Liability-Driven Investors and Monetary Transmission*

Giacomo Carboni[†] European Central Bank

Artur Doshchyn[‡] University of Bristol

Martin Ellison[§] University of Oxford and CEPR

February 1, 2025

Abstract

The ability of monetary policy to influence the term structure of interest rates and the macroeconomy depends on the extent to which financial market participants prefer to hold bonds of different maturities. We introduce such preferred-habitat demand in a fully-specified dynamic stochastic general equilibrium model of the macroeconomy where the term structure is arbitrage-free. The source of preferred habitat demand is an insurance fund that issues annuities and adopts a liability-driven investment strategy to minimise the duration risk on its balance sheet. The behaviour of the insurance fund implies a liability-driven demand function that is upward-sloping in bond prices and downward-sloping in bond yields, especially when interest rates are low. This supports the operation of a recruitment channel at low interest rates, whereby long-term interest rates react strongly to short-term policy rates or unconventional policy because of complementary changes in term premia induced by liability-driven demand. The strong reaction and enhanced monetary transmission extend to inflation and output in general equilibrium.

Keywords: general equilibrium, interest rates, liability-driven investment, preferred habitat, term structure

JEL codes: E43, E44, E52, G21, G22

*The opinions in this paper are those of the authors and do not necessarily reflect the views of the European Central Bank or the Eurosystem. We thank seminar participants at ESSIM, Bank of Chile, Bank of Finland, European Central Bank, Birkbeck, Saïd Business School, Sveriges Riksbank, Keio and Surrey. Charles Brendon, Christoph Görtz, Refet Gürkaynak, Rustam Jamilov, Burçin Kısacıkoğlu, Sang Seok Lee, Alistair Macaulay, Roberto Motto, Andrew Scott, Andreas Tischbirek and Dimitri Vayanos provided helpful comments and suggestions. Arian Kharrazi, Chenchuan Shi and Christopher McGann were excellent research assistants.

[†]European Central Bank, Frankfurt am Main, Germany; email: giacomo.carboni@ecb.europa.eu

[‡]University of Bristol Business School, United Kingdom; email: artur.doshchyn@bristol.ac.uk

[§]Department of Economics, University of Oxford, United Kingdom; email: martin.ellison@economics.ox.ac.uk

1 Introduction

Do the strategic responses of private investors dampen or amplify the impact of monetary policy on the yield curve through to the macroeconomy? Vayanos and Vila (2021) argue for dampening, because preferred-habitat investors reduce their demand for long bonds after a cut in short rates, easing pressure on long bond prices, widening term premia, and long-term yields react less than prescribed by the expectations hypothesis. By contrast, Hanson and Stein (2015) stress amplification through the presence of yield-oriented investors, who allocate their portfolios according to the yield spread between short and long bonds. A relative cut in short-rates hence stimulates preferred-habitat demand for long bonds, bidding up their price, compressing long-term premia, and long-term rates fall even further than predicted by the expectations hypothesis.

We scrutinise the conflicting views of the monetary transmission mechanism in Hanson and Stein (2015) and Vayanos and Vila (2021) by building a general equilibrium model of liability-driven investors. Informing our modelling choices is Figure 1, which uses data from Securities Holding Statistics to display scatter plots for the relationship between long bond prices and the share of long bonds held, for Euro Area insurance companies and pension funds on the left and Euro Area monetary financial institutions on the right. The sample is 2014q2 to 2021q4, with bond prices on the vertical axis referring to Euro Area GDP-weighted 10-year sovereigns and long bonds defined as securities with maturity longer than one year. The contrasting patterns suggests that the objectives of insurance companies and pension funds are fundamentally different from those of monetary financial institutions. In the recent era of low interest rates, it appears that the long bond holdings of insurance companies and pension funds are upward-sloping in bond prices and downward-sloping in bond yields, whereas the long bond holdings of monetary financial institutions slope in the opposite direction.

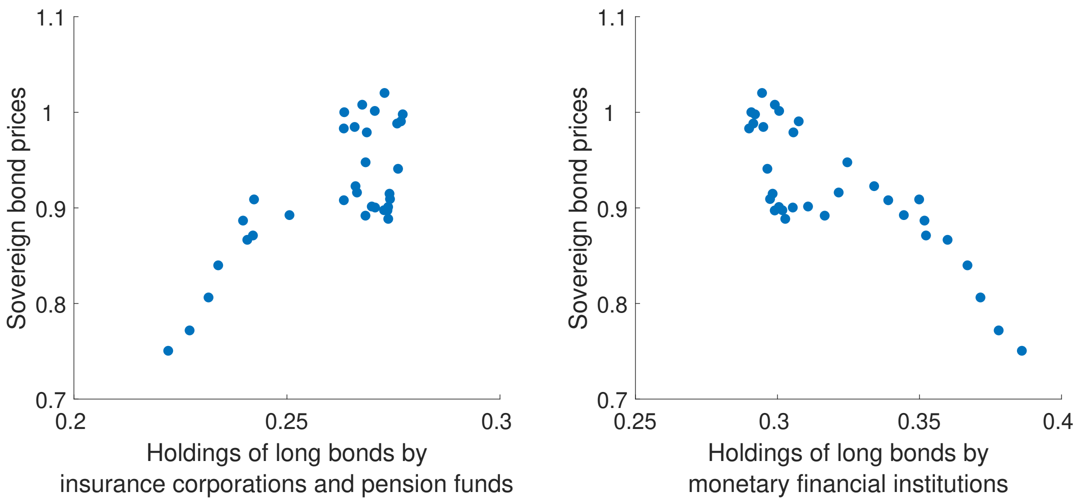

Figure 1: Long bond prices and share of long bonds held

The liability-driven behaviour of insurance companies and pension funds can be rationalised by appealing to the long-term nature of their liabilities and a desire to manage duration risk. The market value of long-term liabilities is more sensitive to market conditions than that of short-term liabilities, so the balance sheets and solvency of insurance companies and pension funds are more exposed to fluctuations in interest rates than those of monetary financial institutions. Insurance companies and pension funds then manage duration risk by holding some of their assets in the form of government bonds, since the market prices of bonds are also sensitive to fluctuations in interest rates. By holding a judicious portfolio of bonds, insurance companies and pension funds can partially immunise their balance sheets and ensure that movements in the market value of their liabilities are partly offset by corresponding changes in the market value of their assets.[1] The lower are interest rates the more sensitive liabilities are to market conditions, so the more bonds that insurance companies and pension funds have to hold to manage duration risk, and the more upward-sloping their demand is in bond prices. In contrast, the short-term nature of liabilities on the balance sheets of monetary financial institutions creates little duration risk and mutes their incentive to increase bond holdings as interest rates fall.

We adopt the "hunt for duration" approach of Domanski et al. (2017) to model the liability-driven investments of insurance companies and pension funds. There, the lower the interest rate the higher the sensitivity of long-term liabilities to interest rate risk, and the greater the demand for long bonds to increase exposure to duration risk on the asset side of the balance sheet. More generally, there are strong empirical and theoretical precedents for believing that demand may be upward-sloping in bond yields for at least some investors. Hanson et al. (2021) attribute this to some combination of yield-oriented investors who refinance mortgage activity, extrapolate recent changes in short rates, or reach for yield when short rates fall.

The management of duration risks in Domanski et al. (2017) creates a liability-driven demand for government bonds that is of the same form as the preferred-habitat demand in Vayanos and Vila (2021). However, in combining these contributions we uncover a new and nuanced interpretation of preferred-habitat demand that provides two significant and important caveats to previous understanding.

(i) The liability-driven investment strategy implied by the hunt for duration suggests that preferred-habitat demand should be thought of as upward-sloping in prices and downward-sloping in yields, especially when interest rates are low, and that the strategic responses of private investors amplify the impact of monetary policy. This contrasts with Vayanos and Vila (2021), who use an OLG model to justify their assumption that the slope of preferred-habitat demand has the opposite sign and that monetary policy actions are dampened rather than amplified by the actions of private investors. It is difficult to construct other examples that support the Vayanos and Vila (2021) interpretation, primarily because preferred-habitat demand is defined in terms of how the market value of bond holdings reacts to a change in bond prices. If bond prices fall then the market value of bond holdings is automatically reduced, at least before any portfolio reallocation. Unless there is a very large offsetting increase in bond holdings, the total market value must fall and preferred-habitat demand will be upward-sloping in prices.

(ii) What matters in Vayanos and Vila (2021) is the total exposure to interest rate risk of arbitrageurs, which we think of in terms of monetary financial institutions. Total exposure in turn depends on the specifics of bond supply and the holdings of preferred-habitat investors, which we equate to insurance

[1] Immunisation is only partial because the only way to completely immunise a long-term liability is to hold a long-term asset with exactly the same cash flow profile.

companies and pension funds. Preferred-habitat demand for bonds is therefore important in terms of defining the residual duration risk that is borne by arbitrageurs, rather than as a demand *per se* for bonds of particular maturities. As we shall see, alternative portfolios of different maturity bonds that are equally good at immunising the balance sheets of insurance companies and pension funds have exactly the same implications for the transmission of monetary policy, since they leave arbitrageurs who price the yield curve with the same residual exposure to duration risk.

The first caveat prompts a re-assessment of how private investors affect the monetary policy transmission mechanism. In Vayanos and Vila (2021), a decline in short-term interest rates bids up bond prices and reduces the demand for long bonds by preferred-habitat investors because of their demand being downward-sloping in bond prices. In equilibrium, the shortfall in demand is met by a fall in the prices of long bonds that incentivises arbitrageurs to hold larger quantities of these bonds. This results in a widening in the term premia that compensate arbitrageurs for holding additional duration risk, and a partial offsetting of the initial decline in the short-term rate. The implication is that the impact of monetary policy is dampened, with forward rates underreacting to the short-term rate and the decline in long rates after an interest rate cut being less than expected because of the higher risk premia.

All this is reversed if preferred-habitat demand is upward-sloping. The impact of monetary policy is amplified in such an economy, where we find that a decline in the short-term interest rate still bids up bond prices but the upward-sloping demand of liability-driven investors means that they increase their demand for long-term bonds. Equilibrium now requires arbitrageurs to hold smaller quantities of these bonds, which implies a narrowing in term premia and an amplification of the reaction of forward rates to a cut in the short-term rate. This is an example of what Stein (2013) terms a "recruitment channel" – by stimulating the demand of liability-driven investors, a reduction in the short-term interest rate reduces the need for arbitrageurs to absorb duration risk and so generates a fall in term premia. Forward rates now overreact to the short-term rate because any changes are compounded by complementary movements in term premia; the policymaker has "recruited" liability-driven investors to help move interest rates in the desired direction. The role of arbitrageurs is crucial here, as in Vayanos and Vila (2021), since they are the residual holders of duration risk and the transmitter of demand pressure from liability-driven investors to the entire term structure. Arbitrageurs ensure that the market price of risk is unique and that the term structure is arbitrage-free, with bonds most exposed to interest rate risk experiencing the largest changes in yields.

The amplification of monetary policy through liability-driven demand being upward-sloping is consistent with a growing body of empirical evidence that documents how long-term nominal interest rates are highly sensitive to movements in short-term rates (Gürkaynak et al., 2005; Giglio and Kelly, 2018; Hanson and Stein, 2015; Hanson and Stein, 2015. It also agrees with accepted wisdom that excess sensitivity in the post 2000 period largely reflects movements in long-term real rates (Beechey and Wright, 2009) and primarily operates through the real term premium (Hanson and Stein, 2015).

Our analysis improves upon Vayanos and Vila (2021) in two complementary respects. Firstly, and as noted above, we use the framework of Domanski et al. (2017) to endogenise the slope of preferred-habitat demand

and show how it becomes steeper when interest rates are low, drawing out implications for the transmission of interest rate policy to the yield curve and the macroeconomy. This rate-amplifying effect has the potential to alleviate limitations faced by interest rate and forward guidance policies in an environment of low equilibrium real interest rates and the proximity of nominal rates to their lower bounds. Secondly, we embed our liability-driven investors and their interactions with arbitrageurs in a fully-specified general equilibrium framework with an affine term structure of interest rates. In doing so, we find that the pricing decisions of insurance companies and pension funds further amplify the reaction of the yield curve to monetary policy actions. We show that the term premia induced by liability-driven demand enter the consumption Euler equation, which implies greater macroeconomic volatility at low interest rates when preferred-habitat demand is upward-sloping in prices. The behaviour of liability-driven investors therefore leads to even greater amplification of monetary policy and a stronger monetary transmission mechanism in general equilibrium.

The paper is organised as follows. Section 2 relates our contribution to the existing literature. In Section 3 we present our model of financial intermediation, whereby a financial intermediary allocates funds between risk-averse arbitrageurs and an insurance fund that follows a liability-driven investment strategy. Section 4 solves for the affine term structure of interest rates in bond market equilibrium. The implications of the term structure for the return to saving are explored in Section 5, allowing Section 6 to develop the general equilibrium implications through the addition of a representative household that decides consumption, savings and labour supply, firms that decide on production and labour demand, and a rule for monetary policy that determines the short-term interest rate. A final section discusses the empirical relevance of our findings.

