Public pensions in the age of automation[*]

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Abstract

We analyze the impact of increased automation on the size and distribution of pension benefits, as well as on the optimal size of public pension systems. To this end, we build an overlapping generations model of a closed economy with heterogeneous agents who make decisions regarding skill formation, consumption/savings, and retirement. Automation is conceptualized either in terms of capital-skill complementarity or in a task-based fashion. We find that any productivity gains from automation, realized as increased returns to savings, disproportionately benefit high-skilled workers who are less dependent on illiquid public pensions. A redistributive pension system can reduce public pension inequality but may increase inequality in private retirement savings. In our calibrated economy, the optimal size of the pension system is larger in the task-based specification, where the displacement effects of automation are accounted for.

Keywords: Automation, General Equilibrium, Overlapping Generations, Public Pensions

JEL Codes: H55, J22, J26

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1 Introduction

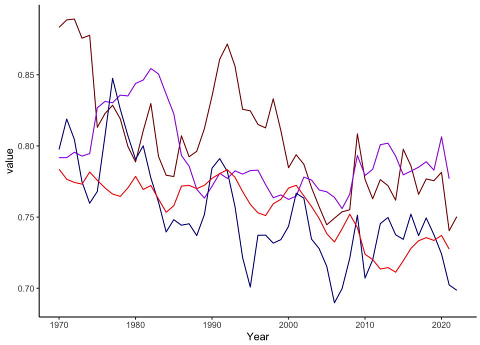
Much empirical evidence indicates that advances in automation have played a significant role in explaining the decline in the labor share of national income, the limited wage growth across routine sectors, and the increased skill premium observed over the past three to four decades (e.g., Acemoglu and Restrepo, 2019, 2022; Autor et al., 2003; Goos et al., 2014; Graetz and Michaels, 2018)[1]. Concurrently, the pension literature has documented a rise in public pension expenditures in many developed economies, mainly driven by aging populations (e.g., Coile et al., 2025). Figure 1 illustrates the recent evolution for a selection of countries that are representative of a broader global pattern. In particular, the decline in fertility rates and the continuous improvements in mortality suggest that the ongoing aging of the population will extend over the next three to four decades. Consequently, the average public pension spending among OECD economies is projected to increase from the current 8.9% to 10.3% by 2060 (OECD, 2023).

We argue that these trends constitute a source of concern for the future provision of public pensions. Indeed, most public pension systems share two fundamental features that make them vulnerable to the impact of extensive automation. First, most systems are pay-as-you-go (PAYG), meaning that the benefits of current pensioners are financed by today's workers. Second, contributions come almost exclusively from payroll taxes or, similarly, social contributions based on labor income. Consequently, if automation leads to a reduced labor share and potentially lower equilibrium wages, ceteris paribus, contributions to the pension system are likely to decline. To maintain solvency, reductions in benefits will be necessary. Such adjustments could have significant negative implications for the welfare of individuals who rely heavily on public pension benefits for their retirement consumption, whether due to behavioral shortcomings which reduces their propensity to save, because their access to capital markets is limited, or because their income barely covers minimum subsistence levels and they cannot save.

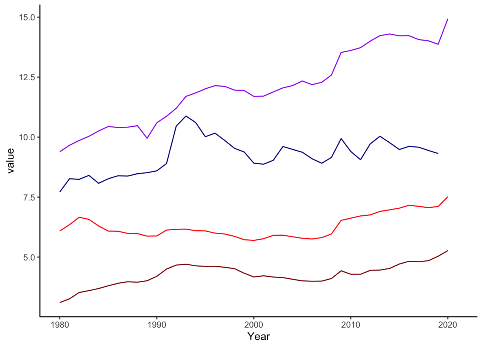
Even in favorable scenarios where the productivity gains of automation boost average

[1]Importantly, these findings are not limited to the United States (see, e.g., Acemoglu et al., 2023, 2020).

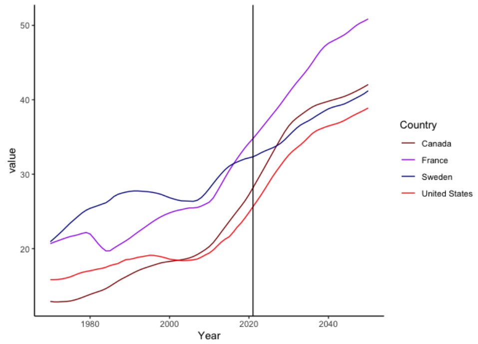
Figure 1: Trends in labor shares, public pension spending,
and old-age dependency ratios for a selection of countries.



