Limited Liability, Asset Price Overvaluation and the Credit Cycle: The Role of Monetary Policy

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Abstract. This paper suggests that the dynamics of the non-fundamental component of asset prices are one of the drivers of the credit cycle. The presented model builds on the financial accelerator literature by including a stock market where investors with limited liability trade stocks of productive firms with stochastic productivities. Investors borrow funds from the banking sector and can go bankrupt. Their limited liability induces a moral hazard problem which shifts demand for risk and drives prices of risky assets above their fundamental value. Embedding the contracting problem in a New Keynesian general equilibrium framework, the model shows that expansionary monetary policy induces loose credit conditions and leads to a rise in both the fundamental and non-fundamental components of stock prices. A positive shock to the non-fundamental component triggers a credit cycle: collateral value rises, and lending and default rates decrease. These effects reverse after several quarters, inducing a credit crunch. The credit boom lasts only while stock market growth maintains sufficient momentum. However, monetary policy does not reduce the volatility of inflation and the output gap by reacting to asset prices.

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1. Introduction

Asset price dynamics are one of the major drivers of the credit cycle. As asset prices enter the balance sheets of both financial and non-financial companies, they determine the collateral constraints of borrowers and contribute to how much banks are willing to lend at a given risk premium. An asset price may consist of both fundamental and non-fundamental components. In this paper, the fundamental value represents the part of the asset price which can be justified by discounted future dividend income under efficient market allocation. The remaining part of the asset price is labeled as non-fundamental and can be interpreted as a bubble. Empirical observations suggest that the non-fundamental component is more volatile than the fundamental one, which tends to be rather stable and is related to the discounted sum of expected future dividends. Consequently, the non-fundamental (or bubble) component may be an important driver of the credit cycle.

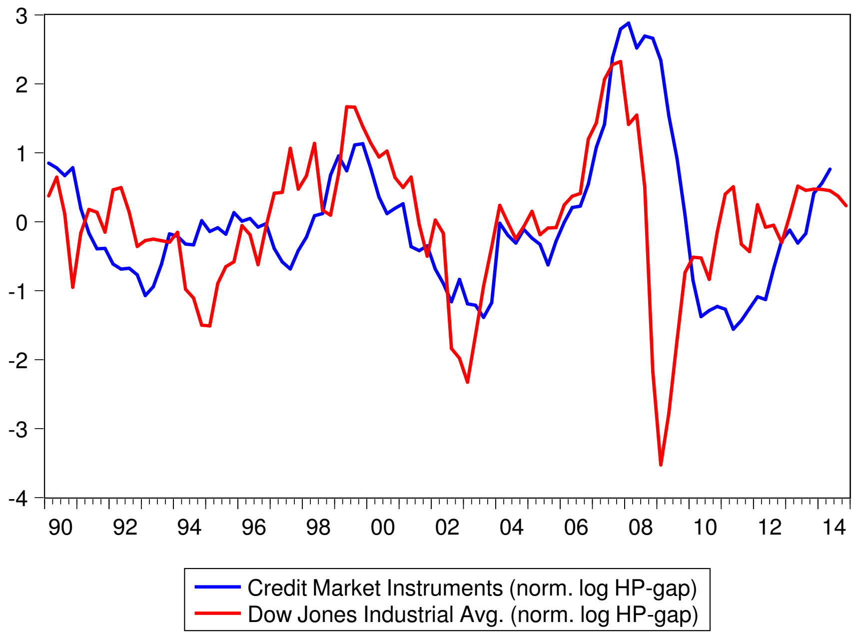
This paper builds a general equilibrium model capable of monetary policy simulations which is based on the financial accelerator literature. It extends the standard model by including an asset (stock) market where assets are endogenously priced above their fundamental values. The assets traded are shares in productive firms, which consist of claims on future returns on capital. These returns (dividends) are stochastic and subject to idiosyncratic productivity shocks. There is information asymmetry between lenders (commercial banks) and borrowers (stock market investors), which induces the costly-state-verification problem and gives rise to debt contracts where the leveraged stock market investors have limited liability for the outcomes of their investment decisions. The limited liability induces excessive risk-taking by the investors and leads to overpricing on the market for risky assets. As the overpriced assets affect the collateral constraints on borrowers, the value of the non-fundamental (as well as fundamental) component has implications for the real economy: it affects the amount of lending, investment, and output. Moreover, the asset price dynamics also affect the loan default rate. Expansionary monetary policy boosts both the fundamental and non-fundamental components. Through the collateral constraints of stock market investors, the higher stock prices induce lower borrowing rates and higher investment. A positive shock to the non-fundamental component of the asset price eases the collateral constraint and temporarily decreases the lending rate. Although the default rate immediately declines with lower interest rates, it picks up later with higher lending rates as the asset price shock fades out, again tightening the collateral constraint. Despite the suggested importance of asset prices for fluctuations in macroeconomic and financial variables, the estimated monetary policy efficiency frontiers show that the central bank achieves the lowest combinations of inflation and output gap volatilities by not reacting to either asset prices or their non-fundamental component.

The paper is organized as follows. Section 2 presents the empirical motivation and Section 3 relates the paper to other literature. Section 4 describes financial intermediation in a partial equilibrium setup, describing the interactions between banks, investors with limited liability, and firms, and shows that this setup leads to inflated prices on the asset market. Section 5 describes an extension where the overpriced assets have longer maturities and can be used as collateral in the periods before maturity. Section 6 extends the model to general equilibrium by describing relations to the remaining sectors of the economy, and Section 7 presents the responses of model variables to shocks and discusses the implications for efficient monetary policy.

2. Empirical Motivation

This paper claims that the credit cycle is to a large extent driven by asset price developments, where the non-fundamental component of asset prices plays a prominent role. This is indeed confirmed by

Figure 1: Asset Prices Leading the Credit Cycle



the observed data. Figure 1 shows the normalized log-deviations (from the HP trend) of the Dow Jones Industrial Average index and the total value of credit market instruments (liabilities) since 1990, both taken from the FRED database of the St. Louis Fed. It is apparent that in this period, the Dow Jones index, used as a proxy for asset prices, was a leading factor for the amount of credit. The credit boom in the late 1990s was preceded by steady growth in asset prices. The asset price bust of the dot-com bubble in 2001 was followed by a negative credit gap, which began to close only after asset prices rebounded in 2003. In 2007, it was again asset prices (both stock and real-estate) which preceded the credit crunch associated with the global financial crisis in 2007–2009. Lending began to pick up only after asset prices returned to growth after 2010.

As another illustrative stylized fact, simple Granger causality tests (Table 1) on a quarterly sample of US data (taken from the FRED database) ranging from 1949Q1 to 2014Q4 suggest that it is the dynamics of asset prices that lead the credit cycle rather than vice versa (all variables in percentage changes). Assuming that the fundamental value of an asset price should reflect dividend income, a proxy for the fundamental component was obtained by regressing the log of dividend income (including dividend income lags and leads of up to the 4th order) on the log of asset prices. The residual of this regression, i.e., the component of prices which cannot be explained by dividend flows, is a naïve estimate of the non-fundamental component of asset prices. The Granger causality tests show that this non-fundamental component (a bubble), rather than the fundamental value, leads the credit cycle.[1]

[1] As a side note, the amount of lending, in turn, predicts the fundamental returns with marginal significance, possibly because credit-financed investment was followed by higher profits and dividends. It is also important to keep in mind the limitations of both the Granger causality test and the estimation of the fundamental component, and these computations should only be viewed as an illustrative stylized fact.

Table 1: Asset Prices Granger Cause the Credit Cycle

Pairwise Granger Causality Tests Sample: 1949Q1 2014Q4 Lags: 4			
Null Hypothesis:	Obs	F-Statistic	Prob.
%Δ(Dow Jones I.A.) does not Granger Cause %Δ(Credit)	246	4.86702	0.0009
%Δ(Credit) does not Granger Cause %Δ(Dow Jones I.A.)		1.72503	0.1451
%Δ(Non-fund. comp.) does not Granger Cause %Δ(Credit)	243	2.48795	0.0442
%Δ(Credit) does not Granger Cause %Δ(Non-fund. comp.)		1.42583	0.2261
%Δ(Fundamental comp.) does not Granger Cause %Δ(Credit)	243	1.28622	0.2761
%Δ(Credit) does not Granger Cause %Δ(Fundamental comp.)		1.80739	0.1281

3. Related Literature

The global financial and European debt crises have pointed out the importance of the financial sector in transmitting and amplifying economic shocks. The literature has reacted to this increased interest by building on the general equilibrium models with financial frictions of the late 1990s. The most important contributions involve works by Carlstrom and Fuerst (1997) and the subsequent synthesis of Bernanke et al. (1999), who integrate the previous models of financial frictions with New Keynesian rigidities and are therefore able to analyze the role of monetary policy. Another stream of literature builds on Kiyotaki and Moore (1997), which establishes a link between collateral value and the business cycle, but does not explicitly model loan defaults. In recent years, enormous work has been done to incorporate other aspects of financial intermediation, such as the role of collateral constraints in the housing market (Iacoviello and Neri, 2010) or the role of unconventional tools of monetary policy (Gertler and Karadi, 2011; Cúrdia and Woodford, 2011; Gertler and Karadi, 2013). A canonical model for the analysis of financial frictions and policy responses during the crisis was established by Gertler and Kiyotaki (2010). The role of liquidity constraints for the possibility of bank runs was analyzed by Gertler and Kiyotaki (2013). The effects of the recently widely adopted tools of macro-prudential regulation have been explored by many, e.g. Kashyap et al. (2014). Similarly to this paper, Farhi and Tirole (2012) concluded that a form of limited liability gives rise to increased risk preference.

