

Mergers and R&D investment: A Unified Approach*

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Comments welcome!

Abstract

We generalize the literature on the implications of mergers on R&D by formulating a model where R&D effort affects both the probability of innovation and the payoff conditional on innovation success, and not just one of these two margins. We identify three channels through which a merger affects R&D investment: anticipation of price coordination that enhances payoffs, internalisation of a direct innovation externality stemming from an enhanced chance of innovation success, and internalisation of an indirect innovation externality arising from business-stealing in the product market. In models of stochastic R&D where R&D increases the probability of success without directly affecting firms' payoffs (conditional on success), the first two channels operate and we show that the common assumption that firms obtain zero payoff upon innovation failure is restrictive. In models where R&D effort impacts the payoff conditional on innovation while keeping the likelihood of innovation success independent of R&D effort, the first and the third channels operate and we show that the usual assumption that R&D effort leads to innovation success with probability one is also restrictive. For both classes of models, we show the new insight that the pre-merger level of innovation, and hence the magnitude of investment costs, may be crucial to determine whether a merger leads to higher or lower incentives to invest in R&D. To the best of our knowledge, our theoretical results nest all the existing results in the literature where R&D is modelled as a long-run variable.

JEL Classification: K21, L13, L40

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1 Introduction

The impact of mergers on innovation has been an important concern for antitrust authorities for at least two decades. For example, before the acquisition of *Sun Microsystems* by *Oracle* was cleared in the US and the EU in 2010, the European Commission investigated innovation concerns in areas such as cloud computing, database management, and open-source software development. Another example is the acquisition of *Monsanto* by *Bayer* in 2018, which underwent extensive antitrust investigation due to concerns about its potential impact on research and development of genetically modified seeds, agricultural chemicals, and digital farming technologies. Finally, the acquisition of *Celgene Corporation* by *Bristol-Myers Squibb* is also a case in point. Before the merger was approved in 2019, the Federal Trade Commission and the European Commission conducted thorough investigations to assess the risk of innovation loss resulting from discontinuation, delay or redirection of overlapping drug development pipelines.

This paper studies the implications of mergers on R&D in a (more) general model of R&D competition. Specifically, we formulate a two-stage game in which firms first invest in R&D and then compete in the market place. A higher effort in R&D may either increase a firm's innovation success probability, a firm's payoff conditional on innovation success, or both. Our model thus nests the results of two classes of existing models. In the first class, R&D is intended for entry into a new market (Federico, Langus and Valletti, 2017; Denicolò and Polo, 2018; Jullien and Lefouili, 2020). In these models, the outcome of R&D is stochastic, R&D effort increases the probability of success and failure to innovate results in a zero payoff. In the second class of models, the R&D process is completely deterministic in the sense that R&D effort surely leads to innovation success (Motta and Tarantino, 2021; Bourreau, Jullien and Lefouili, 2024). In these models, R&D is intended to reduce costs or improve product quality. For a recent survey of the existing literature, see Lefouili and Madio (2025).

The main contribution of our paper is to demonstrate that most of the existing results are incomplete because of two main restrictive assumptions and the heavy reliance on specific simulations. We first show that in models of stochastic R&D where R&D increases the probability of success without directly affecting firms' payoffs (conditional on success), the assumption that firms can only enter the market upon successful innovation is crucial (cf. Federico et al., 2017; Denicolò and Polo, 2018; Jullien and Lefouili, 2020). Specifically, we show that if firms can also obtain positive profits upon an innovation failure, like in Mukherjee (2022) and Federico et al. (2018), in which case they naturally enter (or stay in) the market with the *status quo* marginal cost or product quality, then the pre-merger level of innovation (and hence the level of R&D costs) is crucial for determining whether a merger leads to higher or lower R&D. Secondly, we show that in models in which R&D affects the marginal costs of production or product quality, the assumption that R&D effort leads to innovation success with probability one is restrictive (cf. Motta and Tarantino, 2021, section 3.1; Bourreau, Jullien and Lefouili, 2024, online Appendix).

Specifically, we show that when innovation success is not guaranteed, again the pre-merger level of innovation, and hence the shape of the R&D cost function, matters for the assessment of the impact of a merger on R&D effort. To the best of our knowledge, this result that the impact of mergers depends on the steepness of the R&D cost function is new in the literature. Finally, because in general R&D effort may very well affect both the probability of innovation and the payoff conditional on innovation success, and not just one of these two margins as the literature has so far assumed, our model offers a more complete account of the likely effects of mergers on R&D.

We identify three key channels through which a merger affects R&D investment: (i) anticipation of the price coordination that occurs after merging and enhances firms' payoffs, (ii) internalisation of a *direct* innovation externality stemming from an enhanced chance of innovation success that reduces the rival's expected payoffs, and (iii) internalisation of an *indirect* innovation externality arising from putting on the market a superior product or operating at a lower marginal cost of production that reduces the rival's conditional payoffs due to business-stealing in the product market. The key to our new results is the recognition that when innovation outcomes are stochastic, these three channels bear on the incentives to invest on two accounts, namely, the "conditional on innovation success account" and the "conditional on innovation failure" account. Because the literature so far has focused on the "conditional on innovation success account", it misses an important source of effects that lead to new results. Specifically, determining whether the merged entity has an incentive to invest higher or lower than an individual firm pre-merger boils down to comparing the Arrow replacement effect pre-merger with the Arrow replacement effect net of the externality on the partner firm post-merger. Naturally, these Arrow replacement effects do have different magnitudes depending on whether the partner firm's innovation effort is successful or not.

We now describe more in detail the results we obtain for the different classes of models outlined above. The first class of models refers to models where R&D effort only impacts the likelihood of innovation success, while keeping the payoff conditional on innovation independent of R&D effort. In this class of models, we identify four possible outcomes. In the first outcome, the Arrow replacement effect post-merger net of the externality on the partner firm is smaller than pre-merger, both conditional on partner success and failure. In such a case, the incentives to innovate are stronger post-merger than pre-merger and hence a merger definitely spurs innovation. We show that this case arises naturally in a Hotelling model with quality-differentiated products and price competition (see e.g. Gilbert and Katz, 2022). In such a model, firms first invest to upgrade their products from low quality to high quality. In the second stage, they compete in prices. We show that the price effects associated with the monopolization of the market posterior to the merger are sufficiently strong so as to make the incentives to invest post-merger larger than pre-merger. This result is in line with the result in Jullien and Lefouili

(2020) although in their model failure to innovate results in zero profits.

The second outcome features a situation where the opposite holds, that is, the Arrow replacement effect post-merger net of the externality on the partner firm is larger than pre-merger, both conditional on partner success and failure. In such a case, a merger definitely disincentivizes the partner firms to invest in innovation. We show that this case arises naturally in a model of Cournot competition with the quality-augmented demand system of Sutton (1997, 2001). In such a model, firms first invest to upgrade their products from low quality to high quality. In the second stage, they compete in quantities to serve a representative consumer. We show that when the difference between low and high quality is sufficiently large, the price effects associated with the monopolization of the market posterior to the merger are weak enough so as to make the incentives to invest post-merger smaller than pre-merger. The reason for this is essentially that the Arrow replacement effect post-merger is quite large both conditional on partner success and failure. In fact, facing demand from a representative consumer, conditional on partner success, the Arrow replacement effect is maximal so the merged entity has no incentives to invest while an individual firm pre-merger does. In addition, conditional on partner failure, and being the difference between low and high quality large, the Arrow replacement effect is quite large. This result is in line with the result in Federico et al. (2017) and Denicolò and Polo (2018) although, again, in their model a failure to innovate results in zero profits.

The third type of outcome is one where the merged entity invests more than in the pre-merger situation if and only if the pre-merger level of innovation is sufficiently low, or, equivalently, the marginal cost of effort is high enough. This occurs when the merged entity's incentives to innovate fall too short of the individual firm's conditional on partner success, but too large conditional on partner failure. In such a case, the second effect dominates when the marginal cost of effort is sufficiently high because then the pre-merger level of innovation is low and so partner failure is very likely. We show that this case arises naturally in the Sutton's model of Cournot competition with quality-differentiated products described above but when the difference between low and high quality is small enough. When this is the case, like before, conditional on partner success, the Arrow replacement effect post-merger is maximal so the merged entity has no incentives to invest compared to pre-merger. However, contrary to before, conditional on partner failure, the Arrow replacement effect post-merger is quite limited now because the difference between low and high quality is small. In such a case, the gains from upgrading from low to high quality for the merged entity are larger than for an individual firm pre-merger. As a result, conditional on partner success, the merged entity wishes to reduce investment compared to pre-merger. However, conditional on partner failure, the opposite holds. The second effect dominates when the pre-merger level of innovation is sufficiently low, or the costs of R&D high enough, because in such a case partner failure is very likely.

The last type of outcome is the mirror image of the one just explained before and occurs when the merged entity invests more than in the pre-merger situation if and only if the pre-merger level of innovation is sufficiently high, or, equivalently, the marginal cost of effort is low enough. This type of outcome occurs again due to the existence of conflicting incentives, depending on whether the partner R&D effort is successful or not. We show that this case arises naturally in the Mussa and Rosen (1978) model of vertical product differentiation with price competition (see also Motta, 1993). In this model, firms sell initially a product of low quality and can make investments to upgrade it to high quality. Consumers have heterogeneous valuations for quality. Because of price competition, firms make zero profits when they sell the same quality. This fact has important implications. Conditional on partner success, the merged entity does not have any incentive to invest whatsoever because its profits do not increase; however, an individual firm has even less incentives to do R&D because selling a low-quality product returns some profits while selling a high-quality one does not. As a result, conditional on partner success, a merger neutralises the disincentive an individual firm has to conduct R&D. Conditional on partner failure, however, the replacement effect for the merged entity is larger than for the individual firm pre-merger. This implies that conditional on partner failure, the merged entity has less incentives to conduct R&D than the individual firm pre-merger. The first effect dominates when the pre-merger level of innovation is high, or the costs of R&D low enough, because in such a case partner success is very likely.

The second class of models refers to models where R&D effort impacts the payoff conditional on innovation, often because of lower marginal cost of production or higher product quality, while keeping the likelihood of innovation success independent of R&D effort. In the literature, these models assume a deterministic R&D process by which investment surely results in an innovation (cf. Motta and Tarantino, 2021; Bourreau, Jullien and Lefouili, 2024). We relax this assumption. Essentially, this class of models yields the same results as the class discussed above in detail. The reason for this is that the formula for expected payoffs has essentially the same structure: it continues to be a weighted average of the payoff conditional on partner success and the payoff conditional on partner failure, net of investment costs. Hence, conditional on either partner success or failure, the merged entity has a greater incentive to invest when the *marginal* Arrow replacement effect net of the externality on the partner post-merger is smaller than the *marginal* Arrow replacement effect pre-merger. We show that these two incentives may be aligned or misaligned in the standard model of price competition with Sighn and Vives (1984) system of demands and cost-reducing innovation. In contrast to the result in Motta and Tarantino (2021), we show that the magnitude of the probability of success is crucial. When the likelihood of success is sufficiently high (in Motta and Tarantino it is set equal to 1), the merged entity's incentives to invest both conditional on partner success and failure fall too short from the individual firm pre-merger. Hence, a merger leads to less innovation. However, when

the likelihood of success is low enough, the merged entity’s incentives to invest conditional on partner failure are greater than those of an individual firm. In such a case, we show that the second effect dominates for sufficiently low investment costs, in which case a merger leads to more innovation.

