The Cournot-Awareness Model: Platform Recommendations Within Product Pages*

Armando J. Garcia Pires
Centre for Applied Research at NHH (SNF)

Mohammed Mardan
Norwegian School of Economics (NHH)
CESifo and NoCeT

Mark Tremblay
University of Nevada, Las Vegas
CESifo

Abstract

We develop a simple extension to the standard Cournot model, which we dub the Cournot-Awareness Model, that allows for competition when some consumers may not be aware of all competitors in the market. This is naturally important when market-place platforms like Amazon, Airbnb, Bookings, eBay, etc. are able to impact consumer product awareness in their design. Applying our model to recommendations within a product's page, we find that greater awareness increases competition between firms and that the optimal amount of consumer awareness is ambiguous for the platform. This implies that, depending on consumer awareness priors, the platform may or may not use within page recommendations to increase consumer awareness. We also consider several extensions. First, if the platform sells within page recommendations to sellers as ad slots, then an ad market develops where sellers are willing to pay for ads on competitors pages as well as to block ads on their own pages. Second, by considering the impact of the ads on seller entry by small sellers, we find that platform has an incentive use within page recommendations that promote the small seller to increase competition within the market. Third, we find that the presence of multi-product sellers promotes within page recommendations. Lastly, under a hybrid platform, the platform always promotes its own product and promotes rival products only when the benefit to consumers is sufficiently large.

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1 Introduction

Recommendation systems within online retail are vast, with marketplace recommendations dictating product orderings, displays, and advertisements pre- and post-search. Thus, it is not surprising that the literature on recommendations systems is vast and varies depending on the type of platform, recommendation system, or policy concern under consideration. One area that has largely gone unstudied in the recommendation literature is how platforms often display additional options within a product page for which a consumer arrives.

To better understand a marketplace's use of within page recommendations that include competing products, we develop a model that builds on the well-established Cournot framework, but with several unique adaptations. First, consumers may only observe one of the competing products in the absence of within page recommendations. Second, we allow consumers to be risk averse so that recommendations may also generate an informational gain. We dub this model the Cournot-Awareness Model which generalizes the standard Cournot model in these two respects. The primary goal of the model is to understand how the platform uses within page recommendations to manipulate cross product awareness by consumers which in turn impacts the underlying level of competition and market equilibrium.

In terms of the Cournot-Awareness subgame equilibrium, we find that, in the limit, a lack of consumer awareness approaches the monopoly equilibrium. This implies that, as the platform increases within product recommendations, (i) competition increases, leading to higher quantities and lower prices, and (ii) consumer uncertainty decreases, expanding consumer demand. Altogether, the equilibrium quantity effect from greater recommendations is positive but the price effect is ambiguous.

These results make the platform's recommendation problem all the more interesting as the platform's fee depends on the retail price (as is standard for marketplace platforms) so that both the equilibrium price and quantity impact platform profit. Indeed, we find that both with and without consumer uncertainty, the platform's optimal recommendation system is non-trivial in that it makes recommendations with some probability — that is, don't recommend or always recommend occur only in some cases. The platform's optimal recommendation system conflicts with that which is best for consumers (who always prefer more recommendations).

The Cournot-Awareness Model allows us to consider a variety of extensions that are relevant for marketplaces using within page recommendations. First, we show that the sellers have an incentive to pay an advertising fee so that their product is recommended on their rival's product page; in addition, sellers also have an incentive to block cross-product advertisements on their own page. Notably, because the platform's within page recommendation system is probabilistic, it may prefer to control this system on its own and not develop it into an advertising market.

Second, we find that a platform will use its within page recommendations to ensure the entry of smaller seller at the expense of large sellers. This reemphasizes that the platform has an incentive to use recommendations to promote seller competition. Third, we consider a the impact of a multi-product seller and find that when this diminishes competition, recommendations now only have a consumer awareness effect (not a competitive one) so that more recommendations occur. Lastly, we consider the hybrid platform case and find that prices are higher under a hybrid platform relative to the base model, holding recommendations fixed, and that the hybrid platform will bias its recommendations in favor of its own product, which is confirmed by Chen and Tsai (2023) for Amazon, and will only recommend a rivals product on it's product page if the benefit to consumers from cross-product awareness is sufficiently large; a result inline with much of the literature that hybrid platforms distort markets in their favor at the expense of third-party sellers and consumers.

2 Literature Review

There is a vast literature on upfront recommendation systems (not within page) that is both empirical and theoretical. On the theory side, Hagiu and Jullien (2011) reveal that platform recommendations systems can impact strategic variables for sellers; naturally, this results in the platform biasing search to promote seller behavior that favors the platform. De Corniere and Taylor (2019) consider alternative tradeoffs when consumers may be harmed by recommendation bias while others have focused on monetizing recommendations (through a buybox or advertisements), platform self-preferencing, and/or directing consumption toward one particular seller (Bar-Isaac and Shelegia (2022), Bourreau and Gaudin (2022), Teh and Wright (2022), Calvano et al. (2023), Ciotti and Madio (2023), Zhou and Zou (2023), and Aguiar et al. (2024) to name a few). On the empirical side, Zhou et al. (2024) empirically show that more precision is not always better while Donnelly et al. (2024) reveal that some personalization is better for all agents relative to a "best-selling" recommendation system; a finding that is confirmed by both Dinerstein et al. (2018) and Huang and Xie (2023) who also consider this tradeoff.[1]

Our work also contributes to the literature on a platform's use of advertising. For example, Long and Liu (2024) show how a platform can manipulate competition within the marketplace through sponsored search. In a different vain, Yang et al. (2024) empirically show that advertising slots as recommendations can improve quality sorting in online retail. Furthermore, Aridor et al. (2022) provide empirical evidence that recommendations reduce uncertainty about goods consumers are most uncertain about and induce information acquisition. These findings motivate our assumption that within page recommendations may reduce consumer uncertainty about consumption and therefore expand demand.