This paper presents a general equilibrium model capable of monetary policy simulations which captures the characteristics of the credit cycle. It is inspired by a broadly held view of how large financial crises develop and spread. The idea of Adrian and Shin (2010) and the model of Allen and Gale (2000) are possibly closest to our view. While Allen and Gale (2000) provide a rational explanation (albeit only in a partial equilibrium setup) for the overpricing of risky assets when the pricing is done by investors with limited liability, Adrian and Shin (2010) show how changes in the value of assets used as collateral can immediately lead to large swings in balance sheet sizes (causing a contraction in credit and real activity) when the market is highly leveraged.

This paper suggests that asset price and credit booms may be explained and described using a combination of these two effects. Because the incentive structures faced by the managers of certain types of financial intermediaries (such as investment banks) induce excessive risk-taking, there may be over-investment in risky assets, leading to endogenously inflated prices. If these overpriced assets are allowed to serve as collateral for new loans, the shocks to asset prices may have pronounced and non-trivial effects.

The main contribution of this paper is that it constructs a full New Keynesian general equilibrium model with the above-described mechanism of feedback between asset prices, credit conditions, investment, and capital returns. The model also illustrates how the monetary policy of an inflation-targeting central bank interferes with this mechanism. The simulations suggest that expansionary monetary policy leads to an increase in both the fundamental and non-fundamental element of asset prices and consequently increases collateral value, reduces lending rates, and boosts economic activity. A non-fundamental shock to asset prices can also trigger a credit cycle: shortly after a positive price shock the lending rate falls and the default rate decreases as collateral constraints ease. After several quarters, however, lending rates and the default rate start to rise as investors' wealth shrinks with a stock market slowdown.

Our paper contains a number of other results (contributions). In particular, it finds that the non-fundamental component of asset prices may work as a shock absorber. Similarly to Martin and Ventura (2014), the existence of the non-fundamental component (or a bubble) can have some beneficial aspects. As the proposed model implies, the bubble can partially absorb the effects of exogenous shocks by affecting the wedge between the risk-free rate and capital returns in a counter-cyclical manner. This is most obvious in the case of a monetary policy shock, where the impact of an interest rate shock on capital returns and investment is much smaller compared to the similarly parametrized benchmark of Bernanke et al. (1999).

This paper also shows that if the growth of asset prices and traded volumes maintains a sufficient (precisely defined) momentum, borrowing constraints are relaxed and the dynamics of real economic variables exceed the benchmark of Bernanke et al. (1999). If the growth of the stock market slows below the defined threshold, the dynamics fall below the benchmark model.[2] The paper also shows that under limited liability, leveraged investors prefer risky investment over diversified portfolios.

4. Financial Intermediation Structure and Asset Pricing

Inspired by the model of asset bubbles presented by Allen and Gale (2000), and building on the model of the financial accelerator in the New Keynesian framework of Bernanke et al. (1999), this paper constructs a partial equilibrium model which generates incentives-based overpricing in asset prices. The limited liability of investors (i.e., investors do not suffer the full cost in the event of default) induces them to prefer risky assets and price them above their fundamental value. The fundamental value is defined as the price at which the investors would invest their own resources, which is consistent with pricing based on the present value of future dividend income. In later sections this contracting problem is embedded in the general equilibrium framework of Bernanke et al. (1999) and we show that non-fundamental overpricing of risky assets can emerge within this widely used model framework. This allows us to conduct monetary policy experiments to study the impact of monetary policy shocks on the size of the non-fundamental component of asset prices.

[2] Although this paper studies the effects of overpriced assets on the economy, the model does not feature an explicit mechanism for the burst of the endogenous asset bubble. The bubble is interpreted as a structural, long-term feature of the financial market, where limited liability leads to mispricing of risky assets even in the steady state of the model.

The size of the non-fundamental element of asset prices has real implications: it affects credit availability and consequently the amount of lending, investment, and economic activity.

The model can, however, be interpreted more broadly, as limited liability is present widely in the whole economy as an inherent result of the principal-agent problem. A similar incentive structure applies to the setting where an investment fund is managed by a fund manager with limited liability whose salary is dependent on the fund's performance. When the return on the managed portfolio is sufficiently high, the manager's income increases with the returns. When the returns become negative, the manager can be fired, but does not directly bear the cost of the portfolio loss. By analogy to the model presented below, the limited liability leads to increased demand for risky assets. Further, corporate management is typically partially rewarded in company stock options. In good states of the world managers can execute their options, while in bad states of the world they do not bear the losses. This limited liability leads to more risky projects being undertaken in comparison to the first-best setting (which we will call fundamental), where managers would decide about the investment of funds that they themselves own.

By examining the impacts of limited liability on asset pricing and real activity, this paper suggests that limited liability is at the heart of increased risk-taking in the financial sector.[3]

4.1 Contract Timing and Payoffs

This model builds on Bernanke et al. (1999) and extends it by incorporating a stock market where the shares of productive firms are traded by investors with limited liability. Figure 2 depicts the agents in the contracting problem, the flow of funds, and the respective interest rates. Risk-neutral investors can invest in risky shares S_{t+1} of productive firms, but have only limited own wealth N_{t+1}. These shares of productive firms are traded at an endogenously determined price P_{t+1} and yield a stochastic return equal to the return on the firm's installed capital ωR^K_{t+1}, where ω is an idiosyncratic productivity element, which is i.i.d. with $E[\omega] = 1$. R^K_{t+1} is aggregate capital productivity. The realization of idiosyncratic return ω is not known at the time of the investment decision. It is revealed to the investor ex-post, but it cannot be contracted on, which gives rise to the costly-state-verification problem. The investor needs to borrow $B_{t+1} = P_t S_{t+1} - N_{t+1}$. We initially assume that ownership of shares entitles the holder to capital returns in one period, but later we will relax this assumption to allow for multi-period asset holdings which enter the collateral constraints and lead to more pronounced effects of asset prices on other financial and macroeconomic variables.

The risk-neutral financial intermediaries (banks) are willing to lend B_{t+1} at a contractual rate Z_{t+1} as soon as their expected payoff from the contract exceeds the opportunity cost of investing in a risk-free asset with certain return R_{t+1} (the banking sector is competitive). Lending to investors is generally risky, as an investor can default on the loan whenever the realized return on the portfolio is low enough, such that he is not able to repay the borrowed amount at the contractual rate $Z_{t+1}B_{t+1}$. Let us also assume that investors with limited liability are the only agents in the economy who are capable of investing in risky assets. In general, an investor can also invest positive amounts in a risk-free asset yielding R_{t+1}. However, he will invest a zero amount in this risk-free asset, as his external financing costs (the contractual rate) will generally be higher than the risk-free return

[3] The assets used in this model are claims on the future capital returns of firms and can be interpreted as stocks. However, the same mechanism would apply for other types of assets where returns are uncertain and investors have limited liability, assets such as real estate. The model would imply mispricing of real estate in a similar manner, which could trigger a financial cycle according to Borio (2014) and others. However, the macroeconomic implications may be slightly different (one of the most notable differences being the large exposure of households to housing assets in comparison to stocks), and are beyond the scope of this paper.

Figure 2: Agents, Loans and Repayments if Everything Goes Well

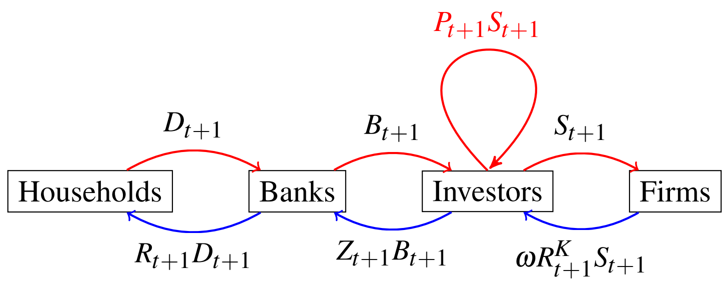
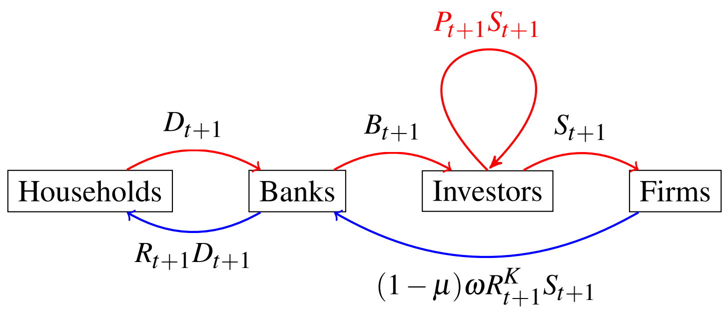


Figure 3: Agents, Loans, and Repayments Under Default



($Z_{t+1} > R_{t+1}$) because the contractual rate Z_{t+1} would need to compensate the bank for any default risk. The riskiness of an investment is not ex-ante observable by the bank, which can only monitor the returns ex-post by paying agency costs (as in the standard costly-state-verification problem). Whenever the realized return on the risky asset is below a certain threshold where the investor is unable to pay the loan back, he declares bankruptcy (Figure 3). The threshold for default is the break-even idiosyncratic return $\bar{\omega}_{t+1}$ on the risky asset, defined by the following constraint:

$$Z_{t+1}B_{t+1} = \bar{\omega}_{t+1}R^K_{t+1}S_{t+1} \tag{1}$$

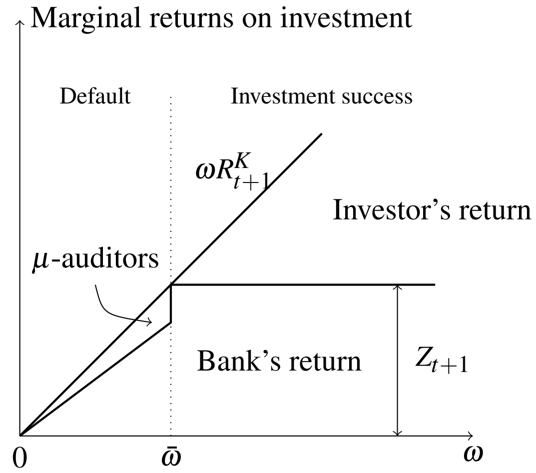
The default-threshold idiosyncratic return $\bar{\omega}_{t+1}$ is such that the borrower will just be able to repay the borrowing B_{t+1} times the contractual rate Z_{t+1}. In the event of default, the borrower (bank) pays a fraction μ as auditing costs to collect whatever remained from the project. The contractual rate has to satisfy the participation constraint for the bank:

$$(1-F(\bar{\omega}_{t+1}))Z_{t+1}B_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega R^K_{t+1}S_{t+1}dF(\omega) \geq R_{t+1}B_{t+1} \tag{2}$$

where the bank's expected payoff from lending to the investors must be higher than the opportunity cost of investing in risk-free bonds. To avoid facing idiosyncratic risk, the banks diversify their loan portfolio among many ex-ante identical investors, charging a flat rate Z_{t+1}. The timing of the financial intermediation contract for a representative investor-bank pair is the following:

1. The bank lends B_{t+1} to the investor at a flat rate Z_{t+1}, which compensates the bank for the ex-ante symmetric risk of default by the investor.
2. The stock market opens and the investor may sell and buy risky assets S_{t+1} for an endogenously determined price P_t.