Related literature

Our work is related to the growing theoretical literature on the impact of mergers on innovation. As mentioned above, the contributions in this literature can be divided into two classes of models. The first class of models looks at settings where the R&D process is uncertain; specifically, firms succeed in R&D with some probability and this probability depends on their R&D effort. The magnitude of the innovation, being it a cost reduction, a quality improvement, or a new product, is however independent of R&D effort. The papers of Federico et al. (2017), Denicolò and Polo (2018) and Jullien and Lefouili (2020) belong in this class. They study the effects of horizontal mergers on the incentives to develop a new product. The papers of Federico et al. (2017), Denicolò and Polo (2018) assume that products are homogeneous, while that of Jullien and Lefouili (2020) allows for horizontal product differentiation *à la* Hotelling. Federico et al. (2017) and Denicolò and Polo (2018) show that a merger between two symmetric duopolists would result in a reduction of R&D effort to develop a new product;¹ by contrast, Jullien and Lefouili (2020) show conditions under which a merger spurs innovation in a model with horizontally differentiated products *à la* Hotelling.

These papers assume that if a firm fails to develop the new product it receives a zero payoff. We relax this assumption and formulate a more general model that nests the above results. More importantly, we show that richer results may obtain in such settings, in particular, that depending on the magnitude of the investment cost function, a merger may lead to an increase or to a decrease in R&D effort. Mukherjee (2022) and Federico et al. (2018) extend the model of Federico et al. (2017) and also relax the assumption that the firms cannot operate in the market in case of innovation failure. Mukherjee (2022) shows that in a model of quantity competition with the Sighn and Vives’ (1984) system of demands and cost-reducing innovation, a merger may enhance innovation incentives. By contrast, Federico et al. (2018) present simulations based on models of price competition with the Sutton’s, nested logit and CES demand systems and quality-enhancing innovation showing that a merger always reduces innovation incentives. These papers, however, do not provide a general characterization of the possible merger outcomes. Our theoretical results in Proposition 1 show that a richer set of outcomes is possible, including that

¹Denicolò and Polo (2018) study further the model of Federico et al. (2017) and notice that a merger can lead to more innovation when the returns to R&D decrease moderately (or when the probability of failure in innovation is log-concave in R&D investments). In that case, the merged entity will focus all its efforts on one research lab. The reason is that under competition, firms may jointly succeed in innovation, which results in innovation duplication. Because the merged entity internalizes this, it may find it optimal to reduce its investment in one research lab and increase it in another.

a merger increases innovation incentives, and we provide micro-founded examples for each of these possible outcomes. We believe that in this way our paper provides a more complete view on the impact of mergers on R&D incentives.

The second broad class of models looks at frameworks where R&D effort impacts the payoff conditional on innovation while keeping the likelihood of innovation success independent of R&D effort. These models, primarily inspired by the work of d’Aspremont and Jacquemin (1988) further explored by Suzumura (1992) as well as Kamien, Muller, and Zang (1992) among others, describe a two-stage process where firms first invest in R&D to lower marginal costs of production, and subsequently engage in market competition. Section 3.1 of Motta and Tarantino (2021) is an example of an analysis of mergers in such a setting. For two specific demand structures (based on Shubik-Levitan and Salop models of demand) they report simulations results showing that equilibrium total investments post merger are lower than in the absence of merger. Also relevant is the extensions section in the online appendix of Bourreau et al. (2024) where firms invest in R&D to shift demand upwards. They show that the incentives to increase demand-enhancing R&D after a merger may be larger than when firms choose prices and R&D simultaneously. A common assumption in this work is that the R&D process is deterministic in the sense that R&D effort leads to innovation success with probability one. We relax this assumption and show in Proposition 2 that the likelihood of success may be very important for the nature of the results so that one gets an incomplete view of the impact of mergers on innovation by fixing this probability to one.²

Our work is also related to the literature on the relation between competition intensity and investments. This literature has pointed out that the Arrow replacement effect for a monopolist need not be larger than for a firm facing competition. Our main results make use of this observation. For example, Chen and Schwartz (2013) analyze the ranking of incentives to introduce new products in the absence of mergers. They show that the gain from such an innovation for a monopolist can be bigger than that for a competitive firm facing competition from sellers of the old product. Greenstein and Ramey (1998) analyze the effects of market structure on the returns from process innovation in a model where new products are vertically differentiated from older products. They show that under conditions competition and monopoly in the old product market can provide identical returns and if monopolist is threatened with entry, monopoly provides strictly greater incentives to innovate.

The remainder of the paper is structured as follows. Section 2 presents the general model and characterizes general solutions for equilibrium investment levels in the non-cooperative benchmark and in the merger scenario, identifies the key externalities and provides the intuition. In

²Both Motta and Tarantino (2021) and Bourreau et al. (2024) main analyses are in one-stage settings where firms invest in R&D and set prices simultaneously. Motta and Tarantino (2021) show that a merger would result in a reduction in merging firms’ investment efforts. The reason is that the merger will internalize pricing externalities and result in higher prices compared to the case with competing firms. This increase in prices reduces quantities, which lowers the returns from cost-reducing investments.

Section 3 we derive conditions under which mergers can spur innovation, or discourage it. Section 3.1 identifies conditions under which mergers can spur innovation in models where probability of success is endogenous, while innovation outcomes are not affected by R&D efforts. Section 3.2 identifies similar conditions for models where innovation outcomes are endogenous, while probability of success is fixed. Section 4 provides several micro-founded examples, which illustrate the novel results obtained in section 3. Section 5 concludes. More detailed derivations and proofs are relegated to Appendix.

2 The model and preliminary intuition

2.1 Model description and assumptions

We consider a duopoly market with symmetric firms, which we denote by i and j . Firms interact in the market during two stages. In the first stage, firms invest in R&D. Let x_i and x_j the amounts firms i and j put in R&D. In the second stage, upon observing the outcomes of their R&D investments, firms compete in the market.

In the innovation stage, we assume that if a firm, say i , invests in R&D an amount $x_i > 0$, then it costs the firm $C(x_i)$, with $C' > 0$ and $C'' < 0$. Investment need not result in innovation success and we denote by $\beta(x_i) \in (0, 1]$ the probability of successful innovation, with $\beta' > 0$ and $\beta'' < 0$.³ In the competition stage, firms compete in the market to sell their products. To keep the model as general as possible, we formulate reduced-form payoffs corresponding to the possible subgames that ensue after the innovation stage is over. The following table describes the possible subgames and the corresponding firms' payoffs:

		Firm 2	
		Success(s)	Failure(f)
Firm 1	Success(s)	$\pi_i^{ss}(x_i; x_j), \pi_j^{ss}(x_j; x_i)$	$\pi_i^{sf}(x_i; x_j), \pi_j^{fs}(x_j; x_i)$
	Failure(f)	$\pi_i^{fs}(x_i; x_j), \pi_j^{sf}(x_j; x_i)$	$\pi_i^{ff}(x_i; x_j), \pi_j^{ff}(x_j; x_i)$

Table 1: Firms' conditional payoffs under product competition

Note that the super-indices that describe a given subgame are ordered by player. For example, in the subgame where firm i 's innovation is successful while firm j 's is not, we index the payoff corresponding to firm i by “ sf ” to indicate that this is the payoff of a successful innovator competing with a failing one, and, likewise, the payoff corresponding to firm j is indexed by “ fs ”. Hence, the first entry of the super-index refers to the innovation outcome of the player in question and the second entry to the innovation outcome of the rival player.

³Further, we assume that $\beta(0) = 0$ and $\lim_{x \rightarrow \infty} \beta(x) = 1$.

An important aspect of this formulation is that it allows for the conditional payoffs to depend on firms' investments. This is often the case in models of cost-reducing innovations as well as product-innovations. We next make some natural assumptions on these conditional payoffs.⁴

Assumption 1. Firms' conditional payoffs.

- i. $\frac{\partial \pi_i^{ss}(\cdot)}{\partial x_i} > 0$ and $\frac{\partial \pi_i^{sf}(\cdot)}{\partial x_i} > 0$. This assumption captures either the impact of cost-reducing innovations, quality-improvements or marketing-related efficiency improvements.
- ii. $\frac{\partial \pi_i^{fs}(\cdot)}{\partial x_i} = 0$ and $\frac{\partial \pi_i^{ff}(\cdot)}{\partial x_i} = 0$. This says that the investment of a firm does not affect its own payoff if its R&D effort proves unsuccessful.
- iii. $\frac{\partial \pi_i^{ss}(\cdot)}{\partial x_j} < 0$ and $\frac{\partial \pi_i^{fs}(\cdot)}{\partial x_j} < 0$. This assumption captures firm rivalry in the product market: conditional on the rival being a successful innovator, an increase in the rival's investment lowers firm i 's payoff no matter whether firm i was successful or not.
- iv. $\frac{\partial \pi_i^{sf}(\cdot)}{\partial x_j} = 0$ and $\frac{\partial \pi_i^{ff}(\cdot)}{\partial x_j} = 0$. This says that when a firm fails to get an innovation, its investment does not bear on the conditional payoff of the rival.
- v. Conditional payoffs rank as follows: $\pi_i^{ss}(x_i, x_j) \geq \pi_i^{fs}(x_j)$ and/or $\pi_i^{sf}(x_i) \geq \pi_i^{ff}(x_j)$ (with one of them being a strict inequality), where we have employed the above assumptions ii. and iv. to describe the dependency of payoffs on investments x_i and x_j .
- vi. $\pi_i^{sf}(x_i)$ and $\pi_i^{ss}(x_i, x_j)$ are strictly concave in x_i .

Assumption 1 imposes some restrictions on the conditional payoffs but these restrictions are rather mild. In particular, the ranking of payoffs in part (v) of Assumption 1 holds in standard models of R&D competition (see examples below).

2.2 Pre-merger market equilibrium

The innovation stage payoff of a firm i investing x_i in R&D is given by:

$$\begin{aligned} \mathbb{E}\pi_i(x_i; x_j) = & \beta_i(x_i) \left[\beta_j(x_j) \pi_i^{ss}(x_i, x_j) + (1 - \beta_j(x_j)) \pi_i^{sf}(x_i) \right] \\ & + (1 - \beta_i(x_i)) \left[\beta_j(x_j) \pi_i^{fs}(x_j) + (1 - \beta_j(x_j)) \pi_i^{ff}(x_j) \right] - C(x_i), \end{aligned}$$

The bracket in the first line of this expression is firm i 's payoff conditional on its innovation project being successful; this payoff depends on the rival's innovation outcome. The bracket in the second line gives firm i 's payoff conditional on failing to innovate.

⁴As an example, suppose we have price competition in the second-stage. Then, anticipating the second-stage Nash equilibrium $\mathbf{p}(\mathbf{x}) = (p_i(x_i; x_j), p_j(x_i; x_j))$, the first-stage payoff is $\pi_i(x_i; \mathbf{p}(\mathbf{x}))$.

The strict concavity of the success probabilities and the conditional payoffs ensure that a pure-strategy equilibrium of the investment subgame exists and is unique. Therefore, assuming the equilibrium is interior, it is given by the solution to the system of first order conditions (FOCs) for profits-maximization:

$$\begin{aligned}
& \underbrace{\frac{\partial \beta_i(\cdot)}{\partial x_i} \left[\beta_j(\cdot) \left[\pi_i^{ss}(x_i, x_j) - \pi_i^{fs}(x_j) \right] + (1 - \beta_j(\cdot)) \left[\pi_i^{sf}(x_i) - \pi_i^{ff} \right] \right]}_{\text{marginal gains from increasing success probability}} \\
& + \underbrace{\beta_i(\cdot) \left[\beta_j(\cdot) \frac{\partial \pi_i^{ss}(x_i, x_j)}{\partial x_i} + (1 - \beta_j(\cdot)) \frac{\partial \pi_i^{sf}(x_i)}{\partial x_i} \right]}_{\text{marginal gains from increasing conditional payoffs}} - C'(x_i) = 0, \quad \text{and similarly for firm } j.
\end{aligned} \tag{1}$$

This FOC says that a firm should continue to increase its R&D investment till the marginal revenue equals the marginal cost of investment. An increase in x_i has two effects on the expected payoff of a firm. On the one hand, it increases the probability of innovation. The first line of this FOC describes this effect, keeping constant conditional payoffs. On the other hand, it increases a firm's payoff conditional on innovation. The second line of this FOC describes this second effect, keeping constant the probability of innovation.⁵ For later use, let x^* denote the pre-merger symmetric equilibrium R&D effort.

