Production Networks and the Wealth Distribution[*]

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Abstract

We document substantial cross-country differences in both wealth inequality and the structure of production networks among European economies. Using household-level survey data and input-output tables, our empirical analysis uncovers a robust negative relationship between the intensity of a country's production network (that is, higher share of off-diagonal entries in the input-output matrix) and its degree of wealth inequality, measured by Gini coefficients or top 10% wealth shares of liquid assets. To explain these findings we build a Heterogeneous-Agent New Keynesian model featuring a multi-sector production network. When sectors rely more extensively on cross-sector inputs, wages decrease, leading to lower wealth inequality. Cross-country simulations, in which we replace only input-output matrices across otherwise identical calibrations, replicate observed patterns: for instance, France's more diagonal network structure aligns with a higher wealth Gini, whereas Spain's denser off-diagonal linkages produce significantly lower inequality. Counterfactual scenarios for Germany confirm that removing input-output linkages raises wage dispersion and thus increases wealth concentration. Production network structures emerge as a key determinant of cross-country variation in wealth inequality. Our paper is the first to suggest that policies that alter sectoral interconnections, such as industrial or trade policy, can have important redistributive consequences.

Keywords: Production Networks; Sticky Wages; HANK; Sectoral Linkages.
JEL Classification: E21; E62; H31

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IMPORTANT: This is a very preliminary version, comments are welcome. Please do not quote or share.

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Non-Technical Summary

We document substantial differences between countries in both wealth inequality and the structure of production networks among European economies. Drawing on household-level survey data and input-output tables, our empirical analysis uncovers a robust negative relationship between a country's production network intensity (i.e., stronger cross-sector linkages) and its degree of wealth inequality. Countries with heavier reliance on their own sectors (i.e., large diagonal terms in the input-output matrix) tend to have higher wealth Gini coefficients, whereas those that source inputs more broadly across different industries (larger off-diagonal shares) exhibit more moderate inequality.

Motivated by these findings, we develop a Heterogeneous-Agent New Keynesian (HANK) model with sticky wages and a multi-sector production network to rationalize this empirical evidence. In our framework, sectors vary in how intensively they buy and sell inputs between each other, creating differential wage and price profiles across industries. When cross-sector linkages are dense, wages are more evenly compressed and profit extremes become more subdued, leading to a more equitable distribution of wealth. Conversely, economies characterized by strong self-sourcing (high diagonal reliance) can experience concentrated profits and steeper wage differentials, resulting in higher wealth inequality.

We calibrate the model to different European countries, matching their observed input-output structures and key macro aggregates. The simulations generate wealth Gini coefficients in line with the data—ranging, for example, from about 0.82 in Germany and 0.81 in France to around 0.65 in Italy and 0.45 in Spain. Our results suggest that the intensity of inter-sectoral linkages is a crucial determinant of how households accumulate and distribute wealth. In countries with denser networks, households face milder income and profit volatility, which lowers precautionary savings and reduces wealth concentration.

These findings highlight the importance of production networks for understanding cross-country heterogeneity in wealth inequality. They further imply that policies that alter sectoral interconnections—whether through trade, energy, or industrial strategies—can have significant distributional ramifications. For policymakers, this project remarks the need for central banks and fiscal authorities alike to account for the structure of production networks when designing policy interventions, especially in highly integrated economies.

1 Introduction

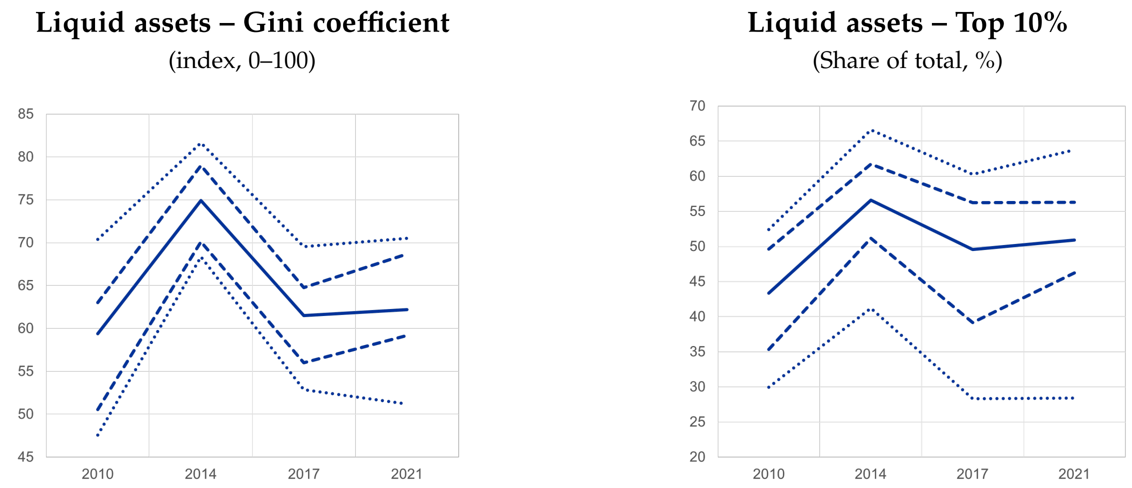
In recent years, the study of wealth distribution has taken center stage in macroeconomic analysis, not least because Heterogeneous-Agent New Keynesian (HANK) models have underscored the critical role of household income and wealth heterogeneity for aggregate demand and policy transmission. At the same time, the literature studying sectoral interdependencies, captured through input-output linkages has gained importance.

However, much of the HANK literature still treats production as a single-sector process, abstracting from the possibility that inter-sectoral connections could shape both macroeconomic dynamics and wealth distribution. In this paper, we argue that the structure of production networks is tightly linked to the extent of wealth inequality. An economy featuring dense cross-sector input sourcing (large off-diagonal shares in their IO matrices) tend to exhibit lower wealth Gini coefficients, whereas those with strong self-sourcing (high diagonal shares) often record higher wealth inequality.

Figures 1 and 2 illustrate two broad patterns among European countries: a stable level of wealth inequality and production network, but heterogeneous across countries. The euro area, in particular, provides an important illustration of how sectoral linkages can profoundly influence macroeconomic and distributional dynamics. For instance, Germany, as a major industrial power, relies heavily on imported energy resources, exposing it to supply disruptions and cost fluctuations driven by geopolitical factors and shifting global markets. These vulnerabilities create linkages throughout its production networks: upstream price hikes or input shortfalls can quickly raise production costs and constrain profit margins in downstream industries. For energy-intensive sectors, the ramifications are especially pronounced, potentially lowering competitiveness and productivity in the short run.

To motivate this paper, a broad literature on production networks shows that the interconnections between firms (and industries) can amplify shocks and affect aggregate outcomes. For example, there is a theoretical literature showing how input–output linkages not only propagate technological improvements (thereby influencing growth) but also create heterogeneity in firm-level performance (which, in turn, affects wages). Although many of these studies focus on the transmission of shocks or growth dynamics per se, the network structure also plays a role in wage formation. A key empirical contribution comes from studies relating a firm's network position to its wage outcomes. For instance, Huneeus et al. (2021) document

