When Preferences Cap Consumption: A Scrooge McDuck Theory of Wealth Dynamics

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Abstract

This paper introduces a new theoretical mechanism to explain the joint evolution of the wealth-to-output ratio, investment, and income and wealth inequality observed in advanced economies over recent decades. Agents derive insatiable utility from wealth, which translates into an upper bound on their optimal consumption. Those at the top of the permanent income distribution, referred to as *Scrooge McDuck*, accumulate unbounded wealth while keeping their consumption levels bounded. A portion of their wealth is held purely for the sake of holding it, with the associated dividends being fully saved. This drives up asset prices and support the existence of rational bubbles that crowd out investment. The aggregate bubble is uniquely determined and grow at a rate exceeding that of the economy. The model predicts a positive correlation between bubble price and both income and wealth inequality, all three diverging over time.

Keywords: Income inequality, Rational bubble, Wealth preference, Wealth-to-output
JEL Classification: E21, E22

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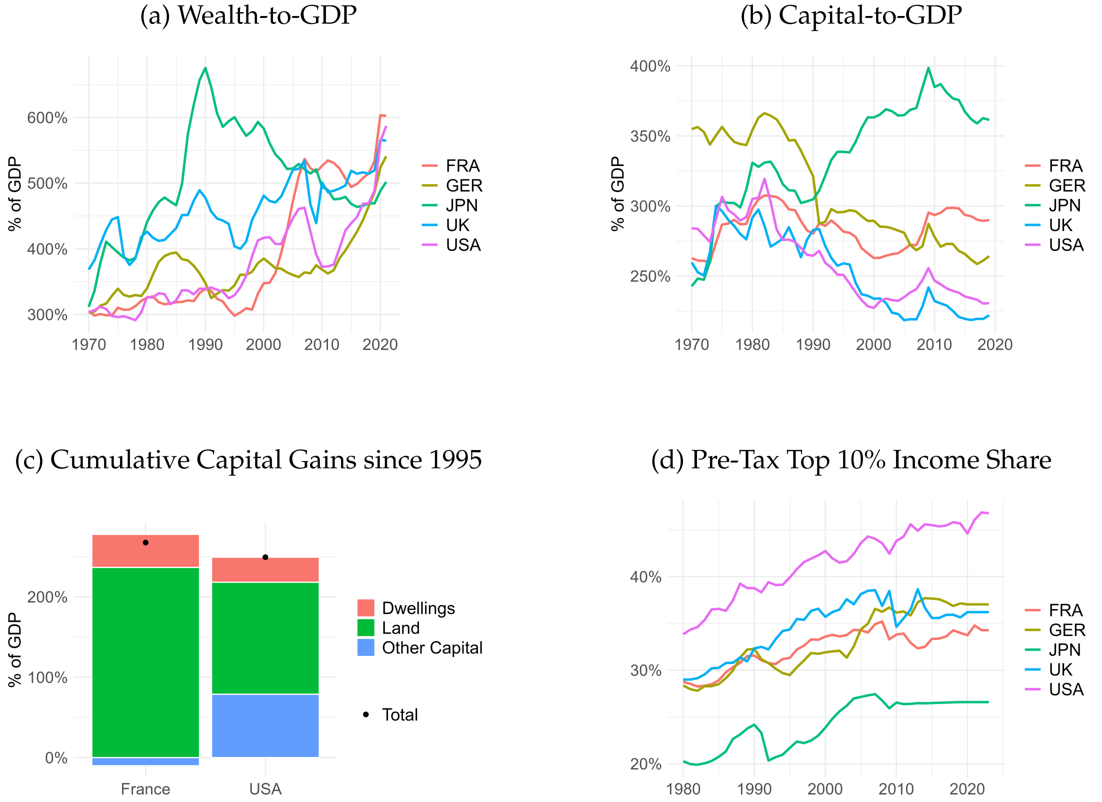
1 Introduction

Over recent decades, advanced economies have seen a rise in their wealth-to-output ratio (Figure 1a). This global trend has been marked by two key features. First, it was largely driven by capital gains rather than increased investment (Figure 1b). Second, the types of assets benefiting from these capital gains varied meaningfully across regions (Figure 1c). The latter aspect has led some lines of research to develop country-specific explanations for a worldwide phenomenon. For instance, in the US, the rise in stock values is often attributed to increasing market power (Farhi and Gourio, 2019; Eggertsson et al., 2021), while the low elasticity of housing supply is frequently cited to explain the rise in real estate prices in Europe (Hilber and Vermeulen, 2016; Muellbauer, 2018). Conversely, another strand of the literature has examined the impact of a global and concomitant phenomenon on the wealth-to-output ratio: the rise in permanent income inequality (Figure 1d). Empirical evidence indicates that saving rates increase with permanent income (Dynan et al., 2004; Straub, 2019), suggesting that a more unequal distribution of income could raise aggregate savings and wealth. However, models that incorporate this mechanism (e.g., Hubmer et al., 2021; Michau et al., 2023b) attribute the rise in the wealth-to-output ratio to a counterfactual rise in investment rather than to capital gains.

In line with this second approach, this paper proposes a framework in which an increase in permanent income inequality raises the wealth-to-output ratio, but not necessarily the capital-to-output ratio. It posits that agents derive utility from holding wealth. This feature has proven effective in generating higher saving rates at the top of the permanent income distribution (e.g., De Nardi, 2004; Mian et al., 2021), but typically only during the transition to a steady state. This is because, in a dynamically efficient economy, an agent maintaining above-average saving rates in every period would accumulate unbounded wealth relative to output. Under standard wealth-in-utility (WIU), such unbounded accumulation is only optimal if consumption also diverges relative to output—a scenario typically precluded by the goods market clearing condition.

This study departs from traditional WIU models by introducing insatiable preferences for wealth, wherein the marginal utility of holding wealth remains bounded below by a strictly positive constant, even as wealth approaches infinity. This generates persistently higher saving rates at the top of the permanent income distribution than at the bottom, even asymptotically. Treating the positive correlation between permanent income and saving rates as a non-transitory feature aligns with its persistent observation in the data over time (e.g. Duesenberry, 1949). In the model, insatiable preferences for wealth implies that the marginal utility of saving for any agent—even one with arbitrarily large wealth—never approaches zero. Consequently, the marginal utility of consumption also does not tend to zero. With utility from consumption satisfying the Inada conditions, it follows that consumption is bounded, with an implicitly defined maximum optimal level of consumption. If the utility from wealth is sufficiently strong, this consumption cap causes agents at the top of the income distribution to accumulate

Figure 1: Wealth dynamics and income inequality



Notes: Panel (a) displays the total market value of national non-financial assets (public and private) relative to GDP (source: World Inequality Database). Panel (b) presents the combined public and private capital stock as a share of GDP (source: IMF Investment and Capital Stock Dataset). Panel (c) depicts capital gains by asset type realized in the private non-financial sector (excluding non-profits), calculated using official national accounting data. Details on the construction of these capital gains are provided in Appendix A.1. Panel (d) presents the pre-tax income share of the top 10% (source: World Inequality Database).

unbounded wealth while maintaining bounded consumption levels, both expressed in terms of output. Such agents are referred to as *Scrooge McDuck* agents, and their consumption paths remain consistent with the goods market clearing condition.

The presence of Scrooge McDuck agents affects asset pricing by generating a rational bubble that grows at a rate exceeding the growth rate of the economy. Agents can save in two types of assets: reproducible capital and an infinitely-lived nonreproducible productive asset, referred to as land. The existence of land prevents excessive capital accumulation, which would otherwise drive the economy's rate of return below the growth rate[1]. The present value of Scrooge McDuck's future consumption is lower than the sum of their current wealth and the present value of their future labor income. Hence, these agents hold a *surplus wealth* that does not finance any future consumption and is held only for the purpose of being held. Dividends from this surplus wealth are fully saved, causing the ratio of surplus wealth to output to diverge. Given a bounded capital-to-output ratio, surplus wealth is asymptotically entirely invested in land, whose price diverges relative to output. Since the fundamental value of land relative to

[1]This argument follows Rhee (1991), assuming that the infinitely-lived productive asset has an asymptotic positive factor share.

output remains bounded, the equilibrium price of land incorporates a rational bubble term. The bubble size coincides with the aggregate surplus wealth with both growing at the economy's rate of return. If the bubble were smaller than the surplus wealth, the diverging discrepancy between them would be saved in capital, leading to dynamic inefficiency, which is ruled out. Conversely, if the bubble were larger than the surplus wealth, it would completely crowd out the capital stock.

By decoupling the dynamics of fundamental and market values, the Scrooge McDuck theory explains two key trends in the data that remain unexplained under standard WIU models. The bubble, by crowding out investment, accounts for the observed increase in the wealth-to-output ratio without implying a counterfactual rise in investment. Furthermore, it can be attached to various categories of real-world assets, not just land, aligning with evidence that capital gains are realized on different asset classes across countries.