While not directly considering within page recommendations, there are recommendation papers similar to ours. Arguably the closest are Li et al. (2018) and Hagiu and Wright (2024) in that each sets up a simple model where consumers make purchases from one of two sellers (as in our model). However, Li et al. (2018) defines recommendations using a scoring method which is difficult to map into this within page recommendation setup that we have

[1] We also see several papers, Foerderer et al. (2018), Wen and Zhu (2019), He et al. (2020), Lam (2023),Farronato et al. (2023) and Waldfogel (2024), that empirically consider the effects of platform self-preferencing.

in mind or consider the impact of recommendations as ads. In Hagiu and Wright (2024), the nature of competition differs (we also differ with Li et al. (2018) in this respect), as due some underlying elements of recommendation, but most importantly they assume that discoverability is symmetric across sellers. We allow for asymmetries in recommendations and find that if sellers differ, then the platform will bias recommendations in favor of the smaller seller to ensure entry.

3 Model

The goal of our model is to develop a simple microfounded environment where consumers derive utility from a product that they are unsure about and firms offer their product through webpages hosted by a platform. Thus by observing a second product page, consumer uncertainty decreases and competition arises between firms. As a starting point, we microfound consumer demand with uncertainty by supposing that consumer v derives the following utility from a product:

$$U(v) = v + \psi - P,$$

where v is the certain stand alone utility earned from the product which also denotes the consumer's type, P is the price, ψ is a random variable that captures the uncertainty over the product's stand alone value. We assume ψ is normally distributed with mean zero, i.e., $\psi \sim \mathcal{N}(0, \sigma^2)$. The assumption of a normal distribution allows us to represent expected utility by the mean-variance approach, i.e.,[2]

$$E(U) = \mathbb{E}(U) - \frac{\gamma}{2} Var(U), \tag{1}$$

[2]We are not the first to use such an approach when consumers dislike uncertainty (Karni and Schmeidler (1991), Balvers and Szerb (1996), Gul and Pesendorfer (2014), Heyes and Martin (2016)).

where $\mathbb{E}(U) = v - P$ and $Var(U) = \sigma^2$. Thus, a consumer's expected utility is given by

$$E(U(v)) = v - P - \frac{\gamma}{2}\sigma^2.$$

Importantly, this implies that higher variance reduces a consumer's willingness to pay for the product.

Consumers differ in their stand alone values and they purchase the product if it generates non-negative utility. Suppose that the stand alone value is distributed across a mass of consumers so that $v \sim U[a-b, a]$ with $a > b > 0$. We assume that consumers purchase the product if their expected utility is non-negative, $E(U(v)) \geq 0$, which implies that the last consumer to purchase the product is given by $E(U(v^*)) = 0$. Consumers being distributed uniformly between a and $a - b$ then implies that $Q = \frac{1}{a-(a-b)} \cdot \int_{v^*}^{a} dv = \frac{1}{b} \cdot \left[a - P - \frac{\gamma}{2}\sigma^2\right]$. This generates an inverse demand curve that is given by:

$$P = a - \frac{\gamma}{2}\sigma^2 - bQ = A(\sigma^2) - bQ,$$

where $A(\sigma^2) = a - \frac{\gamma}{2}\sigma^2$ to simplify notation.

On the product production side, suppose there are two producers (Firm 1 and Firm 2) whose products are only available through the platform. To incorporate market power at the product level, we generalize the Cournot model to allow for consumers to be aware of only one or both products so that "Cournot-Awareness" competition occurs. In practice, consumers observe a variety of products and arrive on product pages in different ways depending on how they search. For example, if a consumer is interested in an exercise bike, they may go to the platform and type "exercise bike" into platform's search bar; instead, they may search for "exercise bike" on a search engine, web browser, AI tool, or social media and be directed to a specific product page within the platform without seeing any other products on the platform. Thus, we assume that consumers are exogenously divided into three masses: a mass ϕ_1 (ϕ_2) only observe Firm 1's (2's) product and a mass $1 - \phi_1 - \phi_2$ observe both so that $\phi_1, \phi_2, \phi_1 + \phi_2 \in [0, 1]$.

We assume that the firms are unable to observe the type of consumer that they are dealing with; however, the platform can distinguish between consumer types and use recommendations on product pages to increase the amount of consumers that see both products. That is, the platform chooses $\lambda_i \in [0, 1]$ so that a mass $\lambda_i \phi_i$ only observe Firm i's product and a mass $1 - \lambda_1\phi_1 - \lambda_2\phi_2$ observe both. In other words, $\lambda_1 = 0$ captures the case where the platform makes Firm 2's product visible on Firm 1's product page for the consumers that only visit Firm 1's product page, $\lambda_1 = 1$ captures the case where the platform does not make Firm 2's product visible on Firm 1's product page for any consumer that only visit Firm 1's product page, and $\lambda_1 \in (0, 1)$ captures the case where the platform makes Firm 2's product visible on Firm 1's product page for a fraction of consumers that only visit Firm 1's product page.

