Modeling the Distribution of Tax Evasion: Implications for the Tax Gap

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Abstract

Understanding the drivers of tax evasion is critical for designing effective compliance-enhancing policies. This paper examines how income heterogeneity influences taxpayers' choices between simple and sophisticated evasion strategies in a dynamic model where sophisticated evasion, characterized by high fixed costs and its capacity to exploit the legal gray areas of a tax system, becomes accessible only as taxpayers accumulate sufficient capital. This creates a threshold effect that disproportionately concentrates tax gap losses—the difference between taxes legally owed and those actually collected—among high-income taxpayers. Calibrated to U.S. data, the model replicates observed simple and sophisticated evasion patterns across the income distribution and its aggregate impact on the tax gap. This model is used to show that traditional tools like audits and fines deter evasion in the short run but lose effectiveness over time as wealthier taxpayers become less sensitive to enforcement risks and adopt sophisticated strategies. Over the long run, fine enforcement capacity—governments' ability to distinguish evasion from avoidance—becomes essential for addressing the disproportionate impact of sophisticated evasion. These findings emphasize the need for dynamic measures to tackle evolving evasion strategies and enhance tax system equity.

Keywords: tax evasion; avoidance; fiscal policy; heterogeneity; tax gap; random audits; enforcement

JEL Codes: H26, H21, H30, E62, D31

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1 Introduction

Improving tax compliance, particularly among high-income individuals, remains a formidable challenge. The tax gap, the share of legally owed taxes that go uncollected, has barely declined, remaining at approximately 15% in the U.S. and 12% in Europe over the past decade (IRS, 2019; Murphy, 2021). While recent global tax enforcement initiatives have had some moderate success in enhancing compliance, recent literature has shown that richer individuals continue to effectively minimize their tax liabilities[1], face total effective income tax rates far below those paid by average taxpayers[2], and still hold large portions of their wealth in tax havens, where offshore flows have remained stagnant at 10% of Global GDP for the past decade.

These observed differences in compliance behavior may rise from high-net-worth individuals' ability to continuously adapt their tax evasion strategy, using costly sophisticated evasion tools that exploit legal gray areas to obscure taxable income instead of relying on simpler evasion strategies, which involves direct under-reporting and is more easily detected (Guyton et al., 2023; Gamannossi degl'Innocenti et al., 2022). The persistent disparities in evasion behavior across income groups underscore a critical weakness in existing enforcement policies: their inability to address the dynamic and heterogeneous nature of evasion practices over time. This highlights the need for a theoretical framework that captures the interplay between income distribution, enforcement mechanisms, and the evolution of evasion strategies to better inform policy design. To tackle these challenges, one must first answer the following more fundamental questions: What drives the differences in tax evasion behavior across the income distribution? How do these differences shape the government's ability to collect revenues over time?

[1] Total evasion as a percentage of true individual income has been documented to increase with income across various regions and countries, including Scandinavia, Colombia, the Netherlands, and the United States (Alstadsæter et al., 2019; Londoño-Vélez and Ávila Mahecha, 2021; Leenders et al., 2023; Johns and Slemrod, 2010; Guyton et al., 2023).

[2] For example, billionaires effective income tax rates are low as 8% in the U.S. (The White House, 2021) and as 2% in France (EU Tax Observatory, 2023)

In order to address these questions, this paper develops a dynamic model economy where risk-averse agents subject to income shocks choose between simple and sophisticated tax evasion strategies to maximize the inter-temporal utility of their consumption. Sophisticated evasion is costly but enables agents to legally avoid taxes when the government fails to distinguish it from illegal evasion. In this framework, we describe how agents' optimal tax evasion decisions vary with their income and how these decisions limit the government's capacity to close their tax gap (or "compliance" gap, Keen and Slemrod, 2017), the difference between the taxes legally owed and those actually collected.

An essential feature of this model is that it incorporates heterogeneity in agents' income dynamics. This allows us to compare the short-term and long-term effects of various tax enforcement tools on both households' decisions and on the tax gap. we illustrate how the aggregate effects of these deterrence tools, such as random audits, fine rates, and government's fine enforcement capacity, defined as the ability to distinguish illegal evasion from legal avoidance, depend on the income distribution and the types of evasion employed by these agents.

For this purpose, we calibrate the model using U.S. data to replicate key empirical moments, such as the aggregate tax gap and its decomposition into sophisticated and simple evasion, as identified by Guyton et al. (2023).[3] we then use the calibrated model as a laboratory to evaluate the time-varying efficacy of enforcement policies, providing insights into how these tools affect agents' evasion strategies and the income distribution over time to assess their aggregate impact.

These exercises demonstrate that tax evasion behavior varies significantly across the income distribution, driven by differences in the type of evasion employed, the effectiveness of deterrence tools, and the time horizon. While total tax evasion increases monotonically with wealth, its impact on the tax gap becomes disproportionately larger when high-income taxpayers transition to sophisticated evasion strategies. These strategies exploit legal "grey

[3]This calibration ensures the model's predictions align with observed tax system patterns.

areas" to obscure taxable income and evade detection, concentrating tax gap losses disproportionately among the wealthiest taxpayers. The fixed costs associated with sophisticated evasion create a threshold effect, making these strategies accessible only to high-income taxpayers. Once these taxpayers transition to sophisticated evasion, their disproportionate contributions to the tax gap further limit the government's capacity to collect revenues.

Classical deterrence tools, such as random audits and fine increases, remain effective in the short run, particularly for deterring simple evasion among low- and middle-income taxpayers. However, their efficacy diminishes for wealthier individuals, who are less responsive to these enforcement measures. Over the long run, the government's capacity to enforce fines emerges as the most critical factor in reducing the disproportionate impact of sophisticated evasion, especially in reducing the fine revenue losses concentrated among high-income taxpayers.

These differences in evasion behavior across the income distribution arise from two key mechanisms: the unequal capital accumulation processes that reduce high-income taxpayers' responsiveness to detection risks, and the threshold effect, whereby wealthier taxpayers disproportionately increase their evasion and capital accumulation through their ability to afford the fixed costs of sophisticated evasion strategies.

First, reduced deterrence efficacy and accelerated capital accumulation drive disproportionate growth in evasion among high-income taxpayers. Wealthier taxpayers are less responsive to classical deterrence tools, such as random audits and fines, because their relative sensitivity to financial risks diminishes with income. This reduced sensitivity, modeled through Hyperbolic Absolute Risk Aversion (HARA) utility preferences, reflects observed behaviors where high-income individuals are more willing to engage in riskier financial activities.[4] As their incomes grow, both currently and in future expectations, the perceived costs of audits and fines become less effective in diminishing their evasion behavior. Additionally, greater initial capital and productivity, coupled with lower marginal consumption needs, enable high-income taxpayers to accumulate capital at an accelerated pace. This accumulated

[4]See (Bucciol and Miniaci, 2014).

capital process finances future riskier portfolio investments, yielding higher returns and creating a feedback loop where increased wealth further facilitates evasion. The mechanism of reduced deterrence efficacy and accelerated capital accumulation thus drives the positive monotonic relationship between evasion and income growth.

Second, the threshold effect caused by the existence of sophisticated evasion strategies that require fixed costs enables wealthier taxpayers to disproportionately expand their evasion and capital growth paths over time. Sophisticated evasion strategies necessitate significant fixed costs, which only wealthier taxpayers with sufficient capital can afford. This threshold allows them to engage in such techniques, leveraging the existence of legal gray areas to obscure taxable income. As these individuals accumulate more capital, their capacity to invest in evasion grows, leading to a compounding effect where their capital and evasion paths diverge sharply from those of lower-income taxpayers. Over time, the legal ambiguities they exploit exacerbate their disproportionate contributions to the net tax gap, as the government's capacity to both reduce their total evasion and offset evasion losses through collecting revenue fines is significantly diminished. The interaction of fixed costs, evasion strategies, and capital growth creates a reinforcing mechanism that entrenches these behaviors. Addressing this dynamic requires reducing legal ambiguities and enhancing fine enforcement capacity to limit the long-term revenue losses caused by sophisticated evasion.

Together, these mechanisms explain the observed differences in tax evasion behavior across the income distribution and their implications for government revenue collection. The declining responsiveness to deterrence tools highlights the limitations of traditional enforcement strategies for high-income taxpayers, while the threshold effect and capital dynamics illustrate how wealthier agents transition to sophisticated evasion, amplifying the challenges for enforcement over time. These findings underscore the importance of fine enforcement capacity and targeted measures to address the disproportionate contributions of high-income taxpayers to the tax gap.

Empirical studies consistently show that tax evasion increases with income, with wealthier individuals disproportionately contributing to the tax gap and highlight the need for better data reporting requirements on rich individuals to adequately estimate the aggregate impact of tax enforcement policies. Despite the implementation of global tax enforcement initiatives such as the Common Reporting Standard (CRS) and the Foreign Account Tax Compliance Act (FATCA), their impact on enhancing tax compliance has often been limited, short-lived, or unclear (De Simone et al., 2020). Offshore wealth still accounts for approximately 10% of global GDP (Guyton et al., 2020), and tax gaps in many countries, such as the United States, remain persistently high (**?**) as its rich citizens continue to circumvent new tax regulations by engaging in more sophisticated evasion strategies. For example, to circumvent FATCA's enhanced reporting requirements on income and assets held abroad, rich U.S. individuals engage in new evasion strategies such as 'round-tripping', where their assets are transferred and hidden to foreign accounts and then re-invested back in US Securities (Hanlon et al., 2015), or by simply investing in other non-financial assets, such as real estate, to circumvent FATCA's new reporting requirements (De Simone et al., 2020). These persistent gaps underscore a critical weakness in existing enforcement policies: their inability to address the dynamic and heterogeneous nature of evasion practices over time. This highlights the need for a theoretical framework that captures the interplay between income distribution, enforcement mechanisms, and the evolution of evasion strategies to better inform policy design. The framework proposed in this paper addresses these dynamics.

Building on these empirical findings, this paper draws on a rich body of work in tax compliance to provide such a framework. Foundational models by Allingham and Sandmo (1972); Yitzhaki (1987); Mayshar (1991); Slemrod and Yitzhaki (2002) established tax evasion as a static trade-off between the potential gains from evasion and the risks of detection and penalties. While these models provided critical insights into compliance behavior, they largely assumed homogeneity among taxpayers and overlooked the evolving nature of evasion decisions. Subsequent work has emphasized the need to analyze how different enforcement tools, such as audits, fines, and third-party reporting, affect taxpayers unevenly across income groups (Kleven et al., 2011; Levaggi and Menoncin, 2016; Keen and Slemrod, 2017; Boning et al., 2023). Complementing this, Alstadsæter et al. (2019); Di Nola et al. (2021); Guyton et al. (2023) highlighted the growing role of offshore wealth and sophisticated evasion strategies among high-income taxpayers, underscoring the challenges these practices pose to enforcement mechanisms. However, much of this research has focused on either static frameworks or specific empirical patterns, leaving key questions about the dynamic evolution of evasion strategies unanswered. The dynamic model presented in this paper addresses these questions.

Recent advancements have sought to address these gaps, particularly in the context of high-income taxpayers. Gamannossi degl'Innocenti et al. (2022) introduced a dynamic model in which wealthier households shift from simple evasion to legal avoidance, showing that total evasion increases with income. While their work links evasion to aggregate tax revenues, it does not explicitly examine the distributional effects of enforcement policies. Similarly, Guyton et al. (2023) demonstrated that sophisticated evasion substitutes for simple evasion as taxpayers surpass a wealth threshold, with theoretical evidence suggesting that higher audit probabilities can trigger a shift toward sophisticated strategies. The framework presented in this paper builds on these findings by incorporating dynamic enforcement mechanisms and modeling the transition to sophisticated evasion as a function of capital accumulation over time. By explicitly linking income heterogeneity, time dynamics, and policy effectiveness, this paper provides a comprehensive understanding of how tax evasion evolves and its implications for enforcement strategies.

2 Model

2.1 Preferences and technologies

Time $t \in [0, \infty)$ is continuous. The optimal choices of a mass of agents who enjoy utility from the inter-temporal consumption of a single private good c_t are modeled with the following Hyperbolic Absolute Risk Aversion (HARA) preferences:

$$U(c_t) =: \mathbb{E}_{t0} \left[\int_{t0}^{\infty} e^{-r(t-t0)} \frac{(c_t - c_m)^{1-\gamma}}{1-\gamma} \, dt \right] \tag{1}$$

in which c_m and γ parameterize a minimum subsistence amount of consumption and risk aversion, and r is the subjective discount rate.

HARA preferences, capturing decreasing relative risk aversion when $c_m > 0$, allows us to reconcile the theoretical predictions that, under decreasing relative risk aversion, evasion demand increases with income (Allingham and Sandmo, 1972; Slemrod and Yitzhaki, 2002; Guyton et al., 2023) with the empirical evidence that evasion rates are higher among the high-income individuals (Johns and Slemrod, 2010; Alstadsæter et al., 2019; Londoño-Vélez and Ávila Mahecha, 2021; Leenders et al., 2023).

Entrepreneurial agents use their capital endowment k_t to produce output (income) with the following AK technology[5]:

$$y_t = a_t k_t, \tag{2}$$

where $a_t \in (\underline{a}, \overline{a})$ is a stochastic random variable capturing Total Factor Productivity (TFP). Agents know their true income realization y_t at time t, but their productivity evolves according to the following Cox-Ingersoll-Ross (CIR) process:

$$da_t = \mu \left(\tilde{a} - a_t \right) dt + \sigma \sqrt{a_t} dZ_t, \tag{3}$$

[5] In line with previous models that incorporate heterogeneous agents and idiosyncratic production risk (Angeletos, 2007).

where Z_t is a standard Brownian motion. μ parametrizes the speed at which the productivity a_t reverts to its long-term level $\tilde{a}$ and σ captures its volatility.