The proposed mechanism has significant implications for the dynamics of income and wealth inequality and their interplay with the wealth-to-output ratio. As in standard WIU models, income and wealth inequality are positively correlated with the wealth-to-output ratio in the Scrooge McDuck framework. Yet, unlike in those models, greater inequality does not necessarily translate into higher investment. Instead, it can lead to a larger surplus wealth held by Scrooge McDuck agents, which, in turn, increases the value of the bubble. Whether the increase in wealth is investment-driven or bubble-driven has important implications for consumption levels: as the economy is dynamically efficient, a larger bubble reduces consumption. Additionally, the presence of a bubble prevents the capital stock from converging to the level that maximizes asymptotic aggregate consumption.

The present model also sheds light on the increase in wealth inequality by predicting ever-growing wealth disparity between the Scrooge McDuck agents and the rest of the distribution. Asymptotically, non-Scrooge McDuck agents tend to hold zero units of the rational bubble; otherwise, they would accumulate unbounded wealth, violating their intertemporal optimization problem. However, these agents continue to benefit from capital gains on their arbitrarily small bubble holdings as the bubble price diverges relative to output. This ensures that their asymptotic absolute wealth converges to a strictly positive level with respect to output. The rise in wealth inequality is hence driven by increasing absolute wealth at the top, rather than by a decline in wealth among the bottom and middle segments of the distribution. This aligns with empirical evidence from the middle 40%, which has benefited from capital gains—particularly in housing—across many advanced economies since the 1980s (Bauluz et al., 2022; Blanchet and Martínez-Toledano, 2023). Despite this positive effect, the bubble's overall impact on the utility of non-Scrooge McDuck agents remains ambiguous, as it is counterbalanced by the negative effects of crowding out.

Related Literature. This paper builds on multiple empirical studies documenting, across advanced economies, rising income and wealth inequality (Katz and Murphy, 1992, Piketty and Saez, 2003, Saez and Zucman, 2016; Batty et al., 2019; Chancel et al., 2022; Smith et al., 2023), an increase in aggregate wealth driven by capital gains (Piketty and Zucman, 2014), and a declining trend in investment (Gutiérrez and Philippon, 2017). It theoretically contributes to three strands of the literature.

First, on empirical evidence that saving rates increase with income and wealth (Carroll, 2000; Dynan et al., 2004; Straub, 2019; Fagereng et al., 2021) and that aggregate savings are primarily driven by the top of the distribution (Mian et al., 2020; Bauluz et al., 2022), a growing theoretical and quantitative literature highlights the importance of non-homothetic preferences for wealth in explaining wealth inequality and the wealth-to-output ratio (Carroll, 2000; De Nardi, 2004; Kumhof et al., 2015; De Nardi and Yang, 2016; Benhabib et al., 2019; Mian et al., 2021; Elina and Huleux, 2023; Gaillard et al., 2023; Michau et al., 2023a). The present framework departs from this literature by assuming insatiable preferences for wealth, a critical assumption for the existence of rational bubbles that drive investmentless increases in wealth. Building on this mechanism, it is the first to predict ever-growing wealth inequality, driven by a non-zero mass of agents holding diverging levels of wealth.[2]

Secondly, this work contributes to the rational bubble literature. Previous research has established that rational bubbles can exist in dynamically inefficient economies (Samuelson, 1958; Diamond, 1965; Tirole, 1985; Michau et al., 2023b) or under financial frictions (Farhi and Tirole, 2012; Martin and Ventura, 2012; Reis, 2021). In both cases, the rate of return on bubbles is below the economy's growth rate, leading to an asymptotically stationary bubble-to-output ratio. In contrast, few studies have explored frameworks with a diverging bubble-to-output ratio. Notable exceptions include Ono (1994) and Kamihigashi (2008), which examine representative agent models with insatiable preferences for liquidity or wealth, finding an infinite set of diverging bubbly equilibria. By introducing heterogeneous agents, the present analysis departs from these approaches and results in a uniquely determined diverging equilibrium.

Finally, this paper contributes to the recent debate on the welfare implications of capital gains that are not driven by changes in expected future payoffs. When capital gains result from a decrease in the discount rate, they mechanically lead to higher wealth inequality without affecting the distribution of capital income. Such gains have been interpreted in various ways: as a pure increase in welfare inequality (Saez et al., 2021), as mere "paper gains" with no welfare effect (Cochrane, 2020; Krugman, 2021), or as a mix of both, depending on net asset sales (Fagereng et al., 2024). This analysis introduces another form of capital gains, arising from rational bubbles. Unlike those generated by a decrease in the discount rate, these gains are persistent, enabling non-Scrooge McDuck agents to steadily benefit from them and sustain their consumption over time.

[2]By featuring an asymptotic rate of return above the economy's growth rate, this paper provides a theoretical foundation for Piketty (2014)'s conjecture, which states that "if the rate of return on capital remains significantly above the growth rate for an extended period, then the risk of divergence in the distribution of wealth is very high" (Piketty, 2014, page 34).

The remainder of the paper comprises five sections. Section 2 develops first the essence of the mechanism in an endowment economy. Section 3 extends the analysis to a production economy, highlighting the crowding out effect of rational bubbles. To assess the empirical relevance of the Scrooge McDuck mechanism, a quantitative analysis is conducted in Section 4. Section 5 concludes.

2 Endowment Economy

2.1 Environment.

To highlight the essence of the Scrooge McDuck mechanism, a deterministic non-growing endowment economy with preferences is considered. Time is discrete and runs from $t = 0$ to ∞. The endowment comes from a single unit of Lucas tree, which delivers one unit of the consumption good each period and is priced at q_t at time t after dividend payment. The rate of return of the Lucas tree is defined as $R_{t+1} \equiv \frac{1+q_{t+1}}{q_t}$.

Households. There is a unit mass of infinitely-lived households, divided into two types that differ only in their initial endowment of Lucas tree units. Agents with a high initial endowment are called high-endowment agents (H) and constitute a share $\lambda \in (0;1)$ of the population. The remaining share, $1-\lambda$, consists of low-endowment agents (L). Agents of type $i \in \{H, L\}$ hold ℓ_t^i units of the Lucas tree at the beginning of period t and consume c_t^i at time t. Their wealth at the end of period t is defined as $a_{t+1}^i \equiv q_t \ell_{t+1}^i$. The initial endowments are given with $\ell_0^H \geq \ell_0^L > 0$.

Both types of agents derive utility from consumption and from holding wealth in every period. They discount the future at a rate $\beta \in [0,1)$ and maximize their intertemporal utility:

$$\sum_{t=0}^{\infty} \beta^t U(c_t^i, a_{t+1}^i) \qquad \text{with} \qquad U(c,a) = u(c) + v(a). \tag{1}$$

Utility from consumption is represented by the term $u(c)$, which satisfies the Inada conditions. Preference for wealth is captured by the term $v(a)$, which is assumed to be twice differentiable, increasing, and concave with the following property:

$$\lim_{a \to \infty} v'(a) = \kappa \qquad \text{with} \qquad \kappa > 0. \tag{2}$$

The only deviation of this paper from standard WIU is the assumption that the parameter κ is strictly above zero. Given that the marginal utility of wealth is then always strictly positive and does not converge toward zero, these preferences are referred to as *insatiable* preferences for wealth. Standard and insatiable preferences for wealth are observationally equivalent, as the asymptotic limit case and a diverging wealth cannot be observed. As will be shown later, asuming κ to be positive should therefore be regarded as a modeling tool to generate persistently increasing saving rates with permanent income, rather than restricting this feature to the transition toward a steady state.

In order to match the empirical evidence that saving rates increase with permanent income (Dynan et al., 2004; Straub, 2019) and in line with the existing literature on preferences for wealth, it is assumed that $v'(a)$ decreases at a slower rate than $u'(c)$. Formally, the non-homotheticity of consumption-saving behavior arises from the following assumption:

Assumption 1 *(Non-homothetic preferences) For any q, $\frac{v'(q\ell)}{u'(\ell)}$ is strictly increasing in ℓ.*

Assumption 1 implies that if agents were to consume exactly their dividend, wealthier agents would exhibit a relatively stronger preference for wealth over consumption compared to less wealthy agents. In equilibrium, this will lead to higher saving rates at the top of the wealth distribution relative to the bottom. Furthermore, households are subject to the following budget constraint:

$$c_t^i + a_{t+1}^i = R_t a_t^i. \tag{3}$$

To ensure the absence of Ponzi schemes, borrowing is ruled out, as all household resources are derived from their Lucas tree units:

$$\ell_t^i \geq 0. \tag{4}$$

Whenever $\{q_t\}_{t=0}^{\infty}$ is defined, the solution to the optimization problem for type i households is characterized by the following Euler equation and the transversality condition[3]:

$$u'(c_t^i) = \beta R_{t+1} u'(c_{t+1}^i) + v'(a_{t+1}^i), \tag{5}$$

$$\lim_{t\to\infty} \beta^t [u'(c_t^i) - v'(a_t^i)] a_t^i = 0. \tag{6}$$

Market Clearing Condition In each period, there is one unit of the Lucas tree and one unit of the non-storable consumption good. The market clearing condition for assets is therefore expressed as:

$$\lambda \ell_t^H + (1-\lambda)\ell_t^L = 1, \tag{7}$$

and the goods market clearing condition is given by:

$$\lambda c_t^H + (1-\lambda) c_t^L = 1. \tag{8}$$

Equilibrium. Given the initial endowment ℓ_0^H, an equilibrium $\{c_t^H, \ell_t^H, q_t\}_{t=0}^{\infty}$ is characterized by the solutions to the household optimization problems and the asset market clearing condition.