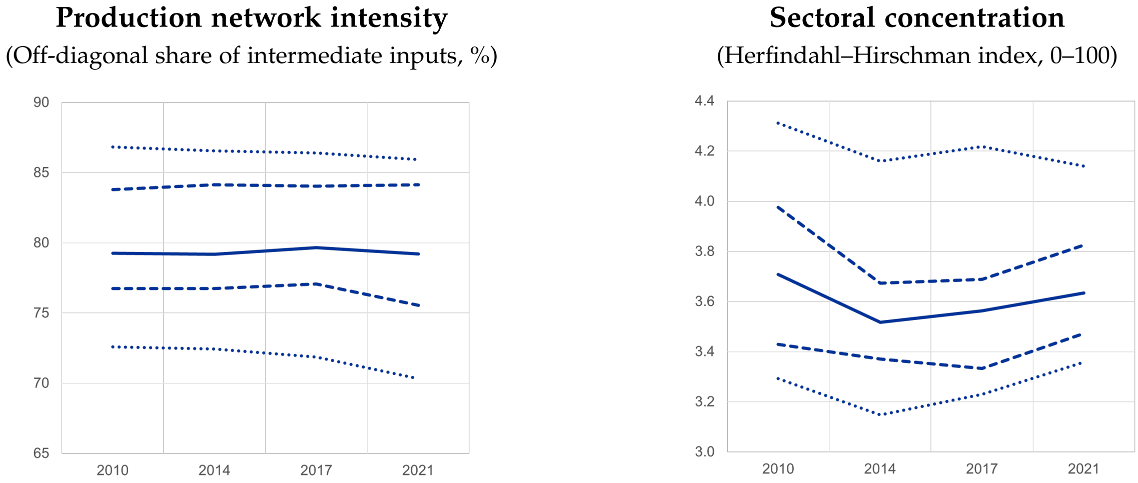
Figure 1: Distribution of liquid assets across euro area countries over time



Source: Household Finance and Consumption Survey. Notes: The chart is based on statistics for a varying panel of countries over time, including—in its largest composition—Germany, France, Italy, Spain, the Netherlands, Austria, Belgium, Finland, Ireland, Portugal, Greece, Slovenia, Croatia, Lithuania, Latvia, Estonia, Sweden, Denmark, Bulgaria, Romania, Poland, Czechia, and Hungary. The solid line depicts the cross-country median, while the dashed (dotted) lines indicate the 25th and the 75th (10th and 90th) percentiles of the cross-country distribution.

that firms with better access to both buyers and suppliers tend to pay higher wages (or generate a higher earnings premium) and simultaneously exhibit lower labor shares. This study shows that the structure of production networks can partly explain differences in wage levels across firms, suggesting that shocks that affect a firm's network connectivity can also shift its wage bill and contribute to overall wage dispersion. Similarly, we are motivated by research on worker mobility along production networks as that in Cardoza et al. (2020), where the authors find that when workers move between firms that are connected through supply-chain relationships, they tend to experience persistent wage growth. These findings imply that network linkages are not just about input–output relationships; they also matter for the evolution of wages at the micro level, with consequences for aggregate inequality. HANK models emphasize that heterogeneous household responses to income shocks—in part driven by differences in wage income—are crucial for understanding the distributional impact of macroeconomic policies. The empirical evidence that production networks influence wage levels (and wage dispersion) motivate us to extend the heterogeneity in coming from the households side by incorporating network-induced wage dynamics. If production networks cause shifts in wages (as seen in the earnings premia or via worker mobility), then these wage changes can be transmitted into the cross-sectional dispersion of income. This supports the idea that a change in

Figure 2: Production network intensity and sectoral concentration



Source: FIGARO Input-Output tables. Notes: The chart is based on statistics for a varying panel of countries over time, including—in its largest composition—Germany, France, Italy, Spain, the Netherlands, Austria, Belgium, Finland, Ireland, Portugal, Greece, Slovenia, Croatia, Lithuania, Latvia, Estonia, Sweden, Denmark, Bulgaria, Romania, Poland, Czechia, and Hungary. The solid line depicts the cross-country median, while the dashed (dotted) lines indicate the 25th and the 75th (10th and 90th) percentiles of the cross-country distribution.

wages, driven by network reconfigurations or shocks, will in turn affect inequality, an insight that has already been central to HANK analyses of monetary and fiscal policy.

This paper combines the insights of a HANK model with a production network framework to examine how heterogeneous sectoral interdependencies shape wealth inequality. By integrating suggestive empirical evidence from household finance data (HFCS) and input-output tables for multiple European countries, we show how features such as sectoral concentration or the extent of cross-sector linkages can have significant implications for wealth distribution. In the case of Germany, for instance, energy dependencies act as a salient channel through which shocks propagate, influencing both productivity and the allocation of economic gains across households. We find that higher sectoral concentration reduces the level of real wages, by increasing the synergy in the production structure. Through the trickling up of the excess savings, this transmits to wealth inequality. When we shut down the production network intensity, this yields higher real wages but less synergy, lower TFP, which ultimately raises wealth inequality in the long run. Zero sectoral concentration also increases wages but does so uniformly, leading to a modest increase in the wealth Gini as top-skilled households save more.

Literature. Literature at the intersection of heterogeneous-agent macroeconomics and production networks is still small, but it might expand rapidly, given the interest in how wealth and income distributions shape policy transmission and aggregate fluctuations. On the one hand, HANK models emphasize the importance of wealth inequality, precautionary savings, and marginal propensities to consume (MPCs) for policy effects. Production networks research highlights how industry-level shocks and technological interdependencies can cascade through input-output (IO) linkages.

Seminal contributions have underscored the significance of household heterogeneity for understanding aggregate outcomes. For instance, Kaplan et al. (2018) illustrate a HANK model that yields realistic distribution of wealth and marginal propensity to consume, identifying how the indirect effects of an unexpected cut in interest rates is stronger than the direct channel through inter-temporal substitution. In a related work, Auclert (2019) demonstrates that redistribution channels, through which monetary or fiscal policy transfers resources across different types of households, can considerably influence aggregate demand. Similarly, Bayer et al. (2019) explores how income risk and incomplete insurance in a heterogeneous-agent framework shape macroeconomic volatility and the effectiveness of policy interventions. In more recent work, Grancini (2024) studies the distributional effects of systematic fiscal policy, showing how changes in interest rates due to high public debt lead to lower state-dependent fiscal multipliers.

In the literature on the production network, Acemoglu et al. (2012) provides early evidence on how microeconomic disturbances at the firm or industry level can become large enough to affect aggregate outcomes through input-output relationships. Alongside, Carvalho and Tahbaz-Salehi (2019) offer a foundational overview of how these sectoral and firm-level connections serve as channels for both shock amplification and propagation. Their work systematically maps out how supply-chain linkages transform local disruptions into macroeconomic fluctuations, emphasizing that network structures - how intensely each sector depends on inputs from others - are crucial for understanding and predicting the size of aggregate spillovers.

Building on these insights, several studies have begun to treat network structures not as fixed, but as the endogenous outcome of firms' strategic decisions. In Acemoglu and Azar (2020), firms choose which inputs to source by weighing economic incentives and constraints, resulting in a model where input linkages evolve. This shift to endogenous networks captures the reality that producers adapt their supply chains in response to profitability, technological change, and policy shifts.

Ghassibe (2021) further refines this perspective by incorporating non-linear monetary policy transmission into a sticky-price model with endogenous input-output linkages. His findings reveal that when firms can reconfigure their supply chains, the overall response of output to a monetary shock is shaped by cycle dependence, path dependence, and size dependence. In particular, the density of the production network can fluctuate endogenously: looser monetary policy may encourage the creation of additional linkages, which then alters how future shocks propagate.

In a related contribution, Dhyne et al. (2023) explores the role of fixed costs in shaping the formation of endogenous production networks. By introducing fixed costs to establishing new supplier-buyer relationships, the paper shows that network structures can become more rigid, potentially reducing the economy's flexibility in adapting to shocks. Firms must balance the costs of forging new linkages against the expected benefits, leading to complex, and sometimes path-dependent, network dynamics.

A smaller body of work explicitly combines HANK frameworks with production-network structures and it is closely related to our paper. Schaab and Tan (2023) propose a HANK-IO model that illustrates how shocks propagate through different sectors once we account for heterogeneity in both households and firms. Their approach underscores that sectoral heterogeneity can matter for the nominal transmission of policy, particularly via differences in price rigidity, but that the real transmission hinges primarily on how household heterogeneity shapes MPCs and consumption responses.

