Non-Fundamental Information in Social Networks: Information Acquisition and Asset Prices

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Abstract

How does the spread of non-fundamental information (e.g., order flow information) affect investor behavior and financial markets? We examine this question in a model in which information about noise traders' demand for an asset can be spread through social networks. When the network connectedness is low, the spread of demand information enables more rational investors to trade against noise traders, reducing mispricing and increasing market efficiency. With the increased market efficiency, investors choose to acquire more private information about noise trading and less private information about asset fundamentals. When the network connectedness is high, the spread of demand information lowers investors' incentives to produce demand information, which decreases market efficiency. Our results suggest that the recent development of online quantitative trading platforms has non-monotonic impacts on market efficiency and the quality of quantitative strategies posted on these platforms.

Keywords: Social network; Information acquisition; Non-fundamental information; Price informativeness; Financial data technology
JEL Classification: G12, G14, G28

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1 Introduction

Empirical literature has shown that information sharing between investors can have significant impacts on their trading behavior and asset prices (see, e.g., Shiller and Pound (1989), Hong et al. (2005), and Ivković and Weisbenner (2007)). Extant theoretical models have also revealed many channels through which the spread of information affects financial markets. Ozsoylev and Walden (2011) show that a higher intensity of information sharing increases information driven price volatility, but has a non-monotonic impact on liquidity driven price volatility. Han and Yang (2013) find that more communication between investors can reduce market efficiency by decreasing their incentives to produce information. Walden (2019) shows that the centrality of social networks can affect investors' profitability and price volatility. Han et al. (2022) demonstrate that active trading strategies can prevail because they are more likely to generate extremely high returns and be shared with others by investors, compared with passive trading strategies.

A limitation of existing social network models of financial markets is that mostly they only consider the spread of fundamental information, i.e., the information about the value of stocks. However, non-fundamental information, e.g., information on market sentiment, random asset supplies, or liquidity traders' order flows, can also affect investors' trading behavior and asset prices (Ganguli and Yang (2009); Farboodi and Veldkamp (2020)). Therefore, the following question naturally arises: What are the impacts of social networks on financial markets when non-fundamental information can also be shared among investors? In this paper, we examine this question with a rational expectations equilibrium model in which investors can share with others information on both asset fundamentals and noise (liquidity) traders' order flows.

Our work builds on the literature on social networks in financial markets (e.g., Han and Yang (2013) and Ozsoylev and Walden (2011)) and information about noisy demands for (or supplies of) risky assets (e.g., Ganguli and Yang (2009) and Farboodi and Veldkamp (2020)). Two features of our model differ our work from others. First, investors' information choices in our model are more flexible. At the beginning of a period, an investor can choose whether to become an informed investor by paying a fixed cost to acquire a data processing technology (see, e.g., Grossman and Stiglitz (1980) and Benhabib et al. (2019)). After becoming informed, the investor can decide how to allocate the data processing capacity to fundamental analysis (i.e., analyzing fundamental information) and demand analysis (i.e., analyzing information about noise traders' demand), as in Farboodi and Veldkamp (2020). Although these two kinds of information choice problems have been extensively studied separately by existing papers, few have analyzed both of them simultaneously in a unified framework. Second, in our model, an informed investor shares with other investors in the same group the information about both an asset's fundamentals and noise traders' demands for the asset. As has been mentioned, allowing for the spread of non-fundamental information differs our model from others in which only fundamental information is shared.

Our analysis starts from a benchmark case in which both the proportion of informed investors and the informed investors' allocation of their data processing capacity are exogenous. In this situation, the price informativeness of a risky asset is increasing in the network connectedness (i.e., the intensity of social communication, measured by the number of other investors each informed investor can share his/her information with).[1] This result is consistent with those in Ozsoylev and Walden (2011) and Han and Yang (2013). When the network connectedness increases, every investor becomes more informed and trade more aggressively, so more information is incorporated into the asset price through their trading. Notably, we find that even if the informed investors only share the order flow (demand) information with others, the asset price still reflects more fundamental information when the network connectedness increases. The reason is that, since the asset price is driven by both the fundamental information and the noise traders' order flow, knowing about the noise traders' order flow helps investors to filter out the noise contained in the asset price and learn about the fundamentals. When the investors can receive more order flow information from others, they can better filter out the noise contained in the asset price and better predict the future value of the asset. Therefore, they will trade more aggressively and thus inject more fundamental information into the asset price.

Our result in the benchmark case can be applied to predict the short-run impact of the spread of quantitative trading strategies on financial market efficiency. In reality, quantitative trading strategies (e.g., statistical arbitrage) largely profit from mispricing. In our model, the noise traders' trading creates mispricing. Learning about noise traders' order flow helps rational investors to profit from mispricing by trading against noise traders. As in Farboodi

[1]Specifically, the price informativeness measures the amount of fundamental information that is reflected by the asset price.

and Veldkamp (2020), it is reasonable to interpret the investors' trading based on order flow information as "quantitative trading". Therefore, in our model, the spread of order flow information resembles the spread of quantitative trading strategies in reality. Recently, many online quantitative trading platforms (e.g., QUANTCONNECT[2] and JoinQuant[3]) are developed. These platforms facilitate the spread of quantitative trading strategies by allowing investors to share their own strategies with others. Based on the result of our benchmark model (in which information is exogenous and only order flow information can be shared), one can predict that, in the short run, the development of such online quantitative trading platforms (i.e., more users sign up for the platforms) allows more investors to adopt quantitative strategies (e.g., statistical arbitrage) and profit from mispricing, accelerating the elimination of mispricing and improving market efficiency.

We then investigate the impacts of network connectedness when the allocation of data processing capacity is optimally determined by the informed investors but the proportion of informed investors is still exogenously given. We find that when the informed investors' information choices are endogenous, the price informativeness still increases in the network connectedness. Han and Yang (2013) argue that when information is endogenous, market efficiency decreases in network connectedness. However, our result indicates that endogenous information does not necessarily lead to the negative impact of network connectedness on market efficiency when multiple dimensions of information can be shared. An increase in network connectedness makes every investor more informed, increasing the price informativeness through the investors' trading. The increased price informativeness allows investors to extract more fundamental information from the price, lowering the informed investors' fundamental analysis (i.e., private learning about the asset's fundamentals). The decreased fundamental analysis does not lead to a lower price informativeness, because more data processing capacity can be used to analyze order flow information. With the spread of more precise information about noise traders' order flow, every investor can better trade against the noise traders, further eliminating mispricing and improving price informativeness. We also find that, even if informed investors only share their order flow information with others, the price informativeness and the quality of order flow information are increasing in network connectedness. This result implies that, when the number of strategy developers is fixed,

[2]https://www.quantconnect.com.
[3]https://www.joinquant.com.

the spread of quantitative trading strategies not only increases market efficiency, but also encourages developers to improve the quality of their quantitative trading strategies.

When both the proportion of informed investors and the informed investors' information choice are endogenous, the impact of network connectedness on price informativeness is non-monotonic. When the network connectedness is low, investors cannot receive enough information from others. Therefore, it is optimal for every investor to become informed (i.e., have access to a data processing technology) by paying a cost. In this situation, the equilibrium proportion of informed investors is endogenously fixed, so the price informativeness and the demand analysis (the fundamental analysis) are increasing (is decreasing) in the network connectedness, similar to the situation when the proportion of informed investors is exogenously given. When the network connectedness is sufficiently high, investors can receive enough information from others. Therefore, a further increase in the network connectedness reduces investors' incentives to become informed, decreasing the equilibrium proportion of informed investors and thus the equilibrium price informativeness. This is similar to the situation in Han and Yang (2013). The decreased price informativeness renders the informed investors to analyze more fundamental information and less order flow information. In summary, when the information is fully endogenous (i.e., both the proportion of informed investors and the informed investors' information choice are endogenous), the price informativeness and the demand analysis (the fundamental analysis) are hump-shaped (is V-shaped) in the network connectedness.

The non-monotonic effects of network connectedness still exist even if only the order flow information is shared. Therefore, our model with endogenous proportion of informed investors can be used to predict the long run impacts of the development of online quantitative trading platforms. Note that in the long run, both the proportion of developers and the quality of strategies can vary. In the early stages of the development of quantitative trading platforms, with more strategy developers and users joining the platforms, more investors can adopt quantitative trading strategies (e.g., statistical arbitrage) to trade in the opposite direction of mispricing, so the market efficiency increases. The increased market efficiency then induces developers to develop quantitative strategies with higher quality. The market efficiency reaches the highest level with an intermediate level of platform development. As the sizes of platforms continue to grow, some developers lose their incentives to develop their own quantitative strategies because they can easily use a lot of strategies by others.

Therefore, the proportion of developers in the population decreases, reducing the market efficiency. The decreased market efficiency further reduces the quality of the quantitative strategies developed by the remaining developers.

Our model also shows that a future increase in network connectedness can create “future information risk”, a concept proposed by Farboodi and Veldkamp (2020). If an increase in the next period’s network connectedness increases the next period’s price informativeness, it makes the next period’s price more sensitive to the next period’s information that is unknown by the current period’s investors, increasing the current period’s investors’ perceived risk of the asset’s resale price. Therefore, the current period’s investors perceive a higher payoff risk and trade less aggressively, reducing the current period’s price informativeness.

Related Literature. Our paper is closely related to the literature on the impacts of social networks on investor behavior and financial markets (Ozsoylev and Walden (2011); Han and Yang (2013); Walden (2019); Hirshleifer (2020); Han et al. (2022)). As has been mentioned, previous works mainly consider the spread of fundamental information (i.e., information that is directly related to asset payoffs). However, the information about noise traders’ asset demands (or random supplies of assets), which is not directly related to asset payoffs, also have crucial impacts on investor behavior and financial markets. Ganguli and Yang (2009) demonstrate that investors’ learning about the supply of an asset can generate multiple equilibria in a financial market, resulting in excess volatility and crashes. Farboodi and Veldkamp (2020) show that switching from fundamental analysis (i.e., learning about fundamental information) to demand analysis (i.e., learning about noise traders’ demand) can increase price informativeness. Marmora and Rytchkov (2018) find that the informativeness of an asset’s price can reach its maximum level when most investors learn about non-fundamental information. We contribute to this strand of literature by showing how the spread of non-fundamental information affects investor behavior and financial markets.

Our paper is also broadly related to the literature on the information acquisition and aggregation in financial markets pioneered by Grossman and Stiglitz (1980) and Verrecchia (1982). Grossman and Stiglitz (1980) study a model in which each investor can decide whether to acquire fundamental information by paying a fixed cost. Verrecchia (1982) study a model in which each investor can decide the precision of their private fundamental information whose cost is continuous, convex, and increasing in the precision. Farboodi and Veldkamp (2020) develop a model in which each investor can continuously decide his/her fundamental information precision and non-fundamental information precision under a data capacity constraint. Many recent papers investigate the determinants of information acquisition and aggregation, such as, transaction cost (Dávila and Parlatore (2021)), commodity financialization (Goldstein and Yang (2022)), government intervention (Brunnermeier et al. (2022)), public disclosure (Goldstein and Yang (2019); Benhabib et al. (2019)), and so on. We contribute to this strand of literature by incorporating the two kinds of information acquisition decisions (i.e., the kind in Grossman and Stiglitz (1980) and the kind in Farboodi and Veldkamp (2020)) into a unified framework and analyze how they are simultaneously affected by social networks in equilibrium.

Our paper is organized as follows. Section 2 presents the model. Section 3 analyzes the financial market equilibrium with exogenous information. Section 4 endogenizes the informed investors' information choice. Section 5 further endogenizes the proportion of informed investors. Section 6 concludes.

2 The model

In this section, we develop a model in which each informed investor can share his/her private information on noise traders' order flow for a firm's security (e.g., stock) and on the firm's fundamentals with other investors in his/her group. Our model can be regarded as an extension of Han and Yang (2013) to a setting in which order flow or asset supply information (see, e.g., Ganguli and Yang (2009) and Farboodi and Veldkamp (2020)) is shared. Compared with social network financial market models in which only fundamental information is shared (see, e.g., Ozsoylev and Walden (2011), Han and Yang (2013), and Walden (2019)), the addition of the sharing of order flow information makes our model more general and more realistic, and can provide richer economic insights.

2.1 Financial assets

There is a risky asset traded in a financial market. At the end of period t, each unit of the risky asset pays a dividend d_{t+1} that follows

$$d_{t+1} - d_t = (1 - G_d)(\mu_d - d_t) + v_{t+1}, \tag{1}$$

where $G_d \in [0,1)$, $\mu_d > 0$, and $v_{t+1} \sim N(0, \tau_v^{-1})$. The (endogenous) risky asset price at the beginning of period t is denoted by p_t, and the per capita net supply of the risky asset is s. There is also a risk-free asset whose (exogenous) gross rate of return is $r \geq 1$.