To the best of our knowledge, the literature has not presented models in which these two effects of increasing innovation effort are in place. There is a group of papers, namely, Federico et al. (2017), Denicolò and Polo (2018), Jullien and Lefouili (2020) and Mukherjee (2022), focusing on stochastic R&D in which the first effect is examined but, owing to their assumptions on the constancy of the conditional payoffs, the second effect is shut down. Likewise, there is a second group of papers of deterministic R&D, namely Motta and Tarantino (2021), Bourreau and Jullien (2018), and Bourreau, Jullien, Lefouili (2024) where success probabilities are exogenously set to 1 and hence the first effect is shut down by construction; the focus then lies on how investment effort impacts conditional payoffs.⁶

2.3 Mergers

We now examine the impact of mergers on R&D investment. Consider now that firms i and j merge. A merger has two important implications. On the one hand, a merger results in the monopolisation of the product market, thereby, as it is by now well known, creating negative

⁵It is also useful to remark that an increase in the rival's R&D investment steals business from firm i on two accounts. First, it increases the rival probability of success, which lowers firm i 's payoff because $\pi_i^{ss}(\cdot) - \pi_i^{sf}(\cdot) < 0$ and $\pi_i^{fs}(\cdot) - \pi_i^{ff} < 0$. Second, it reduces firm i 's conditional payoffs. Hence, from the point of view of the firms as a collective, firms shall invest too much in R&D.

⁶Federico et al. (2018) has both effects. However, their simulations do not identify positive effects of mergers on innovation efforts.

price effects. On the other hand, a merger results in the monopolisation of the innovation market. As we shall see, these two effects are related to one another and a complete understanding of the impact of mergers on innovation ought to take both of them into account.

We capture the price effects of mergers by specifying reduced-form conditional payoffs that are higher than pre-merger and to avoid notation confusion we label the monopoly payoffs with the “hat” symbol. Specifically, the merged-entity’s conditional payoffs are given in the following table:

		<i>Firm 2</i>	
<i>Firm 1</i>		<i>Success(s)</i>	<i>Failure(f)</i>
	<i>Success(s)</i>	$\hat{\pi}_i^{ss}(x_i; x_j) + \hat{\pi}_j^{ss}(x_j; x_i)$	$\hat{\pi}_i^{sf}(x_i) + \hat{\pi}_j^{sf}(x_i)$
	<i>Failure(f)</i>	$\hat{\pi}_i^{fs}(x_j) + \hat{\pi}_j^{fs}(x_j)$	$\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff}$

Table 2: Merged entity’s conditional payoffs

In describing these payoffs in Table 2, we have already assumed assumptions similar to Assumption 1 but for the monopoly conditional payoffs, in particular $\frac{\partial \hat{\pi}_i^{fs}(\cdot)}{\partial x_i} = \frac{\partial \hat{\pi}_i^{ff}}{\partial x_i} = \frac{\partial \hat{\pi}_j^{sf}(\cdot)}{\partial x_i} = \frac{\partial \hat{\pi}_j^{ff}}{\partial x_j} = 0$. The fact that the merged entity coordinates prices implies that $\hat{\pi}_i^{ss}(x_i, x_j) \geq \pi_i^{ss}(x_i, x_j)$, $\hat{\pi}_i^{sf}(x_i) \geq \pi_i^{sf}(x_i)$, $\hat{\pi}_i^{fs}(x_j) \geq \pi_i^{fs}(x_j)$, $\hat{\pi}_i^{ff} \geq \pi_i^{ff}$ for firm i and/or j . Further, we make the following assumptions:

Assumption 2. Monopoly conditional payoffs.

- i. $\frac{\partial \hat{\pi}_i^{ss}(\cdot)}{\partial x_i} > 0$ and $\frac{\partial \hat{\pi}_i^{sf}(\cdot)}{\partial x_i} > 0$.
- ii. $\frac{\partial \hat{\pi}_j^{ss}(\cdot)}{\partial x_i} < 0$ and $\frac{\partial \hat{\pi}_j^{fs}(\cdot)}{\partial x_i} < 0$.
- iii. $\hat{\pi}_i^{ss}(x_i, x_j)$ is strictly concave in x_i and x_j , $\hat{\pi}_i^{sf}(x_i)$ is strictly concave in x_i , and $\hat{\pi}_i^{fs}(x_j)$ is strictly concave in x_j .

Consider now that firms i and j merge and assume that it is optimal for the merged entity to keep the two research labs of the constituent firms running. In such a case, the merged entity chooses investments x_i and x_j to maximize the (joint) payoff:

$$\begin{aligned}
\pi^m(x_i, x_j) = & \beta_i(x_i) \left[\beta_j(x_j) \hat{\pi}_i^{ss}(x_i, x_j) + (1 - \beta_j(x_j)) \hat{\pi}_i^{sf}(x_i) \right] \\
& + (1 - \beta_i(x_i)) \left[\beta_j(x_j) \hat{\pi}_i^{fs}(x_j) + (1 - \beta_j(x_j)) \hat{\pi}_i^{ff} \right] - C(x_i) \\
& + \beta_j(x_j) \left[\beta_i(x_i) \hat{\pi}_j^{ss}(x_i, x_j) + (1 - \beta_i(x_i)) \hat{\pi}_j^{sf}(x_j) \right] \\
& + (1 - \beta_j(x_j)) \left[\beta_i(x_i) \hat{\pi}_j^{fs}(x_i) + (1 - \beta_i(x_i)) \hat{\pi}_j^{ff} \right] - C(x_j)
\end{aligned}$$

The merger payoff is constructed as the sum of the payoffs of the merging parties. Our assumptions imply that the merged entity’s payoff is strictly concave in x_i and x_j . Hence,

assuming that an interior maximum exists, it is given by the solution of the system of FOCs. The FOC for the maximization of the profits of the merged entity with respect to x_i is given by:

$$\begin{aligned}
FOC_i^m(x_i, x_j) \equiv & \underbrace{\frac{\partial \beta_i(\cdot)}{\partial x_i} \left[\beta_j(\cdot) \left[\hat{\pi}_i^{ss}(x_i, x_j) - \hat{\pi}_i^{fs}(x_j) \right] + (1 - \beta_j(\cdot)) \left[\hat{\pi}_i^{sf}(x_i) - \hat{\pi}_i^{ff}(x_j) \right] \right]}_{\text{marginal gains from increasing success probability}} \\
& + \underbrace{\frac{\partial \beta_i(\cdot)}{\partial x_i} \left[\beta_j(\cdot) \left[\hat{\pi}_j^{ss}(x_i, x_j) - \hat{\pi}_j^{sf}(x_j) \right] + (1 - \beta_j(\cdot)) \left[\hat{\pi}_j^{fs}(x_i) - \hat{\pi}_j^{ff}(x_j) \right] \right]}_{\text{direct innovation externality}} \\
& + \underbrace{\beta_i(\cdot) \left[\beta_j(\cdot) \frac{\partial \hat{\pi}_i^{ss}(x_i, x_j)}{\partial x_i} + (1 - \beta_j(\cdot)) \frac{\partial \hat{\pi}_i^{sf}(x_i)}{\partial x_i} \right]}_{\text{marginal gains from increasing conditional payoffs}} \\
& + \underbrace{\beta_i(\cdot) \left[\beta_j(\cdot) \frac{\partial \hat{\pi}_j^{ss}(x_i, x_j)}{\partial x_i} + (1 - \beta_j(\cdot)) \frac{\partial \hat{\pi}_j^{fs}(x_i)}{\partial x_i} \right]}_{\text{indirect innovation externality}} - C'(x_i) = 0, \text{ and similarly for } x_j.
\end{aligned} \tag{2}$$

The FOC for profits maximization of the merged entity with respect to x_j is similar and, to save space, has been omitted.

A comparison between the FOCs pre-merger and post-merger is central to the understanding of the complexity of the impact of mergers on R&D investment. Moreover, it is key to understand why the different assumptions in the literature have led to distinct results. Comparing the post-merger FOC (2) to the pre-merger one in equation (1) leads to three important observations.

- First, because of price coordination in the market stage, the post-merger FOC involves monopoly payoffs rather than competitive payoffs. This means that when the merged entity picks its R&D effort, it factors different conditional payoffs compared to pre-merger. This issue in isolation has a bearing on the choice of R&D effort post-merger.
- Second, the post-merger FOC reflects the internalization of *two* externalities that operate on the incentives to invest in the same direction.
 - The first is a direct and *negative* externality and arises because when the merged entity increases its R&D effort x_i , it decreases the returns from investment of its partner firm (this is the second line of FOC (2)).
 - The second is an indirect and *negative* externality and arises because an increase in R&D effort x_i reduces the conditional payoffs of the partner firm j (this is the last line of FOC (2)).

The standard approach to address the question whether a merger leads to more or less investment compared to the pre-merger equilibrium consists of studying the sign of the FOC (2)

evaluated at the pre-merger symmetric equilibrium x^* . This gives:

$$\begin{aligned}
FOC_i^m(x^*) = & \frac{\partial \beta_i(x^*)}{\partial x_i} \left[\beta_j(x^*) \left[\hat{\pi}_i^{ss}(x^*, x^*) - \hat{\pi}_i^{fs}(x^*) \right] + (1 - \beta_j(x^*)) \left[\hat{\pi}_i^{sf}(x^*) - \hat{\pi}_i^{ff}(x^*) \right] \right] \\
& + \frac{\partial \beta_i(x^*)}{\partial x_i} \left[\beta_j(x^*) \left[\hat{\pi}_j^{ss}(x^*, x^*) - \hat{\pi}_j^{sf}(x^*) \right] + (1 - \beta_j(x^*)) \left[\hat{\pi}_j^{fs}(x^*) - \hat{\pi}_j^{ff}(x^*) \right] \right] \\
& + \beta_i(x^*) \left[\beta_j(x^*) \frac{\partial \hat{\pi}_i^{ss}(x^*, x^*)}{\partial x_i} + (1 - \beta_j(x^*)) \frac{\partial \hat{\pi}_i^{sf}(x^*)}{\partial x_i} \right] \\
& + \beta_i(x^*) \left[\beta_j(x^*) \frac{\partial \hat{\pi}_j^{ss}(x^*, x^*)}{\partial x_i} + (1 - \beta_j(x^*)) \frac{\partial \hat{\pi}_j^{fs}(x^*)}{\partial x_i} \right] - C'(x^*).
\end{aligned} \tag{3}$$

Since the FOC (1) holds (with equality) at the pre-merger market symmetric equilibrium x^* , equation (3) can be simplified to:

$$\begin{aligned}
FOC_i^m(x^*) = & \frac{\partial \beta_i(x^*)}{\partial x_i} \left[\beta_j(x^*) \left[\Delta_i^{ss}(x^*, x^*) - \Delta_i^{fs}(x^*) \right] + (1 - \beta_j(x^*)) \left[\Delta_i^{sf}(x^*) - \Delta_i^{ff}(x^*) \right] \right] \\
& + \frac{\partial \beta_i(x^*)}{\partial x_i} \left[\beta_j(x^*) \left[\hat{\pi}_j^{ss}(x^*, x^*) - \hat{\pi}_j^{sf}(x^*) \right] + (1 - \beta_j(x^*)) \left[\hat{\pi}_j^{fs}(x^*) - \hat{\pi}_j^{ff}(x^*) \right] \right] \\
& + \beta_i(x^*) \left[\beta_j(x^*) \frac{\partial \Delta_i^{ss}(x^*, x^*)}{\partial x_i} + (1 - \beta_j(x^*)) \frac{\partial \Delta_i^{sf}(x^*)}{\partial x_i} \right] \\
& + \beta_i(x^*) \left[\beta_j(x^*) \frac{\partial \hat{\pi}_j^{ss}(x^*, x^*)}{\partial x_i} + (1 - \beta_j(x^*)) \frac{\partial \hat{\pi}_j^{fs}(x^*)}{\partial x_i} \right].
\end{aligned} \tag{4}$$

where $\Delta_i^{ss}(\cdot) \equiv \hat{\pi}_i^{ss}(\cdot) - \pi_i^{ss}(\cdot)$ denotes the extra profits that accrue to the partner firm i in the subgame where both research labs are successful purely stemming from price coordination. Let $\Delta_i^{sf}(\cdot)$, $\Delta_i^{fs}(\cdot)$ and $\Delta_i^{ff}(\cdot)$ be defined analogously.

