Information, Consumer Privacy, and Central Bank Digital Currency*

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Abstract

We develop a general equilibrium framework to examine the informational value of CBDC transaction data. Access to individual transaction data allows the government to guide the economy toward more efficient equilibria through targeted subsidies. Additionally, CBDC data enhances the government's ability to estimate aggregate demand, facilitating more informed consumption and investment decisions. Interestingly, when inflation is high, disclosing the information about aggregate demand tends to improve welfare through both consumption and investment, but the effect reverses under low inflation. Moreover, under certain conditions, consumers' privacy concerns over the use of CBDC can increase the value of the information about aggregate demand, partially offsetting the negative effect of such concerns on welfare.

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1 Introduction

As digital transformation reshapes various sectors, Central Bank Digital Currency (CBDC) has garnered increasing attention from monetary authorities and policymakers around the globe.[1] While extensive research has explored its potential economic implications, such as payment system efficiency, monetary policy transmission, and bank disintermediation, comparatively less attention has been given to the distinct benefits and risks arising from its digital nature: Unlike physical cash, CBDC transactions are recorded on a digital ledger, allowing the central bank, as the ledger manager, to potentially observe transaction data. This capability raises significant public concerns regarding privacy, making data governance a critical consideration in the design of CBDCs (European Central Bank, 2021). Nonetheless, providing complete anonymity may not be desirable due to potential benefits, such as deterring illicit activities, by leveraging transaction data to improve social welfare (Board of Governors of the Federal Reserve System, 2022).

In this paper, we explore a novel dimension of the ongoing research on CBDC: The potential of CBDC transaction data being used to provide insights into aggregate economic conditions and as a tool to steer the economy toward more efficient equilibria.[2] Specifically, we aim to answer the following questions: How do improved forecasts, derived from CBDC transaction data, influence economic activities, and through what mechanism do they take effect? In what ways do public concerns regarding privacy affect the usefulness of such improved forecasts? Can transaction data be leveraged to enhance economic stability, predictability, and welfare by guiding the economy in a more efficient equilibrium? How does monetary policy influence the usefulness of CBDC transaction data?

To answer these questions, we develop a general equilibrium model where aggregate shocks affect the demand in the economy. Specifically, in each period, a subset of buyers want to consume goods produced by sellers. The level of aggregate demand, which we assume cannot be directly observed, is then determined by the number of buyers seeking consumption. The mass of such buyers follows a two-state Markov process, exhibiting inertia in aggregate demand (Ball and Mankiw, 1994; Smets and Wouters, 2007). That is, the likelihood of aggregate demand being high is larger if the demand was high in the previous period, and smaller if it was low. In the model, sellers have the option to invest and improve their

[1]See Auer, Frost, Gambacorta, Monnet, Rice, and Shin (2022) and Auer, Cornelli, and Frost (2023) for an analysis of the economic and institutional drivers of CBDC development and a survey of CBDCs.

[2]Several studies have highlighted the limitations of GDP as a measure of economic activity, emphasizing its inability to capture a wide range of production and unreported economic activities. See Stiglitz, Sen, and Fitoussi (2010) for instance. Additionally, recent advances in microeconometrics demonstrate that incorporating micro-level data into economic forecasting models significantly improves accuracy compared to relying solely on aggregate macroeconomic indicators such as GDP or CPI. See for example Carson, Cenesizoglu, and Parker (2011), Giacomini and Levin (2023), Hong, Huang, Wang, and Zhao (2024), and Perevalov and Maier (2010).

productivity. The level of aggregate demand plays a key role in shaping the investment decisions. We assume that CBDC enables the government to measure accurately aggregate demand in the past, thereby improving its ability to forecast future economic conditions. These improved forecasts, in turn, help economic agents make more informed decisions.

As a benchmark, we assume that a government-issued cash is the only payment instrument. Because aggregate demand cannot be directly observed, buyers' choices of money holdings and sellers' investment choices are based on the unconditional expectation of demand. We find that there exists a strategic complementarity between buyers' and sellers' choices: An increase in buyers' money holdings increases sellers' incentives to invest, and as more sellers improve their productivity, buyers are motivated to hold more money. This strategic complementarity can lead to multiple equilibria. In such cases, welfare is the highest in the equilibrium where all sellers invest to improve their productivity. Thus, this strategic complementarity can cause economic uncertainty and lead to socially inefficient equilibria. Moreover, even when there is a unique equilibrium, sellers may fail to make socially efficient investments because they do not fully internalize the benefits of improved productivity.

When CBDC is introduced as another payment option, the transaction data gathered can be leveraged by the government at both the micro and the macro levels. At the micro level, a targeted government subsidy can be used to compensate sellers who fail to make enough sale to justify their investment costs. This subsidy eliminates the risk sellers face when making investment decisions, and as a result, it ensures the most efficient equilibrium. At the macroeconomic level, CBDC transaction data allows the government to accurately assess each period's aggregate demand. If the information regarding the previous period's demand is disclosed, it will allow agents to make more informed economic decisions, given the inertia inherent in aggregate demand.

Interestingly, the information about past demand can either improve or decrease welfare. There are two channels through which the information takes effect: buyers' money-holding decisions (the demand channel) and sellers' investment decisions (the investment channel). First, due to inflation, holding money is costly, so buyers hold more money when they expect aggregate demand to be high, and less when they expect it to be low. In other words, information disclosure makes buyers' money holdings variable. While buyers' preferences are strictly concave, welfare can be either a concave or convex function of buyers' expectations of aggregate demand. When welfare is concave (convex), disclosing the past demand reduces (increases) welfare through the demand channel. In particular, we find that when inflation is low, consumption is high without information disclosure. Thus, learning that past demand was high has little effect, but disclosing low past demand can significantly reduce consumption. In such case, disclosing the information lowers welfare through the demand channel. Conversely, information disclosure can improve welfare when inflation is sufficiently high.

Similar to buyers, sellers have larger (smaller) incentives to invest and improve their productivity when they expect aggregate demand to be high (low). Suppose sellers choose to invest when the information is not disclosed, and they rely on unconditional expectations. Then, disclosing a low past demand may lead to socially inefficient under-investment. Conversely, if sellers choose not to invest given unconditional expectations, then learning that the past demand was high may increase investment. Furthermore, a decrease in the inflation rate increases trade volume, boosting sellers' incentives to invest. In such case, sellers are likely to invest when the information is not disclosed, so information disclosure can only adversely affect welfare. In conclusion, our model suggests that the government should exercise caution when disclosing the information regarding aggregate demand during periods of low inflation.

Since the government is able to gather transaction data via CBDC, we consider the possibility that buyers may incur a privacy cost. We find that under certain conditions, buyers' privacy concerns can lead to their preferences becoming "less concave", thus making it more likely that welfare exhibits convexity. In such case, information regarding past demand has a positive effect on welfare through the demand channel. We show that this mechanism can partially but not fully offset the negative effect of the privacy cost on welfare.

Literature review Our paper contributes to the growing literature on CBDC. Related to this paper, Barrdear and Kumhof (2022) demonstrate that CBDC injections can help stabilize the business cycle. Williamson (2022b) finds that CBDCs improve welfare by reallocating safe assets from private banks to a narrow banking system. Chiu and Davoodalhosseini (2023) shows that a cash-like CBDC is more effective than a deposit-like CBDC in promoting welfare. Chiu, Davoodalhosseini, Jiang, and Zhu (2023) show that CBDCs can enhance competition in the deposit market and expand bank intermediation, while Fernández-Villaverde, Sanches, Schilling, and Uhlig (2021) warn that CBDC could lead to the central bank becoming a monopoly deposit provider, thereby threatening maturity transformation. Keister and Monnet (2022) argue that CBDC can strengthen financial stability by mitigating bank runs, while Williamson (2022a) finds that CBDC encourages bank panics but reduces their impact. Schilling, Uhlig, and Fernández-Villaverde (2024) suggest that CBDC may not simultaneously achieve efficiency, financial stability, and price stability. While these studies predominantly focus on the economic effects of interest-bearing CBDCs, we shift the attention to the privacy issue – a primary concern for the general public (European Central Bank, 2021) – and the informational value of CBDC transaction data.[3]

Several studies have examined the use of payment data in the private sector for com-

[3]Kahn, Oordt, and Zhu (2024) examine the use of an expiry date for offline CBDCs, focusing on automating personal loss recovery rather than the economic effects of interest payments, and demonstrate that this design can enhance consumer demand and welfare.

mercial purposes, particularly in evaluating consumer preferences. For example, Garratt and van Oordt (2021) explores how payment information can be utilized for price discrimination, while Garratt and Lee (2020) and Kang (2024) investigates the economic implications of using payment data to produce more customized products.[4] In contrast to these studies, our paper highlights the social value of payment data in shaping the economic decisions of the general public and examines the optimal information disclosure.

More pertinently, Wang (2023) demonstrates that CBDCs with low anonymity and high interest rates can curb money laundering and enhance welfare without harming output. While Wang (2023) focuses on the prospective utility of CBDC transaction data at the micro level, our analysis extends to both the micro level (investigating individual transaction data to prevent income misreporting) and the macro level (analyzing aggregate transaction data to assess broader economic conditions and trends).

The macroeconomic implications of information disclosure have been studied in the literature. For instance, Bhaskar and Thomas (2019), Blattner, Hartwig, and Nelson (2023), Elul and Gottardi (2015), and Jang and Kang (2024) examine the impact of default history disclosure on credit markets and its macroeconomic consequences. More related, Andolfatto, Berentsen, and Waller (2014) and Andolfatto and Martin (2013) analyze the effects of news disclosure on asset prices in the context of asset exchange models. Our study contributes to this literature by providing new insights into how the disclosure of aggregate state information influences consumer spending and firm investment decisions.

Lastly, this paper contributes to the growing literature on digital currencies. Chiu and Koeppl (2022) and Kang (2023) explore double-spending incentives within the Bitcoin system. Schilling and Uhlig (2019) and Choi and Rocheteau (2021) examine cryptocurrency pricing in a monetary model with speculative holdings. Kang and Lee (2024) analyze the competition between Bitcoin and central bank-issued currencies, focusing on the impact of monetary policy on welfare and Bitcoin transactions. Chiu and Wong (2015) use a mechanism design approach to show how E-money can enable efficient allocations. While these studies investigate the economic implications of the technical features, such as blockchain, our analysis centers on the informational value of transaction data and privacy concerns surrounding CBDCs.

The rest of this paper is organized as follows. Section 2 outlines the environment. Section 3 characterizes equilibrium and provides a welfare analysis when cash is the sole medium of exchange, while Section 4 examines the model economy by incorporating CBDC. Section 5 explores the impact of privacy concerns on the economic value of CBDC transaction data and the optimal information disclosure policy. Finally, Section 6 concludes the paper.

[4] Chiu and Koeppl (2022) and Gomis-Porqueras and Wang (2024) examine how digital platforms' payment services influence their decisions to monetize user data, although in their models, the platforms acquire user information through platform activities rather than through payment data.

2 Environment

Our framework is built on Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is indexed by $t = 0, 1, 2, \ldots$, and every period is divided into two subperiods: the centralized market (CM) and the decentralized market (DM). The economy consists infinitely-lived buyers and sellers, each with a unit measure. In the CM, all agents can produce and consume CM goods, whereas in the DM, buyers consume DM goods that can only be produced by sellers. We designate the CM goods as the numeraire and assume that one unit of labor is required to produce a unit of either good. Both goods are perishable and must be consumed within a subperiod. All agents share the same discount factor, $\beta \in (0, 1)$, across periods.

A buyer i's instantaneous utility is given by

$$X_t^i - H_t^i + \alpha_t^i u(x_t^i),$$

where X_t^i is the consumption of the CM good, H_t^i is the supply of labor in the CM, and x_t^i is the consumption of the DM goods. We assume $u''(\cdot) < 0 < u'(\cdot)$, $u'(0) = \infty$, $u'(\infty) = 0$, and $-\frac{xu''(x)}{u'(x)} < 1$. The parameter α_t^i is determined by a preference shock that is realized at the beginning of the DM, with the shock being independent across buyers. We assume that $\alpha_t^i \in \{0, 1\}$, and $\alpha_t^i = 1$ (which we refer to as an "active buyer") with probability ω_t, which is a random variable that is also realized at the beginning of the DM. We assume $\omega_t \in \{\omega_G, \omega_B\}$, where $0 < \omega_B < \omega_G < 1$, and its stochastic process elaborated upon below. We refer to the aggregate state of the economy as good if $\omega_t = \omega_G$ and bad if $\omega_t = \omega_B$.

A seller i's instantaneous utility is given by

$$X_t^i - H_t^i - \mathbb{1}_{I,t}^i \kappa - \frac{h_t^i}{\delta_t^i},$$

where X_t^i is the consumption of the CM good, H_t^i is the labor supplied in the CM, and h_t^i is the labor supplied in the DM. Here, δ_t^i represents the seller's productivity, where we assume $\delta_t^i \in \{\delta_H, \delta_L\}$, with $\delta_H > \delta_L > 0$. The indicator function $\mathbb{1}_{I,t}^i$ equals one if the seller invests κ units of effort (measured in disutility) to acquire an improved DM production technology. We assume that $\delta_t^i = \delta_H$ if the seller makes this investment, and $\delta_t^i = \delta_L$ if she does not. Since one unit of labor can be converted into one unit of the DM goods, the improved DM production technology reduces the seller's disutility from production.

In the CM, agents trade the CM good and assets in a centralized Walrasian market. In the DM, buyers and sellers meet bilaterally, and each buyer is randomly matched to a seller with probability one, and vice versa. In these pairwise meetings, the buyer and seller bargain over the terms of trade, determined by the bargaining solution of Kalai (1977), where the buyer's bargaining power is $\theta \in (0,1)$. In each meeting, a buyer (seller) can observe δ_t^i (α_t^i) of the seller (buyer) she is matched with. In other words, bargaining in the DM takes place under complete information between matched agents. However, sellers and buyers cannot observe the δ_t^i and α_t^i of agents they are not matched with.

Ideally, buyers would like to borrow output from sellers in the DM and to repay loans in the next CM. Such credit arrangements are ruled out here because agents are anonymous and no device is available to record credit histories. Consequently, any trades between buyers and sellers must occur on a quid-pro-quo basis through the use of a medium of exchanges. There is government-issued cash that can serve as payment. We assume that the government controls its supply to target an inflation rate of $\gamma \geq \beta - 1$, as equilibrium would not exist otherwise. We also assume that sellers incur a cost c (measured in disutility) in the DM when accepting cash. This cost can be interpreted as the effort spent on storage and safe-keeping, or on identifying counterfeits (Kang, 2017).[5]

Aggregate demand inertia A substantial body of literature has documented the sluggish adjustment of aggregate demand in response to changes in key economic variables such as prices, interest rates, and fiscal or monetary policies (see for example Ball and Mankiw (1994) and Smets and Wouters (2007)). This phenomenon of aggregate demand inertia is important for understanding the dynamics of business cycles and the efficacy of policy interventions. As a result, it has become a central feature in macroeconomic models, serving as a key factor in analyses of economic fluctuations and stabilization policies (see for example Caballero and Simsek (2023, 2024)).