Similar to Tella (2017), there is a complete financial market in which agents can exchange claims written on Z_t and earn the (exogenous) risk premium π. Agents can allocate a fraction $\theta_t \in (0, 1)$ of their capital holdings to these claims to hedge their income risk.

The choice of this setting implies that all types of income are taxed equally and does not distinguish if this income is paid in the form of wages, dividends, or profits. As most unreported income stems from self-reported income from entrepreneurs (Johns and Slemrod, 2010; Kleven et al., 2011; Di Nola et al., 2021)[6], we believe that a setting where future income streams are driven by capital accumulation through risky investments, rather than by increases in labor efforts, are better suited to understand what drives the heterogeneity of tax evasion behavior among entrepreneurial tax payers.

2.2 Taxes

The government levies linear income taxes at the constant $\tau \in [0, 1]$ to finance public spending g_t. As in Gamannossi degl'Innocenti et al. (2022), taxpayers do not consider public spending in their decision, as they do not internalize the link between tax revenues and public good provision (i.e., they are subject to fiscal illusion). As a result of these assumptions, taxpayers' capital holdings with perfect tax compliance evolve with dynamics:

$$dk_t = [a_t k_t (1 - \tau) + \theta_t k_t \pi - c_t] \, dt + \theta_t k_t \sigma \sqrt{a_t} dZ_t. \tag{4}$$

2.3 Simple and Sophisticated Evasion

Taxpayers can minimize their tax liabilities $(y_t \cdot \tau)$ by either using a simple strategy to evade a fraction $e_t \in (0, 1)$ of their taxes or by adopting a more sophisticated strategy that allows

[6] In the US, wage misreporting rates contributed to about 1% of its total Gross Tax Gap from 2014-2016 (?).

them to evade a share $\nu_t \in (0,1)$ of their tax burden, where $0 \leq e_t + \nu_t \leq 1$.

In this setting, simple evasion e_t can be interpreted as taxpayers' choice to knowingly conceal, misreport, and or misvalue assets that they know will be punishable if detected in an audit. On the other hand, sophisticated evasion ν_t is the taxpayers' choice to limit their tax remittances by re-structuring their assets through the use of complex and opaque strategies created with the explicit intention to exploit the tax code's vulnerabilities, as it is unclear that the use of this strategy will be punishable if detected in an audit.

Similar to Lin and Yang (2001); Gamannossi degl'Innocenti et al. (2022); Levaggi and Menoncin (2016), engaging in simple tax evasion e_t is costless. Conversely, in line with previous studies (Allingham and Sandmo, 1972; Slemrod and Yitzhaki, 2002; Yitzhaki, 1987; Jakobsen et al., 2019), sophisticated evasion ν_t is expensive. Formally, an agent who chooses to use sophisticated strategy ν_t faces the following costs:

$$f(\nu_t) = \chi_0 v_t + \frac{\chi_1}{2} v_t^2 \tag{5}$$

per unit of capital k_t, where χ_0 and χ_1 are two positive constants.

Empirically, the parameter χ_0 represents the initial/minimum deposits and payments required to open a financial vehicle to re-arrange the taxpayers' income structure[7]. χ_1 parametrizes the variable costs required to rescale these structures.

2.3.1 Expected Evasion Costs

In line with past models following Allingham and Sandmo (1972), tax evasion demand is strongly determined by the expected costs of being audited and penalized if caught evading. To this end, we assume the following:

[7](E.g., the minimum amount of money required to open an offshore account in a tax haven or the cost to hire the legal and account expertise necessary to conceal part of their income through pass-through businesses (such as S corporations and partnerships in the US)

No Voluntary Compliance Incentives: Tax payers' utility function and subsequent evasion choices are not driven by a sense of civic duty, intrinsic motivation, and/or social norms.[8]

Identical Random Auditing Processes: Tax payers expect both simple and sophisticated forms of evasion (e_t, ν_t) are subject to random audits by the government with the same probability. Auditing events follow a Poisson process Π_t with instantaneous intensity $\mathbb{E}_t[d\Pi_t] = 1 - e^{-\lambda dt} \approx \lambda dt$.

Simple Evasion is always fined: Conditional on being subject to a random audit, simple evasion e_t will always incur a fine η proportional to the amount of tax liabilities attempted to evade: $\eta \cdot e_t \cdot y_t \tau$

Both forms of tax evasion are subject to random audits by the government. After being subjected to a random audit, the government imposes a fine whose magnitude depends on the total amount of evasion considered illegal by the government. More specifically, random audits that detect simple tax evasion e_t always incur a fine η proportional to the total amount of tax liabilities $y_t \cdot \tau$. On the other hand, sophisticated evasion (ν_t) may be considered legal avoidance by the government *after* with probability β and, as a result, is not be fined.

In summary, implementing the evasion strategy (e_t, ν_t) entails the following exposure to auditing events:

$$\eta \left(e_t + \left(1 - \mathbb{I}_{e_t \neq v_t}\right) v_t\right) \tau a_t k_t d\Pi_t,$$

where $\mathbb{I}_x$ is the indicator function taking value one when event x is true. Therefore, the expected fine upon auditing equals

$$\eta \left(e_t + (1 - \beta) v_t\right) \tau a_t k_t \lambda \cdot dt \tag{6}$$

[8]See Luttmer and Singhal (2014) for a further discussion on the relevance of this assumotion.

where we have used the expected value of the indicator function equals the probability of its argument ($\mathbb{E}_t\left[\mathbb{I}_{e_t \neq v_t}\right] = \beta$).

When taking tax evasion into account, the agents' capital holdings evolve according to the following stochastic differential equations

$$\frac{dk_t}{k_t} = \left[a_t\left(1 - \tau(1 - e_t - \nu_t)\right) + \theta_t\pi - f(\nu_t) - \frac{c_t}{k_t}\right]dt + \\ + \theta_t\sqrt{a_t}\sigma dZ_t - a_t\eta\left(e_t + (1 - \mathbb{I}_{e_t \neq \nu_t})\nu_t\right)\tau d\Pi_t. \quad (7)$$

2.3.2 Fine Enforcement Capacity β

Following Gamannossi degl'Innocenti et al. (2022), we can interpret the parameter β as the fine enforcement capacity of the government, capturing the following features: (i) the simplicity of the tax code, (ii) the resources tax authorities have available, and (iii) the efficacy of courts. However, unlike their setting, we consider avoidance an outcome of the sophisticated evasion audit when $\beta = 1$ rather than an optimal tax-minimizing strategy by the agent. Figure 1 below illustrates the sequence of events and the possible outcomes given taxpayer's evasion decisions:

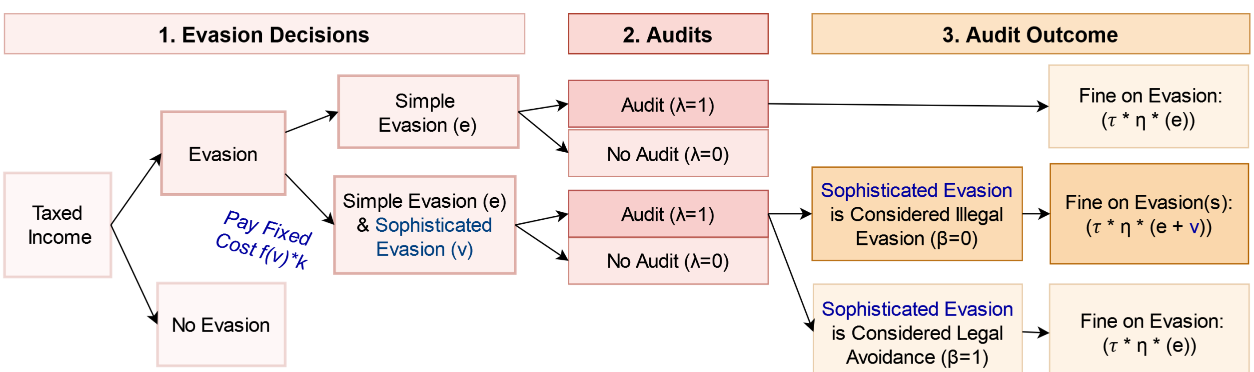


Figure 1: **Flowchart of Taxpayer Evasion Decisions and Audit Outcomes**
This flowchart illustrates the sequential process of taxpayer decisions regarding evasion (Step 1), the audit process (Step 2), and the corresponding audit outcomes (Step 3). It highlights the distinction between simple and sophisticated evasion strategies, the role of audits in detecting evasion, and how enforcement parameters influence the classification of sophisticated evasion as either illegal or legal avoidance.

In this setting, tax payers' expected evasion costs are determined by their perception on how effectively will the government leverage the information acquired by an audit λ to effectively fine them if they choose to evade in a sophisticated manner. Incorporating β allows us to distinguish the effect an increase in the expected consequences of a normal fine would have on agent's evasion behavior compared to an increase in the expectation that the government can successfully use that information to fine them. This distinction will be shown to be especially important for richer tax payers - as any increase in the government's capacity to detect evasion, by increasing the chances they will be randomly audited, may have a much more diminished impact in deterring their evasion choices as long as they expect to be able to afford sophisticated evasion in the future.

Consequently, legal avoidance is an outcome, not a choice made by taxpayers. When $\beta = 1$, no fine is imposed and fraction $\nu \cdot y_t \tau$ is considered legal avoidance. This outcome is critical in modeling taxpayers' optimal evasion decisions across the income distribution and evaluating their aggregate impact on government revenues. From the taxpayers' perspective, β is an institutional parameter that reflects how capable governments are able to impose a fine on their evasion behavior conditional on them being audited. The closer β is to 1, the greater amount of their income can be considered as possible legal avoidance rather than illegal evasion when audited. In other words β directly influences the size of the "gray area" between illegal evasion and legal avoidance a tax payer can abuse.

2.4 Government: Revenues and the Tax Gap

The government does not observe the agents' true income y_t and generates instantaneous revenue T through direct tax remittances and by enforcing fines on evaders who have been audited.

Formally, for an individual taxpayer with income $y_t = a_t k_t$ and a given set of evasion strategies (e_t^*, ν_t^*), the government's total expected total revenues from the agent can be formulated as:

$$\mathbb{E}_t[T_t] = \mathbb{E}_t\Big[k_t a_t \tau \Big((1 - e_t^* - \nu_t^*)dt + \eta(e_t^* + (1 - \mathbb{I}_{e_t \neq v_t})\, \nu_t^*)d\Pi_t\Big)\Big] \tag{8}$$

We can conveniently re-arrange 8 to express the government's expected gross tax gap, G_t^{gross} as the total non-compliance as a share of all tax liabilities:

$$\mathbb{E}_t[G_t^{\text{Gross}}] = \mathbb{E}_t\left[\frac{k_t a_t \tau (e_t^* + \nu_t^*)}{k_t a_t \tau} \cdot dt\right] \tag{9}$$

Similarly, the following equation defines revenue enforcement as the share of revenues collected through successful fines after enforcement of fines and auditing occurs, as the share of all tax liabilities:

$$\mathbb{E}_t[\bar{\eta}_t] = \mathbb{E}_t\left[\frac{k_t a_t \tau \Big(\eta(e_t^* + (1 - \mathbb{I}_{e_t \neq v_t})\, \nu_t^*)\Big)}{k_t a_t \tau} \cdot d\Pi_t\right] \tag{10}$$

Equipped with these equations, the *Net* Tax Gap is the share of all taxes collected as a share of all tax liabilities, net of enforcement:

$$\mathbb{E}_t[G_t^{\text{Net}}] = \mathbb{E}_t\left[G_t^{\text{Gross}}\right] - \mathbb{E}_t\left[\bar{\eta}_t\right]. \tag{11}$$

2.5 Taxpayer's problem

Formally, each taxpayer chooses its consumption (c_t), evasion (e_t, ν_t), and risk-taking (θ_t) strategy to maximize (2.1) subject to (2.1), (7), $k_t > 0$, and $\nu_t \geq 0$, as summarized in the following:

$$\max_{\{c_t, \theta_t, e_t, \nu_t\}_{t\in[t_0,\infty)}} \quad \mathbb{E}_{t0}\left[\int_{t_0}^{\infty} e^{-r(t-t0)}\frac{(c_t-c_m)^{1-\gamma}}{1-\gamma}\,dt\right]$$

s.t.

$$\begin{aligned} dk_t &= \left[a_t k_t - a_t k_t \tau(1-e_t-\nu_t) + \theta_t k_t \pi - f(\nu_t)k_t - c_t\right]dt, \\ &\quad + \theta_t k_t \sigma_{a_t} dZ_{a_t} - \eta\left(e_t + (1-\mathbb{I}_{e\neq\nu})\nu_t\right)\tau a_t k_t d\Pi_t, \\ da_t &= \mu\left(\tilde{a}-a_t\right)dt + \sigma\sqrt{a_t}dZ_t, \\ \nu_t &\geq 0 \end{aligned} \tag{12}$$