3 Results

Evaluating the sign of the FOC (4) is in principle quite difficult because there are many terms with opposite signs. However, it can be rewritten in the following more convenient way:

$$FOC_i^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i} [\beta_j K_1 + (1 - \beta_j) K_2] + \beta_i [\beta_j K_3(x^*) + (1 - \beta_j) K_4(x^*)] \tag{5}$$

where

$$\begin{aligned}
K_1 &\equiv \underbrace{\hat{\pi}_i^{ss} - \hat{\pi}_i^{fs} - (\hat{\pi}_j^{sf} - \hat{\pi}_j^{ss})}_{\text{division } i\text{'s post-merger ARE net of externality on division } j, \text{ conditional on partner success}} - \underbrace{(\pi_i^{ss} - \pi_i^{fs})}_{\text{division } i\text{'s pre-merger ARE, conditional on partner success}} \\
K_2 &\equiv \underbrace{\hat{\pi}_i^{sf} - \hat{\pi}_i^{ff} - (\hat{\pi}_j^{ff} - \hat{\pi}_j^{fs})}_{\text{division } i\text{'s post-merger ARE net of externality on division } j, \text{ conditional on partner failure}} - \underbrace{(\pi_i^{sf} - \pi_i^{ff})}_{\text{division } i\text{'s pre-merger ARE, conditional on partner failure}} \\
K_3(x^*) &\equiv \underbrace{\frac{\partial \hat{\pi}_i^{ss}(x^*, x^*)}{\partial x_i} + \frac{\partial \hat{\pi}_j^{ss}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s post-merger marginal ARE net of externality on division } j, \text{ conditional on partner success}} - \underbrace{\frac{\partial \pi_i^{ss}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s pre-merger marginal ARE, conditional on partner success}} \\
K_4(x^*) &\equiv \underbrace{\frac{\partial \hat{\pi}_i^{sf}(x^*, x^*)}{\partial x_i} + \frac{\partial \hat{\pi}_j^{fs}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s post-merger marginal ARE net of externality on division } j, \text{ conditional on partner failure}} - \underbrace{\frac{\partial \pi_i^{sf}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s pre-merger marginal ARE, conditional on partner failure}}.
\end{aligned}$$

Equation (5) reveals that the incentives of the merged entity to increase or decrease R&D investment relate to the sign of the expressions K_1 , K_2 , $K_3(x^*)$ and $K_4(x^*)$. These expressions all have a similar interpretation. Specifically, the term K_1 represents the difference between division i 's post-merger *Arrow replacement effect* net of the externality on division j , and division i 's pre-merger *Arrow replacement effect*, conditional on partner success. The term K_2 represents the same difference, but this time conditional on partner failure. The terms $K_3(x^*)$ and $K_4(x^*)$ have a similar interpretation but, because they involve marginal changes in the conditional payoffs, we refer to them as *marginal Arrow replacement effects*.

The FOC in (5) implies that a merger results in an increase in R&D if and only if:

$$FOC_i^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i} [\beta_j K_1 + (1 - \beta_j) K_2] + \beta_i [\beta_j K_3(x^*) + (1 - \beta_j) K_4(x^*)] > 0.$$

The first case of interest yields a clear-cut result. Consider a merger in which there aren't price effects, due to for example price regulation, like it occurs for some pharmaceutical products. Alternatively, it could be that while the R&D departments manage to coordinate investments after a merger, the sales departments do not. In such a case, $\hat{\pi}_i^{ss}(\cdot) = \pi_i^{ss}(\cdot)$, $\hat{\pi}_i^{sf}(\cdot) = \pi_i^{sf}(\cdot)$, $\hat{\pi}_i^{fs}(\cdot) = \pi_i^{fs}(\cdot)$ and $\hat{\pi}_i^{ff} = \pi_i^{ff}$, and hence K_1 , K_2 , $K_3(x^*)$, $K_4(x^*) < 0$. As a result, (5) would clearly be negative. Hence, in the absence of gains from price/quantity coordination in the market stage, the merger would alter its investment in order to internalize the direct and indirect innovation externalities (which are negative), thereby reducing investment.

More generally, when price effects are present, the sign of (5) is ambiguous and quite difficult to analyse because there are four terms that need to be evaluated and may potentially have different signs. To do this in a didactic way, we now proceed to discuss two classes of models that have received attention in the literature. The first class of models, which is studied in

Section 3.1, refers to models where R&D effort impacts the success probabilities, while keeping the innovation outcomes exogenous. The second class of models, which is studied in Section 3.2, refers to models where R&D effort impacts the innovation outcomes, while keeping the success probabilities exogenous. For both classes of models, we show conditions under which merges may spur innovation, or discourage it.

3.1 Endogenous probability of success

In this subsection we focus on a class of models where the success probability of a firm's R&D project is endogenous, while innovation outcomes do not depend on R&D efforts. Examples of existing models with similar features are Federico, Langus and Valletti (2017), Denicolò and Polo (2018) and Jullien and Lefouili (2020). However, the models in these three papers can be regarded as models of *R&D for entry* because entry only occurs upon project success. Our next result demonstrates that the predictions from such models are restrictive because they do not allow for the possibility that firms enter the market when their R&D projects fail.

Proposition 1. *In markets where R&D effort increases the probability of project success but does not affect the innovation outcomes:*

- (i) *If $K_1 \geq 0$ and $K_2 \geq 0$, then $x^m \geq x^*$ (with equality if $K_1 = K_2 = 0$).*
- (ii) *If $K_1 \leq 0$ and $K_2 \leq 0$, then $x^m \leq x^*$ (with equality if $K_1 = K_2 = 0$).*
- (iii) *If $K_1 < 0$ and $K_2 > 0$, then $x^m > x^*$ if and only if $x^* < \beta^{-1} \left(\frac{K_2}{K_2 - K_1} \right)$.*
- (iv) *If $K_1 > 0$ and $K_2 < 0$, then $x^m > x^*$ if and only if $x^* > \beta^{-1} \left(\frac{K_2}{K_2 - K_1} \right)$.*

This proposition makes two relevant points. First, a merger may result in an increase or decrease in R&D effort. Second, whether a merger spurs innovation or not may depend on the pre-merger level of R&D investment. We now elaborate on these two observations.

Part (i) of Proposition 1 describes a situation where, irrespective of rival's project success or failure, the pre-merger Arrow replacement effects are stronger than the post-merger ones net of the external effect on the partner. In such situations, a merger will result in an increase in R&D. Part (ii) refers to the opposite situation. Interestingly, parts (iii) and (iv) describe environments where the pre-merger level of innovation matters. Specifically, in part (iii), conditional on rival's project success, the pre-merger Arrow replacement effect is weaker than the post-merger one net of the external effect on the partner, but, conditional on rival's project failure, the opposite holds. In that case, we have two effects of conflicting sign, the first pushing the merged entity to undertake less R&D and the second pressing the firm to do the opposite. Hence, a merger spurs innovation provided that second effect dominates. This naturally occurs when the pre-merger level of innovation is sufficiently low, because in that case the rival's success probability is low

enough. Part (iv) refers to the alternative situation where, conditional on rival's project success, the pre-merger Arrow replacement effect is stronger than the post-merger one net of the external effect on the partner, but, conditional on rival's project failure, the opposite occurs. In that case, for a merger to spur innovation, the pre-merger level of R&D effort should be sufficiently large for otherwise the likelihood of rival's project success would be too low and the second effect would be the dominant one.

Proposition 1 advances the literature by pointing to a broader set of conditions under which mergers may spur innovation. The next question is whether these circumstances can occur in standard models of competition. We now provide a set of examples based on classical micro-founded models, which illustrate the different parts of Proposition 1. The complete derivations are provided in Section 4.1.

- Consider a Hotelling model of price competition with possibly vertically differentiated products *à la* Gilbert and Katz (2021). Initially firms sell products of low quality and can make investments to increase the likelihood of offering high quality. In this model, K_1 and K_2 are strictly positive and mergers spur innovation, thereby illustrating Proposition 1(i). The same results arise if R&D effort is meant to reduce the marginal costs of production.
- Consider a Cournot duopoly with Sutton's (2001) demand system. Initially, firms sell products of low quality and can make investments to increase the likelihood of offering high-quality products. In this model, if the difference between low quality and high quality is sufficiently large, then K_1 and K_2 are strictly negative and mergers discourage innovation. This illustrates Proposition 1(ii). However, if the difference between low quality and high quality is sufficiently small, we have K_1 strictly negative and K_2 strictly positive. In that case, when R&D costs are large, the pre-merger level of innovation is low and a merger spurs innovation; otherwise, a merger discourages it. This illustrates Proposition 1(iii).
- Consider a duopoly model of price competition with demands obtained as in Mussa and Rosen's (1978) model. Initially, firms sell products of low quality and can make investments to increase the likelihood of offering high-quality products. In this model, K_1 is strictly positive and K_2 strictly negative. In that case, when R&D costs are small, the pre-merger level of innovation is high and a merger spurs innovation; otherwise, a merger discourages it. This illustrates Proposition 1(iv).

Compared to the existing literature, Proposition 1, and the examples just provided, identifies a broader range of environments where mergers can spur innovation. To the best of our knowledge, there are no existing results where the pre-merger level of innovation is critical. In fact, in the two well-known results in the literature, namely, the negative effect of mergers in R&D in Federico, Langus and Valletti (2017)⁷ and the positive effect of mergers on R&D identified in

⁷Also observed in Denicolò and Polo's (2018) interior equilibrium.

Jullien and Lefouili (2020), the pre-merger level of innovation is irrelevant because of the restrictive assumption that firms cannot enter the market upon project failure. In that sense, their results are special cases of Proposition 1 where entry only occurs upon successful innovation.