In our model, ω_t represents the mass of buyers seeking consumption in the DM and serves as a parameter that reflects the level of aggregate demand in period t. For instance, $\omega_t = \omega_G$ denotes a state of high aggregate demand, while $\omega_t = \omega_B$ reflects low aggregate demand. To incorporate aggregate demand inertia, we model ω_t as following a two-state Markov process, allowing us to account for the persistence of demand levels across periods:

$$\begin{cases} \mathbb{P}(\omega_t = \omega_G | \omega_{t-1} = \omega_G) = \rho_{GG}, \\ \mathbb{P}(\omega_t = \omega_G | \omega_{t-1} = \omega_B) = \rho_{BG}, \end{cases} \tag{2.1}$$

where $\rho_{GG} > \rho_{BG}$, and we let $\rho_{GB} = 1 - \rho_{GG}$ and $\rho_{BB} = 1 - \rho_{BG}$. Thus, if the economy was in a good state during the previous period, it is more likely to remain in a good state in the following period. Conversely, if the economy was in a bad state, it is more likely to persist in the same bad state in the next period.

[5]Allowing buyers to also incur this cost do not change any of the results in this paper. Therefore, for the ease of exposition, we assume that only sellers incur this cost.

As discussed earlier, the aggregate state ω_t is realized at the beginning of the DM. Therefore, when agents make economic decisions, such as portfolio choices, in the CM, they must rely on rational expectations based on the information available at that time. Given the inertia of ω_t, if agents are aware of the aggregate state in the previous period, they can better infer the aggregate state in the current period. However, a key assumption in the model is that agents, including the government, cannot observe the aggregate state in the past period—unless the government has access to detailed transaction-level data, such as digital transaction records when, for example, a central bank digital currency (CBDC) is used as the medium of exchange.

In practice, data sets like GDP and the industrial production index offer valuable insights into historical aggregate economic conditions. However, these data sets are inherently limited as they rely on sample data rather than comprehensive data collection (Lunsford, 2023), and they also fail to capture informal and unreported economic activities, such as small businesses operating outside formal regulatory frameworks (Coyle, 2014). In contrast, CBDC transaction data may offer detailed micro-level insights, and recent advancements in microeconometrics demonstrate that incorporating such data into economic forecasting models can significantly improve their accuracy compared to relying solely on aggregate macroeconomic indicators (see for example Carson et al. (2011), Giacomini and Levin (2023), Hong et al. (2024), and Perevalov and Maier (2010)).

We would like to acknowledge that the model can be potentially adapted to better capture real-world dynamics by introducing two types of meetings in the DM: formal meetings, where the government can observe and track trade, and informal meetings, where trade volume is no observable. If the masses of active buyers in these two meeting types are imperfectly correlated, the previous period's aggregate trade volume in the informal sector would provide additional information about the current state of that sector. However, since our focus is on the informational value of CBDC transaction data and privacy concerns, we assume that all transactions, including aggregate trade volumes in the DM, are unobservable, as if the formal sector does not exist, to streamline the analysis and emphasize key implications.

3 Benchmark Equilibrium

In this section, we derive a benchmark equilibrium in which cash serves as the only payment instrument. We study central bank digital currency (CBDC) in the next section. Since cash transactions are unobservable, the government cannot ascertain the aggregate state in the previous period. As a result, in the CM, agents must form unconditional expectations for ω_t, which is realized in the following DM. Note that the unconditional distribution of ω_t

corresponds to its steady-state distribution, which is given by

$$\begin{cases} \mathbb{P}(\omega_t = \omega_G) = \frac{\rho_{BG}}{\rho_{GB}+\rho_{BG}}, \\ \mathbb{P}(\omega_t = \omega_B) = \frac{\rho_{GB}}{\rho_{GB}+\rho_{BG}}. \end{cases}$$

Define $\bar{\omega} \equiv \frac{\rho_{BG}\omega_G+\rho_{GB}\omega_B}{\rho_{GB}+\rho_{BG}}$ as the unconditional probability that a buyer has $\alpha_t^i = 1$. A rational agent's expected value of ω_t is then equal $\bar{\omega}$.

Hereafter, we focus on stationary equilibria, where all real variables remain constant over time, and the inflation rate is equal to the money growth rate γ. For simplicity, we omit the time subscript t and the superscript i, provided there is no risk of confusion. Note that when cash is the sole medium of exchange, agents form the same expectation for $\omega_t = \bar{\omega}$, and their economic decisions are state-independent in every period, as in standard money search models (Lagos and Wright, 2005).

3.1 Agents' problems

We begin with the buyers' problems in the CM. Let ξ denote a buyer's expectation about the share of sellers who choose to invest in the improved DM production technology. We define $W^b(z,\xi)$ as the buyer's value of holding z units of cash (in real terms) at the beginning of the CM. Additionally, define $V^b(z',\alpha,\delta)$ to be the buyer's value of holding z' units of cash at the beginning of the subsequent DM, following the realization of the preference shock $\alpha \in \{0,1\}$. Then, the optimal choice of z' solves the following problem:

$$\begin{aligned} W^b(z,\xi) = \max_{X,H,z'} \{ & X - H + \bar{\omega}[\xi V^b(z',1,\delta_H) + (1-\xi)V^b(z',1,\delta_L)] \\ & + (1-\bar{\omega})[\xi V^b(z',0,\delta_H) + (1-\xi)V^b(z',0,\delta_L)]\}, \\ \text{s.t. } & X + z' = z + H. \end{aligned}$$

Next, we similarly define $W^s(z)$ and $V^s(z',\alpha,\delta)$ as the values for sellers in the CM and DM, respectively, where δ depends on $\mathbb{1}_I$, which represents the seller's decision to either invest or not invest to improve productivity in the DM. The optimal choices of z' and $\mathbb{1}_I$ in the CM then solve the following problem:

$$\begin{aligned} W^s(z) = \max_{X,H,z',\mathbb{1}_I} \{ & X - H + \mathbb{1}_I[-\kappa + \bar{\omega}V^s(z',1,\delta_H) + (1-\bar{\omega})V^s(z',0,\delta_H)] \\ & + (1-\mathbb{1}_I)[\bar{\omega}V^s(z',1,\delta_L) + (1-\bar{\omega})V^s(z',0,\delta_L)]\}, \\ \text{s.t. } & X + z' = z + H, \end{aligned}$$

where κ is the investment cost required to improve productivity in the DM.

Both $W^b(z,\xi)$ and $W^s(z)$ are linear in z, which simplifies the solution to the bargaining problems in the DM. Specifically, let x represent the DM goods purchased by a buyer, and d denote the cash transferred from the buyer to the seller. Then, we have

$$V^b(z',\alpha,\delta) = \alpha u(x) + \beta W^b(z'-d,\xi'), \quad \text{s.t. } d \le z',$$
$$V^s(z',\alpha,\delta) = -\frac{h}{\delta} - c + \beta W^s(z'+d), \quad \text{s.t. } h = x.$$

Recall that c is the seller's cost of cash storage and safe-keeping, or identifying counterfeits. Clearly, if $\alpha = 0$, then $x = d = 0$, meaning no trade will occur. When $\alpha = 1$, because $W^b(z,\xi)$ and $W^s(z)$ are linear in z, the terms of trade (x,d) can be derived by solving the following bargaining problem given the seller's productivity δ:

$$\max_{x,d}\{u(x)-d\}, \quad \text{s.t. } \frac{u(x)-\frac{\beta d}{1+\gamma}}{-\frac{x}{\delta}-c+\frac{\beta d}{1+\gamma}} = \frac{\theta}{1-\theta} \text{ and } d \le z'. \tag{3.1}$$

Let the solution to the bargaining problem (3.1) be denoted as $x(z',\delta)$ and $d(z',\delta)$, and define $x^*(\delta)$ such that $u'(x^*(\delta)) = \frac{1}{\delta}$. Then, $x(z',\delta)$ solves

$$\begin{cases} \theta\left(\frac{x}{\delta}+c\right) + (1-\theta)u(x) = \frac{\beta z'}{1+\gamma}, \text{ if } \theta\left[\frac{x^*(\delta)}{\delta}+c\right] + (1-\theta)u(x^*(\delta)) > \frac{\beta z'}{1+\gamma}; \\ x = x^*(\delta), \text{ if otherwise;} \end{cases}$$

and $d(z',\delta)$ is give by

$$\frac{\beta d(z',\delta)}{1+\gamma} = \theta\left[\frac{x(z',\delta)}{\delta}+c\right] + (1-\theta)u(x(z',\delta)).$$

In the CM, given the bargaining solution $(x(z',\delta), d(z',\delta))$ and the buyer's expectation about the probability ξ of encountering a seller with higher productivity, the buyer solves

$$\max_{z'}\left\{-z' + \bar{\omega}\theta\begin{bmatrix} \xi\left[u(x(z',\delta_H)) - \frac{x(z',\delta_H)}{\delta_H} - c\right] \\ +(1-\xi)\left[u(x(z',\delta_L))\frac{x(z',\delta_L)}{\delta_L} - c\right]\end{bmatrix} + \frac{\beta z'}{1+\gamma}\right\}, \tag{3.2}$$
$$\text{s.t. } \frac{\beta z'}{1+\gamma} \ge \theta\left[\frac{x(z',\delta)}{\delta}+c\right] + (1-\theta)u(x(z',\delta)) \text{ for each } \delta \in \{\delta_H,\delta_L\}.$$

The optimal z solves the following first-order condition:

$$\frac{1+\gamma}{\beta} - 1 = \frac{\bar{\omega}\xi\theta[u'(x(z',\delta_H)) - 1/\delta_H]}{\theta/\delta_H + (1-\theta)u'(x(z',\delta_H))} + \frac{\bar{\omega}(1-\xi)\theta[u'(x(z',\delta_L)) - 1/\delta_L]}{\theta/\delta_L + (1-\theta)u'(x(z',\delta_L))}. \tag{3.3}$$

Lemma 1 *There is a unique solution to* (3.3)*, and the optimal* z' *is increasing in* ξ *and* $\bar{\omega}$*.*

Proof: See Appendix A.

Note that buyers need to accumulate cash in the CM without knowing for certain whether they will want to consume the DM goods or not. Consequently, as the probability $\bar{\omega}$ rises, buyers are willing to carry more cash. Furthermore, the total trade surplus generated in a pairwise meeting – and thus the buyer's marginal benefit from holding additional cash – is greater when the matched seller has invested in improved productivity ($\delta = \delta_H$). Hence, a larger ξ incentivizes buyers to hold more cash.

Given the linearity of the seller's value function $W^s(z)$ with respect to z, sellers have no incentive to hold cash, as $\frac{\beta}{1+\gamma} \leq 1$. Then, the sellers' surplus in the DM when they face an active buyer with $\alpha = 1$ and z' units of real cash is given by

$$V^s(z', \delta) \equiv (1-\theta)\left[u(x(z',\delta)) - \frac{x(z',\delta)}{\delta} - c\right], \quad \text{s.t. } x(z',\delta) \text{ solving problem (3.1).}$$

As shown in (3.3), all buyers make the same decision regarding their real cash holdings z'. Similar to buyers, sellers are unaware of the aggregate state in the previous period. Hence, their expectation of the probability of encountering an active buyer in the DM is also given by $\bar{\omega}$. Then, a seller's expected surplus with improved productivity in the DM is $\bar{\omega}V^s(z', \delta_H)$, while the expected surplus without such an investment is $\bar{\omega}V^s(z', \delta_L)$. Hence, the seller's optimal investment decision $\mathbb{1}_I$ is given as follows:

$$\begin{cases} \mathbb{1}_I = 1, \text{ if } \bar{\omega}V^s(z',\delta_H) - \bar{\omega}V^s(z',\delta_L) > \kappa; \\ \mathbb{1}_I \in \{0,1\}, \text{ if } \bar{\omega}V^s(z',\delta_H) - \bar{\omega}V^s(z',\delta_L) = \kappa; \\ \mathbb{1}_I = 0, \text{ if } \bar{\omega}V^s(z',\delta_H) - \bar{\omega}V^s(z',\delta_L) < \kappa. \end{cases} \tag{3.4}$$

Now, define $G(z', \bar{\omega}) = \bar{\omega}[V^s(z', \delta_H) - V^s(z', \delta_L)]$ to be a seller's payoff from investing to improve productivity. The next lemma shows how $\bar{\omega}$ and buyers' cash holdings affect $G(z', \bar{\omega})$.

Lemma 2 *Define z^* to be such that $x(z^*, \delta) = x^*(\delta)$ for each $\delta \in \{\delta_H, \delta_L\}$. Then, $\frac{\partial G(z',\bar{\omega})}{\partial \bar{\omega}} > 0$ and $\frac{\partial G(z',\bar{\omega})}{\partial z'} > 0$ for all $z' < z^*$.*

Proof: See Appendix A.

Lemma 2 says that the payoff from investing in the improved productivity increases with both the buyer's cash holdings z' and and the unconditional probability $\bar{\omega}$ of encountering an active buyer. First, the effect of $\bar{\omega}$ is straightforward: as $\bar{\omega}$ increases, sellers have a greater likelihood of trading in the DM, thereby increasing the value of the investment. Next, note that for all $z' < z^*$, an increase in the buyer's cash holdings z' raises the total trade surplus in the DM meeting. This, in turn, amplifies the positive effects of improved productivity on the seller's trade surplus.

3.2 Equilibrium

A stationary equilibrium consists of the buyers' real cash balance z', the sellers' optimal investment decision $\mathbb{1}_I$, and the mass of sellers ξ who invest in the CM to improve their productivity in the DM such that (1) z' solves (3.3) given ξ, and (2) $\mathbb{1}_I$ is determined by (3.4) given z'. The next proposition characterizes the equilibrium.