2 Related literature

This paper contributes to a growing literature investigating the relationship between asset pricing and changes in the quantity and composition of financial assets held by investors. The mechanisms in play reflect the original intuition of Tobin (1958) and Modigliani and Sutch (1966), with recent contributions offering formulations in which investors have preferences for specific segments and/or operate under limited risk-bearing capacity.

Vayanos and Vila (2021) develop an arbitrage-free framework in which the term structure of interest rates is the result of interactions between preferred-habitat investors and risk-averse arbitrageurs. The preferred-habitat investors have clientele demand over specific maturities, whereas the arbitrageurs engage in carry trade activity and require compensation for exposure to interest rate changes ("duration risk"). One implication is that long rates underreact to changes in short rates, an effect that originates from the assumption that preferred-habitat investors have demand curves that slope downwards in bond prices and upwards in bond yields. A cut in short interest rates reduces preferred-habitat demand at all maturities, which leaves more duration risk to be held by arbitrageurs who then require higher term premia as compensation. The rise in term premia partially offsets the initial cut in interest rates, leading to long interest rates underreacting. Hanson and Stein (2015) question this result by drawing attention to the surprisingly strong effects that monetary policy has on forward real rates in the distant future. Their explanation is that "yield-oriented" investors allocate their portfolios according to the spread between the yields on short and long bonds. A cut in the short rate tends to reduce short rates more than long rates, so stimulates preferred-habitat demand for long bonds. Long bond yields fall still further and so overreact to the initial interest rate cut.

These conflicting perspectives motivate our model of investor behaviour, which endogenises preferred-habitat demand by modelling their decisions in terms of the liability-driven investment strategy in Domanski et al. (2017). The key determinant of liability-driven demand is the duration mismatch between assets and liabilities, which implies a demand curve that is upward-sloping in bond prices and downward-sloping in bond yields. The reason is that the lower are interest rates the greater the duration risk in liabilities than in assets, which means more assets are need to contain duration gaps. Our framework casts this strategy in a model with arbitrageurs and an arbitrage-free term structure. We find that long rates overreact to short rates, because the response of liability-driven demand changes term premia in a direction that amplifies the initial movement in the short rate. This is particularly so when interest rates are already low.

Our results are consistent with Hanson and Stein (2015), Domanski et al. (2017), and several papers in the monetary economic literature. For example, Gilchrist et al. (2015) provide evidence that most of the effect of conventional monetary policy on long-term real borrowing rates is through the reaction of term premia. Gertler and Karadi (2013) augment a standard vector autoregression analysis of conventional monetary policy by incorporating data on the high frequency response of interest rates to policy shocks, finding that shocks have significant effects on the real cost of long-term credit primarily because of changes in term premia and credit spreads, factors that are omitted from standard models of the monetary transmission mechanism. In a model with risk-averse arbitrageurs, King (2019) shows that shocks to the anticipated path of short rates lead to a contraction in term premia when nominal rates are close to the effective lower bound, a result that rests on expectations of reduced interest rate volatility if rates are constrained for a long period. Hanson et al. (2021) add that the excess sensitivity of long rates to movements in short rates appears to be frequency-dependent, as it has strengthened at high frequencies and weakened at low frequencies since 2000. They attribute the presence of rate-amplifying demand to mortgage-refinancing activity, investors extrapolating recent changes in short rates, and investors who "reach for yield" when short rates fall. The claim is that the recruitment channel may be weaker than Stein (2013) suggested, since some of the increase in sensitivity at high frequencies is transitory and likely to be contaminated by macroeconomic effects.

In considering demand-based mechanisms in general equilibrium, our paper relates to the literature investigating the macroeconomic implications of central bank asset purchase programmes. Our work is similar to Gertler and Karadi (2011, 2013) and Carlstrom et al. (2017) in restricting arbitrage across market segments, although their focus is on private assets rather than long-term sovereign bonds. We focus on term premia in bond markets, a perspective we share with Andrés et al. (2004), Chen et al. (2012) and Ellison and Tischbirek (2014). They develop DSGE models with market segmentation between short and long term bonds, capturing the view that some investors are willing to pay a premium for bonds of specific maturities. This results in bond supply mattering, although the assumption that bond markets are segmented means that the term structure in their models is not arbitrage-free. We avoid this problem by allowing our liability-driven investors to interact with arbitrageurs, which ensures that the term structure in our model is arbitrage-free in the sense that risk is priced consistently across bonds with different maturities. A similar approach is taken by King (2022) and Ray (2019), albeit with the latter in a setting where the demand curves of preferred-habitat investors slope downwards in bond prices by assumption, as in Vayanos and Vila (2021).

3 Financial intermediation

This section introduces our model of financial intermediation, which captures the interactions between financial intermediaries, insurance funds, arbitrageurs and government bond supply. We describe the behaviour of each of these financial market agents, before turning to the determination of equilibrium. A schematic overview of the model is in Appendix A.

3.1 Financial intermediaries

The financial intermediary receives deposits d_t from households, using part of their funds to purchase annuities from an insurance fund and investing the remaining funds s_t with arbitrageurs. We prime the model to accommodate financial frictions by restricting the freedom of the financial intermediary to decide how deposits are allocated between annuities and arbitrageurs. For each unit of deposits the financial intermediary channels to arbitrageurs, it is assumed that they have to purchase an annuity from the insurance fund, a requirement that acts as an implicit tax on investing with arbitrageurs.[2] Annuities are not held until maturity, but are bought back by the insurance fund at the end of the period.[3] Financial intermediaries are perfectly competitive, so return all the proceeds from annuities and investment with arbitrageurs to the household in the form of a return to saving $R^s_{t,t+1}$.

3.2 Insurance funds

The insurance fund buys and sells annuities that pay coupons which decay in perpetuity at rate g. This tractable formulation of annuities as perpetuities with exponentially-decaying coupons is taken from Woodford (2001) and allows us to analyse annuities of arbitrary duration by appropriate choice of g. Annuities are commitments to a stream of future cash flow payments, which for an annuity next due to pay coupon c when priced at a benchmark yield of y^*_t implies a present discounted liability of

$$\sum_{j=1}^{\infty} \frac{c(1-g)^{j-1}}{(1+y^*_t)^j} = \frac{c}{y^*_t + g} \tag{1}$$

and duration

$$\sum_{j=1}^{\infty} \frac{jc(1-g)^{j-1}}{(1+y^*_t)^j} \Bigg/ \sum_{j=1}^{\infty} \frac{c(1-g)^{j-1}}{(1+y^*_t)^j} = \frac{1+y^*_t}{y^*_t + g} \tag{2}$$

Fluctuations in the benchmark yield lead to changes in the present value of liabilities, a risk to the insurance fund's balance sheet and solvency they hedge against by following the liability-driven strategy in Domanski et al. (2017). The insurance fund's liabilities L and equity E are equal to cash M and the market value of bond holdings B through the balance sheet identity $L + E = M + B$. The liability-driven strategy partially immunises the balance sheet by matching the interest rate risk in liabilities $L(y^*_t)$ to that in bond holdings,

[2] The requirement to purchase annuities can be interpreted as a risk-adjusted capital requirement, imposed either by the government or financial markets.

[3] The assumption that financial instruments are bought back and reissued each period is often made in the literature on optimal government debt management, for example Angeletos (2002) and Buera and Nicolini (2004).

that is by the insurance fund holding bonds $B(y_t^*)$ such that E and M are independent of small fluctuations in y_t^*. We assume that the benchmark yield for pricing liabilities is that on an n^* period government bond.

The insurance fund partially immunises its balance sheet against interest rate risk by holding sufficient government bonds to close the duration gap between its liabilities and assets. In doing so, what matters is the total exposure of its asset position to interest rate risk. Since in equilibrium it is the duration risk held by arbitrageurs that determines the market price of risk, the yield curve and the reaction of interest rates to monetary policy, we can without loss of generality allow the insurance fund to partially immunise the risk in its liabilities by holding a suitable quantity of bonds of the benchmark maturity. The assumption is without loss of generality because an alternative portfolio that closes the insurance fund's duration gap by holding different maturity bonds must offer exactly the same total exposure to interest rate risk, in which case the residual duration risk held by arbitrageurs is the same and equilibrium outcomes are unaffected. What matters is the insurance fund's demand for exposure to interest rate risk, not how that exposure is structured.

The duration of the benchmark maturity bond is n^*, so closing the duration gap requires holding

$$P_t^{n^*} q_t^{n^*} = \frac{c\left(1+y_t^{n^*}\right)}{n^*(y_t^{n^*}+g)^2} \tag{3}$$

of benchmark bonds for each unit of annuity outstanding, where $P_t^{n^*}$ is the market price of an n^* period bond. The requirement to hedge annuities against interest rate risk hence creates a preferred habitat for the insurance fund, with (3) its demand for bonds of maturity n^*. Domanski et al. (2017) derive the same expression for liability-driven demand, assuming that the insurance fund manages interest rate risk through holding benchmark maturity bonds and imposing a condition that partially immunises the balance sheet by equating the interest rate sensitivity of assets and liabilities. Our derivation in terms of the duration gap highlights the importance of duration for liability-driven demand, and the irrelevance of how that demand is structured. A numerical example with $c = 0.15$, $g = 0.05$ and $n^* = 15$ is in Figure 2.

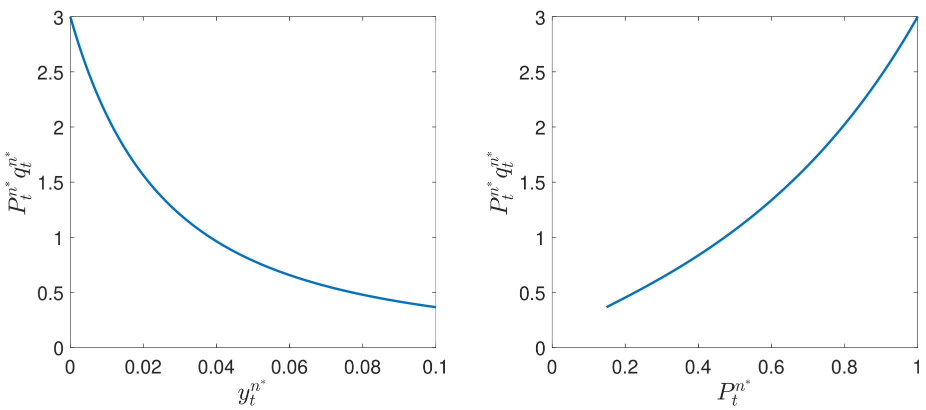

Figure 2: Preferred-habitat demand of insurance funds

When linearised around the steady state benchmark yield $\bar{y}^{n^*}$, the liability-driven demand becomes

$$P_t^{n^*} q_t^{n^*} = \beta + \varphi(y_t^{n^*} - \bar{y}^{n^*}) \tag{4}$$

where

$$\beta = \frac{c(1 + \bar{y}^{n^*})}{n^*(\bar{y}^{n^*} + g)^2} > 0 \tag{5}$$

$$\varphi = \frac{c(g - \bar{y}^{n^*} - 2)}{n^*(\bar{y}^{n^*} + g)^3} < 0 \tag{6}$$

The linearised demand function (4) is the same as that posited for preferred habitat investors in Vayanos and Vila (2021),[4] except in one important respect. They impose the restriction $\varphi \geq 0$, so the demand function of their preferred-habitat investors is always downward-sloping in the price of the benchmark bond (and upward-sloping in its yield). In our model, φ is negative because it is derived from the liability-driven strategy the insurance fund uses to hedge against interest rate risk. The demand curve is also steeper the lower are steady-state interest rates, a state dependency that gives our insurance fund an increasingly prominent role in a low interest rate environment. This is the case emphasised by Domanski et al. (2017) – when steady-state interest rates are low, $\bar{y}^{n*} + g$ is close to zero and changes in yield have a disproportionately large impact on liability-driven demand. Hedging against interest rate risk always entails the insurance fund raising their bond holdings after a fall in yields, but especially so when interest rates are already low.

The price at which annuities are traded does not affect liability-driven demand because the financial intermediary has to purchase annuities in proportion to the deposits it channels to arbitrageurs, irrespective of their price. The price matters in general equilibrium when it influences the return to saving faced by the representative household. We therefore postpone our discussion of annuity pricing until Section 5.1., where it is pinned down by competition in annuity markets.

3.3 Arbitrageurs

Arbitrageurs maximise the expected one-period return $E_t R_{t,t+1}^P$ on their portfolio but have limited risk-bearing appetite that impedes their ability to take on risk. They maximise a mean-variance objective function, where $R_{t,t+1}^n$ is the return on n period bonds and ω_t^n is the proportion of their funds held at that maturity. The first constraint defines the return on the arbitrageur's portfolio, the second ensures that portfolio shares sum to unity. σ measures the risk-bearing appetite of arbitrageurs: the higher is σ the less they are willing to hold a risky portfolio.