Corollary (to Proposition 1). *Assume that firms can only operate in the market upon a successful innovation; otherwise, they exit. Then, we have $K_1 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \pi_i^{ss}$ and $K_2 = 0$. As a result:*

- *If $K_1 > 0$, then $x^m > x^*$. (For an example, see Jullien and Lefouili, 2020)*
- *If $K_1 < 0$, then $x^m < x^*$. (For an example, see Federico, Langus and Valletti, 2017; and Denicolò and Polo, 2020).*

Corollary to Proposition 1 captures the class of models where R&D is intended for entry and entry only occurs upon project success. In that case, the conditional payoffs in case of innovation failure are all equal to zero: $\hat{\pi}_i^{fs}(\cdot) = \pi_i^{fs}(\cdot) = 0$, and $\hat{\pi}_i^{ff}(\cdot) = \pi_i^{ff}(\cdot) = 0$.⁸ Moreover, $\hat{\pi}_i^{sf}(\cdot) = \pi_i^{sf}(\cdot)$ because a firm's freedom to set price/quantity is not constrained by a rival which fails to innovate. Because of this, $K_2 = 0$ and only the value of K_1 has a bearing on R&D incentives. In the setting of Jullien and Lefouili (2020) with horizontally differentiated products, K_1 can be positive, in which case a merger increases R&D.⁹ In Federico, Langus and Valletti (2017) and Denicolò and Polo (2018), K_1 is surely less than zero and a merger decreases R&D.¹⁰

3.2 Endogenous innovation outcomes

In this subsection we focus on a class of models where success probabilities are exogenous while innovation outcomes depend on R&D efforts. Examples of existing models with endogenous innovation outcomes are Motta and Tarantino (2021) and Bourreau, Jullien and Lefouili (2024). However, these models are restrictive because they assume that R&D projects are surely successful, that is, with probability one, and, hence, they ignore the possibility that R&D efforts fail to deliver. We analyze these settings in the following proposition and show that relaxing the assumption that R&D projects have deterministic outcomes allows for a richer set of circumstances under which mergers spur innovation.

Proposition 2. *In markets where innovation outcomes are endogenous while R&D success probabilities are exogenous but not necessarily equal to 1 ($\beta(x) = \mu \in (0, 1]$ for all x):*

- (i) *If $K_3(x^*) > 0$ and $K_4(x^*) > 0$, then $x^m > x^*$.*
- (ii) *If $K_3(x^*) < 0$ and $K_4(x^*) < 0$, then $x^m < x^*$.*

⁸And similarly for firm j .

⁹See Appendix B for a detailed analysis of Jullien and Lefouili's (2020) example.

¹⁰See Appendix B for a detailed analysis of Federico, Langus and Valletti (2017) and Denicolò and Polo (2018).

(iii) Let $\Phi(x) \equiv \frac{K_4(x)}{K_4(x) - K_3(x)}$. Assume it is decreasing in x and \tilde{x} is the solution to $\mu - \Phi(x) = 0$.¹¹ Then:

- If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $x^m > x^*$ if and only if $x^* < \tilde{x}$.
- If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $x^m > x^*$ if and only if $x^* > \tilde{x}$.

(iv) Assume otherwise that $\Phi(x)$ is increasing in x and \tilde{x} is the solution to $\mu - \Phi(x) = 0$. Then

- If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $x^m > x^*$ if and only if $x^* > \tilde{x}$.
- If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $x^m > x^*$ if and only if $x^* < \tilde{x}$.

This proposition shows that also in models where the amount of cost-reduction, or the amount of quality improvement, is endogenous while the success probability is exogenous, a merger may result in an increase or decrease in R&D effort. Moreover, this result identifies the importance of the pre-merger level of R&D investment and the difficulty of project success as key variables. We now provide some additional details on Proposition 2.

Part (i) of Proposition 2 refers to a situation where, irrespective of rival's project success or failure, the pre-merger marginal Arrow replacement effects are stronger than the post-merger ones net of the external marginal effect on the partner. In such situations, a merger will result in an increase in R&D. Part (ii) refers to the opposite situation. Parts (iii) and (iv) describe environments where the magnitude of the success probability and the pre-merger level of innovation matter. When, conditional on rival's project success, the pre-merger marginal Arrow replacement effect is weaker than the post-merger one net of the external marginal effect on the partner, but, conditional on rival's project failure, the opposite holds, then we have two effects of conflicting sign and the impact of mergers on R&D effort is in principle ambiguous. Parts (iii) and (iv) provide conditions under which the first effect is dominant and so the merger spurs innovation. When the pre-merger marginal Arrow replacement effect and the post-merger one net of the external marginal effect on the partner rank the other way around, it is also possible to have mergers than increase innovation effort.

Similarly to Proposition 1, Proposition 2 contributes to the literature by pointing to a broader set of conditions under which mergers may spur innovation, in this case for exogenous success probabilities but endogenous innovation outcomes. The following examples illustrate different parts of Proposition 2. Again, detailed derivations are postponed to Section 4.2.

¹¹ (v) Alternatively, when $\Phi(x)$ is decreasing and $\mu - \Phi(x) = 0$ does not have a solution due to $\mu < \Phi(x)$ for all x . Then:

If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $x^m > x^*$.
If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $x^m < x^*$.

(vi) Otherwise, when $\Phi(x)$ is increasing and $\mu - \Phi(x) = 0$ does not have a solution due to $\mu > \Phi(x)$. Then
If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $x^m < x^*$.
If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $x^m > x^*$.

- Consider a duopoly model of price competition with Singh and Vives (1984) system of demands. Initially, firms produce with marginal costs equal to c . Firms can invest in R&D to reduce these costs. A firm that invests an amount $C(x)$ can reduce its cost by an amount x if the research project is successful; otherwise, the firm continues to operate with cost c . The probability of project success is $\mu \in (0, 1]$. In this model, $K_3(x^*)$ is strictly negative, while $K_4(x^*)$ can be shown to be strictly negative provided that x^* is sufficiently small and positive otherwise. Moreover, the function $\Phi(x)$ can be shown to be increasing in x . As a result, when R&D costs are sufficiently large, the pre-merger level of innovation is sufficiently small and a merger definitely discourages innovation. This result illustrates Proposition 2(ii). Instead, if R&D costs are sufficiently small, a merger spurs innovation if the probability of success is low enough, and discourages it otherwise. This result illustrates Proposition 2(iii).

Proposition 2 thus identifies new scenarios where mergers can spur innovation depending on the pre-merger level of R&D, the magnitude of the success probability, and the relative strength of the pre- and post-merger replacement effects. We now note that some existing results in the literature are special cases of Proposition 2.

Corollary (to Proposition 2). *[Deterministic R&D] Assume that innovation outcomes are endogenous and that R&D projects are surely successful (i.e. $\mu = 1$). Then $K_4(x^*) = 0$ for all x^* and only $K_3(x^*)$ matters. We then have:*

- If $K_3(x^*) > 0$, then $x^m > x^*$.
- If $K_3(x^*) < 0$, then $x^m < x^*$. (For example, Motta and Tarantino, 2021; section 3.1).

This corollary captures the class of models where R&D is deterministic, while the innovation outcome, be it for example a cost reduction or a quality increase, depends on the magnitude of R&D investment. An example of such a model with deterministic R&D process can be found in Motta and Tarantino (2021, section 3.1). Corollary to Proposition 2 shows that whether a merger results in more or less investment by the merging parties is ambiguous. The condition in the proposition says that a merger will result in more investment if the profit boost stemming from price coordination of one of the divisions increases more rapidly in investment compared to the (indirect) innovation externality.

We finish this section by pointing out that in the most general version of our model in Section 2, all K_1 , K_2 , $K_3(x^*)$ and $K_4(x^*)$ will matter. This means that whether mergers spur innovation or discourages it is a highly complex question whose answer will certainly be model dependent.

4 Micro-founded examples

4.1 Micro-founded examples illustrating Proposition 1

In this section we use several micro-founded examples to illustrate the results of the general reduced form model described above in the setting where both firms have research capabilities, success probability is endogenous, but conditional payoffs are independent of R&D effort. We provide three examples that illustrate all sub-cases identified in Proposition 1.

In this section to optimize notation we set $\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} = \hat{\pi}_m^{ss}$, $\hat{\pi}_i^{fs} + \hat{\pi}_j^{sf} = \hat{\pi}_m^{fs}$, $\hat{\pi}_i^{sf} + \hat{\pi}_j^{fs} = \hat{\pi}_m^{sf}$, $\hat{\pi}_i^{ff} + \hat{\pi}_j^{ff} = \hat{\pi}_m^{ff}$.

4.1.1 Hotelling with vertically differentiated products

This example illustrates the result in Proposition 1(i). To characterize the profits in the second stage we use a Hotelling model similar to that Gilbert and Katz (2022) and Houba et al. (2023). Two firms locate at two extremes of a linear city. If a firm successfully innovates, it provides s_s (high quality products), otherwise, it provides low quality products, s_f , and stays in the market. Utility: $u = s_i - tx - p_i$ (buying from i); $u = s_j - t(1-x) - p_j$ (buying from j). We assume that $s_s - s_f < t$. This implies that both firms have positive market shares.¹² Demands are given by

$$D_i = \frac{s_i - s_j - (p_i - p_j)}{2t} + \frac{1}{2} \text{ and } D_j = \frac{1}{2} - \frac{s_i - s_j - (p_i - p_j)}{2t}$$

We normalize the marginal cost of production to zero.¹³ Under these assumptions the corresponding profits are given by¹⁴

$$\begin{aligned} \pi_i^{ss} &= \frac{t}{2}, \quad \pi_i^{sf} = \frac{(3t + (s_s - s_f))^2}{18t}, \quad \pi_i^{fs} = \frac{(3t - (s_s - s_f))^2}{18t}, \quad \pi_i^{ff} = \frac{t}{2} \\ \hat{\pi}_m^{ss} &= s_s - \frac{t}{2}, \quad \hat{\pi}_m^{sf} = \hat{\pi}_m^{fs} = \frac{s_s + s_f}{2} - \frac{4t^2 - (s_s - s_f)^2}{8t}, \quad \hat{\pi}_m^{ff} = s_f - \frac{t}{2}. \end{aligned} \quad (6)$$

Analysis of profits in (6) implies that both $K_1 = \frac{(s_s - s_f)(12t - 5(s_s - s_f))}{72t} > 0$ and $K_2 = \frac{(s_s - s_f)(12t + 5(s_s - s_f))}{72t} > 0$ for all $0 < s_s - s_f < t$. This model illustrates part (i) of

¹²Further, we assume that $s_s + s_f > 3t$, which ensures that the duopolists and the merged firm will cover the market. Also it is assumed that $\min\{s_s, s_f\} > 3t/2$.

¹³This is without loss of generality, due to isomorphic nature of cost advantage and quality advantage in the Hotelling model.

¹⁴The derivations for pre-merger profit level are straightforward and can be found in Table 1 of Houba et al. (2023). In the symmetric post-merger scenario the market is considered fully covered if the most distant consumer receives non-negative utility. Since the merger maintains the same quality level across its divisions, in case the divisions are symmetric each division serves half of the market. Consequently, the farthest consumer incurs a transportation cost of $\frac{t}{2}$. Hence, the monopolist sets the prices $p_i = p_j = s_s - \frac{t}{2}$. Each division earns $\hat{\pi}_i^{ss} = \hat{\pi}_j^{ss} = \frac{s_s}{2} - \frac{t}{4}$, and $\hat{\pi}_m^{ss} = s_s - \frac{t}{2}$. Similarly, each division earns $\hat{\pi}_i^{ff} = \hat{\pi}_j^{ff} = \frac{s_f}{2} - \frac{t}{4}$, and $\hat{\pi}_m^{ff} = s_f - \frac{t}{2}$. Finally, asymmetric post-merger profits $\hat{\pi}_m^{sf} = \hat{\pi}_m^{fs}$ are computed following Appendix in Gilbert and Katz (2021).

