{i=j}$ to exert extra-effort while workers skill $s_{i=j+1}$ do not. For this equilibrium to occur, one needs two conditions. First, inequality (16) must not hold (so that employees with skill $s_{i=j+1}$ do not choose e_h if employees with skill $s_{i=j}$ choose e_h). Rearranging, this implies:

$$d \;>\; s_{i=j+1} \left\{ \left[\epsilon \cdot \mathbb{P}^{j+1}_{i=j\,\cup\, i=j+1} + (1-\epsilon) \cdot \mathbb{P}^{j+1}_{\epsilon,\, i=j+1} \right] \cdot e_h (a_{j+1} - a_j) \;+\; a_j (e_h - 1) \right\} \tag{17}$$

The second condition is that employees with skill $s_{i=j}$ do choose e_h if employees with skill $s_{i=j+1}$ do not choose e_h:

$$\begin{aligned} &\mathbb{P}^{j+1}_{\epsilon,\, i=j} \cdot (a_{j+1}\, s_{i=j}\, e_h \;-\; d) \;+\; \left(1 - \mathbb{P}^{j+1}_{\epsilon,\, i=j}\right) \cdot (a_j\, s_{i=j}\, e_h \;-\; d) \;>\; a_j\, s_{i=j} \iff \\ &d \;<\; s_{i=j} \left[\mathbb{P}^{j+1}_{\epsilon,\, i=j} \cdot e_h (a_{j+1} - a_j) \;+\; a_j (e_h - 1) \right] \end{aligned} \tag{18}$$

In principle, we cannot rule out the hypothesis that the term at the RHS in the second line of (18) is larger than the term at the RHS of (17), so that this equilibrium occurs. This is because $\mathbb{P}^{j+1}_{i=j\,\cup\, i=j+1} \;<\; \mathbb{P}^{j+1}_{\epsilon,\, i=j} \;<\; \mathbb{P}^{j+1}_{\epsilon,\, i=j+1}$. However, under the assumption that ϵ is a small number close to 0, the probability inside the square brackets in (17) is larger than $\mathbb{P}^{j+1}_{\epsilon,\, i=j}$. This is equivalent to saying that, even in presence of noise, the probability of obtaining a promotion for the overqualified is higher than the same probability for the larger group of the rightly qualified ones. Then, the the term at the RHS of (11) is lower than the term at the RHS of (17), and an equilibrium in which only workers with skill $s_{i=j}$ do choose e_h is never possible.

5 Total Output and Welfare

The aim of our work is to check the conditions under which adding a noise may have positive effect on output and welfare. So we compare the perfect meritocracy equilibrium in which only the slightly overqualified workers choose e_h with that obtained in the previous section, when for any job position j both workers with skill $s_{i=j}$ and those with skills $s_{i=j+1}$ exert extra effort. The total amount of output produced by workers with skill i in the perfect meritocracy setting can be written as follows:

$$Y_i = \gamma_i s_i \left\{ L_{j=i} a_{j=i} + L_{j=i+1} a_{j=i+1} + L_{j=i-1} e_h \left[\mathbb{P}_i^{j=i} a_{j=i} \; + \; \left(1 - \mathbb{P}_i^{j=i}\right) a_{j=i-1} \right] \right\} \tag{19}$$

for $i \in [1, ...I]$[8]. Workers with skill i are distributed in jobs with productivity $j = i$, $j = i + 1$, and $j = i - 1$: $\gamma_i L_{j=i}$, $\gamma_i L_{j=i+1}$, and $\gamma_i L_{j=i-1}$ respectively. These last ones are the overqualified workers that, if the condition in Proposition 1 is respected, will exert extra effort e_h. With a little abuse of notation, we denote with $\mathbb{P}_i^{j=i}$ the probability that workers of skill s_i (that are currently employed in a under-qualified position) may get a job with productivity $a_{j=i}$. So, applying a law of large numbers, we can say that a share $\mathbb{P}_i^{j=i}$ of them will get a promotion.

Similarly, the amount of output produced by workers with skill s_i at the equilibrium with noise when for any job position j both workers with skill $s_{i=j}$ and those with skills $s_{i=j+1}$ exert extra effort is equal to

$$\begin{aligned} Y_{\epsilon,i} \; = \; \gamma_i s_i \{ \; & L_{j=i+1} a_{j=i+1} \; + \; L_{j=i} e_h \cdot \left[\; \epsilon \cdot \mathbb{P}_{i \cup i+1}^{j=i+1} \cdot a_{j=i+1} \; + \; \left(1 - \epsilon \cdot \mathbb{P}_{i \cup i+1}^{j=i+1}\right) \cdot a_{j=i} \; \right] + \\ & + L_{j=i-1} e_h \left[\left(\epsilon \cdot \mathbb{P}_{i \cup i-1}^{j=i} + (1-\epsilon) \cdot \mathbb{P}_{\epsilon,\, i}^{j=i} \right) \cdot a_{j=i} \; + \; \left(1 \; - \; \epsilon \cdot \mathbb{P}_{i \cup i-1}^{j=i} - (1-\epsilon) \cdot \mathbb{P}_{\epsilon,\, i}^{j=i}\right) \cdot a_{j=i-1} \right] \} \end{aligned}$$

for $i \in [1, ...I]$[9].