[4] See equation (5) in Vayanos and Vila (2021).

$$\max_{\omega_t^n} E_t R_{t,t+1}^P - \frac{1}{2}\sigma Var_t R_{t,t+1}^P \tag{7}$$

s.t.

$$R_{t,t+1}^P = \sum_{n=1}^{N} \omega_t^n R_{t,t+1}^n \tag{8}$$

$$\sum_{n=1}^{N} \omega_t^n = 1 \tag{9}$$

Government bonds do not pay coupons, so their return depends entirely on the holding period return, i.e., the change in price $R_{t,t+1}^n = P_{t+1}^{n-1}/P_t^n - 1$. The relevant price at $t+1$ is that of an $n-1$ period bond since by then the n period bond is one period closer to maturity.

3.4 Government bond supply

The supply of bonds is not the primary focus of our paper, so we adopt a simple policy rule in which the government adjusts bond supply to fix the market value of debt at each maturity as a proportion of the total market value of debt. It follows that the maturity structure of government debt (but not its total market value) is time-invariant. The government issues zero coupon bonds at N maturities, with $\Sigma\gamma^n = 1$; the maturity structure is uniform if $\gamma^n = 1/N$ for $\forall n$.

$$P_t^n q_t^n = \gamma^n \sum_{n=1}^{N} P_t^n q_t^n \tag{10}$$

We assume that the government buys back and reissues the entire stock of government bonds each period, as in Angeletos (2002) and Buera and Nicolini (2004). Any losses on the government account are recouped through lump-sum taxation of households; gains are distributed as lump-sum credits. It is straightforward to extend our model to more sophisticated settings in which the total quantity and maturity structure of bond supply are determined endogenously by the interplay of government debt issuance policy and a monetary authority that engages in asset purchase programmes.

3.5 Equilibrium

Our model of financial intermediation supports an affine term structure in which the log price of an n period bond is linear in a set of macroeconomic risk factors X_t that follows an AR(1) process, taken as exogenous in bond markets. We take a guess-and-verify approach and introduce an equilibrium where the only risk factors are the short-term interest rate $R_{t,t+1}^1$ and information μ_t about the path of future short-term interest rates, determined jointly by:

$$\underbrace{\begin{pmatrix} R^1_{t,t+1} \\ \mu_t \end{pmatrix}}_{X_t} = \underbrace{\begin{pmatrix} \mu \\ 0 \end{pmatrix}}_{M} + \underbrace{\begin{pmatrix} \phi & 1 \\ 0 & \rho \end{pmatrix}}_{P} \underbrace{\begin{pmatrix} R^1_{t-1,t} \\ \mu_{t-1} \end{pmatrix}}_{X_{t-1}} + \underbrace{\begin{pmatrix} \varepsilon_t \\ \upsilon_t \end{pmatrix}}_{\xi_t \sim N(0,\Sigma)} \tag{11}$$

The path of the short-term interest rate is dictated by two shocks and is not subject to an effective lower bound.[5] The first is a monetary policy shock ε_t, which affects the short-term rate in the usual way and has variance σ^2_ε. The second is a future path shock υ_t, which raises future short-term interest rates and has variance σ^2_υ. The monetary policy shock propagates to the short-term interest rate immediately with persistence ϕ; the forward-path shock with a lag through an AR(2) process with roots ϕ and ρ. The process for the short-term interest rate (11) is endogenised in Section 5 when we consider general equilibrium.

In the proposed affine equilibrium the log price of an n period bond takes the form $p^n_t = -a_n - b_n X_t$, where a_n, b_n are vectors of coefficients to be determined for $n = 0 \dots N$. It is immediately apparent that $a_0 = 0$ and $b_0 = (0, 0)$ because the log price of a 0 period bond is equal to the log of its value at redemption, which is normalised at unity. Similarly, the relationship between the yield and log price of a bond requires $a_1 = 0$ and $b_1 = (1, 0)$ so the log price of a 1 period bond is consistent with the short-term interest rate.[6]

3.5.1 Arbitrageur demand

Arbitrageurs price bonds at each point on the yield curve. Their portfolio is made up of different maturity bonds, the yields of which depend entirely on holding period returns because the bonds do not pay coupons. Holding period returns are log-normally distributed when log bond prices are affine in a set of normally-distributed risk factors, so the expectation and variance of the return on the arbitrageur's portfolio follow from the properties of the multivariate log-normal distribution:[7]

$$\begin{aligned} E_t R^p_{t,t+1} &= E_t \sum_{n=1}^{N} \omega^n_t \left[e^{\left(p^{n-1}_{t+1} - p^n_t\right)} - 1 \right] \\ &= \sum_{n=1}^{N} \omega^n_t \left[-a_{n-1} - b_{n-1} \left(M + P X_t \right) + \frac{1}{2} b_{n-1} \Sigma b'_{n-1} + a_n + b_n X_t \right] \end{aligned} \tag{12}$$

$$Var_t R^p_{t,t+1} = \left(\sum_{n=1}^{N} \omega^n_t b_{n-1} \right) \Sigma \left(\sum_{n=1}^{N} \omega^n_t b'_{n-1} \right) \tag{13}$$

The arbitrageur's demand for n period bonds is obtained by substituting (12) and (13) into the optimisation

[5] Introducing an effective lower bound on the short-term interest rate introduces a non-linearity that breaks the affine nature of the term structure. We follow recent empirical evidence from Debortoli et al. (2020), Swanson (2018) and Gürkaynak et al. (2022) suggesting that the effective lower bound is not a constraint at the short-term interest rates we consider.

[6] The continuously-compounded yield on an n period zero coupon bond is related to the log of its price by the formula $y^n_t = -p^n_t/n$. For $n = 1$, this means $R^1_{t,t+1} = -p^1_t$ and $a_1 = 0, b_1 = (1, 0)$.

[7] The macroeconomic risk factors X_t are normally distributed, as are log bond prices $p^n_t = -a_n - b_n X_t$. It follows that the bond price $P^n_{t+1} = e^{p^n_{t+1}}$ is log-normally distributed, and by the standard formula for the expectation of a log-normally distributed random variable $E_t e^{p^n_{t+1}} = -a_n - b_n E_t X_{t+1} + 0.5 b_n \Sigma b'_n$.

problem of Section 3.3 and taking the first order conditions with respect to the portfolio share ω_t^n:

$$-a_{n-1} - b_{n-1}(M + PX_t) + \frac{1}{2}b_{n-1}\Sigma b'_{n-1} + a_n + b_n X_t - \sigma b_{n-1}\left(\sum_{n=1}^{N}\omega_t^n b'_{n-1}\right)\Sigma + \kappa = 0 \qquad (14)$$

where κ is the Lagrange multiplier on the constraint that portfolio shares have to sum to unity.

3.5.2 Market price of risk

The arbitrageur has a first order condition of the form (14) for each of the N maturities of bond in their portfolio, all but one of which contains a common term in the market price of risk λ_t.[8]

$$\lambda_t = \sigma\sum_{n=1}^{N}\omega_t^n b_{n-1} = \lambda_0 + \lambda_1 X_t \qquad (15)$$

The first order condition for 1 period bonds simplifies to $-(1\ 0)X_t = \kappa$, which identifies κ and allows for an interpretation of arbitrageur demand in terms of excess holding period returns along the yield curve.

$$E_t R_{t,t+1}^n - R_{t,t+1}^1 = b_{n-1}\Sigma\lambda_t \qquad (16)$$

Excess holding period returns are affine in the risk factors, since the first order conditions for ω_t^n define a set of linear relationships between X_t and λ_t.

3.5.3 Bond pricing

Equilibrium bond pricing is characterised by the coefficients $a_n, b_n, \lambda_0, \lambda_1$ for $n = 0 \ldots N$. The pricing coefficients a_n, b_n can be solved recursively as a function of the market price of risk coefficients λ_0, λ_1 through the first order conditions of the arbitrageurs (14). The general form is:

$$a_n = a_1 + a_{n-1} + b_{n-1}(M + \Sigma\lambda_0) - \frac{1}{2}b_{n-1}\Sigma b'_{n-1} \qquad (17)$$

$$b_n = b_1 + b_{n-1}(P + \Sigma\lambda_1) \qquad (18)$$

where the solution has a block recursive structure. Conditional on λ_0 and λ_1, the a_n coefficients depend on the b_n coefficients but the b_n coefficients do not depend on the a_n coefficients. It remains to solve for λ_0, λ_1.

The bond pricing equations confirm that it is the total duration risk borne by arbitrageurs that matters in equilibrium. The b_{n-1} coefficients in the equation for the market price of risk (15) capture the sensitivity of bond prices to the macroeconomic risk factors, i.e., $b_{n-1} = d\log P_t^{n-1}/dX_t$. This is the duration risk associated with holding an $n-1$ period bond, and the sum across maturities $\Sigma\omega_t^n b_{n-1}$ is the total duration risk for each unit of funds in the arbitrageur's portfolio. Alternative hedging strategies for the insurance fund that leave arbitrageurs carrying the same duration risk then have exactly the same implications for equilibrium.

[8]The first order condition for $n = 1$ does not contain a term in the market price of risk because 1 period bonds are risk free.

3.5.4 Market clearing

The bond market clears at prices where government supply at each maturity matches the demand from insurance funds and arbitrageurs. The liability-driven demand of insurance funds can (without loss of generality) be satisfied by only holding bonds of maturity n^*, whose benchmark yield they use to price their liabilities. At that maturity, government bond supply is $\gamma^{n^*}\Sigma P_t^n q_t^n$ and demand from arbitrageurs is $s_t \omega_t^{n^*}$, where s_t are the funds flowing to arbitrageurs. The financial intermediary is required to purchase an annuity for each unit of funding it channels to arbitrageurs, which creates liabilities on the insurance fund balance sheet that they have to partially immunise by demanding $s_t P_t^{n^*} q_t^{n^*}$ of n^* period bonds, where $P_t^{n^*} q_t^{n^*}$ is determined by the insurance fund's linearised liability-driven strategy (4)-(6). Market clearing at each maturity requires

$$\gamma^{n^*} \sum_{n=1}^{N} P_t^n q_t^n \quad = \quad s_t \left(P_t^{n^*} q_t^{n^*} + \omega_t^{n^*} \right) \tag{19}$$

$$\gamma^{n} \sum_{n=1}^{N} P_t^n q_t^n \quad = \quad s_t \omega_t^{n} \tag{20}$$

Summing the market-clearing conditions across maturities implies that the total market value of bonds outstanding is $\Sigma P_t^n q_t^n = s_t \left(P_t^{n^*} q_t^{n^*} + 1 \right)$, because $\Sigma \gamma^n = 1$ and $\Sigma \omega_t^n = 1$. It follows that s_t cancels from each side of the market-clearing conditions to give

$$\gamma^{n^*} \left(P_t^{n^*} q_t^{n^*} + 1 \right) \quad = \quad P_t^{n^*} q_t^{n^*} + \omega_t^{n^*} \tag{21}$$

$$\gamma^{n} \left(P_t^{n^*} q_t^{n^*} + 1 \right) \quad = \quad \omega_t^{n} \tag{22}$$

and equilibrium bond prices are independent of the funds allocated to arbitrageurs, which is a useful property when we extend to general equilibrium in Section 5.

3.5.5 Verifying linearity

The market clearing conditions (21)-(22) imply that the shares of different maturity bonds in the arbitrageur's portfolio are linear in the macroeconomic risk factors X_t, as required to maintain the affine term structure. To see this, substitute in for liability-driven demand $P_t^{n^*} q_t^{n^*}$ from (4) and relate the yield on an n^* period bond to X_t using $y_t^{n^*} = -p_t^{n^*}/n^*$ and $p_t^{n^*} = -a_{n*} - b_{n*} X_t$ to obtain

$$\omega_t^{n^*} = \gamma^{n^*} + (\gamma^{n^*} - 1)\beta + (\gamma^{n^*} - 1)\varphi \left(\frac{a_{n^*}}{n^*} + \frac{b_{n^*} X_t}{n^*} - \bar{y}^{n^*} \right) \tag{23}$$

which confirms that ω_t^{n*} is linear in X_t. The portfolio shares of other bonds also maintain linearity:

$$\omega_t^{n} = \gamma^{n} + \gamma^{n}\beta + \gamma^{n}\varphi \left(\frac{a_{n*}}{n^*} + \frac{b_{n*} X_t}{n^*} - \bar{y}^{n*} \right) \text{ for } n \neq n^* \tag{24}$$

With all portfolio shares linear in the risk factors, the market price of risk (15) is confirmed as linear in X_t. The market clearing conditions (23)-(24) and the definition of the market price of risk (15) determine λ_0 and λ_1 as functions of $\{a_n, b_n\}$. Together with the recursion for the bond pricing coefficients (17)-(18) that determines

$\{a_n, b_n\}$ as a function of λ_0 and λ_1, there is unique fixed point solution for the coefficients $\{a_n, b_n, \lambda_0, \lambda_1\}$.