[3] A proof of the necessity of the transversality condition is provided in Appendix B.1.1

A *steady state* is defined as an equilibrium in which $c_t^i = c^i$ and $l_t^i = \ell^i$ for all periods t. Two sub-cases of steady states are distinguished: the *egalitarian steady state*, where $c^H = c^L$ and $\ell^H = \ell^L$, and the *perfect inequality steady state*, characterized by the absence of wealth for one type of agent $i \in \{L, H\}$, such that $\ell^i = 0$. Finally, a *diverging equilibrium* is characterized by a Lucas tree price q_t that diverges over time.

2.2 Steady State Equilibrium

Consider the steady-state equilibria where, for all t, $\{c_t^L, c_t^H, \ell_t^L, \ell_t^H\} = \{c^L, c^H, \ell^L, \ell^H\}$. To facilitate the derivation of these steady states, the following corollary of Assumption 1 is established:

Corollary 1 *(Saving rates increasing in wealth) Consider two agents i and j with Lucas tree holdings at the beginning of period t, such that $\ell_t^i > \ell_t^j$. Under Assumption 1, whenever $\{q_t\}_{t=0}^{\infty}$ is defined, the following inequality holds:*

$$\frac{c_t^i}{\ell_t^i} < \frac{c_t^j}{\ell_t^j}. \tag{9}$$

A formal proof of Corollary 1 is provided in Appendix B.1.2. Saving rates increase with income or wealth—both captured by the same variable ℓ_t^i—an unsurprising result, as Assumption 1 was explicitly designed to produce this feature.

The steady-state analysis proceeds in two steps: first, focusing on equilibria where all agents hold some wealth, and then examining the perfect inequality steady states. In steady states where $\ell_t^H \in (0, 1/\lambda)$, both types of agents must have the same ratio of marginal utility of wealth to marginal utility of consumption. If this condition is not satisfied, agents with a stronger preference for wealth relative to consumption will acquire more Lucas tree units, while others will reduce their savings. Consequently, the following equation, derived from the combination of the Euler equations for both types of agents, must hold:

$$\frac{v'(q\ell^H)}{u'(c^H)} = \frac{v'(q\ell^L)}{u'(c^L)}, \tag{10}$$

which directly leads to Lemma 1.

Lemma 1 *(Egalitarian Steady State) Under Assumption 1, there exists a unique steady-state equilibrium with $\ell^H \in (0, 1/\lambda)$, corresponding to the egalitarian steady state, which is unstable.*

The uniqueness result follows directly from equation 10 and Assumption 1, while the instability arises as a consequence of Corollary 1. Intuitively, outside the egalitarian steady state, some agents possess greater wealth than others. Wealthier agents display a higher marginal preference for wealth relative to their utility from consumption. They save at higher rates than less wealthy agents, preventing the economy from converging to the egalitarian level. As such, the egalitarian steady state is relevant only when all agents start with the same initial endowment, $\ell_0^H = \ell_0^L$, and is otherwise unattainable.

The case of perfect inequality steady states is now addressed. While such states exist, those in which all wealth is held by type L agents are not considered here, as the economy would not converge to them. More relevant are the perfect inequality steady states in which all wealth is held by type H agents, characterized by the following lemma.

Lemma 2 *(Perfect Inequality Steady State) There exists a unique steady-state equilibrium with $\ell^H = 1/\lambda$, characterized by $c^H = 1/\lambda$. Under Assumption 1, this steady state exhibits the following properties:*

- *it is locally stable, with $q_t = q$ in every period t, if $\kappa \leq \frac{1-\beta}{\beta} u'(1/\lambda)$,*
- *it is locally unstable, with q_t undetermined in every period t, if $\kappa > \frac{1-\beta}{\beta} u'(1/\lambda)$.*

In the perfect inequality steady state, type L agents have neither wealth, $a^L = 0$, nor consumption, $c^L = 0$, resulting in $c^H = 1/\lambda$. Consequently, only two variables remain to be determined: c^H and q. These are derived from the steady-state versions of the budget constraint (11) and the Euler equation (12) for type H agents, with $R \equiv \frac{1+q}{q}$:

$$a^H = \frac{c^H}{R-1}, \tag{11}$$

$$R = \frac{1}{\beta}\Big[1 - \frac{v'(a^H)}{u'(c^H)}\Big]. \tag{12}$$

When $\kappa \leq \frac{1-\beta}{\beta} u'(1/\lambda)$, there is a unique value of $R \in (1, \infty)$ that satisfies both equations with equality, thereby determining the values of q and a^H. Indeed, there always exists a value of R sufficiently close to one that increases the wealth of type H agents (as given by Equation 11) to a level high enough to lower their marginal utility of holding wealth and making them indifferent between saving and consuming (as per Equation 12). The local stability of this equilibrium follows directly from Corollary 1. More generally, given the higher saving rates of type H agents, the economy converges toward the steady state where the entire Lucas tree is held by type H agents whenever $\ell_0^H > \ell_0^L$. As in standard WIU models, steady-state wealth distributions in which agents at the bottom of the distribution hold positive wealth can only be achieved by introducing additional elements, such as borrowing constraints (Mian et al., 2021), mortality (Elina and Huleux, 2023), or exogenous preference shocks (Michau et al., 2023a), for example.

However, when $\kappa > \frac{1-\beta}{\beta} u'(1/\lambda)$, solving equations 11 and 12 yields $R < 1$, which is ruled out as it would imply a negative q. This corresponds to a scenario where, even as the wealth of type H agents diverges, their marginal utility from saving does not decrease sufficiently to make them indifferent between consuming and saving. In this case, insatiable preferences for wealth impose a lower bound on the marginal utility of wealth, $v'(a^H) > \kappa$, and consequently on $u'(c^H)$:

$$u'(c^H) > \sum_{s=0}^{\infty} (\beta R)^s \kappa, \tag{13}$$

where the right-hand side of Equation 13 represents the lower bound of the marginal utility of holding an additional unit of wealth and all its associated returns indefinitely. This leads to the implicit definition of an upper bound on c^H:

$$c^H < \bar{c}(R) \qquad \text{with} \qquad \bar{c}(R) \equiv u'^{-1}\Big(\frac{\beta R}{1-\beta R}\kappa\Big). \tag{14}$$

It follows from Equation 14 that, whenever R is defined and above 1, the optimal consumption of type H agents would be below $1/\lambda$. Consequently, type H agents would receive a dividend of $1/\lambda$ but not consume all of it. Since their marginal utility from consumption is positive, they would only refrain from consuming the entire dividend if they could exchange the unconsumed goods for additional Lucas tree units. However, this exchange is not possible: they cannot trade with type L agents, as $a^L = 0$, nor with other type H agents, as all of them prefer saving over consuming. Therefore, no equilibrium price for the Lucas tree exists, and q is undefined[4].

Under $\kappa > \frac{1-\beta}{\beta} u'(1/\lambda)$, the perfect inequality steady state, in which all wealth is held by type H agents, is locally unstable. When type L agents hold a positive quantity of Lucas tree units, there exists a price q_t at which type H agents exchange consumption goods for Lucas tree units from type L agents. In subsequent periods, the Lucas tree holdings of type L agents remain strictly positive, ensuring that q_t is well-defined for all $t \in [t, \infty)$. As a result, the economy does not revert to the perfect inequality steady state.

2.3 Diverging Equilibria

This section assumes $\kappa \in \Big(\frac{1-\beta}{\beta} u'(1/\lambda), \frac{1-\beta}{\beta} u'(1)\Big)$ and an initial inequality in Lucas tree holdings, $\ell_0^H > \ell_0^L$, for reasons that will become clearer in the analysis. As seen in previous subsection, under these parameter assumptions, the economy does not converge to a steady state. Instead, the equilibrium is diverging and is characterized by Lemma 3.

[4]The price mechanism fails to operate in this context because the equilibrium price of the Lucas tree directly influences the utility derived from it. Higher demand for the Lucas tree increases its price, which, in turn, raises the utility that can be derived from holding it, further driving up its demand and its price. Unlike in standard WIU models, this feedback mechanism persists under insatiable preferences for wealth as the price of the Lucas tree diverges.