[8] Of course, $L_{j=i-1}$ (resp. $L_{j=i+1}$) is equal to 0 if $i = 1$ (resp. $i = I$).

[9] Even in this scenario we have $L_{j=i-1}$ (resp. $L_{j=i+1}$) is equal to 0 if $i = 1$ (resp. $i = I$). Moreover,

Again with a little abuse of notation the term $\epsilon\mathbb{P}_{i\cup i+1}^{j=i+1}$ stands for the probability that the (rightly) qualified workers with skill s_i may get a promotion and reach the position $j = i + 1$. In the previous section we have seen that the pool of applicants is equal to all workers currently employed in a job with productivity $a_{j=i}$ with skill s_i and s_{i+1}. Similarly, the term $\epsilon \cdot \mathbb{P}_{i\cup i-1}^{j=i}$ is the probability that employees with skill s_i working in a position with productivity $a_{j=i-1}$ may get a promotion.

The differences between Y_i and $Y_{\epsilon,i}$ are in the second and the third term inside the graphs at the RHS of the above equation. In this scenario, workers with skill i may compete for a promotion even when they are employed in the right position $j = i$. If the condition in point (3) of Proposition 2 is fulfilled, a measure of workers $L_{j=i}\gamma_i$ will prove extra effort e_h. A fraction $\epsilon \cdot \mathbb{P}_{i\cup i+1}^{j=i+1}$ of them will pass to a job position $j = i + 1$.

If we just consider this change, the introduction of a sufficiently high noise is clearly output enhancing. $L_{j=i}\gamma_i$ workers that in the perfect meritocracy in career advancement equilibrium would not choose extra effort, they exert it once noise is added. In addition, a share of them obtain a position with a higher total factor productivity $a_{j=i+1}$.

Yet, noise also introduces a negative effect on output, captured by the second line of the above equation. While in the equilibrium with perfect meritocracy over-qualified workers with skill s_i employed in a job position $j = i-1$ would have a chance $\mathbb{P}_i^{j=i}$ to reach a position with higher productivity $a_{j=i}$, in the equilibrium with noise such a probability is lower and equal to $\epsilon \cdot \mathbb{P}_{i\cup i-1}^{j=i} + (1-\epsilon) \cdot \mathbb{P}_{\epsilon,i}^{j=i}$. This tends to reduce total production.

A comparison between Y and $Y_{\epsilon,i}$ delivers the following result:

$\mathbb{P}_{i\cup i+1}^{j=i+1}$ is also equal to 0 when $i = I$, as there is no further step to climb for workers with skill I employed in a job with $a_{j=I}$.

Proposition 3 *Consider the perfect meritocracy equilibrium in which only workers with skill $s_{i=j+1}$ exert extra effort and the noise equilibrium in which both $s_{i=j+1}$ and $s_{i=j}$ workers choose e_h:*

1. $Y_{1,\epsilon} > Y_1$ *and* $Y_{I,\epsilon} < Y_I$.

2. *If*
$$\frac{e_h - 1}{e_h} > \frac{a_{j=i} - a_{j=i-1}}{a_{j=i}} \cdot \frac{P_{j=i}}{L_{j=i}\gamma_i} \cdot \left[1 - (1-\epsilon) \cdot \left(\frac{\alpha_\epsilon}{\alpha}\right)^{I-i}\right]$$
then $Y_{\epsilon,i} > Y_i$ *for* $i \in [2, ...I-1]$.

Proof. See Appendix 2. □

Let us focus first on point 1. Recall that noise may have have a negative impact on output because it reduces the probability of the overqualified workers to reach their right position. But there are no over-qualified workers with skill s_1, so the amount of output produced by workers with the lowest level of skill, Y_1, is always greater in a noise equilibrium. Adding ϵ just creates an incentive for workers with skill s_1 to exert extra effort e_h in the hope of an upgrade.

Conversely, $Y_{I,\epsilon}$ is always smaller than Y_I because there is no possibility of an upgrade for workers with skill s_I in position $j = I$. So noise has just a negative impact on output Y_I since it lowers the probability of getting a job with productivity a_I for workers with skill s_I employed in a position $j = I - 1$.