Figure 4: Contract Payoff Distribution Between the Bank, the Investor, and Auditing Costs



3. The idiosyncratic risk ω is realized and the assets S_{t+1} yield ωR^K_{t+1} to the investor.
4. The investor either repays $Z_{t+1}B_{t+1}$ to the bank or defaults. In the case of default, the bank pays auditing costs μ and collects the residual value of the investment.

This is a variant of the standard costly-state-verification problem of Townsend (1979), and the described risky debt contract (including the true reporting of default) was shown to be Pareto-optimal by Gale and Hellwig (1985). The contract payoffs are depicted in Figure 4.

In the case of default, the investor's payoff is zero. In the case of success, the investor is the residual claimant after satisfying the participation constraint of the bank. The bank itself does not face any risk on aggregate, as its assets are perfectly diversified among loans to individual investors, and the loss from the fraction of defaulting loans is ex-ante covered by higher contractual rates. Therefore, households' savings are not subject to any risk either.

4.2 The Investor's Problem and Demand Pricing of the Risky Asset

The investor chooses the amount of risky investment S_{t+1} and the default threshold $\bar{\omega}_{t+1}$ to maximize

$$\max_{S_{t+1},\bar{\omega}_{t+1}} \left[\int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} S_{t+1} dF(\omega) - (1 - F(\bar{\omega}_{t+1})) Z_{t+1} B_{t+1} \right] \tag{3}$$

subject to the bank's participation constraint (2). In other words, the investor only takes into account the "optimistic" part of the return distribution where he has positive profit, and he does not internalize the full cost of the losses. Because of the limited liability, the investor's payoff is zero in the case of default. This causes the investor's subjective return distribution to be more optimistic than the true fundamental return distribution. Next, we show that limited liability increases the investor's appetite for investing in the risky asset, and raises stock market prices.[4]

[4] Several conceptual points are worth noting at this point. First, we can think about the idiosyncratic ω realizations as shocks to distinct sectors of the economy. We conjecture that investors endogenously prefer to fully face the idiosyncratic risk and not to diversify their asset holdings among sectors. The idiosyncratic risk is preferred by

Using $B_{t+1} = P_t S_{t+1} - N_{t+1}$, the investor's problem becomes linear in S_{t+1}. That implies that there is a price of a risky asset P_t above which the demand is infinite and below which the investors would like to short-sell the asset in an infinite amount. As we will show later, the non-degenerate supply ensures a unique and stable finite equilibrium. The no-profit condition for investors requires:

$$\int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} dF(\omega) - (1 - F(\bar{\omega}_{t+1})) Z_{t+1} P_t = 0 \tag{4}$$

This equality will be achieved because of the competitive investors sector and the convex costs of creating investment opportunities (see below). We assume that there are many ex-ante identical investors and they take the universally charged rate Z_{t+1} as exogenous. In other words, the contractual rate Z is not conditioned on the individual characteristics of investors, such as the individual amount of shares invested. The investors' wealth is equalized across households of investors. From condition (4), we can express the price of the risky asset as observed on the stock market:

$$P_t = \frac{1}{Z_{t+1}} \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} dF(\omega)}{(1 - F(\bar{\omega}_{t+1}))} \tag{5}$$

The price reflects investors' valuation of the risky asset, where the expected realization from a truncated return distribution is discounted by the contractual rate multiplied by the probability it will be paid. We claim that this price of risky shares observed on the stock market is overpriced in comparison to the fundamental value as a result of the limited liability of investors. Following Allen and Gale (2000), we define the fundamental value as the price the investor would pay if he invested his own funds without leverage. Such a "fundamental" investor would solve

$$\max_{S_{t+1}} \left[\int_0^{\infty} \left(\omega R^K_{t+1} S_{t+1} + R_{t+1}(N_{t+1} - P^F_t S_{t+1}) \right) dF(\omega) \right] \tag{6}$$

where the investor's own wealth N_{t+1} is sufficient to cover all desired spending on the risky asset and the rest $(N_{t+1} - P_t S_{t+1})$ is left for investing in the risk-free asset. The no-profit condition of fundamental investors is:

$$P^F_t = \frac{1}{R_{t+1}} (E[\omega] R^K_{t+1}) = \frac{R^K_{t+1}}{R_{t+1}} \tag{7}$$

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investors because the limited liability makes the non-diversified risky investment more attractive. Appendix B shows in more detail that investors with limited liability prefer not to diversify their asset holdings.

Second, let us also assume a continuum of sectors of measure 1, so that the probability of default represents the fraction of defaulting investors. There are infinitely many firms in each sector, so that firms do not have any bargaining power vis-à-vis investors, who become residual claimants. To ensure ex-ante symmetry among the wealth of investors at the beginning of each period, we assume that investors gather in households of investors, and each investor's household has one investor covering each sector. The investors in each sector operate individually (importantly, they cannot repay debt using returns from other sectors), coming "home" together and pooling their wealth among household members only after the uncertainty is realized and after returns are paid. Also, the "creative investors" described below hand their profits over to the household pool. Therefore, from the point of view of households of investors, the idiosyncratic risk of individual investment is pooled and vanishes at the household level.

Finally, to prevent arbitrage conducted by banks or households driving the price down to the fundamental level, the model assumes that trading on the stock market is restricted to investors with limited liability, who possess the unique property of controlling and operating the firms which they own.

The fundamental price is no higher than the present value of the one-period-ahead claim on expected returns on installed capital. Comparing the fundamental price with the price with limited liability, we show that

$$P_t \geq P_t^F \tag{8}$$

The full proof can be found in Appendix A. Interestingly, because investors with limited liability are willing to pay a higher price, they effectively drive away any fundamental investors (whose fundamental valuation is lower) from the market.

We have shown that under certain reasonable assumptions (most notably that investors enjoy limited liability) the prices which are observed on the stock market are endogenously inflated compared to their fundamental values. The non-fundamental component is the difference between the observed price of the risky asset and its fundamental value.

4.3 Supply of the Risky Asset

To complete the model of the stock market, we assume that in each household there are investors who "create" investment opportunities (and sell them to other investors on the stock market). The profit from creating an investment opportunity, and the creative investors' objective function, is

$$\max_{S_{t+1}} (P_t S_{t+1} - c(S_{t+1})) \tag{9}$$

where $c(S_{t+1})$ is an increasing convex cost function which links the costs of creating an investment opportunity to the number of investment assets created. To ensure interior equilibrium, assume that $c'(.) > 0$, $c''(.) > 0$. The amount of stocks created S_t is then sold on the stock market at price P_t, which the competitive creative investors take as exogenous and which potentially exceeds their average costs and creates economic profit. The creative investors are evenly distributed among households of investors, so that any profit from trading on the stock market stays in the investors sector and is equalized across households to prevent heterogeneous paths of wealth. The first-order condition of the creative investors' profit maximization problem is

$$P_t = c'(S_{t+1}) \tag{10}$$

This equation describes the supply side of the market for the risky asset.

4.4 Value of the Investment and Wealth Accumulation

Using the substitution for $Z_{t+1}B_{t+1}$ from eq. (1), the investors' objective (3) can be rewritten as

$$\max_{S_{t+1},\bar{\omega}_{t+1}} \left[\int_{\bar{\omega}_{t+1}}^{\infty} \omega R_{t+1}^K S_{t+1} dF(\omega) - (1 - F(\bar{\omega}_{t+1}))(\bar{\omega}_{t+1} R_{t+1}^K S_{t+1}) \right] \tag{11}$$

Substituting the last term from the banks' participation constraint (2), we arrive at the following expression for the value of investment:

$$R_t^K S_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}_{t+1}} \omega R_t^K S_t dF(\omega)}{P_{t-1} S_t - N_t} \right) (P_{t-1} S_t - N_t) \tag{12}$$

As noted above, some of the investors are "creative" and they actively generate investment opportunities, establishing contact with firms. They are able to generate shares of firms, which they will be able to sell for P_t, by paying convex costs $c(S_{t+1})$. Because we have not yet established any particular functional form for $c()$, let us normalize it such that $c(S_{t+1}) \approx 1$ and that the deviations around this value resulting from the positive derivative are of second-order importance for investors' wealth. Then the investors' net profit can be expressed as $S_{t+1}(P_t - 1)$, which they store on a risk-free account for the rest of the period. We assume that these creative investors are uniformly distributed across households of investors and pool their gain in the wealth of a representative household of investors. Further imposing the market-clearing condition that $S_t = Q_{t-1}K_t$ (i.e., stocks of firms entitle their holders to a share of firms' installed capital), we get

$$\begin{aligned} V_t &= R_t^K Q_{t-1}K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}_{t+1}} \omega R_t^K Q_{t-1}K_t dF(\omega)}{P_{t-1}Q_{t-1}K_t - N_t} \right) (P_{t-1}Q_{t-1}K_t - N_t) + R_t Q_{t-1}K_t(P_{t-1} - 1) \\ &= R_t^K Q_{t-1}K_t - \left(R_t + \frac{\mu \int_0^{\bar{\omega}_{t+1}} \omega R_t^K Q_{t-1}K_t}{Q_{t-1}K_t - N_t} dF(\omega) Q_{t-1}K_t - N_t \right) (Q_{t-1}K_t - N_t) \end{aligned} \tag{13}$$

which is identical to the wealth accumulation equation of Bernanke et al. (1999) (although here it results from a different financial market structure). In other words, we consider a sector of limited liability investors between firms and banks, and we show that the prices of assets traded on the financial market are inflated. Because of a different incentive structure leading to different investment decisions, this leads to a different allocation of real resources in comparison to Bernanke et al. (1999).