3.5.6 Consistency

One important issue remains, in that the intercept and slope of the insurance fund's demand (4) are both functions of the steady-state benchmark yield through equations (5) and (6). The benchmark yield incorporates a term premium and so depends on all the parameters of the model, most notably (but not exclusively) the constant μ in the exogenous process for the short-term interest rate. We ensure consistency in equilibrium by identifying a unique fixed point at which the triple $(\beta, \varphi, \bar{y}^{n*})$ is mutually compatible. A simple iterative procedure that loops to convergence over $\beta(\bar{y}^{n*}), \varphi(\bar{y}^{n*})$ from (5) and (6) and $\bar{y}^{n*}(\beta, \varphi)$ in equilibrium suffices.

3.5.7 Term premia

The equilibrium dynamics of the term structure are completely characterised by the stochastic process for the risk factors and the coefficients mapping that process to the market price of risk and bond prices. The link between the market price of risk and term premia can be derived by writing the first order condition of the arbitrageurs (14) in terms of yields:

$$-(n-1)E_t y_{t+1}^{n-1} + n y_t^n - b_{n-1}\Sigma\lambda_t = R_{t,t+1}^1$$

which when solved forward implies

$$y_t^n = \underbrace{\frac{1}{n}E_t \sum_{i=1}^{n} R_{t+i-1,t+i}^1}_{\text{expectations}} + \underbrace{\frac{1}{2n}\sum_{i=1}^{n} b_{n-i}\Sigma b_{n-i}'}_{\text{convexity adjustment}} + \underbrace{\frac{1}{n}E_t \sum_{i=1}^{n} b_{n-i}\Sigma\lambda_{t+i-1}}_{\text{term premium}} \tag{25}$$

The first component of the yield is the average expected future short rate over the horizon n of the bond, the premium a risk-neutral investor require as compensation for holding the bond. The second term is a standard convexity adjustment and the third component is the term premium that arbitrageurs demand for holding the bond, given their limited risk-bearing appetite.

4 Term structure of interest rates

In this section, we illustrate the behaviour of our model of financial intermediation through a numerical example. We continue to assume that the set of risk factors follows an exogenous AR(1) process, leaving general equilibrium considerations to Section 5.

4.1 Parameterisation

There are limited degrees of freedom in our tightly parameterised model, which raises challenges when constructing a numerical example. The parameterisation in Table 1 follows Vayanos and Vila (2021) where possible, and supports a quantitatively acceptable equilibrium in which the period is a year and each financial market agent plays a significant role.

Parameter	Value	Description
$\frac{\mu}{1-\phi}$	0.00 - 0.10	Unconditional mean of short-term interest rate
ϕ	0.875	Persistence of short-term interest rate
ρ	0.8	Persistence of future path variable
σ_{ε}^2	0.01^2	Variance of monetary policy shocks
σ_{υ}^2	0.0001^2	Variance of future path shocks
N	30	Number of bond maturities issued
γ^n	1/30	Proportion of bonds issued at maturity n
σ	6.78	Risk-bearing capacity of arbitrageurs
n^*	15	Benchmark maturity for pricing liabilities
c	0.15	Coupon payment on annuity
g	0.05	Decay rate of annuity coupon payments

Table 1: Parameterisation with exogenous risk factors

The exogenous process for the short-term interest rate is parameterised for a range of μ so its unconditional mean varies from 0% to 10% in steps of 2%, in each case the rate being both persistent and volatile to generate quantitatively reasonable term premia along the yield curve. The importance of insurance funds in equilibrium is regulated by the coupon payment c on the annuity that financial intermediaries have to purchase. Our parameterisation gives them a significant role by requiring the financial intermediary to purchase an annuity paying $0.15 for each $1 they invest with arbitrageurs. The decay rate g of the annuity coupon payments implies that annuities issued by the insurance fund have a duration of approximately 15 years when the benchmark yield is 2%, higher than that in Carlstrom et al. (2017) and Sims and Wu (2021), to ensure sufficient differentiation between the appetite for duration of the insurance fund and arbitrageurs. The benchmark maturity n^* for pricing liabilities is set to match the duration of annuities.

4.2 Preferred-habitat demand

The dynamics of the term structure are obtained by iterating until the steady-state benchmark yield is consistent with its implications for the intercept and slope of the insurance fund's liability-driven demand. Table 2 summarises the steady-state properties of the resulting equilibrium.

Short-term interest rate %	Benchmark yield %	Demand of preferred-habitat investors	
$\bar{R}^1$	$\bar{y}^{n*}$	β	φ
0	1.31	2.54	-78.04
2	3.37	1.48	-33.84
4	5.43	0.97	-17.65
6	7.48	0.69	-10.42
8	9.51	0.52	-6.70
10	11.53	0.41	-4.57

Table 2: Steady-state properties with short-term interest rate exogenous

The steady-state benchmark yield is increasing in the steady-state short-term interest rate, albeit at only a mildly increasing pace that reflects the gradual upticking of term premia. The intercept β and slope φ of liability-driven demand have the largest magnitude at low steady-state interest rates, reflecting the insurance fund's greater need to hold bonds and manage duration risk when yields on benchmark bonds are low. This is the strengthening of the "recruitment" channel at low interest rates trailed in the introduction.

4.3 Response of yield curve to future path shock

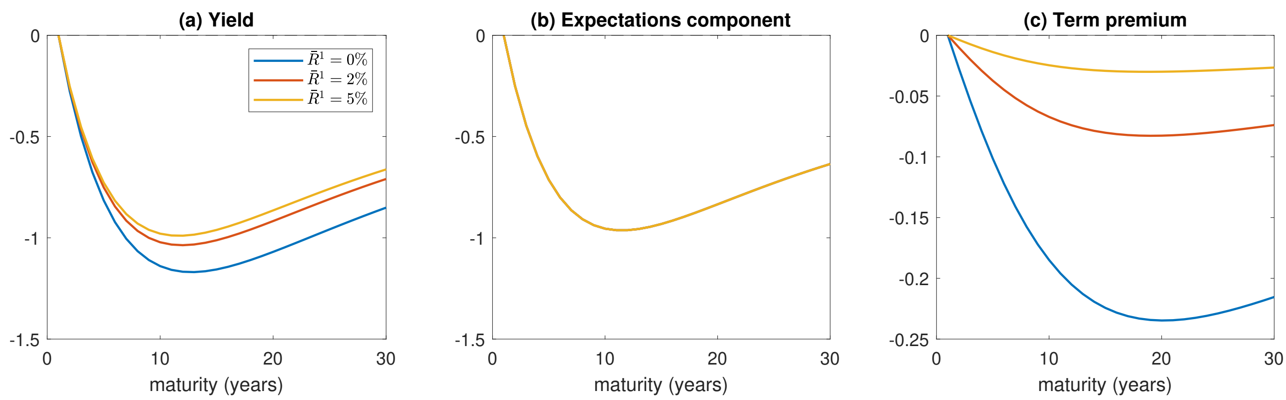

Figure 3: Impact response of yield curve to a future path shock

Figure 3 shows the immediate impact on the yield curve of a 50 basis points negative shock to the future path of the short-term interest rate. There is little movement at the short end of the curve because it is a shock to the future path, but the knowledge that rates will rise in the future has a significant effect at the long end. The dependency of the strength of the responses on steady-state short-term interest rates is entirely due to differential movements in term premia, since the response of the expectations component of the term structure is independent of steady-state short-term interest rates. The narrowing of the term premium is largest when steady-state interest rates are low. This is the "recruitment" channel in operation. It results in yields overreacting to future path shocks, especially when interest rates are low. Private investors hence amplify the impact of monetary policy, as suggested by Hanson and Stein (2015).

4.4 Response of yield curve to monetary policy shock

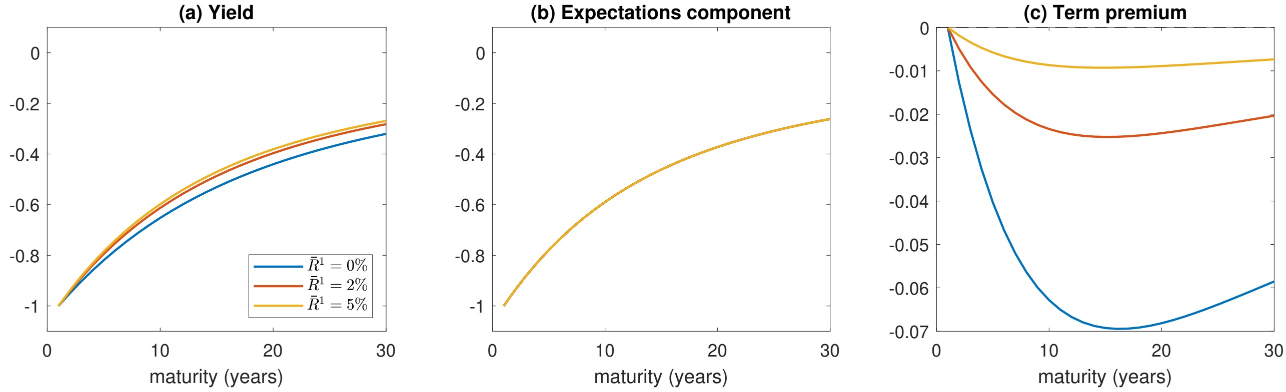

Figure 4: Impact response of yield curve to a monetary policy shock

The monetary policy shock by construction has the greatest impact at the short end of the yield curve, especially in our parameterisation of the model where the monetary policy shock has only limited persistence. In Figure 4 a 100 basis points cut in the short-term rate reduces the benchmark yield by less than 50 basis points, almost all of which is due to movements in the expectations component of yields. The term premium reacts as expected, narrowing most in response to a negative monetary policy shock when interest rates are low. The overall effect is negligible at less than ± 10 basis points, not surprising given that monetary policy shocks have only small effects at longer maturities where liability-driven investors operate.

5 Annuity pricing and return to saving

The first challenge in moving to general equilibrium is to explain how the insurance fund prices annuities given their liability-driven strategy, since what happens to annuity prices affects the return to saving that the financial intermediary offers the household. This section begins by specifying the way annuities are priced and by deriving the return to saving, after which we show how the response of the return to saving depends on steady-state short-term interest rates. We continue to assume that the short-term interest rate follows an exogenous process, leaving the discussion of general equilibrium to the next section.

5.1 Annuity pricing

The insurance fund trades annuities with the financial intermediary, selling at the beginning of the period and buying back at the end. It receives P_t^c when it sells the annuity, which leaves $P_t^c - P_t^{n^*} q_t^{n^*}$ to invest after purchasing the requisite quantity of benchmark bonds. We assume that the insurance fund has access to outside investment opportunities that pay a fixed premium ξ over the steady-state short-term rate. The steady-state term premium on benchmark maturity bonds in Table 2 is in the range $1.31\% - 1.53\%$, so by setting $\xi = 0.05$ we ensure that it is always costly for the insurance fund to partially immunise its liabilities. If ξ was lower then the insurance fund would at times prefer to hold government bonds over alternative investments, and would not always be constrained by its liability-driven strategy.

The duration risk on the insurance fund's balance sheet is unaffected by investing in outside opportunities that pay a fixed premium over a constant steady-state short-term rate, so the preferred-habitat demand for government bonds to manage duration risk is unchanged. More loosely, we can interpret the constant return on outside investments as a constant risk-adjusted return, provided the risk involved is uncorrelated with duration risk. Such would not be the case, for example, if outside investment opportunities paid a fixed premium over the short-term rate itself rather than the steady-state short-term rate.

There are three terms in the proceeds the insurance fund expects to make on each annuity it issues (26). The first is the difference between the selling price of the annuity at the beginning of the period and the buying price at the end, adjusted for the decay in coupons; the second is the coupon payment; the third is the sum of the expected holding period return on benchmark bonds and the return on investments in outside opportunities.

$$[P_t^c - E_t(1-g)P_{t+1}^c] - c + \left[P_t^{n^*} q_t^{n^*} E_t R_{t,t+1}^{n^*} + (P_t^c - P_t^{n^*} q_t^{n^*}) \left(\bar{R}^1 + \xi\right)\right] \tag{26}$$

We assume the annuity market is perfectly competitive, so insurance funds issue annuities until the expected net proceeds from further issuance are zero.[9] It follows that the annuity price satisfies the recursion

$$\left(1 + \bar{R}^1 + \xi\right) P_t^c = \left[c + P_t^{n^*} q_t^{n^*} (\bar{R}^1 + \xi - E_t R_{t,t+1}^{n^*})\right] + E_t(1-g)P_{t+1}^c \tag{27}$$

where the term in square brackets is the marginal cost of issuing an annuity. The marginal cost is the coupon payment, plus the opportunity cost of holding the benchmark bonds required to partially immunise the insurance fund's balance sheet. $E_t R_{t,t+1}^{n^*}$ and $P_t^{n^*} q_t^{n^*}$ are both linear functions of the macroeconomic risk factors X_t, so to a first-order approximation the annuity price satisfies the linear recursion

$$\left(1 + \bar{R}^1 + \xi\right) P_t^c = \Omega_0 + \Omega_1 (X_t - \bar{X}) + (1-g) E_t P_{t+1}^c \tag{28}$$

where expressions for the coefficient matrices Ω_0 and Ω_1 are in Appendix B.[10] It can be solved by iterating forward if $\left|(1-g)/(1+\bar{R}^1+\xi)\right| < 1$ and X_t is stationary, conditions that are satisfied in our numerical

[9] Although perfect competition drives the *ex ante* return to zero, this does not guarantee that the *ex post* return will also be zero. We assume that any *ex post* gains or losses are covered by transfers between the insurance fund and the household.