As we show in Appendix A, the optimal taxpayers' strategy is given by:

$$c_t^* = c_m + F(a,t)^{-\frac{1}{\gamma}}\left(k_t - H(\,a,t)\right), \tag{13}$$

$$\theta_t^* k_t = \left(k_t - H(a,t)\right)\left(\frac{\pi}{\gamma a_t \sigma^2} + \frac{\partial F(a,t)}{\partial a}\frac{1}{F(a,t)}\right) + \frac{\partial H(a,t)}{\partial a}, \tag{14}$$

$$e_t^* = \frac{(k_t - H(a,t))}{a_t \eta k_t}\left(1 - \left(\frac{\eta\lambda}{\tau}\right)^{\frac{1}{\gamma}}\right) - (1-\beta)\,v_t^*, \tag{15}$$

$$v_t^* = \max\left\{\frac{\tau a_t \beta - \phi_t - \chi_0}{\chi_1}, 0\right\} \tag{16}$$

where ϕ_t is such that $\nu_t^*\phi_t = 0$ and the complementary functions $H(a,t)$ and $F(a,t)$ are given by

$$H(a,t) = c_m \mathbb{E}_t^{\gamma}\left[\int_t^{\infty} e^{-\int_t^s \rho_H(v_u^*)du} ds\right], \tag{17}$$

and

$$F(a,t) = \mathbb{E}_t^{\gamma}\left[\left(\int_t^{\infty} e^{-\frac{1}{\gamma}\int_t^s \rho_F(v_u^*)du} ds\right)^{\gamma}\right] \tag{18}$$

where $\rho_H(\cdot)$ and $\rho_F(\cdot)$ are deterministic functions of v^* that are described in the appendix, and expectations are taken under the following probability measure:

$$dZ_t^{\gamma} = \frac{\pi}{\sqrt{a_t}\sigma}dt + dZ_t. \tag{19}$$

Moreover, the agents' value function equals

$$V(t, a, k) = F(a, t)\frac{(k_t - H(a, t))^{1-\gamma}}{1-\gamma}. \tag{20}$$

The optimal consumption and risk-taking strategies are those obtained from a standard consumption/asset-portfolio problem under HARA preferences[9]. Simple tax evasion (e_t^*) increases with capital k_t and decreases directly with productivity a_t. Its overall relationship with a_t, however, is ambiguous, as it depends on the slope of $H(t, a)$. This endogenous relationship with a_t requires us to examine $H(t, a)$'s derivatives through numerical approximations. we explore its behavior aspect numerically after calibrating the model in the next section.

Conditional on being positive, the optimal level of sophisticated evasion ν_t^* also increases directly with productivity. Moreover, it rises with higher taxes (τ) and decreases with greater enforcement capacity (β). Perhaps surprisingly, ν_t^* is not directly affected by simple evasion deterrence parameters, such as the audit probability (λ) and fines (η). Notably, these parameters influence ν_t^* only if its productivity is large enough. Consequently, productivity a_t is key in determining both the total evasion demand. Unlike, Menoncin et al. (2022), total evasion demand strongly depends on the stochastic process referenced 2.1. This observation is key as total evasion varies both by time and the productivity distribution of a_t due to stochastic process da.

Consequently, the optimal strategies in (16) and (15) reveal that simple and sophisticated evasion are substitutes - mirroring a documented empirical phenomenon seen in studies such as in Guyton et al. (2023); EU Tax Observatory (2023); Londoño-Vélez and Ávila Mahecha (2021); Leenders et al. (2023).

[9]See Merton (1969).

2.6 Comparative Statics

To understand the impact of a change in a policy parameter on both optimal evasion and on the government's tax gap in the short run, we analyze individual taxpayer's reactions to changes in policy parameter $X \in (\eta, \lambda, \tau, \beta)$ depending on their income $y_t = a_t k_t$. For this exercise, we can take the following assumptions:

Assumption 1: The policy parameters satisfy:

$$\tau, \beta, \lambda \in (0,1), \quad \gamma, \eta > 1, \quad \text{and} \quad c_m, \chi_1, \chi_0 > 0.$$

Assumption 2: Capital exceeds the threshold:

$$k_t > H(a,t).$$

Where the combination of both assumptions ensures that all tax payers' optimal evasion is $e_t^* > 0$ under a realistic range of policy parameters.

I define

$$k_t - H(a,t) \tag{21}$$

as the disposable capital that remains after saving enough for financing the future streams of subsistence consumption. Consequently, Eq.(21) determines how much future income can be evaded.

2.6.1 Evasion

Tax payer's excess capital gained through its total evasion choices, $\bar{E}_t = \nu_t + e_t$, alongside with risky asset choices $\theta_t k_t$, can be re-invested to increase their income process y_t.

Evasion Increases with income and capital: The dynamics behind capital accumulation process of k, and its relationship with $H(a,t)$, determine their evasion process. This process accelerates with net-worth k_t, as taxpayers' utility function is less responsive to increases in the risks behind the enforcement policies such as λ, η. Total evasion demand is further exacerbated by the drastic reduction in risks associated with evasion once they can afford to pay ν's costs. Thus, under the stated assumptions above, total evasion $\bar{E}_t$ grows with both capital k_t and income y_t.

The impact different policies have on tax payer's total evasion strategies vary on their income level. Table 1 summarizes how individual taxpayer's optimal evasion strategies react to a change in these policies by analyzing e_t^*, ν_t^* and $\bar{E}_t$'s derivative with respect to an increase in $(\eta, \lambda, \tau, \beta)$.

However, to understand how the changes in policies unevenly affect taxpayer's total evasion strategy $\bar{E}$ depending on their level of income y_t, we take the second derivative of $\bar{E}$ with respect to their income y_t. The derivations for these are detailed in the numerical appendix (6) and are shown under the $\frac{\partial \bar{E}}{\partial y \partial X}$.

This analysis shows that taxpayer's total evasion behavior, $\bar{E}$, increases with their net worth k_t and income y_t, as $\frac{\partial(\bar{E}_t)}{\partial y} > 0$. Under this setting, this holds true even when we assume the minimum consumption level $c_m = 0$[10]. However, the size and magnitude of the policy parameter's effect on agent total evasion behavior may change depending if they can engage in sophisticated evasion ν_t^* or not.

Sophisticated Evasion's Threshold: Additionally, the threshold where agents can start engaging in sophisticated evasion:

$$\tau a \beta \geq \chi_0 \tag{22}$$

exacerbates their reactions to market or policy changes. For these reasons, understanding how different agents' optimal capital accumulation processes vary over time and their income level can illustrate how these policies may induce uneven effects across agents and, in turn, have different aggregate effects on the government's revenue functions.

[10]As $H(a,t) \approx \frac{c_m}{a_t(1-\tau) + \frac{(\tau a \beta - \phi - \theta_0)^2}{2\theta_1}}$

The threshold (22) plays an essential role in the total evasion dynamics of taxpayers. Once crossed, it decreases the efficacy of enforcement policies λ, η even further, the higher the income y_t or capital k_t the taxpayer has. In the short run, as individual tax payer's capital stock k_t grows through stochastic changes in their productivity a_t and θ_t investments in risky capital, the deterrence capacity of random audits λ in reducing total evasion is strongly diminished at the threshold where agents can afford to pay χ_0, and exacerbated by lower variable costs χ_1. This illustrates the importance enforcement β in reducing the tax gap contributions of richer tax payers, relative to tax payer's capital accumulation process. This does not necessarily mean that in the long run audits and fines are completely inefficient in reducing total evasion. Their deterrence capacity will depend on the drivers of agents' capital accumulation process - such as their initial productivity a_0, the risk premium of π and volatility σ_{at} - and on the government's capacity to distinguish simple from sophisticated evasion; β.

To better understand how taxpayer's evasion choices respond to parameter changes, we summarize their expected change by displaying their derivatives in table 1 below:

Table 1: **Short-run effect of enforcement/policy parameters on individual tax-payer's optimal evasion choices.**
This table shows the sign of the derivatives of the function in the column with respect to the parameter in the row, as detailed in this section. Column **"Threshold Amplification?"** denotes if a change in parameter X has a disproportionately large impact on tax payers' total Evasion $\bar{E}$ past threshold $\tau a \beta \geq \chi_0$.

Parameter (X)	ν_t^*	e_t^*	$\bar{E} = \nu_t^* + e_t^*$	$\frac{\partial \bar{E}}{\partial y_t \partial X}$	**Threshold Amplification?**
η	0	$-$	$-$	$-$	No
λ	0	$-$	$-$	$-$	No
β	$+$	*und.*	*und.*	$+$	Yes
τ	$+$	*und.*	*und.*	$+$	Yes

2.6.2 Government

We can first note that the gross tax gap is equal to the total amount of taxpayer's evasion, where $G_t^{Gross} = \bar{E}_t$. We can then see that the net tax gap G_t^{net} contribution by agents can be re-formulated by inserting agents' optimal evasion choices e_t^* and ν_t^* into Eq.(11) as:

$$E_t[G_t^{net}] = [\beta \cdot \max\left\{\frac{\tau a_t \beta - \phi_t - \chi_0}{\chi_1}, 0\right\} + \frac{(k_t - H(a,t))}{k_t a_t \eta}(1 - \eta\lambda)\left(1 - \left(\frac{\eta\lambda}{\tau}\right)^{\frac{1}{\gamma}}\right)]dt \quad (23)$$

Although the gross tax gap follows similar patterns described above, we can highlight the impact of changes in total direct revenues $\bar{\eta}$ on affecting the total net tax gap. The presence of λ, η in the fine revenue function dampens the revenues collected by the government for tax payer's above threshold 22 on the total impact in the net tax gap. As λ, η have diminished efficacy in collecting revenues from high-income tax payers, the loss in revenue collection through the use of fines is amplified.

To understand the impact of a change in a policy parameter on the expected net tax gap, and on its components 23 in the short-run, we can take its derivatives with respect to policy parameters $\eta, \lambda, \tau, \beta$. Table 2 summarizes our results below.

Table 2: **Short-run effect of enforcement/policy parameters on individual tax-payer's contribution to net tax gap and its components.**
This table shows the sign of the derivatives of the function in the column with respect to the parameter in the row, as detailed in this section. Column **"Threshold Amplification?"** denotes if a change in parameter X has a disproportionately large impact on tax payers' contributions to the government's tax gap past threshold $\tau a\beta \geq \chi_0$.

Parameter (X)	$G_t^{\mathbf{Gross}}$	$-\bar{\eta}_t$	$G_t^{\mathbf{Net}}$	$\frac{\partial G_t^{\mathbf{Net}}}{\partial y_t \partial X}$	**Threshold amplification?**
η	$-$	$+$	$-$	$-$	Yes
λ	$-$	*und.*	$-$	$-$	Yes
β	$+$	0	$+$	$+$	Yes
τ	*und.*	$-$	*und.*	$+$	Yes

Through these exercises we can see that policy parameter changes have disproportionate effects on the individual's tax payers' contribution to the tax gap and its components depending on their income level.

2.6.3 Aggregation

To estimate the aggregate effects a policy parameter $X \in (\eta, \lambda, \tau, \beta)$ has on the net tax gap (G_t^{net}) in the short-run, we must integrate all behavioral changes, $\{e_t^*(k_t, a_t), v_t^*(k_t, a_t), \theta_t^*(k_t, a_t), c_t^*(k_t, a_t)\}$, across the continuum of tax payers in distribution $f(a, k)$ in response to this change. However, these aggregate effects may depend on the initial distribution of agents $f(a, k)$.

Uneven Enforcement Effects across the Income Distribution. In the short-run, under fixed stationary distribution $f(a, k)$, higher levels of income $y_t = a_t \cdot k_t$ lead to diminished efficacy of random audits and fines, λ, η, in reducing the the net-tax gap, as $\frac{\partial dG_t^{net}}{\partial \lambda \partial y} > 0$ and $\frac{\partial dG_t^{net}}{\partial \lambda \partial y} < 0$, while the opposite happens for increased enforcement β. This effect is exacerbated for $f(a, k)^{\nu_t^* > 0}$. Conversely, higher enforcement β and taxes τ depend on a and k. For these reasons, we can summarize the short-run effects in table 3 below and later corroborate them with our numerical simulations.

Threshold effects due to distribution $f(a, k)$ on the aggregate Net Tax Gap (G_t^{net}). Given the discontinuity present in Eq. 16 due to ν_t^*, we must analyze the impact of a change in a policy parameter for the amount of tax payers where threshold Eq. 22 holds or not. We can define $f(a, k)^{\nu_t^* > 0}$ as the mass of tax payers who can afford to pay sophisticated evasion and $f(a, k)^{\nu_t^* = 0}$ as the ones who cannot, where $f(a, k)^{\nu_t^* > 0} + f(a, k)^{\nu_t^* = 0} = f(a, k)$. However, the expected sign change of as change in random audits and fines are always found to be positive and do not depend on the initial distribution - even though their magnitude may be impacted by it. The third column in Table 3 summarizes this.

However, these aggregate effects may depend both on the distribution of agents $f(a, k)$ and on the time horizon. In the long-run, distribution $f(a, k)$ may change depending on the distribution of capital income accumulation processes k_t. Both k_t and and disposable excess capital that can be evaded $k - H(a, t)$, and the dispersion created by different agents initial

Table 3: **Short-run Aggregate effect of enforcement/policy parameters on the Net Tax Gap.** This table shows the signs of the derivatives of the Net Tax Gap function, along with dependency on the distribution $f(a,k)$. Column **"Sign Depends on Distribution $f(a,k)$?"** summarizes if G_t^{net}'s sign caused by an increase in parameter X may change depending on initial distribution $f(a,k)$.

Parameter (X)	**Net Tax Gap** (G_t^{net})	**Sign Depends on Distribution** $f(a,k)$?	$\frac{\partial G_t^{net}}{\partial y_t \partial X}$
η	$-$	No	$-$
λ	$-$	No	$-$
β	$+$	Yes	$+$
τ	und.	Yes	und.

levels of productivity $a_0 \in (\underline{a}, \overline{a})$ determine the long-run effects of a policy change. We rely on the numerical methods described in the next section to estimate them.