Proposition 1 (1) *Given $\bar{\omega}$, there exist κ_1 and κ_2 such that:*

(i) *if $\kappa > \kappa_2$, there is a unique equilibrium with $\xi = 0$;*

(ii) *if $\kappa < \kappa_1$, there is a unique equilibrium with $\xi = 1$; and*

(iii) *if $\kappa_1 \leq \kappa \leq \kappa_2$, multiple equilibria exist with $\xi = 0$, $\xi = 1$, or $\xi \in (0, 1)$.*

(2) *Given κ, there exist $\bar{\omega}_1$ and $\bar{\omega}_2$ such that:*

(i) *if $\bar{\omega} > \bar{\omega}_2$, there is a unique equilibrium with $\xi = 1$;*

(ii) *if $\bar{\omega} < \bar{\omega}_1$, there is a unique equilibrium with $\xi = 0$; and*

(iii) *if $\bar{\omega}_1 \leq \bar{\omega} \leq \bar{\omega}_2$, multiple equilibria exist with $\xi = 0$, $\xi = 1$, or $\xi \in (0, 1)$.*

Proof: See Appendix A.

The first takeaway from Proposition 1 is fairly intuitive: in equilibrium, sellers do not invest to improve productivity when the cost of investment, κ, is sufficiently large, or the expected number of active buyers, $\bar{\omega}$, is sufficiently small, both of which lowers sellers' surplus from the investment. If κ is sufficiently small or if $\bar{\omega}$ is sufficiently large, then sellers always invest to improve productivity in equilibrium.

The second takeaway from the proposition is that multiple stationary equilibria can arise when κ and $\bar{\omega}$ are neither too large nor too small (see Figure 1 for an illustration). In such cases, if buyers believe the share of sellers who invest to improve productivity, ξ, is low, they will reduce their cash holdings, z', by the result of Lemma 1. This reduction in buyers' cash holdings limit the amount of DM goods that can be traded, thus lowering the total surplus in the DM and sellers' incentives to invest (see Lemma 2). This in turn validates the buyers' initial expectation. Conversely, more optimistic beliefs about ξ lead to higher z', and consequently, higher ξ in equilibrium. This strategic complementarity results in multiple equilibria, akin to outcomes in platform models with two-sided markets, as seen in Rochet and Tirole (2003) and Hagiu and Spulber (2013).[6]

[6]Similarly, Lester, Postlewaite, and Wright (2012) and Kang (2024) derive the emergence of multiple stationary equilibria stemming from strategic complementarities in the money search models.

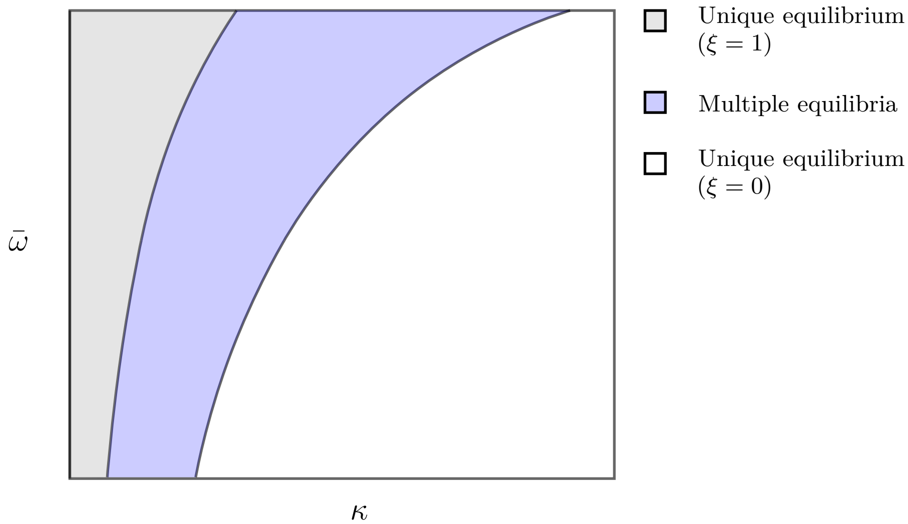


Figure 1: Equilibria given κ and $\bar{\omega}$

To further illustrate this multiplicity result, we plot the best response functions of sellers and buyers in Figure 2. The black line denotes buyers' optimal choice of z' given their beliefs regarding ξ, while the red line indicates sellers' best response given z'. Since the benefit of the investment increases with buyers' cash holdings, there exists a value of z' at which sellers are indifferent between investing and not investing (indicated by the dashed line in the figure). In this example, alongside the two pure strategy equilibria, a mixed strategy equilibrium also arises with $\xi \in (0, 1)$. However, note that the mixed strategy equilibrium is unstable in the sense that a slight deviation in either buyers' beliefs or sellers' investment decisions will lead to either $\xi = 0$ or $\xi = 1$, both of which are part of a stable equilibrium.

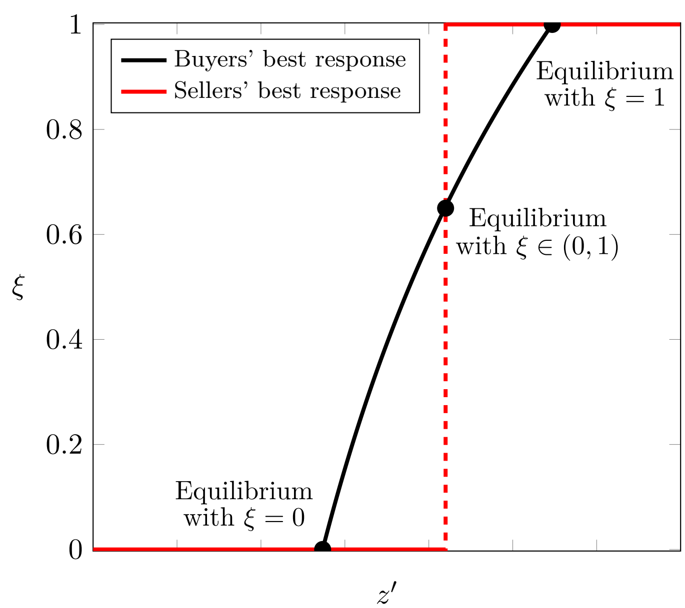


Figure 2: Buyers' and sellers' best responses

3.3 Aggregate welfare

In this section, we analyze how aggregate welfare, defined as the equally weighted sum of all agents' utility, is affected by sellers' investment decisions, ξ, and the expected value of the share of active buyers, $\bar{\omega}$. Note that while $\bar{\omega}$ is constant over time, the realized value, ω_t, is stochastic. In what follows, we focus on expected aggregate welfare, which, in a stationary equilibrium, is given by

$$\begin{aligned} W &\equiv \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\{-\xi\kappa + \omega_t[\xi(u(x_H) - x_H/\delta_H - c) + (1-\xi)(u(x_L) - x_L/\delta_L - c)]\}\right] \\ &= \sum_{t=0}^{\infty}\beta^t w_t(\bar{\omega}). \end{aligned} \tag{3.5}$$

where $x_H = x(z', \delta_H)$, $x_L = x(z', \delta_L)$, and

$$w_t(\omega) \equiv -\xi\kappa + \omega[\xi(u(x_H) - x_H/\delta_H - c) + (1-\xi)(u(x_L) - x_L/\delta_L - c)]. \tag{3.6}$$

Note that the production and consumption of the CM good cancel each other out due to agents' linear preferences in the CM. Therefore, they do not appear in the welfare expression.

3.3.1 Sellers' investment decisions

The proposition below demonstrates that when multiple equilibria exist, aggregate welfare is highest in the equilibrium where all sellers invest to improve their productivity.

Proposition 2 *Suppose the conditions are such that multiple equilibria exist. Then, aggregate welfare W is highest when $\xi = 1$ and lowest when $\xi = 0$.*

Proof: See Appendix A.

The intuition for this result is as follows. If more sellers invest to improve their productivity, buyers will hold more real balances, as shown by Lemma 1. This increase in cash holdings raises the amount of DM goods traded and therefore the total trade surplus in the DM. Although the seller's investment cost κ represents a social cost, sellers invest to improve their productivity only if the increase in their surplus from the investment exceeds the investment cost κ. Therefore, whenever it is profitable for sellers to invest, it is also socially optimal.

The result in Proposition 2 also implies that the equilibrium with the highest aggregate welfare, where $\xi = 1$ (i.e., all sellers invest to improve productivity), may not be realized in the economy due to strategic complementarity. However, even when κ and $\bar{\omega}$ are such that a unique equilibrium exists, sellers' investment decisions may not be socially efficient.

Specifically, due to the proportional bargaining rule, sellers receive a fraction $(1-\theta)$ of the total surplus generated in the DM, meaning they do not fully internalize the benefits of the investment. As a result, sellers may refrain from investing to improve their productivity even when such investment increases aggregate welfare. In other words, sellers may under-invest (but not over-invest) in the technology, as stated in the next proposition.

Proposition 3 *There exists $\kappa > \kappa_2$ such that* (1) *there is a unique equilibrium with $\xi = 0$; and* (2) *aggregate welfare is higher if instead all sellers invest to improve their productivity (i.e., $\xi = 1$).*

Proof: See Appendix A.

3.3.2 The expectation regarding the aggregate state of the economy

In this section, we study how agents' expectation regarding the share of active buyers, $\bar{\omega}$, affects aggregate welfare. In order to isolate the effect of $\bar{\omega}$, we assume for now that $\kappa = 0$ so $\xi = 1$ for all $\bar{\omega}$. From Lemma 1, we know that both x_H and x_L are increasing in $\bar{\omega}$. Hence, it is straightforward to show that $w_t'(\omega) > 0$, so aggregate welfare is increasing in $\bar{\omega}$.

The second derivative, $w_t''(\omega)$, becomes important if agents receive information about the previous aggregate state of the economy (i.e., ω_{t-1}) *before* they make decisions regarding investment and money holdings. Because ω_t follows a two-state Markov process, if the previous state is bad (i.e., $\omega_{t-1} = \omega_B$), then the current state is also likely to be bad, and vice versa. In this case, agents' expectation regarding the aggregate state is no longer constant but depends on the previous state of the economy. Specifically, if such information is available, the conditional expectation of ω_t is given by $\bar{\omega}_G \equiv \rho_{GG}\omega_G + \rho_{GB}\omega_B$ when the previous state was good, and $\bar{\omega}_B \equiv \rho_{BG}\omega_G + \rho_{BB}\omega_B$ when the previous state was bad. Recall that by assumption, $\rho_{BG} < \rho_{GG}$ (thereby $\rho_{GB} < \rho_{BB}$). It follows that $\bar{\omega}_G > \bar{\omega} > \bar{\omega}_B$.

Now, suppose the information is not available in period 0 but becomes available from $t = 1$ onward. At $t = 0$, the expected probability of $\omega_{t-1} = \omega_G$ is given by $\tilde{\rho} \equiv \frac{\rho_{BG}}{\rho_{GB}+\rho_{BG}}$ for all $t \geq 1$. Hence, we have

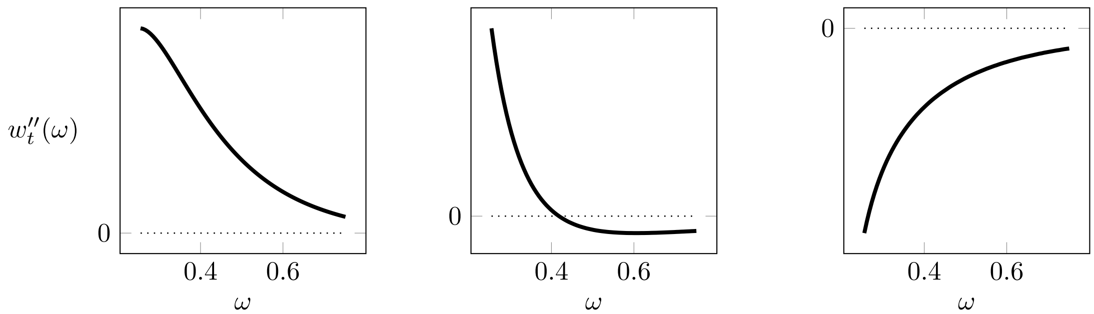
$$\mathbb{E}_0[w_t] = \tilde{\rho} w_t(\bar{\omega}_G) + (1-\tilde{\rho}) w_t(\bar{\omega}_B) \text{ for all } t \geq 1.$$

If such information is not available, agents' expectation of ω_t is $\bar{\omega}$ for all future periods. In other words, we have

$$\mathbb{E}_0[w_t] = w_t(\bar{\omega}) \text{ for all } t \geq 1.$$

Note that $\tilde{\rho}\bar{\omega}_G + (1-\tilde{\rho})\bar{\omega}_B = \bar{\omega}$. Hence, how such information affects expected aggregate welfare depends on $w_t''(\omega)$: by Jensen's inequality, if w_t is convex in ω, the information improves welfare; if w_t is concave in ω, the information hurts welfare.

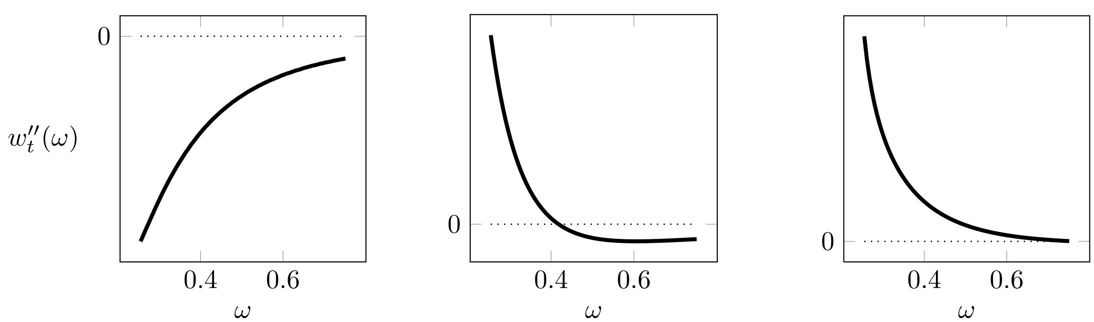
While $u(x)$ is assumed to be strictly concave, $w_t''(\omega)$ may be positive or negative depending on parameter values. In the numerical example below, we assume that $u = \frac{x^{1-\sigma}}{1-\sigma}$ and fix inflation to $\gamma = 0.1$. We show that $w_t''(\omega)$ may be positive or negative depending on the values of σ.



(a) $\sigma = 0.2$ (b) $\sigma = 0.4$ (c) $\sigma = 0.6$

Figure 3: The effect of $\bar{\omega}$ on aggregate welfare ($\gamma = 0.1$)

We repeat the exercise by fixing σ and varying γ instead. Again, we show that $w_t(\omega)$ may be convex or concave in ω. As we discuss in Section 4.2, the concavity/convexity of $w_t(\omega)$ plays an essential role in determining whether the information gathered from CBDC transaction data improves aggregate welfare.