Point (2) of Proposition 3 tells us that noise has a positive impact on the output produced by all workers with skill s_i, $i \in [2, ...I-1]$, provided that a sufficient condition is fulfilled. This condition is more likely to be respected if e_h is large (the term at the LHS of the inequality in point (2) is increasing in e_h). A high value for extra effort implies that the increase in output generated by the hope condition (i.e. the possibility for workers with skill s_i employed in a job $j = i$ to get one step further and reach a position $j = i+1$) is large, for any given level of technology. This tends to increase $Y_{\epsilon,i}$. On the other hand, noise is associated with misallocation of resources, because it assigns a non zero probability to employees with skill i to end up working in a job with productivity $a_{j=i+1}$, while lowering the odds for the overqualified employees with skill i to get a position $j = i$. A large gap between productivity $a_{j=i}$ and productivity $a_{j=i-1}$ worsens this misallocation problem and reduces the likelihood of satisfying the inequality presented in point 2 of Proposition 3.

From the same inequality we also obtain that the odds that noise is output enhancing negatively depend on promotions under the perfect meritocracy scenario, $P_{j=1}$, and are positively affected by the measure of workers perfectly qualified for the job position j, $L_{j=i}\gamma_i$. With a lot of career advancements in the perfect meritocracy scenario, output Y_i is large, because many workers with skill i employed in position $j = i - 1$ have the opportunity to produce more. Inequality in point 2 of Proposition 3 is less likely to hold. On the contrary, a large value for $L_{j=i}\gamma_i$ means that many employees with skill i in a job position $j = i$, that in the perfect meritocracy setting will never find convenient to provide extra effort, will choose e_h in the noisy scenario. This tends to raise $Y_{\epsilon,i}$.

Finally, recall that we have assumed that the share of replacements occurring via career advancements is different in the two scenarios: $\alpha_\epsilon \neq \alpha$. A low value for α_ϵ compared to α may capture the fact that employers are more willing to hire workers entering in the labour market for the first time if noise enters in the career advancement process. But with a low value for the α_ϵ/α the condition in point 2 of Proposition 3 is less likely to be respected. Indeed, increasing the share of "newcomers" to fill job vacancies means adding more slightly (under- or over-) qualified to the labour market. This increases misallocation. The exponent $I - i$ also implies that such misallocation effect is stronger for low values of i, that is for job positions at the bottom of the skill ladder. This stems from the steady-state conditions on labour market positions presented in section 2.2. As equation 2 makes clear, the number of promotions at level i positively depends on how many workers get promoted at the higher positions $i + 1$, $i + 2$, ..., I, because this means that more job vacancies must be filled at level i. This "cascading" mechanism implies that a low value for α_ϵ compared to α at higher job positions generates misallocation at the lower levels of the ladder.

Now we focus on the welfare implications of our two models by considering an utilitarian function: aggregate welfare is just the sum of the utilities of all the workers in the economy. In the perfect meritocracy model in which only workers with skill $s_{i=j+1}$ employed in a position j put extra effort the welfare function for workers with skill i can be written as follows:

$$W_i = \gamma_i s_i \left\{ L_{j=i} a_{j=i} + L_{j=i+1} a_{j=i+1} + L_{j=i-1} \left[e_h \mathbb{P}_i^{j=i} a_{j=i} \; + \; \left(1 - \mathbb{P}_i^{j=i}\right) a_{j=i-1} \right] \right\} \; - \; \gamma_i L_{j=i-1} \cdot d$$
$$= \; Y_i \; - \; \gamma_i L_{j=i-1} \cdot d$$

The sum of the utilities of all workers with skill s_i is just equal to total output Y_i minus the disutility of extra effort e_h exerted only by workers employed in position $j = i - 1$.

Similarly, the utilitarian welfare function in case we are in equilibrium with noise in which both workers with skill $s_{i=j}$ and those with skills $s_{i=j+1}$ exert extra effort takes the following form:

$$W_{\epsilon,\, i} \; = \; \gamma_i s_i \{ \; L_{j=i+1} a_{j=i+1} \; + \; L_{j=i} e_h \cdot \left[\; \epsilon \cdot \mathbb{P}_{i \cup i+1}^{j=i+1} \cdot a_{j=i+1} \; + \; \left(1 - \epsilon \cdot \mathbb{P}_{i \cup i+1}^{j=i+1}\right) \cdot a_{j=i} \; \right] +$$
$$+ \, L_{j=i-1} e_h \left[\left(\epsilon \cdot \mathbb{P}_{i \cup i-1}^{j=i} + (1 - \epsilon) \cdot \mathbb{P}_{\epsilon,\, i}^{j=i} \right) \cdot a_{j=i} \; + \; \left(1 \; - \; \epsilon \cdot \mathbb{P}_{i \cup i-1}^{j=i} - (1 - \epsilon) \cdot \mathbb{P}_{\epsilon,\, i}^{j=i} \right) \cdot a_{j=i-1} \right] \} +$$
$$- \; \gamma_i L_{j=i-1} \cdot d \, - \, \gamma_i L_{j=i} \cdot d \, =$$
$$= \; Y_{\epsilon,\, i} \; - \; \gamma_i L_{j=i-1} \cdot d \; - \; \gamma_i L_{j=i} \cdot d$$

Again the only difference between $W_{i,\,\epsilon}$ and $Y_{i,\,\epsilon}$ is that in the former they are also present the disutility costs for extra effort (the two terms in the third line of the equation). Of course in this equilibrium the disutility is larger because it involves not just workers employed in position $j = i - 1$ but also those working in position $j = i - 1$. In comparing W_i with $W_{i,\,\epsilon}$, we get the following results.