3 Numerical Methods and Calibration

This section describes the strategies used to derive this paper's main results. To do this, we describe the numerical solution methods used, how we use them to validate our results show in the comparative statics section, and calibration strategies.

3.1 Methodology

All results presented are resolved numerically. To do this, we can resort to numerical simulation methods to endogenously determine the joint stationary $f(a,k)$ using a set of initial parameters, whose choice is described in our calibration methods below. This allows us to then compute optimal policies $\{e_t^*(k_t,a_t), v_t^*(k_t,a_t), \theta_t^*(k_t,a_t), c_t^*(k_t,a_t)\}$ and their aggregate effects on the Net Tax Gap G_t^{net} as well as its components.

Specifically, we first approximate the objects $F(a,t)$ and $H(a,t)$ by using Monte Carlo simulations of dZ_t realizations over a discretized productivity grid a_M, where $M = 10$ are the number of linearly spaced intervals ranging from $(\underline{a}, \overline{a})$. Using these objects, we then compute the optimal policy function in each state space point and simulate the dynamics of the controlled state (capital k_t and productivity a) process over a long time horizon; $T = 25,000$. we then discretize capital grid k_N, where $N = 50$, spaced according to the computed distribution of k, with intervals ranging from the minimum and maximum k_t simulated $\in (\underline{k}, \overline{k})$. Under the assumption that the joint dynamics of these processes are ergodic, we use the obtained (empirical) density function to approximate $f(a,k)$. By multiplying all optimal policies times the stationary distribution $f(a,k)$ yields the optimal policies of the continuum of agents across the distribution. Finally, the aggregate results at the stationary distribution are computed by integrating the optimal policies over state space $f(a,k)$.

I validate our main long-run results by calibrating the model to match the empirical observations found by Guyton et al. (2023) on the aggregate and distribution of tax evasion by income group and type of evasion seen in the U.S. during 2011-13. The main results are presented in Section 4.3.

I then do policy experiments in this setting. The short-run experiment computes the tax gap by changing optimal policies but keeping the distribution constant; the long-run adjusts both. Then, we compare the outcomes.

3.2 Calibration

The following parameters are externally calibrated: The nominal tax rate (τ), the audit intensity (λ), and the auditing fine (η). These are consistent with **?**. In particular, the values of τ and λ are taken as the average rates across all taxpayers during 2011-13 in the US. The maximum evasion fine, η, is the maximum penalty rate. The risk aversion γ and the discount rate r take standard values from the literature. Table 4 summarizes these parameter values and their sources.

Table 4: Externally Calibrated Parameters

Parameter	**Description**	**Value**	**Source**
μ, σ	Mean-reversion time, productivity risk	(0.13, 0.8)	-
$\bar{a}$	Long term Aggregate Productivity	1.00	-
π	Premium of Risky Assets	0.06	Graham and Harvey (2010)
λ	Average Probability of being Audited	0.03	**?**
η	Max. Evasion Fine of 75%	1.75	**?**
τ	Tax Rate	0.20	**?**
γ	Risk Aversion	2.50	Standard
r	Discount Rate	0.015	Standard

Government's fine enforcement capacity β, the cost parameters χ_0 and χ_1, and the subsistence consumption level c_m are chosen to match a few key moments on the long-term distribution in the steady steady state of tax evasion across income groups, according to Guyton et al. (2023), as well as their total contribution to the tax gap. Table 5 reports the internally calibrated parameters and the corresponding target moments.

Where $Y_t\tau$ denotes all the true tax liabilities summed across all agents and $Y_t^{\bar{a}}\tau$ is the true tax liabilities of the riches income percentile of income group.

Table 5: Internally Calibrated Parameters

Parameter (X)	Description	Value	Moment	Target	Unit
β	Fine Enforcement Capacity	0.8	Soph. Evasion Contribution to Tax Gap	2	% of all tax liabilities $Y_t\tau$
χ_0	Fixed Cost of Soph. Evasion	0.2	Income Ptile Where Soph. Evasion > 0	30	Percentile of Income
χ_1	Variable Costs of Soph. Evasion	0.3	Soph. Evasion by Top 1%	6	% of tax liabilities $Y_t^{\bar{a}}\tau$
c_m	Minimum Consumption Level	0.6	Minimum Income Ptile for Simple Evasion	10	Percentile of Income

Specifically, we can interpret the long-term productivity $\tilde{a}$ as the nominal median salary in the U.S. around that that time period, of approximately 50,000$ (?). This choice of minimum consumption level $c_m = 0.6$ approximately replicates the ratio of this median income over the minimum wage plus average government transfers estimated at around 27,000$ nominally where $\frac{\tilde{c_m}}{\tilde{a}} \approx \frac{27{,}000}{50{,}000}$. This calibration allows for tax payers' to start evading $e_t > 0$ at the 10th lowest level of income levels we simulate.

Fixed and variable costs χ_0, χ_1 are harder to match empirically. Loosely, χ_0 corresponds to the minimum costs of talking to a tax consultant to re-shuffle one's assets or the minimum deposit required to keep in a haven. There exists a wide range of these minimum estimates in the literature[11]. However, we target these values to proportionally increase with households net-worth as a fraction of taxpayer's wealth as fixed costs are $k_t \cdot f(\nu^*) = k_t\chi_0\nu_t + k_t\frac{\chi_1}{2}\nu_t^2$. χ_0 is chosen to match the percentile of income where sophisticated evasion propensity seen in the income distribution, and χ_1 is calibrated to match the maximum sophisticated evasion behavior of the richest agents, as a share of all their true tax liabilities, as found by Guyton et al. (2023). Finally, β, the fine enforcement capacity of the government, is set to target the aggregate sophisticated evasion contributions to the Net Tax Gap; $\int_{\underline{a}}^{\bar{a}} \int_0^{\infty} \nu_t^* \cdot f(a,k) \cdot da \cdot dk$.

[11]These may range from 5,000-100,000$, where this maximum value stems from the minimum deposit required to declare assets under FATCA (?)

4 Main results

This section presents the main results on the long-term stationary distribution of tax evasion distribution and the resulting aggregate tax gap. We rely on numerical solutions to compute both the stationary distribution $f(a,k)$ and aggregate outcomes G_t^{net} and its decomposition. To analyze what effect policies $\beta, \tau, \eta, \lambda$, we first establish a baseline model of the long-term stationary distribution of tax evasion and validate its accuracy with the empirical regularities seen in the data.

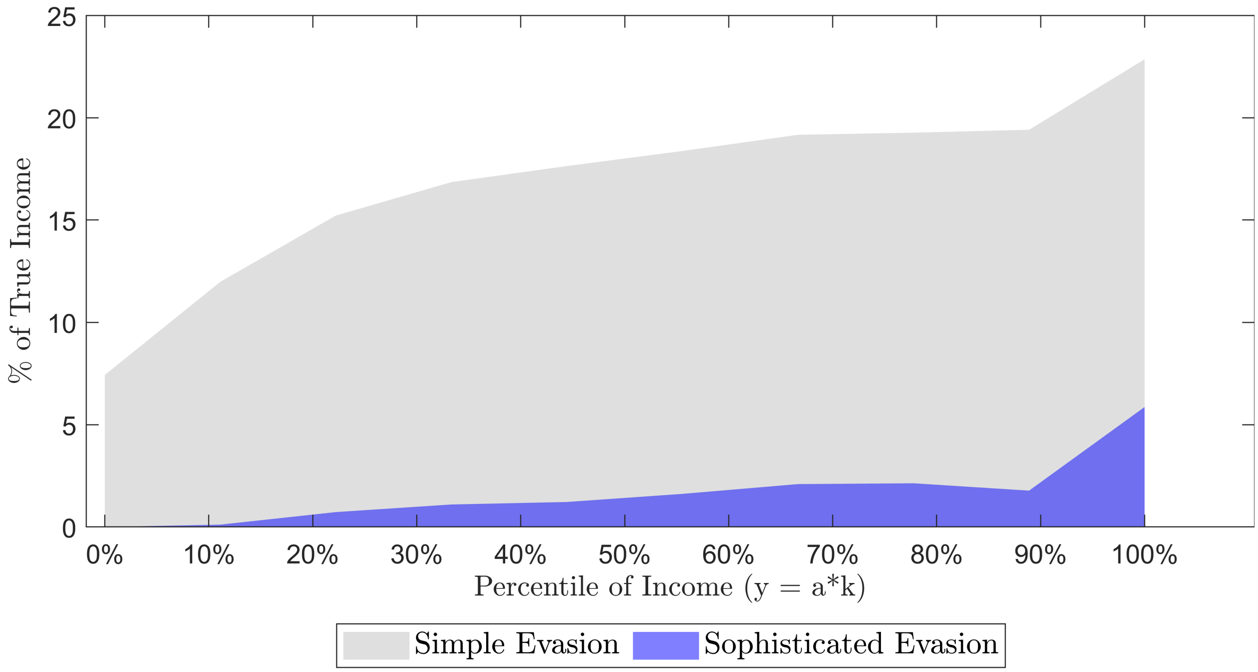
4.1 Benchmark and Validation: US Tax Gap

I first simulate the long-term stationary distribution of tax evasion and the resulting aggregate tax gap in the US found by Guyton et al. (2023) using our internal calibration strategy described in the previous section. I find that these benchmark numerical results closely resemble their estimates. Table 6 contrasts the simulated aggregate results with their findings, while figure 2 shows the model's simulated distribution of total simple e_t and sophisticated ν_t evasion across the income $y_t = a_t \cdot k_t$ distribution that correspond to the aggregate results shown in 6. Additionally, figure 8 in the appendix compares these results on the distribution with Guyton et al. (2023)'s estimates.

Table 6: **Model Results: Untargeted Moments compared to previous Estimates.** Decomposition of aggregate tax gap moments, expressed as percentages of sum of all tax liabilities.

Parameter	Description	Model	Data	Source
Gross Tax Gap	Taxes paid, as % of all Tax Liabilities	11.1	12.8	Guyton et al. (2023)
Net Tax Gap	Gross Tax Gap - Enforcement Revenues, as % of all Tax Liabilities	10.6	11.3	Guyton et al. (2023)
Enforcement Revenues	Revenues from evasion fines, as % of all Tax Liabilities	0.5	1.5	IRS
Total Evasion Rate by Rich	Total Evasion by Richest Income Group, as % of their Income	23.9	21.4	Guyton et al. (2023)

Figure 2: **Model Estimates of U.S. Unreported Incom by Percentile of Income, Average 2006-13 (as % of True Income y_t).** This figure plots the models simulated estimates of tax payers' optimal under-reported income by evasion strategy, optimal evasion type (e_t^*, ν_t^*) and over their position in the simulated stationary income distribution.



The simulated results closely resemble those found by Guyton et al. (2023) as both total evasion rates and contributions to the total net tax-gap increase at higher income levels. The inflection point seen in the total evasion behavior around the 90th percentile of agents shows how the inclusion of an alternative, but costly, to simple evasion e_t^* drastically increases the total evasion of behavior of richest tax payers. It is important to note that this inflection point, unlike threshold $\beta a \tau \geq \chi_0$, occurs as sophisticated evasion gains dominate simple evasion gains as $k_t - c_m \cdot H(a,t)^{-1}$ grows at a much faster rate as c_m becomes insignificant compared to total k_t. This result is in line with the past theoretical predictions: richer tax payer's lower risk aversion induces higher levels of total evasion as their demand for both risky assets, in the form of θ_t and evaded assets subject to audits λ, increases linearly with their net worth k_t.

4.2 Short-Term

I now test how a 1% increase in policy parameter $X \in (\eta, \lambda, \tau, \beta)$ would affect the total evasion and net tax gap aggregate estimates compared to the baseline simulated distribution $f(a,k)$ computed for the benchmark US tax gap in the section above. Table 7 summarizes such results[12].

Table 7: **Short-run effect of enforcement/policy parameters on Aggregate Values (as % increase).** This table shows the simulated impact of a 1% increase in any of policy parameters on the aggregate tax gap as well its components, assuming a constant distribution $f(a,k)$ as displayed in the previous section.

Parameter	**Simple Evasion** e^*	**Soph. Evasion** ν^*	**Fine Revenues** $\bar{\eta}$	**Gross Tax Gap** $G^{\textbf{Gross}}$	**Net Tax Gap** $G^{\textbf{Net}}$
η	-0.149	0.000	-0.003	-0.152	-0.149
λ	-0.057	0.000	0.002	-0.055	-0.057
$1-\beta$	0.000	-0.594	0.000	-0.494	-0.494
τ	0.052	0.482	0.003	0.441	0.438

This table corroborates most of the short-term analytical derivations and predictions shown in Section 2. However, under this baseline scenario, an increase in fine enforcement $1-\beta$ leads to the greatest decrease in the net tax gap. This is driven by both the large number of agents who choose sophisticated evasion $\nu_t^* > 0$ and due to the large part of income and its corresponding tax liabilities being concentrated in higher levels of income distribution.