(a) Labor share of GDP



(b) Pension spending (% of GDP)



(c) Actual and projected old-age dep. ratios

Note: Data collected from The World Inequality Database (a), the OECD Social Expenditure Database (b), and the World Bank (c).

wages and total contributions to the pension system, automation can still negatively impact the distribution of pension benefits. For instance, in countries like Sweden, Italy, or Latvia, pension systems display a strong link between contributions and benefits. If automation mainly favors the wage growth of high-skilled workers, the increased pension contributions from high-skilled workers relative to those of lower-skilled workers could exacerbate income inequality, as the size of the earnings differential systematically carries over to the relative pension payments between skill groups. If, moreover, the use of robots causes a decrease in the demand for low-skilled workers, this effect would be further amplified.

Furthermore, the design of PAYG pension systems implies that the contributions, and therefore the pension income they generate, only appreciate with population and wage growth. In contrast, advances in automation are expected to boost investment returns.

Consequently, in a dynamically efficient economy, the disparity between pensioners who save privately for retirement and those who depend on the unfunded public pension system is likely to widen. This, in turn, exacerbates the automation-driven inequality between capital owners and non-capital owners, as discussed in Moll et al. (2022).

Policymakers therefore face a potentially complex dilemma: to curb increased income inequality, policymakers might be tempted to reduce the earnings-dependence of public pensions in favor of a more redistributive system. However, such a reform would create larger disincentives for both labor supply and private savings among low-income workers as their pension's replacement rate increases (see, e.g., Gustafsson, 2023a).

Ultimately, this paper performs an ex-ante evaluation of the impact of advances in automation on the provision and distribution of pension benefits. To this end, we develop a general equilibrium overlapping generations (OLG) model of a closed economy that incorporates automation on the production side. The OLG framework is particularly suitable for analyzing retirement behavior and pension policy, while the general equilibrium mechanism is essential for capturing changes to the skill premium and investment returns.

To understand how automation affects wage and pension benefit levels, and their distributions, we explore both the capital-skill-complementarity (CSC) framework (as in Krusell et al. (2000)) and the task-based (TB) framework (as in Acemoglu and Restrepo (2018b)). We argue that the way technology is modeled matters. The CSC framework allows for capital to replace low-skill workers at the margin while the TB approach allows both an adjustment between labor and capital at the margin and an additional channel where automation shifts low-skill labor-intensive tasks to capital-intensive ones. The significance of the second channel is supported by studies showing that automation has "hollowed out" the wage distribution by eliminating or reducing the share of routine jobs (e.g., Acemoglu and Restrepo, 2020; Autor et al., 2003). To the best of our knowledge, we are the first to (i) introduce the task-based production function to an OLG model, and (ii) make a systematic quantitative comparison of these two model types in general equilibrium.

For the model to constitute an environment in which public pensions play a meaningful role from a welfare perspective, we populate the model with agents that differ both in terms of savings behavior—where some save rationally and some are hand-to-mouth consumers—and in their skill level. In this context, the pension system can potentially enhance welfare by mandating savings for hand-to-mouth consumers and by reducing economic inequality between high- and low-skilled workers, as suggested by World Bank (1994). Both skill formation and retirement timing are endogenous decisions, where the former allows us to capture how the interactions between the pension system and the level of automation affect the share of high-skilled workers in the long run.

We calibrate the model to match the macroeconomic regularities of an average, representative OECD economy. The analysis proceeds in two steps. First, we analyze the comparative static effects of increased automation on the levels and distribution of pension benefits and other income types. We also decompose the relative importance of public vs. private pension income for retirement consumption across different worker types. In the second step, we assess whether improved automation warrants changes to the size of the public pension system by solving the social planner's problem to determine the optimal pension contribution rate.