Proposition 4 *Consider the perfect meritocracy equilibrium in which only workers with skill* $s_{i=j+1}$ *exert extra effort and the noise equilibrium in which both* $s_{i=j+1}$ *and* $s_{i=j}$ *workers choose* e_h*:*

1. $W_{1,\epsilon} > W_1$ *and* $W_{I,\epsilon} < W_I$.

2. *If*

$$\frac{e_h - 1}{e_h} > \frac{a_{j=i} - a_{j=i-1}}{a_{j=i}} \cdot \frac{P_{j=i}}{L_{j=i}\gamma_i} \cdot \left[1 - (1-\epsilon) \cdot \left(\frac{\alpha_\epsilon}{\alpha}\right)^{I-i} \right]$$

AND

$$d < s_{i=j} \cdot \mathbb{P}^{j=i+1}_{i \cup i+1} \cdot e_h(a_{j+1} - a_j)$$

then $W_{\epsilon,i} > W_i$ *for* $i \in [2, ...I-1]$.

Proof. See Appendix 3. □

The first point has an interesting Rawlsian implications. Introducing noise improves the welfare of the least skilled, while reduces the utility of those on the top of the skill ladder. The rationale for these results is the same explained when discussing the effects of noise on output for the most and the least skilled workers. For the latter, an equilibrium in which they decide to exert extra effort to get a chance to reach job position of level $i = 2$ is always better than an equilibrium in which this possibility is not present. Moreover, the disadvantage of reducing the chances for the over-qualified does not concern them, since they are never over-qualified (there is no level 0 in which employees of level 1 are employed). Conversely, workers with the highest skill s_I will be worse off in an equilibrium in they must compete with workers with skill s_{I-1} to reach the top position I. And there is no no possibility of an upgrade for workers with skill s_I in position $j = I$. In a word, the hope effect that noise introduces in the system is a redistributive force, that takes from the most skilled to give to the least ones.

Apart from these two extreme cases, point 2 of Proposition 4 presents the two conditions that are sufficient to ensure that the noise equilibrium in which both workers with skill $s_{i=j+1}$ and those with skill $s_{i=j}$ exert extra effort e_h is also welfare enhancing. The first condition is the same presented in Proposition 3 and does not need any further elaboration. The second condition can be better understood by comparing it with the inequality in point 3 of Proposition 2. That inequality ensures that the Nash equilibrium of the game with noise in career advancements is such that workers with skill $s_{i=j+1}$ and those with skill $s_{i=j}$ exert extra effort e_h. This occurs if the disutility d is not too large. The second condition in point 2 of Proposition 4 is even more restrictive on the value of d. If d is sufficiently low, then it is possible that the greater total disutility of effort that a noise equilibrium implies is smaller than the expected gains of getting a promotion for workers with skill $s_{i=j}$.

6 Conclusions

In this paper we argue that if asymmetric information exists in the recruitment process, a system of perfect meritocracy in career advancement may not be desirable, either from an efficiency standpoint or a welfare perspective.

In our simple one-good economy we assume that the outcome of the production process only depends on skills, individual effort, and a parameter that reflects total factor productivity. In such a setting human capital does not play any role, as the nature of the product and the technology involved do not demand any particular level of individual education or training. Of course, in a more complex and realistic production setting the process of accumulation of human capital should be modeled and the equilibrium level of human capital endogenously determined. This could be a subject for future research. We note however that the impact of meritocracy in a model in which agents have to decide the level of human capital accumulation on the basis of their expected earnings and future life cycle welfare could be ambiguous, and for the same type of arguments we have been discussing in this paper. Indeed, on the one hand a recruitment process in which employers are able to infer the true level of human capital of any applicant would boost human capital accumulation and thus overall income. On the other hand, individuals that know that they are poorly endowed in terms of learning skills could decide not to invest in human capital. In other words, a bit of noise in the recruitment process could enhance human capital accumulation of the less performing students, thus generating an improvement in aggregate income and welfare.

Our model does not allow for skill and effort complementarities in the production process. Future research could also address this issue. In principle, one could expect that the level of production for each level of skill is positively affected by the output of workers in a lower job position. Provided that the conditions in Proposition 3 are met, the effects of noise on aggregate output could be even greater.

As concerns the policy implications, our work does not want to challenge the overall value of meritocracy. But in a world where perfect meritocracy is not totally present in one step of the working life, it may not be optimal to devote resources for achieving perfect meritocracy in the subsequent steps. Broadly speaking, we could say that economic policy could face situations that recall the second best literature, even in the peculiar topic of the optimal level of meritocracy.