However, if the inflated assets are held for more than one period and can be used as collateral for further loans, the overpricing will have an even stronger impact on credit availability, lending, investment, and real activity. In the next section we explain how the model economy behaves when the assets (with inflated prices as a result of the mechanism described above) are held for two periods before they mature.

5. Extension: Multi-Period Assets and the Collateral Constraint

In this section we consider an extension of the presented model where the agents hold and trade the assets for multiple periods until the assets mature. In that case, the prices of assets held for more than one period will affect the evolution of investors' wealth, which is used as collateral. If the prices of these assets are inflated similarly to the single-period case described above, the investors' wealth (which serves as collateral) is inflated as well. Most importantly, the results suggest that if the asset prices maintain a sufficient (precisely defined) growth momentum, investors' wealth is higher than in the Bernanke et al. (1999) benchmark. If the asset price growth slows down, the wealth decreases below the Bernanke et al. (1999) benchmark.

5.1 Financial Intermediation Contract with Two-Period Assets

Assume there is an asset which is purchased in the first period, held in the second, and transforms into a claim on productive capital in the third. As a result, in every period t two types of asset are traded: the old ones S_{t+1}^{old} (issued in the previous period $t-1$ and maturing in $t+1$) with price P_t^{old} and the newly issued ones S_{t+1}^{new} with price P_t^{new}. While S_{t+1}^{old} yield R_{t+1}^K at the beginning of the next period, S_{t+1}^{new} yield nothing in period $t+1$, but can be traded as S_{t+2}^{old}, which in turn yield R_{t+2}^K at the beginning of period $t+2$. The cash-flow constraint, i.e., the relationship between the contractual rate Z_{t+1} and the threshold idiosyncratic productivity $\bar{\omega}_{t+1}$ (formerly eq. (1)), in this case becomes

$$Z_{t+1}B_{t+1} = \bar{\omega}_{t+1}R^K_{t+1}S^{old}_{t+1} + P^{old}_{t+1}S^{new}_{t+1} \tag{14}$$

because in addition to capital returns on maturing assets, the investors will in $t+1$ own previously purchased assets maturing in the following period. The bank participation constraint (2) changes to

$$(1-F(\bar{\omega}_{t+1}))Z_{t+1}B_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \left(\omega R^K_{t+1}S^{old}_{t+1} + P^{old}_{t+1}S^{new}_{t+1}\right) dF(\omega) \geq R_{t+1}B_{t+1} \tag{15}$$

5.2 Investors' Objective and Asset Pricing

The investors' objective (previously eq.(3)) is to maximize expected profit, which, in the presence of two-period assets, is defined as

$$\max_{S^{new}_{t+1}, S^{old}_{t+1}, \bar{\omega}_{t+1}} \left[\int_{\bar{\omega}_{t+1}}^{\infty} \left(\omega R^K_{t+1}S^{old}_{t+1} + P^{old}_{t+1}S^{new}_{t+1}\right) dF(\omega) - (1-F(\bar{\omega}_{t+1}))Z_{t+1}B_{t+1}\right] \tag{16}$$

The amount of borrowing is given by $B_{t+1} = P^{new}_t S^{new}_{t+1} + P^{old}_t S^{old}_{t+1} - N_{t+1}$. When substituted into (16), the no-profit conditions of investors can be derived, which lead to the following (demand) pricing equations:

$$P^{old}_t = \frac{1}{Z_{t+1}} \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} dF(\omega)}{1-F(\omega)} \tag{17}$$

$$P^{new}_t = \frac{P^{old}_{t+1}}{Z_{t+1}} = \frac{1}{Z_{t+1}Z_{t+2}} \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+2} dF(\omega)}{1-F(\omega)} \tag{18}$$

Both of these can be shown to be higher with respect to their corresponding fundamental values defined as if there were no information asymmetry (analogously to the case of single-period assets).

5.3 Investors' Wealth Accumulation Under Two-Period Assets

Combining the investors' objective (16) with the bank participation constraint (15), substituting for $Z_{t+1}B_{t+1}$ using (14), and imposing the market-clearing condition that $S^{old}_t = Q_{t-1}K_t$, one can obtain the evolution of the aggregate value of investors' assets.

$$\begin{aligned} V_t &= R^K_t Q_{t-1}K_t + P^{old}_t S^{new}_t \\ &- \left(R_t + \frac{\mu \int_0^{\bar{\omega}_{t+1}} \left(\omega R^K_t Q_{t-1}K_t + P^{old}_t S^{new}_t\right) dF(\omega)}{P^{new}_{t-1}S^{new}_t + P^{old}_{t-1}Q_{t-1}K_t - N_t}\right)(P^{new}_{t-1}S^{new}_t + P^{old}_{t-1}Q_{t-1}K_t - N_t) \end{aligned} \tag{19}$$

The value of investment is now different in comparison to single-period assets and to the Bernanke et al. (1999) benchmark. This is because (overpriced) assets can be used as collateral in the meantime before maturity. However, the purchase of overpriced assets also constitutes extra costs with respect to the Bernanke et al. (1999) benchmark. Comparing the wealth accumulation equation with the benchmark, we are able to establish the conditions under which the mispriced risky assets have boosting effects on the economy, and when their effect is restrictive.

The question is whether the difference between investors' wealth in Bernanke et al. (1999) and that in the present model is positive or negative:

$$V_t^{INF} - V_t^{BGG} = P_t^{old} S_t^{new}(1 - \mu F(\bar{\omega}_{t+1})) - R_t(P_{t-1}^{old} Q_{t-1} K_t) \lesseqgtr 0 \tag{20}$$

This inequality translates (using the fact that new assets will become old in the next period, $S_t^{new} = S_{t+1}^{old}$) into the question of whether the nominal growth of the stock market has sufficient momentum:

$$\frac{P_t^{old} S_{t+1}^{old}}{P_{t-1}^{old} S_t^{old}} \lesseqgtr \frac{R_t}{1 - \mu F(\bar{\omega}_{t+1})} \tag{21}$$

If the growth of asset prices and/or the volume of assets traded remains above this threshold, the investors' wealth and the amount of borrowing, investment, and economic activity exceeds the Bernanke et al. (1999) benchmark. When the growth of the asset market loses momentum, the effects of non-fundamental prices on the real economy become negative. The threshold implies that for the non-fundamental pricing to have an expansionary effect on the economy, the growth of the volume of the risky asset market multiplied by the fraction which is not lost to auditing costs needs to cover the risk-free interest rate.

6. Other Sectors of the New Keynesian General Equilibrium Model

Now we embed the contracting problem described above in a general equilibrium model. We follow the framework of Bernanke et al. (1999) closely. In addition to investors and firms, there are retailers, households, the central bank, and the government.

6.1 Investors and Banks

The households of investors (as distinct from the ordinary households described below) are risk-neutral, but leave the system at a rate of γ. This ensures that investors always demand credit and do not accumulate enough wealth to be eventually fully self-financing. After departure, they consume the remaining part of their wealth. Investors accumulate wealth according to

$$N_{t+1} = \gamma V_t + W_t^i \tag{22}$$

where V_t is the value of investment in firms' shares as defined above by (13). When a household of investors dies, it consumes all its wealth and departs the scene. This process creates the investors' consumption C_t^e. W_t^i is investors' wages.

The demand price of assets P_t was defined by (5), and the supply side by (10). Dividing the banks' participation constraint (2) by B_{t+1} and using (1) for substitution of the term inside the integral of after-default asset recovery, we can express the risk premium (the difference between Z_{t+1} and R_{t+1}) as a function of the default threshold $\bar{\omega}_{t+1}$:

$$\frac{R_{t+1}}{Z_{t+1}} = 1 - F(\bar{\omega}_{t+1}) + \frac{1-\mu}{\bar{\omega}_{t+1}} \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega) \equiv \Psi(\bar{\omega}_{t+1}) \tag{23}$$

where $\frac{\partial \Psi(\bar{\omega}_{t+1})}{\partial \bar{\omega}_{t+1}} < 0$. Similarly, the demand price of risky assets is a function of the aggregate capital returns R_{t+1}^K, the contractual rate Z_{t+1}, and the default threshold $\bar{\omega}_{t+1}$ eq. (5), and can be transformed such that

$$\frac{P_t Z_{t+1}}{R^K_{t+1}} = \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega)}{(1-F(\bar{\omega}_{t+1}))} \equiv \Theta(\bar{\omega}_{t+1}) \tag{24}$$

where $\frac{\partial \Theta(\bar{\omega}_{t+1})}{\partial \bar{\omega}_{t+1}} > 0$, i.e., the price of the risky asset increases with the risk, which is a result of investors' elevated risk preference induced by the limited liability. Combining equations (23) and (24), one can see that the wedge between the risk-free rate R_{t+1} and capital returns R^K_{t+1} can be expressed as a function of $\bar{\omega}_{t+1}$ and the risky asset price P_t:

$$\frac{R^K_{t+1}}{R_{t+1}} = \frac{P_t}{\Psi(\bar{\omega}_{t+1})\Theta(\bar{\omega}_{t+1})} \tag{25}$$

where P_t is in turn determined by the increasing supply-side marginal costs, which link it to S_{t+1} (10).