Lemma 3 *(Diverging Equilibrium) For $\kappa \in \left(\frac{1-\beta}{\beta}u'(1/\lambda), \frac{1-\beta}{\beta}u'(1)\right)$ and $\ell_0^H > \ell_0^L$, there exists a unique equilibrium, which is a diverging equilibrium. The variables $\{c_t^L, c_t^H, a_t^L, R_t\}$ converge to their asymptotic values $\{c^L, c^H, a^L, R\}$ as $t \to \infty$.*

Lemma 3 can be both proved and better understood by rewriting the budget constraint for type H agents as:

$$q_t \times (\ell_{t+1}^H - \ell_t^H) = \ell_t^H - c_t^H. \tag{15}$$

Agents of type H receive a dividend ℓ_t^H from their Lucas tree units at the beginning of period t. Following Corollary 1, they do not consume their full dividend but instead exchange the unconsumed portion, $\ell_t^H - c_t^H$, with type L agents for additional Lucas tree units, $\ell_{t+1}^H - \ell_t^H > 0$. On the one hand, since the economy does not converge to either the egalitarian steady state or the perfect inequality steady state, the quantity of goods that type H agents are willing to exchange does not approach zero. On the other hand, given that the quantity of Lucas tree units in the economy is bounded, it must hold that $\lim_{t\to\infty}(\ell_{t+1}^H - \ell_t^H) = 0$. Asymptotically, type H agents exchange a quantity of goods significantly greater than zero for an arbitrarily small quantity of Lucas tree units. In other words, the price of the Lucas tree diverges,

$$\lim_{t\to\infty} q_t = \infty. \tag{16}$$

Asymptotic Consumption and Wealth As c_t^H is increasing and bounded above, consumption levels are converging with $\lim_{t\to\infty} c_t^H = c^H$ and $\lim_{t\to\infty} c_t^L = c^L$. Agents H asympotically hold a diverging wealth and their marginal utility from wealth tends to κ. It follows from the asymptotic version of their Euler equation (17) that R_t converges,

$$R = \frac{1}{\beta}\left[1 - \frac{\kappa}{u'(c^H)}\right] \qquad \text{with} \qquad \lim_{t\to\infty} R_t = R. \tag{17}$$

In contrast, for type L agents, holding diverging wealth would violate their intertemporal optimization problem, as they would have an incentive to increase consumption at some point. Consequently, their absolute wealth is bounded and converges,

$$a^L = \frac{c^L}{R-1}. \tag{18}$$

The diverging equilibrium is characterized not only by a diverging wealth-to-output ratio but also by increasing wealth inequality, as the ratio a_t^H/a_t^L diverges over time. Asymptotically, the relative strength of the preference for wealth over consumption must be equalized across both types of agents, as given by Equation 19.

$$\frac{v'(a^L)}{u'(c^L)} = \frac{\kappa}{u'(c^H)}. \tag{19}$$

The values of $\{c^L, c^H, a^L, R\}$ are uniquely determined by the goods market clearing condition (8), the asymptotic budget constraint of type L agents (18), the asymptotic Euler equation of type H agents (17), and the combination of the Euler equations for both types of agents (19).

Consumption Cap and Scrooge McDuck Agents Insatiable preferences for wealth implicitly define an upper bound on optimal consumption. In any period, every agent has a marginal utility of wealth greater than κ. Consequently, the marginal utility of holding an additional unit of wealth and the associated future dividends and capital gains indefinitely is bounded below by,

$$\sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} R_{t+j} \kappa, \tag{20}$$

which establishes a lower bound on the marginal utility of saving. As the marginal utilities of consumption and saving equalize from the agents' optimization problem, it follows that there exists a lower bound on the marginal utility of consumption. This translates into an upper bound on optimal consumption, $\bar{c}_t$, above which no agent, even one with unbounded wealth, would consume in period t:

$$\bar{c}_t = u'^{-1} \left(\sum_{s=0}^{\infty} \beta^s \prod_{j=1}^{s} R_{t+j} \kappa \right). \tag{21}$$

The price of the Lucas tree can diverge only because agents H optimally choose to accumulate unbounded wealth. However, since consumption is constrained by the goods market clearing condition, agents H must simultaneously maintain bounded consumption levels. Any agent whose consumption-saving behavior involves diverging wealth and bounded consumption is referred to as a *Scrooge McDuck* agent. Agents H are Scrooge McDuck agents due to the consumption cap (21), which prevents them from consuming above $\bar{c}_t$. Asymptotically, it follows from Equation 17 that the consumption of agents H converges to the asymptotic consumption cap, $\bar{c}$:

$$\lim_{t \to \infty} c_t^H = \bar{c} \qquad \text{with} \qquad \bar{c} \equiv u'^{-1} \left(\sum_{s=0}^{\infty} (\beta R)^s \kappa \right). \tag{22}$$

Under standard WIU, corresponding to a subcase of $\kappa \leq \frac{1-\beta}{\beta} u'(1/\lambda)$, the presence of Scrooge McDuck agents is precluded. The accumulation of diverging wealth is only optimal if consumption c_t also diverges[5]. This conclusion holds even when the marginal utility of consumption, $u'(c)$, decreases at a much faster rate than the marginal utility of wealth, $v'(a)$, as is often assumed in the literature. The hypothesis of insatiable preferences for wealth fundamentally differs from the assumption of a higher curvature

[5]Moreover, the goods market clearing condition typically rules out the possibility of diverging consumption levels, thereby preventing the existence of diverging equilibria. Exceptions include Bourguignon (1981) and Michau et al. (2023b), where an arbitrarily small fraction of the population holds diverging wealth and achieves arbitrarily high levels of consumption.

of $u(c)$ compared to $v(a)$ in its ability to generate Scrooge McDuck consumption-saving behavior, leading to the existence of diverging equilibria.

By definition, capital income and wealth inequality between Scrooge McDuck agents and non-Scrooge McDuck agents diverge over time, as the latter do not accumulate unbounded wealth. Note that the case $\kappa \geq \frac{1-\beta}{\beta} u'(1)$ is not considered, as it corresponds to a parametrization where both types of agents could exhibit Scrooge McDuck behavior. In this case, there exists certain levels of wealth inequality such that all agents marginally prefer saving over consuming for any path $\{q_t\}_{t=0}^{\infty}$, rendering q_t indeterminate for all t, as in the perfect inequality steady state, where all wealth is held by type H agents.

Rate of Return. Lemma 4 examines the asymptotic value of the rate of return.

Lemma 4 *(Asymptotic Rate of Return) In a diverging equilibrium, the asymptotic value of the rate of return R satisfies the following inequality:*

$$1 < R < \frac{1}{\beta}. \tag{23}$$

Preferences for wealth increase the incentives to save, thereby reducing the asymptotic rate of return below the level it would have reached in their absence, $1/\beta$. This can be observed, for instance, in the asymptotic Euler equation of type H agents (Equation 17). More interestingly, the rate of return exceeds the (zero) growth rate of the economy. The condition $R > 1$ allows type L agents to maintain asymptotically a strictly positive level of consumption without accumulating diverging wealth. This feature will remain robust in the production economy, resulting in dynamic inefficiency.

Rational Bubble. The fundamental value of the Lucas tree, denoted by f_t, corresponds to the present value of future dividends, discounted at the rate R_t at the end of period t:

$$f_t \equiv \sum_{j=1}^{\infty} \frac{1}{\prod_{s=t+1}^{t+j} R_s}. \tag{24}$$

Given that agents are rational and have perfect foresight in the present deterministic and frictionless framework, any positive discrepancy between the market value of the Lucas tree and its fundamental value must, by definition, constitute a rational bubble. The bubbly component of the Lucas tree price, b_t, is defined as:

$$b_t \equiv q_t - f_t. \tag{25}$$

As R_t tends to R, the fundamental value of the Lucas tree also converges, implying that any diverging equilibrium must feature a strictly positive and unbounded bubble component:

$$\lim_{t\to\infty} f_t = \frac{1}{R-1} \qquad \text{and} \qquad \lim_{t\to\infty} b_t = \infty. \tag{26}$$

The existence of this rational bubble creates a divergence between the evolution of aggregate wealth and its fundamental value. This divergence is a key prediction of the model and will help explain the investmentless increase in the wealth-to-output ratio, as discussed in Section 3. Moreover, linking this theoretical result to the stylized facts that motivate this paper, rational bubbles can attach to various categories of real-world assets, not just land. The Scrooge McDuck theory could therefore provide a general explanation for capital gains realized across different asset classes, varying by country (Figure 1c).