References

Andre, P. (2024). Shallow meritocracy. *The Review of Economic Studies*, accepted.

Cappelen, A. W., A. D. Hole, E. Sørensen, and B. Tungodden (2007). The pluralism of fairness ideals: An experimental approach. *American Economic Review 97*(3), 818–827.

Cappelen, A. W., K. O. Moene, S.-E. Skjelbred, and B. Tungodden (2022). The merit primacy effect. *The Economic Journal 133*(651), 951–970.

Carvalho, J.-P. (2022). Markets and communities: the social cost of the meritocracy. *Journal of Institutional Economics 18*(3), 501–519.

Chan, J. (2013). Political Meritocracy and Meritorious Rule: A Confucian Perspective. In D. A. Bell and C. Li (Eds.), *The East Asian Challenge for Democracy: Political Meritocracy in Comparative Perspective*. New York: Cambridge University Press.

Fang, D. and T. Noe (2022). Less competition, more meritocracy? *Journal of Labor Economics 40*(3), 669–701.

Fong, C. (2001). Social preferences, self-interest, and the demand for redistribution. *Journal of Public Economics 82*(2), 225–246.

Frank, R. H. (2016). *Success and Luck: Good Fortune and the Myth of Meritocracy.* Princeton University Press.

Keyt, D. (1991). Aristotle's theory of distributive justice. In D. Keyt and F. Miller (Eds.), *A Companion to Aristotle's Politics*. Oxford: Blackwell.

Konow, J. (2000). Fair shares: Accountability and cognitive dissonance in allocation decisions. *The American Economic Review 90*(4), 1072–1091.

Mijs, J. J. B. (2019). The paradox of inequality: income inequality and belief in meritocracy go hand in hand. *Socio-Economic Review 19*(1), 7–35.

Morgan, J., J. Tumlinson, and F. Várdy (2022). The limits of meritocracy. *Journal of Economic Theory 201*.

Sandel, M. J. (2020). *The tyranny of Merit : What's Become of the Common Good?* New York: Farrar, Straus and Giroux.

Sen, A. (2000). Merit and justice. In K. J. Arrow and et al (Eds.), *Meritocracy and Economic Inequality*. Princeton: Princeton University Press.

Young, M. (1958). *The Rise of the Meritocracy.* Thames and Hudson.

Appendix 1: Proof of Proposition 2

Let us focus first on point 3 of Proposition 2. Inequality (15) provides the necessary and sufficient condition under which workers employed in a position a_j with skill $s_{i=j}$ choose

e_h if workers with skill $s_{i=j+1}$ also choose e_h. We can re-express it by isolating d:

$$d < s_{i=j} \cdot \left[\epsilon\, \mathbb{P}^{j+1}_{i=j\,\cup\, i=j+1} \cdot e_h(a_{j+1} - a_j) + a_j(e_h - 1)\right] \tag{20}$$

Moreover, inequality (16) is the necessary and sufficient condition for workers with skill $s_{i=j+1}$ to choose e_h if workers employed in a position a_j with skill $s_{i=j}$ do the same. Rewriting (16) in terms of d we get:

$$d \;<\; s_{i=j+1} \left\{e_h \left[\epsilon \cdot \mathbb{P}^{j+1}_{i=j\,\cup\, i=j+1} + (1-\epsilon) \cdot \mathbb{P}^{j+1}_{\epsilon,\, i=j+1}\right] (a_{j+1} - a_j) \;+\; a_j(e_h - 1)\right\} \tag{21}$$

It is easy to see that the term at the RHS of (20) is lower than the term at the RHS of (21), because $s_{i=j} < s_{i=j+1}$ and $\mathbb{P}^{j+1}_{i=j\,\cup\, i=j+1} < \mathbb{P}^{j+1}_{\epsilon,\, i=j+1}$. So, there exists an equilibrium in which both workers with skill $s_{i=j+1}$ and those with skill $s_{i=j}$ choose e_h if and only if condition (20) is verified. This proves point 3 of Proposition 2.

Consider now point 1 of Proposition 2. If workers with skill $s_{i=j}$ do not choose e_h, workers with skill $s_{i=j+1}$ also do not choose e_h if and only if:

$$\mathbb{P}^{j+1}_{\epsilon,\, i=j+1} \cdot (a_{j+1}\, s_{i=j+1}\, e_h \;-\; d) \;+\; \left(1 - \mathbb{P}^{j+1}_{\epsilon,\, i=j+1}\right) \cdot (a_j\, s_{i=j+1}\, e_h \;-\; d) \;>\; a_j\, s_{i=j+1} \tag{22}$$

Expressing in terms of d we get:

$$d \;>\; s_{i=j+1} \cdot \left[\, \mathbb{P}^{j+1}_{\epsilon,\, i=j+1} \cdot e_h(a_{j+1} - a_j) \;+\; a_j(e_h - 1)\,\right] \tag{23}$$