Finally, the market-clearing condition links the financial sector to the production sector:

$$S_t = Q_t K_t \tag{26}$$

6.2 Firms

A representative firm produces output Y_t using the production function with capital K_t and aggregate labor L_t as inputs.

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{27}$$

where A_t is stochastic total factor productivity following an autoregressive process. The capital share is denoted by α. Labor consists of workers' labor H_t and investors' labor H_t^i (consider venture capitalists):

$$L_t = H_t^{\Omega} (H_t^i)^{1-\Omega} \tag{28}$$

where Ω is the share of workers' labor. New capital creation involves installment costs, while old capital depreciates at rate δ.

$$K_{t+1} = \Phi\left(\frac{I_t}{K_t}\right) - (1-\delta)K_t \tag{29}$$

The installment costs can be thought of as a competitive sector of capital producers who purchase investment and rent capital stock to produce new capital using the production function $\Phi\left(\frac{I_t}{K_t}\right)$ to sell it at price Q_t. The FOC of their problem determines the "replacement cost" component of the price of capital: $Q_t = \left[\Phi'\left(\frac{I_t}{K_t}\right)\right]^{-1}$.

Firms produce wholesale goods, which are sold to monopolistically competitive retailers at a relative price $\frac{1}{X_t}$. The firms sector is assumed to be competitive. Return on capital R^K is equal to the marginal product of capital multiplied by the price of wholesale goods produced, augmented by the change in the value of capital (consisting of the change in Q_t and depreciation). In expectation terms:

$$E[R^K_{t+1}] = E\left[\frac{\frac{1}{X_{t+1}}\frac{\alpha Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)}{Q_t}\right] \tag{30}$$

Wages in the workers and investors sectors of the labor market are competitive and follow the marginal products of labor:

$$W_t = (1-\alpha)\Omega\frac{1}{X_t}\frac{Y_t}{H_t} \tag{31}$$

$$W^e_t = (1-\alpha)(1-\Omega)\frac{1}{X_t}\frac{Y_t}{H^e_t} \tag{32}$$

6.3 Households and Retailers

In addition to the agents involved in the contracting problem, the model features a standard New Keynesian general equilibrium setup. Households derive utility from consumption C_t, leisure $1-H_t$, and real money holdings $\frac{M_t}{P^C_t}$. This gives rise to demand for consumption (the Euler equation), labor supply, and demand for real money balances. Household savings are deposited (D_t) at a risk-free rate R_t in banks, which use them to lend to investors (market clearing implies $D_t = B_t$). The expected utility

$$E_t\sum_{k=0}^{\infty}\beta^k[\ln(C_{t+k}) + \zeta\ln(\frac{M_{t+k}}{P^C_{t+k}}) + \xi\ln(1-H_{t+k})] \tag{33}$$

where ζ is a preference parameter of real money holdings and ξ is a preference parameter of leisure, is maximized subject to the budget constraint

$$C_t = W_tH_t - T_t + \Pi_tR_tD_t - D_{t+1} + \frac{(M_{t-1}-M_t)}{P^C_t} \tag{34}$$

where T_t are taxes and $\Pi_t = P^C_t/P^C_{t-1}$ is consumer price inflation. The first-order conditions of this problem form the Euler equation, labor supply, and money demand:

$$\frac{1}{C_t} = E_t\{\beta\frac{1}{C_{t+1}}R_{t+1}\} \tag{35}$$

$$\frac{W_t}{C_t} = \xi\frac{1}{1-H_t} \tag{36}$$

$$\frac{M_t}{P^C_t} = \zeta C_t\left(\frac{R^n_{t+1}-1}{R^n_{t+1}}\right) \tag{37}$$

Monopolistically competitive retailers buy wholesale goods from the producers and costlessly diversify products to establish market power. Retailers set prices according to Calvo pricing, where only a fraction of retailers change prices each period. The final product is sold to households. Monopolistically competitive retailers face the Dixit-Stiglitz demand functions for the final product varieties

$$Y_t(z) = \left(\frac{P_t^C(z)}{P_t^C}\right)^{\varepsilon} \tag{38}$$

where $Y_t(z)$ is the quantity demanded, $P_t^C(z)$ is the price of consumption good z, and ε is the elasticity of substitution. The consumption goods are aggregated into final consumption bundles using

$$Y_t = \left[\int_0^1 Y_t(z)^{(\varepsilon-1)/\varepsilon} dz\right]^{\varepsilon/(\varepsilon-1)} \tag{39}$$

and the consumption price index is

$$P_t^C = \left[\int_0^1 P_t^C(z)^{(1-\varepsilon)} dz\right]^{1/(1-\varepsilon)} \tag{40}$$

In each period, only a fraction θ of retailers chooses prices $P_t^{\star}$ to maximize expected profits until the next expected price change. Retailers transfer profits back to workers' households.

$$P_t^C = [\theta {P_{t-1}^C}^{1-\varepsilon} + (1-\theta)(P_t^{C\star})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{41}$$

The optimal price-setting of monopolistically competitive retailers leads to a New Keynesian Phillips curve. In log-linear form, where lower-case letters define log-deviations from the steady state, this is expressed as:

$$\pi_t = E_{t-1}[\kappa(-x_t) + \beta \pi_{t+1}] \tag{42}$$

where β is the discount factor for workers' households and $\kappa = (1-\theta)(1-\theta\beta)/\theta$.

6.4 Government Policies and the Resource Constraint

The government consumes a fraction of output, financing it by taxes collected and seigniorage received.

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t \tag{43}$$

The central bank sets the nominal interest rate according to an inflation-targeting monetary policy rule.

$$R_{t+1}^n = (R_t^n)^{\rho}\, \Pi_t^{\psi} \varepsilon_{t+1}^{R^n} \tag{44}$$

where ε_t^{rn} is a monetary policy shock.

The resource constraint is the national accounts identity in a closed-economy setting

$$Y_t = C_t + I_t + G_t + C_t^e + \phi_t^y \tag{45}$$

where ϕ_t^y represents the resources devoted to monitoring costs, which are lost. The complete log-linearized model can be found in Appendix C, which pays special attention to describing the financial sector block.

Table 2: Calibrated Parameter Values

Parameter	Note	Value
β	quarterly discount factor	0.99
η	labor supply elasticity	3
α	capital share	0.35
$\Omega(1-\alpha)$	workers' labor share	0.64
$(1-\Omega)(1-\alpha)$	investors' labor share	0.01
δ	quarterly capital depreciation	0.025
G/Y	s.s. share of government expenditures	0.2
ϕ	elasticity of Q to capital-to-investment	0.25
K/N	capital-to-worth ratio	2
Ξ	s.s. default threshold (3% default rate)	0.19
γ	investors' quarterly departure rate	0.0272
μ	asset recovery (auditing) costs	0.12
θ	quarterly Calvo parameter	0.75
ψ	monetary policy sensitivity to inflation	0.11
ρ	monetary policy shock smoother	0.9
ρ_g	autocorrelation of government expenditure shock	0.95
ρ_a	autocorrelation of technology shock	0.999
ρ_p	autocorrelation of asset price shock	0.9

7. Model Simulations

7.1 Calibration

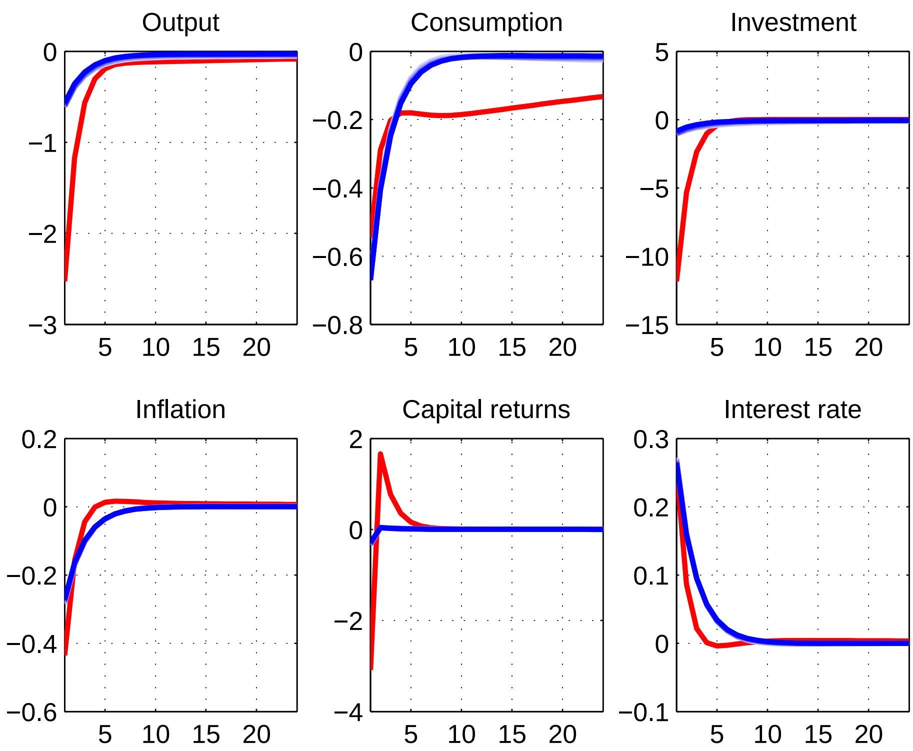
The parameters are calibrated similarly to Bernanke et al. (1999). The values are summarized in Table 2.