(a) $\gamma = 0.02$ (b) $\gamma = 0.10$ (c) $\gamma = 0.20$

Figure 4: The effect of $\bar{\omega}$ on aggregate welfare ($\sigma = 0.4$)

4 Central Bank Digital Currency

In this section, we assume that the government issues a Central Bank Digital Currency (CBDC). Throughout this section, we assume that the CBDC is exchangeable one-for-one with cash in the CM and bears no interest. Additionally, we assume that CBDC eliminates cash-related costs, such as storage, safe-keeping, and the identification of counterfeits. In other words, sellers do not incur the cost c when accepting CBDC in the DM. In this section, we also assume there are no privacy costs associated with using CBDC. These assumptions imply that buyers will only use CBDC. Buyers' problem in the CM (3.2) can be then rewritten as follows:

$$\max_{z'} \left\{ -z' + \bar{\omega}\theta \left[\xi \left[u(x(\delta_H, z')) - \frac{x(\delta_H, z')}{\delta_H} \right] + (1-\xi) \left[u(x(\delta_L, z')) - \frac{x(\delta_L, z')}{\delta_L} \right] \right] + \frac{\beta z'}{1+\gamma} \right\},$$
$$\text{s.t. } \frac{\beta z'}{1+\gamma} \geq \theta \frac{x(z', \delta)}{\delta} + (1-\theta) u(x(z', \delta)), \text{ where } \delta \in \{\delta_H, \delta_L\}.$$

where, with some abuse of notation, z' represents the choice of CBDC holding. The optimal choice solves the following first-order condition

$$\frac{1+\gamma}{\beta} - 1 = \frac{\bar{\omega}\xi\theta[u'(x(\delta_H, z')) - 1/\delta_H]}{\theta/\delta_H + (1-\theta)u'(x(\delta_H, z'))} + \frac{\bar{\omega}(1-\xi)\theta[u'(x(\delta_L, z')) - 1/\delta_L]}{\theta/\delta_L + (1-\theta)u'(x(\delta_L, z'))}. \tag{4.1}$$

Unlike most of the existing literature on central bank digital currencies, we abstract from the problem of optimal interest rate on the digital currency, focusing instead on its informational aspect.[7] Specifically, we assume that, unlike cash, CBDC enables the government to observe transactions in the DM whenever it is used. In the following analysis, we show that such information can be useful for two purposes: (1) eliminating inferior equilibria when multiple equilibria exist, and (2) enabling the government to identify the aggregate state of the economy.

4.1 Unique equilibrium with central bank digital currency

In our environment, multiple equilibria exist when the cost of investment to improve sellers' productivity, κ, and the expected share of active buyers in the DM, are neither too large nor too small (see Proposition 1). In such case, equilibria exist in which no seller invests to improve their productivity ($\xi = 0$), all sellers invests to improve productivity ($\xi = 1$), or sellers play mixed strategies ($\xi \in (0,1)$). We show that the equilibrium with $\xi = 1$ yields the highest surplus for both buyers and sellers (see Proposition 2). However, strategic complementarity may prevent such equilibrium from happen if sellers do not expect buyers to hold sufficient amounts of real balances to purchase sufficient amount of the DM goods so that their investments are worthwhile.

In what follows, we consider a government policy that can ensure $\xi = 1$ happens in equilibrium. First, we assume that the government can verify whether a seller has invested to improve their productivity. Let $z'(1)$ denote the solution to (4.1) when $\xi = 1$. In other words, $z'(1)$ is the amount of digital currency (in real term) a buyer will opt to hold under the belief

[7] For papers that discuss the optimal interest rate on CBDC, see for example Chiu et al. (2023), Keister and Sanches (2023), Williamson (2022a,b), and Wang (2020, 2023).

that all sellers will invest to improve productivity. The following proposition demonstrates that the government can eliminate equilibria where $\xi < 1$ by committing to compensate sellers for their investment costs if the buyers they are matched with do not carry sufficient amounts of digital currency (i.e., $z' < z'(1)$).

Proposition 4 *Suppose κ and $\bar{\omega}$ satisfy the conditions in* Proposition 1 *such that multiple equilibria exist. Consider the following policy: in the CM, the government offers a transfer T to sellers who have invested to improve their productivity, where T is given by*

$$\begin{cases} T = \kappa/\beta, \text{ if } z' < z'(1); \\ T = 0, \text{ if } z' = z'(1), \end{cases}$$

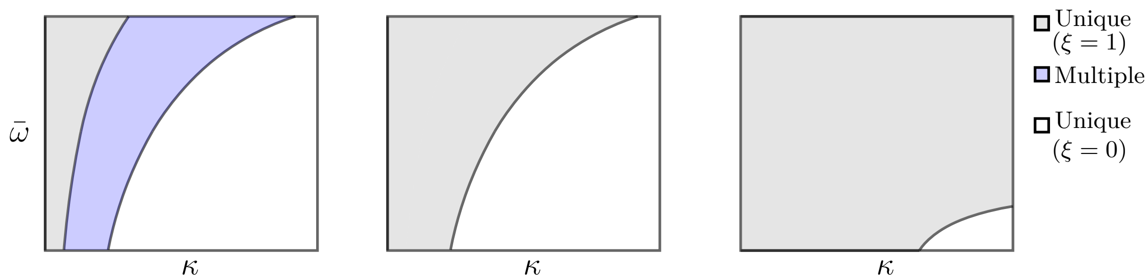
with z' representing the CBDC holdings of the buyer the seller is matched with. Under this policy, there is a unique equilibrium with $\xi = 1$.

Proof: See Appendix A.

The proposed policy ensures the efficient equilibrium with $\xi = 1$ by eliminating the uncertainty sellers face regarding the returns on their investments, namely the possibility that buyers do not carry enough digital currency and, as a result, not enough DM goods are purchased to justify sellers' investment. With this policy, all sellers choose invest in the technology (i.e., $\xi = 1$), and as a result, all buyers carry $z' = z'(1)$. To verify this is indeed an equilibrium, we need to check whether buyers and sellers have any incentive to collude and exploit the policy. For such collusion to be profitable, buyers would need to carry less than $z'(1)$, allowing sellers to later receive the government transfer, after which the sellers would compensate buyers for reduced DM consumption. However, since buyers and sellers are anonymous and do not meet again, sellers have no incentive to compensate buyers. As a result, buyers have no incentive to carry less than $z'(1)$.

Since all buyers carry $z' = z'(1)$, the government do not make any transfers to sellers in equilibrium. In essence, this policy works as a coordination mechanism that supports the efficient equilibrium, similar to how deposit insurance eliminates bank run equilibrium (Diamond and Dybvig, 1983). Nevertheless, for this policy to work, both sellers and buyers must believe that the government has the necessary tools (e.g., access to taxation) to make such transfers along off-equilibrium paths.

The proposed policy takes advantage of the digital nature of the currency, enabling the government to observe the currency holdings of each individual buyer and their transactions. Such a policy would be challenging to implement if cash were the only payment instrument available. However, we do not assert that this is the sole policy capable of eliminating multiple equilibria. For instance, the government could also subsidize sellers' investments in the technology and finance these subsidies through a lump-sum tax in the CM. The advantage of the proposed policy is that the government does not need to make any actual transfers or impose any taxes. Finally, this policy also does not eliminate sellers' under-investment problem when $\xi = 0$ is the unique equilibrium (see Proposition 3), as illustrated by the following example. Since this problem arises because sellers do not fully internalize benefit of their investments, subsidies funded by taxes are needed to address it.



(a) Without transfer policy (b) With transfer policy (c) Socially efficient investment

Figure 5: Equilibria with CBDC

In what follows, we assume that the policy described in Proposition 4 is always active, so there is always a unique equilibrium.

4.2 Information regarding the previous aggregate state

The digital nature of CBDC allows the government to observe transactions in DM and calculate ω_t, the realized value of the share of active buyers in t, when all buyers opt to use CBDC. Recall that ω_t follows a two-state Markov process as described in (2.1). Knowing the aggregate state of the previous period, ω_{t-1}, enables agents to infer the current aggregate state more accurately due to the inertia of ω_t. In this section, we assume that the government introduces the CBDC in the CM of period $t = 0$. The information regarding ω_{t-1} is therefore available for all $t \geq 1$. We examine the welfare effects of such information.

Recall that we define aggregate welfare to be $W = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t w_t(\omega)\right]$, where $w_t(\omega)$ is the sum of buyers' and sellers' utility in a period given agents' expectation of ω:

$$w_t(\omega) \equiv -\xi\kappa + \omega[\xi(u(x_H) - x_H/\delta_H) + (1 - \xi)(u(x_L) - x_L/\delta_L)],$$

where x_H, x_L, and $\xi \in \{0, 1\}$ solve buyers' and sellers' problems given ω. Suppose the government discloses the value of ω_{t-1} to agents in all $t \geq 1$. The conditional expectation of ω is $\bar{\omega}_G = \rho_{GG}\omega_G + \rho_{GB}\omega_B$ when the previous state was good, and $\bar{\omega}_B = \rho_{BG}\omega_G + \rho_{BB}\omega_B$ when the previous state was bad. Because by assumption $\rho_{BG} < \rho_{GG}$, we have $\bar{\omega}_G > \bar{\omega} > \bar{\omega}_B$. At $t = 0$, the expected probability of the aggregate state being good is given by $\tilde{\rho} \equiv \frac{\rho_{BG}}{\rho_{GB}+\rho_{BG}}$

for all $t \geq 1$. Hence, we have

$$\mathbb{E}_0[w_t(\omega)] = \tilde{\rho} w_t(\bar{\omega}_G) + (1 - \tilde{\rho}) w_t(\bar{\omega}_B) \text{ for all } t \geq 1.$$

If such information is not disclosed, the unconditional expectation of ω is $\bar{\omega} \equiv \frac{\rho_{BG}\omega_G + \rho_{GB}\omega_B}{\rho_{GB} + \rho_{BG}}$. In this case, we have

$$\mathbb{E}_0[w_t(\omega)] = w_t(\bar{\omega}) \text{ for all } t \geq 1.$$

The disclosure of ω_{t-1} influences welfare through two distinct channels:
(1) Investment channel: the disclosure of ω_{t-1} can affect sellers' decisions to invest in improving their productivity. If sellers anticipate a higher likelihood of encountering active buyers (i.e., a larger ω), their incentive to invest increases. In particular, denote $\xi(\omega)$ as sellers' investment decisions given sellers' expectation ω. Using Propositions 1 and 4, it is straightforward to derive the following lemma.

Lemma 3 *There exist κ_1, κ_2, and κ_3 such that* (1) $\xi(\bar{\omega}_B) = \xi(\bar{\omega}) = 0$ *and* $\xi(\bar{\omega}_G) = 1$ *if* $\kappa \in (\kappa_2, \kappa_3]$; (2) $\xi(\bar{\omega}_B) = 0$ *and* $\xi(\bar{\omega}) = \xi(\bar{\omega}_G) = 1$ *if* $\kappa \in (\kappa_1, \kappa_2]$; (3) $\xi(\bar{\omega}_B) = \xi(\bar{\omega}) = \xi(\bar{\omega}_G) = 1$ *if* $\kappa \leq \kappa_1$; *and* (4) $\xi(\bar{\omega}_B) = \xi(\bar{\omega}) = \xi(\bar{\omega}_G) = 0$ *if* $\kappa > \kappa_3$.

Proof: See Appendix A.

Recall that sellers may under-invest in improving their productivity due to sellers not fully internalizing the benefit of the investment. Hence, if the disclosure of ω_{t-1} leads to more investment, which happens when $\kappa \in (\kappa_2, \kappa_3]$, it improves aggregate welfare. In contrast, if $\kappa \in (\kappa_1, \kappa_2]$, the disclosure of ω_{t-1} leads to less investment and exacerbates the under-investment problem.

(2) Demand channel: for any given $\xi \in \{0, 1\}$, buyers' expectations of ω affect their choice of digital currency holdings, which in turn determine the amounts of goods that can be purchased in the DM. To focus only on this channel, assume for a moment that $\xi(\bar{\omega}_B) = \xi(\bar{\omega}_G) = \xi(\bar{\omega})$ (i.e., the investment channel is shut down). As discussed in Section 3.3, because $\tilde{\rho}\bar{\omega}_G + (1 - \tilde{\rho})\bar{\omega}_B = \bar{\omega}$, the concavity/convexity of $w_t(\omega)$ plays an essential role in deciding the welfare effect of such information: the information improves welfare if w_t is convex but hurts welfare if w_t is concave. Define $x(\omega)$ to be the DM consumption conditional on buyers' expectation ω. The following lemma describes conditions for $w_t(\omega)$ to be concave/convex.

Lemma 4 *Assume that* $\xi(\bar{\omega}_B) = \xi(\bar{\omega}_G) = \xi(\bar{\omega})$. *Then,*
(1) $w_t(\omega)$ *is convex on* $[\bar{\omega}_B, \bar{\omega}_G]$ *if* $\frac{[u''(x)]^2}{u'''(x)\left(u'(x) - 1/\delta\right)} < \frac{1}{3}$ *for all* $x \in [x(\bar{\omega}_B), x(\bar{\omega}_G)]$ *and concave on* $[\bar{\omega}_B, \bar{\omega}_G]$ *if* $\frac{[u''(x)]^2}{u'''(x)\left(u'(x) - 1/\delta\right)} > 1$ *for all* $x \in [x(\bar{\omega}_B), x(\bar{\omega}_G)]$.

(2) *Assume* $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ *where* $\sigma \in (0,1)$*. There exist* $\underline{\theta}$*,* $\bar{\theta}$*,* $\underline{\sigma}$*, and* $\bar{\sigma}$ *such that* $w_t(\omega)$ *is convex if* $\sigma < \underline{\sigma}$ *and concave if* $\sigma > \bar{\sigma}$ *and* $\theta \in (\underline{\theta}, \bar{\theta})$*.*

Proof: See Appendix A.

If the utility function takes the CRRA form, the condition for convexity boils down to a simple condition on σ, which determines the curvature of $u(x)$. If the conditions in the lemma are not met, then $w_t(\omega)$ may not be convex nor concave on the interval $[\bar{\omega}_B, \bar{\omega}_G]$.

Similar to the benchmark case, we focus on stationary equilibria where the real value of digital currency remains constant over time. This implies that when ω_{t-1} is disclosed to agents, the government must adjust the money supply based on the prior aggregate state. The following proposition summarizes the overall effect of information disclosure.