Regardless of the model specifications, our comparative statics analysis reveals that advances in automation lead to increased wage and pension inequality, higher returns on investments, and therefore a greater appreciation of private pension savings. Consequently, this results in a larger capital share and a reduced low-skilled labor share of national income. In our preferred calibration, the task-based specification generates more inequality and a higher interest rate following the displacement effect of automation. In a sensitivity analysis, where the skill level is set exogenously, the equilibrium wage for low-skilled workers decreases. This suggests that labor mobility across skill groups is an important, yet previously overlooked, margin of adjustment in determining the net result of the productivity and displacement effects of automation on wages in the long run.

The intra-generational redistribution inherent in the pension system mitigates wage-driven inequality but also generates larger disincentives for private savings among low-skilled workers. Consequently, the higher equilibrium interest rate exacerbates inequality between workers who save privately for retirement and those who live hand-to-mouth.

This confirms the potential policy dilemma: if automation increases wage inequality, policymakers might be tempted to reduce inequality by making the contribution-benefit formula of the public pension system more redistributive. However, such a reform reduces the incentives for low-skilled workers to save privately.

The welfare analysis suggests that the optimal size of the public pension system is larger in the model economy with task-based production. Based on our baseline calibration, when welfare is evaluated across a cross-section of the population, the optimal contribution rate is approximately 21% for the CSC case and 27.5% for the TB economy. Conversely, when welfare is assessed based on the present value of the lifetime utility of a newly born generation, the optimal size is 9% for the CSC case and 14% for the TB economy. Interestingly, despite the inclusion of a basic income component in the pension system, which works to redistribute income intragenerationally, we do not find that the optimal size of the pension system is influenced by automation-driven growth, even though automation increases lifetime inequality under both technology specifications. However, as the interest rate increases, ceteris paribus, the opportunity cost of contributing to illiquid PAYG pension systems increases due to the additional foregone compound interest from being forced to save in a scheme that offers returns below those of financial markets.

Our findings demonstrate that technology can have important implications for welfare analyses of pension reform. The increased returns to private savings suggest additional long-term benefits of capitalizing pension systems, as the inequality between capital owners and hand-to-mouth households grows. Finally, although we do not find that automation demands a change in the size of the pension system, it may still warrant redesigning specific aspects of public pensions unrelated to their size, such as the degree of redistribution and the sources of funding.

The rest of the paper is organized as follows. We review the literature in Section 2. The model is introduced and solved in Section 3. Section 4 describes the calibration exercises and Section 5 present the analysis of a number of simulations. Section 6 concludes.

2 Literature review

The present paper contributes to a mature literature on the economic implications of improved automation. Within this literature, automation has been conceptualized in the production process in various ways. The traditional view, based on neoclassical production theory, suggests that improved automation, as a general part of technological progress, is either Hicks neutral, leading to a proportional increase in marginal products, or factor augmenting, increasing the marginal product of capital or labor (see, e.g., Bessen, 2019; Graetz and Michaels, 2018; Nordhaus, 2021). For both labor- and capital-augmenting options, automation always increases labor demand and the equilibrium wage—outcomes that are contested by some recent evidence suggesting the opposite (see, e.g., Acemoglu and Restrepo, 2020).

Others have conceptualized improvements to automation as part of skill-biased technological change à la Krusell et al. (2000), where automation complements high-skilled workers more than their low-skilled counterparts (see, e.g., Prettner, 2019; Prettner and Strulik, 2020). The capital-skill complementarity hypothesis has gained popularity as it can explain the recent evolution of the wage structure, particularly the increased skill premium observed in the US[2], and also a non-trivial share of the observed decline in the labor share of US national income (Prettner, 2019).

A third, and currently very prominent approach, is to model automation within a task-based framework (see, e.g., Acemoglu and Restrepo, 2018b; Kına et al., 2024; Zeira, 1998). Improved automation increases the number of assembly tasks where capital (or robots) have a comparative advantage over human labor, thus replacing or displacing workers on the margin. In the latter, as shown in Acemoglu and Restrepo (2018a), automation always reduces the labor share but can also reduce labor demand and equilibrium wages if the productivity gains are not large enough to dominate the displacement effect. The task-based framework has been shown to explain several labor market trends that have

[2]See Autor et al. (2008) for a discussion of the wider question.

arisen since at least the 1980s in several developed economies, such as decreasing labor shares and wage growth across mainly manufacturing sectors, and also an increased skill premium (e.g., Acemoglu et al., 2023; Acemoglu and Restrepo, 2019, 2020, 2022).