Overall, the literature indicates that the distribution of wealth and the structure of production networks might be deeply interconnected. While a canonical HANK models underline how household heterogeneity can amplify or dampen the effects of monetary and fiscal policy, the more recent focus on network formation demonstrates that the production side of the economy is equally dynamic. Firms respond to technological conditions, policy shifts, and cost structures by adjusting their input-output linkages, influencing both the propagation of shocks and the distribution of income.

By building on these insights, with our study we aim to deepen our understanding of how sectoral linkages, shape both aggregate dynamics and the wealth distribution, with a particular focus on the euro area. We show that production networks have fundamental distributional consequences. Our model shows that network transformations have tangible distributional effects: changes in sectoral prices and wages transmit across supply chains, influencing household incomes and ultimately wealth distribution. We show that policy makers must account not just for household heterogeneity but also for the evolution of production linkages, to design effective stabilization and distributive policies.

Roadmap The remainder of the paper is organized as follows. We begin with a brief presentation of the data and the empirical strategy in section 2. Then we present our model in section 5 and the related calibration in section 4. In section 5 we discuss the results of the model. Section 7 concludes.

2 Data and Empirical Evidence

We estimate the relationship between production network intensity and wealth inequality using micro-level household data from the HFCS (Household Finance and Consumption Survey) and national input-output tables. Our empirical approach consists of measuring sectoral interdependencies through the off-diagonal input-output shares. Moreover, we calculate wealth inequality indicators, including the Gini coefficient. We conduct cross-country regressions using production network measures and income inequality coefficients to assess the correlation between network intensity and wealth dispersion.

Our findings indicate that countries with denser inter-sectoral linkages exhibit lower wealth inequality. In contrast, economies with high self-sourcing levels tend to display higher Gini coefficients, suggesting that sectoral isolation reinforces wealth concentration by exacerbating wage and profit disparities.

Data Sources and Variables. We use data from the HFCS, which provides household-level data on liquid assets (checking, savings, etc.), income, and demographics, to compute the Gini coefficient and the top 10% share of liquid assets for the a panel of 23 countries, covering up to 4 waves (2010, 2014, 2017, 2021).[1]

For the production network, we use the FIGARO Input-Output Tables (from Eurostat), data

[1]We use data for 23 European countries, namely Germany, France, Italy, Spain, the Netherlands, Austria, Belgium, Finland, Ireland, Portugal, Greece, Slovenia, Croatia, Lithuania, Latvia, Estonia, Sweden, Denmark, Bulgaria, Romania, Poland, Czechia, Hungary. Hence, among the countries available in the HFCS database, we do not consider Slovakia, Cyprus, Luxembourg and Malta, which stand out as outliers in one or more variables included in our analysis.

that offer industry-by-industry input-output matrices for 45 countries, including all European Union member countries, as well as the rest of the world. We compute the production network intensity as the sum of off-diagonal shares $\sum_{j \neq k} \omega_{jk}$ (or the fraction of total intermediate inputs that is off-diagonal) and an index of sectoral concentration, the Herfindahl-Hirschman index, measuring how concentrated a country's total output is within a few sectors.

Methodology. We estimate the following panel specification:

$$y_{i,t} \; = \; \alpha_i + \beta \, \text{PNint}_{i,t} \; + \; \gamma \, \text{Conc}_{i,t} \; + \; \delta \, Z_{i,t} \; + \; \epsilon_{i,t}, \tag{1}$$

where $y_{i,t}$ measures wealth inequality in country i and year t (e.g. Gini of liquid assets or top-10% share), $\text{PNint}_{i,t}$ is the production network intensity (off-diagonal share), $\text{Conc}_{i,t}$ is sectoral concentration (Herfindahl index). $Z_{i,t}$ includes other controls (size of economy, average length of input-output paths, etc.). Morever, α_i is a country fixed effect capturing time-invariant unobserved heterogeneity. We use heteroskedasticity-robust standard errors.

2.1 Results

Table 1 summarizes our main findings. For liquid-asset Gini and top-10% share we find that production network intensity is strongly negatively correlated with wealth inequality. Quantitatively, a 1 p.p. increase in cross-sector usage predicts around a 3.3 p.p. drop in the top 10% wealth share.

Sectoral concentration also shows a significant negative correlation with liquid-asset Gini measures (in the range of 23–24 p.p. for top 10% share), suggesting that more "concentrated" economies may also exhibit lower measured inequality. We discuss plausible channels in Section 6.

The negative coefficient on the production network intensity aligns with the premise that "denser" cross-sector linkages moderate extremes in wages/profits, mitigating top-end wealth concentration. The strong negative coefficient on sectoral concentration is more surprising at first glance; we discuss in Section 6 how certain types of concentration might reflect large but stable dominant sectors that do not necessarily foster extreme wealth accrual at the top.

Overall, these results underscore a robust empirical correlation: more intense cross-sector linkages and certain concentration patterns are associated with lower measured inequality.

Table 1: Production Network Intensity, Sectoral Concentration, and Wealth Inequality

	Liquid Assets		**Total Gross Income**	
	Gini	Top 10%	Gini	Top 10%
PN intensity	-2.133**	-3.264***	-0.245	-0.171
	(0.815)	(0.814)	(0.180)	(0.152)
Sectoral conc.	-23.055**	-23.788***	-0.464	-0.896
	(10.796)	(7.643)	(2.447)	(2.371)
Controls	Y	Y	Y	Y
Fixed effects	Y	Y	Y	Y
Observations	63	63	63	63
R^2	0.22	0.15	0.06	0.08

Notes: Controls include other characteristics of the production structure (labor intensity in terms of total output, average length of path, and size of the economy). Numbers in parentheses refer to robust standard errors.

In the next section, we develop a HANK model with a multi-sector network to rationalize this relationship and examine how sector-level wage and profit formation transmits to the distribution of wealth.

3 The Model

The model embeds a production network with input-output linkages between sectors in an otherwise standard Heterogeneous Agent New Keynesian (HANK) model with sticky wages and flexible prices. Specifically, following Auclert et al. (2018), the model economy is populated by households who consume and work, wage-setting labor unions with adjustment costs, price-setting sectoral producers, a monetary authority following a standard Taylor rule, and a fiscal authority borrowing from net saving households to finance unproductive spending, repay outstanding debt and rebate the rest to all households. On the household side, the model incorporates heterogeneous agents with idiosyncratic income shocks and incomplete markets. On the firm side, the model includes a representative firm for each sector producing a goods or services with labor and intermediate inputs.

3.1 Households

Households face idiosyncratic risk and aggregate uncertainty. Each household i decides how much to consume and save given their state. The utility function of household i at time t

depends on consumption, c_{it} and work time, n_{it} and it is given by:

$$U(c_{it}, n_{it}) = \frac{c_{it}^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{n_{it}^{1+\nu}}{1+\nu}, \tag{2}$$

where σ denotes the intertemporal elasticity of substitution, φ regulates the disutility of work, and ν is the inverse of the Frisch labor elasticity.