[10] The expected holding period return $E_t R_{t,t+1}^{n^*}$ depends on $R_{t,t+1}^1$ and X_t through the excess holding-period return (16) and the market price of risk (15), with $R_{t,t+1}^1 \in X_t$. The benchmark bond holdings $P_t^{n^*} q_t^{n^*}$ depend on the benchmark yield through preferred-habitat demand (4), which itself is a function of X_t through the definition of the benchmark yield $y_t^{n^*} = -p_t^{n^*}/n^*$ and affine bond pricing $p_t^{n^*} = -a_{n^*} - b_{n^*} X_t$.

parameterisation. We obtain

$$P_t^c = \left(\frac{1}{1+\bar{R}^1+\xi}\right) E_t \sum_{i=0}^{\infty} \left(\frac{1-g}{1+\bar{R}^1+\xi}\right)^i \left[\Omega_0 + \Omega_1(X_{t+i} - \bar{X})\right] \tag{29}$$

The expected future values of the macroeconomic risk factors are $E_t(X_{t+i} - \bar{X}) = P^i(X_t - \bar{X})$ from the exogenous process (11), so the annuity price can be derived by applying the standard formula for the sum of an infinite geometric progression.

$$P_t^c = \left(\frac{1}{\bar{R}^1+\xi+g}\right)\Omega_0 + \left(\frac{1}{1+\bar{R}^1+\xi}\right)\left(I - P\left(\frac{1-g}{1+\bar{R}^1+\xi}\right)\right)^{-1} \Omega_1(X_t - \bar{X}) \tag{30}$$

5.2 Return to saving

The financial intermediary invests the deposits it receives from households into annuities and with arbitrageurs. From their perspective, the expected return to annuities is

$$E_t R_{t,t+1}^c = \frac{(1-g)E_t P_{t+1}^c - P_t^c + c}{P_t^c} \tag{31}$$

and the expected return from investing with arbitrageurs is the one-period portfolio holding return (12). The requirement that the financial intermediary purchases an annuity for every \$1 it invests with arbitrageurs means it offers households an expected return to saving that is a weighted average of these returns.

$$E_t R_{t,t+1}^s = \frac{E_t R_{t,t+1}^p + P_t^c E_t R_{t,t+1}^c}{(1+P_t^c)} \tag{32}$$

The steady-state relationship between expected returns and the level of short-term interest rates is in Table 3. The expected return to arbitrageurs tracks short rates well, with relative returns increasing at higher rates. The same is true of the return to annuities, so in steady state the return to savings rises more than proportionally with the short rate.

Short-term interest rate %	Return to arbitrageurs %	Price of annuity \$	Return to annuities %	Return to savings %
$\bar{R}^1$	$\bar{R}^p$	$\bar{P}^c$	$\bar{R}^c$	$\bar{R}^s$
0	1.39	2.17	1.92	1.75
2	3.65	1.57	4.54	4.20
4	5.85	1.25	7.03	6.50
6	7.98	1.04	9.36	8.68
8	10.06	0.90	11.59	10.79
10	12.12	0.80	13.77	12.85

Table 3: Steady-state expected returns and price of annuity with short-term interest rate exogenous

Annuities perform relatively poorly when short rates are low because low benchmark yields increase the duration of liabilities that insurance funds have to partially immunise by holding bonds. The cost of holding these additional bonds is a reduction in funds available for outside investment opportunities that pay a higher return. With only limited access to high returns, the proceeds to the insurance fund from issuing an annuity are lower, which in perfectly competitive annuity markets means that the price of annuities has to be high and the return on annuities offered to the financial intermediary is low.

5.3 Response to future path shock

The response of the return to saving to a negative 50 basis points future path shock is in Figure 5. Panel (b) confirms that the yield on benchmark bonds overreacts to the short-term interest rate most when interest rates are low. This is driven by the liability-driven demand in panel (c), which increases with the fall in benchmark yield, compressing term premia via the "recruitment channel" as in Section 4.3. The return to arbitrageurs in panel (d) falls in accordance with decreasing yields at all maturities, inheriting the overreaction due to liability-driven investors.

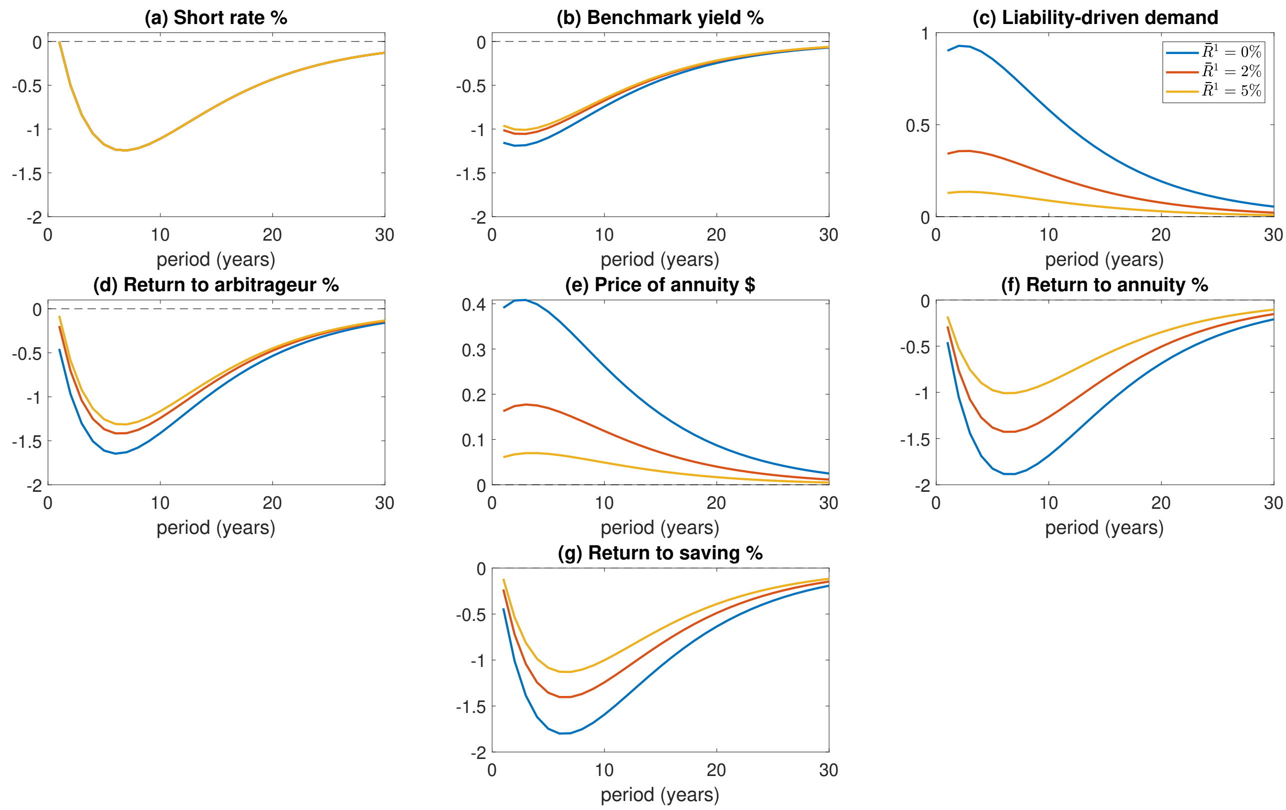

Figure 5: Response of returns to a future path shock

The fall in benchmark yield reduces the income the insurance fund receives on the benchmark bonds it holds to partially immunise its balance sheet. This increases the marginal cost of issuing annuities and leads to the increase in annuity prices in panel (e). The size of the price rise depends on the insurance fund's exposure to changes in the benchmark yield, being larger at low interest rates where the liability-driven strategy of the insurance fund requires large holdings of relatively expensive benchmark bonds. At higher interest rates the exposure is less, given that benchmark bonds are cheaper and the insurance fund needs to hold less of them. The greater the exposure, the more that a fall in the benchmark yield leads to a rise in the marginal cost of issuing annuities and the more that the annuity price has to rise in perfectly competitive annuity markets.

The price of the annuity gradually returns to steady state as the future path shock dissipates. From the perspective of the financial intermediary, falling annuity prices imply that the return to annuities narrows. It is narrowest in panel (f) when interest rates are low because of the larger initial rise in the annuity price being unwound. The return to saving in panel (g) is a weighted average of the returns to arbitrageurs and annuities, the latter receiving more weight when the price of the annuity is elevated. Both returns fall in response to the future path shock and fall most when interest rates are low, so our model of financial intermediation features a strong “recruitment channel”.

5.4 Response to monetary policy shock

The corresponding responses to a negative 100 basis points monetary policy shock are in Figure 6. In keeping with the results in Section 4.4, there is very little effect on the benchmark yield in panel (b) and only minor deviations in liability-driven demand in panel (c). The response of the return to the arbitrageur in panel (d) closely mirrors the dynamics of the short rate.

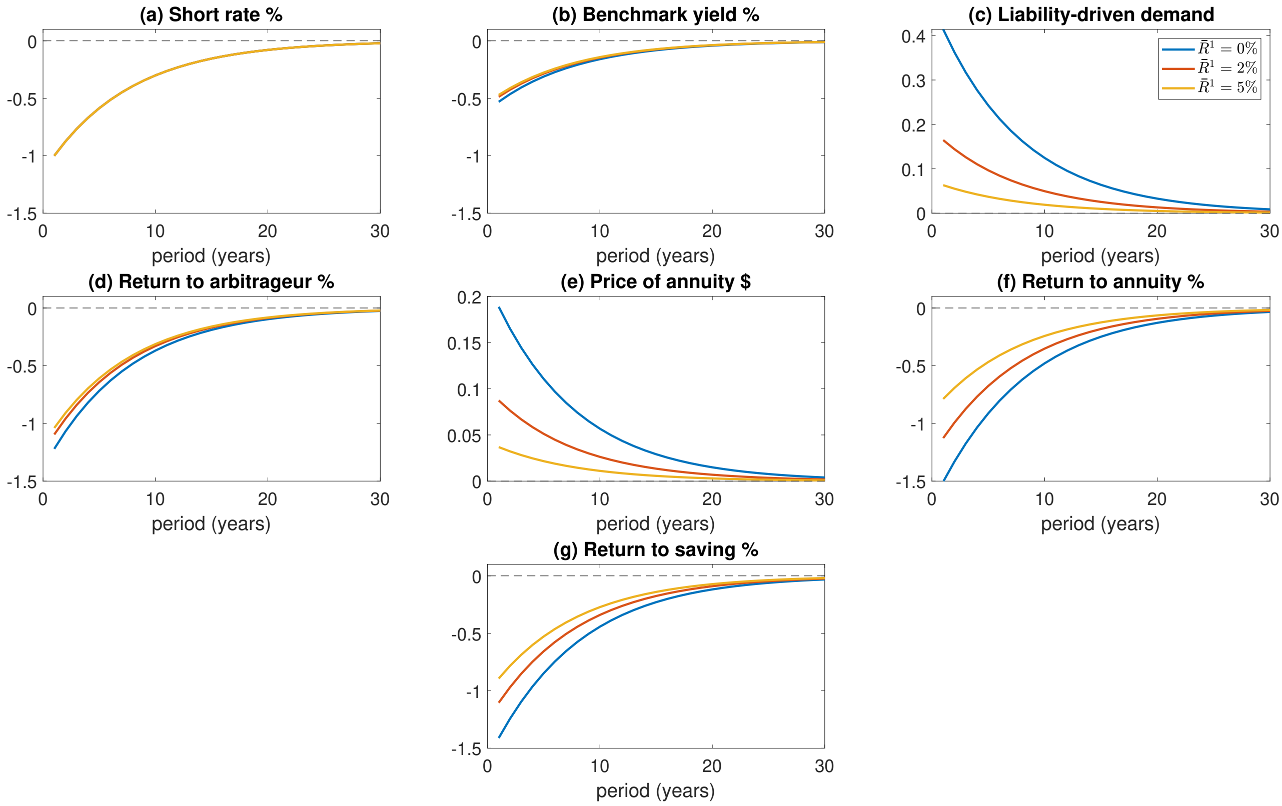

Figure 6: Response of returns to monetary policy shock

Although small, the fall in the benchmark yield meaningfully impedes the ability of the insurance fund to use outside investment opportunities to subsidise the marginal cost of issuing annuities. The price of the annuity has to rise, as before increasing most in panel (e) at low interest rates when the insurance fund is most exposed to changes in the benchmark yield. The return to annuities in panel (f) again narrows, with clear differentiation according to the level of interest rates mirrored in the return to saving in panel (g). Our model hence features a "recruitment channel" also in response to monetary policy shocks, with the return to saving reacting most when interest rates are low. Its quantitative importance derives predominantly from the behaviour of the insurance fund, with the compression of term premia due to liability-driven demand only playing a minor role.