Following this empirical evidence, the taxation literature has studied the potential role of a robot tax to reduce automation-driven inequality. Under the conventional assumption that the government cannot use lump-sum or skill-indexed income taxes to achieve redistribution, a general conclusion of this literature is that automation should be subject to a positive marginal tax to compress the wage structure and reduce automation-driven inequality (Guerreiro et al., 2022; Kına, 2024; Thuemmel, 2023). Guerreiro et al. (2022) find that this “robot tax” should be positive as long as the current generations of routine workers, who are unable to move to non-routine occupations, are active in the labor force. Once these generations are retired, the tax should be abandoned. Kına (2024) find that the optimal tax policy should not only include a positive tax on automation that mainly substitutes for low-skilled workers, such as industrial robots, but also a small subsidy on automation that replaces high-skilled workers, such as artificial intelligence and machine learning. Akar et al. (2023) show that a transition from an economy with a traditional production structure to an automated economy is characterized by reduced wages and declining labor shares. Taxing capital and robots can slow down this process, but not halt it as long as the exogenous growth rate is positive.

The role of automation and robot taxation has also been explored in other public finance contexts, as well as in the growth literature. Within an overlapping generations model, Prettner and Strulik (2020) find that automation increases educational attainment and inequality, and leads to a declining labor share. As the skill premium increases, tertiary education becomes more attractive. However, due to ability constraints, enrolling in higher education is only a feasible option for high-ability workers. Ultimately, low-skilled workers ”lose the race” against automation. This evolution merits educational subsidies to reduce inequality.

However, the consequences of automation for public pensions have been largely overlooked in the modeling and quantitative literature. A notable exception is Kim and Lee

(2024), who study how improvements in longevity or decreased fertility affect pension benefits in an economy with automation capital. In the case of increased longevity, both direct and indirect subsidies targeting low-skilled workers adversely affected by automation can be Pareto improving. In the case of reduced fertility, however, only indirect subsidies are found to achieve this improvement. There are, however, several important differences between our paper and theirs. First, we endogenize labor supply and consider the effects of automation on retirement behavior. This is an important feature since we want to capture that a reform to the design of the pension system will change the implicit taxation of contributing to the pension system. Indeed, it is well-known that the efficiency costs of public pensions follow from their degree of actuarial fairness (see, e.g., Gustafsson, 2023a). Our focus on retirement timing, as opposed to hours worked, follows from micro-econometric evidence suggesting larger labor supply elasticities along the extensive margin compared to the intensive margin (see, e.g., Chetty et al., 2011; Keane and Wasi, 2016).

Second, our analysis includes individuals that deviate from the rational, forward-looking agent paradigm, as we postulate that a fraction of the population lives hand-to-mouth. This aligns with the current frontier of heterogeneous agent macroeconomic modeling (see, e.g., Krueger et al. (2016), Kaplan and Violante (2018), Aguiar et al. (2024)). Third, while Kim and Lee (2024) include automation through a capital-skill-complementarity specification, their analysis focuses on the effects of aging demographics. The main focus of the current paper is instead on the implications of the technology specification for the provision and distribution of pension benefits. In particular, we also study the effects of a task-based production function, allowing for displacement effects of automation.

3 Model

Time is continuous and denoted by t. At each moment t, the time endowment is normalized to one. Consider a closed economy in steady state with a constant population size of unit mass, populated by overlapping generations. The reason we abstract from population growth is to isolate the effects of automation on the performance of public pensions from those of population aging, which has been extensively explored.[3]

On the production side, a representative firm produces a final good that can be either consumed or saved, in which case it is realized one-to-one as physical capital. Any savings appreciate by the real rate of return to capital investments r.

On the consumer side, each worker type lives for a fixed duration T with certainty and replicates itself identically. There are no inter-generational links and, therefore, no bequest motive exists.

Upon entering the model, a worker first draws an optimization type j. Some are born as rational savers ($j = RS$), and others as hand-to-mouth ($j = HTM$) and do not save. Let Λ_H^{RS} denote the share of high-skilled workers, Λ_L^{RS} the share of rational low-skilled workers, and the complement $\Lambda^{HTM} = 1 - \Lambda_H^{RS} - \Lambda_L^{RS}$ the share of low-skilled workers that live hand-to-mouth. We assume that all HTM workers are low-skilled, allowing us to drop the skill indexation for these workers. Rational saving workers make an irreversible decision to remain low-skilled or pay a fixed cost to become high-skilled. The life cycle can then be divided into two distinct phases: working life and retirement. Throughout the working life, the worker supplies labor inelastically. Let w_i denote the skill-specific wage rate. The retirement age R_j^i is endogenous. During the retirement phase, workers are full-time pensioners. Retirement is an absorbing state.