2.2 Social networks and information

In each period t, $G_t > 0$ groups of rational investors enter the financial market. Each group contains $N_t > 0$ investors, $\mu_t N_t$ of which are informed, and $(1-\mu_t)N_t$ of which are uninformed, where $\mu_t \in [0,1]$. There are also noise traders whose per capita demand for the risky asset is $n_{t+1} \sim N(0, \tau_n^{-1})$. If investor i in group g is informed, then the investor can observe a private signal about the noise traders' demand, $z_{igt} = n_{t+1} + \varepsilon_{igt}^z$, $\varepsilon_{igt}^z \sim N(0, (\tau_{igt}^z)^{-1})$, and a private signal about the asset payoff, $x_{igt} = v_{t+1} + \varepsilon_{igt}^x$, $\varepsilon_{igt}^x \sim N(0, (\tau_{igt}^x)^{-1})$. Uninformed investors cannot observe private signals.

2.2.1 Sending information to others

A period-t informed investor i in group g shares his/her private signals about both the noise traders' demand and the asset payoff with the remaining $N_t - 1$ investors in the same group. As in Han and Yang (2013), we call N_t, which is the number of investors in a group, as the "network connectedness", because investors within a group can communicate with each other. We assume that the sequence of the network connectedness, $\{N_t\}$, is common knowledge. The two signals from investor i received by another investor $j \neq i$ in group g are $y_{igt}^x = x_{igt} + \eta_{igt}^x$ and $y_{igt}^z = z_{igt} + \eta_{igt}^z$, where $\eta_{igt}^x \sim N(0, (\tau^{\eta x})^{-1})$ and $\eta_{igt}^z \sim N(0, (\tau^{\eta z})^{-1})$. We assume that $(\{v_t\}, \{n_t\}, \{\varepsilon_{igt}^x\}, \{\varepsilon_{igt}^z\}, \{\eta_{igt}^x\}, \{\eta_{igt}^z\})$ are mutually independent. If all informed investors choose the same level of precision of their private signals x_{igt} (resp., z_{igt}) about the dividend innovation v_{t+1} (resp., noise traders' demand n_{t+1}), i.e., $\tau_{igt}^x = \tau_t^x$ (resp., $\tau_{igt}^z = \tau_t^z$), $\forall i, g$, then the precision of the signal y_{igt}^x (resp., y_{igt}^z) sent from an informed investor is $\tau_t^{yx} = (Var[\varepsilon_{igt}^x + \eta_{igt}^x])^{-1} = [(\tau_t^x)^{-1} + (\tau^{\eta x})^{-1})]^{-1}$ (resp., $\tau_t^{yz} = (Var[\varepsilon_{igt}^z + \eta_{igt}^z])^{-1} = [(\tau_t^z)^{-1} + (\tau^{\eta z})^{-1})]^{-1}$).

2.2.2 Information received from others

Denote the set of informed investors from group g in period t by I_{gt}. Then the information about the dividend innovation v_{t+1} received by an informed investor j through communicating with others in group g can be summarized by the average of the signals sent from others, i.e.,

$$Y_{jgt}^{IN,x} = \frac{1}{\mu_t N_t - 1} \sum_{i \in I_{gt} \setminus \{j\}} y_{igt}^x = v_{t+1} + \frac{1}{\mu_t N_t - 1} \sum_{i \in I_{gt} \setminus \{j\}} (\varepsilon_{igt}^x + \eta_{igt}^x), \tag{2}$$

which is also an unbiased noisy signal about v_{t+1}. The precision of $Y_{jgt}^{IN,x}$ is thus

$$\left(Var\left[\frac{1}{\mu_t N_t - 1} \sum_{i \in I_{gt} \setminus \{j\}} (\varepsilon_{igt}^x + \eta_{igt}^x) \right] \right)^{-1} = (\mu_t N_t - 1)\tau_t^{yx}. \tag{3}$$

Similarly, the information about the noise traders' demand n_{t+1} received by this informed investor from others can be summarized by

$$Y_{jgt}^{IN,z} = \frac{1}{\mu_t N_t - 1} \sum_{i \in I_{gt} \setminus \{j\}} y_{igt}^z = n_{t+1} + \frac{1}{\mu_t N_t - 1} \sum_{i \in I_{gt} \setminus \{j\}} (\varepsilon_{igt}^z + \eta_{igt}^z), \tag{4}$$

and the precision of $Y_{jgt}^{IN,z}$ is thus

$$\left(Var\left[\frac{1}{\mu_t N_t - 1} \sum_{i \in I_{gt} \setminus \{j\}} (\varepsilon_{igt}^z + \eta_{igt}^z) \right] \right)^{-1} = (\mu_t N_t - 1)\tau_t^{yz}. \tag{5}$$

In contrasts to the informed investors, the uninformed investors born at the beginning of period t do not have private signals, but they can receive information from all $\mu_t N_t$ informed investors in their same groups. Therefore, the information about the dividend innovation v_{t+1} received by an informed investor j through communicating with others in group g can be summarized by

$$Y_{jgt}^{UN,x} = \frac{1}{\mu_t N_t} \sum_{i \in I_{gt}} y_{igt}^x = v_{t+1} + \frac{1}{\mu_t N_t} \sum_{i \in I_{gt}} (\varepsilon_{igt}^x + \eta_{igt}^x), \tag{6}$$

and the precision of this signal is

$$\left(Var\left[\frac{1}{\mu_t N_t} \sum_{i \in I_{gt}} (\varepsilon_{igt}^x + \eta_{igt}^x) \right] \right)^{-1} = \mu_t N_t \tau_t^{yx}. \tag{7}$$

Similarly, the information about the noise traders' demand n_{t+1} received by the uninformed investors from others can be summarized by

$$Y_{jgt}^{UN,z} = \frac{1}{\mu_t N_t} \sum_{i \in I_{gt}} y_{igt}^z = n_{t+1} + \frac{1}{\mu_t N_t} \sum_{i \in I_{gt}} (\varepsilon_{igt}^z + \eta_{igt}^z), \tag{8}$$

and the precision of $Y_{jgt}^{UN,z}$ is thus

$$\left(Var \left[\frac{1}{\mu_t N_t} \sum_{i \in I_{gt}} (\varepsilon_{igt}^z + \eta_{igt}^z) \right] \right)^{-1} = \mu_t N_t \tau_t^{yz}. \tag{9}$$

The investors born at the beginning of period t can observe all the past dividends $\{d_s\}_{s \le t}$, as well as the past and the current asset prices $\{p_s\}_{s \le t}$. Therefore, an informed investor's information set when trading in the financial market is $\mathcal{F}_{igt}^{IN} = \{\ x_{igt},\ z_{igt},\ Y_{igt}^{IN,x},\ Y_{igt}^{IN,z},\ \{d_s\}_{s \le t},\ \{p_s\}_{s \le t}\ \}$, and an uninformed investor's information set is $\mathcal{F}_{igt}^{UN} = \{\ Y_{igt}^{UN,x},\ Y_{igt}^{UN,z},\ \{d_s\}_{s \le t},\ \{p_s\}_{s \le t}\ \}$.

2.3 Investors' preference and portfolio choices

An investor born at the beginning of period t invests in period t and consumes in period $t+1$. The investor's consumption can be expressed as

$$c_{ig,t+1} = (e_{igt} - m_{igt} p_t) r + m_{igt}(d_{t+1} + \psi p_{t+1}) - \mathbb{1}_{\{i \in I_{gt}\}} C_F, \tag{10}$$

where e_{igt} is the investor's initial endowment, m_{igt} is the investor's risky asset holding, C_F is the fixed cost of being informed, and $\mathbb{1}_{\{.\}}$ is the indicator function (i.e., $\mathbb{1}_{\{i \in I_{gt}\}} = 1$ if $i \in I_{gt}$ and $\mathbb{1}_{\{i \in I_{gt}\}} = 0$ if $i \notin I_{gt}$). Note that $\psi \in \{0, 1\}$ is an indicator about whether investors care about future resale prices of the risky asset. When $\psi = 1$ ($\psi = 0$), our model can be considered as a dynamic (static) model. This modelling technique is in the spirit of Farboodi and Veldkamp (2020).

Both the informed and uninformed investors have CARA utility, i.e., their utility function has the form $U(c) = -\exp(-\gamma c)$, where $\gamma > 0$ is the absolute risk aversion coefficient. When trading in the financial market, an investor chooses the risky asset holding to maximize the conditional expected utility of the consumption,

$$\max_{m_{igt}} E\left[U(c_{ig,t+1})|\mathcal{F}_{igt}\right], \tag{11}$$

subject to the budget constraint (10), where $\mathcal{F}_{igt} = \mathcal{F}_{igt}^{IN}$ if the investor is informed, and $\mathcal{F}_{igt} = \mathcal{F}_{igt}^{UN}$ if the investor is uninformed.

2.4 Definition of financial market equilibrium

The financial market equilibrium is a competitive noisy rational expectations equilibrium. Similar to Han and Yang (2013), we analyze a large economy in which there are infinitely many groups of investors (i.e., $G_t \to +\infty$, $\forall t$) for tractability. The formal definition of the financial market equilibrium is as follows.

Definition 1 (Financial market equilibrium)**.** *Given the sequence of the proportion of informed investors* $\{\mu_t\}$*, and the sequences of informed investors' private signal precision* $\{\tau_{igt}^x\}$ *and* $\{\tau_{igt}^z\}$*, a financial market equilibrium consists of a sequence of investors' risky asset holdings* $\{m_{igt}\}$ *and a sequence of the risky asset prices* $\{p_t\}$*, such that (i) each investor's risky asset holding* m_{igt} *solves the utility maximization problem* (11) *subject to the budget constraint* (10)*, and (ii) in each period* t*, the asset price* p_t *clears the financial market, i.e.,*

$$\lim_{G_t \to +\infty} \frac{1}{G_t} \sum_{g=1}^{G_t} \left[\frac{1}{N_t} \left(\sum_{i \in I_{gt}} m_{igt} + \sum_{i \in U_{gt}} m_{igt} \right) \right] + n_{t+1} = s, \tag{12}$$

where U_{gt} *is the set of uninformed investors from group* g *in period* t*.*

2.5 Characterization of financial market equilibrium

We use the standard "conjecture and verify" method which is common in the rational expectations equilibrium literature (e.g., Han and Yang (2013), Benhabib et al. (2019), and Farboodi and Veldkamp (2020)) to solve for the financial market equilibrium. First, conjecture that the risky asset price in period t has the form

$$p_t = \beta_{0t} + \beta_{1t} v_{t+1} + \beta_{2t} n_{t+1} + \beta_{3t}(d_t - \mu_d), \tag{13}$$

where β_{0t}, β_{1t}, β_{2t}, and β_{3t} are deterministic (endogenous) price coefficients. An informed investor of period t can transform p_t into a signal about the dividend d_{t+1},

$$\begin{aligned}\tilde{p}_{igt} &= \frac{p_t - \beta_{0t} - \beta_{2t}E[n_{t+1}|\mathcal{F}_{igt}^{IN}] - \beta_{3t}(d_t - \mu_d)}{\beta_{1t}} \\ &= v_{t+1} + \frac{\beta_{2t}}{\beta_{1t}}(n_{t+1} - E[n_{t+1}|\mathcal{F}_{igt}^{IN}]).\end{aligned} \tag{14}$$

The precision of the signal $\tilde{p}_{igt}$ is thus $\tau_{igt}^{\tilde{p}} = (\frac{\beta_{1t}}{\beta_{2t}})^2(\tau_n + \tau_{igt}^z + (\mu_t N_t - 1)\tau_t^{yz})$. Similarly, an uninformed investor of period t can also transform p_t into a signal about the dividend d_{t+1} using the information about the noise traders' order flow,

$$\begin{aligned}\hat{p}_{igt} &= \frac{p_t - \beta_{0t} - \beta_{2t}E[n_{t+1}|\mathcal{F}_{igt}^{UN}] - \beta_{3t}(d_t - \mu_d)}{\beta_{1t}} \\ &= v_{t+1} + \frac{\beta_{2t}}{\beta_{1t}}(n_{t+1} - E[n_{t+1}|\mathcal{F}_{igt}^{UN}]).\end{aligned} \tag{15}$$

The precision of the signal $\hat{p}_{igt}$ is thus $\tau_{igt}^{\hat{p}} = (\frac{\beta_{1t}}{\beta_{2t}})^2(\tau_n + \mu_t N_t \tau_t^{yz})$.