6 General equilibrium

The second challenge in moving to general equilibrium is to introduce households, firms and government in a way that is consistent with the process for the macroeconomic risk factors taken as exogenous in financial intermediation. In this section we adopt a standard New Keynesian DSGE specification that generates an endogenous process for the macroeconomic risk factors of the form (11), as required. The notation follows Ellison and Tischbirek (2014), which is also our reference for detailed derivations of the first order conditions.

6.1 Households

The preferences of the representative household are

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{\delta}}{1-\delta} - \chi L_t \right) \tag{33}$$

where $C_t \equiv [\int_0^1 C_t(i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$ is a cost-minimising CES consumption index and L_t is labour supply. β, δ, χ are preference parameters and θ is the elasticity of substitution between consumption goods. The household maximises expected utility subject to the budget constraint

$$P_t C_t + T_t + d_t = R^s_{t-1,t} d_{t-1} + W_t L_t + \pi^h_t \tag{34}$$

in which $P_t \equiv [\int_0^1 P_t(i)^{1-\theta} di]^{1/1-\theta}$ is the price of the composite consumption good, W_t is the nominal market wage, T_t is a lump-sum transfer paid to the government, and d_t are deposits with the financial intermediary remunerated at return to saving $R^s_{t,t+1}$. Households own the insurance fund and firms that produce and sell consumption goods, so receive profit π^h_t which they treat as lump-sum income. All prices are measured in units of the numeraire good "money", which is not modelled. The first order conditions of the household's optimisation problem are

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\delta} \frac{R^s_{t,t+1}}{\Pi_{t+1}} \right] \tag{35}$$

$$\frac{W_t}{P_t} = \chi C_t^{\delta} \tag{36}$$

with $\Pi_{t+1} \equiv P_{t+1}/P_t$. (35) is the intertemporal Euler equation that characterises the optimal consumption-savings decision; (36) is the intratemporal condition for optimality between consumption and labour supply.

6.2 Firms

There is a continuum of monopolistically competitive firms indexed by $i \in [0,1]$. Firm i's production function is $Y_t(i) = L_t(i)^{1/\eta}$, where $L_t(i)$ is labour employed. As in Calvo (1983), only a fraction $1-\alpha$ of firms can adjust the price of their respective good in any given period. Denoting by $P_t^*(i)$ the price chosen by a firm able to reset its price in period t, the evolution of the aggregate price level is described by

$$P_t = \left[(1-\alpha) P_t^*(i)^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{1/(1-\theta)} \tag{37}$$

All firms that can change their price in period t choose $P_t(i)$ to maximise the expected discounted stream of their future profits

$$E_t \sum_{T=0}^{\infty} \alpha^{T-t} \Gamma_{t,T} \left(P_t(i) Y_T(i) - W_T L_T(i) \right) \tag{38}$$

subject to the relevant demand constraints and the firm's stochastic discount factor

$$\Gamma_{t,T} = \beta^{T-t} \left(\frac{C_T}{C_t} \right)^{-\delta} \frac{P_t}{P_T} \tag{39}$$

The first order condition for price setting and the price adjustment process imply that inflation satisfies[11]

$$\begin{aligned} \frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} &= \left(\frac{F_t}{K_t}\right)^{(\theta-1)/(\theta(\eta-1)+1)} && (40) \\ F_t &= C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta-1} F_{t+1} && (41) \\ K_t &= \frac{\theta\eta}{\theta-1} Y_t^{\phi} + \alpha\beta E_t \Pi_{t+1}^{\theta\eta} K_{t+1} && (42) \end{aligned}$$

where F_t and K_t are auxiliary variables. Equations (40)-(42) are the standard recursive formulation of the New Keynesian Phillips Curve. They collapse to the single equation form when linearised.

6.3 Government

The government supplies bonds and uses monetary policy to set the short-term nominal interest rate. We continue to assume that the government adjusts bond supply to fix the market value of debt at each maturity as a proportion of total market value of debt, and that the entire stock of government bonds is bought back and reissued each period. As before, the time-invariant market shares are given by (10).

6.3.1 Monetary policy

Monetary policy is consistent with the short-term nominal interest rate satisfying a Taylor-type policy rule.[12]

$$\frac{1+R_{t,t+1}^1}{1+\bar{R}^1} = \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\gamma_\Pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\gamma_Y} e^{[1\ 0] z_t} \tag{43}$$

where $\bar{R}^1, \bar{\Pi}, \bar{Y}$ are steady-state values and γ_Π, γ_Y are policy parameters chosen to ensure that the model is determinate in general equilibrium. The disturbance term z_t has an AR(1) structure that incorporates normally-distributed monetary policy shocks ε_t and future path shocks υ_t with variance-covariance matrix Σ. The parameters ϕ, ρ, Σ are structural.

$$z_t = \begin{pmatrix} \phi & 1 \\ 0 & \rho \end{pmatrix} z_{t-1} + \begin{pmatrix} \varepsilon_t \\ \upsilon_t \end{pmatrix} \tag{44}$$

The lump-sum tax on households guarantees that the government budget constraint holds each period.

6.4 Market clearing

The model is completed by conditions for clearing in bond, goods and labour markets. Demand and supply in the market for bonds are equated through the affine term structure in Section 3; the aggregate resource

[11] See Section A.1 in the appendix of Ellison and Tischbirek (2014) for details.

[12] The implicit policy instrument for monetary policy is the money supply, the dynamics of which are adjusted so the Taylor-type rule (43) for the short-term nominal interest rate holds in equilibrium each period.

constraint in goods markets is $Y_t = C_t$. Labour market clearing requires that hours supplied by the representative household L_t are equal to aggregate hours demanded by the firms $\int_0^1 L_t(i)di$, implying that aggregate production is $Y_t = (L_t/D_t)^{1/\phi}$, where $D_t \equiv \int_0^1 [P_t(i)/P_t]^{-\theta\phi} di$ is a measure of price dispersion. However, the assumption that household utility is linear in hours worked gives the model a block recursive structure with equilibrium consumption, inflation, output, hours worked and financial market variables all independent of the dynamics of the price dispersion term.[13]

6.5 Summary

The specification of the general equilibrium model is collected, for reference, in (45)-(53). The consumption Euler equation is (45) and the intratemporal optimality condition for labour supply is (46). The recursive form of the New Keynesian Phillips Curve is described by (47)-(49), with the Taylor-type rule for monetary policy defining (50)-(51). Market clearing is (52).

$$1 = \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\delta} \frac{R^s_{t,t+1}}{\Pi_{t+1}} \right] \tag{45}$$

$$\frac{W_t}{P_t} = \chi C_t^{\delta} \tag{46}$$

$$\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha} = \left(\frac{F_t}{K_t} \right)^{(\theta-1)/(\theta(\eta-1)+1)} \tag{47}$$

$$F_t = C_t^{-\delta} Y_t + \alpha\beta E_t \Pi_{t+1}^{\theta-1} F_{t+1} \tag{48}$$

$$K_t = \frac{\theta\eta}{\theta-1} Y_t^{\phi} + \alpha\beta E_t \Pi_{t+1}^{\theta\eta} K_{t+1} \tag{49}$$

$$\frac{1+R^1_{t,t+1}}{1+\bar{R}^1} = \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\gamma_\Pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\gamma_Y} e^{[1\ 0]z_t} \tag{50}$$

$$z_t = \begin{pmatrix} \phi & 1 \\ 0 & \rho \end{pmatrix} z_{t-1} + \begin{pmatrix} \varepsilon_t \\ \upsilon_t \end{pmatrix} \tag{51}$$

$$Y_t = C_t \tag{52}$$

$$E_t R^s_{t,t+1} = E_t R^s_{t,t+1} \left(R^1_{t,t+1}; \mu_t \right) \tag{53}$$

The final equation (53) is the link between the return to saving and the macroeconomic risk factors that emerges from our model of financial intermediation in Section 5.

6.6 Macroeconomic parameterisation

The additional parameters that appear in the general equilibrium model are calibrated in Table 4, which where possible follow standard values found in Ellison and Tischbirek (2014). The exceptions are $\beta, \alpha, \theta, \gamma_\Pi, \gamma_Y$, which are adjusted to produce a quantitatively acceptable Phillips curve trade-off whilst maintaining equilibrium determinacy when household utility is linear in hours worked. We reduce the persistence of the disturbance

[13] See Section A.4 in the appendix of Ellison and Tischbirek (2014) for details. A first-order approximation of the model is similarly block recursive even if disutility is increasing in hours worked.

terms in general equilibrium to avoid perverse reactions of inflation and output.[14]

Parameter	Value	Description
β	0.99	Household discount factor
δ	2	Inverse elasticity of intertemporal substitution in consumption
χ	1	Weight on disutility of labour
θ	1.2	Intratemporal elasticity of substitution
η	1.1	Inverse of returns to scale in goods production
α	0.85	Degree of price rigidity
ξ	0.05	Return on outside investment opportunities
γ_Π	3.5	Reaction of short-run interest rate to inflation
γ_Y	0	Reaction of short-run interest rate to output
ϕ	0.25	Persistence of short-term interest rate disturbance
ρ	0.5	Persistence of future path disturbance

Table 4: Macroeconomic parameterisation

6.7 Consistency

The behavioural equations of the model are completely forward-looking, so to first order all macroeconomic variables are linear functions of the monetary policy disturbance term z_t. Since the disturbance term is defined by the exogenous process (44), it follows that macroeconomic variables can be written as an AR(1) process. In particular, the short-term interest rate $R^1_{t,t+1}$ and information about the future path μ_t are AR(1), confirming the assumption in bond markets that macroeconomic risks follow an autoregressive process, and ensuring consistency between bond market equilibrium and macroeconomic dynamics. The equilibrium law of motion for the risk factors inherits its persistence from the disturbance term, but its volatility is an equilibrium object that depends on the structural parameters of the model. We iterate to obtain an equilibrium law of motion for the macroeconomic risk factors that is consistent with the affine term structure model in general equilibrium.

6.8 Response to shocks

The general equilibrium response to shocks differs from those in Sections 5.3 and 5.4 because the short-term nominal interest rate is no longer exogenous. Shocks to the short rate have partial equilibrium effects as before, but they also have general equilibrium effects as monetary policy sets the short rate based on current macroeconomic conditions. A shock to the short rate affects the return to saving, which has an impact on aggregate demand via the consumption Euler equation and inflation through the New Keynesian Phillips Curve. With monetary policy reacting to inflation and output by the Taylor-type rule, it follows that the short-term nominal interest rate is endogenous.

[14] If the disturbance terms are very persistent then an expansionary monetary policy shock that puts downward pressure on rates can have such a large positive effect on inflation and output in general equilibrium that interest rates overall have to rise to satisfy the Taylor Rule. See Galí (2008) p. 51.

6.8.1 Response to future path shocks

The general equilibrium response to a future path shock is in Figure 7, with the steady-state short-term interest rate set at 2% and 5%. Panels (a) and (b) show the mechanics of a 50 basis points expansionary future path shock in terms of what happens to the disturbance term (44) in the Taylor-type rule for monetary policy (43). The component related to the future path z_t^2 is persistently reduced, with the component related to the short-term interest rate z_t^1 falling with a delay. The shock is expansionary at both levels of steady-state interest rates, increasing inflation and output in panels (c) and (d).