The government administers a public pension system financed by payroll taxes. In the absence of mortality risk and aggregate risk, the pension system can serve two meaningful purposes within this model framework. The first is to mandate savings, given that HTM workers do not save privately at all. As such, redistribution at any age prior to retirement will not help these individuals to smooth consumption. The second is to reduce intragenerational inequality, as promoted by World Bank (1994). To accommodate both these purposes, we follow Casamatta et al. (2000) and model a two-pillar pension system: one earnings-related (Bismarckian) pillar to mandate savings, and one common

[3]For papers that study the effects of aging populations on public pensions, see, e.g., Marchand and Pestieau (1991), Disney (2000), Heijdra and Romp (2009), Poterba (2014), Kudrna et al. (2022).

benefit (Beveridgean) pillar that aims to reduce intragenerational inequality.

3.1 Aggregate output

Aggregate output is produced by a representative firm using a CES technology with constant returns to scale. Inputs are high-skilled labor L_H, and a intermediate composite G that is assembled by low-skilled labor L_L and capital/machines K:

$$Y = \Omega \Big[L_H^{\rho_F} + B\, G^{\rho_F} \Big]^{\frac{1}{\rho_F}}, \tag{1}$$

where Ω is total factor productivity (TFP) and B the factor productivity that is specific to G. $\rho_F = \frac{\sigma_F - 1}{\sigma_F}$, where σ_F is the elasticity of substitution between high-skilled labor and the intermediate composite.

We now proceed to fully specify the production function for two different cases. First, we consider the case when G is takes the form of an another CES function between K and L_L, resulting in an aggregate production function with capital-skill complementarity. In the second case, we assume that G is the output resulting from the assembly of a range of tasks produced by either capital or low-skilled labor.

3.1.1 Case A: capital-skill-complementarity

First, we follow the capital-skill-complementarity specification in Prettner and Strulik (2020) and specify G as follows:

$$G = G(K, L_L) = (AK^{\rho_G} + L_L^{\rho_G})^{\frac{1}{\rho_G}}, \tag{2}$$

so that aggregate output becomes:

$$Y = \Omega \Big[L_H^{\rho_F} + B \Big(AK^{\rho_G} + L_L^{\rho_G} \Big)^{\frac{\rho_F}{\rho_G}} \Big]^{\frac{1}{\rho_F}}. \tag{3}$$

Here, A is the capital factor productivity which we, like Prettner and Strulik (2020), interpret as the state of automation. $\rho_G = \frac{\sigma_G - 1}{\sigma_G}$, where σ_G is the elasticity of substitution between capital and low-skilled labor.

Operating under perfect competition, the firm employs high- and low-skilled labor, and rents capital, so that their marginal products equal their factor prices respectively:

$$L_H^d : \Omega L_H^{\rho_F - 1}\left[L_H^{\rho_F} + B\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1-\rho_F}{\rho_F}} = MP_{L_H} = w_H, \tag{4}$$

$$L_L^d : \Omega B L_L^{\rho_g - 1}\left[L_H^{\rho_F} + B\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1-\rho_F}{\rho_F}} \Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F-\rho_G}{\rho_G}} = MP_{L_H} = w_L, \tag{5}$$

$$\begin{aligned} K^d : \Omega BAK^{\rho_G - 1}\left[L_H^{\rho_F} + B\Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1-\rho_F}{\rho_F}} \Big(AK^{\rho_G} + L_L^{\rho_G}\Big)^{\frac{\rho_F-\rho_G}{\rho_G}} - \delta = \\ MP_K - \delta = r, \end{aligned} \tag{6}$$

where δ is the depreciation rate of capital.

3.1.2 Case B: task-based

In this section we show how the task based approach leads to an aggregate between low-skill labor and capital which we can express as a function of aggregate capital K and low-skill labor L_L.