Solving for problem (11), the optimal risky asset holding for an (informed or uninformed) investor in period t is

$$m_{igt} = \frac{E[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}] - rp_t}{\gamma Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}]}, \tag{16}$$

where $\mathcal{F}_{igt}$ is the information set of the investor. The expressions for the expectation and variance are provided in the Appendix. Substituting investors optimal decisions (16) into the market clearing condition (12), and rearranging terms, we can calculate the implied price function,

$$\begin{aligned}p_t = &\lim_{G_t\to\infty} \frac{\omega_t^{IN}}{\omega_t^{IN} + \omega_t^{UN}} \frac{1}{r\mu_t N_t G_t} \sum_{g=1}^{G_t} \sum_{i\in I_{gt}} E[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}] \\ &+ \lim_{G_t\to\infty} \frac{\omega_t^{UN}}{\omega_t^{IN} + \omega_t^{UN}} \frac{1}{r(1-\mu_t)N_t G_t} \sum_{g=1}^{G_t} \sum_{i\in U_{gt}} E[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{UN}] + \frac{n_{t+1} - s}{r(\omega_t^{IN} + \omega_t^{UN})},\end{aligned} \tag{17}$$

where $\omega_t^{IN} = \mu_t/(\gamma Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}])$ and $\omega_t^{UN} = (1-\mu_t)/(\gamma Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{UN}])$. Further calculations can show that the implied price function p_t is linear in v_{t+1}, n_{t+1}, and

$(d_t - \mu_d)$. Comparing the implied price function (17) and the conjectured price function (13), we can derive a system of difference equations that determines the price coefficients. The following proposition summarizes the above discussion and characterizes the financial market equilibrium.

Proposition 1 (Characterization of financial market equilibrium)**.** *The equilibrium risky asset price in period t is expressed as*

$$p_t = \beta_{0t} + \beta_{1t} v_{t+1} + \beta_{2t} n_{t+1} + \beta_{3t}(d_t - \mu_d),$$

where β_{0t}, β_{1t}, β_{2t}, and β_{3t} satisfy the following system of difference equations,

$$\begin{aligned}
\beta_{0,t} &= \frac{-s}{r(\omega_t^{IN} + \omega_t^{UN})} + \frac{1}{r}(\psi\beta_{0,t+1} + \mu_d),\\
\beta_{1,t} &= \frac{\omega_t^{IN}}{r(\omega_t^{IN} + \omega_t^{UN})}(1 + \psi\beta_{3,t+1})V_t^{IN}(\tau^x + (\mu_t N_t - 1)\tau_t^{yx} + \tau_t^{\tilde{p}})\\
&\quad + \frac{\omega_t^{UN}}{r(\omega_t^{IN} + \omega_t^{UN})}(1 + \psi\beta_{3,t+1})V_t^{UN}(\mu_t N_t \tau_t^{yx} + \tau_t^{\hat{p}}),\\
\beta_{2,t} &= \frac{\omega_t^{IN}}{r(\omega_t^{IN} + \omega_t^{UN})}(1 + \psi\beta_{3,t+1})V_t^{IN}\tau_t^{\tilde{p}}\frac{\beta_{2,t}}{\beta_{1,t}}\frac{\tau_n}{\tau_n + (\mu_t N_t - 1)\tau_t^{yz} + \tau^z}\\
&\quad + \frac{\omega_t^{UN}}{r(\omega_t^{IN} + \omega_t^{UN})}(1 + \psi\beta_{3,t+1})V_t^{UN}\tau_t^{\hat{p}}\frac{\beta_{2,t}}{\beta_{1,t}}\frac{\tau_n}{\tau_n + \mu_t N_t \tau_t^{yz}},\\
\beta_{3,t} &= \frac{G_d}{r - \psi G_d},
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
V_t^{IN} &= Var[\psi p_{t+1} + d_{t+1} | \mathcal{F}_{igt}^{IN}]\\
&= \psi^2(\beta_{1,t+1}^2 \tau_v^{-1} + \beta_{2,t+1}^2 \tau_n^{-1}) + (1 + \psi\beta_{3,t+1})^2(\tau_v + \tau_t^x + (\mu_t N_t - 1)\tau_t^{yx} + \tau_t^{\tilde{p}})^{-1},\\
V_t^{UN} &= Var[\psi p_{t+1} + d_{t+1} | \mathcal{F}_{igt}^{UN}]\\
&= \psi^2(\beta_{1,t+1}^2 \tau_v^{-1} + \beta_{2,t+1}^2 \tau_n^{-1}) + (1 + \psi\beta_{3,t+1})^2(\tau_v + \mu_t N_t \tau_t^{yx} + \tau_t^{\tilde{p}})^{-1}.
\end{aligned}$$

Proof. See Appendix A. □

3 Analysis of the financial market equilibrium

In this section, we analyze the impacts of social communication on financial markets, taking the proportion of informed investors and the informed investors' information choice (i.e., the precision of their private signals) as given. We first investigate the static situation in which investors do not care about the risky asset's resale price (i.e., $\psi = 0$), and then explore the dynamic situation in which investors care about the asset's resale price (i.e., $\psi = 1$).

3.1 The static case

In this subsection, we assume that $\psi = 0$. Without loss of generality, we also assume that $\mu_d = G_d = 0$. With these assumptions, the model developed in Section 2 becomes a one-period model. Therefore, in this subsection, we drop the time subscript for each variable. Moreover, by the assumption that $G_d = 0$ and Proposition 1, we can see that $\beta_3 = 0$. Therefore, the risky asset price in the static situation is

$$p = \beta_0 + \beta_1 v + \beta_2 n. \tag{19}$$

As is standard in the rational expectations equilibrium literature (see, e.g., Farboodi and Veldkamp (2020)), we define the risky asset's price informativeness as its signal-to-noise ratio, β_1/β_2. We find that the price informativeness increases with the network connectedness N (i.e., the number of investors in a group).

Proposition 2 (Price informativeness in the static case with exogenous information)**.** *Assume that (i) $\psi = G_d = \mu_d = 0$, and (ii) the proportion of informed investors μ, fundamental analysis τ^x, and demand analysis τ^z are given. If $\gamma^2 > 4\mu^2[\tau^z + (N-1)\tau^{yz}][\tau^x + (N-1)\tau^{yx}]$, then the equilibrium asset price is given by Eq.* (19) *where β_0, β_1, and β_2 are endogenous coefficients. Particularly, the equilibrium price informativeness is*

$$\frac{\beta_1}{\beta_2} = \frac{\gamma - \sqrt{\gamma^2 - 4\mu^2[\tau^z + (N-1)\tau^{yz}][\tau^x + (N-1)\tau^{yx}]}}{2\mu[\tau^z + (N-1)\tau^{yz}]}. \tag{20}$$

Moreover, the price informativeness increases in the network connectedness: $\frac{\partial(\beta_1/\beta_2)}{\partial N} > 0$.

Proof. See Appendix A. □

Proposition 2 shows that when information acquisition decisions are exogenous, the price informativeness is increasing in the network connectedness. The reason is that when every informed investor share his/her own information to more other investors, every investor becomes more informed, and more information is injected into the price through their trading. Proposition 2 generalizes the result with exogenous information in Han and Yang (2013) to a situation with demand analysis. In the following proposition, we show that the price informativeness increases with demand analysis. Moreover, the demand analysis strengthens the impact of social communication on price informativeness.

Proposition 3 (Impacts of demand analysis)**.** *Assume that (i) $\psi = G_d = \mu_d = 0$, (ii) the proportion of informed investors μ, fundamental analysis τ^x, and demand analysis τ^z are given, and (iii) $\gamma^2 > 4\mu^2[\tau^z + (N-1)\tau^{yz}][\tau^x + (N-1)\tau^{yx}]$. Then a higher level of demand analysis leads to a higher price informativeness, i.e., $\frac{\partial(\beta_1/\beta_2)}{\partial\tau^z} > 0$. Moreover, the sensitivity of the price informativeness to network connectedness increases with the demand analysis, i.e., $\frac{\partial^2(\beta_1/\beta_2)}{\partial N\partial\tau^z} > 0$.*

Proof. See Appendix A. □

The demand analysis (i.e., learning about noise traders' order flow) helps investors better filter out the noise contained in the price signal (see Eq. (14)). Therefore, with a higher level of demand analysis, investors can extract more information about the fundamentals from the asset price, so the price informativeness is increased through the investors' trading. More interestingly, the demand analysis affects the sensitivity of price informativeness to network connectedness through two channels. First, when demand analysis increases, investors' communication about the noise traders' demand becomes more effective (i.e., every investor can receive more precise information about the noise traders' demand from other investors within the same group). Therefore, the marginal effect of network connectedness on the information received by each investor increases, so the price informativeness becomes more sensitive to the network connectedness. Second, even if investors within a same group do not communicate the order-flow information with each other, the sensitivity of price informativeness to network connectedness still increases with informed investors' own demand analysis. When the informed investors have more precise information about the order flow, their perceived risk decreases, so they respond more aggressively to the fundamental information received from others, strengthening the impact of network connectedness on the price informativeness.

3.2 The dynamic case

In this subsection, we investigate the impacts of network connectedness and demand analysis on price informativeness under the dynamic setting (i.e., $\psi = 1$) with exogenous information (i.e., the proportion of informed investors μ, fundamental analysis τ^x, and demand analysis τ^z are exogenously given). We plot the price informativeness as functions of network connectedness and demand analysis in Figure 1.

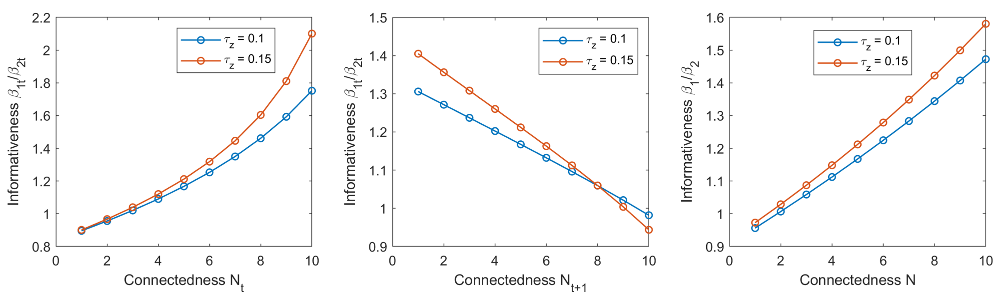

Figure 1: Network connectedness, demand analysis, and price informativeness in the dynamic setting with exogenous information. The left panel plots the period-t price informativeness β_{1t}/β_{2t} as a function of period-t network connectedness N_t. The middle panel plots the period-t price informativeness β_{1t}/β_{2t} as a function of period-$t+1$ network connectedness N_{t+1}. The right panel plots the stationary price informativeness β_1/β_2 as a function of the stationary network connectedness N. Parameter values: $\mu_d = 0.04$, $G = 0.98$, $r = 1.02$, $\gamma = 0.05$, $\mu = 0.73$, $\tau_v = 80.08$, $\tau_n = 19.75$, $\tau^x = 2$, $\tau^{\eta x} = 0.1$, $\tau^{\eta z} = 0.1$, and $\psi = 1$.

The left panel of Figure 1 plots the period-t price informativeness β_{1t}/β_{2t} as a function of period-t network connectedness N_t, assuming that $N_s = 5$, $\forall s > t$. It shows that the current period's price informativeness β_{1t}/β_{2t} increases with the current period's network connectedness N_t. The intuition is the same with that in the static setting (see Proposition 2). When the investors can immediately communicate with more other investors, each investor will become more informed, and their trading will immediately inject more information into the asset price, increasing the price informativeness. In contrast, the middle panel of Figure 1 shows that the current period's price informativeness β_{1t}/β_{2t} decreases with the next period's network connectedness N_{t+1}. The reason is that, when investors in the next period are more connected, the price informativeness in the next period will increase. In other words, the next period's price will be more sensitive to the next period's information. However, investors in the current period are uncertain about the realization of information the next period's investors will acquire. Therefore, a higher sensitivity of the next period's asset price to the next period's information increases the risk perceived by the current period's investors. Specifically, the current period's investors are more uncertain about their asset's resale price. As a result, the current period's investors will trade less aggressively, decreasing the current period's price informativeness. Our result that a future increase in the network connectedness can decrease current price informativeness provides another example for the "future information risk" concept proposed by Farboodi and Veldkamp (2020).

The right panel of Figure 1 shows the impact of network connectedness on price informativeness under the situation in which the network connectedness in all periods are equal. In other words, an overall increase in the network connectedness leads to a higher price informativeness in every period. The reason is that the impact of future network connectedness on the current price informativeness is less pronounced than that of the current network connectedness. Therefore, when there is an overall increase in the network connectedness, the positive impact of the higher current network connectedness dominates, resulting in a higher price informativeness in the current period.[4]

Figure 1 also shows that more demand analysis can lead to a price informativeness and strengthen the impact of network connectedness on price informativeness (see the left panel and the right panel), consistent with Proposition 3. Interestingly, the middle panel of Figure 1 demonstrates that, when the next period's network connectedness is much higher than the current period's network connectedness (e.g., $N_{t+1} = 10$ and $N_t = 5$), more demand analysis can result in a lower price informativeness β_{1t}/β_{2t} in the current period. The reason is that, when the demand analysis τ_z increases, the next period's price informativeness $\beta_{1,t+1}/\beta_{2,t+1}$ also increases. Therefore, the investors in period t face a higher risk of their resale price $Var[p_{t+1}|\mathcal{F}_{igt}]$ and trade less aggressively, decreasing the period-t price informativeness.