Given that the firms are unaware of a consumer's type, we have that Firm i chooses the single output q_i which is the sum of Firm i's output sold to exclusive consumers, q_i^E, and the output sold to consumers that consider both competing firms, q_i^C, so that the endogenous q_i is broken down by $q_i = q_i^E + q_i^C$ which are each determined by market clearing conditions. We also assume that the variance in ψ is decreasing in the number of products that the consumer observes with σ_L^2 if they observe both products and $\sigma_H^2 > \sigma_L^2$ if they only observe one product. Altogether, this implies that demands are given by:

$$\begin{aligned} q_i^E &= [A(\sigma_H^2) - P] \cdot \frac{\lambda_i \phi_i}{b} \text{ for } i = 1, 2, \\ q_1^C + q_2^C &= [A(\sigma_L^2) - P] \cdot \frac{1 - \lambda_1\phi_1 - \lambda_2\phi_2}{b}. \end{aligned} \tag{2}$$

This implies that if $\lambda_1\phi_1 = \lambda_2\phi_2 = 0$, then we have a standard Cournot model as all consumers observe both products; alternatively, as $\lambda_1\phi_1 + \lambda_2\phi_2$ approaches 1, firms become individual monopolists with limited contact between the two products.

Firms are symmetric in their marginal costs, c, and face the same ad valorem fee charged by the platform, f. Thus, Firm i maximizes profit with respect q_i where profit is given by

$$\pi_i = [(1-f)P - c] \cdot q_i. \tag{3}$$

The platform takes its fee as given when selecting its within product recommendation system.[3] Thus, the platform maximizes profit from the two products with respect to λ_1 and λ_2 where profit is given by

$$\Pi = f \cdot PQ, \tag{4}$$

where $Q = q_1 + q_2$.

4 Equilibrium

We solve the game backwards by first considering the problem of the firms, taking λ_1 and λ_2 as given, and then we consider the platform's problem. Hence, our solution concept is the Subgame Perfect Nash Equilibrium.

4.1 The Product Subgame Equilibrium

Given Equation (2) and the fact that $q_i = q_i^E + q_i^C$, we have that

$$q_i = [A(\sigma_L^2) - P] \cdot \frac{1 - \lambda_j \phi_j}{b} - \Delta A \cdot \frac{\lambda_i \phi_i}{b} - q_j^C,$$

where $\Delta A = A(\sigma_L^2) - A(\sigma_H^2) \geq 0$ captures the demand shift from the reduction in consumer uncertainty upon observing both products. This, along with Equation (2), generate three inverse demand equations that allow us to determine firm profit as a function of endogenous variables and the market clearing levels of q_1^C and q_2^C. More specifically, we have the three

[3]In practice, marketplace platforms set fees at an aggregated level (Tremblay (2021)); while recommendations within product pages are personalized (Chen and Tsai (2023)).

inverse demand equations given by:

$$
\begin{aligned}
P &= A(\sigma_L^2) - \frac{b}{1-\lambda_j\phi_j}(q_i + q_j^C) - \frac{\lambda_i\phi_i}{1-\lambda_j\phi_j}\Delta A \text{ for } i = 1,2 \text{ and } j \neq i, \\
P &= A(\sigma_L^2) - \frac{b}{1-\lambda_1\phi_1-\lambda_2\phi_2}(q_1^C + q_2^C).
\end{aligned} \tag{5}
$$

Solving the product page Cournot-Awareness Game implies the following result:

Lemma 1. *The equilibrium quantities and price are given by*

$$
\begin{aligned}
q_i^* &= \frac{1-\lambda_j\phi_j}{b(3-\lambda_1\phi_1-\lambda_2\phi_2)} \cdot \left[A(\sigma_H^2) - C + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)\right], \\
Q^* &= q_1^* + q_2^* = \frac{(2-\lambda_1\phi_1-\lambda_2\phi_2)}{b(3-\lambda_1\phi_1-\lambda_2\phi_2)} \cdot \left[A(\sigma_H^2) - C + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)\right], \\
P^* &= \frac{A(\sigma_H^2) + (2-\lambda_1\phi_1-\lambda_2\phi_2)C + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)}{3-\lambda_1\phi_1-\lambda_2\phi_2},
\end{aligned}
$$

where $C = \frac{c}{1-f}$.

The solution of the Cournot-Awareness Game is a generalization of the standard Cournot solution which occurs when $\lambda_1\phi_1 = \lambda_2\phi_2 = 0$ and $\Delta A = 0$. Looking at each new feature in turn, we first see that individual quantities and the market price are increasing as more consumers are only aware of each firm's product; that is, if $\Delta A = 0$, then $\frac{dq_i^*}{d\lambda_i\phi_i} > 0$ and $\frac{dP^*}{d\lambda_i\phi_i} > 0$. Second, we see that a reduced variance from learning about additional products shifts demand outward and therefore increases quantities and price: $\frac{dq_i^*}{d\Delta A}, \frac{dP^*}{d\Delta A} > 0$.