Scrooge McDuck Surplus Wealth Up to this point, Scrooge McDuck agents have been defined by their accumulation of diverging wealth while maintaining asymptotically bounded consumption. An equivalent characterization is that their wealth persistently exceeds the present value of their future consumption. The excess of wealth over future consumption at the end of period t is referred to as *surplus wealth*, denoted by s^H_{t+1}, and is expressed as:

$$s^H_{t+1} \equiv a^H_{t+1} - \sum_{j=1}^{\infty} \frac{c^H_j}{\prod_{s=t+1}^{t+j} R_s}. \tag{27}$$

Agents of type H never use this wealth to finance consumption, and the dividends it generates are fully reinvested to maximize their utility from holding wealth. As a result, surplus wealth grows at rate R_t and diverges. Lemma 5 characterizes the value of the surplus wealth[6].

Lemma 5 *In any diverging equilibrium, the surplus wealth of type H agents is given by:*

$$s^H_{t+1} = \frac{b_t}{\lambda}. \tag{28}$$

Why does the bubble size coincides with the aggregate surplus wealth? This result follows directly from the definitions of the fundamental value (24) and the rational bubble (25). In this model, where all consumption stems from the Lucas tree, aggregate consumption must equal the Lucas tree dividend in each period. Consequently, the present value of future aggregate consumption corresponds to the fundamental value of the Lucas tree, which is defined as the present value of future dividends (24):

$$f_t = \sum_{j=1}^{\infty} \frac{\lambda c^H_j + (1-\lambda) c^L_t}{\prod_{s=t+1}^{t+j} R_s}. \tag{29}$$

[6]The surplus wealth of type L agents is not considered here, as it is equal to zero for agents with bounded wealth.

When each unit of wealth is held to finance some future consumption, aggregate wealth equals the present value of future consumption and the Lucas tree is priced at its fundamental value. However, if agents hold a surplus wealth that is retained solely for the sake of being held, aggregate wealth corresponds to the sum of the present value of future consumption and the surplus wealth:

$$q_t = \lambda a_{t+1}^H + (1-\lambda)a_{t+1}^L = \sum_{j=1}^{\infty} \frac{\lambda c_j^H + (1-\lambda)c_t^L}{\prod\limits_{s=t+1}^{t+j} R_s} + \lambda s_t^H. \tag{30}$$

Whenever $s_t^H > 0$, Equations 29 and 30 indicate that the value of the Lucas tree exceeds its fundamental value. The discrepancy between the equilibrium price and the fundamental value represents, by definition, a rational bubble (25), which corresponds precisely to the aggregate surplus wealth. Lemma 5 holds in every period since both the aggregate surplus and the bubble grow at the rate of return, R_t. The aggregate surplus follows this growth pattern because its associated dividends are fully saved, while the bubble must yield the same rate of return as the Lucas tree's fundamental due to non-arbitrage. If any portion of the surplus wealth were consumed, the resulting demand for the Lucas tree would be insufficient to sustain a (non-dominated) bubbly component in its price. By fueling demand for additional Lucas tree units each period, the full allocation of surplus wealth dividends to asset accumulation drives q_t to diverge due to a bubbly component.

Inequality and Asset Pricing. Lemma 6 describes how the degree of inequality affects the pricing of the Lucas tree.

Lemma 6 *In any diverging equilibrium, the price of the Lucas tree, q_t, is strictly increasing in the Lucas tree holdings of type H agents, ℓ_t^H.*

Given the concavity of both components of the utility function, the consumption of both types of agents is strictly increasing in q_t. However, under Assumption 1, type H agents have a lower marginal propensity to consume out of wealth than type L agents. As inequality, captured by ℓ_t^H, increases, aggregate consumption decreases for a given q_t. Since aggregate consumption is fixed by the Lucas tree dividend, it follows that the equilibrium price of the Lucas tree must rise as inequality increases.

Asymptotically, type H agents hold the entire Lucas tree and exhibit a zero marginal propensity to consume. The equilibrium price of the Lucas tree is then determined to ensure that type L agents retain sufficient wealth to sustain their asymptotic consumption level, c^L:

$$\lim_{t\to\infty} q_t \ell_t^L = a^L, \qquad \text{with} \qquad \lim_{t\to\infty} \ell_t^L = 0. \tag{31}$$

The asymptotic limit case clarifies why the present model does not lead to an indeterminacy of the bubble size, unlike other models incorporating insatiable preferences for liquidity or wealth within a representative agent framework (Ono, 1994;

Kamihigashi, 2008). In those models,when the marginal propensity to consume of the representative agent reaches zero, any increase in the bubble size translates directly into a one-to-one increase in surplus wealth, thereby justifying the initial rise in the bubble's value. This implies the empirically less plausible proposition that any asset price level exceeding the fundamental value could still be consistent with agents' optimization behavior. On the contrary, in the presence of non-Scrooge McDuck agents, even when the marginal propensity to consume of Scrooge McDuck agents tends to zero, any increase in the bubble size raises the wealth and consumption of non-Scrooge McDuck agents. As a result, the increase in the bubble size is not accompanied by a one-to-one increase in surplus wealth, ensuring a uniquely determined equilibrium. Asset prices increase with inequality in Lucas tree holdings, consistent with the long-term empirical joint rise in the wealth-to-output ratio and permanent income and wealth inequality.

Capital Gains In every period, low-endowment agents sell a portion of their Lucas tree holdings to high-endowment agents to finance their consumption. As their Lucas tree holdings tend toward zero (31), they rely on capital gains to sustain their strictly positive asymptotic levels of consumption and wealth. Asymptotically, the impact on their wealth of their Lucas tree sales at time t, given by $q_t(\ell^L_{t+1} - \ell^L_t)$, is exactly offset by the appreciation of the Lucas tree from period $t-1$ to t, $(q_t - q_{t-1})\ell^L_t$:

$$\lim_{t\to\infty} q_t(\ell^L_{t+1} - \ell^L_t) - (q_t - q_{t-1})\ell^L_t = 0. \tag{32}$$

There are capital gains on the rational bubble each period, as its value increases at the rate of return. In contrast, the literature addressing capital gains focus on those arising from fundamental value. Such gains typically manifest as a one-time increase in the price-to-dividend ratio, driven by either a decline in the discount rate or a rise in future expected dividends. This distinction—whether new capital gains arise each period or not—has implications for at least two strands of the literature on capital gains.

First, this model relates to the ongoing debate on whether the ex-ante average rate of return on wealth has declined. If assets are assumed to be priced at their fundamental value, the ex-ante expected rate of return appears to have declined in recent decades, as indicated by declining rent- or dividend-to-price ratios (Kuvshinov and Zimmermann, 2021) and quantitative analyses (Eggertsson et al., 2021). This decline in the discount rate would have contributed to the observed capital gains, temporarily increasing the ex-post rate of return on wealth. In contrast, if these capital gains had been observed on a rational bubble, agents would anticipate future capital gains, implying that the ex-ante expected rate of return would be higher than under the assumption of no bubble. This insight is particularly relevant in light of studies arguing that the ex-ante expected rate of return may have remained stable or only slightly declined (Duarte and Rosa, 2015; Caballero et al., 2017; Reis, 2022).

Secondly, the distinction between capital gains driven by fundamental value and those arising from a rational bubble also carries significant welfare implications. When capital gains stem from a decline in the discount rate, they mechanically increase wealth inequality without altering the distribution of capital income. Such gains have been interpreted in various ways: as a pure increase in welfare inequality (Saez et al., 2021), as mere "paper gains" with no welfare effect (Cochrane, 2020; Krugman, 2021), or as a mix of both, depending on net asset sales (Fagereng et al., 2024). In the Scrooge McDuck framework, capital gains on the rational bubble cause wealth to diverge at the top of the distribution, resulting in unbounded wealth inequality. However, they also enable non-Scrooge McDuck agents to sustain higher consumption levels over time, leading to a persistent reduction in consumption inequality.

This result is especially relevant given empirical evidence from the middle 40% of the wealth distribution, which has experienced substantial capital gains—particularly in housing—across many advanced economies since the 1980s (Bauluz et al., 2022; Blanchet and Martínez-Toledano, 2023). Furthermore, Mian et al. (2020) shows that, in the US, capital gains realized by the bottom 90% have not led to wealth accumulation but have instead been used to finance consumption through increased collateralized debt. This rising indebtness can be interpreted as a real-world illustration of the theoretical mechanism outlined above, in which agents use capital gains to finance consumption.

3 Production Economy

This section introduces insatiable preferences for wealth in a growing production economy. The objective is threefold. First, it demonstrates that the Scrooge McDuck mechanism remains robust in the presence of reproducible capital, in which agents can invest. Second, it enables the analysis of the crowding-out effect of the rational bubble on the capital stock. Third, it provides the framework for the quantitative analysis conducted in Section 4.