[4] Note that under the stationary equilibrium, the values of the price informativeness in all periods are equal.

3.3 Application: the short-run impacts of sharing quantitative trading strategies

In recent years, many online quantitative trading platforms (e.g., QUANTCONNECT and JoinQuant) are developed. On these platforms, investors not only develop and implement their own quantitative trading strategies, but also share their strategies with others (or learn the strategies developed by other users). Therefore, such platforms facilitate the spread of trading strategies. In this subsection, we apply our model to analyze the consequences of the development of online quantitative trading platforms.

As in Farboodi and Veldkamp (2020), we interpret the investors' trading based on the order flow information as "quantitative trading". A private signal about the noise trader's order flow $z_{ig} = n + \varepsilon_{ig}^{z}$ can be regarded as being generated by the quantitative trading algorithm developed by an informed investor. To further understand why the trading based on the order flow information can be thought of as quantitative trading, recall that (in the static case) the asset price can be expressed as $p = \beta_0 + \beta_1 v + \beta_2 n$, where $\beta_0 + \beta_1 v$ reflects the asset's fundamental value, and $\beta_2 n$ reflects the "mispricing" caused by the noise traders' trading. Therefore, analyzing the noise traders' order flow n helps investors profit from the mispricing. In other words, trading based on the signal z_{ig} resembles quantitative strategies like "statistical arbitrage" and "factor investing" in reality.

To focus on the spread of quantitative trading strategies, we assume that investors do not share fundamental information with others, i.e., $\tau^{\eta x} = 0$. Moreover, since we are focusing on the "short-run" effect, we assume that μ, τ_x, and τ_z are exogenously given. Intuitively, investors only share existing trading strategies, without developing new strategies. The following corollary of Proposition 2 shows the impact of the spread of quantitative trading strategies on market efficiency.

Corollary 1 (Short-run impact of sharing quantitative trading strategies under the static setting)**.** *Assume that $G_d = \mu_d = \psi = 0$, and $\tau^{\eta x} = 0$. The price informativeness is increasing in the network connectedness, i.e.,* $\frac{\partial(\beta_1/\beta_2)}{\partial N}|_{\tau^{\eta x}=0} > 0$.

Proof. This result follows directly from Proposition 2. □

When $\tau^{\eta x} = 0$, investors do not communicate fundamental information with others, so an increase in the network connectedness N only means that a quantitative trading strategy is shared with more investors. Corollary 1 shows that under the static setting, the sharing of quantitative trading strategies has a positive short-run impact on financial price informativeness. In reality, as online quantitative trading platforms develop, more people can use strategies like statistical arbitrage to profit from mispricing, which accelerates the elimination of mispricing, making the financial market more efficient in the short run.

4 Equilibrium with endogenous fundamental and demand analysis

In this section, we first extend the model in Section 2 to a setting with endogenous fundamental analysis and demand analysis (i.e., an informed investor can optimally choose the signal precision τ_{igt}^x and τ_{igt}^z), while exogenously fixing the proportion of informed investors μ_t. Then we investigate how the network connectedness affects informed investors' information acquisition decisions.

4.1 Information choice problem

As in Farboodi and Veldkamp (2020), before receiving information in period t, an investor chooses τ_{igt}^x and τ_{igt}^z to maximize the ex ante expected utility,

$$\max_{\tau_{igt}^x,\tau_{igt}^z} E[U(c_{ig,t+1})|\{d_s\}_{s\leq t}], \tag{21}$$

subject to the constraint $(\tau_{igt}^x)^2+\chi(\tau_{igt}^z)^2 \leq H_t$, where $H_t > 0$ is the data processing capacity in period t, and $\chi > 0$ determines the cost of demand analysis relative to fundamental analysis. Following Farboodi and Veldkamp (2020), one can also interpret H_t as the level of financial data technology. The utility maximization problem in (21) can be transformed into minimizing the ex post payoff variance $Var[d_{t+1} + \psi p_{t+1}|\mathcal{F}_{igt}^{IN}]$, which can further be expressed as maximizing $\tau_{igt}^x + (\frac{\beta_{1t}}{\beta_{2t}})^2\tau_{igt}^z$. This problem can be solved using the method of Lagrange's multipliers. Moreover, since the informed investors are ex ante identical, all of them will choose the same level of signal precision. Therefore, we focus on the symmetric equilibrium in which $\tau_{igt}^x = \tau_t^x$ and $\tau_{igt}^z = \tau_t^z$, $\forall i, g$. We have the following proposition that characterizes the informed investors' optimal information choice.

Proposition 4 (Information choice)**.** *Assume that in period t the proportion of informed investors μ_t and the level of financial data technology H_t are given. Then every informed investor's optimal fundamental analysis τ_t^x (i.e., the precision of the private signal about the asset payoff) and demand analysis τ_t^z (i.e., the precision of the private signal about the noise traders' order flow), and the equilibrium price coefficients β_{jt}, $j = 0, 1, 2, 3$, must satisfy Eq.* (18) *as well as the following system of equations,*

$$\tau_t^x = \frac{\sqrt{H_t}}{\sqrt{1 + (\frac{\beta_{1t}}{\beta_{2t}})^4 \frac{1}{\chi}}}, \qquad \tau_t^z = \frac{1}{\chi} \left(\frac{\beta_{1t}}{\beta_{2t}} \right)^2 \frac{\sqrt{H_t}}{\sqrt{1 + (\frac{\beta_{1t}}{\beta_{2t}})^4 \frac{1}{\chi}}}. \tag{22}$$

Proof. See Appendix A. □

One can derive the equilibrium fundamental analysis, demand analysis, and price coefficients by solving Eqs. (22) and (18) simultaneously.

4.2 The static case

In this subsection, we consider the static case (i.e., $\psi = 0$) of the model with endogenous fundamental analysis and demand analysis. As in Subsection 3.1, we also assume that $G_d = \mu_d = 0$ and drop the time subscript. We find that when fundamental analysis and demand analysis can be endogenously chosen by informed investors, the price informativeness still increases with a higher network connectedness if the proportion of informed investors is fixed. Moreover, a higher network connectedness increases the informed investors' demand analysis and decreases their fundamental analysis.

Proposition 5 (Price informativeness in the static case with endogenous fundamental and demand analysis)**.** *Consider the situation in which $\psi = G_d = \mu_d = 0$. Also assume that the proportion of informed investors $\mu \in [0, 1]$ is given. Then the equilibrium price informativeness is increasing in the network connectedness, i.e., $\frac{\partial(\beta_1/\beta_2)}{\partial N} > 0$. Moreover, the equilibrium fundamental analysis is decreasing in the network connectedness, i.e., $\frac{\partial \tau^x}{\partial N} < 0$, and the equilibrium demand analysis is increasing in the network connectedness, i.e., $\frac{\partial \tau^z}{\partial N} > 0$.*

Proof. See Appendix A. □

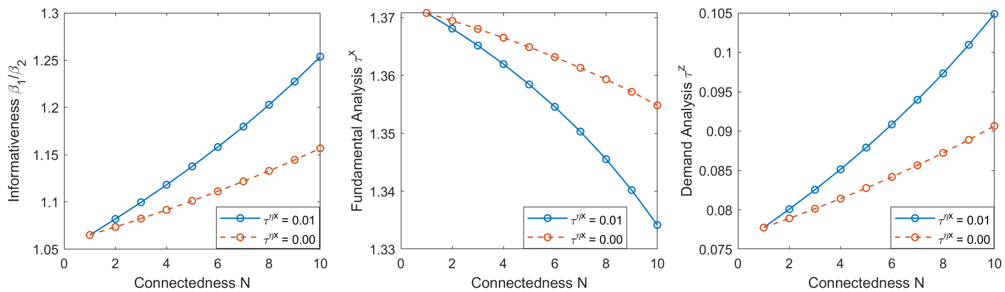

Figure 2: Impact of network connectedness when the proportion of informed investors is fixed (static case). This figure plots the price informativeness β_1/β_2, fundamental analysis τ^x, and demand analysis τ^z as functions of network connectedness N. Parameter values: $\tau_v = 1$, $\tau_n = 1$, $\tau^{\eta z} = 0.01$, $\gamma = 1$, $\chi = 20$, $H = 2$, $\mu = 0.73$, and $G_d = \mu_d = \psi = 0$.

Han and Yang (2013) find that when information is endogenous, price informativeness decreases in network connectedness. However, Proposition 5 and Figure 2 show that in our setting, even if information is endogenous, the price informativeness is increasing in the network connectedness. Proposition 5 and Figure 2 also show that the fundamental (demand) analysis is decreasing (increasing) in network connectedness. When the proportion of informed investors is fixed, an increase in network connectedness makes all investors more informed, injecting more information into the asset price. The increased price informativeness allows informed investors to extract more fundamental information from the price and lowers their incentives to conduct fundamental analysis by themselves. However, the decrease in fundamental analysis does not lead to a lower price informativeness, because more data processing capacity are available for demand analysis, which also increases the price informativeness (see Proposition 3).

4.3 The dynamic case

In this subsection, we investigate the impacts of network connectedness on price informativeness and informed investors' allocation of their data processing capacity under the dynamic setting (i.e., $\psi = 1$) when the proportion of informed investors μ is exogenously given. We plot the equilibrium price informativeness, fundamental analysis, and demand analysis as functions of network connectedness in Figure 3.

The upper left and lower left panels of Figure 3 show that an increase in the current

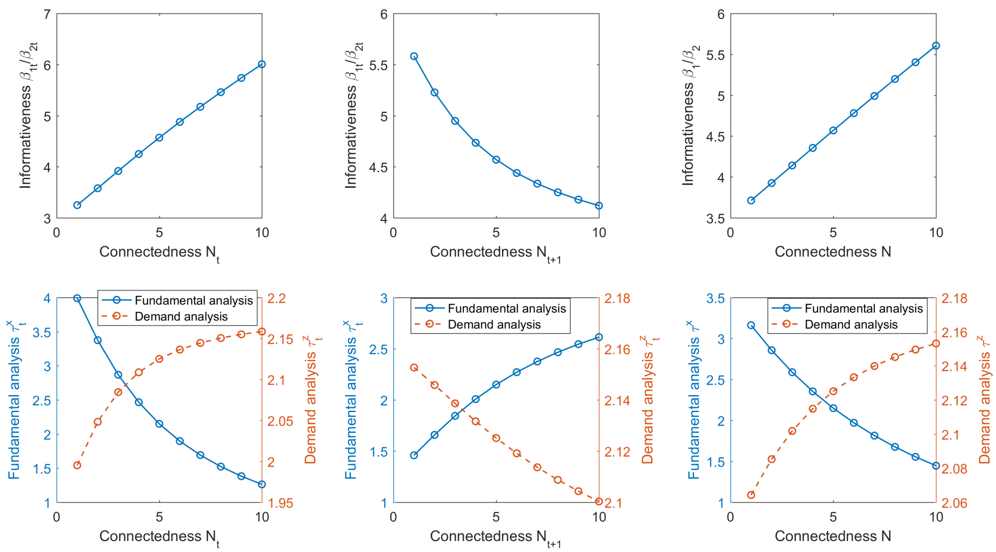

Figure 3: Impacts of network connectedness when the proportion of informed investors is fixed (dynamic case). The upper left panel plots the period-t price informativeness β_{1t}/β_{2t} as a function of period-t network connectedness N_t. The lower left panel plots the period-t fundamental analysis τ_t^x and demand analysis τ_t^z as functions of period-t network connectedness N_t. The upper middle panel plots the period-t price informativeness β_{1t}/β_{2t} as a function of period-$t+1$ network connectedness N_{t+1}. The lower middle panel plots the period-t fundamental analysis τ_t^x and demand analysis τ_t^z as functions of period-$t+1$ network connectedness N_{t+1}. The upper right panel plots the stationary price informativeness β_1/β_2 as a function of the stationary network connectedness N. The lower right panel plots the stationary fundamental analysis τ^x and demand analysis τ^z as functions of the stationary network connectedness N. Parameter values: $\mu_d = 0.04$, $G_d = 0.98$, $r = 1.02$, $\gamma = 0.05$, $\mu = 0.73$, $\tau_v = 80.08$, $\tau_n = 19.75$, $\tau^{\eta x} = 0.1$, $\tau^{\eta z} = 0.1$, $\chi = 21.12$, $H = 100$, and $\psi = 1$.

period's network connectedness results in a higher price informativeness, a lower fundamental analysis, and a higher demand analysis in the current period. The intuitions are the same as those of Proposition 5. In contrast, an increase in the next period's network connectedness results in a lower price informativeness, a higher fundamental analysis, and a lower demand analysis in the current period. The reason is the same as that described in Section 3: A higher next period's network connectedness increases the next period's asset price's response to the next period's information, which increases the current period's investors' perceived risk of the resale price, reducing informed trading and lowering the price informativeness in the current period. A lower price informativeness induces investors to conduct more fundamental analysis. However, the increase in fundamental analysis does not lead to a higher price informativeness, because demand analysis decreases due to the limitation of data processing capacity. Since the impacts of network connectedness in the future on the current period's equilibrium variables are less pronounced than the impacts of network connectedness in current period, an overall increase in the network connectedness increases the stationary price informativeness, decreases the stationary fundamental analysis, and increases the stationary demand analysis (see the upper right and lower right panels of Figure 3).