We can also determine how the platform recommendations impact the market by noting that the comparative statics reveal that the platform may face a tradeoff when selecting recommendations since the equilibrium price is ambiguous with respect to λ_1 and λ_2:

Corollary 1. *The equilibrium quantity is decreasing in the* λ_i *while the equilibrium price is ambiguous in the* λ_i*. More specifically,* $\frac{dQ^*}{d\lambda_i} < 0$ *always holds, but* $\frac{dP^*}{d\lambda_i} > 0$ *if and only if* $A(\sigma_H^2) - C > 2\Delta A$.

Note that both of these effects make sense since a decrease in λ_i generates two effects: (1)

firm competition increases and (2) demand shifts outward through less consumer uncertainty. Both effects expand quantity but the first effect reduces the price while the second effect increases the price generating the ambiguity of the price effect.

From a consumers perspective, more recommendations is always beneficial as prices decrease and sales expand. Note that we consider the impact of recommendations on sellers in the next section where we develop an extension for within page recommendations as ads. Before that, consider the platform's problem.

4.2 The Platform Subgame Equilibrium

We now turn to the more interesting subgame where the platform considers how it's product page recommendations can increase it's profit by promoting greater competition through more recommendations ($\lambda_1, \lambda_2 \to 0$) or by limiting consumer awareness of other substitutes ($\lambda_1, \lambda_2 \to 1$). The platform's profit depends on both price and quantity, since they use an ad valorem fee (as is the case in practice), and the comparative statics in Corollary 1 then imply that a variety of recommendation systems are possible in equilibrium. To refine this result more precisely, first consider the case where product demand is unaffected by consumers learning so that $\Delta A = 0$. In other words, there is no uncertainty about the product's benefit, just consumers are potentially unaware of the availability of a competing product. In this case, the only use of λ_1 and λ_2 are for the platform to influence the amount of competition within the marketplace. In this case, we see that recommendations occur but may be limited:

Proposition 1. *If $\Delta A = 0$, then the platform prefers competition between product sellers when market surplus is not too large so that they always make recommendations: $\lambda_1^*, \lambda_2^* = 0$ if $A(\sigma_H^2) < 4C$; instead, the platform prefers to limit recommendations when surplus is large: $\lambda_1^*, \lambda_2^* \in (0, 1)$ if $A(\sigma_H^2) > 4C$.*

Thus, we find that, even without informational benefits from consumer cross-product awareness, the platform may have an incentive to use within page recommendations to manipulate competition between products to favor their own profits (depending on the surplus generated from the product market). But, importantly, we see that the platform often selects partial recommendations where a recommendation occurs with some probability. In other words, we see that the platform's recommendation system need not be trivial.[4]

It is also worth noting that, for the cases where $\lambda_1^*, \lambda_2^* \in (0, 1)$, we have a continuum of equilibria. That is, the platform is indifferent across the λ_1^*, λ_2^* so long as they are such that $\lambda_1^*\phi_1 + \lambda_2^*\phi_2 = \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$. This is since $\lambda_1\phi_1$ and $\lambda_2\phi_2$ are not independent in the equilibrium price and quantity found in Lemma 1. Indeed, so long as both firms earn sufficient profits to enter, the platform need not distinguish between the two in recommendations. However, as we show in Section 5.2 the platform will distort recommendations in favor of a smaller seller to promote entry.

Unfortunately, solving the for optimal λ_i^* when $\Delta A > 0$ does not offer explicit results. Luckily, as shown in Proposition 1, we can write the equilibrium price and market quantity as a function of $\Lambda = \lambda_1\phi_1 + \lambda_2\phi_2$. This captures the amount of consumer cross-page awareness, where small (large) Λ coincides with greater (lesser) consumer cross-page awareness. Tying this to the platform's endogenous use of recommendations (the λ_1 and λ_2), a preference for smaller Λ corresponds to smaller λ_1 and λ_2 or more recommendations. Considering this from the platform's perspective, we see that the platform's optimal recommendation system is non-trivial:

Proposition 2. *The optimal amount of consumer cross-page awareness for the platform is hill-shaped in ΔA. That is, $\frac{d\Lambda^*}{d\Delta A} > 0$ if and only if $2\Delta A < A(\sigma_H^2) - C$.*

Mapping this to the recommendation story, holding ϕ_1 and ϕ_2 fixed, as the benefit to consumers from greater product information increases (ΔA increases from zero), the platform initially prefers less cross-page awareness (Λ^* increases in ΔA), corresponding to less recommendations, until the threshold is reached in which case more cross-page awareness is

[4]By trivial, we mean that the platform either always recommends ($\lambda_1^* = \lambda_2^* = 0$) or never recommends ($\lambda_1^* = \lambda_2^* = 1$).

preferred (Λ^* decreases in ΔA). This reveals that (i) the platform's recommendation system is non-trivial, which aligns with our results from Proposition 1, and that (ii) there is a disconnect with consumers who always prefer recommendations.

Note that greater ΔA always increases the platform's profit. It is just that it distorts this non-trivial recommendation system where an interior solution is often preferred by the platform (i.e., the platform often prefers to recommend with some probability that is non zero and non one.