Similarly, if workers with skill $s_{i=j+1}$ do not choose e_h, workers skill $s_{i=j}$ also do not choose e_h if and only if:

$$\mathbb{P}^{j+1}_{\epsilon,\, i=j} \cdot (a_{j+1}\, s_{i=j}\, e_h \;-\; d) \;+\; \left(1 - \mathbb{P}^{j+1}_{\epsilon,\, i=j}\right) \cdot (a_j\, s_{i=j}\, e_h \;-\; d) \;>\; a_j\, s_{i=j} \tag{24}$$

Expressing in terms of d we get:

$$d \;>\; s_{i=j} \cdot \left[\, \mathbb{P}^{j+1}_{\epsilon,\, i=j} \cdot e_h(a_{j+1} - a_j) \;+\; a_j(e_h - 1)\,\right] \tag{25}$$

Notice that the term at the RHS of (23) is larger than the term at the RHS of (25), because $s_{i=j+1} > s_{i=j}$ and $\mathbb{P}^{j+1}_{\epsilon,\, i=j+1} > \mathbb{P}^{j+1}_{\epsilon,\, i=j}$ (recall that $\gamma_{i=j+1} \cdot L_j < \gamma_{i=j} \cdot L_j$). This means that inequality (23) is the necessary and sufficient condition for an equilibrium in which nobody exerts any extra effort e_h.

It is also easy to see that the term at the RHS of (23) is larger than the the term at the RHS of (20). So it remains to show which equilibrium may exist in the interval:

$$s_{i=j} \cdot \left[\epsilon\, \mathbb{P}^{j+1}_{i=j \,\cup\, i=j+1} \cdot e_h(a_{j+1} - a_j) + a_j(e_h - 1)\right] < d < s_{i=j+1} \cdot \left[\,\mathbb{P}^{j+1}_{\epsilon,\, i=j+1} \cdot e_h(a_{j+1} - a_j) \,+\, a_j(e_h - 1)\,\right]$$

If condition (20) is not respected (i.e. d is higher than the term at the lower bound of the interval above), that means that workers with skill $s_{i=j}$ do not choose e_h if workers with skill $s_{i=j+1}$ choose it. At the same time, if condition (23) is not fulfilled (i.e. d is lower than the term at the lower bound of the interval above), this implies that workers workers with skill $s_{i=j+1}$ choose e_h when those with skill $s_{i=j}$ do not choose it. So inside this interval an equilibrium implies that only workers with $s_{i=j+1}$ choose e_h.

Finally, as we have explained by presenting equations (17) and (18), we can rule out the hypothesis that an equilibrium in which only workers with skill $s_{i=j}$ exert extra effort e_h if ϵ is sufficiently small.

Appendix 2: Proof of Proposition 3

To prove the results of Proposition 3 it is first convenient to present again the promotion probabilities in both career advancements scenarios:

$$\begin{aligned}
\mathbb{P}^{j=i}_{\epsilon,\, i} &= \frac{P_{\epsilon,\, j=i}}{L_{j=i-1}\,\gamma_i} \\
\mathbb{P}^{j=i}_{i \,\cup\, i-1} &= \frac{P_{\epsilon,\, j=i}}{L_{j=i-1}\,(\gamma_i + \gamma_{i-1})} \\
\mathbb{P}^{j=i}_{i} &= \frac{P_{j=i}}{L_{j=i-1}\,\gamma_i}
\end{aligned}$$

The first two equations concern the noisy scenario. First, $\mathbb{P}^{j=i}_{\epsilon,\, i}$ is the probability that workers with skill i employed in a job position $j = i - 1$ may get a promotion to reach position $j = i$ when noise ϵ does not hit the process. Moreover, $\mathbb{P}^{j=i}_{i \cup i-1}$ is the probability in the that a worker of skill s_i or skill s_{i-1} employed in a job position $j = i - 1$ may get to position $j = i$ if a noise ϵ hits the process.

The last equation concerns the perfect meritocracy scenario, in which $\mathbb{P}^{j=i}_{i}$ is the probability that workers with skill i employed in a job position $j = i - 1$ may get a promotion to reach position $j = i$.

Using these expressions and the definition for Y_i and $Y_{\epsilon,i}$ presented in section 5 , for $i \in [2, ...I - 1]$ we get:

$$\begin{aligned} Y_{\epsilon,i} - Y_i \; = \; & L_{j=i}\, \gamma_i s_i\, a_{j=i} \cdot \, (e_h - 1) \, + \\ & + \; L_{j=i}\, \gamma_i s_i\, e_h\, \epsilon\, \mathbb{P}^{j=i+1}_{i \cup i+1} \; (\, a_{j=i+1} \; - \; a_{j=i}\,) \; + \\ & - \; L_{j=i-1}\, \gamma_i s_i\, e_h \, \frac{P_{j=i} - P_{\epsilon,\, j=i}}{L_{j=i-1}\gamma_i} \; (\, a_{j=i} \; - \; a_{j=i-1}\,) + \\ & - \; L_{j=i-1}\, \gamma_i s_i\, e_h \cdot (\, a_{j=i} \; - \; a_{j=i-1}\,)\, \epsilon\, P_{\epsilon,\, j=i} \left(\frac{1}{L_{j=i-1}\gamma_i} \; - \; \frac{1}{L_{j=i-1}(\gamma_i + \gamma_{i-1})} \right) \end{aligned} \tag{26}$$