Proposition 5 *Given* γ*, the disclosure of the previous state affects welfare as follows:*
(1) *Both the investment channel and the demand channel improve aggregate welfare if* $\kappa \in (\kappa_2, \kappa_3]$ *and* $w_t(\omega)$ *is convex under the conditions in Lemma 4.*
(2) *Both the investment channel and the demand channel lower aggregate welfare if* $\kappa \in (\kappa_1, \kappa_2]$ *and* $w_t(\omega)$ *is concave under the conditions in Lemma 4.*
(3) *When* $\kappa > \kappa_3$ *or* $\kappa \leq \kappa_1$*, the investment channel has no effect, while the demand channel improves aggregate welfare if* $w_t(\omega)$ *is convex and lowers aggregate welfare if* $w_t(\omega)$ *is concave.*

(4) *In the other cases, the effect is ambiguous.*

Proof: See Appendix A.

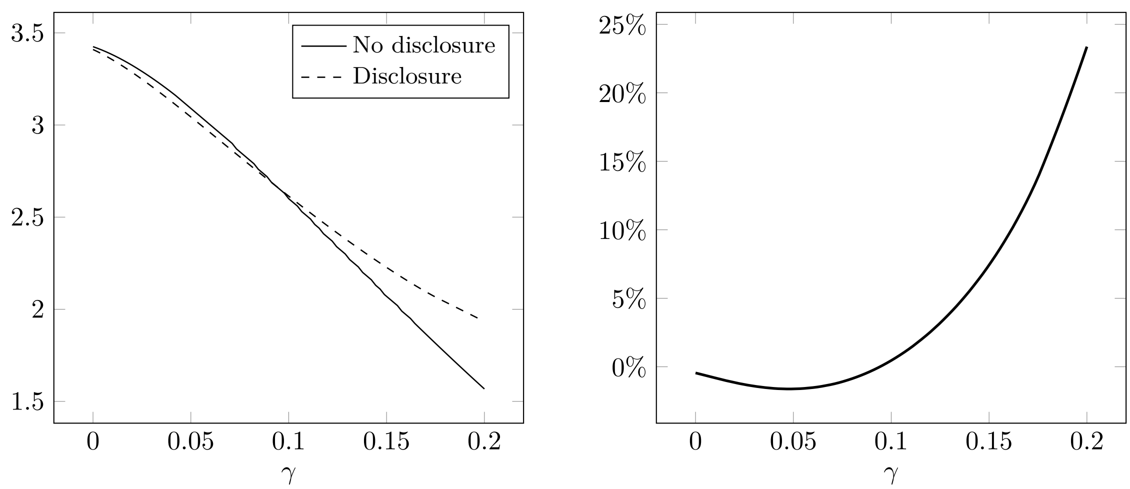
To understand the proposition, first note that if $\kappa \in (\kappa_2, \kappa_3]$, the disclosure of ω_{t-1} increases investment when the previous aggregate state is good but otherwise has no effect on investment. In contrast, $\kappa \in (\kappa_1, \kappa_2]$, the disclosure of ω_{t-1} decreases investment when the previous aggregate state is bad but has no effect on investment if otherwise. The investment channel is only active in these two cases, since when κ is sufficiently large or small, the information regarding the previous aggregate state has no effect on investment. In the absence of the investment channel, the demand channel determines the welfare effect of information disclosure. However, if the demand channel operates in the opposite direction of the investment channel, which happens when $\kappa \in (\kappa_2, \kappa_3]$ and $w_t(\omega)$ is concave or when $\kappa \in (\kappa_2, \kappa_3]$ and $w_t(\omega)$ is convex, then the overall effect is ambiguous.

In addition to the above cases, it is also possible for $w_t(\omega)$ to be neither concave nor convex on the internal $[\bar{\omega}_B, \bar{\omega}_G]$ (see Figures 3 and 4 in Section 3.3). In such case, $w_t(\omega)$ may be still *locally* concave or convex depending on the values of x, which in turn depend on γ. The following proposition explains the results.

Proposition 6 *Suppose $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ where $\sigma \in (0,1)$. Then, there exist $\underline{\gamma}$, $\bar{\gamma}$, and $\hat{\sigma} \in (0,1)$ such that if $\sigma < \hat{\sigma}$, the demand channel lowers aggregate welfare when $\gamma < \underline{\gamma}$ and improves aggregate welfare when $\gamma > \bar{\gamma}$.*

Proof: See Appendix A.

The intuition behind this result is as follows. The amount of DM goods traded in each bilateral meeting, x, is increasing in ω but decreasing in γ. When γ is sufficiently low, x is relatively high, so updating agents' expectations from $\bar{\omega}$ to $\bar{\omega}_G$ only marginally increases x, as it is bounded above by $x^*(\delta)$, which solves $u'(x^*(\delta)) = \frac{1}{\delta}$. However, shifting expectations from $\bar{\omega}$ to $\bar{\omega}_B$ lowers x more significantly. Consequently, $w_t(\omega)$ is concave in ω, and thus information disclosure lowers aggregate welfare. Conversely, when γ is sufficiently high, x is low, and adjusting expectations from $\bar{\omega}$ to $\bar{\omega}_G$ increases x substantially. Consequently, $w(\omega)$ is convex, and information disclosure improves welfare. Figure 6 provides a numerical example.



(a) Aggregate welfare

(b) Welfare gain from disclosure

Figure 6: The effect of inflation on aggregate welfare

Inflation also affects sellers' investment incentives. When inflation is low, buyers' money holdings are large, so trade surplus in the DM is high. This means that sellers may opt to invest in improving their productivity even when the previous aggregate state is not disclosed. Disclosing ω_{t-1}, therefore, can only lead to lower investment when the previous aggregate state is bad. This finding, combined with the results in Proposition 6, suggests that the government should exercise greater caution in disclosing the previous state when inflation is low.

The results in this section are related to findings in Andolfatto et al. (2014) and Andolfatto and Martin (2013), who examine the macroeconomic implications of information disclosure in an asset exchange framework. In their models, assets functioning as a medium of exchange generate stochastic dividends, and the government determines whether to disclose information about the future state of these dividends. Since the quality of the asset influences trade volume – higher (lower) dividends leading to higher (lower) trade volumes – nondisclosure is generally desirable as it facilitates consumption smoothing across states.

In contrast, we consider the government's disclosure of information regarding aggregate demand, which influences buyers' choices of money holdings. Moreover, information disclosure in our framework also shapes sellers' investment decisions. As a result, the welfare implications of information disclosure are more nuanced: it can either increse or decrease welfare depending on the circumstances.

5 CBDC and Privacy

In the last section, we explore how CBDC allows the government to collect more information regarding the aggregate state of the economy, and how such information can improve aggregate welfare under certain conditions. However, when the government has the ability to collect buyers' transactional data, privacy concerns may arise. In this section, we assume that whenever CBDC is used in a transaction, the buyer incurs a privacy cost $v(x_t)$, where x_t represents the DM goods purchased by buyers. The privacy cost is assumed to depend on x_t because we interpret x_t as measuring not just the quantity but also the variety of the DM goods. Hence, the larger x_t is, the more information about the buyer is revealed to the government via transaction data. Buyers' instantaneous utility is then given by

$$X_t - H_t + \alpha_t u(x_t) - \mathbb{1}_p v(x_t),$$

where $\mathbb{1}_p$ is an indicator function that is equal to one if CBDC is used as payment in the DM and zero if cash is used instead. In what follows, we assume that $v'(x_t) > 0$ and define $\tilde{u}(x) = u(x) - v(x)$. To ensure an equilibrium exists, we also assume that $\tilde{u}''(\cdot) < 0 < \tilde{u}'(\cdot)$ and $-\frac{x\tilde{u}''(x)}{\tilde{u}'(x)} < 1$ for all $x \in [0, x^*]$, where x^* solves $\tilde{u}'(x) = 1/\delta_H$.

We first derive conditions under which it is optimal for buyers to use CBDC. Recall that if buyers choose to use cash, they solve the following problem in the CM

$$V^c(\xi) \equiv \max_{z'} \left\{ -z' + \bar{\omega}\theta \left[\begin{array}{c} \xi \left[u(x(z', \delta_H)) - \frac{x(z',\delta_H)}{\delta_H} \right] \\ +(1-\xi) \left[u(x(z', \delta_L)) - \frac{x(z',\delta_L)}{\delta_L} \right] - c \end{array} \right] + \frac{\beta z'}{1+\gamma} \right\},$$
$$\text{s.t. } \frac{\beta z'}{1+\gamma} \geq \theta \frac{x(z', \delta)}{\delta} + (1-\theta) u(x(z', \delta)) \quad \text{for each } \delta \in \{\delta_H, \delta_L\},$$

where $x(z', \delta)$ and $d(z', \delta)$ are from the bargaining solution and ξ is the buyer's expectation about the share of sellers with high productivity. If buyers choose to use CBDC, they instead solve the following problem

$$V^d(\xi) \equiv \max_{z'} \left\{ -z' + \bar{\omega}\theta \begin{bmatrix} \xi \left[\tilde{u}(z', \delta_H) - \frac{x(z',\delta_H)}{\delta_H} \right] \\ +(1-\xi) \left[\tilde{u}(x(z', \delta_L)) - \frac{x(z',\delta_L)}{\delta_L} \right] \end{bmatrix} + \frac{\beta z'}{1+\gamma} \right\},$$

$$\text{s.t. } \frac{\beta z'}{1+\gamma} \geq \theta \frac{x(z', \delta)}{\delta} + (1-\theta)\tilde{u}(x(z', \delta)) \text{ for each } \delta \in \{\delta_H, \delta_L\}.$$

Recall that sellers only incur the cost c (interpreted as the cost of storage, safe-keeping, and fraud prevention) when they accept cash. Buyers choose CBDC over cash if and only if $V^d(\xi) \geq V^c(\xi)$. We assume that CBDC is chosen if buyers are indifferent. Note that due to the existence of the fixed cost c, buyers do not hold CBDC and cash simultaneously. In the next section, we consider a scenario where cash and CBDC coexist.

Before continuing the analysis, we would like to emphasize that buyers incur the privacy cost due to the government's capability to observe DM transactions when CBDC is used. This means that even if the government chooses not to actively collect transaction information, the existence of this capability may result in perceived risks of abuse and security concerns among buyers. However, if CBDC could be designed to fully eliminate privacy concerns, the optimal policy would involve a trade-off between the benefits of such information and the associated privacy cost. In what follows, we assume that privacy concerns cannot be eliminated through the design of CBDC.

5.1 Optimal disclosure policy with the privacy cost

In this section, we assume that c, which is interpreted as the cost of storing cash or identifying counterfeits, is sufficiently large so that buyers prefer CBDC to cash despite the privacy cost. In the next section, we discuss the case where this assumption is not satisfied so that at least some buyers choose cash. The goal of this section is to analyze how $v(x)$ affects the optimal policy regarding the disclosure of the previous aggregate state (i.e., ω_{t-1}).

Recall from Section 4.2 that disclosing ω_{t-1} influences welfare through a demand channel and an investment channel. In particular, the effect of the demand channel is determined by the concavity or convexity of the welfare function, which itself depends on the shape of buyers' DM utility function, $u(x)$, which becomes $\tilde{u}(x) = u(x) - v(x)$ once the privacy cost is incorporated. We are particularly interested in checking whether it is possible for information disclosure to be optimal only when the privacy cost is taken into account. In other words, the information becomes valuable *because of* the privacy cost, thereby mitigating the adverse welfare effects resulted from buyers' privacy concerns.

We begin by assuming that κ, sellers' investment cost, is such that the equilibrium is described by either Case (2) or Case (3) in Proposition 5. Recall that in these cases, as long as $w_t(\omega)$, which is defined by (3.6), is concave under the conditions in Lemma 4, the disclosure of ω_{t-1} unambiguously lowers aggregate welfare (unlike the other cases). The following proposition shows the results.

Proposition 7 *Assume $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$ and $v(x) = Ax^{\epsilon}$ where $\sigma \in (0, 1)$. Assume also that* (a) *κ is such that the equilibrium is described by either Case (2) or Case (3) in Proposition 5, and* (b) *when $A = 0$, $w_t(\omega)$ is concave under the conditions in Lemma 4. Then, when $A > 0$, the optimal policy switches from no disclosure to the disclosure of ω_{t-1} only if $\epsilon < 1$.*

Proof: See Appendix A.

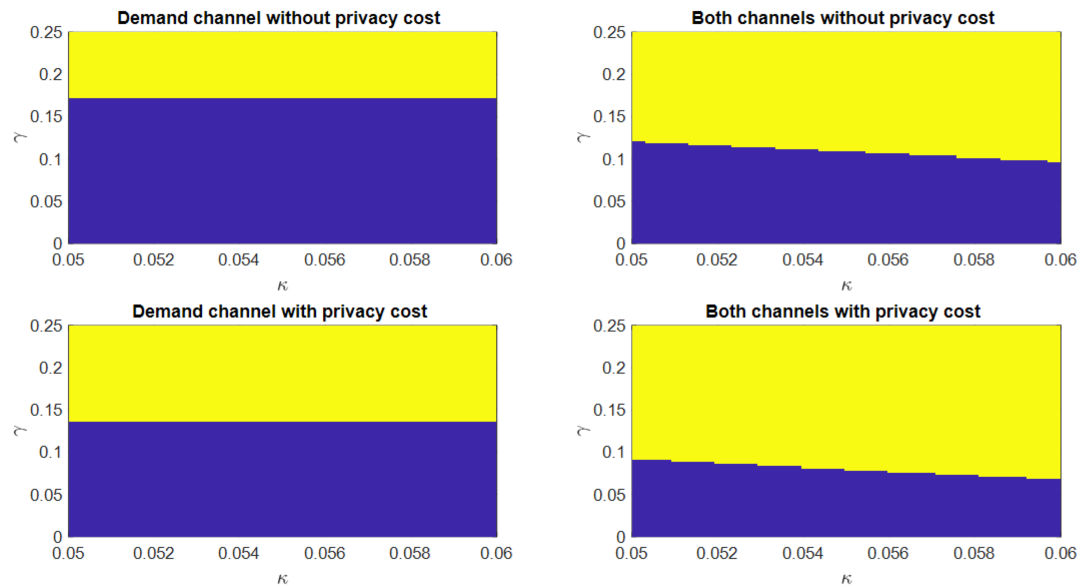
The condition $\epsilon < 1$ means that the privacy cost function is concave. That is, when a buyer has already lost some privacy, the marginal cost of losing privacy becomes lower. Intuitively, if the government already has extensive knowledge about a buyer through her existing transactions, she may not care about purchasing even more goods with CBDC. A similar assumption can be found in Ichihashi (2023), who studies privacy policies of digital platforms.