Recursive formulation of the household problem. At each point in time t, household i is characterized by the vector (e_{it}, a_{it}), where e_{it} denotes the idiosyncratic income shock and a_{it} the specific holdings of liquid assets. At any point in time, each household is in one of n_e idiosyncratic states and they transition between any two such states e_{it} and e_{it+1} with exogenous probability $P(e_{it}, e_{it+1})$. Each household has a fixed transition matrix Π, and the mass of households in state s is equal to π_s, such that $\sum_{s_t} \pi_s e(s_t) = 1$. A union chooses for the agents the hours worked n_{it}. The household's optimization problem over consumption and future asset holdings recursively is defined as follows:

$$\begin{aligned} V_t(e_{it}, a_{it-1}) = \max_{c_{it}, a_{it}} & \left\{ \frac{c_{it}^{1-\sigma}}{1-\sigma} - \phi \frac{n_{it}^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t V_{t+1}(e_{it+1}, a_{it}) \right\} \\ \text{s.t.} \quad & c_{it} + a_{it} = (1+r_t) a_{it-1} + \frac{W_t}{P_t} e_{it} N_t - \tau \\ & a_{it} \geq \underline{a}, \end{aligned} \tag{3}$$

3.2 Government

The government issues bonds, B^g, to issue transfers, τ and spend on goods and services, G_t, to balance its budget constraint period by period:

$$\tau_t = B_t + \frac{P_t^G}{P_t^C} G_t - (1+r_t) B_{t-1} \tag{4}$$

3.3 Monetary authority

The monetary authority follows a standard Taylor rule to set the nominal interest rate:

$$i_t = r_t^* + \phi_\pi \pi_t + \epsilon_t \tag{5}$$

where r_t^* is the optimal real interest rate, and ϕ_π and ϕ_y are the inflation Taylor rule coefficient and the Taylor rule coefficient on output, respectively.

3.4 Labor Unions

Following Auclert et al. (2018), and as in a standard New Keynesian model with sticky wages, household labor hours, n_{it}, are determined by union labor demand. A continuum of unions exists, k, and a different labor union settles each labor type wage. Firms use labor in their production function, which is a CES bundle of type-specific labor inputs. This is each union, k, aggregates efficient units of work into a union-specific task $N_{kt} = \int e_{it} n_{ikt} di$. A competitive labor packer then includes these tasks into aggregate employment, using a CES bundle of type-specific labor inputs. At a given time each union asks their members to supply hours according to, $n_{ikt} = N_{ikt}$, and setting wages to maximize the average utility of households, taking as given their consumption-savings decisions. Setting a nominal wage, W_{kt}, involves a quadratic adjustment cost similar to the price adjustment cost incurred by the firm:

$$\psi_t^w \left(W_{kt}, W_{kt-1} \right) = \left(\frac{\mu_w}{\mu_w - 1} \right) \left(\frac{1}{2\kappa_w} \right) \left[\log \left(W_{kt} / W_{kt-1} \right) \right]^2 .$$

The union maximization problem leads to a Phillips curve[2] for wage inflation:

$$\pi_t^w = k_w \left(\phi N_t^\nu - \frac{\epsilon - 1}{\epsilon} \frac{W_t}{P_t} c_t^{-\sigma} \right) + \beta(\pi_{t+1}^w) \tag{6}$$

3.5 Firms: Production Network

The economy is characterized by N production sectors. Each sector comprises intermediates firms whose dynamics pricing problem can create in transition sectoral Phillips curves. Firms in sector j produce output using labor N_{jit} and a vector of intermediate goods X_{jkt}, sourced from other sectors. The production function for sector j is:

$$Y_{jt} = A_{jt} N_{jt}^{1 - \sum_{k=1}^{S} \omega_{jk}} \prod_{k=1}^{S} X_{jkt}^{\omega_{jk}}, \tag{7}$$

where A_{jt} is the productivity mapping.

Assumption 1. (Productivity mapping): For every sector $j = 1, 2, \ldots, J$, the productivity

[2] Check Appendix XX for the complete derivation.

mapping A_{jt} takes the following form:

$$A_{jt} = Z_t \left(1 - \sum_{k=1}^{S} \omega_{jk}\right)^{-(1-\sum_{k=1}^{S}\omega_{jk})} \prod_{k=1}^{S} \omega_{jk}^{-\omega_{jk}}, \tag{8}$$

and Z_t is aggregate productivity following an AR(1) process:

$$\log Z_t = \rho_z \log Z_{t-1} + \zeta_t.$$

Profit Maximization leads to optimal labor demand:

$$W_t = P_{jt} \left(1 - \sum_{k=1}^{S} \omega_{jkt}\right) A_{jt} N_{jt}^{-\sum_{k=1}^{S}\omega_{jkt}} \prod_{k=1}^{S} X_{jkt}^{\omega_{jkt}}.$$

The optimal demand for intermediate inputs is:

$$P_{kt} = P_{jt}\omega_{jkt} A_{jt} N_{jt}^{1-\sum_{k=1}^{S}\omega_{jkt}} \prod_{k=1}^{S} X_{jkt}^{\omega_{jkt}-1}.$$

$$Y_{jt} = A_{jt} N_{jt}^{1-\sum_{k=1}^{S}\omega_{jkt}} \prod_{k=1}^{S} X_{jkt}^{\omega_{jkt}}.$$

Input Demand. Cost minimization implies that firms minimize costs, leading to input demand:

$$X_{jk} \;=\; \omega_{jk} \, \frac{P_j \, Y_j}{P_k}, \tag{9}$$

where P_j and P_k are sectoral prices.

Labor Demand. The labor share θ_j means the wage bill is $W N_j$. In equilibrium, the ratio $\frac{W N_j}{P_j Y_j} \approx \theta_j$ up to markups or rigidities.

3.6 Market Clearing Conditions and Aggregation

Equilibrium requires clear markets for each sectoral good j and for bonds.

Goods market clearing in sector j requires that total production be equal to total use. The output of each sector j, Y_{jt}, is used as intermediate goods by other sectors and for final consumption. The goods market clearing condition for each sector is:

$$Y_{jt} = C_{jt} + G_{jt} + \sum_{k=1}^{S} X_{jkt},$$

where C_t is the final consumption of goods, and X_{kjt} are intermediate goods used by sector k.

Consumption indices are CES aggregates of consumption of sectoral goods:

$$C_t = \prod_{k=1}^{S} C_{jt}^{\omega_j^C}, \qquad G_t = \prod_{k=1}^{S} G_{jt}^{\omega_j^G}$$

The sum of sectoral consumption is equal to aggregate consumption:

$$\sum_{k=1}^{S} P_{jt} C_{jt} = P_t^C C_t, \qquad \sum_{k=1}^{S} P_{jt} Y_{jt} = \tilde{Y}_t \tag{10}$$

The bond market clears when the net bond position of the household sector is equal to the government's outstanding debt, that is:

$$B_t = \int a dD$$

Outputs of the Model

To capture the overall economic activity, we consider an aggregate resource constraint or GDP identity.

Aggregate Goods Market Clearing (Aggregate Resource Constraint):

$$Y_t = C_t + G_t + \sum_{j=1}^{S} \sum_{k=1}^{S} X_{jkt}. \tag{11}$$

Value Added in Sector *j*:

$$\text{Value Added}_{jt} = Y_{jt} - \sum_{k=1}^{S} P_{kt} X_{jkt}. \tag{12}$$

Aggregate Nominal GDP as Sum of Value Added. Nominal GDP is the sum of all expenditures on goods for final use, that is:

$$GDP_t = \sum_{j=1}^{S} \left(Y_{jt} - \sum_{k=1}^{S} P_{kt} X_{jkt} \right) \tag{13}$$

$$= C_t + G_t. \tag{14}$$

Final Aggregate Resource Constraint:

$$GDP_t = C_t + G_t. \tag{15}$$

3.7 Stationary Equilibrium

Definition (Competitive Equilibrium). Let $\mu_0(a,e)$ be the initial distribution of households over assets and idiosyncratic shocks, and B_{-1} the initial government debt. Suppose we are given exogenous sequences for sectoral productivities $\{A_{j,t}\}_{j=1}^{J}$, government spending $\{G_{j,t}\}_{j=1}^{J}$, lump-sum taxes $\{\tau_t\}$, and a monetary policy rule (24). A *competitive equilibrium* is a collection of sequences

$$\{r_t,\, i_t,\, \pi_t^w,\, W_t,\, P_{j,t},\, Y_{j,t},\, X_{jk,t},\, N_{j,t},\, \Pi_{j,t}\}_{t=0}^{\infty}$$

for sectoral prices and quantities, together with

$$\{c_{it}(a,e),\; a_{it}(a,e)\}_{t=0}^{\infty} \quad \text{and} \quad \{\mu_t(a,e)\}_{t=0}^{\infty},$$

such that:

1. **Household Optimization:** For each (a,e) and every t, $c_{it}(a,e)$, $n_{it}(a,e)$, and $a_{it}(a,e)$ solve the household problem (17) given prices and taxes, yielding $V_t(a,e)$.
2. **Wage-Setting/Unions:** Wages satisfy the sticky-wage Phillips curve (22), derived from union optimization, and labor is allocated according to firms' labor demands.
3. **Firm Optimization:** In each sector j, firms minimize costs subject to (19) and choose $N_{j,t}$, $X_{jk,t}$, and $p_{j,t}$ (or $P_{j,t}$) to maximize profits $\Pi_{j,t}$, taking W_t and $P_{k,t}$ as given.