4.4 Application: the sharing and quality of quantitative trading strategies

In this subsection, we apply our model with endogenous allocation of data processing capacity and fixed proportion of informed investors to analyze the impact of the development of online quantitative trading platforms on the quality of quantitative trading strategies. As has been explained in Section 3, we follow Farboodi and Veldkamp (2020) to interpret a private signal about the noise trader's order flow $z_{ig} = n + \varepsilon^z_{ig}$ as a quantitative trading strategy developed by an informed investor, so the precision of this signal τ^z can be regarded as the quality of the quantitative trading strategy. Intuitively, when τ^z becomes higher, the quantitative strategy can identify the mispricing n more precisely, resulting in a higher trading profit. The following corollary of Proposition 5 shows that, when the proportion of informed investors is fixed, the spread of quantitative trading strategies encourages investors to develop better quantitative trading strategies.

Corollary 2 (Spread of quantitative trading strategies, market efficiency, and quality of strategies)**.** *Assume that $\psi = G_d = \mu_d = 0$, and μ is exogenously given. Also assume that $\tau^{\eta x} = 0$, so that no fundamental information is shared. Then the share of quantitative trading strategies improves both the market efficiency and the quality of quantitative strategies, i.e., $\frac{\partial(\beta_1/\beta_2)}{\partial N}|_{\tau^{\eta x}=0} > 0$ and $\frac{\partial \tau^z}{\partial N}|_{\tau^{\eta x}=0} > 0$. Moreover, the sharing of quantitative strategies lowers the fundamental analysis, i.e., $\frac{\partial \tau^x}{\partial N}|_{\tau^{\eta x}=0} < 0$.*

Proof. This result follows directly from Proposition 5. □

As has been explained in Section 3, the spread of quantitative trading strategies allows more investors to engage in statistical arbitrage, which accelerates the elimination of mispricing and improves the market efficiency. A more efficient financial market that reveals more fundamental information lowers the potential profits from fundamental trading, reducing the investors' incentives to produce private fundamental information. Therefore, more resources are allocated to developing quantitative trading strategies, increasing their quality.

5 The full equilibrium: endogenous proportion of informed investors

In this section, we consider the full equilibrium in which both the proportion of informed investors (i.e., μ_t) and the informed investors' information choice (i.e., τ_t^x and τ_t^z) are endogenously determined. Assume that at the beginning of period t, each investor decides whether to become informed or not. If an investor decides to become informed, then the investor pays a fixed cost C_F to gain access to a data processing technology with capacity H_t. In the spirit of Grossman and Stiglitz (1980), the equilibrium proportion of informed investors μ_t should be such that any investor is indifferent between being informed and being uninformed. Formally, define

$$\Delta U_t(\mu_t) = E[E[U(c_{ig,t+1})|\mathcal{F}_{igt}^{IN}]|\{d_s\}_{s\leq t}]|_{\mu_t} - E[E[U(c_{ig,t+1})|\mathcal{F}_{igt}^{UN}]|\{d_s\}_{s\leq t}]|_{\mu_t}. \tag{23}$$

Intuitively, $\Delta U_t(\mu_t)$ measures the benefit of being informed when the proportion of informed investor is μ_t. If $\Delta U_t(0) > 0$ and $\Delta U_t(1) < 0$, then the equilibrium proportion of informed investors is determined by $\Delta U_t(\mu_t) = 0$. If $\Delta U_t(0) \leq 0$ (i.e., being informed is worse than being uninformed even if no one is informed), the equilibrium proportion of informed investors is $\mu_t = 0$. If $\Delta U_t(1) \geq 0$ (i.e., being informed is better than being uninformed even if everyone is informed), the equilibrium proportion of informed investors is $\mu_t = 1$. We have the following proposition that characterizes the equilibrium proportion of informed investors.

Proposition 6 (The proportion of informed investors)**.** *In period t, (a) if $\Delta U_t(0) > 0$ and $\Delta U_t(1) < 0$, the proportion of informed investors μ_t must satisfy the following equation,*

$$\sqrt{\frac{Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{UN}]|_{\mu_t}}{Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}]|_{\mu_t}}} = e^{\gamma C_F}. \tag{24}$$

(b) If $\Delta U_t(0) \leq 0$, then the equilibrium proportion of informed investors is $\mu_t = 0$. (c) If $\Delta U_t(1) \geq 0$, then the equilibrium proportion of informed investors is $\mu_t = 1$.

Proof. See Appendix A. □

One can derive the equilibrium proportion of informed investors, fundamental analysis, demand analysis, and price coefficients by solving Eqs. (24), (22), and (18) simultaneously.

5.1 The static case

In this subsection, we analyze the static case (i.e., $\psi = 0$) of the full model to investigate the impacts of network connectedness on the price informativeness, informed investors' information choices, and the proportion of informed investors. We find that when both the two kinds of information choice decisions (i.e., μ and (τ^x, τ^z)) are endogenous, the effect of network connectedness on the price informativeness can be non-monotonic.

Proposition 7 (The impacts of network connectedness when both the proportion of informed investors and their information choice are endogenous)**.** *Assume that $\psi = \mu_d = G_d = 0$. Define $\Delta U(\mu) = E[E[U(c_{ig})|\mathcal{F}_{ig}^{IN}]]|_{\mu} - E[E[U(c_{ig})|\mathcal{F}_{ig}^{UN}]]|_{\mu}$. (a) If $\Delta U(1) \geq 0$, then the proportion of informed investors is fixed at $\mu = 1$. Moreover, a higher network connectedness increases the price informativeness, decreases the fundamental analysis, and increases the demand analysis, i.e., $\frac{\partial(\beta_1/\beta_2)}{\partial N} > 0$, $\frac{\partial \tau^x}{\partial N} < 0$, and $\frac{\partial \tau^z}{\partial N} > 0$. (b) If $\Delta U(0) > 0$ and $\Delta U(1) < 0$,* *then the proportion of informed investors μ satisfies the following equation,*

$$\mu = \frac{[\tau^{x} - \tau^{yx} + (\frac{\beta_1}{\beta_2})^2(\tau^{z} - \tau^{yz})] - (e^{2\gamma C_F} - 1)(\tau_v + (\frac{\beta_1}{\beta_2})^2\tau_n)}{N(e^{2\gamma C_F} - 1)[\tau^{yx} + (\frac{\beta_1}{\beta_2})^2\tau^{yz}]}. \tag{25}$$

Moreover, a higher network connectedness decreases the price informativeness, increases the fundamental analysis, and decreases the demand analysis, i.e., $\frac{\partial(\beta_1/\beta_2)}{\partial N} < 0$, $\frac{\partial \tau^{x}}{\partial N} > 0$, *and* $\frac{\partial \tau^{z}}{\partial N} < 0$.

Proof. See Appendix A. □

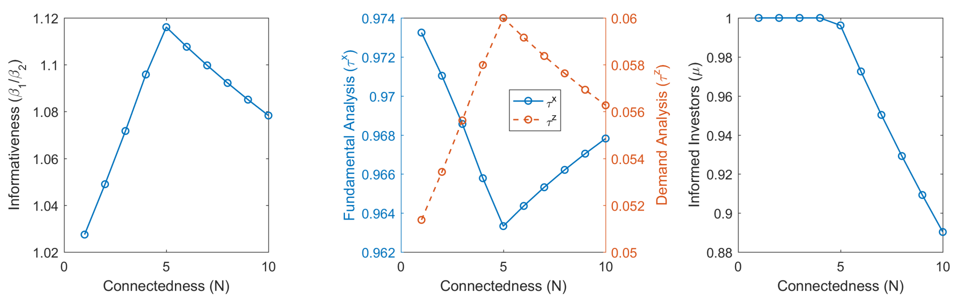

Figure 4: Impact of network connectedness when the proportion of informed investors is endogenous (static case). This figure plots price informativeness $\frac{\beta_1}{\beta_2}$, fundamental analysis τ^x, demand analysis τ^z, and proportion of informed investors μ as functions of network connectedness N. Parameter values: $\tau_v = 1$, $\tau_n = 1$, $\tau^{\eta x} = 0.01$, $\tau^{\eta z} = 0.01$, $\gamma = 1$, $\chi = 20$, $\bar{v} = 1$, $H = 1$, $C_F = 0.17$, and $\psi = G_d = \mu_d = 0$.

Proposition 7 suggests that when the proportion of informed investors μ is endogenously determined, the price informativeness β_1/β_2 is hump-shaped in the network connectedness N. When the network connectedness N is low, investors can get little information from others. Therefore, even if others are all informed, it is still optimal for an investor to pay a cost to become informed (i.e., $\Delta U(1) > 0$), so the proportion of informed investors is fixed at $\mu = 1$. Since the proportion of informed investors is fixed, by Proposition 5, the price informativeness is increasing in the network connectedness N. In contrast, when the network connectedness N is sufficiently large, every investor can learn enough information from others. In this situation, if all other investors are informed, it is optimal for an investor not to be informed (i.e., $\Delta U(1) < 0$). Therefore, the proportion of informed investors μ is not fixed at 1. Moreover, a further increase in N reduces investors' incentives to become informed and thus the equilibrium proportion of informed investors μ, lowering the price informativeness. Therefore, as is shown in Figure 4, the price informativeness is hump-shaped in the network connectedness N. Correspondingly, the demand analysis τ^z is hump-shaped in the network connectedness N, and the fundamental analysis is V-shaped in N.

5.2 The dynamic case

In this subsection, we investigate the impacts of network connectedness on price informativeness and informed investors' allocation of their data processing capacity under the dynamic setting (i.e., $\psi = 1$) when the proportion of informed investors μ_t is endogenous. We plot the equilibrium price informativeness, fundamental analysis, demand analysis, and the proportion of informed investors as functions of network connectedness in Figure 5.

The upper left and upper middle panels of Figure 5 show that the current period's price informativeness and demand analysis are hump-shaped (is V-shaped) in the current period's network connectedness. Moreover, the upper right panel of Figure 5 shows that when the current period's network connectedness is low, an increase in it has no impact on the current period's proportion of informed investors. In contrast, when the current period's network connectedness is sufficiently large, an increase in it decreases the proportion of informed investors. These results are consistent with those in the static case (see Proposition 7 and Figure 4).

As has been explained in Section 3, a higher price informativeness in the next period leads to a higher risk perceived by the current period's investors and thus a lower price informativeness in the current period. Therefore, as is shown in the lower left panel of Figure 5, the current period's price informativeness is V-shaped in the next period's network connectedness. Correspondingly, the lower middle panel of Figure 5 shows that the current period's fundamental analysis (demand analysis) is hump-shaped (V-shaped) in the next period's network connectedness. Moreover, the current period's proportion of informed investors is hump-shaped in the next period's network connectedness (see the lower right panel of Figure 5), because when the price informativeness becomes lower, the investors cannot extract enough information from the price, so it is optimal for more of them to pay a cost to become informed, and vice versa.