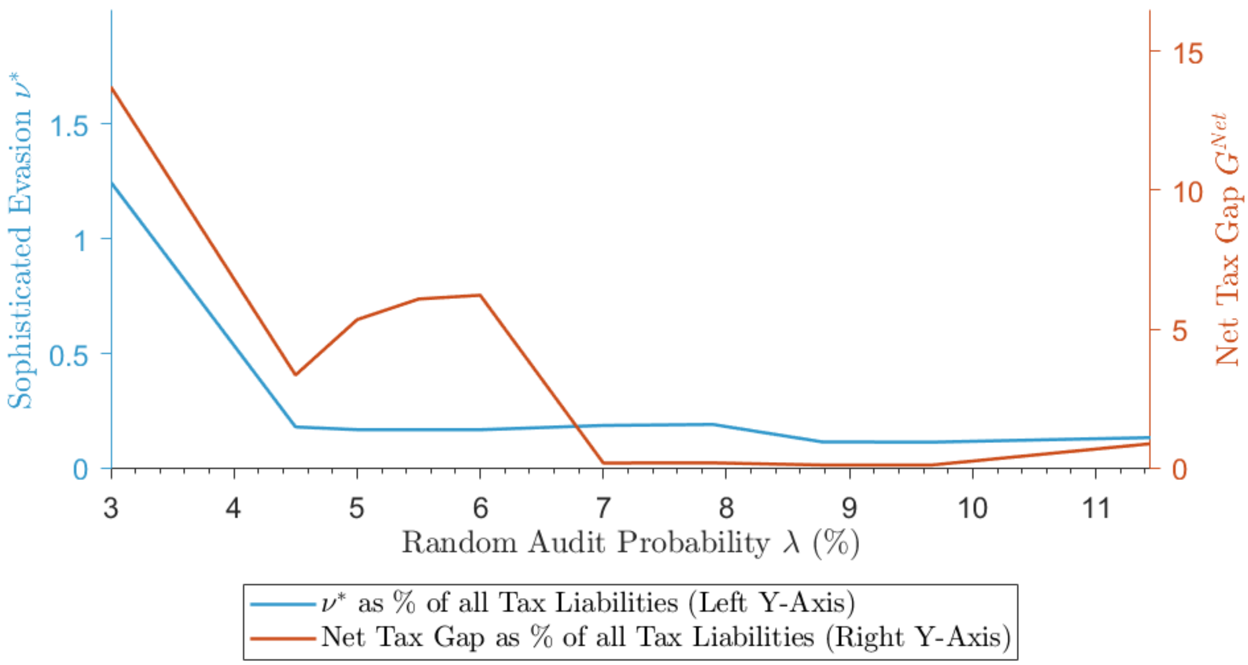
4.3 Long-Term

The aggregate effect of changes in policy parameters $\eta, \lambda, \tau, \beta$ in the tax-gap and on total evasion in the long-run are not as clear as in the short-run as stationary distribution $f(a,k)$ may change.

To show this, we can simulate the aggregate net tax gap and total sophisticated evasion ν_t^* for different values of auditing λ and enforcement β, holding them constant.

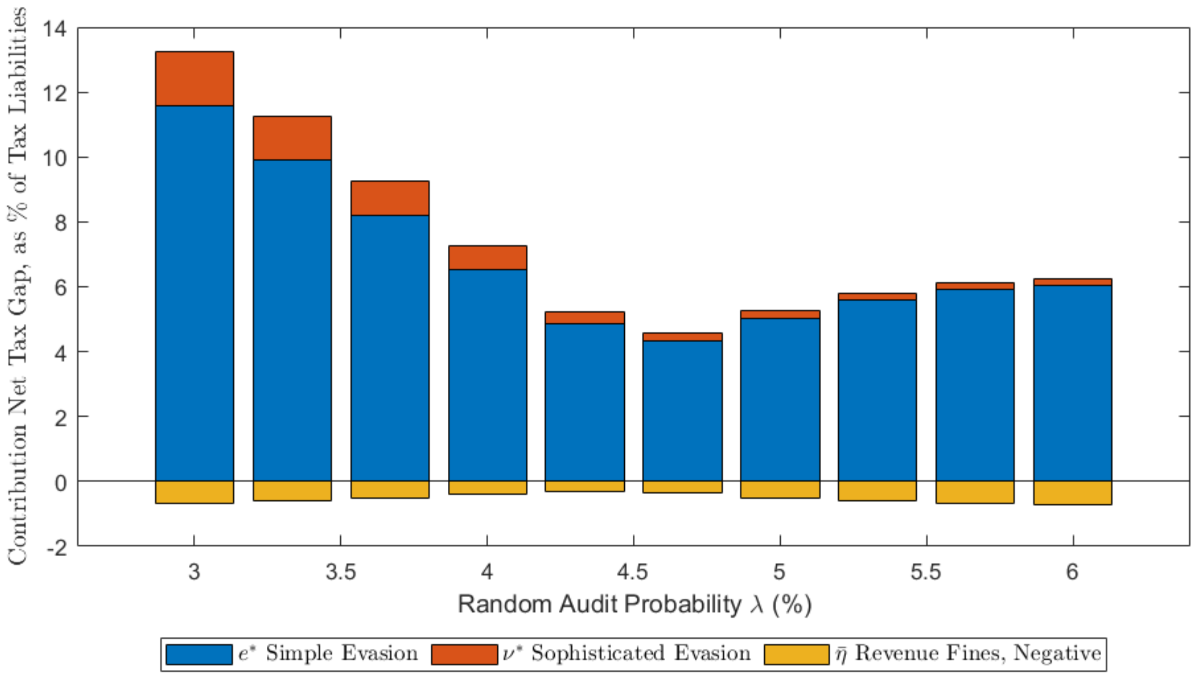
[12] An increase in fine enforcement would be a decrease in β, thus we present the results of and increase in $1-\beta$ as it is equivalent.

Figure 3: **Aggregate Sophisticated Evasion and Net Tax Gap across all Random Audit Probability in the Long Term** as % of all Tax Liabilities ($\beta = 0.8$)



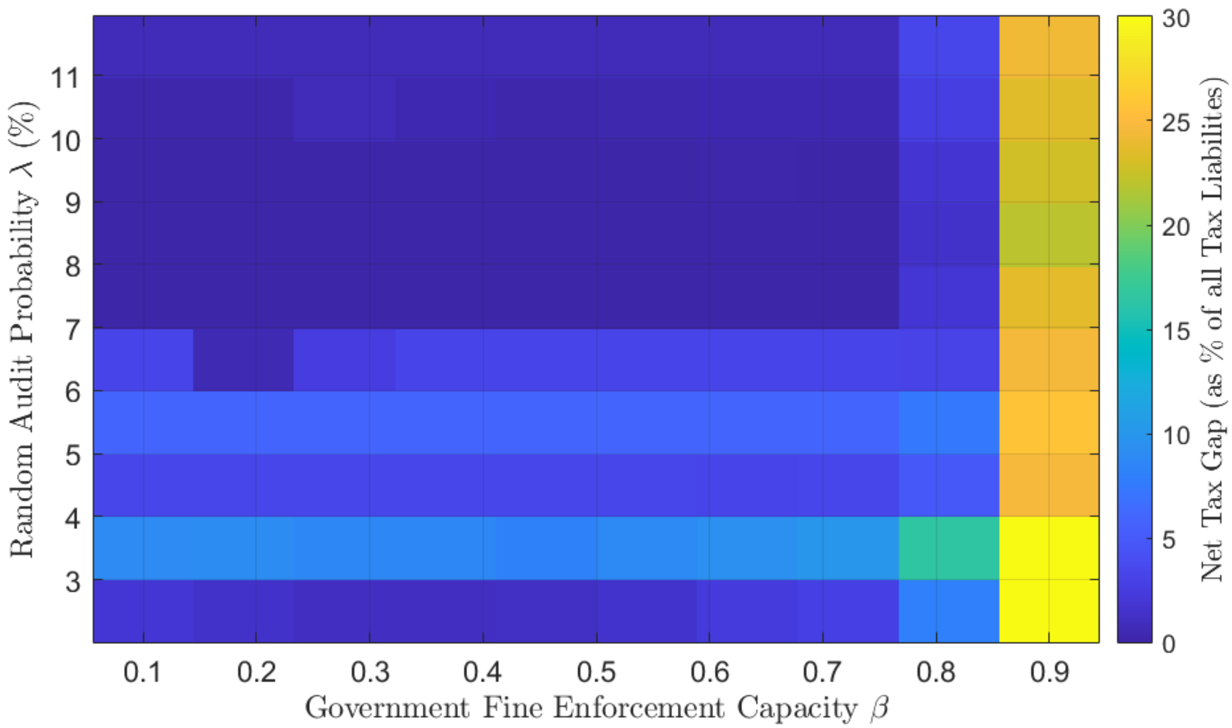
As evidenced in figure 3, an increase in random audits does not necessarily lead to a decrease in the total net tax gap, contrary to the effect it has in the short run (see Section 2). This non-linear relationship between aggregate sophisticated avoidance ν^* and the tax gap reflects changes in several aggregate evasion responses. As expected, low levels of random audits induce higher levels of total tax evasion, increasing the tax gap. An increase in the probability of being audited may initially decrease the total tax gap as simple evasion is deterred, but may eventually induce several high income tax payers to engage in higher levels of sophisticated evasion as they shift to its use. Consequently, the governments' ability to close the tax gap through the use of fines is diminished, as richer agents who engage in ν^* are subject to less fines proportional to their income y_t compared than the others. Its ability to close the tax gap relies on deterring evasion and reducing total gross tax gap G^{Gross}, rather than making up the potential tax liabilities lost through increased fine revenues $\bar{\eta}$. Figure 4 below decomposes the effects increased random audits λ has on closing the tax gap and how revenues from fines are limited in its capacity to reduce the total tax gap.

Figure 4: **Decomposition of Aggregate Net Tax Gap** as % of all Tax Liabilities (β=0.8)



This decrease in the ability of random audits decreases as long as $f(a,k)^{\nu_t^*>0} > 0$. This highlights the importance of enforcement β in reducing the tax gap. However, an improvement in fine enforcement quality, or a reduction in β, is only useful in reducing the tax gap if $f(a,k)^{\nu_t^*>0} > 0$ in the long-run and may also reduce the total capital accumulation growth of agents above threshold Eq.22. At higher levels of the quality of fine enforcement $\beta \to 0$, total sophisticated evasion and the net tax gap remains relatively stable as its driven by the total simple evasion demand across all tax payers. Under very low levels of quality of enforcement $\beta \to 1$, the tax gap increases linearly with total sophisticated ν^*, as seen in figure 5.

Figure 5: **Average Long-Term Net Tax Gap estimates across different Random Audit Probability Rates and Government's Fine Enforcement Capacity**



This figure further emphasizes the importance of distinguishing threshold Eq. (22). It can be observed that once $\beta > 0.8$, holding all fixed costs and distributional estimates of a constant, the capacity of random audits to deter the net tax gap is strongly diminished. Even at very high levels of random audits $\lambda > 10\%$, tax gaps remain unable to be closed once threshold Eq. (22) is trespassed. This threshold denotes the point where there exists a mass of agents where $f(a, k)^{\nu_t^{*}>0} > 0$.

5 Discussion

The results in Section 4 underscore the critical role that the income distribution of taxpayers $f(a, k)$ plays in shaping tax evasion behavior and its implications for government revenue collection over time. This section delves into the mechanisms driving these results, the policy implications, broader insights for tax enforcement, and the limitations of the model.

5.1 Main Mechanisms

5.1.1 Unequal Capital Growth Dynamics.

The unequal capital growth dynamics observed across agents within the joint productivity and capital distribution $f(a, k)$ arise from their higher propensity to optimally choose riskier strategies, such as increased evasion and greater investment in risky portfolio shares, to maximize their future income streams y_t. These dynamics are largely driven by agents' HARA utility preferences, where risk aversion decreases with income. Consequently, agents' optimal strategies to maximize consumption through future income streams y_t are highly dependent on their initial productivity a_0 and capital k_0. Agents with higher a_0 and k_0 can afford to engage in both higher levels of total evasion and riskier portfolio investments. As a result, their total evasion strategy, $\nu_t^* + e_t^*$, increases monotonically with income y_t.

The factors driving these capital growth dynamics, and their implications for optimal evasion paths, are further influenced by productivity risks σ and the premium on risky assets π. Moreover, understanding the differences in productivity processes a_t is essential to predicting agents' future evasion and capital accumulation paths, based on their initial productivity a_0. The varied evasion trajectories and their effects on income processes and the government's tax gap can be seen in Figure 6.

Figure 6: **Evasion time dynamics by initial productivity.**
These charts depict different evasion paths chosen by agents starting at various initial productivity levels a_0 in two economies: one with low fine enforcement capacity $\beta = 0.9$ and another with high fine enforcement capacity $\beta = 0.5$. Initial productivity $a_0 \in \left(a_{Init}^{Low} = 0.7, \bar{a}_{Init}^{Average} = 1, a_{Init}^{High} = 1.45\right)$ corresponds to the 10th, 50th, and 90th percentile productivity simulated, with minimum and maximum productivity of $a_0 \in \left(\underline{a_t} = 0.6, \overline{a_t} = 1.6\right)$ over a 15-year period.