4. **Goods Market Clearing:** For each sector j, $Y_{j,t}$ satisfies

$$Y_{j,t} \;=\; \sum_{k=1}^{J} X_{kj,t} \;+\; C_{j,t} \;+\; G_{j,t}.$$

5. **Bond Market Clearing:** The aggregate amount of bonds held by households equals government debt,

$$\int_0^1 a_{it}\, d\mu_t(a,e) \;=\; B_t.$$

6. **Government Budget Constraint:** Government debt evolves according to

$$B_t = (1+r_t)B_{t-1} + \sum_j P_{j,t}\, G_{j,t} \;-\; \int_0^1 \tau_t\, d\mu_t(a,e).$$

7. **Distribution Consistency:** μ_{t+1} is generated by households' optimal asset choices and the Markov process for e_{it}.

8. **Monetary Policy Rule:** The nominal interest rate i_t follows (24), given inflation π_t and output y_t.

When these conditions are satisfied for all $t \geq 0$ given the initial state (B_{-1}, μ_0) and the exogenous paths $\{A_{j,t}, G_{j,t}, \tau_t\}$, the economy is in a competitive equilibrium.

4 Calibration

The model calibration is mainly focused on Germany. We calibrate the model to match the Germany economy with moments following the literature, in particular, Kaplan et al. (2018) and Auclert et al. (2024). We also report endogenously calibrated parameters, for the main calibrated benchmark economy. These change when transitioning from one steady-state to another. All aggregate variables are in relation to GDP.

4.1 Preferences and Labor

We set the standard Frisch elasticity of labor supply to 0.5, similar to what is used in the literature. The disutility of work and the discount factor are among the parameters calibrated to match key moments in the data. The coefficient of risk aversion is set to be equal to 0.5 as in Bayer et al. (2019). As standard in the literature, the levels of β and ϕ are calibrated to hit a target for the level of government bonds in the economy. For the standard calibration, β is 0.972 and the disutility of labor is 1.69. These parameters are obtained to match the German level of the IO table, data publicly available in the Eurostat database (Figaro).

4.2 Government, Monetary Policy and other arameters

We set government spending, G, to around 0.20% of GDP and government bonds, as in the data (Eurostat-average 2000-2024). For monetary policy, we use the same parameters as in Auclert et al. (2020), that is, we set the response of the central bank to be equal to 1.5. Also, the price markup is 1.1, as in Auclert et al. (2020).

The nominal rigidity of the New Keynesian Phillips Curve is set to be 0.1. Productivity and labor demand (when the economy is in full employment) are both set to 1. The factor prices are endogenously calibrated, for each calibration for the different levels of government debt. The Markov chain points are 7, one for each income state. There are 500 points on the asset grids. The rest of the parameters are in table 2

Table 2: Calibrated Parameters for the HANK Model calibrated to the German Economy

Parameter	Description	Value
Household & Preferences		
β	Discount factor	0.967
EIS	Elasticity of intertemporal substitution	0.5
Frisch	Frisch elasticity of labor supply	0.5
χ	Disutility of labor	0.1
W	Wage level	2.0
ρ_e	Autocorrelation of earnings shock	0.966
σ_e	Std. dev. of earnings shock	0.5
n_e	Earnings shock grid points	7
$\underline{a}$	Borrowing constraint (min. asset level)	0.0
Government & Policy		
π	Steady-state inflation	0.0
G	Government spending (fraction of GDP)	0.2034
B	Bond supply / government debt	70%
ϕ_π	Taylor rule coefficient on inflation	1.5
ϕ_T	Fiscal policy parameter	0.1
r	Real interest rate	0.005
ϕ_τ	Tax parameter	0.2
T^{ss}	Steady-state taxes	0.0
Grid & Other Model Parameters		
a_{min}	Minimum asset level	0.0
a_{max}	Maximum asset level	150.0
n_a	Number of asset grid points	500
$A_{j,\text{shock}}$	Asset shock parameter	0.1
Z	Technology normalization	1.0
μ	Markup parameter	1.1
κ	Price adjustment cost parameter	0.1
$v\varphi$	Utility/Adjustment parameter	1.0
γ	Curvature parameter	0.2
α	Additional parameter (should be ≤ 0)	-0.01
$markup_{ss}$	Steady-state markup	1.015
$markup_w$	Wage markup	1.015
κ_w	Wage rigidity parameter	0.2
$ishock$	Income shock indicator	0.0
C	Consumption normalization	1.0

5 Quantitative Results

We calibrate the model primarily to Germany, then conduct two main exercises: a cross - country comparison, and counterfactual scenarios for Germany.

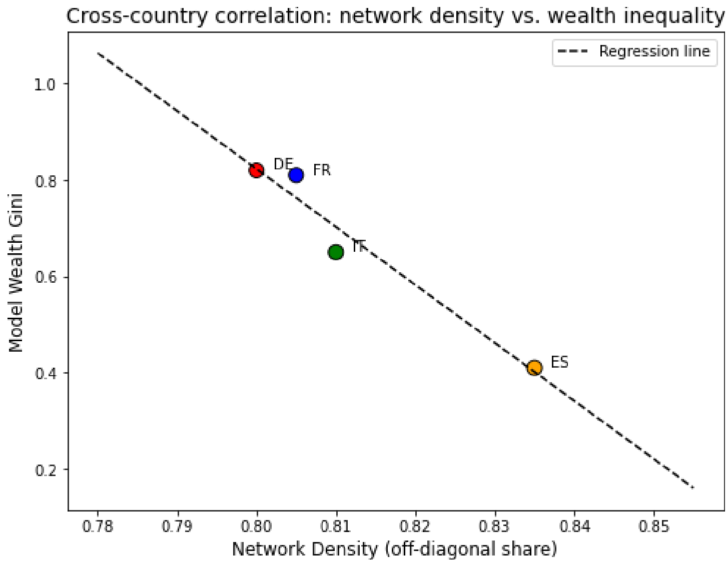


Figure 3: Cross-Country Variation in Network Density vs. Wealth Gini (Model)

5.1 Cross-Country Comparison

We calibrate the model to data from Germany and to compare it with other countries, France, Italy and Spain, we change only the input-output matrix $\{\omega_{jk}\}$ and the production shares. In other words, we hold household preferences, wage rigidity parameters, and fiscal policy rules fixed at 'Germany-like' values. We then replace only the IO matrix $\{\omega_{jk}\}$ and sectoral shares with those from other European countries (France, Italy, Spain, etc.). Figure 3 and Table 3 summarize the main results. We find that countries with a more concentrated matrix $\{\omega_{jk}\}$ (diagonal) exhibit higher wealth Gini, because sector wages respond differently. Countries with heavier off-diagonal shares see more stable real wages and lower inequality.

Countries with higher off-diagonal IO shares (e.g. Spain) exhibit lower wealth Gini in the model, consistent with the data. Heavier diagonal reliance (e.g. France) leads to higher wealth Gini, as certain sectors earn higher markups or pay higher wages, increasing the saving of top earners. This channel that goes from wages to savings and then to inequality is called the 'Trickling up of Excess Savings', first seen in Auclert et al. (2023): the poorest households with higher MPCs spend their excess savings down the fastest, increasing the incomes of other households and their excess savings. This leads to a long-lasting in aggregate demand until excess savings have "trickled up" to the richest savers with the lowest MPCs, raising wealth inequality.