5 Extensions

5.1 Within Page Recommendations as Ads

While not directly in charge of the within page recommendation system controlled by the platform, sellers may be able to impact recommendations through side payments or a more formal advertising market setup by the platform. As we showed in Propositions 1 and 2, the platform may not want to always display a within page recommendation; however, we see that sellers always have an incentive to advertise on their rivals page:

Proposition 3. *For all $\Delta A \geq 0$, seller i prefers to have its product recommended on seller j's page:* $\frac{d\pi_i}{d\lambda_j} < 0$.

This result is not surprising as more recommendations on seller j's page increases demand for product i. Notably, we can also show that each seller is willing to pay to block advertisements on their own page. This could be done explicitly through the platform (if the platform allows for blocking ads on pages) or by becoming a multi-seller for the sole purpose of blocking rival recommendations (we consider a multi-product seller more seriously in the next subsection):

Proposition 4. *If the benefit to consumers from cross-product information is sufficiently small ($2\Delta A < A(\sigma_H^2) - C$), then seller i does not want recommendations on its page:* $\frac{d\pi_i}{d\lambda_i} > 0$

if and only if $2\Delta A < A(\sigma_H^2) - C$.

Interestingly, we see that both the threshold and direction of recommendations in Proposition 4 aligns with the results in Proposition 2 with less (more) recommendations being optimal when ΔA is small (large) suggesting that the platform and the sellers may be aligned in an advertising market. However, we also see from Proposition 1 that the platform's recommendations can come in the form of probabilities while Propositions 3 and 4 reveal that sellers recommendation decisions are discrete. Combined, this suggests that the platform may prefer to control a within page recommendation system on its own instead of developing a recommendation system as an advertising market.

5.2 A Recommendation System That Promotes Seller Entry

As we showed in Proposition 1, the equilibrium price and quantity found in Lemma 1 reveal that the platform cares solely about the level of cross-product awareness in the form or $\lambda_1\phi_1 + \lambda_2\phi_2$, opposed to the specific levels of λ_1 and λ_2. This story however changes when each seller incurs a nonzero fixed cost to enter the market. With a fixed cost of entry, a smaller seller, say Firm 2 with a low ϕ_2, will not earn enough revenue for entry to be worthwhile without the help of the platforms recommendation system promoting its product (through a small λ_1 which increases visability of Firm 2's product on the page of Product 1). In this setting, the platform may want to use its recommendation system to deter entry of the smaller seller. To see if this is the case, note that the equilibrium when entry is deterred is given by the following:

Lemma 2. *If the entry of Firm 2 is foreclosed, then demand, the equilibrium quantity, and* *the equilibrium price are given by*

$$\begin{aligned} q_1 &= \frac{1-\phi_2}{b}[A(\sigma_H^2) - P], \\ q_1^F = Q^F &= \frac{1-\phi_2}{2b}[A(\sigma_H^2) - C], \\ P^F &= \frac{A(\sigma_H^2) + C}{2}, \end{aligned}$$

where the $1-\phi_2$ *captures the mass of consumers that are aware of Product 1.*

Proposition 1 reveals that when $\Delta A = 0$ (which makes foreclosure more attractive than when $\Delta A > 0$), we see that recommendations are the least desirable for the platform when $A(\sigma_H^2) > 4C$, and in this case we have that the platform selects $\lambda_1^*\phi_1 + \lambda_2^*\phi_2 = \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$. Thus, to see if the platform desires foreclosure of a small seller, we compare this duopoly case to the case of foreclosure which reveals that the platform will not use it's recommendation system to foreclose small sellers:

Proposition 5. *If Firm 2 is a small seller (*ϕ_2 *close to zero), then the platform will use its recommendation system to promote Firm 2 at the expense of Firm 1 by setting* $\lambda_2^F = 1$ *and minimizing* λ_1*.*[5]

Thus, we see that the platform has an incentive to use within page recommendations to promotes smaller sellers at the expense of larger ones.

5.3 A Multi-Product Seller

In practice, there are many similar products sold on marketplace platforms (certainly more than two). However, for simplicity (and because the intuition here extends), we continue to assume that two products are available but that they are offered by a single seller. Indeed, many sellers on marketplace platforms are multi-sellers and this extension offers some nice

[5] In particular, if $A(\sigma_H^2) < 4C$ so that $\lambda_1^*\phi_1 + \lambda_2^*\phi_2 = \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$, then $\lambda_2^F = 1$ and $\lambda_1^F = \frac{1}{\phi_1}\left[\frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C} - \phi_2\right]$.

insights into how minimizes the competitive effect impacts optimal recommendations for the platform and the seller. In this case, the market equilibrium is given by

Lemma 3. *The equilibrium quantity and price under a multi-product seller are given by*

$$\begin{aligned} Q^M &= \frac{A(\sigma_H^2) - C + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)}{2b}, \\ P^M &= \frac{A(\sigma_H^2) + C + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)}{2}. \end{aligned}$$

In this setting, the competition effect is minimized so that the primary effect from recommendations is to expand demand. Thus, it is not surprising that the platform is more inclined to use recommendations since it can no longer manipulate competition to its liking and recommendations can generate more surplus. Similarly for the seller, in the absence of a competition effect, the seller benefits from recommendation when the consumer benefit from cross-product knowledge exists. Formally, we have the following:

Proposition 6. *With a multi-product seller, we have that*

1. *The platform always makes recommendations within pages:* $\lambda_1^M = \lambda_2^M = 0$.
2. *The multi-product seller is willing to pay for recommendations when cross-page recommendations benefit consumers:* $\frac{d\pi}{d\lambda_i} < 0$ *if and only if* $\Delta A > 0$.