Production The production function follows a Cobb-Douglas specification with three factors of production, labor, reproducible capital, and non-reproducible land:

$$Y_t = K_t^{\alpha} L_t^{\gamma} (Z_t N_t)^{1-\alpha-\gamma}, \tag{33}$$

where K_t denotes the capital stock, L_t the land stock, N_t the labor supply and Z_t the labor productivity. Land is in fixed supply, normalized to $L_t = 1$ for all t, and each unit of land is priced at Q_t. The capital stock depreciates at a rate δ.

The presence of an infinitely-lived, non-reproducible asset—land[7]—is necessary in this framework for a rational bubble to exist. Indeed, in the absence of debt claims and Ponzi schemes, the only other asset, capital, is traded at its fundamental value, as it cannot be priced above its reproduction cost. Moreover, whenever land is productive, $\gamma > 0$, it prevents the economy from becoming dynamically inefficient. This argument follows Rhee (1991). Since the land factor share does not converge to zero, the return on land remains above the growth rate asymptotically. By arbitrage, the return on capital must also exceed the growth rate, ensuring that the economy remains dynamically efficient.

The growth rates of productivity and labor supply from period t to $t+1$ are denoted by g^Z_{t+1} and g^N_{t+1}, respectively:

$$g^Z_{t+1} \equiv \frac{Z_{t+1} - Z_t}{Z_t}, \qquad g^N_{t+1} \equiv \frac{N_{t+1} - N_t}{N_t}. \tag{34}$$

The growth rate of the economy along a balanced growth path for $\{K_t, Y_t\}$, denoted by g_t, is given by:

$$g_{t+1} \equiv [(1 + g^Z_{t+1})(1 + g^N_{t+1})]^{\frac{1-\alpha-\gamma}{1-\alpha}} - 1. \tag{35}$$

On a balanced growth path, the economy grows at a lower rate than the effective labor force due to the presence of a production factor that does not scale with output, land. To account for growth, each capital-letter variable X_t has a normalized counterpart x_t, defined as:

$$x_t \equiv \frac{X_t}{(Z_t N_t)^{\frac{1-\alpha-\gamma}{1-\alpha}}}. \tag{36}$$

Since $(Z_t N_t)^{\frac{1-\alpha-\gamma}{1-\alpha}}$ grows at rate g_t, any variable that exhibits balanced growth has a constant normalized counterpart. Factors of production are paid at their their marginal productivity. At time t, one unit of labor is paid a wage W_t. By arbitrage, the rates of return on land and capital from period t to $t+1$ are equalized and denoted by R_{t+1}:

$$R_{t+1} = 1 + \alpha k^{\alpha-1}_{t+1} - \delta = (1 + g_{t+1})\frac{q_{t+1} + \gamma y_t}{q_t}. \tag{37}$$

Households The population consists of a unit mass of households, each comprising N_t agents. There are three types of household, $i \in \{L, M, H\}$, categorized by their labor productivity, with each type representing a share λ^i of the total population. Type L agents are assumed to be hand-to-mouth, while those of types M and H save in capital and land. A household of type $i \in \{M, H\}$ holds K^i_{t+1} units of capital and L^i_{t+1} units of land at the end of period t. Households differ in their initial capital and land endowments, with $K^H_0 \geq K^M_0$ and $L^H_0 > L^M_0$. Their normalized end-of-period total

[7] Land is interpreted as encompassing all non-reproducible, long-lived assets. Its real-world definition is subject to debate, depending on how one defines reproducibility and longevity—whether, for instance, it includes long-lived intangible assets such as brands.

wealth, denoted a^i_{t+1}, is defined as:

$$a^i_{t+1} \equiv k^i_{t+1} + \frac{q_t L^i_{t+1}}{1+g_{t+1}}. \tag{38}$$

Households of types M and H derive utility from consumption and holding wealth and maximize the following utility:

$$\sum_{t=0}^{\infty} \beta^t U(c^i_t, a^i_{t+1}) \qquad \text{with} \qquad U(c,a) = \frac{c^{1-\theta}}{1-\theta} + \psi\Big(\frac{(a-\underline{a})^{1-\eta}}{1-\eta} + \kappa a\Big). \tag{39}$$

In line with standard preferences for wealth, parameters are set such that $\eta < \theta$ and $\underline{a} > 0$. The condition $\eta < \theta$ implies that the marginal utility of holding wealth declines more slowly than that of consumption, making wealth a luxury good and ensuring that saving rates increase with wealth. Moreover, since $\underline{a} > 0$, the marginal utility of holding wealth remains finite as a^i_t approaches zero. This specification deviates from standard WIU by introducing a linear term in preferences for wealth, $\psi\kappa a$. As a result, the marginal utility of holding wealth has a lower bound of $\psi\kappa$, making preferences for wealth insatiable:

$$\lim_{a\to\infty} \frac{\partial U(c,a)}{\partial a} = \psi\kappa. \tag{40}$$

The variables entering the utility function are not in absolute levels but are instead normalized. This ensures that, as the economy grows, saving rates do not continuously increase, which would be counterfactual. As discussed in Mian et al. (2021), the purpose of preferences for wealth is to break individual scale invariance—ensuring that wealthier households have higher saving rates—without breaking aggregate scale invariance, so that a wealthier economy does not exhibit higher saving rates[8].

Each agent $i \in \{L, M, H\}$ supplies ζ^i_t units of labor inelastically each period, where ζ^i_t captures productivity differences and satisfies $\zeta^L_t \le \zeta^M_t \le \zeta^H_t$ for all t. Productivity levels are set such that $\lambda^L\zeta^L_t + \lambda^M\zeta^M_t + \lambda^H\zeta^H_t = 1$. Consequently, labor supply coincides with population size and is equal to N_t. The normalized consumption of type L households, c^L_t, is given by,

$$c^L_t = \zeta^L_t w_t. \tag{41}$$

The budget constraint for type i households is given by,

$$c^i_t + a^i_{t+1}(1+g_{t+1}) = R_t a^i_t + \zeta^i_t w_t, \tag{42}$$

and agents are subject to a no-borrowing constraint,

$$a^i_{t+1} \ge 0. \tag{43}$$

[8]In Mian et al. (2021) and Michau et al. (2023a), utility is derived from the absolute value of consumption and the normalized value of wealth. This specification ensures aggregate scale invariance under log-utility of consumption. However, since this paper allows θ to deviate from 1, consumption must also be normalized.

Whenever the no-borrowing constraint is not binding, the solution to the optimization problem for type $i \in \{M, H\}$ households is characterized by the following Euler equation (44) and the transversality condition (45):

$$c_t^{i\,-\theta} = \beta R_{t+1} \frac{(c_{t+1}^i)^{-\theta}}{1+g_{t+1}} + \psi \frac{(a_{t+1}^i)^{-\eta} + \kappa}{1+g_{t+1}}, \tag{44}$$

$$\lim_{t\to\infty} \beta^t \left[u'(c_t^i) - \frac{v'(a_t^i)}{1+g_{t+1}} \right] a_t^i = 0. \tag{45}$$

Equilibrium The goods market clearing condition writes as,

$$k_{t+1}(1+g_{t+1}) + \lambda^L c_t^L + \lambda^M c_t^M + \lambda^H c_t^H = (1-\delta)k_t + y_t. \tag{46}$$

The asset market clearing conditions for land and capital are given by:

$$1 = \lambda^H L_t^H + \lambda^M L_t^M, \tag{47}$$

$$k_t = \lambda^H k_t^H + \lambda^M k_t^M. \tag{48}$$

Given the initial endowment $\{k_0^M, k_0^H, L_0^H\}$, an equilibrium $\{c_t^L, c_t^M, c_t^H, k_t^M, k_t^H, L_t^H, q_t\}_{t=0}^{\infty}$ is characterized by the solutions to the household optimization problems, the type L consumption equation and the land and capital market clearing conditions. The goods market clearing condition holds by Walras's law.