The idiosyncratic risk ω is log-normally distributed with $\mathrm{var}[\omega] = 0.28$ and $\mathrm{E}[\omega] = 1$. With an assumption of a 3% default rate, this implies a default threshold of 0.19 and the p.d.f. at this point is approximately equal to 0.1.

7.2 Policy Simulations

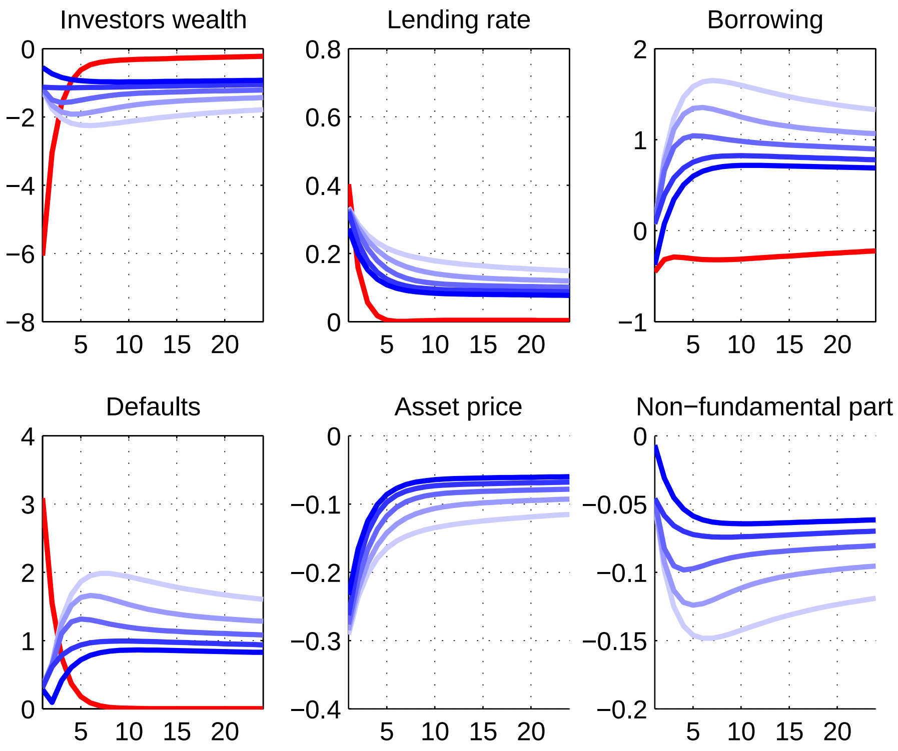
Figures 5 and 6 show the impulse responses of the model variables (expressed in log-deviations from their steady-state values) to a 0.1 percentage point increase in the nominal monetary policy rate. In all the graphs, the red line shows the responses of the benchmark Bernanke et al. (1999) model, the darker blue line shows the responses of the single-period assets version of the present model, and the lighter blue lines relate to longer maturities in the multiple-period assets version of the model. Because of the standard New Keynesian features of the model (monopoly power of retailers, price rigidities), the nominal interest rate hike transfers to an increase in the real interest rate. Consumption falls as households' optimal allocation shifts towards savings. Further, output additionally falls and inflation drops as marginal costs decrease. Investment also falls, but in a much smaller magnitude in comparison to the Bernanke et al. (1999) benchmark. The reason is that a large part of the shock is absorbed by the financial sector, most notably by the prices of risky assets, while capital returns and investment are affected much less. In a sense, the financial sector in this model works as an immediate shock absorber, as the non-fundamental price reacts to the shocks in a counter-cyclical manner by narrowing the wedge between capital returns and the risk-free rate

Figure 5: Impulse Responses of Macro Variables to a Restrictive MP Shock



Note: Impulse responses to a monetary policy shock (to the nominal interest rate) of size 0.1. The responses are in percentage deviations from the respective steady states. The red line represents the responses from the Bernanke et al. (1999) benchmark model. The darkest blue line represents the present model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).

Figure 6: Impulse Responses of Financial Variables to a Restrictive MP Shock



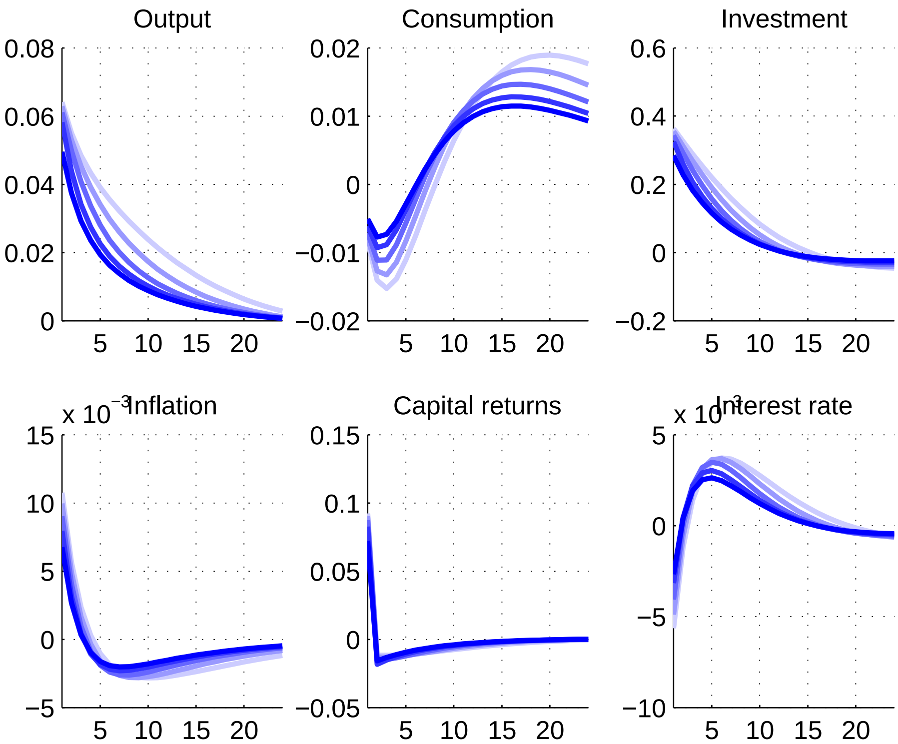
Note: Impulse responses to a monetary policy shock (to the nominal interest rate) of size 0.1. The responses are in percentage deviations from the respective steady states. The red line represents the responses from the Bernanke et al. (1999) benchmark model. The darkest blue line represents the present model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).

(25) in the case of restrictive shocks (and extending it in the case of expansionary shocks), thereby mitigating the impact of the shocks on capital returns and investment.

With a nominal interest rate hike, the financing costs of loans rise, inducing an elevated default threshold. Asset prices, including their non-fundamental component, fall (as they are discounted by the lending rate). As the cost of borrowing increases, investors' wealth falls. Unlike in the benchmark model, these reactions are weaker on impact but more persistent, giving rise to momentum of the credit cycle. In the present model, borrowing reacts with an increase after several quarters, as the investors' wealth falls and the remaining funds are borrowed, partially absorbing the shock and smoothing the investment cycle. This effect occurs because the elevated idiosyncratic risk of default makes investors demand more risky assets (which are claims on installed capital) and thus mitigates the fall of investment observed in the benchmark model.

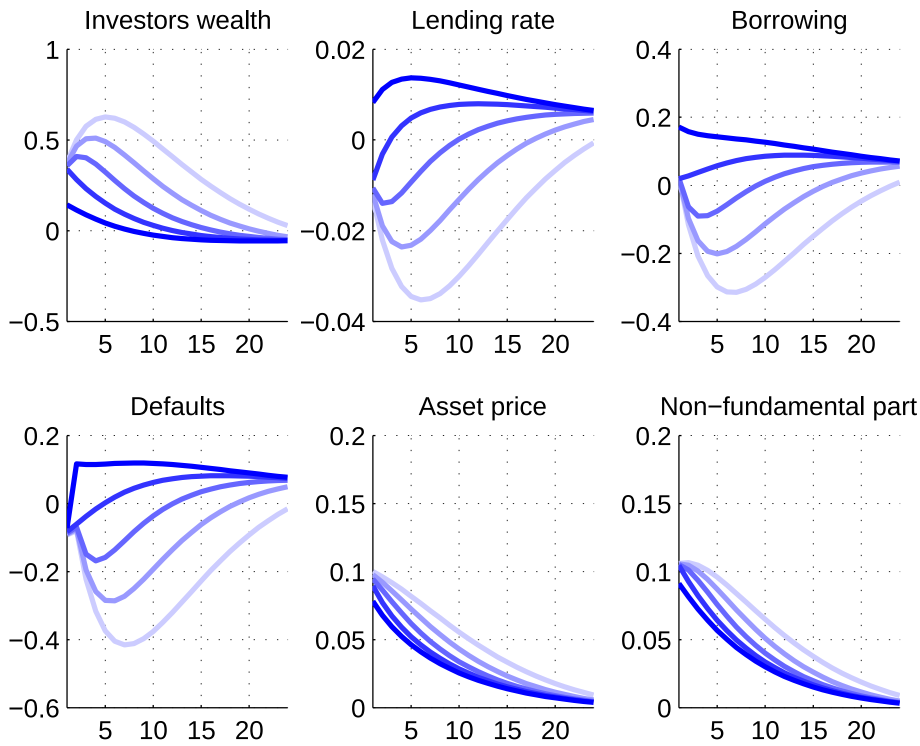
Figures 7 and 8 show the responses of the model variables to a shock to the non-fundamental component of asset prices. This shock is just an autoregressive term added to the log-linearized pricing equation (5). As asset prices rise, investors' wealth increases, inducing more investment. Consumption temporarily falls, as it is optimal to postpone consumption and invest. Because of higher marginal costs, inflation rises too. The responses of the lending rate, the amount of borrowing, and the default threshold heavily depend on whether asset prices are treated as collateral as a part of