5.4 A Hybrid Platform: The Platform is a Seller

Cite in this section: $https://www.sciencedirect.com/science/article/pii/S0167718724000535$

Many online retailers are marketplaces and sellers (e.g., Amazon). Naturally, this creates perverse incentives for platform to direct recommendations towards its products and keep other products away. Indeed, Chen and Tsai (2023) show that frequently bought together recommendations on Amazon product pages are more likely to be products sold by Amazon. To study this in our setting, suppose that Product 1 is sold by the platform. For the Cournot-Awareness subgame, this implies that q_1 is determined by maximizing the platform's total profit, $\Pi = [(1-f)P - c]q_1 + f \cdot P(q_1 + q_2)$, taking Firm 2's actions and the inverse demands given by Equation (5) as given. Solving for the Cournot-Awareness subgame equilibrium under the hybrid platform generates the following result:

Lemma 4. *If Product 1 is sold by the hybrid platform, then the Cournot-Awareness subgame equilibrium quantity and price are given by*

$$\begin{aligned} Q^H &= \frac{[A(\sigma_H^2) + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)][2-\lambda_1\phi_1-\lambda_2\phi_2+F(1-\lambda_2\phi_2)] - C(2-\lambda_1\phi_1-\lambda_2\phi_2)}{b[3-\lambda_1\phi_1-\lambda_2\phi_2+F(2-\lambda_2\phi_2)]}, \\ P^H &= \frac{[A(\sigma_H^2) + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)](1+F) + (2-\lambda_1\phi_1-\lambda_2\phi_2)C}{[3-\lambda_1\phi_1-\lambda_2\phi_2+F(2-\lambda_2\phi_2)]}, \end{aligned}$$

where $F = \frac{f}{1-f}$.

Comparing Lemmas 1 and 4 it is straightforward to show that, holding recommendations and consumer arrivals fixed across the base and hybrid models, prices are higher under the hybrid platform: $P^H > P^*$. This is not surprising as the hybrid platform benefits from Firm 2 and this dampens competition.

Turning to the hybrid's platform optimal recommendation system, the asymmetry between sellers results in the platform having asymmetric preferences in terms of recommendations. Not surprisingly, we find that the platform favors itself:

Proposition 7. *With a hybrid platform, we have that*

1. *if there is no benefit to consumers from cross-product awareness ($\Delta A = 0$), then it always recommends it's Product 1 on Product 2's page ($\lambda_2^H = 0$) and it never makes a recommendation on it's product page ($\lambda_1^H = 1$).*

2. *there exists a sufficiently large cross-product awareness ($\Delta A >> 0$) so that the platform recommends it's rival's product on it's Product 1 page ($\lambda_1^H < 1$).*

Thus, without any benefits from cross-product awareness, the platform fully distorts recommendations in it's favor. Furthermore, we see that this persists when consumers do benefit from cross-product awareness but to a lesser degree.

6 Conclusion

Platforms often make product recommendations within a specific products webpage as the platform does not have complete control over consumers arrivals. For example, the consumer could have selected this product after searching on the platform or arrived on a product page from external recommendations. Thus, within product recommendations allow the platform to increase consumer awareness about the product options available on their platform.

In doing so, we find that the platform may or may not use recommendations, depending on consumer priors, to increase competition between products. In addition, this naturally creates the potential for ad revenues as sellers have an incentive to advertise on rival pages and block rivals from advertising on their's. As a result, within page recommendations can create a variety of dynamic elements that are relevant for managers and policy makers alike.

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A Appendix of Proofs

Proof of Lemma 1: Equations (3) and (5) imply that Firm's profit is given by

$$\pi_i = [(1-f)P - c] \cdot q_i = \left[(1-f)\left\{A(\sigma_L^2) - \frac{b}{1-\lambda_j\phi_j}(q_i + q_j^C) - \frac{\lambda_i\phi_i}{1-\lambda_j\phi_j}\Delta A\right\} - c\right] \cdot q_i.$$

The first-order condition implies that

$$q_i = \frac{A(\sigma_L^2) - \frac{c}{1-f}}{2b} \cdot (1-\lambda_j\phi_j) - \frac{q_j^C}{2} - \frac{\lambda_i\phi_i}{2b}\Delta A.$$

Substituting this back into the inverse demand implies that

$$P = \frac{1}{2}[A(\sigma_L^2) + \frac{c}{1-f} - \frac{b}{1-\lambda_j\phi_j}q_j^C - \frac{\lambda_i\phi_i}{1-\lambda_j\phi_j}\Delta A],$$

which must hold for $i = 1$, $j = 2$ and $i = 2$, $j = 1$. In addition, we know from Equation (5) that $P = A(\sigma_L^2) - \frac{b}{1-\lambda_1\phi_1-\lambda_2\phi_2}(q_1^C + q_2^C)$. These three pricing equations, combined with the two first-order conditions, implies that