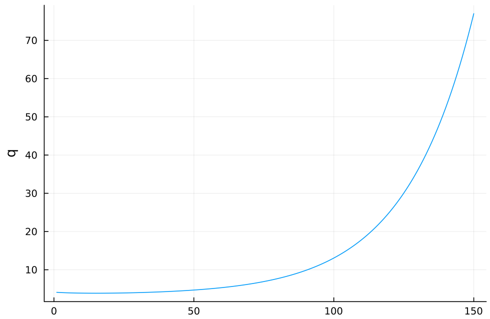
Scrooge McDuck Mechanism The model is solved numerically, and a unique equilibrium exists. For sufficiently low values of κ, the economy converges to a steady state in which agents of type H hold all the wealth and asymptotically consume all net output. However, for sufficiently high values of κ, the land price diverges, leading to a diverging wealth-to-output ratio (Figure 2a). In this case, the Scrooge McDuck mechanism is at work. Insatiable preferences for wealth lead type H agents to accumulate unbounded wealth (Figure 2c) while maintaining a bounded consumption level (Figure 2f). Type H agents are Scrooge McDuck agents and hold a *surplus wealth*, s_{t+1}^H, which does not finance any future consumption:

$$s_{t+1}^H > 0 \qquad \forall t, \text{with} \qquad s_{t+1}^H \equiv a_{t+1}^H + \sum_{j=1}^{\infty} \frac{\zeta^H w_j - c_j^H}{\prod_{s=t+1}^{t+j} R_s/(1+g_s)}. \tag{49}$$

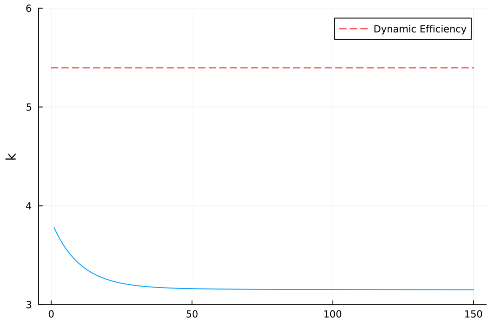
As in Section 2, the land price can be decomposed into two components: a fundamental term, f_t,

$$f_t \equiv \sum_{j=1}^{\infty} \frac{\gamma y_t}{\prod_{s=t+1}^{t+j} R_s/(1+g_s)}, \tag{50}$$

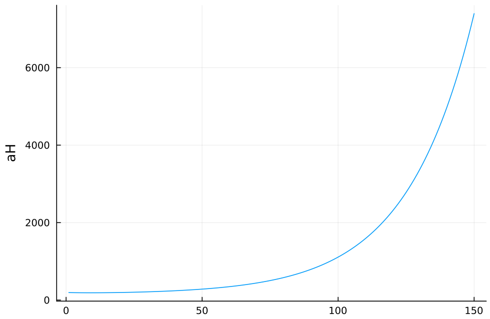
Figure 2: Transition Dynamics in the Production Economy



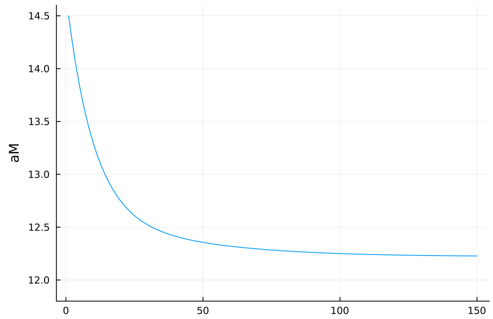
(a) Land value



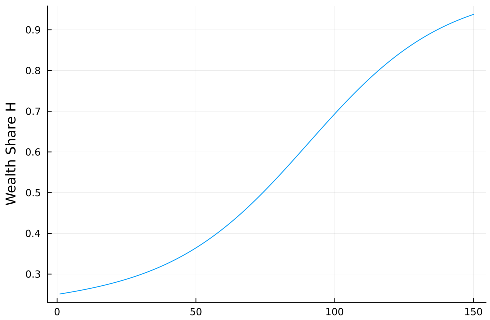
(b) Capital Stock



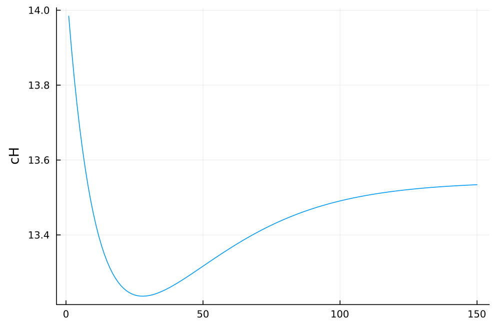
(c) Wealth of Type H



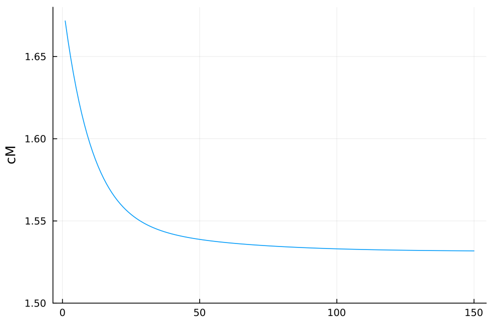
(d) Wealth of Type M



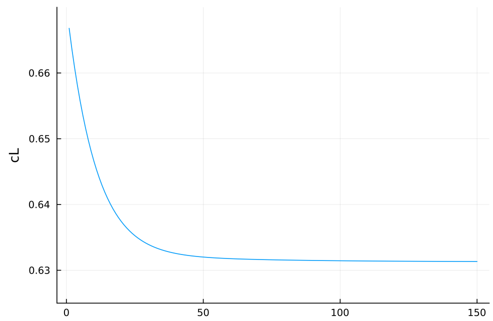
(e) Wealth Share held by Type *H*



(f) Consumption of Type H



(g) Consumption of Type M



(h) Consumption of Type L

Notes: Purely illustrative graphs; a proper calibration is forthcoming. In Panel (a), the red line represents the capital level that maximizes asymptotic consumption. The economy is dynamically efficient when asymptotic capital is below this threshold and dynamically inefficient when it exceeds it.

and a rational bubble, b_t,

$$b_t \equiv q_t - f_t. \tag{51}$$

One can then show that the rational bubble coincides with the aggregate surplus wealth:

$$b_t = \lambda s^H_{t+1}. \tag{52}$$

The Scrooge McDuck mechanism is robust to allowing agents to invest in reproducible capital. As in the endowment economy, wealth inequality between Scrooge McDuck and non-Scrooge McDuck agents diverges, as the wealth of type M agents remains bounded (Figure 2d). In contrast, consumption inequality converges asymptotically (Figures 2f, 2g and 2h). A non-bubbly equilibrium, in which demand for the bubble would instead be directed toward reproducible capital, is ruled out. Such an equilibrium would be a dynamically inefficient one, which is impossible in the presence of land with $\gamma > 0$[9].

Capital Crowding Out The production economy allows for the study of the crowding-out effect of the rational bubble. Two remarks are particularly worth emphasizing here. First, despite a diverging wealth-to-output ratio, the economy asymptotically remains at a capital level that is too low to maximize asymptotic consumption. The capital level that would maximize consumption, represented by the dotted red line in Figure 2b, corresponds to the point at which the rate of return on capital equals the growth rate of the economy.

Secondly, since $R_t > 1 + g_t$ for all t, the economy is dynamically efficient. Consequently, a smaller rational bubble would lead to higher future consumption. However, a lower bubble cannot be an equilibrium outcome. If the bubble were smaller than surplus wealth, the diverging discrepancy between them would be saved in capital, leading to dynamic inefficiency, which is ruled out[10]. The Scrooge McDuck mechanism thus prevents consumption from being maximized. Further investigation is needed to determine whether redistribution from agents H to type L could reduce surplus wealth and, consequently, the bubble, thereby enhancing consumption.

Relation to Piketty (2014) By featuring diverging wealth inequality alongside an asymptotic rate of return exceeding the growth rate of the economy, the Scrooge McDuck theory contributes to the discussion initiated by Piketty (2014) on the role of the $r - g$ gap in driving wealth inequality. A key conjecture of Piketty (2014) (p. 34) posits that "if the rate of return on capital remains significantly above the growth rate for an extended period, the risk of divergence in the distribution of wealth is very high".

[9]When $\gamma = 0$, multiple equilibria are conjectured to exist. One is the diverging bubbly equilibrium that features Scrooge McDuck agents and dynamic efficiency. Another one is a non-bubbly equilibrum, where the economy ends up in dynamic inefficiency. There exists also an infinity of equilibira with a lower bubble than in the diverging case converging to a perfectly inegalitarian steady-state that is either dynamically inefficient or such that $R = 1 + g$.

[10]Conversely, if the bubble were larger than the surplus wealth, it would completely crowd out the capital stock.

Addressing this conjecture in a model with standard preferences for wealth, Michau et al. (2023a) refutes the possibility of r remaining asymptotically above g when wealth inequality is diverging. As wealth at the top of the distribution diverges, aggregate savings should rise, resulting in greater capital accumulation and a corresponding decline in the rate of return. In contrast, by breaking the link between higher wealth inequality and higher capital stock, this paper provides theoretical foundations for Piketty's conjecture, suggesting that wealth inequality can diverge while r remains significantly above g. Moreover, one can remark that the surplus wealth, which drives the diverging wealth inequality, requieres $r > g$ to diverge relative to output.

4 Quantitative Analysis

[In progress]

5 Conclusion

This paper proposes a parsimonious explanation for the rise in the wealth-to-output ratio, the stagnation (or decline) of the capital-to-output ratio, and the increase in wealth inequality observed across advanced economies in recent decades. The core assumption of this framework is that agents derive *insatiable* utility from wealth. As a result, agents at the top of the income distribution accumulate a *surplus wealth* solely for the sake of holding it. They are defined as *Scrooge McDuck* agents, as they ultimately hold unbounded wealth while maintaining bounded consumption. Their surplus wealth drives asset prices above their fundamental value, leading to a rational bubble. The latter grows at a rate exceeding the rate of return, enabling a disconnect between the wealth-to-output ratio and the capital-to-output ratio.