Table 3: Cross-Country Exercise output of the model. Starting from the benchmark economy, we substitute the IO matrix of France, Italy, and Spain.

Parameter	DE	FR	IT	ES
β	0.9507	0.9522	0.9600	0.9521
w	0.1300	0.1210	0.0360	0.0100
C	8.3500	7.3100	1.9400	0.2900
MPC	0.5600	0.5400	0.1500	0.1100
HTM	0.5000	0.4800	0.0700	0.1100
T	0.2300	0.2400	0.3300	0.3500
GINI	0.8200	0.8100	0.6500	0.4160

5.2 Counterfactual Scenarios in Germany

Starting from the German calibration, we systematically alter only one dimension of the input-output structure, by changing the IO matrix $\{\omega_{jk}\}$ only. We perform three different separated experiments: the zero production network intensity, the no sectoral concentration and the half labor intensity.

For the zero production network intensity, we set all off-diagonal $\omega_{j\neq k} = 0$ and reallocate that mass to $\omega_{j=j}$, preserving total input share. The economy becomes highly self-sufficient in each sector. This means removing cross-sector linkages, so that each sector primarily relies on its own inputs. It leads to a higher real wage in most sectors, because intermediate inputs are not used. That can raise the labor share slightly as well. The labor share is slightly higher in nearly every sector relative to the baseline (for example, sector 0 rises from about 20.68% to 20.71%). The real wage $\frac{W}{P_j}$ is also higher. Output per worker $\frac{Y_j}{N_j}$ goes slightly up for some sectors. We might expect that removing cross-sector linkages would make the economy "less efficient" (because it forgoes beneficial input diversification), but in equilibrium that raises the marginal product of labor in certain sectors, driving up the real wage. Meanwhile, the total output per worker at the sector level can be higher or lower depending on how the reallocation of labor and intermediate inputs. .

Furthermore, by eliminating cross-sector linkages, the economy reallocates labor and intermediate inputs differently. The total real wages and the output per workerrker increase. We interpret that as the marginal product of labor in each sector being higher on average—though whether this is "less efficient". In general equilibrium, some reallocation can produce slightly higher measured real wages and slightly higher GDP, even if we might consider the economy "less efficient" in a broad sense. In general equilibrium, some reallocation can produce slightly higher measured real wages and slightly higher GDP, even if we might consider the economy "less efficient" in a broad sense (e.g., there might be less variety of inputs, or more price distortions). The data show that aggregate consumption is also up a bit, suggesting the net effect is not obviously "negative" from a purely real wage or consumption standpoint.

The no sectoral concentration scenario can cause identical results across every sector, because all sectors have the same input composition, producing the exact same W/P_j or Y_j/N_j ratio. The production network is completely symmetric: every sector has the same structure and cost shares. The sectoral outcomes are identical across all 62 sectors: labor share is 20.5390% in every single sector. The real wage remains stable at 20.9690 in every single sector. Output per worker is 1.0209 in every single sector. This indicates a completely symmetric solution. Removing sectoral concentration means $\omega_j k$ is spread evenly, so each sector has an identical cost structure, identical markups, and so forth. In equilibrium, they converge to the exact same labor share, real wage, and productivity. Economically, this scenario can be "less efficient" or "more efficient" depending on how the production function aggregates across the symmetrical network. Here, the real wage is actually a bit higher than in the baseline (20.96 vs. around 20.88), and productivity Y_j/N_j is also slightly higher (1.0209 vs. 1.01). Because all sectors are identical, the system ends up in a uniform "balanced" equilibrium across sectors. In this completely symmetric input-output setting, each sector ends up with identical labor share, real wage, and productivity. The real wage and average productivity are higher compared to the baseline. Because the production structure is uniform across sectors, we see a single equilibrium level in each sector that yields a small bump in real wage and output per worker.

Half labor intensity means a smaller wage bill share θ_j, so we see $\frac{W N_j}{P_j Y_j}$ either fall or remain near the new θ_j. Labor share is mostly slightly lower than the baseline. For instance, sector 0 goes from 20.68% down to 20.63%. Real wage are meaningfully lower in many sectors (sector 0: 20.88 to 20.74). Output per worker is also lower in most sectors (sector 0: 1.0099 to 1.0054). With half the original labor intensity, a smaller fraction of costs is dedicated to labor, pushing down the equilibrium real wage and measured labor share. Because labor is 'less important', the value of any given sector can decrease if the reduced wage bill no longer supports the same level of production or if other inputs become relatively more expensive. With a smaller share of factor payments going to wages, total income that labor receives is

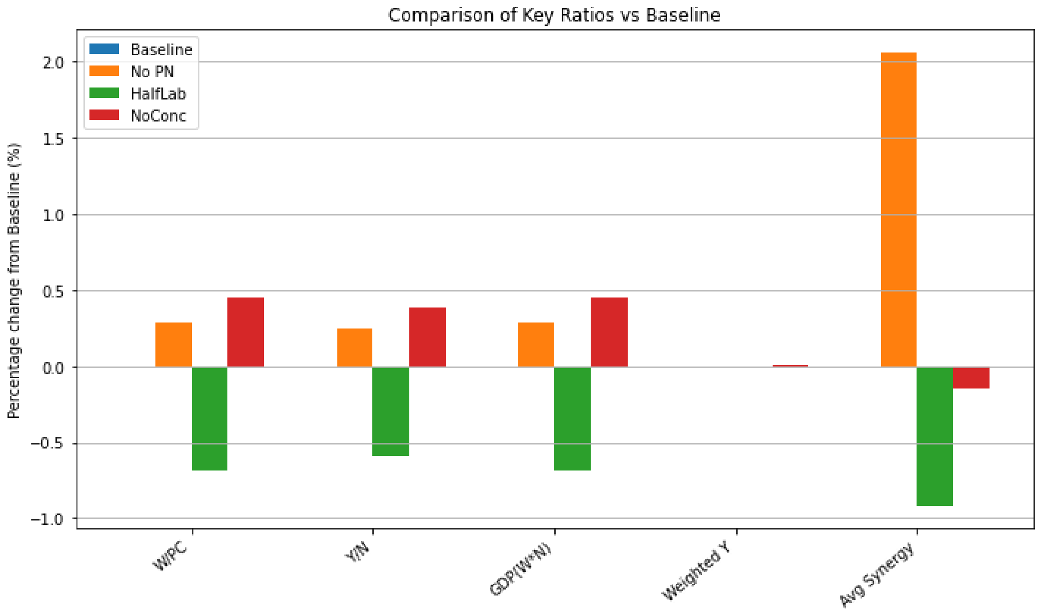


Figure 4: Comparison of percentage changes in the different scenarios. From the left, we plot the real wage, the real output, the total level of GDP, the level of output weighted by production shares and the level of synergy (X_{jk}). These are percentage changes from the baseline.

smaller; thus, consumption and GDP both go down. On the margin, intermediate inputs become more significant, and we see a slight decline in measured productivity per worker. Furthermore, reducing labor's share in production lowers both the aggregate labor share and the real wage. With labor being "less important," equilibrium output per worker also declines. This is consistent with lower labor payments and possibly a smaller fraction of production going to wages.

These results confirm that stronger cross-sector linkages reduce wealth inequality while self-sourcing or uniform structures can amplify top-end concentration under plausible wage responses.

5.3 The Core Channel: From Production Networks to Wages to Distribution

Production networks determine how sectors source intermediate inputs. If a sector heavily relies on external inputs (large off-diagonals in omega) it typically gains synergy or "productivity boost" from cheaper or specialized suppliers. On the other side, if it mostly sources from itself (large diagonal in ω_{jk}), it is more self-sufficient but can lose synergy from cross-sector integration.