$$\begin{aligned}
q_j^{C*} &= \frac{1}{b(3-\lambda_1\phi_1-\lambda_2\phi_2)}\left[\left(A(\sigma_L^2) - C\right)(1-\lambda_1\phi_1-\lambda_2\phi_2)(1-\lambda_j\phi_j) - X \cdot \Delta A\right], \\
q_i^* &= \frac{1-\lambda_j\phi_j}{b(3-\lambda_1\phi_1-\lambda_2\phi_2)}\left[A(\sigma_H^2) - C + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)\right], \\
Q^* &= q_1^* + q_2^* = \frac{(2-\lambda_1\phi_1-\lambda_2\phi_2)}{b(3-\lambda_1\phi_1-\lambda_2\phi_2)} \cdot \left[A(\sigma_H^2) - C + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)\right], \\
P^* &= \frac{A(\sigma_H^2) + (2-\lambda_1\phi_1-\lambda_2\phi_2)C + \Delta A(1-\lambda_1\phi_1-\lambda_2\phi_2)}{3-\lambda_1\phi_1-\lambda_2\phi_2},
\end{aligned}$$

where $X = [\lambda_i\phi_i(1-\lambda_1\phi_1-\lambda_2\phi_2) - 2\lambda_j\phi_j(1-\lambda_j\phi_j)]$ and $C = \frac{c}{1-f}$. □

Proof of Proposition 1: The derivatives from Corollary 1 are explicitly given by

$$\begin{aligned}
\frac{dP^*}{d\lambda_i} &= \frac{\phi_i}{(3-\lambda_1\phi_1-\lambda_2\phi_2)^2}\cdot[A(\sigma_H^2)-C-2\Delta A],\\
\frac{dQ^*}{d\lambda_i} &= \frac{-\phi_i}{b(3-\lambda_1\phi_1-\lambda_2\phi_2)^2}\\
&\quad\cdot\left[A(\sigma_H^2)-C+\Delta A(7+\lambda_1^2\phi_1^2-6\lambda_1\phi_1+2\lambda_1\phi_1\lambda_2\phi_2-6\lambda_2\phi_2+\lambda_2^2\phi_2^2)\right]<0.
\end{aligned}$$

Differentiating Equation (4) with respect to λ_1, when $\Delta A = 0$, yields:

$$\frac{d\Pi}{d\lambda_1} = f\cdot\frac{(A(\sigma_H^2)-C)\phi_1(A(\sigma_H^2)(1-\lambda_1\phi_1-\lambda_2\phi_2)-2C(2-\lambda_1\phi_1-\lambda_2\phi_2))}{b(3-\lambda_1\phi_1-\lambda_2\phi_2)^3}.$$

By simplifying and noticing the symmetry between the first-order conditions for λ_1 and λ_2 we have that

$$\frac{d\Pi}{d\lambda_i} = 0 = A(\sigma_H^2)(1-\lambda_1\phi_1-\lambda_2\phi_2)-2C(2-\lambda_1\phi_1-\lambda_2\phi_2).$$

This implies that λ_1^* and λ_2^* must be in $(0,1]$ whenever $A(\sigma_H^2) > 4C$. More specifically, we have that $\lambda_1^*, \lambda_2^* \in (0,1)$ with $\lambda_1^*\phi_1+\lambda_2^*\phi_2 = \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$ whenever $\phi_1+\phi_2 \geq \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$ and $\lambda_1^*, \lambda_2^* = 1$ whenever $\phi_1+\phi_2 < \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$. If $A(\sigma_H^2) < 4C$, then $\frac{d\Pi}{d\lambda_1} < 0$ so that $\lambda_1^*, \lambda_2^* = 0$ □

Proof of Proposition 2: Using the Implicit Function Theorem we have that

$$\frac{d\Lambda^*}{d\Delta A} = \frac{(3-\Lambda)[2A(\sigma_H^2)(5-3\Lambda)+(C+2\Delta A)(11-19\Lambda+9\Lambda^2-\Lambda^3)]}{2(A(\sigma_H^2)-C-2\Delta A)(A(\sigma_H^2)\Lambda+(C+2\Delta A)(3-2\Lambda))},$$

where $\Lambda = \lambda_1\phi_1+\lambda_2\phi_2$ and Λ^* is the platform's optimal choice in Λ given by $\frac{d\Pi}{d\Lambda} = 0$.

Note that this is greater than zero if and only if $A(\sigma_H^2)-C > 2\Delta A$. Thus, the platform prefers larger Λ (less recommendations) when ΔA increases for low levels of ΔA ($A(\sigma_H^2)-C > 2\Delta A$); otherwise, the platform prefers smaller Λ (more recommendations) when ΔA is large. □

Proof of Proposition 3: Differentiating π_i with respect to λ_j reveals that $\frac{d\pi_i}{d\lambda_j} < 0$ if and only if

$$(A(\sigma_H^2) - C)(1 - \lambda_i\phi_i + \lambda_j\phi_j) + \Delta A(7 + \lambda_i^2\phi_i^2 + \lambda_j^2\phi_j^2 - 8\lambda_j\phi_j - 4\lambda_i\phi_i + 2\lambda_i\phi_i\lambda_j\phi_j) > 0,$$

which holds. $\square$

Proof of Lemma 2: Without Product 1, Firm 2 has a mass of $1 - \phi_2$ consumers interested in its product so that Equation (2) becomes $q_1 = \frac{1-\phi_2}{b}[A(\sigma_H^2) - P]$ with Firm 1 being a monopolist. Maximizing Firm 1's monopoly profit implies that we have equilibrium quantity and price given by

$$\begin{aligned} q_1^F = Q^F &= \frac{1-\phi_2}{2b}[A(\sigma_H^2) - C], \\ P^F &= \frac{A(\sigma_H^2) + C}{2}, \end{aligned}$$