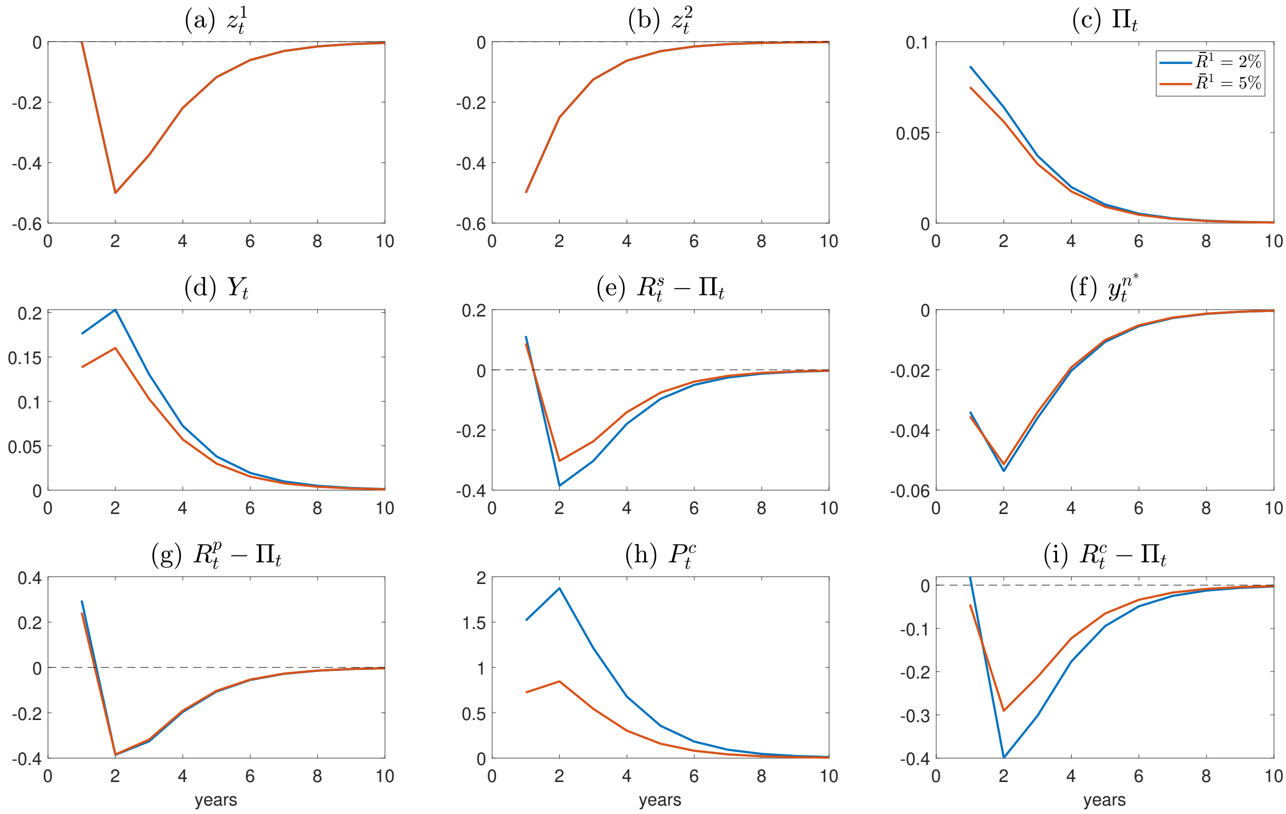

Figure 7: Response to future path shock in general equilibrium

The impact of the future path shock is amplified at low steady-state interest rates. To see why, consider panel (e) which plots the response of the real return to saving. The expansionary nature of the shock creates immediate inflationary pressure that the Taylor-type rule reacts to by endogenously raising the short-term nominal interest rate. Since the rule satisfies the Taylor Principle, there must be a corresponding increase in the short-term real interest rate. This upward pressure is subsequently ameliorated at low steady-state interest rates, via a reduction in term premia and a squeeze on insurance funds that reduces the return to annuities. With the real return to saving falling lower for longer, the impact on inflation and output is larger and the future path shock is more expansionary.

The remaining panels in Figure 7 illustrate the mechanisms that drive the negative response of the real return to saving. In panel (f) the yield to maturity on benchmark n^* period bonds falls, triggering a rise in the duration of liabilities on insurance fund balance sheets. Insurance funds react by following their liability-driven demand and increasing their bond holdings, a "recruitment channel" response that reduces the residual duration risk borne by arbitrageurs and hence compresses term premia. This contributes to reducing the real return to arbitrageurs in panel (g), which falls most at longer horizons once the future path shock starts offsetting the endogenous short-term interest rate increases prescribed by the Taylor Rule. The need to increase bond holdings as the benchmark yield falls also affects the income of insurance funds, which through perfect competition in annuity markets requires the annuity price to rise in panel (h) and the real return to saving to fall in panel (i). The maximum impact occurs after two years. Both mechanisms contribute to the overall fall in the real return to saving, with reduced real returns to arbitrageurs playing a larger role at longer horizons and the squeeze on insurance funds more important at shorter horizons.

6.8.2 Response to monetary policy shocks

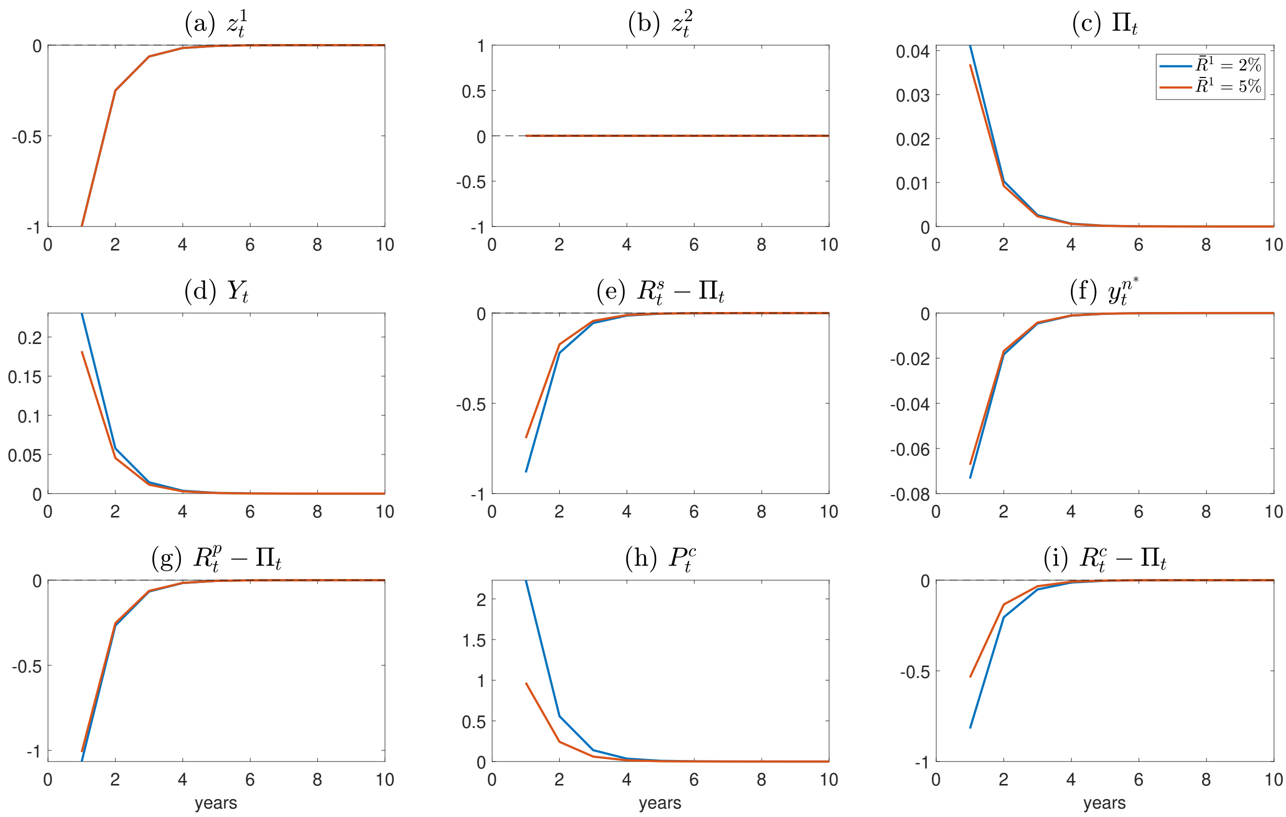

Figure 8: Response to monetary policy shock in general equilibrium

The corresponding general equilibrium responses to a monetary policy shock are in Figure 8. As before, the shock is more expansionary at low steady-state interest rates when liability-driven demand and insurance fund income have more of an effect on the real return to saving. Also as before, the differences in responses by steady-state interest rate are primarily driven by the return to annuities.

7 Concluding remarks

Our claim that monetary policy is more effective at low interest rates depends on the behaviour of arbitrageurs and liability-driven investors. In particular, it relies on the demand curves of insurance funds being notably upward-sloping in bond prices when interest rates are low, with the corresponding demand curves of arbitrageurs downward-sloping. The evidence in Figure 1 in the introduction is consistent with this, with insurance companies and pension funds increasing their relative exposure to long bonds as interest rates fall and long bond prices rise, at the expense of monetary and financial institutions. Further support for this view is provided by Koijen et al. (2017), who report that insurance companies and pension funds increased their holdings of long bonds as interest rates fell in a “recruitment channel” response to the asset purchase programme of the European Central Bank started in 2015.

The way our insurance fund amplifies the actions of monetary policy is compatible with the idea of yield-oriented investors in Hanson and Stein (2015). It is less supportive of the Vayanos and Vila (2021) claim that private investors dampen the impact of monetary policy. Acting through changes in real term premia, our model also speaks to the empirical evidence that excess sensitivity of long-term nominal rates to short-term rates mostly reflects movements in long-term real rates and primarily operates through the real term premium, as argued for the post 2000 period by Beechey and Wright (2009) and Hanson and Stein (2015).

Extending our analysis to general equilibrium reveals an addition channel through which monetary policy actions are amplified. At low interest rates, insurance funds have to purchase a lot of bonds to partially immunise their balance sheets against duration risk. Doing so comes at a price - the opportunity cost of not being able to invest in more lucrative outside opportunities. The income of insurance funds is reduced, which raises the price of annuities above that warranted by the increased duration of their liabilities. Rates of return on annuities fall, which acts as a drag on the return to savings offered to households. This mechanism adds to the way insurance funds amplify future path shocks, and is sufficiently strong to differentiate the response to monetary policy shocks by steady-state interest rates. It suggests that the balance sheet position of insurance companies and pension funds may have important implications for the transmission of monetary policy.

In modelling liability-driven demand, we uncovered new state-dependency in the transmission of monetary policy.[15] The lower are interest rates the more the demand of liability-driven investors strengthens monetary policy. Our model predicts that the potency of monetary policy increases as interest rates get lower, a sharp empirical state-dependent conditionality that has received only limited attention to date.[16] One promising approach is in Rostagno et al. (2021), who looked at whether reducing the policy rate by 10 basis points has a different effect on the yield curve depending on whether interest rates are currently in positive or negative territory. Figure 9 reproduces their results.

[15] Eichenbaum et al. (2022) examine state-dependency of monetary policy that depends on the distribution of savings from refinancing mortgages.

[16] A few empirical papers show that monetary policy effectiveness is at least not reduced at the zero lower bound, e.g., Debortoli et al. (2020), Swanson (2018) and Gürkaynak et al. (2022). These papers use different methods and metrics, but all conclude that monetary policy is no weaker at the zero lower bound than above it.

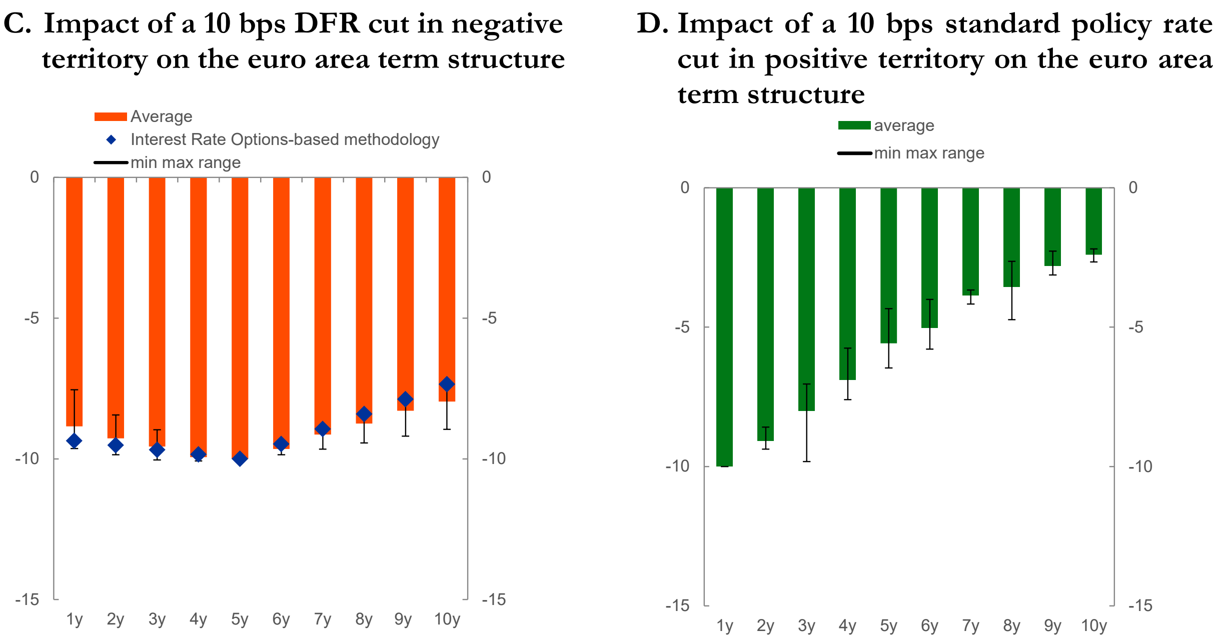

Figure 9: Impact of 10 basis points policy rate cut on euro area term structure, from Rostagno et al. (2021)

The right panel of Figure 9 shows the impact of cutting the policy rate when rates are positive. The largest effect is at a maturity of 1 year, with the impact at longer maturities geometrically declining. This is consistent with liability-driven investors having demand curves at long maturities that are only mildly upward-sloping in bond prices. As monetary policy cuts rates, bond prices rise and yields fall as liability-driven investors hardly react. The left panel of Figure 9 show the effect of the same cut in the policy rate when rates are negative. We now see significant falls along the whole yield curve, compatible with liability-driven investors having demand curves at long maturities that slope strongly upwards in bond prices. As the policy rate falls and bond prices rise, liability-driven investors increase their demand and yields fall even further.