Effect on Equilibrium Wages. When a sector (or the entire economy) becomes less efficient (for instance, removing cross-sector intensities or forcing uniform usage when previously some specialized advantage existed), we generally see lower TFP in the final production. If the total final consumption is the same, our model requires a different wage to maintain the equilibrium with bond-clearing and lumpsum taxes. The wage can go up (to compensate for lost TFP, so labor has to work harder or at higher cost), or it might go down (if that scenario reduces labor demand). Overall, changing the production network structure shifts the marginal product of labor in each sector, which changes aggregate labor demand and hence the wage needed to clear the bond market or match final consumption.

In our model, household inequality depends heavily on labor incomes (since lumpsum taxes, partial equilibrium interest rates, etc., are stable). If the real wage rises significantly, higher-skill workers might gain proportionally more income. This can increase or slightly reduce inequality, depending on how progressive or lumpsum the taxes are, and the skill distribution.

Our results show that a higher wage correlates with higher top incomes (especially if the skill distribution is persistent), which in turn can push up the top's saving.

6 Discussion and Relation to the Literature

Our findings that dense production networks (large off-diagonal IO shares) correlate with lower wealth inequality relate to two main strands of literature, in particular the one on heterogeneous agents and the one on production networks. Previous work shows how the marginal propensities of heterogeneous agents to consume magnify the impact of policy (Auclert, 2019; Bayer et al., 2019). We add the insight that a multi-sector production structure can shape who receives income gains (which sectors expand) and thus how savings are distributed. Production network density reduces sectoral markups and wages, dampening the extreme incomes that otherwise accumulate at the top. Moreover, studies such as Acemoglu et al. (2012); Carvalho and Tahbaz-Salehi (2019) highlight that IO links amplify or transmit shocks. Our contribution is to show these linkages have a distributional impact, beyond the usual aggregate concerns. The close connection between synergy effects and wage formation underscores how the production side can systematically influence wealth outcomes.

An unexpected result is our empirical regressions' negative coefficient on sectoral concentration. One possible explanation is that a few large, heavily unionized, or regulated sectors enforce stable wage structures, preventing the sharp extremes in labor income that might appear when many small competitive sectors exist. Alternatively, certain concentrated industries might exhibit stable profit flows that do not necessarily concentrate wealth at the very top. Further micro-level evidence distinguishing these scenarios would clarify whether concentration truly lowers top wealth shares or if other institutional factors confound the analysis.

Overall, our model-based experiments align well with the cross-country patterns: heavily diagonal IO structures (e.g. France) coincide with higher measured Gini, while economies with broad interconnections among sectors (e.g. Spain) see more moderate wealth concentration. From a policy perspective, these results suggest that structural reforms influencing how sectors source inputs—or how open they are to cross-sector trade—carry significant distributional implications in addition to their efficiency effects.

7 Conclusion

We show that production network structures play a critical role in explaining cross-country heterogeneity in wealth inequality within Europe. Empirically, higher off-diagonal IO shares (i.e. stronger cross-sector linkages) correlate with significantly lower top-10% wealth shares. Our HANK model with a multi-sector production network rationalizes this relationship through a wage-change channel: dense inter-sectoral input sourcing reduces profit and wage differentials across industries, tempering wealth concentration at the top.

Our results imply that policies shaping sectoral interconnections—such as industrial clustering programs, trade integration, or energy supply reforms—have important distributional as well as efficiency ramifications. An economy structured to foster cross-sector sourcing may dampen wealth inequality by preventing extreme wage or profit spikes in a few heavily specialized industries.

Other extensions remain. First, introducing physical capital or capital-intensive production processes may alter the way network linkages matter for top-end wealth accumulation. Second, sectoral skill differentiation (i.e. workers employed in different sectors earning distinct wages) could sharpen distributional consequences. Finally, endogenizing ω_{jk} in a dynamic setting—along the lines of Ghassibe (2021)—would be a natural step to see how network formation itself responds to policy or macro shocks, further reshaping inequality over time.

Despite these open questions, the core insight stands: production networks are not merely a technical detail. They systematically influence real wages and the evolution of wealth inequality. As European economies reconfigure their supply chains for reasons ranging from energy security to industrial policy, understanding these distributional channels becomes ever more relevant.

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A Model Setup and Competitive Equilibrium

This appendix formally defines the HANK model with a multi-sector production network and sticky wages. We begin by laying out the environment and the agents' problems, and conclude with the definition of a competitive equilibrium.

A.1 Households

Preferences. The economy is populated by a continuum of ex-ante identical households indexed by $i \in [0,1]$. Each household faces idiosyncratic labor-productivity shocks e_{it}, drawn from a finite support. Households maximize expected utility:

$$\max_{\{c_{it}, a_{it}, n_{it}\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_{it}^{1-\sigma}}{1-\sigma} - \phi \, \frac{n_{it}^{1+\varphi}}{1+\varphi} \right], \tag{16}$$

where c_{it} is consumption, n_{it} is hours worked, $\sigma > 0$ is the coefficient of relative risk aversion, $\varphi > 0$ is the inverse Frisch elasticity of labor supply, $\beta \in (0,1)$ is the discount factor, and $\phi > 0$ is a disutility parameter.

Budget Constraint. Households can save in one-period nominal bonds a_{it} (with a net supply B_t coming from government debt). Let r_t denote the real interest rate. Each period, households face:

$$c_{it} + a_{it} = (1 + r_t) \, a_{it-1} + w_t \, e_{it} \, n_{it} - \tau_t, \tag{17}$$

$$a_{it} \geq \underline{a}, \tag{18}$$

where $w_t \equiv \frac{W_t}{P_t}$ is the real wage (with W_t the nominal wage and P_t the aggregate price level), and τ_t is a lump-sum tax (or transfer). The variable $\underline{a}$ captures a borrowing limit or no-borrowing constraint.

Household Optimization. Each household solves its dynamic problem taking as given prices $\{r_t, w_t\}$, policy $\{\tau_t\}$, and initial asset holdings $a_{i,-1}$. This yields optimal decision rules $c_{it}(a,e)$, $n_{it}(a,e)$, and $a_{it}(a,e)$ and a corresponding value function $V_t(a,e)$.

A.2 Firms and Production Network

Sectors and Firms. There are J sectors, each producing a distinct final good $Y_{j,t}$. Within each sector j, there is a continuum of monopolistically competitive firms that combine labor $N_{j,t}$ and intermediate inputs $\{X_{jk,t}\}_{k=1}^{J}$ to produce output:

$$Y_{j,t} \;=\; A_{j,t}\, N_{j,t}^{1-\sum_{k=1}^{J}\omega_{jk}} \prod_{k=1}^{J} X_{jk,t}^{\omega_{jk}}, \tag{19}$$

where $A_{j,t}$ is sectoral TFP, and $\omega_{jk} \geq 0$ measures the share of inputs from sector k used by sector j. We denote $\Omega = [\omega_{jk}]$ as the *IO matrix*.

Cost Minimization and Input Demand. Each firm j chooses $N_{j,t}$ and $X_{jk,t}$ to minimize costs, taking prices W_t (nominal wage) and $P_{k,t}$ (price of sector-k output) as given. The cost-minimization conditions imply:

$$N_{j,t} = \frac{\left(1 - \sum_k \omega_{jk}\right) P_{j,t}\, Y_{j,t}}{W_t}, \quad X_{jk,t} \;=\; \frac{\omega_{jk}\, P_{j,t}\, Y_{j,t}}{P_{k,t}}, \tag{20}$$

under constant elasticity of substitution among inputs (or a Cobb-Douglas specification, as in (19)).