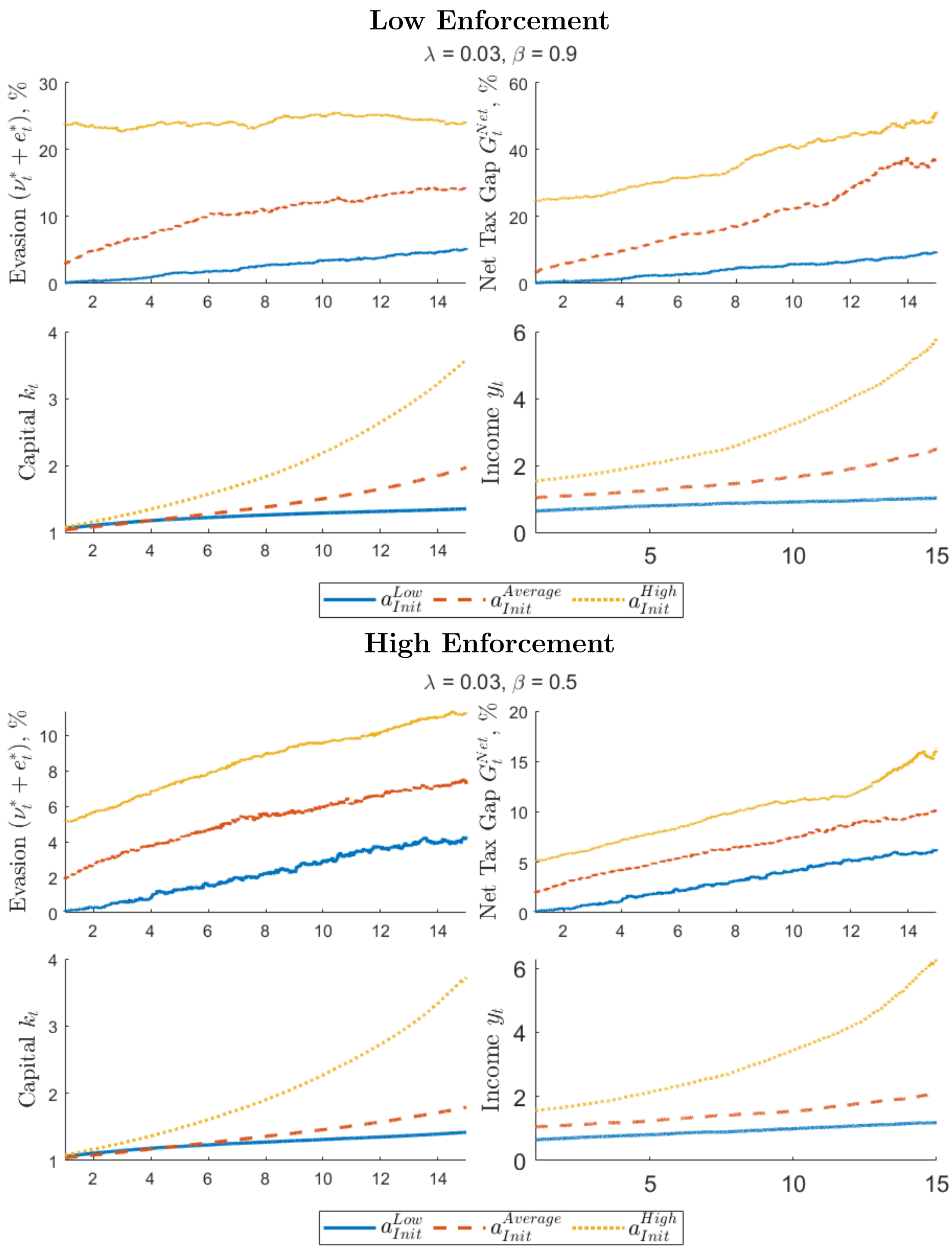


Figure 6 illustrates that capital k_t, income y_t, total evasion $e_t^* + \nu_t^*$, and their initial contributions to the net tax gap G_t^{Net} grow at much higher rates for agents starting with higher initial productivity levels, $a_0 \in \left(a_{Init}^{Low}, \bar{a}_{Init}^{Average}, a_{Init}^{High}\right)$. As a result of these differing capital accumulation processes, the expected costs of fines due to random audits diminish with taxpayers' income, even for those not engaging in sophisticated evasion. Increased perceptions of audit probability or fines lead to less significant reductions in total evasion for wealthier agents. Consequently, total evasion increases with income as wealthier taxpayers' optimal strategies become less sensitive to the expected costs of being audited.

This further reinforces riskier behavior among wealthier agents. Increased evasion creates a feedback loop where disposable capital, after financing future consumption streams, leads to riskier investments, higher returns, and further increases in future evasion. Figure 6 shows how total evasion, capital, and income rates converge at much faster rates for agents with higher levels of productivity in the high enforcement scenario.

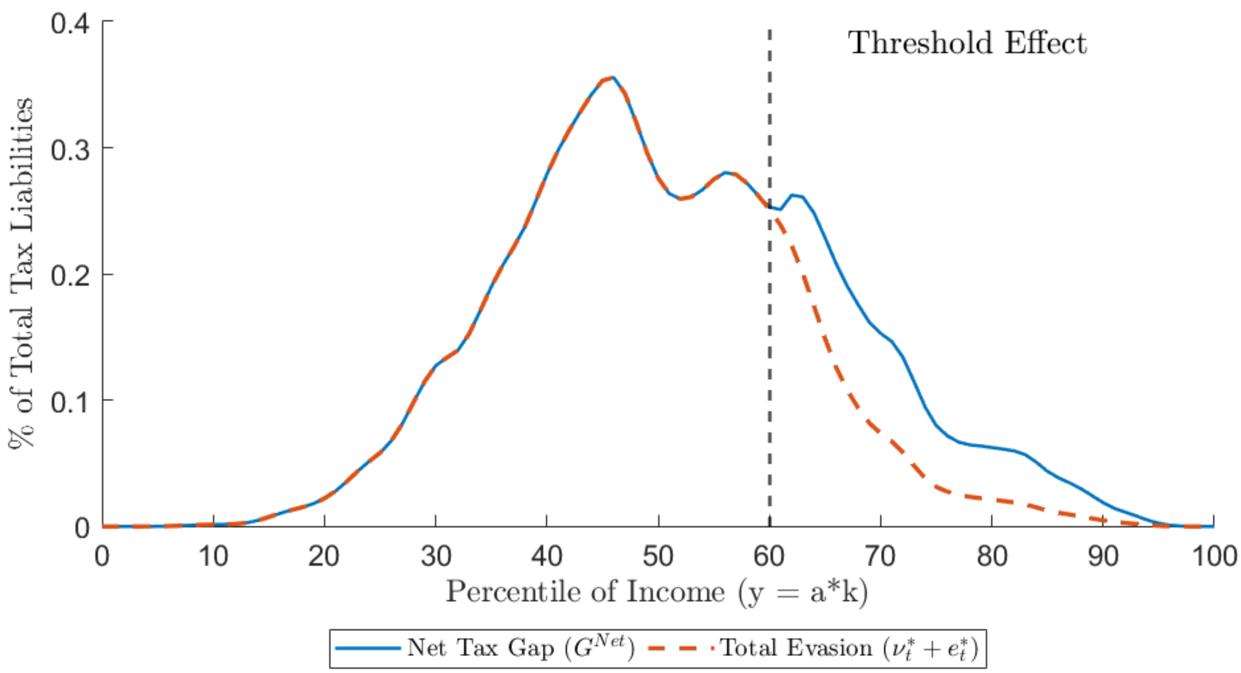
Nonetheless, the mean-reversion process driven by μ ensures that total evasion does not grow indefinitely in the long run and eventually converges. This ensures that all optimal paths converge to a stationary joint stationary distribution for all agents. Thus, for a given set of optimal evasion paths, random audits will always remain effective at reducing evasion in the short term across all income groups.

5.1.2 Threshold Effects Caused by Fixed Costs Lead to Disproportionate Effects on the Tax Gap.

Taxpayers with sufficient productivity (a_t) and capital (k_t) to surpass the threshold $a_t \geq \frac{\chi_0}{\tau\beta}$ disproportionately contribute to the tax gap. This threshold stems from the fixed costs (χ_0, χ_1) required for sophisticated evasion strategies, creating a discontinuity in evasion behavior across income levels and further limiting the government's capacity to close the tax gap through increased random audits or fine rates.

As more capital is allocated to cover fixed costs (χ_0, χ_1), taxpayers above the threshold significantly increase their total evasion. This results in disproportionately greater impacts on the net tax gap, as a larger share of the income generated by these agents is not collected by the government.

Figure 7: **Distribution of Net Tax Gap and Evasion Contributions** This figure depicts the amount of evasion and net tax gap contributed to the total amounts by income group, weighted by the amount of wealth held by agents across the distribution.



While total evasion may grow monotonically with income, contributions to the net tax gap do not. The effectiveness of collecting fines or reducing evasion decreases for wealthier agents. This phenomenon is illustrated in Figure 7, which shows the long-run distribution of agents' contributions to both total evasion and the net tax gap under baseline assumptions (Section 4.3).

The threshold effect introduces a non-monotonic decrease in the ability of governments to close the tax gap among wealthier taxpayers. Traditional deterrence policies, such as random audits (λ) and fines (η), have no impact on sophisticated evasion strategies in the short term, leading to increased evasion behaviors despite greater detection risks. This further diminishes their effectiveness in reducing total evasion among the wealthy.

The amount of taxable income subject to possible abuse, quantified as $\beta \nu_t y_t \tau$, reduces the government's capacity to reduce the total tax gap. The amount of income in an economy subject to this, the legal gray area depends on the distribution of a_t agents in an economy, the fixed costs to engage in sophisticated evasion χ_0, χ_1, the tax rate τ, and more importantly, the government's capacity to enforce fines β. This can be seen in figure 6 - in the low enforcement scenario, agents with average initial productivity $\bar{a}_{Init}^{Average}$ contribute disproportionately to the net tax gap and follow income and evasion paths similar to those agents who started with high productivity. This happens as in the high enforcement setting, agents $\bar{a}_{Init}^{Average}$ do not cross threshold $a_t \geq \frac{\chi_0}{\tau\beta}$, but do so in the low enforcement one. On the other hand, agents who start low levels of productivity a_{Init}^{Low}, follow very similar evasion, capital and income paths in both enforcement settings. More importantly, they do not contribute disproportionately to the tax gap as the other agents who cross this threshold do.

Thus, it can be seen that higher fine enforcement capacity (β) reduces the legal gray area where taxpayers exploit ambiguous boundaries between evasion and avoidance, strengthening the government's ability to collect revenues from wealthier taxpayers over time.

5.2 Policy and Broader Implications

The model's mechanisms underscore critical insights into how enforcement policies influence tax compliance across different income groups and over time. The effectiveness of tax deterrence policies, such as random audits, fines, and fine enforcement capacity (β), are intricately tied to the underlying distribution of agents ($f(a,k)$) and their expected capital accumulation trajectories.

Heterogeneous Effects of Enforcement Policies: The aggregate size and magnitude of deterrence depend heavily on the initial distribution of agents and the uneven impact of risk determinants such as $\mu, \sigma, \pi, \lambda$. Wealthier agents, with greater capital and productivity levels, are uniquely positioned to engage in riskier strategies, accumulating more wealth over time. This dynamic reduces the overall efficacy of conventional policies like random audits (λ) and fines (η) as income inequality rises.

Threshold effects from fixed costs ($a_t \geq \frac{\chi_0}{\tau\beta}$) concentrate sophisticated evasion and disproportionate net tax gap losses among high-income taxpayers and make traditional enforcement less effective for this group. The more amount of total income in the economy is concentrated in the hands of tax payers who are above this threshold, the higher the legal gray area of an economy may be, reducing the the efficacy of classic evasion deterrence tools to collect income over time and in the short-run.

Balancing Short-Term and Long-Term Policy Trade-offs (λ and β): While random audits (λ) remain effective in reducing simple evasion among lower-income taxpayers, their short-term impact diminishes when addressing wealthy individuals who rely on sophisticated evasion strategies. The effectiveness of enforcement depends on the government's ability to influence expectations about future income streams. Specifically, increasing β alters agents' perceptions that illegal income, particularly that exploiting legal gray areas, will face higher fines and reduced loopholes. This adjustment reduces the incentive to engage in sophisticated evasion. However, higher β may also dampen capital accumulation among high-income taxpayers in the long run. Additionally, the choice of enforcement strategy must consider the expected implementation costs, which are not explored within this paper. These costs, ranging from administrative expenses to the economic impact of altered capital accumulation, are pivotal in determining the overall feasibility and effectiveness of policy measures. Thus, the optimal policy lies in balancing these trade-offs—using λ and fines effectively in the short run, especially for taxpayers unlikely to cross the β threshold, while strengthening institutional and legal enforcement to achieve sustained compliance in the long run.

5.2.1 Policy Recommendations

The insights from the model suggest several key recommendations for addressing tax evasion and closing the tax gap more effectively.

Increase random audits to deter simple evasion, but complement them with stronger fine enforcement for high-income individuals. In the short term, increasing random audits (λ) can effectively reduce simple evasion, particularly among lower-income taxpayers who are responsive to detection risks and lack the resources for sophisticated strategies, as demonstrated in Table 1. However, Figure 3 shows diminishing returns when applying random audits to high-income individuals who exploit sophisticated evasion strategies, both individually and in aggregate. This underscores the necessity of coupling random audits with policies that enhance the government's fine enforcement capacity or demonstrate the ability to utilize new reporting mechanisms effectively. As corroborated in the past literature, increased detection rates that improve credible third-party information reporting, combined with increased random audits rates, can still be particularly effective; as low-income tax payers do not expect their income processes to lead them to a point where they can exploit the legal gray areas available past the threshold where sophisticated evasion becomes available (Kleven et al., 2011). As such, credible increases in the likelihood that detection will lead to a fine through random audits may be especially impactful in countries with low levels of third-party information reporting or where many agents cannot afford the fixed costs of sophisticated evasion.