The Scrooge McDuck theory has several implications. It suggests that wealth inequality may follow a diverging path, that an increase in income or wealth inequality does not necessarily lead to higher investment, and that the rate of return could be permanently sustained above the dividend-to-price ratio as a result of capital gains. The next research steps involve a quantitative exercise based on the production model and the introduction of capital income and wealth taxes to analyze their effects on the Scrooge McDuck mechanism.

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A Data Appendix

A.1 Cumulative Capital Gains

Cumulative capital gains displayed in Figure 1c are calculated from national accounting data for non-financial firms and households (excluding NPISH). This extends the national wealth accumulation decomposition of Piketty and Zucman (2014) to different asset types, expressed as:

$$W_{t+1}^k = W_t^k + S_t^k + KG_t^k,$$

where W_{t+1}^k is the market value of wealth in asset type k at time $t+1$, S_t^k is the net-of-depreciation saving flow in asset type k between time t and $t+1$ (volume effect), and KG_t^k is the capital gain or loss between time t and $t+1$, calculated as a residual. All calculations are performed for total wealth, land, and dwellings. Other domestic capital results are then computed as a residual.

The market value of household's nonfinancial wealth, as well as the market value of land and dwellings of nonfinancial business, are obtained from the nonfinancial assets balance sheets. To account for the fact that Tobin's Q may differ significantly from one (Gutiérrez and Philippon, 2017), the total value of nonfinancial businesses is calculated as total liabilities (including equity) minus total financial assets. Depending on the availability of country-level data, the wealth of the unincorporated sector is either calculated using this methodology or derived directly from the nonfinancial balance sheet.

The net-of-depreciation saving flow of asset k is first calculated separately for non-financial businesses and households as gross formation of fixed capital - consumption of fixed capital + other volume changes + acquisitions minus disposals[11]. S_t^k then corresponds to the sum of the net-of-depreciation saving flows from households and nonfinancial businesses.

B Proofs

B.1 Toy Economy

B.1.1 Proof of the necessity of the transversality condition

The proof follows Kamihigashi (2002), which identifies five conditions under which the transversality condition is a necessary condition.

[11] I am currently in the process of obtaining data on acquisitions minus disposals for the US, which are assumed to be zero for now. However, their inclusion should have a moderate overall effect, as acquisitions minus disposals within the considered institutional sectors (nonfinancial firms and households) do not impact the capital gains calculation.

The maximization problem of household i is rewritten as:

$$\begin{cases} \max\limits_{\{a_t^i\}_{t=0}^{\infty}} & \sum\limits_{t=0}^{\infty} g_t^i(a_t^i, a_{t+1}^i) \\ \text{s.t.} & \forall t \in \mathbb{Z}_+, (a_t^i, a_{t+1}^i) \in X_t, \qquad \text{with } a_0^i \text{ given,} \end{cases} \tag{53}$$

with $g_t(\ell_t^i, \ell_{t+1}^i) \equiv \beta^t \Big[u\Big(a_t^i R_t - a_{t+1}^i\Big) + v(a_{t+1}^i) \Big]$. X_t corresponds to the set of combinaison (a_t^i, a_{t+1}^i) satisying the budget constraint, that is such that: $a_{t+1}^i < R_t a_t$.

Solely interior solution of this problem are considered, as it can be easily shown that solution with $a_t^i = 0$ for some t are dominated by interior solutions. The five conditions identified by Kamihigashi (2002) to prove the necessity of the transversality condition are then satisfied.

Condition 1. $\exists n$, such that $a_0^i \in \mathbb{R}_+^n$ and $\forall t \in \mathbb{Z}_+$, $X_t \subset \mathbb{R}_+^n \times \mathbb{R}_+^n$

This assumption is fulfilled for $n = 1$.

Condition 2. $\forall t \in \mathbb{Z}$, X_t is convex and $(0, 0) \subset X_t$

It can be shown that if (y, z), $(y', z') \in X_t$, then, for all $\gamma \in [0; 1]$, $(\gamma y + (1-\gamma)y', \gamma z + (1-\gamma)z') \in X_t$

Condition 3. $\forall t \in \mathbb{Z}$, $g_t : X_t \to \mathbb{R}$ is C^1 on $\overset{\circ}{X}_t$ and concave.

As $u(c)$ and the function $a_t^i :\to v(a_{t+1}^i)$ are C^1, g_t is C^1. Moreover, the consumption level implied by $(a_t^i, a_{t+1}^i) = (y, z)$ is defined as $c(y, z) = yR_t - z$. As $u(c)$ is concave, for all $(y, z) \in \overset{\circ}{X}_t$, it can be shown that $\forall t \in \mathbb{Z}$ and $\forall \gamma \in [0; 1]$

$$\gamma u\Big(c(y,z)\Big) + (1-\gamma)u\Big(c(y',z')\Big) \leq u\Big(\gamma c(y,z) + (1-\gamma)c(y',z')\Big) \tag{54}$$

$$= u\bigg(c\Big(\gamma y + (1-\gamma)y', \gamma z + (1-\gamma)\Big)\bigg) \tag{55}$$

Given the concavity of both preference terms for consumption and wealth, it follows that:

$$\gamma g_t(y,z) + (1-\gamma)g_t(y',z') \leq g_t\Big(\gamma y + (1-\gamma)y', \gamma z + (1-\gamma)z'\Big), \tag{56}$$

and hence that g_t is concave.

Condition 4. $\forall t \in \mathbb{Z}$, $\forall (y, z) \in \overset{\circ}{X}_t$, $g_{t,1}(y, z) \geq 0$.

It follows from $u'(c) \geq 0 \; \forall c$.

Condition 5. For any feasible path a_t^i,

$$\sum_{t=0}^{\infty} g_t(a_t^i, a_{t+1}^i) \equiv \lim_{T\to\infty} \sum_{t=0}^{T} g_t(a_t^i, a_{t+1}^i), \tag{57}$$

exists in $(-\infty, \infty)$.

The wealth of an agent cannot grow at a rate above R_t. Given that $R_t < 1/\beta$ and does not converge to $1/\beta$, $\lim_{T\to\infty} \sum_{t=0}^{T} g_t(a_t^i, a_{t+1}^i)$ is not diverging and the assumption is fulfilled.

The five conditions of Kamihigashi (2002) being fulfilled, the transversality condition is a necessary condition of the household maximization problem in the endowment economy ■

B.1.2 Proof of Corollary 1

Corollary 1 is proved by contradiction. Suppose $\ell_t^i > \ell_t^j$ and $\frac{c_t^i}{\ell_t^i} \geq \frac{c_t^j}{\ell_t^j}$. Given the goods market clearing condition and the fact that all agents of the same type behave identically at the optimum, it follows that $c_t^i \geq \ell_t^i$ and $c_t^j \leq \ell_t^j$, which implies:

$$\frac{v'(a_{t+1}^i)}{u'(c_t^i)} \geq \frac{v'(q_t\ell_t^i)}{u'(\ell_t^i)} \quad \text{and} \quad \frac{a_{t+1}^j)}{u'(c_t^j)} \leq \frac{v'(q_t\ell_t^j)}{u'(\ell_t^j)}. \tag{58}$$

Moreover, under Assumption 1, there exists a sufficiently small $\epsilon > 0$ such that:

$$\frac{v'(q_t\ell_t^i)}{u'(\ell_t^i)} > \frac{v'(q_t\ell_t^j)}{u'(\ell_t^j)} + \epsilon. \tag{59}$$

Combining the Euler equations (5) for the two types of agents, the following expression is obtained:

$$\frac{u'(c_t^i)}{u'(c_{t+1}^i)} - \frac{u'(c_t^j)}{u'(c_{t+1}^j)} = \underbrace{\frac{v'(a_t^i)}{u'(c_{t+1}^i)} - \frac{v'(a_t^j)}{u'(c_{t+1}^j)}}_{\geq \epsilon}. \tag{60}$$

This indicates that the consumption of agent i is increasing, $c_{t+1}^i > c_t^i$, while their holdings of Lucas tree units are decreasing, $\ell_{t+1}^i < \ell_t^i$. It follows by induction that, for every period $s > t$, $c_{s+1}^i > c_s^i$ and $\ell_{s+1}^i < \ell_s^i$. To satisfy Equation 60, c_t^i must increase sufficiently in each period (and, conversely, c_t^j must decrease sufficiently). Given that c_t^i is bounded above by the goods market clearing condition, this is only possible if c_t^j converges to zero. It can be shown that having c_t^j converge to zero without ℓ_t^j also converging to zero is not optimal for agent j. Therefore, $\ell_t^i > \ell_t^j$ implies $\frac{c_t^i}{\ell_t^i} < \frac{c_t^j}{\ell_t^j}$ ■