The evidence in Rostagno et al. (2021) is consistent with our model's predictions, but it remains to verify whether we have uncovered a causal relationship. An alternative reading of the evidence is that the cuts in the policy rate when rates were negative revealed information about the location of the effective lower bound on nominal rates and signalled that policy was less likely to be constrained in the future. It should be possible to distinguish between the two explanations as central banks gain more experience with negative interest rates. The longer that monetary policy retains potency at negative interest rates the less likely signalling is important and the more likely a liability-driven mechanism is responsible.

References

Andrés, J., Lopez-Salido, D., & Nelson, E. (2004). Tobin's imperfect asset substitution in optimizing general equilibrium. *Journal of Money, Credit, and Banking*, *36*(4), 655–690.

Angeletos, G.-M. (2002). Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure. *Quarterly Journal of Economics*, *117*(3), 1105–1131.

Beechey, M., & Wright, J. (2009). The high-frequency impact of news on long-term yields and forward rates: Is it real? *Journal of Monetary Economics*, *56*(4), 535–544.

Buera, F., & Nicolini, J. P. (2004). Optimal maturity of government debt without state contingent bonds. *Journal of Monetary Economics*, *51*(3), 531–554.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, *12*, 383–398.

Carlstrom, C. T., Fuerst, T. S., & Paustian, M. (2017). Targeting Long Rates in a Model with Segmented Markets. *American Economic Journal: Macroeconomics*, *9*(1), 205–242.

Chen, H., Ferrero, A., & Curdia, V. (2012). The Macroeconomic Effects of Large-scale Asset Purchase Programmes. *Economic Journal*, *122*(564), F289–F315.

Debortoli, D., Forni, M., Gambetti, L., & Sala, L. (2020). *Asymmetric Effects of Monetary Policy Easing and Targeting* (Working Paper Series No. 1205). Barcelona GSE.

Domanski, D., Shin, H. S., & Shusko, V. (2017). Hunt for Duration: Not Waving but Drowning? *MF Economic Review*, *65*, 113–153.

Eichenbaum, M., Rebelo, S., & Wong, A. (2022). State Dependent Effects of Monetary Policy: the Refinancing Channel. *American Economic Review*, *112*(3), 721–761.

Ellison, M., & Tischbirek, A. (2014). Unconventional government debt purchases as a supplement to conventional monetary policy. *Journal of Economic Dynamics and Control*, *43*, 199–217.

Galí, J. (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

Gertler, M., & Karadi, P. (2011). A model of unconventional monetary policy. *Journal of Monetary Economics*, *58*(1), 17–34.

Gertler, M., & Karadi, P. (2013). QE 1 vs. 2 vs. 3. . . : A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool. *International Journal of Central Banking*, *9*(1), 5–53.

Giglio, S., & Kelly, B. (2018). Excess Volatility: Beyond Discount Rates. *Quarterly Journal of Economics*, *133*(1), 71–127.

Gilchrist, S., López-Salido, D., & Zakrajšek, E. (2015). Monetary Policy and Real Borrowing Costs at the Zero Lower Bound. *American Economic Journal: Macroeconomics*, *7*(1), 77–109.

Gürkaynak, R. S., Karasoy-Can, H. G., & Lee, S. S. (2022). Stock Market's Assessment of Monetary Policy Transmission: The Cash Flow Effect. *Journal of Finance*, *77*(4), 2375–2421.

Gürkaynak, R. S., Swanson, E. T., & Sack, B. P. (2005). The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models. *American Economic Review*, *95*(1), 425–436.

Hanson, S. G., Luca, D. O., & Wright, J. W. (2021). Rate-Amplifying Demand and the Excess Sensitivity of Long-Term Rates. *Quarterly Journal of Economics*, *136*(3), 1719–1781.

Hanson, S. G., & Stein, J. C. (2015). Monetary Policy and Long-Term Real Rates. *Journal of Financial Economics*, *115*(3), 429–448.

King, T. B. (2019). Expectation and duration at the effective lower bound. *Journal of Financial Economics*, *134*(3), 736–760.

King, T. B. (2022). *Real Yields and the Transmission of Central Bank Balance-Sheet Policies* (mimeo). Federal Reserve Bank of Chicago.

Koijen, R. S. J., Koulischer, F., B., N., & Yogo, M. (2017). Euro-Area Quantitative Easing and Portfolio Rebalancing. *American Economic Review: Papers & Proceedings*, *107*(5), 621–627.

Modigliani, F., & Sutch, R. (1966). Innovations in Interest-Rate Policy. *American Economic Review*, *56*, 178–197.

Ray, W. (2019). *Monetary Policy and the Limits to Arbitrage: Insights from a New Keynesian Preferred Habitat Model* (mimeo).

Rostagno, M., Altavilla, C., Carboni, G., Lemke, W., & Motto, R. (2021). *Monetary Policy in Times of Crisis: A Tale of Two Decades of the European Central Bank*. Oxford University Press.

Stein, J. C. (2013). Yield-oriented investors and the monetary transmission mechanism [Speech at "Banking, Liquidity and Monetary Policy," a symposium sponsored by the Center for Financial Studies, Frankfurt, Germany].

Swanson, E. (2018). The Federal Reserve is not very constrained by the lower bound on nominal interest rates. *Brookings Papers on Economic Activity*, *2018*(2), 555–572.

Tobin, J. (1958). Liquidity Preference as Behavior Towards Risk. *Review of Economic Studies*, *25*(2), 65–86.

Vayanos, D., & Vila, J.-L. (2021). A Preferred-Habitat Model of the Term Structure of Interest Rates. *Econometrica*, *89*(1), 77–112.

Woodford, M. (2001). Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking*, *33*(3), 669–728.

A Schematic overview of financial intermediation

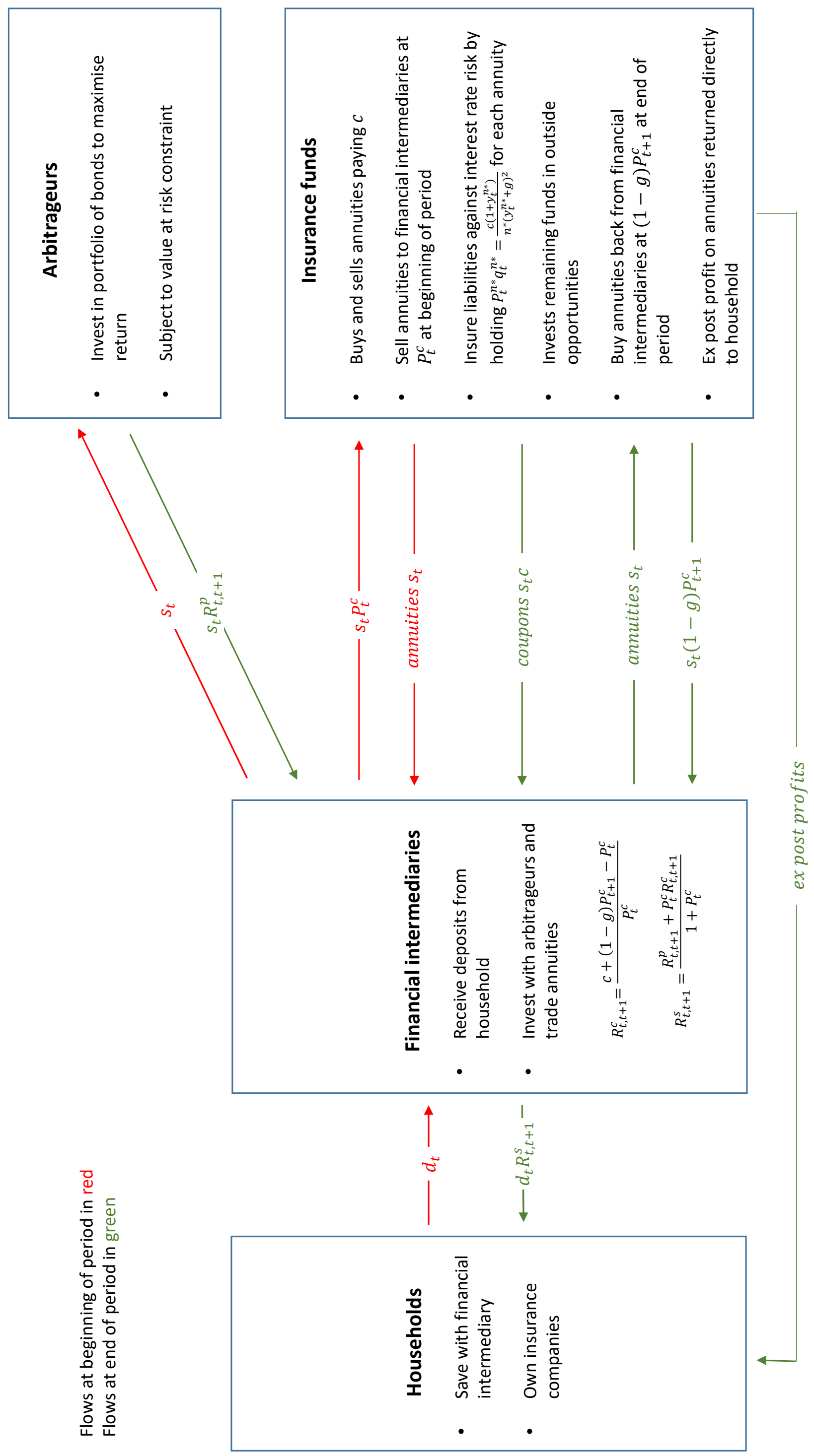

Figure A.1: Schematic overview of financial intermediation

B Linearisation for annuity pricing

The standard linear expansion of the term in square brackets in annuity price recursion (27) is

$$c + P_t^{n^*} q_t^{n^*} (\bar{R}^1 + \xi - E_t R_{t,t+1}^{n^*}) \approx c + \bar{P}^{n^*} \bar{q}^{n^*} (\bar{R}^1 + \xi - \bar{R}^{n^*}) + \begin{bmatrix} -\bar{P}^{n^*} \bar{q}^{n^*} \nabla E_t \bar{R}^{n^*} \\ \left(\bar{R}^1 + \xi - \bar{R}^{n^*}\right) \nabla \bar{P}^{n^*} \bar{q}^{n^*} \end{bmatrix} (X_t - \bar{X}) \quad (54)$$

where ∇ denotes vector derivative with respect to X_t, evaluated at steady state. The excess holding period return on the benchmark bond satisfies (16) and liability-driven demand is (4), hence in steady state

$$\bar{R}^{n^*} = \bar{R}^1 + b_{n^*-1} \Sigma \bar{\lambda} \quad (55)$$

$$\bar{P}^{n^*} \bar{q}^{n^*} = \beta \quad (56)$$

and the vector derivatives are

$$\nabla E_t \bar{R}^{n^*} = \nabla \bar{R}^1 + b_{n^*-1} \Sigma \lambda_1 \quad (57)$$

$$\nabla \bar{P}^{n^*} \bar{q}^{n^*} = \varphi \nabla \bar{y}^{n^*} \quad (58)$$

$\nabla \bar{R}^1$ and $\nabla \bar{y}^{n^*}$ are the vector derivatives of the short rate and benchmark yield, evaluated at steady state. Since the former is the first macroeconomic risk factor and the latter is $y_t^{n^*} = \frac{a_{n^*}}{n^*} + \frac{b_{n^*}}{n^*} X_t$, it follows that

$$\nabla \bar{R}^1 = (1\ 0) \quad (59)$$

$$\nabla \bar{y}^{n^*} = \frac{b_{n^*}}{n^*} \quad (60)$$

The expansion to derive the linearised annuity pricing recursion (28) is then

$$E_t \left[c + P_t^{n^*} q_t^{n^*} (\bar{R}^1 + \xi - E_t R_{t,t+1}^{n^*}) \right] \approx \Omega_0 + \Omega_1 (X_t - \bar{X}) \quad (61)$$

$$\Omega_0 = c + \beta \left(\bar{R}^1 + \xi - \bar{R}^{n^*} \right) \quad (62)$$

$$\Omega_1 = \left[-\beta \left((1\ 0) + b_{n^*-1} \Sigma \lambda_1 \right) + \left(\bar{R}^1 + \xi - \bar{R}^{n^*} \right) \varphi \frac{b_{n^*}}{n^*} \right] \quad (63)$$