To understand why $\epsilon < 1$ is necessary, first recall from Lemma 4 that $w_t(\omega)$ is convex if $u(x)$ exhibits sufficiently low concavity. When $\epsilon < 1$, the privacy cost makes the buyer's preference, $\tilde{u}(x) = u(x) - v(x)$, "less concave". If the concavity of $\tilde{u}(x)$ is sufficiently low, the disclosure of ω_{t-1} improves aggregate welfare through the demand channel. To illustrate this mechanism, we provide a numerical example in Figure 7's first column.[8] In this example, we shut down the investment channel by assuming $\delta_L = \delta_H$. The first takeaway is that information disclosure improves welfare through the demand channel only when the inflation rate is sufficiently high, which is consistent with our findings in Proposition 6. Second, a concave privacy cost function increases the size of the yellow area, where the disclosure of ω_{t-1} improves aggregate welfare. This shows that there exist scenarios where information disclosure is optimal only when the privacy cost is taken into account.

To complete the analysis, we turn on the investment channel in the second column of Figure 7 by assuming that $\delta_L < \delta_H$. This allows the privacy cost to also affect sellers' investment decisions. In this example, the disclosure ω_{t-1} has a more positive effect on welfare when κ is larger. This is because in such scenarios, sellers tend to refrain from investing to improve their productivity in the absence of information on ω_{t-1}. Consequently, disclosing a good prior state generally shifts the sellers' decisions from non-investment to investment, whereas disclosing a bad prior state does not alter the sellers' investment choices. As a result, the investment channel strengthens the positive impact of information disclosure, which is demonstrated by the larger yellow areas in the second column compared to the first.

[8] We assume $u(x) = 2\sqrt{x}$, $v(x) = 0.2x^{0.3}$, $\beta = 0.95$, $\theta = 0.5$, $\tilde{\rho} = 0.5$, $\omega_G = 0.6$, and $\omega_B = 0.3$. In addition, in the first column, $\delta_L = \delta_H = 1$, while in the second column, $\delta_L = 1$ and $\delta_H = 1.5$.

Nevertheless, the effect of the privacy cost remains the same: the parameter space in which information is beneficial is larger when the cost is taken into account than when it is not.



Note: yellow area = info disclosure improves welfare; blue area = info disclosure lowers welfare.

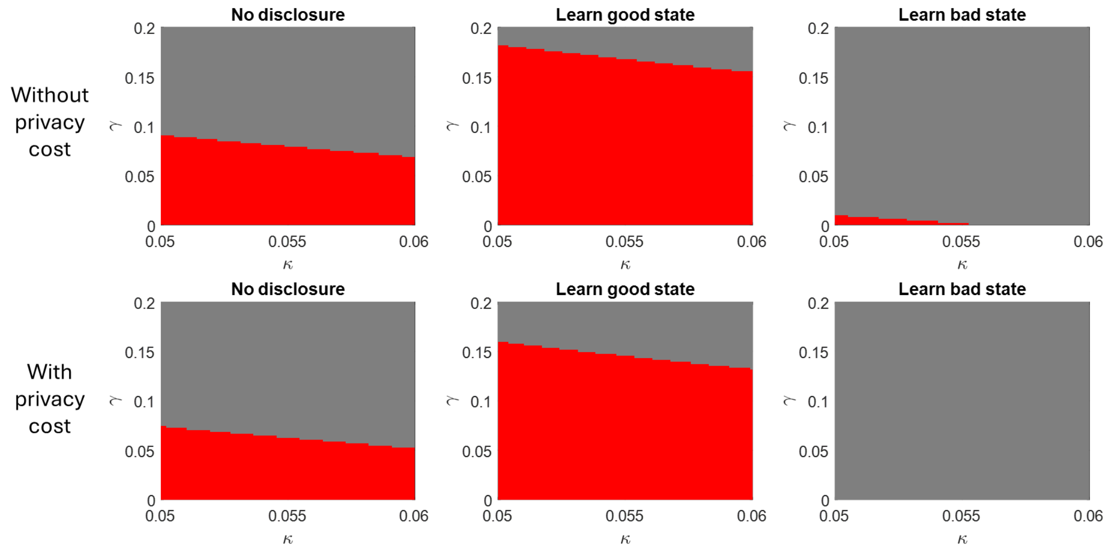
Figure 7: Optimal information disclosure in (κ, γ) space

To further explain the investment channel and how it interacts with the privacy cost, we plot sellers' investment decisions in the (κ, γ) space in Figure 8. First, due to the aggregate shock being persistent, sellers tend to invest when the previous state is good but not when the previous state is bad. Second, the privacy cost shrinks the parameter space where sellers choose to invest across all cases. This is because the cost reduces the total surplus in the DM, thereby reducing sellers' incentives to invest. Nevertheless, not disclosing ω_{t-1} may lead to even lower aggregate welfare, as shown by the second column of Figure 7.

It is worth noting that the benefit of disclosing ω_{t-1} cannot fully offset the effect of the privacy cost. To see this, first note that conditional on ω_{t-1} being disclosed, the privacy cost unambiguously reduces welfare. Now suppose non-disclosure is optimal when the privacy cost is absent. In this case, disclosure leads to lower welfare. Thus, even if accounting for the privacy cost shifts the optimal policy from non-disclosure to disclosure, it still unambiguously reduces welfare.

5.2 Privacy and the co-existence of CBDC and cash

In this section, we assume that a fraction $\eta \in (0, 1)$ of buyers incur a sufficiently large privacy cost when they use CBDC so that they only use cash in DM transactions. The rest of the



Note: red area = sellers invest to improve productivity; gray area = sellers do not invest.

Figure 8: Seller's investment decision in (κ, γ) space

buyers incur a privacy cost that is sufficiently small so that they prefer CBDC. We maintain all the other assumptions. In particular, we assume the preference shock that determines whether a buyer is active or not in the DM is independent across all buyers. This means the information regarding the previous aggregate state can be used to predict the preferences of not only cash buyers but also CBDC buyers.

We first check if the transfer policy discussed in Section 4.1 is still able to eliminate inferior equilibria when multiple equilibria exist. Recall that the policy takes advantage of the digital nature of CBDC and bases the transfer on CBDC holdings of the buyer, z', that the seller is matched with (see Proposition 4). When all buyers use CBDC, this policy eliminates the risk that buyers do not carry enough digital currency and, as a result, not enough purchases are made to justify sellers' investment. Consequently, all sellers opt to invest to improve their productivity in equilibrium.

Now, suppose some buyers prefer cash for privacy reasons. When a seller encounters such a buyer, the seller will not receive the transfer even if the buyer does not carry enough cash. Let $V^s(\delta, z^{c\prime})$ denote the expected surplus of a seller who encounters a cash buyer, where $z^{c\prime}$ is the buyer's cash holding (in real term). Similarly, define $V^s(\delta, z^{d\prime})$ to be the expected surplus of a seller who encounters a CBDC buyer who holds $z^{d\prime}$ units of CBDC. Recall that when a seller invest to improve her productivity, her $\delta = \delta_H$, otherwise her $\delta = \delta_L$. First, suppose there is no transfer policy. Then, the expected benefit of investing for a seller is given by the following

$$\eta[V^s(\delta_H, z^{c\prime}) - V^s(\delta_L, z^{c\prime}) - \kappa] + (1-\eta)[V^s(\delta_H, z^{d\prime}) - V^s(\delta_L, z^{d\prime}) - \kappa].$$

Depending on $z^{c\prime}$ and $z^{d\prime}$, a seller's investment decision, $\mathbb{1}_I$, is given by

$$\begin{cases} \mathbb{1}_I = 1, \text{ if } \eta[V^s(\delta_H, z^{c\prime}) - V^s(\delta_L, z^{c\prime}) - \kappa] + (1-\eta)[V^s(\delta_H, z^{d\prime}) - V^s(\delta_L, z^{d\prime}) - \kappa] > 0; \\ \mathbb{1}_I \in \{0,1\}, \text{ if } \eta[V^s(\delta_H, z^{c\prime}) - V^s(\delta_L, z^{c\prime}) - \kappa] + (1-\eta)[V^s(\delta_H, z^{d\prime}) - V^s(\delta_L, z^{d\prime}) - \kappa] = 0; \\ \mathbb{1}_I = 0, \text{ if } \eta[V^s(\delta_H, z^{c\prime}) - V^s(\delta_L, z^{c\prime}) - \kappa] + (1-\eta)[V^s(\delta_H, z^{d\prime}) - V^s(\delta_L, z^{d\prime}) - \kappa] < 0. \end{cases}$$

Now, suppose we introduce the transfer policy. Recall that we use $z'(1)$ to denote the amount of digital currency a buyer will choose to hold if she believe all sellers will invest to improve productivity. The government then offers a transfer $T = \kappa/\beta$ to sellers who have invested in the previous period if $z' < z'(1)$ and no transfer if otherwise. Then, the expected benefit of investing becomes

$$\begin{cases} \eta[V^s(\delta_H, z^{c\prime}) - V^s(\delta_L, z^{c\prime}) - \kappa] + (1-\eta)[V^s(\delta_H, z^{d\prime}) - V^s(\delta_L, z^{d\prime})], \text{ if } z^{d\prime} < z^{d\prime}(1); \\ \eta[V^s(\delta_H, z^{c\prime}) - V^s(\delta_L, z^{c\prime}) - \kappa] + (1-\eta)[V^s(\delta_H, z^{d\prime}) - V^s(\delta_L, z^{d\prime}) - \kappa], \text{ if } z^{d\prime} = z^{d\prime}(1). \end{cases}$$

The following proposition shows that under certain conditions, the transfer policy still ensures the most efficient outcome.

Proposition 8 *Suppose conditions in* Proposition 1 *are satisfied so that multiple equilibria exist. There exists $\bar{\eta}$ such that as long as $\eta < \bar{\eta}$, the transfer policy ensures that there is a unique equilibrium where all sellers invest in the technology.*

Proof: See Appendix A.

The intuition is as follows. The presence of cash buyers prevents the transfer policy from fully eliminating the investment risk, because a seller may encounter a buyer with a relatively small cash holding and not be compensated for her investment cost. However, as long as the likelihood of such encounters is low, the risk will be smaller than the benefit that the seller receives when she is matched with a CBDC buyer. Consequently, all sellers choose to invest in equilibrium.

Next, we study the optimality of disclosing ω_{t-1} if some buyers use cash. Recall from that the effect of information disclosure through the demand channel depends on whether $w_t(\omega)$ is concave or convex, where $w_t(\omega)$ is the sum of buyers' and sellers' utility in a period conditional on agents' expectation of ω. When some buyers use cash, $w_t(\omega)$ is given by

$$\begin{aligned} w_t(\omega) \equiv & -\xi\kappa + \omega\eta[\xi(u(x_H^c) - x_H^c/\delta_H - c) + (1-\xi)(u(x_L^c) - x_L^c/\delta_L - c)] \\ & + \omega(1-\eta)[\xi(\tilde{u}(x_H^d) - x_H^d/\delta_H) + (1-\xi)(\tilde{u}(x_L^d) - x_L^d/\delta_L)], \end{aligned}$$

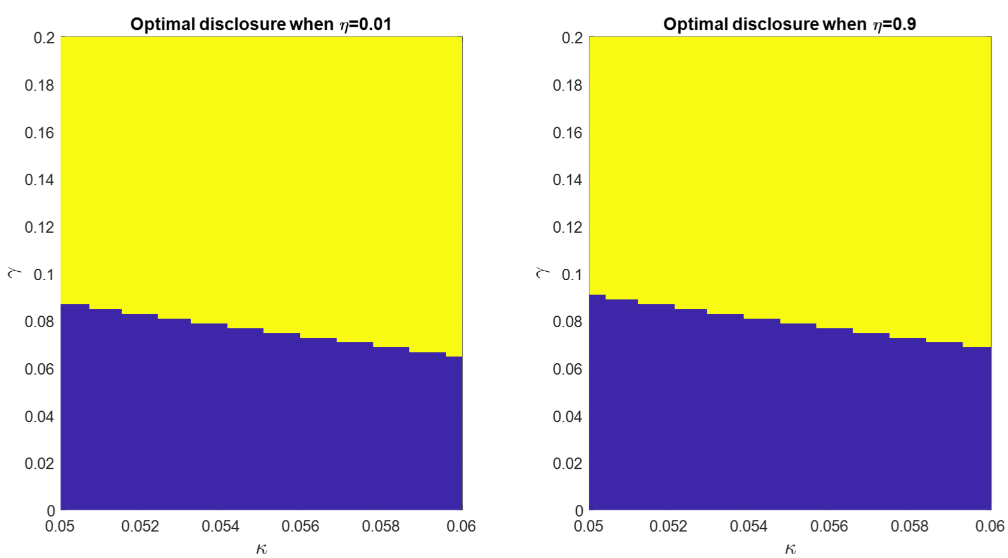
where x_H^c, x_L^c, x_H^d, and x_L^d denote buyers' DM consumption condition on the payment instrument they use and sellers' investment decisions. In what follows, assume that η is sufficiently small so that the transfer policy ensures there is always a unique equilibrium.

Recall from Section 5.1 that the concavity/convexity of $w_t(\omega)$ depends on the shape of buyers' DM utility function, $u(x)$. Although the presence of cash buyers reduces the proportion of CBDC users, it does not change how the privacy cost, $v(x)$, affects the optimality of information disclosure via the demand channel. As a result, the conditions necessary for the optimal policy to switch from no disclosure to disclosure remain the same.

Proposition 9 *Proposition 7 holds for all* $\eta \in [0, 1)$.

Proof: See Appendix A.

It is worth noting that as η increases, the effect of $v(x)$ on the optimality of information disclosure via the demand channel diminishes, as the privacy cost only affects CBDC users. Additionally, the presence of cash buyers influences the investment channel by altering sellers' incentives. Since cash buyers do not incur the privacy cost, they hold larger quantities of real balances compared to CBDC users. Consequently, as η increases, sellers are more likely to invest and improve their productivity when there is no information disclosure. This reduces the positive effect of disclosing the good state on investment and increases the possibility that disclosing the bad state will deter investment. Thus, the presence of cash buyers weakens the positive effects of information disclosure through both channels, ultimately reducing the value of CBDC transaction data. Figure 9 provides a numerical example. When η is larger (the right panel), the parameter space where information disclosure improves welfare (i.e, the yellow area) is smaller.



Note: yellow area = info disclosure improves welfare; blue area = info disclosure lowers welfare.

Figure 9: Optimal information disclosure in (κ, γ) space

6 Conclusion

In this paper, we develop a general equilibrium framework to explore the economic value of central bank digital currency (CBDC) transaction data. We assume that aggregate shocks are persistent, and therefore information regarding the previous aggregate state of the economy is useful for predicting future aggregate states. In a benchmark model, we assume that there is no CBDC, and that the aggregate state of the economy is not directly observable. We find that due to strategic complementarity, there may exist multiple equilibria. In addition, investments tend to inefficiently low.