Market Clearing (Goods). For each sector j, output is absorbed by final consumption $C_{j,t}$, government spending $G_{j,t}$, and intermediate usage $\sum_k X_{kj,t}$. Hence,

$$Y_{j,t} \;=\; C_{j,t} + G_{j,t} + \sum_{k=1}^{J} X_{kj,t}. \tag{21}$$

Aggregate consumption is $C_t = \sum_j C_{j,t}$, and government spending is $G_t = \sum_j G_{j,t}$.

Sticky Wages (Union Setting). We assume a wage-setting union à la Erceg et al. (2000), which sets the nominal wage W_t subject to nominal rigidities. Let $\pi_t^w = \frac{W_t}{W_{t-1}} - 1$ denote wage inflation. The union's problem typically yields a *wage Phillips curve* of the form:

$$\pi_t^w \;=\; \kappa_w \left(\hat{w}_t - \hat{mrt}_t\right) \;+\; \beta\, \mathbb{E}_t[\pi_{t+1}^w], \tag{22}$$

where $\hat{w}_t$ is the log wage deviation from its flexible-wage counterpart, and $\hat{mrt}_t$ is the marginal rate of substitution between consumption and leisure, capturing household labor supply.

A.3 Government and Monetary Policy

Government. The government issues one-period bonds B_t to finance an exogenous path of spending $\{G_{j,t}\}$ and lump-sum taxes $\{\tau_t\}$. The government budget constraint is:

$$B_t \;=\; (1+r_t)\,B_{t-1} + \sum_j P_{j,t}\,G_{j,t} - \int \tau_t\,d\mu_t(a,e), \tag{23}$$

where $\mu_t(a,e)$ denotes the cross-sectional distribution of households over assets and idiosyncratic shocks.

Monetary Policy. A conventional Taylor-type rule sets the nominal interest rate $i_t \approx r_t + \pi_t$ with deviations responding to inflation and output:

$$i_t \;=\; \rho_i\,i_{t-1} + (1-\rho_i)\big(\bar{i} + \phi_\pi\,\pi_t + \phi_y\,y_t\big) + \varepsilon^i_t, \tag{24}$$

where π_t is aggregate inflation, y_t measures an output gap, and ε^i_t is a monetary shock.

A.4 Aggregation and Market Clearing

Bond Market. Aggregate bond holdings by households must equal government debt:

$$\int_0^1 a_{it}\,d\mu_t(a,e) \;=\; B_t. \tag{25}$$

Distribution Dynamics. The cross-sectional distribution $\mu_t(a,e)$ evolves according to households' optimal asset choice:

$$\mu_{t+1}(A,E) \;=\; \int_{\{(a,e)\,:\,a_{i,t}(a,e)\in A,\,\Gamma_e(e)\in E\}} d\mu_t(a,e), \tag{26}$$

where $\Gamma_e(e)$ is the Markov process for idiosyncratic shocks e_{it}.

A.5 Competitive Equilibrium Definition

Definition (Competitive Equilibrium). Let $\mu_0(a, e)$ be the initial distribution of households over assets and idiosyncratic shocks, and B_{-1} the initial government debt. Suppose we are given exogenous sequences for sectoral productivities $\{A_{j,t}\}_{j=1}^J$, government spending $\{G_{j,t}\}_{j=1}^J$, lump-sum taxes $\{\tau_t\}$, and a monetary policy rule (24). A *competitive equilibrium* is a collection of sequences

$$\{r_t,\ i_t,\ \pi_t^w,\ W_t,\ P_{j,t},\ Y_{j,t},\ X_{jk,t},\ N_{j,t},\ \Pi_{j,t}\}_{t=0}^{\infty}$$

for sectoral prices and quantities, together with

$$\{c_{it}(a, e),\ n_{it}(a, e),\ a_{it}(a, e)\}_{t=0}^{\infty} \quad \text{and} \quad \{\mu_t(a, e)\}_{t=0}^{\infty},$$

such that:

1. **Household Optimization:** For each (a, e) and every t, $c_{it}(a, e)$, $n_{it}(a, e)$, and $a_{it}(a, e)$ solve the household problem (17) given prices and taxes, yielding $V_t(a, e)$.
2. **Wage-Setting/Unions:** Wages satisfy the sticky-wage Phillips curve (22), derived from union optimization, and labor is allocated according to firms' labor demands.
3. **Firm Optimization:** In each sector j, firms minimize costs subject to (19) and choose $N_{j,t}$, $X_{jk,t}$, and $p_{j,t}$ (or $P_{j,t}$) to maximize profits $\Pi_{j,t}$, taking W_t and $P_{k,t}$ as given.
4. **Goods Market Clearing:** For each sector j, $Y_{j,t}$ satisfies
$$Y_{j,t} \;=\; \sum_{k=1}^{J} X_{kj,t} \;+\; C_{j,t} \;+\; G_{j,t}.$$
5. **Bond Market Clearing:** The aggregate amount of bonds held by households equals government debt,
$$\int_0^1 a_{it}\, d\mu_t(a, e) \;=\; B_t.$$
6. **Government Budget Constraint:** Government debt evolves according to
$$B_t = (1 + r_t)B_{t-1} + \sum_j P_{j,t}\, G_{j,t} \;-\; \int_0^1 \tau_t\, d\mu_t(a, e).$$

7. **Distribution Consistency:** μ_{t+1} is generated by households' optimal asset choices and the Markov process for e_{it}.

8. **Monetary Policy Rule:** The nominal interest rate i_t follows (24), given inflation π_t and output y_t.

When these conditions are satisfied for all $t \geq 0$ given the initial state (B_{-1}, μ_0) and the exogenous paths $\{A_{j,t}, G_{j,t}, \tau_t\}$, the economy is in a competitive equilibrium.

B Production Block Algorithm Details.

Model Setup: Each sector $j = 1, \dots, S$ has cost shares $\{\omega_{jk}\}_{k=1}^S$ for intermediate inputs and labor share $1 - \sum_{k=1}^S \omega_{jk}$. Prices are $\{P_j\}_{j=1}^S$, and W is the common wage. Output is Y_j, labor usage is N_j, and intermediate inputs X_{jk}. The production function is Cobb-Douglas:

$$Y_j \;=\; A_j \;\; N_j^{1-\sum_k \omega_{jk}} \prod_{k=1}^{S} (X_{jk})^{\omega_{jk}}, \quad j = 1, \dots, S.$$

Cost minimization implies

$$X_{jk} \;=\; \omega_{jk}\,\frac{P_j\,Y_j}{P_k}, \quad N_j \;=\; \left(1 - \sum_{k=1}^{S} \omega_{jk}\right) \frac{P_j\,Y_j}{W}.$$

Fixed-Point Iteration on $\{Y_j\}$*:* We want Y_j consistent with these cost shares and the production function. We do the following steps:

1. **Initialize** a guess $Y_j^{(0)}$ for each sector j.

2. **Compute inputs**:
$$X_{jk} \;=\; \omega_{jk}\,\frac{P_j\,Y_j}{P_k}, \quad N_j \;=\; \left(1 - \sum_k \omega_{jk}\right) \frac{P_j\,Y_j}{W}.$$

3. **Compute synergy:**
$$\text{synergy}_j \;=\; \exp\Big[\sum_k \omega_{jk} \ln\big(X_{jk}\big)\Big],$$

4. **Update output** from the Cobb-Douglas formula:
$$Y_j^{\text{new}} \;=\; A_j\,(N_j)^{\,1-\sum_k \omega_{jk}} \times \text{synergy}_j.$$

5. **Damp** or relax the update:

$$Y_j \;\leftarrow\; \alpha\, Y_j^{\text{new}} \;+\; (1-\alpha)\, Y_j,$$

   for some damping factor $\alpha \in (0, 1]$ (e.g. $\alpha = 0.2$).

6. **Repeat** until $\|Y_j^{\text{new}} - Y_j\|$ is below tolerance, or until a maximum number of iterations is reached.