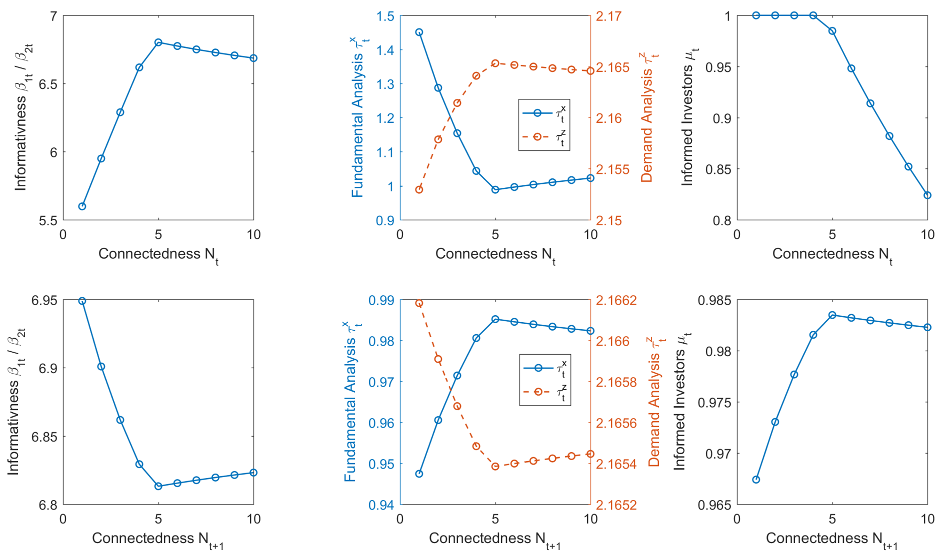

Figure 5: Impacts of network connectedness when the proportion of informed investors is endogenous (dynamic case). The upper left panel plots the period-t price informativeness β_{1t}/β_{2t} as a function of period-t network connectedness N_t. The upper middle panel plots the period-t fundamental analysis τ_t^x and demand analysis τ_t^z as functions of period-t network connectedness N_t. The lower left panel plots the period-t price informativeness β_{1t}/β_{2t} as a function of period-$t+1$ network connectedness N_{t+1}. The lower middle panel plots the period-t fundamental analysis τ_t^x and demand analysis τ_t^z as functions of period-$t+1$ network connectedness N_{t+1}. The upper right panel plots the period-t proportion of informed investors μ_t as a function of period-t network connectedness N_t. The lower right panel plots the period-t proportion of informed investors μ_t as a function of period-$t+1$ network connectedness N_{t+1}. Parameter values: $\mu_d = 0.04$, $G_d = 0.98$, $r = 1.02$, $\gamma = 0.05$, $\mu = 0.73$, $\tau_v = 80.08$, $\tau_n = 19.75$, $\tau^{\eta x} = 0.1$, $\tau^{\eta z} = 0.1$, $\chi = 21.12$, $H = 100$, $C_F = 0.07$, and $\psi = 1$.

5.3 Application: the long-run impacts of sharing quantitative trading strategies

In this subsection, we apply our model with endogenous information choice and endogenous proportion of informed investors to analyze the long-run impact of sharing quantitative trading strategies. As before, when analyzing the spread of quantitative trading strategies, we assume that no fundamental information is shared, i.e., $\tau^{\eta x} = 0$. In the long-run, both the quality of quantitative trading strategies τ^z and the proportion of strategy developers μ are determined in equilibrium. We have the following corollary of Proposition 7.

Corollary 3 (Long-run impacts of sharing quantitative trading strategies)**.** *Assume that* $\psi = G_d = \mu_d = 0$. *Also assume that* $\tau^{\eta x} = 0$ *so that no fundamental information is shared. (a) If* $\Delta U(1) \geq 0$, *then* μ *is fixed at* 1. *Moreover, we have* $\frac{\partial(\beta_1/\beta_2)}{\partial N}|_{\tau^{\eta x}=0} > 0$, $\frac{\partial \tau^x}{\partial N}|_{\tau^{\eta x}=0} < 0$, *and* $\frac{\partial \tau^z}{\partial N}|_{\tau^{\eta x}=0} > 0$. *(b) If* $\Delta U(0) > 0$ *and* $\Delta U(1) < 0$, *then* μ *satisfies the following equation,*

$$\mu = \frac{[\tau^x + (\frac{\beta_1}{\beta_2})^2(\tau^z - \tau^{yz})] - (e^{2\gamma C_F} - 1)(\tau_v + (\frac{\beta_1}{\beta_2})^2 \tau_n)}{N(e^{2\gamma C_F} - 1)(\frac{\beta_1}{\beta_2})^2 \tau^{yz}}. \tag{26}$$

Moreover, we have $\frac{\partial(\beta_1/\beta_2)}{\partial N}|_{\tau^{\eta x}=0} < 0$, $\frac{\partial \tau^x}{\partial N}|_{\tau^{\eta x}=0} > 0$, *and* $\frac{\partial \tau^z}{\partial N}|_{\tau^{\eta x}=0} < 0$.

Proof. This result follows directly from Proposition 7. □

Corollary 3 indicates that the spread of quantitative trading strategies has non-monotonic impacts on market efficiency, the quality of strategies, and the number of developers in the long run. When each strategy is not shared with many investors, it is optimal for every investor to develop his/her own strategies. When quantitative strategies are shared with more investors, the investors can implement better statistical arbitrage, reducing mispricing and improving market efficiency. When the market becomes more efficient, the investors can profit less from trading on fundamental information, so more (less) resources are allocated to developing quantitative (fundamental) strategies, increasing the quality of quantitative (fundamental) strategies.

When each strategy can be shared with many investors, the investors do not have high incentives to develop their own strategies. Therefore, when the strategies are shared with even more investors, it is optimal for a proportion of investors not to develop their own strategies but instead rely on the strategies developed by others, so the number of developers (i.e., the proportion of informed investors in our model) decreases. As a result, the number of quantitative strategies decreases, and the statistical arbitrage implemented by the investors is less effective, which exacerbates mispricing, reducing market efficiency. Since the market efficiency decreases, the investors can profit more from trading on fundamental information, so more (less) resources are allocated to developing fundamental (quantitative) strategies, decreasing (increasing) the quality of quantitative (fundamental) strategies.

6 Conclusion

This paper presents a model of financial market in which information on both noise traders' order flows and asset fundamentals can be shared among investors. We demonstrate that when both the proportion of informed investors and the informed investors' choice between fundamental analysis and demand analysis are endogenous, the financial price informativeness and demand analysis (fundamental analysis) are hump-shaped (is V-shaped) in the network connectedness. These results still hold even if only the demand information is shared. We also find that investor social networks can generate the "future information risk" effect. An application of our model predicts that the development of online quantitative trading platforms can improve market efficiency in the short run, but have non-monotonic long-run impacts on market efficiency and the quality of quantitative strategies. Our model can be further extended to investigate questions that are not answered in this paper. For example, adding a real sector to the model (i.e., endogenizing the asset payoff) allows people to study how the sharing of information between financial investors affects firm managers' real decisions through the market feedback channel. Moreover, one can investigate the determinants of network connectedness by endogenizing investors' sharing decisions. We leave these questions for future research.

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Appendix

A Proofs of Propositions

A.1 Proof of Proposition 1

Proof. By direct calculation, the expectation of the asset payoff $\psi p_{t+1} + d_{t+1}$ conditional on the information set $\mathcal{F}_{igt}$ can be decomposed as

$$E[\psi p_{t+1}+d_{t+1}|\mathcal{F}_{igt}] = \psi\beta_{0,t+1}+\mu_d+(1+\psi\beta_{3,t+1})G(d_t-\mu_d)+(1+\psi\beta_{3,t+1})E[v_{t+1}|\mathcal{F}_{igt}]. \tag{A.1}$$

For informed investors, the conditional expectation of v_{t+1} can be calculated using Bayes' rule,

$$E[v_{t+1}|\mathcal{F}^{IN}_{igt}] = \frac{\tau^x_t x_{igt} + (\mu_t N_t - 1)\tau^{yx}_t Y^{IN,x}_{igt} + \tau^{\tilde{p}}_{igt}\tilde{p}_{igt}}{\tau_v + \tau^x_t + (\mu_t N_t - 1)\tau^{yx}_t + \tau^{\tilde{p}}_{igt}}. \tag{A.2}$$

For uninformed investors, the conditional expectation of v_{t+1} is

$$E[v_{t+1}|\mathcal{F}^{UN}_{igt}] = \frac{(\mu_t N_t)\tau^{yx}_t Y^{UN,x}_{gt} + \tau^{\hat{p}}_{igt}\hat{p}_{igt}}{\tau_v + (\mu_t N_t)\tau^{yx}_t + \tau^{\hat{p}}_{igt}}. \tag{A.3}$$

By direct calculation, the variance of the asset payoff $\psi p_{t+1} + d_{t+1}$ conditional on the information set $\mathcal{F}_{igt}$ can be decomposed as

$$\begin{aligned} Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}] &= \psi^2 Var[p_{t+1}|\mathcal{F}_{igt}] + Var[d_{t+1}|\mathcal{F}_{igt}] + 2\psi Cov[p_{t+1}, d_{t+1}|\mathcal{F}_{igt}] \\ &= \psi^2(\beta_{1,t+1}^2\tau_v^{-1} + \beta_{2,t+1}^2\tau_n^{-1}) + (1+\psi\beta_{3,t+1})^2 Var[d_{t+1}|\mathcal{F}_{igt}]. \end{aligned} \tag{A.4}$$

An informed investor's perceived variance of d_{t+1} is

$$Var[d_{t+1}|\mathcal{F}_{igt}^{IN}] = Var[v_{t+1}|\mathcal{F}_{igt}^{IN}] = \frac{1}{\tau_v + \tau_t^x + (\mu_t N_t - 1)\tau_t^{yx} + \tau_{igt}^{\tilde{p}}}. \tag{A.5}$$

An uninformed investor's perceived variance of d_{t+1} is

$$Var[d_{t+1}|\mathcal{F}_{igt}^{UN}] = Var[v_{t+1}|\mathcal{F}_{igt}^{UN}] = \frac{1}{\tau_v + (\mu_t N_t)\tau_t^{yx} + \tau_{igt}^{\hat{p}}}. \tag{A.6}$$

With the conditional variances calculated above, we can derive the expressions for ω_t^{IN} and ω_t^{UN}.

Without loss of generality, assume that the first $\mu_t N_t$ investors in each group are informed. Then the implied price function (17) can also be expressed as

$$\begin{aligned} p_t = &\frac{\omega_t^{IN}}{\omega_t^{IN} + \omega_t^{UN}} \frac{1}{r\mu_t N_t} \sum_{i=1}^{\mu_t N_t} \left(\lim_{G_t \to \infty} \frac{1}{G_t} \sum_{g=1}^{G_t} E[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}] \right) \\ &+ \frac{\omega_t^{UN}}{\omega_t^{IN} + \omega_t^{UN}} \frac{1}{r(1-\mu_t)N_t} \sum_{i=\mu_t N_t + 1}^{N_t} \left(\lim_{G_t \to \infty} \frac{1}{G_t} \sum_{g=1}^{G_t} E[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{UN}] \right) + \frac{n_{t+1} - s}{r(\omega_t^{IN} + \omega_t^{UN})}. \end{aligned} \tag{A.7}$$

Using the expression for the conditional expectations calculated above, we can show that

$$\begin{aligned} &\lim_{G_t \to \infty} \frac{1}{G_t} \sum_{g=1}^{G_t} E[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}] \\ &= C_t + \frac{1+\psi\beta_{3,t+1}}{\tau_v + \tau_t^x + (\mu_t N_t - 1)\tau_t^{yx} + \tau_{igt}^{\tilde{p}}} \lim_{G_t \to \infty} \frac{1}{G_t} \sum_{g=1}^{G_t} \left[\tau_t^x x_{igt} + (\mu_t N_t - 1)\tau_t^{yx} Y_{igt}^{IN,x} + \tau_{igt}^{\tilde{p}} \tilde{p}_{igt} \right], \end{aligned} \tag{A.8}$$

where $C_t = \psi\beta_{0,t+1} + \mu_d + (1+\psi\beta_{3,t+1})G(d_t - \mu_d)$. Recall that $x_{igt} = v_{t+1} + \varepsilon_{igt}^x$. By the Law of Large Numbers, we can calculate that

$$\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}x_{igt}=v_{t+1}+\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}\varepsilon^x_{igt}=v_{t+1}. \tag{A.9}$$

Similarly, we can show that

$$\begin{aligned}\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}Y^{IN,x}_{igt}&=\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}\left(\frac{1}{\mu_tN_t-1}\sum_{j\in I_{gt}\setminus\{i\}}y^x_{jgt}\right)\\&=v_{t+1}+\frac{1}{\mu_tN_t-1}\sum_{j\le\mu_tN_t,j\neq i}\left[\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}(\varepsilon^x_{jgt}+\eta^x_{jgt})\right]\\&=v_{t+1}.\end{aligned} \tag{A.10}$$

Moreover, we have

$$\begin{aligned}\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}\tilde{p}_{igt}&=\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}\left[v_{t+1}+\frac{\beta_{2t}}{\beta_{1t}}(n_{t+1}-E[n_{t+1}|\mathcal{F}^{IN}_{igt}])\right]\\&=v_{t+1}+\frac{\beta_{2t}}{\beta_{1t}}\left[n_{t+1}-\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}\frac{\tau^z_tz_{igt}+(\mu_tN_t-1)\tau^{yz}_tY^{IN,z}_{igt}}{\tau_n+\tau^z_t+(\mu_tN_t-1)\tau^{yz}_t}\right]\\&=v_{t+1}+\frac{\beta_{2t}}{\beta_{1t}}\left[n_{t+1}-\frac{\tau^z_tn_{t+1}+(\mu_tN_t-1)\tau^{yz}_tn_{t+1}}{\tau_n+\tau^z_t+(\mu_tN_t-1)\tau^{yz}_t}\right]\\&=v_{t+1}+\frac{\beta_{2t}}{\beta_{1t}}\left[\frac{\tau_nn_{t+1}}{\tau_n+\tau^z_t+(\mu_tN_t-1)\tau^{yz}_t}\right].\end{aligned} \tag{A.11}$$