When CBDC is introduced as a new payment instrument, targeted subsidies, enabled by CBDC, can eliminate multiple equilibria and increase investment. Information collected via CBDC also enables the government to observe the aggregate state of the economy. However, disclosing such information is not always beneficial, though it tends to improve welfare when inflation is relatively high. If agents incur a privacy cost when using CBDC, then under certain conditions, such a cost can increase the value of the information collected via CBDC, thereby partially offsetting the privacy cost.

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Appendix A Proofs

Proof of Lemma 1: First, note that for $\delta \in \{\delta_H, \delta_L\}$, $x(\delta)$ is strictly increasing in z' until $x(\delta) = x^*(\delta)$, where $u'(x^*(\delta)) = \frac{1}{\delta}$. Hence, $\frac{\theta[u'(x(\delta))-1/\delta]}{\theta/\delta+(1-\theta)u'(x(\delta))}$ is strictly decreasing in z' until $x(\delta) = x^*(\delta)$, in which case it is equal to 0. This means that the right-hand side of (3.3) is strictly decreasing in z' until z' is such that $x(\delta) = x^*(\delta)$ for all $\delta \in \{\delta_H, \delta_L\}$. Hence, there is a unique solution to (3.3). This also means that the optimal z' is increasing in ω. To prove that the optimal z' is increasing in ξ, we only need to show that

$$\frac{\theta[u'(x(\delta_H)) - 1/\delta_H]}{\theta/\delta_H + (1-\theta)u'(x(\delta_H))} > \frac{\theta[u'(x(\delta_L)) - 1/\delta_L]}{\theta/\delta_L + (1-\theta)u'(x(\delta_L))}.$$

To show this, consider the equation $\frac{\beta z'}{1+\gamma} = \theta\frac{x(\delta)}{\delta} + (1-\theta)u(x(\delta))$. We have $\frac{\mathrm{d}x}{\mathrm{d}\delta} = \frac{\theta x/\delta^2}{\theta/\delta+(1-\theta)u'(x)}$. Define $f(\delta) = \frac{u'(x(\delta))-1/\delta}{\theta/\delta+(1-\theta)u'(x(\delta))}$. Then

$$f'(\delta) \propto 1 + \frac{\frac{\theta x u''(x)}{u'(x)}}{\theta + (1-\theta)\delta u'(x)} \geq 1 + \frac{\theta x u''(x)}{u'(x)} \geq 1 - \theta > 0.$$

The first inequality is because $u'(x) \geq \frac{1}{\delta}$, and the second is because $-\frac{xu''(x)}{u'(x)} < 1$ by assumption (see Section 2). □

Proof of Lemma 2: To show $\frac{\partial G(z',\bar{\omega})}{\partial z'} > 0$, it suffices to show that $V^s(\delta_H, z') - V^s(\delta_L, z')$ is increasing in z'. Define $g(z') = V^s(\delta_H, z') - V^s(\delta_L, z')$. We have

$$g'(z') \propto \frac{u'(x(\delta_H)) - 1/\delta_H}{\theta/\delta_H + (1-\theta)u'(x(\delta_H))} - \frac{u'(x(\delta_L)) - 1/\delta_L}{\theta/\delta_L + (1-\theta)u'(x(\delta_L))}.$$

Following the proof of Lemma 1, we have $g'(z') > 0$. □

Proof of Proposition 1: Let $\tilde{z}_1$ and $\tilde{z}_2$ solve (3.3) when $\xi = 0$ and when $\xi = 1$, respectively. We know that $\tilde{z}_1 < \tilde{z}_2 \leq z^*$. Hence, $G(\tilde{z}_1, \bar{\omega}) < G(\tilde{z}_2, \bar{\omega})$ follows from Lemma 2. Define $\kappa_1 \equiv G(\tilde{z}_1, \bar{\omega})$ and $\kappa_2 \equiv G(\tilde{z}_2, \bar{\omega})$. Suppose $\kappa_1 < \kappa < \kappa_2$. Define $\hat{z}$ to be such that $G(\hat{z}, \bar{\omega}) = \kappa$. Then, from Lemma 1, we can conclude there exists a unique $\xi \in (0, 1)$ such that $\hat{z}$ solves (3.3). If $\kappa < \kappa_1$, then for all ξ, buyers' choice of z' is such that sellers will always invest in the technology. If $\kappa > \kappa_2$, then for all ξ, buyers' choice of z' is such that sellers will always not invest in the technology. Finally, if $\kappa = \kappa_1$ or if $\kappa = \kappa_2$, there are two equilibria with $\xi = 0$ and $\xi = 1$, respectively.

Regarding the second result, define $\bar{\omega}_2$ to be such that $G(z'(\bar{\omega}_2, 0), \bar{\omega}_2) = \kappa$, where $z'(\bar{\omega}_2, 0)$ solves (3.3) with $\xi = 0$ and $\bar{\omega} = \bar{\omega}_2$, provided that $G(z'(1, 0), 1) \geq \kappa$. Otherwise, let $\bar{\omega}_2 = 1$. Define $\bar{\omega}_1$ to be such that $G(z'(\bar{\omega}_1, 1), \bar{\omega}_1) = \kappa$, where $z'(\bar{\omega}_1, 1)$ solves (3.3) with $\xi = 1$ and $\bar{\omega} = \bar{\omega}_1$, provided that $G(z'(0, 1), 0) \leq \kappa$. Otherwise, let $\bar{\omega}_1 = 0$. If $\bar{\omega} > \bar{\omega}_2$, then for all ξ, buyers' choice of z' is such that sellers will always invest in the technology. Similarly, if $\bar{\omega} < \bar{\omega}_1$, then for all ξ, buyers' choice of z' is such that sellers will never invest. If $\bar{\omega}_1 < \bar{\omega} < \bar{\omega}_2$, define $\hat{z}$ to be such that $G(\hat{z}, \bar{\omega}) = \kappa$. Based on Lemma 1, there exists a unique $\xi \in (0,1)$ such that $\hat{z}$ solves (3.3). Finally, if $\bar{\omega} = \bar{\omega}_1$ or if $\bar{\omega} = \bar{\omega}_2$, there exist two equilibria with $\xi = 0$ and $\xi = 1$ if $G(z'(1,0),1) \geq \kappa$ and $G(z'(0,1),0) \leq \kappa$. If $G(z'(1,0),1) < \kappa$ and $G(z'(0,1),0) > \kappa$, then in such cases there also exists a equilibrium with $\xi \in (0,1)$. □

Proof of Proposition 2: Define $W(1)$, $W(\xi)$, and $W(0)$ to be the aggregate welfare when $\xi = 1$, $\xi \in (0,1)$, and $\xi = 0$, respectively. Recall that sellers invest in the DM production technology if

$$\kappa \leq (1-\theta)\{\bar{\omega}[u(x(\delta_H)) - x(\delta_H)/\delta_H - (u(x(\delta_L)) - x(\delta_L)/\delta_L)]\}.$$

Let $\xi \in (0,1)$. Substitute the inequality into (3.5) to get

$$\begin{aligned} W(1) \geq & \bar{\omega}[\theta(u(x(\delta_H)) - x(\delta_H)/\delta_H - c) + (1-\theta)(u(x(\delta_L)) - x(\delta_L)/\delta_L - c)] \\ > & \bar{\omega}[\xi\theta(u(x(\delta_H)) - x(\delta_H)/\delta_H - c) + (1-\xi\theta)(u(x(\delta_L)) - x(\delta_L)/\delta_L - c)] > W(\xi). \end{aligned}$$

The second inequality is because $u(x(\delta)) - x(\delta)/\delta$ is increasing in δ. The third inequality is because z' is increasing in ξ. Similarly, we have

$$\begin{aligned} W(\xi) = & \bar{\omega}[\xi\theta(u(x(\delta_H)) - x(\delta_H)/\delta_H - c) + (1-\xi\theta)(u(x(\delta_L)) - x(\delta_L)/\delta_L - c)] \\ > & \bar{\omega}(u(x(\delta_L)) - x(\delta_L)/\delta_L - c) > W(0). \end{aligned}$$

Again, the first inequality is because $u(x(\delta)) - x(\delta)/\delta$ is increasing in δ. The second inequality is because z' is increasing in ξ. □

Proof of Proposition 3: Recall that from Proposition 1 that if $\kappa > \kappa_2$, then for all for ξ, buyers' choice of z' is such that sellers will not invest in the technology. This means that

$$\kappa > (1-\theta)\{\bar{\omega}[u(x(\delta_H)) - x(\delta_H)/\delta_H - (u(x(\delta_L)) - x(\delta_L)/\delta_L)]\}.$$

However, aggregate welfare is higher with $\xi = 1$ as long as

$$\kappa < \bar{\omega}[u(x(\delta_H)) - x(\delta_H)/\delta_H - (u(x(\delta_L)) - x(\delta_L)/\delta_L)].$$

Hence, there exists κ such that $\xi = 1$ implies higher aggregate welfare. □

Proof of Proposition 4: First, note that under the proposed policy, sellers do not gain from not investing in the technology. If $z' < z'(1)$, sellers are fully compensated for the investment costs. If $z' = z'(1)$, then it is optimal for sellers to invest in the technology. Conditional on all sellers investing in the technology, all buyers will choose $z' = z'(1)$. Now, we consider potential collusion between buyers and sellers. For collusion to be profitable, buyers will have to carry less than $z'(1)$ in order for sellers to obtain the transfer later. However, there is no incentive for buyers to do so, because it means less consumption for buyers, and since buyers and sellers do not meet again, there is no incentive for sellers to compensate buyers. □

Proof of Lemma 3: For any given ω, let $\tilde{z}(\omega)$ solve (3.3) when $\xi = 1$. Define $\kappa_1 \equiv G(\tilde{z}(\bar{\omega}_B), \bar{\omega}_B)$, $\kappa_2 \equiv G(\tilde{z}(\bar{\omega}), \bar{\omega})$, and $\kappa_3 \equiv G(\tilde{z}(\bar{\omega}_G), \bar{\omega}_G)$. Then, it is clear that if $\kappa \leq \kappa_1$, all sellers invest to improve their productivity for all $\omega \in [\bar{\omega}_B, \bar{\omega}_G]$. If $\kappa > \kappa_3$, no seller invests to improve their productivity for any $\omega \in [\bar{\omega}_B, \bar{\omega}_G]$. If $\kappa_1 < \kappa \leq \kappa_2$, sellers invest to improve their productivity if $\omega = \bar{\omega}_G$ or if $\omega = \bar{\omega}$. If $\kappa_2 < \kappa \leq \kappa_3$, sellers invest if $\omega = \bar{\omega}_G$. $\square$

Proof of Lemma 4: With some abuse of notation, define $w(\omega)$ to be the per-period utility of buyers and sellers (except for the cost of investment, κ) for a given ξ. That is, $w(\omega) = \omega[u(x) - x/\delta]$. Recall that x is given by $\frac{1+\gamma}{\beta} - 1 = \frac{\omega\theta[u'(x)-1/\delta]}{\theta/\delta+(1-\theta)u'(x)}$. We have

$$\frac{\partial x}{\partial \omega} = -\frac{(\delta u'(x) - 1)\,(\delta(1-\theta)u'(x) + \theta)}{\delta\omega u''(x)},$$

$$\frac{\partial^2 x}{\partial \omega^2} = \frac{(\delta u'(x) - 1)\,(\delta(1-\theta)u'(x) + \theta)^2 \left(2\delta[u''(x)]^2 + u'''(x)\,(1 - \delta u'(x))\right)}{\delta^2\omega^2[u''(x)]^3}.$$

Hence,

$$w''(\omega) = \left[\delta[u''(x)]^2 \left(3\delta(1-\theta)u'(x) + 3\theta - 2\right) - u'''(x)\left(\delta u'(x) - 1\right)\left(\delta(1-\theta)u'(x) + \theta\right)\right] \times \left(\delta u'(x) - 1\right)^2 \left(\delta(1-\theta)u'(x) + \theta\right)/(\delta^2\omega^2[u''(x)]^3).$$

Then, the sign of $w''(\omega)$ is positive if and only if

$$\frac{[u''(x)]^2}{u'''(x)} < \frac{(u'(x) - 1/\delta)\,(\delta(1-\theta)u'(x) + \theta)}{3\delta(1-\theta)u'(x) + 3\theta - 2}. \tag{A.1}$$

Note that

$$\frac{(u'(x) - 1/\delta)\,(\delta(1-\theta)u'(x) + \theta)}{3\delta(1-\theta)u'(x) + 3\theta - 2} = \frac{u'(x) - 1/\delta}{3 - \frac{2}{\delta(1-\theta)u'(x)+\theta}} < \frac{u'(x) - 1/\delta}{3 - 2} = u'(x) - 1/\delta,$$

and

$$\frac{(u'(x) - 1/\delta)\,(\delta(1-\theta)u'(x) + \theta)}{3\delta(1-\theta)u'(x) + 3\theta - 2} = \frac{u'(x) - 1/\delta}{3 - \frac{2}{\delta(1-\theta)u'(x)+\theta}} > \frac{u'(x) - 1/\delta}{3}.$$

We can then conclude that

$$\begin{cases} \frac{[u''(x)]^2}{u'''(x)} > \frac{\left(u'(x)-1/\delta\right)\left(\delta(1-\theta)u'(x)+\theta\right)}{3\delta(1-\theta)u'(x)+3\theta-2}, \text{ if } \frac{[u''(x)]^2}{u'''(x)\left(u'(x)-1/\delta\right)} > 1; \\ \frac{[u''(x)]^2}{u'''(x)} < \frac{\left(u'(x)-1/\delta\right)\left(\delta(1-\theta)u'(x)+\theta\right)}{3\delta(1-\theta)u'(x)+3\theta-2}, \text{ if } \frac{[u''(x)]^2}{u'''(x)\left(u'(x)-1/\delta\right)} < 1/3. \end{cases}$$

Now, we derive the results with the assumption that $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$. In such case, (A.1) becomes

$$\sigma x^{-3\sigma-2}\left(\theta(\sigma+1)x^{2\sigma} + \delta(\theta(\sigma-2) - \sigma + 1)x^{\sigma} + \delta^2(-2\theta\sigma + \theta + 2\sigma - 1)\right) < 0.$$

Note that the quadratic expression in the bracket has roots given by

$$x^{\sigma} = \frac{\delta\left(2\theta - 1 + \sigma(1-\theta) \pm \sqrt{1 - \sigma(1-\theta)(9\theta\sigma - \sigma + 2)}\right)}{2\theta(\sigma+1)}.$$

Note that the derivatives of the roots with respect to σ is given by

$$\text{(Lager root)}\ \frac{2\delta\left((\theta-1)(9\theta-2)\sigma+\theta-2+(2-3\theta)\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}\right)}{\theta(\sigma+1)^2\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}}<0,$$

$$\text{(Smaller root)}\ -\frac{2\delta\left((\theta-1)(9\theta-2)\sigma+\theta-2-(2-3\theta)\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}\right)}{\theta(\sigma+1)^2\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}}>0.$$

The inequalities hold because if $\theta \geq 2/3$, then

$$(\theta-1)(9\theta-2)\sigma+\theta-2+(2-3\theta)\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}<0,$$
$$(\theta-1)(9\theta-2)\sigma+\theta-2-(2-3\theta)\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}<(\theta-1)(9\theta-2)\sigma+4\theta-4<0.$$

If $\theta < 2/3$, we have

$$(\theta-1)(9\theta-2)\sigma+\theta-2+(2-3\theta)\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}<(\theta-1)(9\theta-2)\sigma-2\theta<0,$$
$$(\theta-1)(9\theta-2)\sigma+\theta-2-(2-3\theta)\sqrt{1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)}<0.$$

Next, note that the smaller root becomes negative when $\sigma \to 0$ and the larger root converges to δ when $\sigma \to 0$. Since $x^\sigma \in [0,\delta]$, there are three scenarios. (a) If both roots are between 0 and δ, then $w(\omega)$ is convex when ω has intermediate values so that $x(\delta, z')^\sigma \in [0,\delta]$ and concave when ω is large or small. (b) If only one root is between 0 and δ, then $w(\omega)$ is concave when ω is small and convex when ω is large. (c) If σ is sufficiently large, then a root does not exist, in which case $w(\omega)$ is always concave. Now, define $f(\sigma) \equiv 1-\sigma(1-\theta)(9\theta\sigma-\sigma+2)$. For σ that is sufficiently large, there exist $\underline{\theta}$ and $\bar{\theta}$ such that $f(\sigma)<0$ if and only if $\theta \in (\underline{\theta}, \bar{\theta})$.