Proposition 1. In this model merger will always result in an increase in R&D. As mentioned above, Jullien and Lefouili (2020) observed similar result in the Hotelling model in the absence of vertical differentiation, under quadratic transportation cost and no entry in case of failure.

4.1.2 Sutton's (2001) model of horizontally and vertically differentiated products

This example illustrates the results in Proposition 1 (ii) and (iii). To characterize the profits in the second stage, we use the demand structure which stems from the model with the quality-augmented quadratic utility function and Cournot competition.¹⁵ For the sake of exposition, we assume away horizontal product differentiation by setting $\sigma = 2$ and set marginal cost of production to zero.¹⁶ The basic product has low quality $s_f > 0$. If the firm's investment turns out successful, we assume that the firm is able to offer a product of higher quality s_s , with $s_f < s_s < 2s_f$.¹⁷ Otherwise, the firm continues offering basic low quality product. We normalize the marginal cost of production to zero. Under these assumptions utility maximization yields the system of demands for the (possibly) vertically differentiated products of the two players i and j :

$$p_i = \alpha - \frac{2\beta^2 q_i}{s_i^2} - \frac{2\beta^2}{s_i} \sum_{i \neq j} \frac{\beta q_j}{s_j}.$$

The corresponding profits are given by¹⁸

$$\begin{aligned} \pi_i^{ss} &= \frac{\alpha^2 s_s^2}{18\beta^2}, \quad \pi_i^{sf} = \frac{\alpha^2 (2s_s - s_f)^2}{18\beta^2}, \quad \pi_i^{fs} = \frac{\alpha^2 (2s_f - s_s)^2}{18\beta^2}, \quad \pi_i^{ff} = \frac{\alpha^2 s_f^2}{18\beta^2} \\ \hat{\pi}_m^{ss} &= \hat{\pi}_m^{sf} = \hat{\pi}_m^{fs} = \frac{\alpha^2 s_s^2}{8\beta^2}, \quad \hat{\pi}_m^{ff} = \frac{\alpha^2 s_f^2}{8\beta^2}. \end{aligned} \quad (8)$$

Analysis of the profits in (8) implies that the following two scenarios are possible. Either we have $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} > \pi_i^{sf} - \pi_i^{ff}$ for $1 < \frac{s_s}{s_f} < \frac{9}{7}$, which corresponds $K_2 > 0$ and $K_1 < 0$. Alternatively, we can have $\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} < \pi_i^{sf} - \pi_i^{ff}$ for $\frac{9}{7} < \frac{s_s}{s_f} < 2$, which corresponds to $K_2 < 0$ and $K_1 < 0$. Note that in this example in the absence of horizontal differentiation we have $\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs} = 0 < \pi_i^{ss} - \pi_i^{fs}$,

¹⁵See Sutton (1997, 2001):

$$U = \sum_{i=1}^2 \left[\alpha q_i - \left(\frac{\beta q_i}{s_i} \right)^2 \right] - \sigma \sum_{i=1}^2 \sum_{i < j} \frac{\beta q_i}{s_i} \frac{\beta q_j}{s_j} - \sum_{i=1}^2 p_i q_i$$

¹⁶In a more general setting with both vertical and horizontal product differentiation we would have more complicated expressions for corresponding profits (see (7)). However, this will not effect the main insights presented in this example.

$$\begin{aligned} \pi_i^{ss} &= \frac{2\alpha^2 s_s^2}{(4 + \sigma)^2 \beta^2}, \quad \pi_i^{sf} = \frac{2\alpha^2 (4s_s - \sigma s_f)^2}{(16 - \sigma^2)^2 \beta^2}, \quad \pi_i^{fs} = \frac{2\alpha^2 (4s_f - \sigma s_s)^2}{(16 - \sigma^2)^2 \beta^2}, \quad \pi_i^{ff} = \frac{2\alpha^2 s_f^2}{(4 + \sigma)^2 \beta^2} \\ \hat{\pi}_m^{ss} &= \frac{\alpha^2 s_s^2}{2(2 + \sigma)\beta^2}, \quad \hat{\pi}_m^{sf} = \hat{\pi}_m^{fs} = \frac{\alpha^2 (s_s^2 + s_f^2 - \sigma s_s s_f)}{2(4 - \sigma^2)\beta^2}, \quad \hat{\pi}_m^{ff} = \frac{\alpha^2 s_f^2}{2(2 + \sigma)\beta^2}. \end{aligned} \quad (7)$$

¹⁷The restriction $s_s < 2s_f$ rules out drastic innovations.

¹⁸See Dijk et al. (2024) for detailed derivations.

implying that $K_1 < 0$ for all parameters. Hence, in the second scenario we always have $x^m < x^*$. While in the first scenario the outcome depends on the level of x^* (which in turn may depend on the shape of investment cost function). For some parameter values we can have $x^m > x^*$. This happens for $x^* < \hat{x}$. Where \hat{x} is the solution to $\beta(x) = \frac{K_2}{K_2 - K_1}$ and under linear probability is given

by $\hat{x} = \frac{\left[\frac{\alpha^2 s_s^2}{8\beta^2} - \frac{\alpha^2 s_f^2}{8\beta^2} - \left(\frac{\alpha^2 (2s_s - s_f)^2}{18\beta^2} - \frac{\alpha^2 s_f^2}{18\beta^2} \right) \right]}{\left[\frac{\alpha^2 s_s^2}{8\beta^2} - \frac{\alpha^2 s_f^2}{8\beta^2} - \left(\frac{\alpha^2 (2s_s - s_f)^2}{18\beta^2} - \frac{\alpha^2 s_f^2}{18\beta^2} \right) \right] - \left[0 - \left(\frac{\alpha^2 s_s^2}{18\beta^2} - \frac{\alpha^2 (2s_f - s_s)^2}{18\beta^2} \right) \right]} = \frac{7s_s - 9s_f}{7s_s - 25s_f}$. These two scenarios illustrate the differences between Proposition 1(ii) and Proposition 1(iii).

Further, assuming linear probability of success and quadratic investment costs $c(x_i) = \frac{\gamma x_i^2}{2}$, where γ is the parameter which reflects the steepness of the cost function, we can compute closed form solutions for both pre-merger and post-merger investment levels. They are given by

$$x^* = \frac{(\pi_i^{sf} - \pi_i^{ff})}{\gamma + (\pi_i^{sf} - \pi_i^{ff}) - (\pi_i^{ss} - \pi_i^{fs})}$$

$$x^m = \frac{(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff})}{\gamma + (\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}) - (\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs})}$$

Comparing these two expressions gives **lower** bound on parameter γ , for which merger can result in higher R&D investments:

$$\bar{\gamma} = \frac{(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}) (\pi_i^{ss} - \pi_i^{fs}) - (\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs}) (\pi_i^{sf} - \pi_i^{ff})}{(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}) - (\pi_i^{sf} - \pi_i^{ff})} = \frac{2\alpha^2 s_f (s_f^2 - s_s^2)}{\beta^2 (7s_s - 9s_f)}.$$

Hence, for all $\gamma < \bar{\gamma}$ merger will result in a reduction in R&D investments compared to pre-merger equilibrium.

This example illustrates the scenarios identified in Proposition 1, parts (ii) and (iii).¹⁹ To the best of our knowledge, a situation of part (iii) has not been identified in previous contributions. It shows the importance of the shape of the R&D cost function in combination with conditions on the second stage profits. We show that the new result, where mergers can enhance innovation incentives, can be obtained also for sharply increasing innovation cost in environments where a failure to develop a new higher quality product does not result in a zero payoff and price effects are present.

4.1.3 Mussa and Rosen (1978) model of vertical product differentiation.

This example illustrates the results in Proposition 1(iv). To characterize the profits in the second stage we adopt the Mussa and Rosen's (1978) model of vertical product differentiation, further

¹⁹As was mentioned above scenarios similar to (ii) have been discussed in FLV(2018) and DP(2020).

studied by Motta (1993). Two firms offer vertically differentiated products. They start with low-quality products (s_f). If a firm successfully innovates, it sells high-quality products ($s_s > s_f$). Utility is given by $u_i = \theta s_i - p_i$. θ is the quality taste, which follows uniform distribution $[0, 1]$. This market is always uncovered. We assume zero marginal cost.²⁰ Under these assumptions the system of demands for asymmetric configurations is given by

$$D_s = 1 - \frac{p_s - p_f}{s_s - s_f} \text{ and } D_f = \frac{p_s - p_f}{s_s - s_f} - \frac{p_f}{s_f}$$

The corresponding profits are given by²¹

$$\begin{aligned} \pi_i^{ss} &= 0, \quad \pi_i^{sf} = \frac{4s_s^2(s_s - s_f)}{(4s_s - s_f)^2}, \quad \pi_i^{fs} = \frac{s_s s_f(s_s - s_f)}{(4s_s - s_f)^2}, \quad \pi_i^{ff} = 0 \\ \hat{\pi}_m^{ss} &= \hat{\pi}_m^{sf} = \hat{\pi}_m^{fs} = \frac{s_s}{4}, \quad \hat{\pi}_m^{ff} = \frac{s_f}{4}. \end{aligned} \quad (9)$$

Analysis of the profits in (9) implies that $K_1 = \hat{\pi}_m^{ss} - \hat{\pi}_m^{fs} - (\pi_i^{ss} - \pi_i^{fs}) = \frac{s_s s_f(s_s - s_f)}{(4s_s - s_f)^2} > 0$ and $K_2 = \hat{\pi}_m^{sf} - \hat{\pi}_m^{ff} - (\pi_i^{sf} - \pi_i^{ff}) = \frac{s_f(s_s - s_f)(s_f - 8s_s)}{4(4s_s - s_f)^2} < 0$ for all $0 < s_s - s_f$. This example illustrates the scenario identified in part (iv) of Proposition 1. To the best of our knowledge, this case has not been identified in previous literature on the effects of mergers. In this scenario the outcome depends on the level of x^* (which in turn may depend on the shape of investment cost function). For some parameter values it can be possible that $x^m > x^*$. This happens for $x^* > \hat{x}$.

Where \hat{x} is the solution to $\beta(x) = \frac{K_2}{K_2 - K_1} = \frac{\frac{s_f(s_s - s_f)(s_f - 8s_s)}{4(4s_s - s_f)^2}}{\frac{s_s s_f(s_s - s_f)}{(4s_s - s_f)^2} - \frac{s_s s_f(s_s - s_f)}{(4s_s - s_f)^2}} = \frac{8s_s - s_f}{12s_s - s_f}$ and under linear probability is given by $\hat{x} = \frac{8s_s - s_f}{12s_s - s_f}$.

Further, assuming linear probability of success and quadratic investment costs $c(x_i) = \frac{\gamma x_i^2}{2}$, we can compute closed form solutions for both pre-merger and post-merger investment levels. They are given by

$$\begin{aligned} x^* &= \frac{(\pi_i^{sf} - \pi_i^{ff})}{\gamma + (\pi_i^{sf} - \pi_i^{ff}) - (\pi_i^{ss} - \pi_i^{fs})} \\ x^m &= \frac{(\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff})}{\gamma + (\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}) - (\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs})} \end{aligned}$$

²⁰With positive marginal cost results are qualitatively similar.