Point 1 of Proposition 3 is easy to prove. Indeed, as $i = 1$, the two negative terms in the third and fourth line of equation (26) disappear because there are not positions at $j = 0$, so $L_{j=i-1} \; = \; L_{j=0} \; = \; 0$. So $Y_{\epsilon,1} - Y_1 \; > \; 0$. Similarly, $Y_{\epsilon,I} - Y_I \; < \; 0$ because there are not positions $j = I + 1$, so $L_{j=I}\gamma I$ workers never exert extra effort e_h (i.e. the terms at the RHS in the first two lines in equation (26) are equal to 0).

As concerns point 2 of Proposition 3, we compare the positive term at the RHS in the first line of equation (26) with the two negative terms at the third and the fourth lines. We get that a sufficient condition for $Y_{\epsilon,i} \; - \; Y_i \; > \; 0$ is

$$L_{j=i}\gamma_i\, a_{j=i} \cdot \, (\, e_h \; - \; 1\,) \; > \; e_h\, (\, a_{j=i} \; - \; a_{j=i-1}\,)\, [P_{j=i} - P_{\epsilon,\, j=i} \; + \epsilon P_{\epsilon,\, j=i}] \tag{27}$$

Rearranging, we have:

$$\frac{e_h \; - \; 1}{e_h} \; > \; \frac{a_{j=i} \; - \; a_{j=i-1}}{a_{j=i}} \cdot \frac{P_{j=i} - (1 - \epsilon) P_{\epsilon,\, j=i}}{L_{j=i}\gamma_i} \tag{28}$$

Recall from equation (2) that the only difference between promotions $P_{j=i}$ and promotions

$P_{\epsilon,\, j=i}$ is in the different parameters, α and α_ϵ respectively. More precisely:

$$P_{j+1} \;=\; \alpha\,\delta \sum_{x=1}^{J-j} \alpha^{x-1} \cdot L_{j+x} \;=\; \alpha\delta\left[L_{j+1} \;+\; \alpha \cdot L_{j+2} \;+\; \alpha^2 \cdot L_{j+3} \;+\; ... \;+ \alpha^{I-(j+1)} \cdot L_I\right]$$

and

$$P_{\epsilon,j+1} \;=\; \alpha_\epsilon\,\delta \sum_{x=1}^{J-j} \alpha_\epsilon^{x-1} \cdot L_{j+x} \;=\; \alpha_\epsilon\delta\left[L_{j+1} \;+\; \alpha_\epsilon \cdot L_{j+2} \;+\; \alpha_\epsilon^2 \cdot L_{j+3} \;+\; ... \;+ \alpha_\epsilon^{I-(j+1)} \cdot L_I\right]$$

It is easy to see that P_{j+1} (resp. $P_{\epsilon,j+1}$) is increasing in α (resp. α_ϵ). So, if $\alpha < \alpha_\epsilon$, then $P_{j+1} < P_{\epsilon,j+1}$ and a sufficient condition for (28) to be respected is:

$$\frac{e_h \;-\; 1}{e_h} \;>\; \epsilon \cdot \frac{a_{j=i} \;-\; a_{j=i-1}}{a_{j=i}} \cdot \frac{P_{\epsilon,\, j=i}}{L_{j=i}\gamma_i} \tag{29}$$

Conversely, inequality (28) is less likely to be respected if $\alpha > \alpha_\epsilon$, so that $P_{j+1} > P_{\epsilon,j+1}$. In this case, we can rewrite $P_{\epsilon,j+1}$ as follows:

$$P_{\epsilon,j+1} \;=\; \alpha_\epsilon\,\delta \sum_{x=1}^{J-j} \alpha_\epsilon^{x-1} \cdot L_{j+x} \;=\; \mathbb{A}\,\alpha\delta\left[L_{j+1} \;+\; \mathbb{A}\,\alpha \cdot L_{j+2} \;+\; (\mathbb{A}\alpha)^2 \cdot L_{j+3} \;+\; ... \;+ (\mathbb{A}\alpha)^{I-(j+1)} \cdot L_I\right]$$

in which $\mathbb{A} \;\equiv\; \frac{\alpha_\epsilon}{\alpha}$. Notice also that:

$$\begin{aligned} P_{\epsilon,j+1} \;&=\; \mathbb{A}\,\alpha\delta\left[L_{j+1} \;+\; \mathbb{A}\,\alpha \cdot L_{j+2} \;+\; (\mathbb{A}\alpha)^2 \cdot L_{j+3} \;+\; ... \;+ (\mathbb{A}\alpha)^{I-(j+1)} \cdot L_I\right] \\ &> \mathbb{A}^{I-(j+1)} \cdot \alpha\delta\left[L_{j+1} \;+\; \alpha \cdot L_{j+2} \;+\; \alpha^2 \cdot L_{j+3} \;+\; ... \;+ \alpha^{I-(j+1)} \cdot L_I\right] \;\;=\;\; \mathbb{A}^{I-(j+1)} \cdot P_{j+1} \end{aligned}$$