The above results show that it is not possible for $w(\omega)$ to be convex for all ω. However, we can check if $w(\omega)$ is convex for $x \in [x(\bar{\omega}_B), x(\bar{\omega}_G)]$. Note that the larger root is decreasing in σ, and it converges to δ when $\sigma \to 0$. Also,

$$x^\sigma = \frac{\alpha\delta[\beta\theta(\omega-1)+\beta+(\gamma+1)(\theta-1)]}{\theta[\beta(\omega-1)+\gamma+1]}<\delta.$$

This means that as long as σ is sufficiently small, then the smaller root is smaller than $[x(\bar{\omega}_B)]^\sigma$, while the larger root is larger than $[x(\bar{\omega}_G)]^\sigma$. This implies that $w''(\omega)>0$ for all $\omega \in [\bar{\omega}_B, \bar{\omega}_G]$. □

Proof of Proposition 5:

Finally, based on Proposition 1, we know that $\xi = 1$ for $\omega \in \{\bar{\omega}_B, \bar{\omega}_G, \bar{\omega}\}$ if κ is sufficient small, and $\xi = 0$ for $\omega \in \{\bar{\omega}_B, \bar{\omega}_G, \bar{\omega}\}$ if κ is sufficient large. In such cases, the expected per-period utility of buyers and sellers satisfies

$$\frac{\rho_{BG}w(\bar{\omega}_G)+\rho_{GB}w(\bar{\omega}_B)}{\rho_{GB}+\rho_{BG}} > w\left(\frac{\rho_{BG}\bar{\omega}_G+\rho_{GB}\bar{\omega}_B}{\rho_{GB}+\rho_{BG}}\right) = w(\bar{\omega})$$

if $w(\omega)$ is convex, and

$$\frac{\rho_{BG}w(\bar{\omega}_G) + \rho_{GB}w(\bar{\omega}_B)}{\rho_{GB} + \rho_{BG}} < w\left(\frac{\rho_{BG}\bar{\omega}_G + \rho_{GB}\bar{\omega}_B}{\rho_{GB} + \rho_{BG}}\right) = w(\bar{\omega})$$

if $w(\omega)$ is concave. That is, information increases welfare if $w(\omega)$ is convex and decreases welfare if $w(\omega)$ is concave.

Next, if $\kappa \in (\kappa_2, \kappa_3]$, the disclosure of ω_{t-1} increases investment when the previous aggregate state is good but otherwise has no effect on investment. In contrast, $\kappa \in (\kappa_1, \kappa_2]$, the disclosure of ω_{t-1} decreases investment when the previous aggregate state is bad but has no effect on investment if otherwise. Hence, when $\kappa \in (\kappa_2, \kappa_3]$ and $w_t(\omega)$ is convex or when $\kappa \in (\kappa_2, \kappa_3]$ and $w_t(\omega)$ is concave, both channels work in the same direction. However, if the demand channel operates in the opposite direction of the investment channel, which happens when $\kappa \in (\kappa_2, \kappa_3]$ and $w_t(\omega)$ is concave or when when $\kappa \in (\kappa_2, \kappa_3]$ and $w_t(\omega)$ is convex, then the overall effect is ambiguous. $\square$

Proof of Proposition 6: Recall that when $u(x) = \frac{x^{1-\sigma}}{1-\sigma}$, (A.1) becomes

$$\sigma x^{-3\sigma-2}\left(\theta(\sigma+1)x^{2\sigma} + \delta(\theta(\sigma-2) - \sigma + 1)x^{\sigma} + \delta^2(-2\theta\sigma + \theta + 2\sigma - 1)\right) < 0.$$

Let y_H and y_L be the larger and smaller root, respectively, of the quadratic expression in the bracket. Following the proof of Lemma 4, we know that there exists $\hat{\sigma}$ such that if $\sigma < \hat{\sigma}$, we have $y_L < 0 < y_H$. Now, let $\tilde{\kappa} \equiv V^s(\delta_H) - V^s(\delta_L)$ when $z = z^*$. Suppose $\kappa < \tilde{\kappa}$. If γ is sufficiently small, then for all $x \in \{x(\bar{\omega}_B), x(\bar{\omega}), x(\bar{\omega}_G)\}$, we have $x > y_H$ and $\xi = 1$. If $\kappa \geq \tilde{\kappa}$, then for all $x \in \{x(\bar{\omega}_B), x(\bar{\omega}), x(\bar{\omega}_G)\}$, we have $x > y_H$ and $\xi = 0$. Similarly, if γ is sufficiently large and $\kappa > 0$, then for all $x \in \{x(\bar{\omega}_B), x(\bar{\omega}), x(\bar{\omega}_G)\}$, we have $x < y_H$ and $\xi = 0$. In the former case, $w_t(\omega)$ is concave, so information disclosure hurts welfare, while in the latter case, $w_t(\omega)$ is convex, so information disclosure benefits welfare. $\square$

Proof of Proposition 7: Recall from Proposition 5 that in Cases (2) and (3), the disclosure of ω_{t-1} is optimal only if $w_t(\omega)$ is concave under the conditions in Lemma 4. It should be noted that when $w_t(\omega)$ is concave given the conditions in Lemma 4, it is concave for all $x \in [0, x^*]$, where x^* solves $u'(x) = 1/\delta$ (see the proof of Lemma 4).

First, recall that the sign of $w''(\omega)$ is positive if and only if

$$\frac{[u''(x)]^2}{u'''(x)} < \frac{(u'(x) - 1/\delta)\left(\delta(1-\theta)u'(x) + \theta\right)}{3\delta(1-\theta)u'(x) + 3\theta - 2}.$$

For the optimal policy to switch from no disclosure to disclosure, we need

$$\frac{[u''(x)]^2}{u'''(x)} > \frac{(u'(x) - 1/\delta)\left(\delta(1-\theta)u'(x) + \theta\right)}{3\delta(1-\theta)u'(x) + 3\theta - 2} \tag{A.2}$$

but

$$\frac{[u''(x)-v''(x)]^2}{u'''(x)-v'''(x)} < \frac{(u'(x)-v'(x)-1/\delta)\left(\delta(1-\theta)[u'(x)-v'(x)]+\theta\right)}{3\delta(1-\theta)[u'(x)-v'(x)]+3\theta-2}.$$

Note that the right-hand side of (A.2) is increasing in $u'(x)$. Hence, a necessary condition for both inequalities to hold is that

$$\frac{[u''(x)]^2}{u'''(x)} > \frac{[u''(x)-v''(x)]^2}{u'''(x)-v'''(x)}.$$

Now, suppose $u(x)=\frac{x^{1-\sigma}}{1-\sigma}$ and $v(x)=Ax^{\epsilon}$. Given x, define $F(A)$ to be the following

$$F(A)\equiv\frac{[u''(x)-v''(x)]^2}{u'''(x)-v'''(x)}=\frac{[-\sigma x^{-\sigma-1}-A(\epsilon-1)\epsilon x^{\epsilon-2}]^2}{-A(\epsilon-2)(\epsilon-1)\epsilon x^{\epsilon-3}-(-\sigma-1)\sigma x^{-\sigma-2}}.$$

We have

$$F'(A)=\frac{(\epsilon-1)\epsilon x^{\epsilon}\left[A(\epsilon-1)\epsilon x^{\sigma+\epsilon}+\sigma x\right]\left[\sigma(2\sigma+\epsilon)-A(\epsilon-2)(\epsilon-1)\epsilon x^{\sigma+\epsilon-1}\right]}{\left(A(\epsilon-2)(\epsilon-1)\epsilon x^{\sigma+\epsilon}-\sigma(\sigma+1)x\right)^2},$$

which is non-negative when $\epsilon\in[1,2]$. This implies that $\frac{[u''(x)]^2}{u'''(x)}\le\frac{[u''(x)-v''(x)]^2}{u'''(x)-v'''(x)}$. Next, suppose $\epsilon>2$. Consider $G(x)\equiv\sigma(2\sigma+\epsilon)-A(\epsilon-2)(\epsilon-1)\epsilon x^{\sigma+\epsilon-1}$. First, $G'(x)<0$. Second, to ensure an equilibrium exists, we need $H(x)\equiv\frac{-x\tilde{u}''(x)}{\tilde{u}'(x)}\le 1$ (see Lemma 1). Note that

$$H'(x)=\frac{A\epsilon(\sigma+\epsilon-1)^2x^{\sigma+\epsilon}}{\left(x-A\epsilon x^{\sigma+\epsilon}\right)^2}>0.$$

If $H(x)=1$, then $\hat{x}=\left(\frac{A\epsilon^2}{1-\sigma}\right)^{\frac{1}{1-\sigma-\epsilon}}$, and

$$G(\hat{x})=\frac{(\sigma+\epsilon-1)[(2\sigma-1)\epsilon+2]}{\epsilon}.$$

Then, if $\sigma>0.5$, $G(x)>0$ and therefore $F'(A)>0$, which means $\frac{[u''(x)]^2}{u'''(x)}<\frac{[u''(x)-v''(x)]^2}{u'''(x)-v'''(x)}$ when $\epsilon>2$. Note that $\sigma>0.5$ is always satisfied as long as $w_t(\omega)$ is concave under the conditions in Lemma 4. In conclusion, a necessary condition for the policy to switch from no disclosure to disclosure is $\epsilon<1$. $\square$

Proof of Proposition 8: First, note that $z^{c\prime}\in[z^{c\prime}(0),z^{c\prime}(1)]$, where $z^{c\prime}(0)$ is a cash buyer's money holding when she expects no seller to invest in the technology, while $z^{c\prime}(0)$ is her money holding when she expects all seller to invest in the technology. Similarly, $z^{d\prime}\in[z^{d\prime}(0),z^{d\prime}(1)]$. Furthermore, we know from Lemma 2 that $[V^s(\delta_H,z^{c\prime})-V^s(\delta_L,z^{c\prime})]$ is increasing in $z^{c\prime}$, and $[V^s(\delta_H,z^{d\prime})-V^s(\delta_L,z^{d\prime})]$ is increasing in $z^{d\prime}$. Next, note that $V^s(\delta_H,z^{d\prime})-V^s(\delta_L,z^{d\prime})>0$ for all $z^{d\prime}\in[z^{d\prime}(0),z^{d\prime}(1)]$. Now, define $\bar{\eta}$ to be such that

$$\bar{\eta}[V^s(\delta_H,z^{c\prime}(0))-V^s(\delta_L,z^{c\prime}(0))-\kappa]=(1-\bar{\eta})[V^s(\delta_H,z^{d\prime}(0))-V^s(\delta_L,z^{d\prime}(0))]=0.$$

It is straightforward to see that so long as $\eta<\bar{\eta}$, sellers will always find it optimal to invest in the technology regardless of the values of $z^{c\prime}$ and $z^{d\prime}$. $\square$

Proof of Proposition 9: Using the proof of Proposition 4, it is straightforward to derive the following result regarding $w(\omega)$:

$$
\begin{aligned}
&w''(\omega) = \\
&\quad \eta \left[\delta[u''(x)]^2 \left[3\delta(1-\theta)u'(x) + 3\theta - 2\right] - u'''(x) \left[\delta u'(x) - 1\right] \left[\delta(1-\theta)u'(x) + \theta\right]\right] \\
&\quad \times \left[\delta u'(x) - 1\right]^2 \left[\delta(1-\theta)u'(x) + \theta\right] / (\delta^2\omega^2[u''(x)]^3) \\
&\quad +(1-\eta) \left[\delta[\tilde{u}''(x)]^2 \left[3\delta(1-\theta)\tilde{u}'(x) + 3\theta - 2\right] - \tilde{u}'''(x) \left[\delta\tilde{u}'(x) - 1\right] \left[\delta(1-\theta)\tilde{u}'(x) + \theta\right]\right] \\
&\quad \times \left[\delta\tilde{u}'(x) - 1\right]^2 \left[\delta(1-\theta)\tilde{u}'(x) + \theta\right] / (\delta^2\omega^2[\tilde{u}''(x)]^3).
\end{aligned}
$$

It is straightforward to see that similar to Proposition 7, for the policy to switch from no disclosure to disclosure, we need

$$\frac{[u''(x)]^2}{u'''(x)} > \frac{(u'(x) - 1/\delta)\,(\delta(1-\theta)u'(x) + \theta)}{3\delta(1-\theta)u'(x) + 3\theta - 2}$$

but

$$\frac{[u''(x) - v''(x)]^2}{u'''(x) - v'''(x)} < \frac{(u'(x) - v'(x) - 1/\delta)\,(\delta(1-\theta)[u'(x) - v'(x)] + \theta)}{3\delta(1-\theta)[u'(x) - v'(x)] + 3\theta - 2}.$$

Hence, the necessary condition remains unchanged. $\square$