Enhance fine enforcement capacity (β) to address sophisticated evasion and reduce future evasion opportunities. Increasing fine enforcement capacity β is critical for reducing the legal gray area exploited by high-income individuals, as shown in Figures 7 and 5. Enhanced fine enforcement reduces opportunities for high-income taxpayers to avoid detection and increases the penalties for non-compliance. Achieving this requires simplifying tax codes and allocating resources to the tax collecting agencies and the judiciary for processing complex evasion cases more effectively Gamannossi degl'Innocenti et al. (2022). Additionally, addressing information asymmetries through targeted audits and third-party reporting requirements on the rich, improves the quality of fine enforcement revenues while preventing future income from being diverted into sophisticated evasion. By reducing the size of the economy subject to legal gray areas, these measures also enhance the proportional deterrence effect of random audits and fines across all income groups.

Integrate domestic and global enforcement mechanisms for a cohesive tax compliance framework. Effective tax enforcement requires a comprehensive strategy that aligns global transparency efforts with strengthened domestic legal frameworks and auditing capacity. Global reforms that enhance reporting requirements should be designed to support domestic auditing processes, ensuring that the newly acquired information is effectively leveraged to improve fine enforcement on evaders. Without a credible domestic threat that this information will lead to tangible penalties, high-income taxpayers may perceive audits as low-risk and continue evading taxes, as the expected gains from evasion in the future outweigh the potential costs of being fined. To address this, reforms must prioritize increasing transparency, enhancing fine enforcement, and closing legal ambiguities. By reducing the gray areas that enable sophisticated evasion strategies, these measures ensure the government can sustain revenue collection and strengthen its ability to deter evasion across all income groups.

These policy recommendations may have direct implications when analyzing the effects of global initiatives like FATCA and the CRS. If rich tax payers do not believe that increased in third party information reporting can be effectively used to fine sophisticated evasion strategies, and a massive amount of tax liabilities in the economy are subject to legal grey areas, then the rich tax payers evasion behavior will not be proportionately curtailed by a threat of an increase in auditing.

5.2.2 Policy Discussion through the Model: FATCA, CRS, and U.S. Proposals

Global Initiatives: FATCA and CRS were landmark initiatives designed to address cross-border tax evasion by requiring foreign financial institutions to report information on non-resident account holders. While these frameworks successfully reduced offshore evasion, they left domestic enforcement gaps unaddressed - IRS estimates of the total tax gap and its composition, have remain relatively unchanged over the years (See Figure 10 in the Appendix). Third party-information acquired by the newly required reporting standards enacted by them were perhaps not as effective in reducing the tax gap as the threat of domestic fine enforcement at home was not leveraged. Specifically:

- **FATCA and CRS focus on information sharing**, improving governments' ability to detect cross-border evasion. However, they do not target domestic evasion strategies or effectively enhance fine enforcement (β) for high-income individuals detected in new cross-border domestic flow data acquired through the reporting mechanisms enacted by them.

- **Threshold Effects and Legal Gray Areas**: Figures 7 and 6 show how sophisticated evasion strategies dominate once taxpayers surpass the fixed cost threshold. FATCA and CRS may be less effective in curtailing evasion, as richer tax payers' can keep relying on exploiting domestic legal ambiguities rather than hiding income offshore, or rely on the government's inability to enforce fines under these new reporting mechanisms. These findings corroborate recent government assessments that the Internal Revenue service is still not prepared to enforce compliance with FATCA Treasury Inspector General for Tax Administration (TIGTA) (2018), as the IRS could not corroborate the data provided to them on U.S. taxpayers.

5.2.3 Proposed Domestic Legislation

The Biden administration's proposed financial reporting reforms, as outlined in "The American Families Plan Tax Compliance Agenda" U.S. Department of the Treasury (2021), may address these shortcomings by:

- **Increasing Fine Enforcement Capacity (β) through better Funding**: By improving the IRS's ability to enforce fines domestically through improved funding and the ability to leverage third-party information to successfully target their audit efforts, this proposed legislation reduces the legal gray area that sophisticated evasion exploits. Third-party information that is already available through enhanced reporting established by FATCA, could finally be used to pose a more credible threat to richer tax payers, making the threat of random audits increasingly effective in deterring their use of both simple and sophisticated evasion strategies.

- **Improving Domestic Third-Party Reporting**: The proposal mandates that domestic banks and financial institutions report aggregate inflows and outflows for individual accounts, rather than detailing income by type. Additionally, extending reporting requirements to encompass new financial assets, such as cryptocurrency transactions, would enable the IRS to monitor compliance among high-value accounts over time. Discrepancies between bank-reported aggregates and taxpayer filings could be utilized to enhance audit effectiveness and fine enforcement on individuals.

- **Focusing on Domestic High-Income Taxpayers**: Unlike FATCA and CRS, which target cross-border evasion, this proposal prioritizes high-income taxpayers at home. By implementing improved reporting on financial flows and establishing exceptions for accounts below a low de minimis gross flow threshold, the IRS can focus its audit efforts on significant misreporting. This strategy addresses information asymmetries that facilitate under-reporting by high-income taxpayers, who often have the means to restructure income sources through professional services and absorb associated fixed costs. Consequently, the focus remains on wealthier individuals, aligning with the model's emphasis on the necessity for stronger fine enforcement.

These measures may be particularly impactful in light of the model's findings, as they improve the quality of fine enforcement and address critical gaps in existing frameworks. Figures 7 and 6 underscore the importance of β in closing the tax gap, demonstrating how enhanced fine enforcement capacity and reduced ambiguity in domestic tax reporting complement global transparency efforts. Additionally, the new legislation effectively targets wealthy individuals financial flows domestically that exceed the fixed cost threshold for sophisticated evasion, thereby addressing a significant shortfall left by FATCA and CRS.

5.3 Limitations

Despite its contributions, the model has several limitations that warrant discussion:

- **Restrictive Assumptions:** The model assumes linear tax rates (τ) and uniform audit probabilities (λ). In reality, high-income taxpayers are often subject to higher tax rates and more frequent audits, which, in this framework, have opposing effects on evasion behavior. While incorporating these dynamics would improve realism, the core insights of the model are unlikely to change significantly.

- **Limited Corroborating Data:** The model relies heavily on U.S. data, particularly estimates from Guyton et al. (2023). Although similar evasion patterns have been documented by Alstadsæter et al. (2019); Leenders et al. (2023); Keen and Slemrod (2017); Londoño-Vélez and Ávila Mahecha (2021) in Scandinavia, Norway, and Colombia, more comparable cross-country evidence is needed to generalize the findings.

- **Partial Equilibrium Setting:** The absence of supply-side modeling, such as the role of firms and financial intermediaries in facilitating evasion, limits the model's ability to conduct a full welfare analysis. Incorporating these elements could provide a more comprehensive understanding of evasion dynamics.

- **Government Inaction:** The model does not account for the costs of auditing or increasing β, nor does it consider redistribution benefits. While these omissions limit the analysis of optimal government policies, the framework can be extended to include such considerations.

6 Conclusion

The old adage that "the poor evade and the rich avoid" remains partially true, but the paper's findings suggest a more nuanced view: "the poor and the rich evade, but the rich also avoid when they can"—placing the responsibility on governments to define and enforce the boundaries between legal avoidance and illegal evasion. This framework explains why tax deterrence policies, such as increasing random audits, may have limited or short-lived effects, particularly on high-income taxpayers who can exploit sophisticated evasion strategies. The model also highlights the critical role of fine enforcement capacity in sustaining compliance, especially among the wealthy. The U.S. calibration exercise validates the model's capacity to replicate observed patterns in the tax gap and its decomposition, providing a solid foundation to study the impacts of short- and long-term tax deterrence policies.

The results emphasize the importance of income heterogeneity and time dynamics in shaping the effectiveness of tax enforcement. While random audits can effectively deter simple evasion in the short run, their long-term impact diminishes as wealthy agents adapt by exploiting legal gray areas. Strengthening fine enforcement capacity emerges as a key lever for reducing the tax gap in the long term. Future research should explore the model in a general equilibrium setting, incorporating optimal government decisions that account for enforcement costs and redistribution effects. Additionally, examining the role of progressive taxation and auditing would provide further insights into designing equitable and efficient tax systems. This framework lays the groundwork for understanding how policy interventions can address persistent tax gaps and promote sustainable revenue generation.

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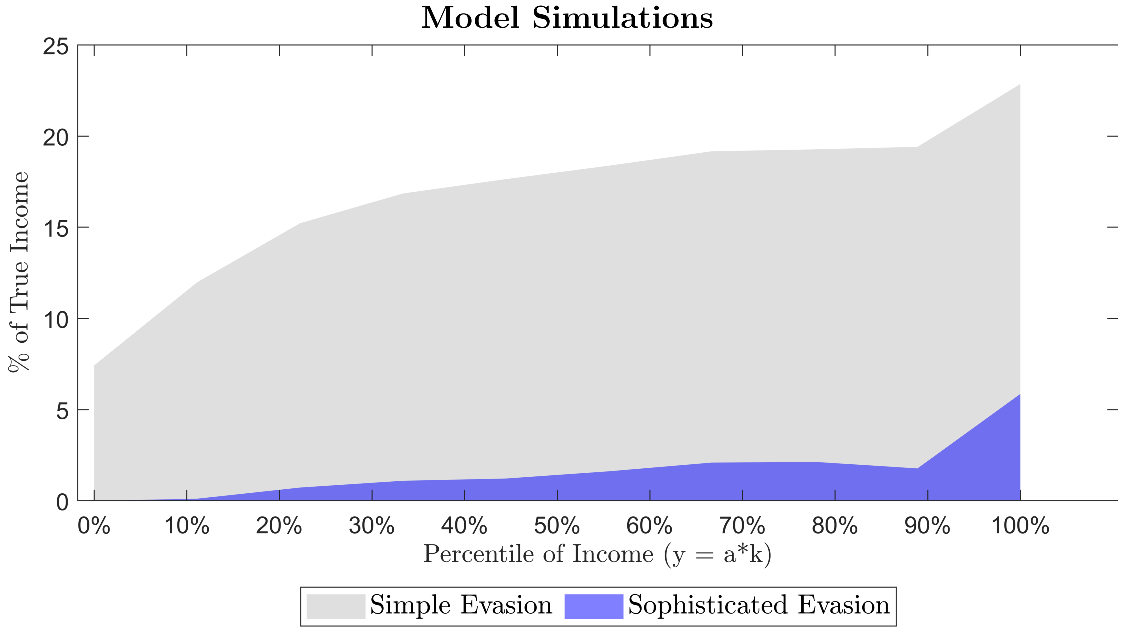
Figures

Figure 8: **Comparison of Model Simulations and Estimates on US Unreported Income, by Percentile of Income (as % of True Income y_t), Average 2006-2013**. The top panel shows the model estimates of unreported income, while the bottom panel presents the corresponding real data as estimated by Guyton et al. (2023). Both plots display the distribution of simple and sophisticated evasion across percentiles of income.



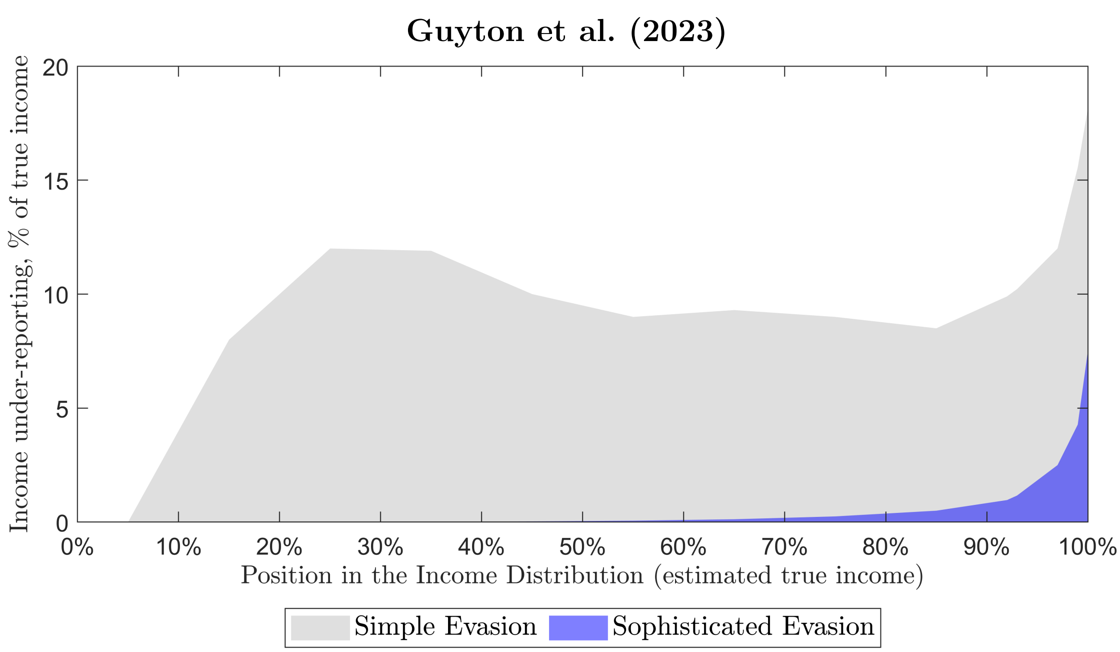


Figure 9: **U.S. Tax Gap Decomposition Estimates over Time**

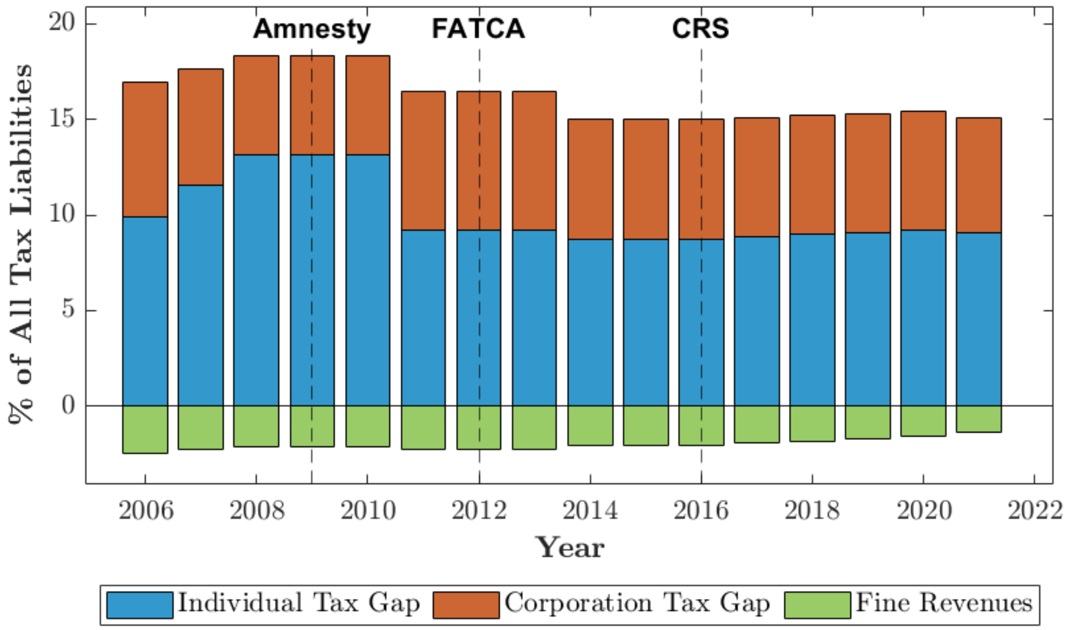


Figure 10: **Source: ?** Estimates.