Therefore, we can calculate that

$$\begin{aligned}\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}E[\psi p_{t+1}+d_{t+1}|\mathcal{F}^{IN}_{igt}]=C_t+\frac{1+\psi\beta_{3,t+1}}{\tau_v+\tau^x_t+(\mu_tN_t-1)\tau^{yx}_t+\tau^{\tilde{p}}_t}\times\\\left[(\tau^x_t+(\mu_tN_t-1)\tau^{yx}_t+\tau^{\tilde{p}}_t)v_{t+1}+\tau^{\tilde{p}}_t\frac{\beta_{2t}}{\beta_{1t}}\frac{\tau_nn_{t+1}}{\tau_n+\tau^z_t+(\mu_tN_t-1)\tau^{yz}_t}\right].\end{aligned} \tag{A.12}$$

We can also calculate that

$$\begin{aligned}&\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}E[\psi p_{t+1}+d_{t+1}|\mathcal{F}_{igt}^{UN}]\\&=C_t+\frac{1+\psi\beta_{3,t+1}}{\tau_v+(\mu_tN_t)\tau_t^{yx}+\tau_{igt}^{\hat{p}}}\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}\left[(\mu_tN_t)\tau_t^{yx}Y_{gt}^{UN,x}+\tau_{igt}^{\hat{p}}\hat{p}_{igt}\right],\end{aligned}\tag{A.13}$$

and that

$$\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}Y_{gt}^{UN,x}=v_{t+1}+\frac{1}{\mu_tN_t}\sum_{i=\mu_tN_t+1}^{N_t}\left[\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}(\varepsilon_{igt}^x+\eta_{igt}^x)\right]=v_{t+1},\tag{A.14}$$

and that

$$\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}\hat{p}_{igt}=v_{t+1}+\frac{\beta_{2t}}{\beta_{1t}}\left(n_{t+1}-\frac{\mu_tN_t\tau_t^{yz}n_{t+1}}{\tau_n+\mu_tN_t\tau_t^{yz}}\right)=v_{t+1}+\frac{\beta_{2t}}{\beta_{1t}}\left(\frac{\tau_nn_{t+1}}{\tau_n+\mu_tN_t\tau_t^{yz}}\right).\tag{A.15}$$

Therefore, we have

$$\begin{aligned}&\lim_{G_t\to\infty}\frac{1}{G_t}\sum_{g=1}^{G_t}E[\psi p_{t+1}+d_{t+1}|\mathcal{F}_{igt}^{UN}]\\&=C_t+\frac{1+\psi\beta_{3,t+1}}{\tau_v+(\mu_tN_t)\tau_t^{yx}+\tau_{igt}^{\hat{p}}}\left[(\mu_tN_t\tau_t^{yx}+\tau_t^{\hat{p}})v_{t+1}+\tau_t^{\hat{p}}\frac{\beta_{2t}}{\beta_{1t}}\frac{\tau_nn_{t+1}}{\tau_n+\mu_tN_t\tau_t^{yz}}\right].\end{aligned}\tag{A.16}$$

Substituting Equations (A.12) and (A.16) into Equation (A.7), and collecting terms, we have

$$
\begin{aligned}
p_t = & \frac{-s}{r(\omega_t^{IN}+\omega_t^{UN})} + \frac{1}{r}(\psi\beta_{0,t+1}+\mu_d) \\
& + \left[\frac{\omega_t^{IN}}{r(\omega_t^{IN}+\omega_t^{UN})}(1+\psi\beta_{3,t+1})V_t^{IN}(\tau^x+(\mu_t N_t-1)\tau_t^{yx}+\tau_t^{\tilde{p}})\right. \\
& \quad \left. + \frac{\omega_t^{UN}}{r(\omega_t^{IN}+\omega_t^{UN})}(1+\psi\beta_{3,t+1})V_t^{UN}(\mu_t N_t\tau_t^{yx}+\tau_t^{\hat{p}})\right] v_{t+1} \\
& + \left[\frac{\omega_t^{IN}}{r(\omega_t^{IN}+\omega_t^{UN})}(1+\psi\beta_{3,t+1})V_t^{IN}\tau_t^{\tilde{p}}\frac{\beta_{2,t}}{\beta_{1,t}}\frac{\tau_n}{\tau_n+(\mu_t N_t-1)\tau_t^{yz}+\tau^z}\right. \\
& \quad \left. + \frac{\omega_t^{UN}}{r(\omega_t^{IN}+\omega_t^{UN})}(1+\psi\beta_{3,t+1})V_t^{UN}\tau_t^{\hat{p}}\frac{\beta_{2,t}}{\beta_{1,t}}\frac{\tau_n}{\tau_n+\mu_t N_t\tau_t^{yz}}\right] n_{t+1} \\
& + \left[\frac{G_d}{r-\psi G_d}\right](d_t-\mu_d),
\end{aligned}
\tag{A.17}
$$

where $V_t^{IN} = Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}]$ and $V_t^{UN} = Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{UN}]$. Comparing the implied price function (A.17) and the conjectured price function (13), we can derive Equation (18), which completes the proof. □

A.2 Proof of Proposition 2

Proof. In the static case we have $G_d = \mu_d = \psi = 0$. From Proposition 1 we know that

$$
\frac{\beta_1}{\beta_2} = \frac{\mu\tau_x + \mu(N-1)\tau^{yx} + \mu\tau^{\tilde{p}} + (1-\mu)\tau^{\hat{p}}}{\mu\tau^{\tilde{p}}\frac{\beta_2}{\beta_1}\frac{\tau_n}{\tau_n+\tau^z+(\mu N-1)\tau^{yz}} + (1-\mu)\tau^{\hat{p}}\frac{\beta_2}{\beta_1}\frac{\tau_n}{\tau_n+\mu N\tau^{yz}} + \gamma}.
$$

Rearranging terms, we have

$$
[\mu\tau^z + \mu(N-1)\tau^{yz}](\tfrac{\beta_1}{\beta_2})^2 - \gamma(\tfrac{\beta_1}{\beta_2}) + \gamma[\mu\tau^x + \mu(N-1)\tau^{yx}] = 0. \tag{A.18}
$$

Solving this equation of $\frac{\beta_1}{\beta_2}$, and following Farboodi and Veldkamp (2020) to choose from the two solutions, we can derive (20).

Notice that $\frac{\beta_1}{\beta_2}$ can be viewed as a function of N. Taking derivative with respect to N on both sides of (A.18), we have $\mu[\tau^{yz}(\frac{\beta_1}{\beta_2})^2 + (\tau^z+(N-1)\tau^{yz})2(\frac{\beta_1}{\beta_2})\frac{\partial(\beta_1/\beta_2)}{\partial N}] - \gamma\frac{\partial(\beta_1/\beta_2)}{\partial N} + \mu\tau^{yx} = 0.$

Collecting terms, we have

$$\frac{\partial(\beta_1/\beta_2)}{\partial N} = \frac{\mu\tau^{yz}(\frac{\beta_1}{\beta_2})^2 + \mu\tau^{yx}}{\gamma - 2\mu(\tau^z + (N-1)\tau^{yz})(\frac{\beta_1}{\beta_2})}. \tag{A.19}$$

From (20) we can calculate that

$$2\mu(\tau^z + (N-1)\tau^{yz})(\tfrac{\beta_1}{\beta_2}) = \gamma - \sqrt{\gamma^2 - 4\mu^2[\tau^z + (N-1)\tau^{yz}][\tau^x + (N-1)\tau^{yx}]} < \gamma.$$

Therefore, we have $\frac{\partial(\beta_1/\beta_2)}{\partial N} > 0$. □

A.3 Proof of Proposition 3

Proof. Denote $\Omega_x = \tau^x + (N-1)\tau^{yx}$ and $\Omega_z = \tau^z + (N-1)\tau^{yz}$. Then from Eq. (20) the price informativeness β_1/β_2 can be written as

$$\frac{\beta_1}{\beta_2} = \frac{\gamma - \sqrt{\gamma^2 - 4\mu^2\Omega_z\Omega_x}}{2\mu\Omega_z} = \frac{2\mu\Omega_x}{\gamma + \sqrt{\gamma^2 - 4\mu^2\Omega_z\Omega_x}}. \tag{A.20}$$

Notice that $\frac{\partial(\beta_1/\beta_2)}{\partial\Omega_z} > 0$. Moreover, since $\tau^{yz} = [(\tau^z)^{-1} + (\tau^{\eta z})^{-1}]^{-1}$, we have $\frac{\partial\tau^{yz}}{\partial\tau^z} > 0$ and thus $\frac{\partial\Omega_z}{\partial\tau^z} > 0$. Therefore, by the chain rule, we have

$$\frac{\partial(\beta_1/\beta_2)}{\partial\tau^z} = \frac{\partial(\beta_1/\beta_2)}{\partial\Omega_z}\frac{\partial\Omega_z}{\partial\tau^z} > 0.$$

Combining this result with Eq. (A.19), we immediately have $\frac{\partial^2(\beta_1/\beta_2)}{\partial N\partial\tau^z} > 0$. □

A.4 Proof of Proposition 4

Proof. If an investor employs the optimal trading strategy (16), then her consumption can be written as

$$c_{ig,t+1} = e_{igt}r + \frac{E[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}] - rp_t}{\gamma Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}]}(d_{t+1} + \psi p_{t+1} - rp_t) - \mathbb{1}_{\{i\in I_{gt}\}}C_F, \tag{A.21}$$

and her expected utility conditional on $\mathcal{F}_{igt}$ is thus (by the moment generating function of a normal random variable)

$$E[U(c_{ig,t+1})|\mathcal{F}_{igt}] = -e^{-\gamma[e_{igt}r - \mathbb{1}_{\{i\in I_{gt}\}}C_F] - \frac{1}{2}\frac{(E[\psi p_{t+1}+d_{t+1}-rp_t|\mathcal{F}_{igt}])^2}{Var[\psi p_{t+1}+d_{t+1}|\mathcal{F}_{igt}]}}. \tag{A.22}$$

Therefore, the investor's ex ante expected utility is (by the moment generating function of non-central chi-squared random variable)

$$E[E[U(c_{ig,t+1})|\mathcal{F}_{igt}]|\{d_s\}_{s\leq t}] = -\sqrt{\frac{V_t}{V_t+V_{0t}}}e^{-\gamma[e_{igt}r - \mathbb{1}_{\{i\in I_{gt}\}}C_F] - \frac{1}{2}\frac{(E[\psi p_{t+1}+d_{t+1}-rp_t|\{d_s\}_{s\leq t}])^2}{V_t+V_{0t}}}, \tag{A.23}$$

where $V_t = Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}]$ and $V_{0t} = Var[E[\psi p_{t+1} + d_{t+1} - rp_t|\mathcal{F}_{igt}]|\{d_s\}_{s\leq t}]$. Notice that

$$V_t + V_{0t} = V_t^o = Var[\psi p_{t+1} + d_{t+1} - rp_t|\{d_s\}_{s\leq t}]. \tag{A.24}$$

Also notice that (by the Law of Iterated Expectations),

$$E[U(c_{ig,t+1})|\{d_s\}_{s\leq t}] = E[E[U(c_{ig,t+1})|\mathcal{F}_{igt}]|\{d_s\}_{s\leq t}]. \tag{A.25}$$

We can calculate that

$$\ln\left(-E[U(c_{ig,t+1})|\{d_s\}_{s\leq t}]\right) = \ln\sqrt{\frac{V_t}{V_t^o}} - \gamma[e_{igt}r - \mathbb{1}_{\{i\in I_{gt}\}}C_F] - \frac{1}{2}\frac{(E[\psi p_{t+1}+d_{t+1}-rp_t|\{d_s\}_{s\leq t}])^2}{V_t^o}. \tag{A.26}$$