Figure 7: Impulse Responses of Macro Variables to a Shock to the Non-Fundamental Part of the Asset Price



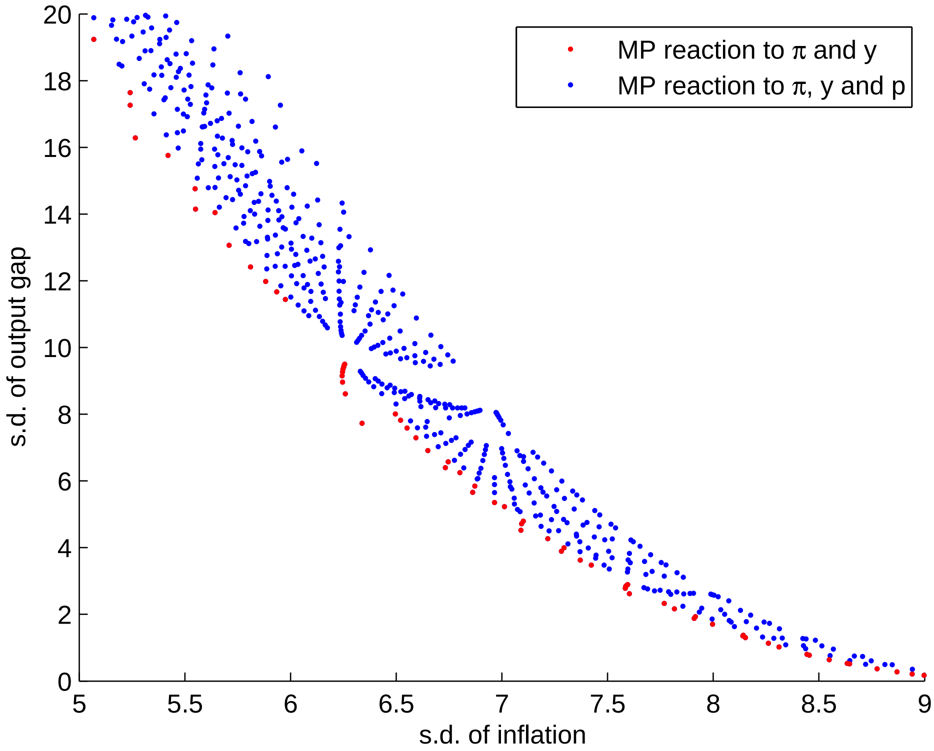
Note: Impulse responses to an asset price shock of size 0.1. The responses are in percentage deviations from the respective steady states. The darkest blue line represents the model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).

Figure 8: Impulse Responses of Financial Variables to a Shock to the Non-Fundamental Part of the Asset Price



Note: Impulse responses to an asset price shock of size 0.1. The responses are in percentage deviations from the respective steady states. The darkest blue line represents the model with single-period assets; lighter colors represent models with longer asset maturities (2 to 5 periods).

Figure 9: Monetary Policy Efficiency with a Backward-Looking Rule



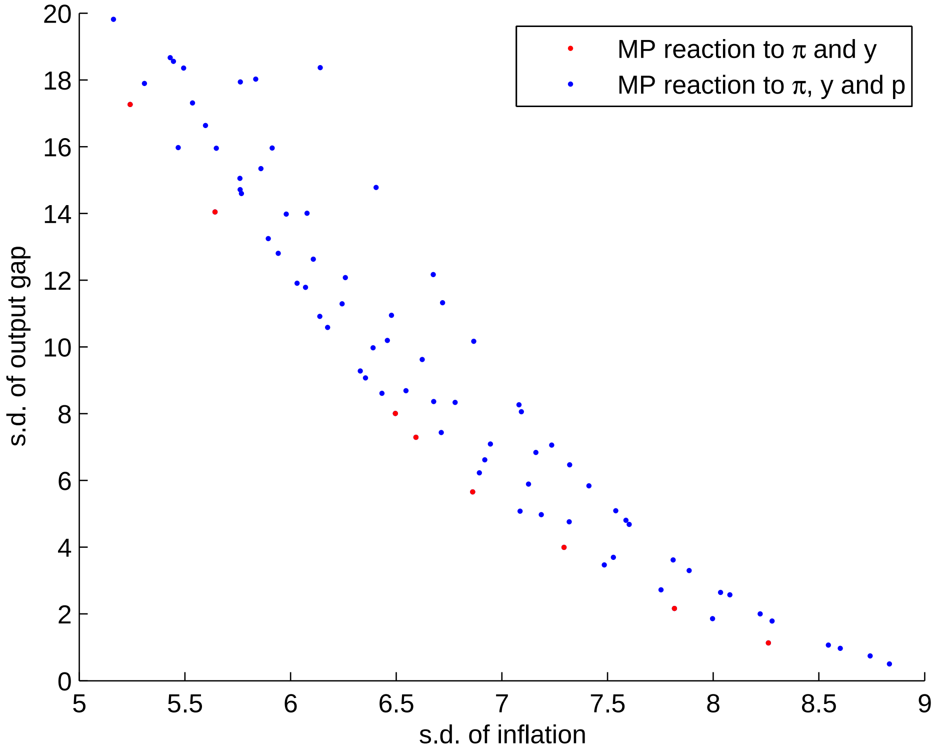
Note: The vertical and horizontal lines show the s.d. of the output gap and inflation, respectively, both in percentage deviations from the steady state. The reaction parameters are in the range (0,1) except for the low values of the inflation reaction parameter, which do not ensure determinacy. Based on 1,000 simulations.

investors' wealth and on how long the inflated prices stay in the portfolio. When portfolio turnover is fast and assets mature quickly, wealth increases only temporarily and expenditure on more expensive risky assets is financed by borrowing, which increases the default threshold and lending rate. When longer maturities dominate, wealth rises more and the lending rate falls, as do the amount of borrowing and the default threshold.

7.3 Should Monetary Policy React to Asset Prices?

For the purposes of this exercise, the assumption of a strict inflation-targeting monetary authority was relaxed, allowing the central bank also to react to the output gap and asset prices, in both the backward-looking and forward-looking policy rules. A grid search among various combinations of reaction parameters to inflation, the output gap, and asset prices was conducted to locate the combinations of the lowest implied standard deviations of the two target variables, inflation and the output gap, under shocks to technology, government spending, asset prices, and the monetary policy rate. The combinations of minimized standard deviations define the monetary policy efficiency frontier from which the central bank can choose its optimal reaction function based on its preferences (the relative disutility from output and inflation fluctuations). Figure 9 illustrates the monetary policy frontier when reacting to inflation and the output gap only (in red) in comparison the outcome when monetary policy can also react to asset prices (in blue). The version with 2-period asset maturity is used for this exercise, so that asset prices do affect the collateral constraints. The simulations

Figure 10: Monetary Policy Efficiency with a Shock to the Non-Fundamental Component of Asset Prices Only



Note: The vertical and horizontal lines show the s.d. of the output gap and inflation, respectively, both in percentage deviations from the steady state. The reaction parameters are in the range (0,1), except for the low values of the inflation reaction parameter, which do not ensure determinacy. Based on 1,000 simulations.

suggest that there is virtually no gain from reacting to asset prices, as the lower envelope of the minimized combinations of the standard deviations of inflation and the output gap are achieved when the backward-looking central bank does not react to asset prices. The reaction to the non-fundamental component of asset prices was also tested, with a similar result: a monetary policy reaction to the non-fundamental component does not lead to lower volatility of output and inflation.

The analysis for the forward-looking monetary policy rule shows the same results: efficient combinations of the standard errors of inflation and output on the monetary policy frontier are achieved when forward-looking monetary policy does not explicitly react to current asset prices. The same holds for the reaction to the non-fundamental component of asset prices. Still, this analysis does not imply that monetary policy should not react to asset prices at all. More precisely, it suggests that the reaction should not go beyond the effects which asset prices have on the inflation and output forecast. This result is broadly in line with previous research (a notable example being (Bernanke and Gertler, 1999)). Figure 10 shows the simulation in which all shocks except the shock to the non-fundamental component of asset prices were switched off. Even in this case, where asset prices should be most informative about the future evolution of the target variables, the numerical grid search was not able to find a parametrization under which adding asset prices would dominate the standard monetary policy reaction function.

The explanation for this may be that asset prices, and most importantly their non-fundamental component, work as a shock absorber that cushions exogenous shocks (including monetary policy). Interfering in the endogenous absorber mechanism then does not lead to lower volatilities of the target variables.

8. Concluding Remarks

This paper analyzed the role of asset prices and their non-fundamental component in the dynamics of the credit cycle. We presented a model of financial intermediation where the limited liability of investors gives rise to overpricing on the market for risky assets such as shares in productive firms. The model is based on the established framework of Bernanke et al. (1999), but is altered substantially to include a stock market where stocks of firms with stochastic idiosyncratic returns are traded by investors with limited liability. We show a number of results. First, the prices of assets on this market exceed their fundamental values (which are defined as in the case of the absence of the principal-agent problem, i.e., if investors did not borrow but only used their own funds for trading). Second, investors prefer to face idiosyncratic, sector-specific risk rather than diversify their portfolios, because of their limited liability. Third, we show that when the nominal amount of assets traded (i.e., the product of the asset price and the number of assets traded) maintains a sufficient growth momentum, there is also a boom in investors' wealth, credit, investment, and output. When the growth of asset prices (or traded volumes) slows down, wealth, credit, investment, and output fall below the benchmark allocations – a credit crunch occurs.