$\square$

Proof of Proposition 5: To see if the platform prefers foreclosure over a duopoly setting, note that the duopoly equilibrium quantity and price in Lemma 1 reveal that the platform's profit from the duopoly is smallest when $\Delta A = 0$. From there, Proposition 1 that the platform only has an incentive to limit recommendations in the duopoly case when $A(\sigma_H^2) > 4C$ so that $\lambda_1\phi_1 + \lambda_2\phi_2 = \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$. Thus, we compare the platform's duopoly profit under this worst case scenario to that of foreclosure and find that the duopoly profit is greater than the foreclosure profit if and only if

$$\frac{[A(\sigma_H^2) + (2 - \lambda_1\phi_1 - \lambda_2\phi_2)C](2 - \lambda_1\phi_1 - \lambda_2\phi_2)}{(3 - \lambda_1\phi_1 - \lambda_2\phi_2)^2} > \frac{1-\phi_2}{4} \cdot (A(\sigma_H^2) + C).$$

Using that $\lambda_1\phi_1 + \lambda_2\phi_2 = \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$, this simplifies to

$$A(\sigma_H^2)^2 > (1 - \phi_2) \cdot (A(\sigma_H^2)^2 - C^2),$$

which clearly holds. This implies that the platform prefers duopoly over foreclosure so that it will use its recommendation system to promote Firm 2 at the expense of Firm 1 by setting $\lambda_2^F = 1$ and minimizing λ_1. In particular, if $A(\sigma_H^2) < 4C$ so that $\lambda_1^*\phi_1 + \lambda_2^*\phi_2 = \frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C}$, then $\lambda_2^F = 1$ and $\lambda_1^F = \frac{1}{\phi_1}\left[\frac{A(\sigma_H^2)-4C}{A(\sigma_H^2)-2C} - \phi_2\right]$. □

Proof of Lemma 3: Equation (2) implies that multi-product seller demand is given by

$$Q = \frac{A(\sigma_H^2) - P + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)}{2b}.$$

Given that seller profit is now $\pi = [(1-f)P - c]Q$, solving the seller's problem yields:

$$\begin{aligned} Q^M &= \frac{A(\sigma_H^2) - C + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)}{2b}, \\ P^M &= \frac{A(\sigma_H^2) + C + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)}{2}. \end{aligned}$$

□

Proof of Proposition 6: Differentiating $\Pi^M = f \cdot P^M Q^M$ with respect to λ_i reveals a first-order condition that is negative:

$$-\frac{\Delta A \cdot \phi_1(A(\sigma_H^2) + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)}{2b} < 0,$$

so that setting $\lambda_1^M = \lambda_2^M = 0$ is optimal.

From Lemma 3, we see that both P^M and Q^M are decreasing in λ_1 and λ_2 when $\Delta A > 0$ so that $\frac{d\pi}{d\lambda_i} < 0$ if and only if $\Delta A > 0$. □

Proof of Lemma 4: Substituting P in Equation (2) into $\Pi = [(1-f)P-c]q_1 + f \cdot P(q_1+q_2)$ yields a first-order condition of $\frac{d\Pi}{dq_1} = 0$. This, along with the first-order condition for q_2 from the proof of Lemma 1 and the three inverse demand equations given by Equation (2) gives five equations and five unknowns $(q_1, q_2, q_1^C, q_2^C, P)$. Solving this system of equations gives

$$Q^H = \frac{[A(\sigma_H^2) + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)][2 - \lambda_1\phi_1 - \lambda_2\phi_2 + F(1 - \lambda_2\phi_2)] - C(2 - \lambda_1\phi_1 - \lambda_2\phi_2)}{b[3 - \lambda_1\phi_1 - \lambda_2\phi_2 + F(2 - \lambda_2\phi_2)]},$$

$$P^H = \frac{[A(\sigma_H^2) + \Delta A(1 - \lambda_1\phi_1 - \lambda_2\phi_2)](1 + F) + (2 - \lambda_1\phi_1 - \lambda_2\phi_2)C}{[3 - \lambda_1\phi_1 - \lambda_2\phi_2 + F(2 - \lambda_2\phi_2)]},$$

where $Q^H = q_1^H + q_2^H$ and $F = \frac{f}{1-f}$. □

Proof of Proposition 7: Plugging in the solutions from Lemma 4 into $\Pi = [(1-f)P - c]q_1 + f \cdot P(q_1 + q_2)$ and differentiating with respect to λ_1 (λ_2) yields $\frac{d\Pi}{d\lambda_1} > 0$ $\left(\frac{d\Pi}{d\lambda_2} < 0\right)$ if $\Delta A = 0$. This implies that $\lambda_1^H = 1$ and $\lambda_2^H = 0$ if $\Delta A = 0$. Furthermore, we have that $\frac{d\Pi^2}{d\lambda_1 d\Delta A} < 0$ so that $\lambda_1^H < 1$ when ΔA is sufficiently large. □