²¹Because firms compete in prices, they each earn zero profit in case both have the same quality level. Hence, $\pi_i^{ss} = \pi_j^{ss} = \pi_i^{ff} = \pi_j^{ff} = 0$. Asymmetric pre-merger profits, π_i^{sf} and π_i^{fs} , are derived following Motta (1993). Symmetric post-merger profits are straightforward and given by $\hat{\pi}_m^{ss} = \frac{s_s}{4}$, $\hat{\pi}_i^{ss} = \hat{\pi}_j^{ss} = \frac{s_s}{8}$ and $\hat{\pi}_m^{ff} = \frac{s_f}{4}$, $\hat{\pi}_i^{ff} = \hat{\pi}_j^{ff} = \frac{s_f}{8}$. In asymmetric post-merger case, monopolist always sells only high quality if both qualities are available. Hence, $\hat{\pi}_m^{sf} = \hat{\pi}_m^{fs} = \frac{s_s}{4}$, $\hat{\pi}_i^{sf} = \frac{s_s}{4}$ and $\hat{\pi}_j^{fs} = 0$.

Comparing these two expressions gives **upper** bound on parameter γ , for which merger can result in higher R&D investments:

$$\bar{\gamma} = \frac{(\hat{\pi}_m^{ss} - \hat{\pi}_m^{fs}) (\pi_i^{sf} - \pi_i^{ff}) - (\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff}) (\pi_i^{ss} - \pi_i^{fs})}{(\pi_i^{sf} - \pi_i^{ff}) - (\hat{\pi}_m^{sf} - \hat{\pi}_m^{ff})} = \frac{(s_s^2 - s_f s_s)}{8s_s - s_f}$$

For all $\gamma > \bar{\gamma}$ merger will result in a reduction in R&D investments compared to pre-merger equilibrium.

4.2 Micro-founded examples illustrating Proposition 2

In this section we provide micro-founded examples to illustrate the results of the general reduced form model described above in the setting where both firms have research capabilities, conditional payoffs dependent on R&D effort and success probabilities are exogenous.

4.2.1 Singh and Vives's (1984) model of price competition, horizontally differentiated products and cost-reducing innovation

We consider the Singh and Vives's (1984) system of demands, investments that decrease the marginal cost of the products and price competition.

The demands are given by

$$\begin{aligned} q_1 &= a - bp_1 + dp_2 \\ q_2 &= a - bp_2 + dp_1; \end{aligned}$$

and we need the usual assumption that $b > d$.

In the pre-merger market, the second-stage equilibrium is:

$$\begin{aligned} p_1 &= \frac{2b(a + bc_1) + d(a + bc_2)}{4b^2 - d^2} \\ p_2 &= \frac{d(a + bc_1) + 2b(a + bc_2)}{4b^2 - d^2} \end{aligned}$$

The profits of firm 1 (and similarly of firm 2) are:

$$\pi_1 = \frac{b(2b(a - bc_1) + d(a + bc_2) + c_1 d^2)^2}{(d^2 - 4b^2)^2}$$

We use this expression to build our payoff for stage 1, where the firms decide on R&D efforts to reduce costs. For this we assume that initially firms have a basic cost, denoted c , with $a > (b - d)c$. If a firm invests an amount $C(x_i) = \gamma \frac{x_i^2}{2}$, then it gets a cost decrease of x_i .

Hence the following conditional payoffs:

$$\pi_i^{ss} = \frac{b(2b(a - b(c - x_1)) + d(a + b(c - x_2)) + (c - x_1)d^2)^2}{(d^2 - 4b^2)^2}$$

$$\pi_i^{sf} = \frac{b(2b(a - b(c - x_1)) + d(a + bc) + (c - x_1)d^2)^2}{(d^2 - 4b^2)^2}$$

$$\pi_i^{fs} = \frac{b(2b(a - bc) + d(a + b(c - x_2)) + (cd^2)^2)}{(d^2 - 4b^2)^2}$$

$$\pi_i^{ff} = \frac{b(2b(a - bc) + d(a + bc) + (cd^2)^2)}{(d^2 - 4b^2)^2}$$

In the pre-merger the problem of a firm i is to maximize:

$$E\pi_i = (1 - \mu)^2 \pi_i^{ff} + (1 - \mu)\mu \pi_i^{fs} + \mu(1 - \mu)\pi_i^{sf} + \mu^2 \pi_i^{ss} - \alpha \frac{x_i^2}{2}$$

Taking the FOC wrt x_i and applying symmetry we get the equilibrium investment in the pre-merger market:

$$x^* = \frac{2b\mu(2b + d)(2b^2 - d^2)(a - (b - d)c)}{2b^2 d \mu^2 (2b^2 - d^2) + \gamma (d^2 - 4b^2)^2 - 2b\mu(2b^2 - d^2)^2}.$$

In the post-merger market, the second-stage optimal prices are:

$$p_1 = \frac{1}{2} \left(\frac{a}{b - d} + c_1 \right),$$

$$p_2 = \frac{1}{2} \left(\frac{a}{b - d} + c_2 \right)$$

Following the slides, we shall need the division i 's profits, which are:

$$\pi_i^m = \frac{(a + c_1(d - b))(a - bc_1 + c_2 d)}{4(b - d)}$$

Hence, conditional on the success or failure of the R&D projects we have the following payoffs:

$$\hat{\pi}_i^{ss} = \frac{(a + (c - x_1)(d - b))(a - b(c - x_1) + (c - x_2)d)}{4(b - d)},$$

$$\hat{\pi}_i^{sf} = \frac{(a + (c - x_1)(d - b))(a - b(c - x_1) + cd)}{4(b - d)},$$

$$\hat{\pi}_i^{fs} = \frac{(a + c(d - b))(a - bc + (c - x_2)d)}{4(b - d)},$$

$$\hat{\pi}_i^{ff} = \frac{(a + c(d - b))(a - bc + cd)}{4(b - d)},$$

In the post-merger market the joint entity maximizes joint profits. The actual investment post-merger is then given by

$$x^m = \frac{\mu(a - c(b - d))}{2\gamma - \mu(b - d\mu)}.$$

We now compute the relevant expressions K_3 and K_4 .

$$\begin{aligned}
K_3 &\equiv \underbrace{\frac{\partial \hat{\pi}_i^{ss}(x^*, x^*)}{\partial x_i} + \frac{\partial \hat{\pi}_j^{ss}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s post-merger marginal ARE net of externality on division } j, \text{ conditional on partner success}} - \underbrace{\frac{\partial \pi_i^{ss}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s pre-merger marginal ARE, conditional on partner success}} \\
K_4 &\equiv \underbrace{\frac{\partial \hat{\pi}_i^{sf}(x^*, x^*)}{\partial x_i} + \frac{\partial \hat{\pi}_j^{sf}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s post-merger marginal ARE net of externality on division } j, \text{ conditional on partner failure}} - \underbrace{\frac{\partial \pi_i^{sf}(x^*, x^*)}{\partial x_i}}_{\text{division } i\text{'s pre-merger marginal ARE, conditional on partner failure}}.
\end{aligned}$$

$$\begin{aligned}
K_3 &= -\frac{1}{4} \left(a \left(\frac{b}{d-b} - 1 \right) + 2b(c-x^*) - (c-x^*)d \right) - \frac{d(a + (c-x^*)(d-b))}{4(b-d)} + \\
&\quad \frac{2b(d^2 - 2b^2)(2b(a-b(c-x^*)) + d(a+b(c-x^*)) + (c-x^*)d^2)}{(d^2 - 4b^2)^2} \\
&= \frac{d(-4b^2 + 2bd + d^2)(a - (b-d)(c-x^*))}{2(2b-d)^2(2b+d)} < 0 \text{ because } b > d \text{ and } a - (b-d)(c-x^*) > 0.
\end{aligned}$$

So K_3 is always negative. This demonstrates the result in Motta-Tarantino: a merger always reduces investment.

$$\begin{aligned}
K_4 &= -\frac{1}{4} \left(a \left(\frac{b}{d-b} - 1 \right) + 2b(c-x^*) - cd \right) - \frac{d(a + c(d-b))}{4(b-d)} \\
&\quad + \frac{2b(d^2 - 2b^2)(d(a+bc) + 2b(a-b(c-x^*)) + d^2(c-x^*))}{(d^2 - 4b^2)^2} \\
&= \frac{d[(-8b^3 + 4bd^2 + d^3)(a - c(b-d)) + bdx^*(8b^2 - 3d^2)]}{2(d^2 - 4b^2)^2}
\end{aligned}$$

whose sign is in principle ambiguous. Note however that if investment pre-merger is very low ($x^* \rightarrow 0$) the sign of K_4 will be negative (the same as K_3) and in that case we know that investment post-merger will be definitely less than pre-merger. So when the cost of investment is sufficiently high, the merger will result in less investment. Note also that $K_3(x)$ is decreasing in x , while $K_4(x)$ is increasing in x . Further, in this micro-founded example $\frac{K_4(x)}{K_4(x) - K_3(x)}$ is increasing in x .

So, we conclude that there are two relevant cases here: $K_3 < 0$ and $K_4 < 0$ and $K_3 < 0$ and $K_4 > 0$. First case implies unambiguous reduction in R&D efforts post-merger. This illustrates part (ii) of Proposition 2. Second case illustrates part (iv) of Proposition 2. In this scenario the outcome depends on the level of x^* (which in turn may depend on the shape of investment cost function). For some parameter values it can be possible that $x^m > x^*$. This happens for $x^* > \tilde{x}$. Where \tilde{x} is the solution to

$$\mu = \frac{K_4(x)}{K_4(x) - K_3(x)} = \frac{(ad^3 - 8ab^3 + 8b^4c + cd^4 + 4abd^2 + 3bcd^3 - 8b^3cd - 3bd^3x + 8b^3dx - 4b^2cd^2)}{(8b^4 + d^4 - 4b^2d^2)x},$$

and is given by

$$\tilde{x} = \frac{(-8ab^3 + ad^3 + 8b^4c + cd^4 + 4abd^2 + 3bcd^3 - 8b^3cd - 4b^2cd^2)}{3bd^3 - 8b^3d + (8b^4 + d^4 - 4b^2d^2)\mu}$$

To identify conditions on parameters of the R&D cost function we compare expressions for pre-merger and post-merger investment levels. Recall that they are given by

$$x^* = \frac{2b\mu(2b+d)(2b^2-d^2)(a-(b-d)c)}{2b^2d\mu^2(2b^2-d^2) + \gamma(d^2-4b^2)^2 - 2b\mu(2b^2-d^2)^2}$$

$$x^m = \frac{\mu(a-c(b-d))}{2\gamma - \mu(b-d\mu)}$$

Comparing these two expressions gives upper bound on parameter γ , for which merger can result in higher R&D investments:²²

$$\gamma < \bar{\gamma} = \frac{(-4b^4\mu + 2bd^3\mu - 4b^3d\mu + 4b^4\mu^2 + 2b^2d^2\mu - 2bd^3\mu^2 + 4b^3d\mu^2 - 2b^2d^2\mu^2)}{4bd^2 - 8b^3 + d^3}.$$

Hence, for all $\gamma > \bar{\gamma}$ merger will result in a reduction in R&D investments compared to pre-merger equilibrium.²³ In the numerical example with $a = 2, b = 0.76, d = 0.7, c = 1, \mu = 0.2$ this threshold is $\bar{\gamma} = 0.140657913969$.

5 Conclusions

We formulate a general two-stage game to study the implications of mergers on R&D. We identify three channels through which a merger affects R&D investment: anticipation of price coordination that enhances payoffs, internalisation of a direct innovation externality stemming from an enhanced chance of innovation success, and internalisation of an indirect innovation externality arising from business-stealing in the product market. Under price regulation, only the latter two effects play a role and mergers result in an unambiguous decrease in R&D. Our model captures the results of existing models with stochastic innovation intended to enter a new market and deterministic innovation intended to reduce costs or improve quality. We argue that the literature so far has restricted the analysis to settings where the payoffs from innovation failure are equal to zero (and hence the Arrow replacement effect is absent) and illustrate the importance of the pre-merger level of innovation, and hence the magnitude of investment costs.