The financial sector can therefore be source of real economic fluctuations. A positive shock to asset prices gives rise to a credit cycle: first, the lending rate and default rate decrease and wealth increases. After several quarters (depending on the maturity structure of the portfolios) the lending rate and the default rate rise as the asset price falls and shrinks collateral value. At the same time, however, the financial sector also functions as a shock absorber, because in periods of elevated risk the non-fundamental component of asset prices rises as investors prefer higher risk. This, in turn, stabilizes investors' wealth and encourages funding for capital investment.

The model illustrates that an expansionary monetary policy shock temporarily boosts the financial market by reducing the lending rate and the fraction of defaulting investors and increasing asset prices (including the non-fundamental component), which in turn inflates investors' wealth.

Finally, our estimates of monetary policy efficiency frontiers suggest that reacting to asset prices or the non-fundamental component does not help achieve more favorable combinations of inflation and output gap volatilities.

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Appendix A: Proof of Overpricing of Risky Assets

Proof. We need to show that

$$P_t = \frac{1}{Z_{t+1}} \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R_{t+1}^K dF(\omega)}{(1-F(\bar{\omega}_{t+1}))} \geq \frac{R_{t+1}^K}{R_{t+1}} = P_t^F \tag{A1}$$

We define $\tilde{Z}_{t+1}$ and $\tilde{P}_t$ such that

$$\tilde{P}_t = \frac{1}{\tilde{Z}_{t+1}} \frac{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R_{t+1}^K dF(\omega)}{(1-F(\bar{\omega}_{t+1}))} = \frac{R_{t+1}^K}{R_{t+1}} = P_t^F \tag{A2}$$

By showing that $\tilde{Z}_{t+1} > Z_{t+1}$ we will prove that $P_t > P_t^F$. Using $\tilde{P}_t = P_t^F = \frac{R_{t+1}^K}{R_{t+1}}$:

$$\tilde{Z}_{t+1}(1-F(\bar{\omega}_{t+1})) = \frac{R_{t+1}}{R_{t+1}^K} \int_{\bar{\omega}_{t+1}}^{\infty} \omega R_{t+1}^K dF(\omega) \tag{A3}$$

Eliminating R_{t+1}^K and multiplying by B_{t+1}:

$$\tilde{Z}_{t+1}B_{t+1}(1-F(\bar{\omega}_{t+1})) = R_{t+1}B_{t+1} \underbrace{\int_{\bar{\omega}_{t+1}}^{\infty} \omega dF(\omega)}_{>1} > R_{t+1}B_{t+1} \tag{A4}$$

Now using the banks' participation constraint (2):

$$Z_{t+1}B_{t+1}(1-F(\bar{\omega}_{t+1})) = R_{t+1}B_{t+1} \underbrace{-(1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega R_{t+1}^K S_{t+1} dF(\omega)}_{<0} < R_{t+1}B_{t+1} \tag{A5}$$

therefore $\tilde{Z}_{t+1} > Z_{t+1}$ and $P_t > P_t^F$. □

Therefore, the risky asset is overpriced in comparison to the fundamental price.

Appendix B: Investors Prefer Not to Diversify

We show that everything else being equal, investors prefer non-diversified over diversified portfolios in the presence of limited liability. Therefore, investors do not have incentives to deviate from the no-diversification equilibrium and the equilibrium is stable.

The expected return on shares net of financing costs under full diversification (no risk) equals

$$R_{t+1}^K - Z_{t+1}B_{t+1} = \int_0^{\infty} \omega R_{t+1}^K - Z_{t+1}B_{t+1} dF(\omega) \tag{B1}$$

The expected return on shares net of expected financing costs when there is no diversification and the investor with limited liability fully faces the idiosyncratic risk is

$$E[R_{t+1}^K - Z_{t+1}B_{t+1} | \omega > \bar{\omega}_{t+1}] = \int_{\bar{\omega}_{t+1}}^{\infty} \omega R_{t+1}^K - Z_{t+1}B_{t+1} dF(\omega) \tag{B2}$$

Now it is obvious that $E[R^K_{t+1} - Z_{t+1}B_{t+1}|\omega > \bar{\omega}_{t+1}] > R^K_{t+1} - Z_{t+1}B_{t+1}$ as

$$\underbrace{\int_0^{\infty} \omega R^K_{t+1} - Z_{t+1}B_{t+1} dF(\omega)}_{R^K_{t+1} - Z_{t+1}B_{t+1}} = \underbrace{\int_0^{\bar{\omega}_{t+1}} \omega R^K_{t+1} - Z_{t+1}B_{t+1} dF(\omega)}_{<0} + \underbrace{\int_{\bar{\omega}_{t+1}}^{\infty} \omega R^K_{t+1} - Z_{t+1}B_{t+1} dF(\omega)}_{E[R^K_{t+1} - Z_{t+1}B_{t+1}|\omega > \bar{\omega}_{t+1}]} \tag{B3}$$

Therefore, the investor prefers idiosyncratic risk over diversification. Because of the limited liability, the investor enjoys profits in the case of success, while he does not internalize the costs in the case of default. Limited liability thus shifts investors' demand for risk.

Appendix C: Log-Linearized Model

This section of the appendix presents the log-linearized model including the extension of two-period assets as it enters the simulations. The financial sector block is discussed in greater detail than other parts, which come directly from Bernanke et al. (1999). The lower-case letters denote log-deviations from steady-state values. The ratios of capital letters denote the steady-state values of the respective ratios. Further, we define the nominal interest rate $r^n_{t+1} = r_{t+1} + E[\pi_{t+1}]$.

C.1 Firms

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t \tag{C1}$$

$$k_{t+1} = \delta i_t + (1-\delta)k_t \tag{C2}$$

$$r^k_{t+1} = (1-\varepsilon)(y_{t+1} - k_{t+1} - x_{t+1}) + \varepsilon q_{t+1} - q_t \tag{C3}$$

$$q_t = \phi(i_t - k_t) \tag{C4}$$

C.2 Retailers and Households

$$\pi_t = E_{t-1}[\kappa(-x_t) + \beta \pi_{t+1}] \tag{C5}$$

$$c_t = -r_{t+1} + E_t[c_{t+1}] \tag{C6}$$

$$c^e_t = n_{t+1} + \psi^{c^e}_t \tag{C7}$$

$$y_t - h_t - x_t - c_t = \eta^{-1} h_t \tag{C8}$$

C.3 Financial Intermediation

Although some of the log-linearized equations and variables could be eliminated in this block, the full set of equations is presented and the dynamics of all corresponding variables are shown, as the financial sector is a key part of the general equilibrium model. In addition to the main text, we introduce a shock to the (non-fundamental) asset price, υ_t, with a standard deviation of 0.1, corresponding to the empirical moment of the log-deviation of the quarterly Dow Jones index from its HP trend. Ξ stands for the steady-state value and ϖ for the log-deviation from the steady state of the default threshold $\bar{\omega}_{t+1}$, while $f(\Xi)$ is the probability density function of the idiosyncratic return distribution evaluated at the steady-state default threshold.

$$z_{t+1} - r_{t+1} = \frac{1}{2}(1 - \mu f(\Xi))\Xi\varpi_{t+1}; \tag{C9}$$

$$p_t = E\left[r^k_{t+1}\right] - z_{t+1} + f(\Xi)\Xi E\left[\varpi_{t+1}\right] + \upsilon_t; \tag{C10}$$

$$p^f_t = p_t - (r^k_{t+1} - r_{t+1}) \tag{C11}$$

$$\varpi_t + r^k_t + k_t = z_t + b_t \tag{C12}$$

$$p_t + q_t + k_{t+1} = \frac{N}{QK} n_{t+1} + (1 - \frac{N}{QK}) b_{t+1} \tag{C13}$$

$$n_{t+1} = \frac{\gamma RK}{N}(r^k_t - r_t) + r_t + n_t \tag{C14}$$

$$p_t = \iota(q_t + k_{t+1}) \tag{C15}$$

Equation (C9) is the bank participation constraint, linking the risk premium to the default threshold. Equation (C10) is the log-linear version of the demand for risky assets, which, together with the previous equations, defines the wedge between the risk-free rate and capital returns as a function of the default threshold and asset prices. When we examine the determinacy conditions of the model, it turns out that the term involving the expectation of the default threshold in equation (C10) has to be lower than the right-hand side of equation (C9) for the model to be determined, i.e., default has to be a sufficiently improbable (tail) event. Equation (C11) defines the fundamental price, equation (C12) defines the default threshold, and equation (C13) defines borrowing. Equation (C14) is the law of motion of investors' wealth. In the case where assets are held for multiple periods, this equation changes to

$$n_{t+1} = \frac{\gamma RK}{N}(r^k_t + p_t - r_t - pcost_{t-1}) + r_t + n_t \tag{C16}$$

where the asset price dynamics enter the investors' wealth. The cost of previously purchased assets $pcost_{t-1}$ equals p_{t-1} in the case of two-period assets, but can include longer lags of asset purchasing costs to create the desired mix of asset maturities in the portfolio. Similarly to Bernanke et al. (1999), we assume that the composition of portfolios is stable over time and the shares of assets held (including different maturities) stay close to their steady-state values. In the plots presented in the main text, maturities from one to four quarters ahead are combined in the respective portfolios with equal weights. Finally, equation (C15) describes the supply side of the investment asset market, linking its price to the quantity of investment opportunities produced via increasing marginal costs.

C.4 Monetary Policy, the Resource Constraint, and Shocks

$$r_t^n = \rho r_{t-1}^n + \psi \pi_{t-1} + \varepsilon_t^{r^n} \tag{C17}$$

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t + \frac{C^e}{Y} c_t^e + \psi_t^y \tag{C18}$$

$$g_t = \rho_g g_{t-1} + \varepsilon_t^g \tag{C19}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \tag{C20}$$

$$\upsilon_t = \rho_p \upsilon_{t-1} + \varepsilon_t^p \tag{C21}$$