Appendix A. Solution

Omitting time subscripts, the taxpayer's constrained optimization problem stated in eq.(12) satisfies the following Hamilton–Jacobi–Bellman equation (HJBE)[13]:

$$0=\frac{\partial V}{\partial t}-(r+\lambda)V+\frac{\partial V}{\partial k}ak(1-\tau)+\frac{\partial V}{\partial a}\mu_a+\frac{1}{2}\frac{\partial^2 V}{\partial a^2}\sigma_a^2+$$

$$+\sup_{e,v}\left\{\frac{\partial V}{\partial k}k\left[\tau a\left(e+v\right)-\phi v-f(v)\right]+\lambda V(k-\tau ak\eta\left(e+(1-\beta)v\right))\right\}$$

$$+\sup_{\theta}\left\{\frac{\partial V}{\partial k}k\theta\pi+\frac{1}{2}\frac{\partial^2 V}{\partial k^2}\theta^2k^2\sigma_a^2+\theta k\frac{\partial^2 V}{\partial a\partial k}\sigma_a^2\right\}+\sup_{c}\left\{\frac{(c-c_m)^{1-\gamma}}{1-\gamma}-\frac{\partial V}{\partial k}c\right\}$$

with complementary slackness condition $\phi v=0$,where ϕ is the Lagrangian Multiplier. Subsequently, its FOCs are:

$$c: c=c_m+\left(\frac{\partial V}{\partial k}\right)^{-\frac{1}{\gamma}}$$

$$e:\frac{\partial V}{\partial k}=\eta\lambda\frac{\partial V(k-\tau ak\eta\left(e+(1-\beta)v\right))}{\partial(k-\tau ak\eta\left(e+(1-\beta)v\right))}$$

$$v:\frac{\partial V}{\partial k}\left(\tau a-\phi-\frac{\partial f(v)}{\partial v}\right)=\lambda\frac{\partial V(k-\tau ak\eta\left(e+(1-\beta)v\right))}{\partial(k-\tau ak\eta\left(e+(1-\beta)v\right))}\tau A\eta(1-\beta)$$

$$\theta:\frac{\partial V}{\partial k}k\pi+\frac{\partial^2 V}{\partial k^2}\theta k^2\sigma_a^2+k\frac{\partial^2 V}{\partial a\partial k}\sigma_a^2=0$$

Substituting the second FOC in the third and rearranging yields:

$$e:\frac{\partial V}{\partial k}=\eta\lambda\frac{\partial V(k-\tau ak\eta\left(e+(1-\beta)v\right))}{\partial(k-\tau ak\eta\left(e+(1-\beta)v\right))}$$

$$\frac{\frac{\partial V}{\partial k}\left(\tau a-\phi-\frac{\partial f(v)}{\partial v}\right)}{\tau a(1-\beta)}=\lambda\frac{\partial V(k-\tau ak\eta\left(e+(1-\beta)v\right))}{\partial(k-\tau ak\eta\left(e+(1-\beta)v\right))}\eta$$

Solving the model: Next, we guess (and verify) that the value function takes the following form:

$$V=F(t,a)^b\frac{(k-H(t,a))^{1-\gamma}}{1-\gamma}$$

[13] See Karatzas and Shreve (1998).

where b is a free coefficient to be determined at convenience. Omitting functional dependence, the remaining FOCs entail the following optimal policies:

$$c^* = c_m + F^{-\frac{b}{\gamma}} (k - H)$$

$$e^* = \frac{(k-H)}{\tau a \eta k}\left(1 - (\eta\lambda)^{\frac{1}{\gamma}}\right) - (1-\beta)\, v^*$$

$$\theta^* k = (k - H)\left(\frac{\pi}{\gamma\sigma_a^2} + b\frac{\partial F}{\partial a}\frac{1}{F}\right) + \frac{\partial H}{\partial a}$$

Note that tax avoidance and evasion are substitutes (see e^*). Evasion is increasing in net worth (k) but decreasing in productivity a (directly); unclear indirectly (through $H(a)$). Substituting the guesses, the optimal policies in the HJBE, setting $b = 1$ and rearranging yields the following PDE:

$$(r+\lambda)\, F\frac{(k-H)^{1-\gamma}}{1-\gamma} = \frac{\partial F}{\partial t}\frac{(k-H)^{1-\gamma}}{1-\gamma} - F(k-H)^{-\gamma}\frac{\partial H}{\partial t} + F(k-H)^{1-\gamma}A(1-\tau) +$$

$$+F(k-H)^{-\gamma}HA(1-\tau) + \frac{\partial F}{\partial a}\frac{(k-H)^{1-\gamma}}{1-\gamma}\mu_a - F(k-H)^{-\gamma}\frac{\partial H}{\partial a}\mu_a +$$

$$+F(k-H)^{1-\gamma}\left(\tau A\beta v^* - \phi v^* - f(v^*)\right) + \lambda F\,(k-H)^{1-\gamma}\,\frac{(\eta\lambda)^{\frac{1-\gamma}{\gamma}}}{1-\gamma} +$$

$$+F(k-H)^{-\gamma}H\left(\tau A\beta v^* - \phi v^* - f(v^*)\right) + F(k-H)^{1-\gamma}\left(1 - (\eta\lambda)^{\frac{1}{\gamma}}\right) +$$

$$+\frac{1}{2}b\frac{\partial^2 F}{\partial a^2}\frac{(k-H)^{1-\gamma}}{1-\gamma}\sigma_a^2 - \frac{1}{2}F(k-H)^{-\gamma}\frac{\partial^2 H}{\partial a^2}\sigma_a^2 +$$

$$+F(k-H)^{1-\gamma}\frac{\pi^2}{\gamma\sigma_a^2} + +F^b(k-H)^{-\gamma}\pi\frac{\partial H}{\partial a} +$$

$$+\frac{\pi}{\gamma\sigma_a^2}\frac{\partial F}{\partial a}(k-H)^{1-\gamma}\sigma_a^2 + \frac{\pi}{\gamma\sigma_a^2}\gamma F^b(k-H)^{-\gamma}\frac{\partial H}{\partial a}\sigma_a^2 +$$

$$+F^{1-\frac{1}{\gamma}}\,(k-H)^{1-\gamma}\,\frac{\gamma}{1-\gamma} - \left(F(k-H)^{-\gamma}c_m\right) +$$

$$-\frac{\gamma}{2}F(k-H)^{1-\gamma}\left(\frac{\pi}{\gamma\sigma_a^2}\right)^2\sigma_a^2-\frac{\gamma}{2}F(k-H)^{-\gamma}\frac{\pi}{\gamma\sigma_a^2}\frac{\partial H}{\partial a}\sigma_a^2$$

Similar to Menoncin and Vergalli (2021), separating the PDE into two equations containing $(k-H)^{1-\gamma}$ and $(k-H)^{-\gamma}$ yields the following two PDEs:

$$\rho_F F = F^{1-\frac{1}{\gamma}}\gamma + \frac{\partial F}{\partial t} + \left(\mu_a + \frac{1-\gamma}{\gamma}\pi\right)\frac{\partial F}{\partial a} + \frac{1}{2}\frac{\partial^2 F}{\partial a^2}\sigma_a^2$$

where
$\rho_F \coloneqq (1-\gamma)\left(\frac{r+\lambda}{1-\gamma} - a(1-\tau) - (\tau a\beta v^* - \phi v^* - f(v^*)) - (1-\gamma)\,\lambda\,(\eta\lambda)^{\frac{1-\gamma}{\gamma}} + \left(1-(\eta\lambda)^{\frac{1}{\gamma}}\right) + \frac{1}{2\gamma}\frac{\pi^2}{\sigma_a^2}\right)$
and

$$H\rho_H = c_m + \frac{\partial H}{\partial t} + \frac{\partial H}{\partial a}(\mu_a - \pi) + \frac{1}{2}\frac{\partial^2 H}{\partial a^2}\sigma_a^2$$

where $\rho_H \coloneqq A(1-\tau) + \tau A\beta v^* - \phi v^* - f(v^*)$, with boundary conditions $\lim_{t\to\infty} H_t < \infty$ and $\lim_{t\to\infty} F_t < \infty$.

The solutions of these two PDEs have the following Feynman-Kac equations:[14]

$$H_t = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\infty} c_m e^{\int_t^s \rho_{H,u} du} ds\right]$$

with

$$dZ_a^{\mathbb{Q}} = \frac{\pi}{\sigma_a}dt + dZ_a$$

and

$$F_t = \mathbb{E}_t^{\gamma}\left[G_t\right]$$

where G_t satisfies the following ODE:

$$\frac{dG_t}{dt} = \rho_{F,t}G_t - G_t^{1-\frac{1}{\gamma}}\gamma$$

with boundary condition $\lim_{t\to\infty} G_t = 0$. This last equation is a Bernoulli equation, which can be linearized using the following transform:

$$U = G^{\frac{1}{\gamma}}$$

which implies

[14] See Yong and Zhou (1999); Oksendal (2000).

$$\frac{dU_t}{dt} = \frac{1}{\gamma} U_t \rho_{F,t} - 1$$

Integrating yields

$$U_t = \int_t^\infty e^{-\frac{1}{\gamma}\int_t^s \rho_{F,u} ds} dt$$

and thus

$$F_t = \mathbb{E}_t^{\mathbb{Q}_\gamma}\left[\left(\int_t^\infty e^{-\frac{1}{\gamma}\int_t^s \rho_{F,u} ds} dt\right)^\gamma\right]$$

where

$$dZ_a^{\mathbb{Q}_\gamma} = \frac{\gamma - 1}{\gamma}\frac{\pi}{\sigma_a} dt + dZ_a.$$

Numerical Solution: We compute $F(a,t)$ and $H(a,t)$ and thus the joint distribution of (k_t, a_t) and the optimal policies $\{e_t^*(k_t,a_t), v_t^*(k_t,a_t), \theta_t^*(k_t,a_t), c_t^*(k_t,a_t)\}$ by numerical (Monte Carlo) simulation. To do this, we follow the following steps:

1. **Step 1:** Solve F and H as functions of a over a suitable grid.

2. **Step 2:** Simulate (k, a) using the optimal policy functions over a long time horizon. Discard the first Nsimulations and use the ergodic theorem to interpret time averages as cross-sectional net-worth productivity distributions (space averages).

Deriving Evasion and Income: Taxpayer's total evasion behavior, $\bar{E} = \nu_t + e_t$, increases with their net worth k_t and income $y_t = a_t k_t$. Total evasion increases with total income as:

$$\frac{\partial(\bar{E})}{\partial y} = \frac{(1-\eta\lambda)\left(1 - \left(\frac{\eta\lambda}{\tau}\right)^{1/\gamma}\right)}{\eta y}\left[\underbrace{\frac{H(a,t)}{y}}_{\text{A) Simple Evasion Gains}} + \underbrace{\frac{H(a,t)\left(\frac{\beta\tau\left(\frac{\beta\tau y}{k} - \chi_0\right)}{k\chi_1} + \frac{1-\tau}{k}\right)}{\left((y/k)(1-\tau) + \frac{(\tau(y/k)\beta - \phi - \chi_0)^2}{2\chi_1}\right)}}_{\text{B) Sophisticated Evasion Gains}}\right] + \underbrace{\frac{\beta^2\tau}{k\chi_1}}_{\substack{\text{C) Additional Sophisticated} \\ \text{Evasion Growth}}} \tag{24}$$

Where $\frac{\partial(\bar{E})}{\partial y} > 0$, even when we assume the minimum consumption level $c_m = 0$[15]. However, the size and magnitude of the policy parameter's effect on agent total evasion behaviour are unclear.

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[15]As $H(a,t) \approx \frac{c_m}{a_t(1-\tau) + \frac{(\tau a\beta - \phi - \theta_0)^2}{2\theta_1}}$