²²One can also show that γ that solves $x^* = \tilde{x}$ is given by the following expression $\frac{1}{(d^2-4b^2)^2} \left(2b\mu(2b^2-d^2)^2 - 2b^2d\mu^2(2b^2-d^2) + \frac{2b\mu(2b+d)(a-c(b-d))(2b^2-d^2)(3bd^3-8b^3d+8b^4\mu+d^4\mu-4b^2d^2\mu)}{-8ab^3+ad^3+8b^4c+cd^4+4abd^2+3bcd^3-8b^3cd-4b^2cd^2} \right)$. After simplifying it can be shown that it is identical to $\bar{\gamma}$.

²³There are cases where merger leads to more investment. For example, when the parameters are $a = 2, b = 0.76, d = 0.7, c = 1, \mu = 0.2$ and $\gamma = 0.14$ (low investment costs) post-merger investment is greater than pre-merger. However, when $\gamma = 4$ (high investment costs) we get the opposite (less investment post-merger). Numerically we can also identify the threshold on γ , where investment incentives flip for given parameters. For $a = 2, b = 0.76, d = 0.7, c = 1, \mu = 0.2$ this threshold is $\gamma < \bar{\gamma} = 0.140657913969$. See desmos link: <https://www.desmos.com/calculator/ychi4xybbi>

6 Appendix A: Proofs

6.1 Proof of Proposition 1:

Proof. In this setting, where conditional payoffs are independent of R&D effort and probability of success is endogenous and depends on x , expression in (5) becomes:

$$FOC_i^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i} [\beta_j K_1 + (1 - \beta_j) K_2].$$

Then since $\beta' > 0$, the inequality $\frac{\partial \beta_i(x^*)}{\partial x_i} [\beta_j K_1 + (1 - \beta_j) K_2] > 0$ is equivalent to

$$\beta(x^*) K_1 + (1 - \beta(x^*)) K_2 > 0 \quad \text{or} \quad \beta(x^*) (K_1 - K_2) + K_2 > 0.$$

Further, denote

$$\hat{x} = \beta^{-1} \left(\frac{K_2}{K_2 - K_1} \right).$$

Suppose $K_1 > 0$ and $K_2 > 0$, then $FOC_i^m(.)$ evaluated at x^* is positive. The same conclusion holds for $K_1 \geq 0$, $K_2 > 0$ and $K_1 > 0$ and $K_2 \geq 0$. This proves result in (i).

Suppose $K_1 < 0$ and $K_2 < 0$, then $FOC_i^m(.)$ evaluated at x^* is negative. The same conclusion holds for $K_1 \leq 0$, $K_2 < 0$ and $K_1 < 0$ and $K_2 \leq 0$. This proves result in (ii).

Suppose $K_1 < 0$ and $K_2 > 0$, then $FOC_i^m(x^*)$ is positive for all $x^* < \hat{x}$. This proves result in (iii).

Suppose $K_1 > 0$ and $K_2 < 0$, then $FOC_i^m(x^*)$ is positive for all $x^* > \hat{x}$. This proves result in (iv).

Note that $x^m = x^*$ can only hold when $K_1 = 0$ and $K_2 = 0$ or when $x^* = \hat{x}$. This is reflected in parts (i) and (ii). □

6.2 Proof of Corollary to Proposition 1

Proof. In the setting, where conditional payoffs are independent of R&D effort, probability of success depends on x and firms can only operate in the market upon a successful innovation FOC (5) becomes

$$FOC^m(x^*) = \frac{\partial \beta_i(x^*)}{\partial x_i} \beta_j(x^*) \left[\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \pi_i^{ss} \right].$$

Note that $K_1 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \pi_i^{ss}$, since $\hat{\pi}_i^{fs} = \pi_i^{fs} = 0$.

Further, $K_2 = 0$, since $\hat{\pi}_i^{sf} = \pi_i^{sf}$ and $\hat{\pi}_i^{ff} = \hat{\pi}_j^{ff} = \hat{\pi}_j^{fs} = \pi_i^{ff} = 0$.

Then it is straightforward to conclude that $FOC^m(x^*) \geq 0$, when $K_1 \geq 0$. □

6.3 Proof of Proposition 2:

Proof. In this setting, where conditional payoffs are endogenous and depend on R&D effort while probability of success is exogenous and is given by $\mu \in (0, 1]$, expression in (5) becomes:

$$FOC_i^m(x^*) = \mu [\mu K_3(x^*) + (1 - \mu)K_4(x^*)].$$

Since $\mu > 0$, the inequality $\mu [\mu K_3(x^*) + (1 - \mu)K_4(x^*)] > 0$ is equivalent to

$$\mu K_3(x^*) + (1 - \mu)K_4(x^*) > 0 \quad \text{or} \quad \mu < \Phi(x), \quad \text{where } \Phi(x) = \frac{K_4(x)}{K_4(x) - K_3(x)}.$$

Then it is straightforward to see that

(i) If $K_3(x^*) > 0$ and $K_4(x^*) > 0$, then $FOC_i^m(.)$ evaluated at x^* is positive. This proves result in (i).

(ii) If $K_3(x^*) < 0$ and $K_4(x^*) < 0$, then $FOC_i^m(.)$ evaluated at x^* is negative. This proves result in (ii).

(iii) Next, we analyze the setting where $\Phi(x)$ is decreasing in x and \tilde{x} is the solution to $\mu = \Phi(x)$.

If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $FOC_i^m(x^*)$ is positive for all $x^* < \tilde{x}$.

If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $FOC_i^m(x^*)$ is positive for all $x^* > \tilde{x}$.

This proves result in (iii).

(iv) Next, we analyze the setting where $\Phi(x)$ is increasing in x and \tilde{x} is the solution to $\mu = \Phi(x)$.

If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $FOC_i^m(x^*)$ is positive for all $x^* > \tilde{x}$.

If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $FOC_i^m(x^*)$ is positive for all $x^* < \tilde{x}$.

This proves result in (iv).

Note that $x^m = x^*$ can only hold when $K_3(x^*) = 0$ and $K_4(x^*) = 0$.

Alternatively, when $\Phi(x)$ is decreasing and $\mu - \Phi(x) = 0$ does not have a solution due to $\mu < \Phi(x)$ for all x . Then:

If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $FOC_i^m(x^*)$ is positive for all x^* .

If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $FOC_i^m(x^*)$ is negative for all x^* .

Otherwise, when $\Phi(x)$ is increasing and $\mu - \Phi(x) = 0$ does not have a solution due to $\mu > \Phi(x)$. Then

If $K_3(x^*) < 0$ and $K_4(x^*) > 0$, then $FOC_i^m(x^*)$ is negative for all x^* .

If $K_3(x^*) > 0$ and $K_4(x^*) < 0$, then $FOC_i^m(x^*)$ is positive for all x^* . □

6.4 Proof of Corollary to Proposition 2

Proof. In the setting, where conditional payoffs are endogenous, probability of success is exogenous and equal to 1, FOC (5) becomes:

$$FOC_i^m(x^*) = K_3(x^*).$$

Then it is straightforward to conclude that $FOC_i^m(x^*) \geq 0$, when $K_3(x^*) \geq 0$. \square

7 Appendix B: Detailed analysis of JL (2020), FLV (2017) and DP(2018)

7.1 Detailed analysis of Jullien and Lefouili (2020)

In this section we provide a more detailed analysis of the result in Jullien and Lefouili (2020) and show that it is a special of Proposition 1(i).

Jullien and Lefouili (2020) consider horizontally differentiated products and also assume that unsuccessful innovation results in entry failure. In Jullien and Lefouili (2020) Π_2 is the monopoly payoff of a two-product seller, thus in our notation $\Pi_2 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss}$. Π_1 is the monopoly payoff of a single-product seller (because the other innovation has failed), thus in our notation $\Pi_1 = \hat{\pi}_i^{sf}$. π_2 is the profit of a duopolistic firm when both innovate, thus $\pi_2 = \pi_i^{ss} = \pi_j^{ss}$. The payoffs are summarized in Tables 1' and 2'. Hence, the condition for merger to unambiguously increase the R&D in Proposition 1 of Jullien and Lefouili (2020), $\Pi_2 - \Pi_1 > \pi_2$, is identical to $\hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} > \pi_i^{ss}$ in our notation. Which can be rewritten as condition in our Corollary to Proposition 1 (i) $K_1 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \pi_i^{ss} > 0$.

		Firm 2	
		Success(s)	Failure(f)
Firm 1	Success(s)	π_2, π_2	$\Pi_1, 0$
	Failure(f)	$0, \Pi_1$	$0, 0$

Table 1': Firms' conditional payoffs under product competition: Jullien and Lefouili (2020)

		Firm 2	
		Success(s)	Failure(f)
Firm 1	Success(s)	Π_2	Π_1
	Failure(f)	Π_1	0

Table 2': Merged entity's conditional payoffs: Jullien and Lefouili (2020)

7.2 Detailed analysis of Federico et al. (2017)

Results in Federico et al. (2017) are captured by part (ii) of Proposition 1. With homogeneous products, price competition and no entry in case of failure the relevant profits, using notation of Federico et al. (2017), are given by expressions in Tables 1'' and 2'' below.

		Firm 2	
		Success(s)	Failure(f)
Firm 1	Success(s)	δ, δ	1,0
	Failure(f)	0,1	0,0

Table 1'': Firms' conditional payoffs under product competition: Federico et al. (2017)

		Firm 2	
		Success(s)	Failure(f)
Firm 1	Success(s)	$\frac{1}{2} + \frac{1}{2}$	1+0
	Failure(f)	0+1	0+0

Table 2'': Merged entity's conditional payoffs: Federico et al. (2017)

Hence, the condition in Corollary to Proposition 1 $K_1 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \pi_i^{ss} > 0$ can easily be checked after noting that $\hat{\pi}_i^{ss} - \pi_i^{ss} = \frac{1}{2} - \delta$, which can be positive or negative, $\hat{\pi}_j^{ss} = \frac{1}{2}$ and $\hat{\pi}_j^{sf} = 1$. It follows that this condition never holds and a merger always results in less R&D, which is the result Federico et al. (2017) get.

7.3 Detailed analysis of Denicolò and Polo (2018)

Furthermore, part (ii) of Proposition 1 also captures Denicolò and Polo's (2018) interior equilibrium. They have a homogeneous products price competition setting and no entry in case of innovation failure. The relevant profits before and after the merger are given in Tables 1''' and 2'''.

		Firm 2	
		Success(s)	Failure(f)
Firm 1	Success(s)	$\frac{V}{2}, \frac{V}{2}$	V,0
	Failure(f)	0,V	0,0

Table 1''': Firms' conditional payoffs under product competition: Denicolò and Polo (2018)

		Firm 2	
		Success(s)	Failure(f)
Firm 1	Success(s)	$\frac{V}{2} + \frac{V}{2}$	V+0
	Failure(f)	0+V	0+0

Table 2''': Merged entity's conditional payoffs: Denicolò and Polo (2018)

Noting that $\hat{\pi}_i^{ss} = \hat{\pi}_j^{ss} = \frac{V}{2}$, $\hat{\pi}_j^{ss} = \frac{V}{2}$ and $\hat{\pi}_j^{sf} = V$. Condition in Corollary to Proposition 1, $K_1 = \hat{\pi}_i^{ss} + \hat{\pi}_j^{ss} - \hat{\pi}_j^{sf} - \pi_i^{ss} > 0$ becomes $0 > \frac{V}{2}$, which never holds. Hence, a merger reduces R&D.

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