This inequality is verified since we are considering the case $\mathbb{A} < 1$. So, inequality (28) holds a fortiori if

$$\frac{e_h \;-\; 1}{e_h} \;>\; \frac{a_{j=i} \;-\; a_{j=i-1}}{a_{j=i}} \cdot \frac{P_{j=i} - (1-\epsilon)\mathbb{A}^{I-i} \cdot P_{j=i}}{L_{j=i}\gamma_i} \;=\; \frac{a_{j=i} \;-\; a_{j=i-1}}{a_{j=i}} \cdot \frac{P_{j=i}}{L_{j=i}\gamma_i} \cdot \left[\,1 \;-\; (1-\epsilon) \cdot \mathbb{A}^{I-i}\,\right]$$

This proves point 2 of Proposition 3.

Appendix 3: Proof of Proposition 4

We use the definition for W_i and $W_{\epsilon,i}$ presented in section 5 and we employ the same steps illustrated in the previous Appendix to write the equation $Y_i - Y_{\epsilon,i}$. We get:

$$\begin{aligned}
W_{\epsilon,i} - W_i \;=\; & L_{j=i}\,\gamma_i s_i\, a_{j=i} \cdot (e_h - 1) \;+ \\
& +\; L_{j=i}\,\gamma_i s_i\, e_h\, \epsilon\, \mathbb{P}_{i \cup i+1}^{j=i+1}\,(\,a_{j=i+1} \;-\; a_{j=i}\,) \;+ \\
& -\; L_{j=i-1}\,\gamma_i s_i\, e_h\, \frac{P_{j=i} - P_{\epsilon,\, j=i}}{L_{j=i-1}\gamma_i}\,(\,a_{j=i} \;-\; a_{j=i-1}\,) \;+ \qquad (30)\\
& -\; L_{j=i-1}\,\gamma_i s_i\, e_h \cdot (\,a_{j=i} \;-\; a_{j=i-1}\,)\; \epsilon\, P_{\epsilon,\, j=i}\left(\frac{1}{L_{j=i-1}\gamma_i} \;-\; \frac{1}{L_{j=i-1}(\gamma_i + \gamma_{i-1})}\right) + \\
& -\; L_{j=i}\gamma_i \cdot d
\end{aligned}$$

Point 1 of Proposition 4 is easy to prove. Indeed, as $i = 1$, the two negative terms in the third and the fourth line of equation (30) do not exist because there are not positions at $j = 0$. So we have:

$$W_{\epsilon,1} - W_1 \;=\; L_{j=1}\,\gamma_1\,\left\{s_1 \cdot \left[\,a_1 \cdot (e_h - 1) \;+\; e_h\, \epsilon\, \mathbb{P}_{1 \cup 2}^{j=2}\,(\,a_2 \;-\; a_1\,)\right] \;-\; d\right\}$$

The term inside the graphs is positive in the noisy scenario in which workers with skill 1 employed in a position $j = 1$ exert extra effort (i.e. inequality in point 3 of Proposition 2 is respected). So $W_{\epsilon,1} - W_1 \;>\; 0$.

Similarly, $W_{\epsilon,I} - W_I \;<\; 0$ because there are not positions $j = I + 1$, so $L_{j=I}\gamma I$ workers never exert extra effort e_h (i.e. the terms at the RHS in the first, the second, and the fifth line in equation (30) are all equal to 0).

As concerns the cases $i \;\in\; [2, ...I - 1]$, in Appendix 3 we have proved the sufficient condition that ensures the positive term in the first line of equation (30) is greater than the negative terms in the third and fourth lines:

$$\frac{e_h \;-\; 1}{e_h} \;>\; \frac{a_{j=i} \;-\; a_{j=i-1}}{a_{j=i}} \cdot \frac{P_{j=i}}{L_{j=i}\gamma_i} \cdot \left[\,1 \;-\; (1 - \epsilon) \cdot \mathbb{A}^{I-i}\,\right]$$

This is the first condition presented in point 2 of Proposition 4. To get $W_{\epsilon,i} - W_i \;>\; 0$

one also needs that the positive term in the second line in equation (30) is greater than the negative term in the fifth line:

$$d \; < \; s_{i=j} \cdot \mathbb{P}^{j=i+1}_{i \cup i+1} \cdot e_h(\, a_{j+1} \; - \; a_j \,)$$

This is the second inequality presented in point 2 of Proposition 4.