Notice that both V_t^o and $E[\psi p_{t+1} + d_{t+1} - rp_t|\{d_s\}_{s\leq t}]$ are not affected by a specific investor's information choice. Therefore, for a specific informed investor, the information choice problem (21) can be converted to minimizing $V_t^{IN} = Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}]$. Recall that

$$\begin{aligned}&Var[\psi p_{t+1} + d_{t+1}|\mathcal{F}_{igt}^{IN}]\\ =\ &\psi^2(\beta_{1,t+1}^2\tau_v^{-1} + \beta_{2,t+1}^2\tau_n^{-1}) + (1+\psi\beta_{3,t+1})^2(\tau_v + \tau_{igt}^x + (\mu_t N_t - 1)\tau_t^{yx} + \tau_{igt}^{\tilde{p}})^{-1},\end{aligned} \tag{A.27}$$

where $\tau_{igt}^{\tilde{p}} = (\frac{\beta_{1t}}{\beta_{2t}})^2(\tau_n + \tau_{igt}^z + (\mu_t N_t - 1)\tau_t^{yz})$. Since the specific investor can only choose τ_{igt}^x and τ_{igt}^z, the information choice problem can be expressed as

$$\max_{\tau_{igt}^x, \tau_{igt}^z} \tau_{igt}^x + (\frac{\beta_{1t}}{\beta_{2t}})^2 \tau_{igt}^z, \quad \text{s.t. } (\tau_{igt}^x)^2 + \chi(\tau_{igt}^z)^2 \leq H_t. \tag{A.28}$$

Since the objective function is increasing in both choice variables, we know that the constraint is binding. The Lagrangian function is

$$L_t = \tau_{igt}^x + (\frac{\beta_{1t}}{\beta_{2t}})^2 \tau_{igt}^z + \lambda_t(H_t - (\tau_{igt}^x)^2 - \chi(\tau_{igt}^z)^2), \tag{A.29}$$

where λ_t is the Lagrangian multiplier. The first-order conditions give the following system of equations,

$$\frac{\partial L_t}{\partial \tau_{igt}^x} = 1 - 2\lambda_t \tau_{igt}^x = 0, \ \frac{\partial L_t}{\partial \tau_{igt}^z} = \left(\frac{\beta_{1t}}{\beta_{2t}}\right)^2 - 2\chi\lambda_t \tau_{igt}^z = 0, \ \frac{\partial L_t}{\partial \lambda_t} = H_t - (\tau_{igt}^x)^2 - \chi(\tau_{igt}^z)^2 = 0. \tag{A.30}$$

Solving this system of equations for τ_{igt}^x, τ_{igt}^z, and λ_t, and using the symmetric equilibrium conditions (i.e., $\tau_{igt}^x = \tau_t^x$ and $\tau_{igt}^z = \tau_t^z$, $\forall i, g$), we can derive Equation (22), which completes the proof. □

A.5 Proof of Proposition 5

Proof. Recall that the equilibrium price informativeness β_1/β_2 is determined by (A.20). Notice that when the fundamental analysis and the demand analysis are endogenously determined by the investors, both $\Omega_x = \tau^x + (N-1)\tau^{yx}$ and $\Omega_z = \tau^z + (N-1)\tau^{yz}$ are functions of β_1/β_2 and N (see Eq. (22)). Denote $\xi = \beta_1/\beta_2$. Then Eq. (A.20) can be written as $\xi = f(\xi, N)$, where

$$f(\xi, N) = \frac{2\mu\Omega_x(\xi, N)}{\gamma + \sqrt{\gamma^2 - 4\mu^2\Omega_z(\xi, N)\Omega_x(\xi, N)}}. \tag{A.31}$$

It is obvious that $\frac{\partial\Omega_x(\xi,N)}{\partial N} > 0$ and $\frac{\partial\Omega_z(\xi,N)}{\partial N} > 0$. Therefore, we can notice that $\frac{\partial f(\xi,N)}{\partial N} > 0$. Moreover, from Eq. (22), we can calculate that $\tau^x|_{\xi=0} = \sqrt{H} > 0$ and $\tau^z|_{\xi=0} = 0$, so we know that $f(0, N) > 0$.

Let $N_1 \geq 0$, and $\Delta N > 0$. Denote $N_2 = N + \Delta N > N_1$. Let $\xi(N_2)$ be the solution to $\xi = f(\xi, N_2)$. Since $\frac{\partial f(\xi,N)}{\partial N} > 0$, we have $f(\xi(N_2), N_1) < f(\xi(N_2), N_2) = \xi(N_2)$, so $f(\xi(N_2), N_1) - \xi(N_2) < 0$. Recall that $f(0, N_1) - 0 > 0$. Therefore, by the intermediate value theorem, we know that there exists a $\xi(N_1) \in (0, \xi(N_2))$, such that $f(\xi(N_1), N_1) - \xi(N_1) = 0$, or equivalently, $\xi(N_1) = f(\xi(N_1), N_1)$.

Now we have already proved that $\xi(N_1 + \Delta N) > \xi(N_1)$, where $\xi(N)$ is the solution to $\xi = f(\xi, N)$. Since the selection of $\Delta N > 0$ is arbitrary, we have

$$\xi'(N_1) = \lim_{\Delta N \to 0} \frac{\xi(N_1 + \Delta N) - \xi(N_1)}{\Delta N} > 0. \tag{A.32}$$

Since the selection of N_1 is also arbitrary, we know that $\xi'(N) > 0$, $\forall N \geq 0$. In other words, $\frac{\partial(\beta_1/\beta_2)}{\partial N} > 0$. With this result and Eq. (22), one can immediately observe that $\frac{\partial \tau^x}{\partial N} < 0$ and $\frac{\partial \tau^z}{\partial N} > 0$. $\square$

A.6 Proof of Proposition 6

Proof. The results when $\Delta U_t(0) \leq 0$ and $\Delta U_t(1) \geq 0$ are trivial. Now we consider the situation when $\Delta U_t(0) > 0$ and $\Delta U_t(1) < 0$. Denote

$$\phi_t = -\gamma e_{igt} r - \frac{1}{2} \frac{(E[\psi p_{t+1} + d_{t+1} - r p_t | \{d_s\}_{s \leq t}])^2}{V_t^o}. \tag{A.33}$$

From Equation (A.23) we know that, given μ_t, an investor's ex ante expected utility can be written as

$$E[E[U(c_{ig,t+1})|\mathcal{F}_{igt}]|\{d_s\}_{s \leq t}]|_{\mu_t} = -\sqrt{\frac{V_t}{V_t^o}} e^{\phi_t + \gamma \mathbb{1}_{\{i \in I_{gt}\}} C_F}. \tag{A.34}$$

Therefore, we have

$$\Delta U_t(\mu_t) = -\sqrt{\frac{V_t^{IN}}{V_t^o}} e^{\phi_t + \gamma C_F} + \sqrt{\frac{V_t^{UN}}{V_t^o}} e^{\phi_t} = \sqrt{\frac{V_t^{IN}}{V_t^o}} e^{\phi_t} \left(\sqrt{\frac{V_t^{UN}}{V_t^{IN}}} - e^{\gamma C_F} \right). \tag{A.35}$$

By inspection, we have

$$\Delta U_t(\mu_t) = 0 \Leftrightarrow \sqrt{\frac{V_t^{UN}}{V_t^{IN}}} = e^{\gamma C_F}, \tag{A.36}$$

which completes the proof. $\square$

A.7 Proof of Proposition 7

Proof. **Part (a):** As has been shown in Proposition 6, when $\Delta U(1) > 0$, the proportion of informed investors is $\mu = 1$. Therefore, as long as N is within the range such that $\Delta U(1) > 0$, μ is fixed at 1, so the impacts of N on equilibrium variables are the same as those when μ is exogenously set to be 1. From Proposition 5 we know that $\frac{\partial(\beta_1/\beta_2)}{\partial N} > 0$, $\frac{\partial \tau^x}{\partial N} < 0$, and $\frac{\partial \tau^z}{\partial N} > 0$.

Part(b): Now we consider the situation in which $\Delta U(1) < 0$ and $\Delta U(0) > 0$. Denote $\xi = \beta_1/\beta_2$. From Eq. (25), we can notice that when $\psi = 0$, the equilibrium proportion of informed investors μ must satisfy the following equation,

$$\sqrt{\frac{\tau_v + \tau^x + (\mu N - 1)\tau^{yx} + \xi^2(\tau_n + \tau^z + (\mu N - 1)\tau^{yz})}{\tau_v + \mu N \tau^{yx} + \xi^2(\tau_n + \mu N \tau^{yz})}} = e^{\gamma C_F}, \tag{A.37}$$

Therefore, we can express the proportion of informed traders as a function of ξ and N,

$$\mu = \mu(\xi, N) = \frac{[\tau^x - \tau^{yx} + \xi^2(\tau^z - \tau^{yz})] - (e^{2\gamma C_F} - 1)(\tau_v + \xi^2 \tau_n)}{N(e^{2\gamma C_F} - 1)(\tau^{yx} + \xi^2 \tau^{yz})}. \tag{A.38}$$

Notice that τ^x, τ^{yx}, τ^z, and τ^{yz} are also functions of ξ, but are not directly affected by N. From Eq. (A.20) we know that the equilibrium price informativeness is determined by the equation $\xi = h(\xi, N)$, where

$$h(\xi, N) = \frac{2\mu(\xi, N)\Omega_x(\xi, N)}{\gamma + \sqrt{\gamma^2 - 4[\mu(\xi, N)\Omega_z(\xi, N)][\mu(\xi, N)\Omega_x(\xi, N)]}}. \tag{A.39}$$

We can calculate that

$$\begin{aligned} &\mu(\xi, N) \times \Omega_x(\xi, N) \\ &= \underbrace{\frac{[\tau^x - \tau^{yx} + \xi^2(\tau^z - \tau^{yz})] - (e^{2\gamma C_F} - 1)(\tau_v + \xi^2 \tau_n)}{(e^{2\gamma C_F} - 1)(\tau^{yx} + \xi^2 \tau^{yz})}}_{\text{Not affected by } N} \times \underbrace{\left[\frac{1}{N}(\tau^x - \tau^{yx}) + \tau^{yx}\right]}_{\text{Decreasing in } N}. \end{aligned} \tag{A.40}$$

Recall that $\tau^{yx} = [(\tau^x)^{-1} + (\tau^{\eta x})^{-1}]^{-1}$, so we can notice that $\tau^x > \tau^{yx}$. Therefore, we can see that $\frac{\partial[\mu(\xi,N)\Omega_x(\xi,N)]}{\partial N} < 0$. Similarly, we can prove that $\frac{\partial[\mu(\xi,N)\Omega_z(\xi,N)]}{\partial N} < 0$. Therefore, we have $\frac{\partial h(\xi,N)}{\partial N} < 0$.

Now we prove that $h(0,N) > 0$, $\forall N \geq 1$. Suppose that μ is fixed at 0. Then from Eq. (A.20) we know that the equilibrium price informativeness is $\frac{\beta_1}{\beta_2} = 0$. From Eq. (22) we know that the equilibrium information choices are $\tau^x = \sqrt{H}$ and $\tau^z = 0$. Correspondingly, we have $\tau^{yx} = [(\sqrt{H})^{-1} + (\tau^{\eta x})^{-1}]^{-1}$. By assumption, we have $\Delta U(0) > 0$, i.e., $V^{UN}/V^{IN} > e^{2\gamma C_F}$, which leads to the following relationship between exogenous parameters,

$$\sqrt{H} - [(\sqrt{H})^{-1} + (\tau^{\eta x})^{-1}]^{-1} > (e^{2\gamma C_F} - 1)\tau_v. \tag{A.41}$$

Notice that $\mu(0,N)$ can be expressed as

$$\mu(0,N) = \frac{\sqrt{H} - [(\sqrt{H})^{-1} + (\tau^{\eta x})^{-1}]^{-1} - (e^{2\gamma C_F} - 1)\tau_v}{N(e^{2\gamma C_F} - 1)[(\sqrt{H})^{-1} + (\tau^{\eta x})^{-1}]^{-1}}. \tag{A.42}$$

Therefore, by Eq. (A.41), we know that $\mu(0,N) > 0$, and thus $h(0,N) > 0$, $\forall N \geq 1$.

Now we prove that $\xi(N)$ is decreasing in N, where $\xi(N)$ is the solution to equation $\xi = h(\xi,N)$. Let $N_2 > N_1 \geq 1$. Let $\xi(N_1)$ be the solution to equation $\xi = h(\xi,N_1)$. Since $\frac{\partial h(\xi,N)}{\partial N} < 0$, we know that $h(\xi(N_1),N_2) < h(\xi(N_1),N_1) = \xi(N_1)$, so $h(\xi(N_1),N_2) - \xi(N_1) < 0$. Recall that we have proved that $h(0,N_2) - 0 > 0$. Therefore, by the intermediate value theorem, there exists a $\xi(N_2) \in (0,\xi(N_1))$, such that $h(\xi(N_2),N_2) - \xi(N_2) = 0$. In other words, $\xi(N_2)$, which is the solution to $\xi = h(\xi,N_2)$, is less than $\xi(N_1)$. Since the selection of $N_2 > N_1 \geq 1$ is arbitrary, we know that $\xi(N)$ is decreasing in N, i.e., $\xi'(N) < 0$. With this result and Eq. (22), we can derive that $\frac{\partial \tau^x}{\partial N} > 0$ and $\frac{\partial \tau^z}{\partial N} < 0$. $\square$