We assume that capital and low skill labour can be combined in a large number of tasks. To simplify we identify the set of tasks to the interval $[0, 1]$. Following Acemoglu and Restrepo (2018b) we assume that labor and capital are perfect substitutes in the production of any possible task . Let $k(u)$ and $l_L(u)$ denote the amount of capital and low-skill labor used for a specific task $u \in [0, 1]$ so that the output $y(u)$ produced by task u is:

$$y(u) = \psi(u)k(u) + l_L(u). \tag{7}$$

To ensure an interior solution where a fraction $a \in (0, 1)$ of tasks are automatized, we specify the marginal productivity of capital $\psi(u)$ such that capital has a comparative advantage over labor for a range $u \in [0, a)$ of tasks, and where labor has a comparative advantage for $u \in (a, 1]$. Following Kına (2024), we specify $\psi(u)$:

$$\psi(u) = D\,(u^{-\eta} - 1), \tag{8}$$

for some parameters $\eta > 0$ and $D > 0$. This functional form ensures that (i) $\psi(0) \approx \infty$, (ii) $\psi(u)$ is continuous and decreasing in u over the interval $[0, 1]$, and (iii) $\psi(1) = 0$[4]. The extensive margin of automation a is then the unique solution to the no-arbitrage condition $\psi(a) = \frac{r+\delta}{w_l}$ [5]. Importantly, anytime a change in $\psi(u)$ (either resulting from a change in D or η) leads to a change in the $(r+\delta)/w_l$, the extensive margin of automation is also affected in general equilibrium.

The production process allows for some complementarity or substitution between tasks. We assume that the intermediate aggregate production, $\mathcal{G}$ that results from combining capital and low-skill labor across all tasks $u \in [0, 1]$ takes the form

$$\mathcal{G} \equiv \left(\int_0^1 y(u)^{\rho_{\mathcal{G}}} du \right)^{\frac{1}{\rho_{\mathcal{G}}}}, \tag{9}$$

where $\rho_{\mathcal{G}} \equiv \frac{\sigma_{\mathcal{G}}-1}{\sigma_{\mathcal{G}}}$ and $\sigma_{\mathcal{G}}$ can now be understood as the elasticity of substitution between different tasks in production.

Following Acemoglu and Restrepo (2018b), we show in Appendix A.3 that the intermediate production can be written in terms of the aggregate use of the different sources of input in the following fashion :

$$\mathcal{G}(K, L_L, a) = (1-a)^{\frac{1-\rho_{\mathcal{G}}}{\rho_{\mathcal{G}}}} S_{\mathcal{G}}(K, L_L, a)^{\frac{1}{\rho_{\mathcal{G}}}}, \tag{10}$$

[4] If we denote $\bar{F}(x)$ the survival function for some distribution function, then $\psi(u)$ can be defined as the inverse of $\bar{F}(x)$ so that $\psi(u) \equiv \bar{F}(u)^{-1}$. Kına (2024)'s choice correspond to the productivity of capital being distributed according to a specific Lomax distribution, Lomax (1954), such that $F(x) = 1 - (x+1)^{-k}$. Obviously, other choices are possible.

[5] $r + \delta$ is the gross interest rate, including depreciation.

with $S_{\mathcal{G}}(K, L_L, a)$ such that:

$$S_{\mathcal{G}}(K, L_L, a) \equiv (1-a)^{\rho_{\mathcal{G}}-1}\Big(\int_0^a \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}} du\Big)^{1-\rho_{\mathcal{G}}} K^{\rho_{\mathcal{G}}} + L_L^{\rho_{\mathcal{G}}}.$$

The aggregate production function then takes the form:

$$Y = \Omega\left[L_H^{\rho_F} + \tilde{B}\left[(1-a)^{\rho_{\mathcal{G}}-1}\Big(\int_0^a \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}} du\Big)^{1-\rho_{\mathcal{G}}} K^{\rho_{\mathcal{G}}} + L_L^{\rho_{\mathcal{G}}}\right]^{\frac{\rho_F}{\rho_{\mathcal{G}}}}\right]^{\frac{1}{\rho_F}}, \tag{11}$$

where a can be conceptualized as the extensive margin of automation and where $\tilde{B} \equiv B(1-a)^{\frac{\rho_F}{\rho_{\mathcal{G}}}(1-\rho_{\mathcal{G}})}$ [6]. The quantities K and L_L are defined as the aggregated input uses across the relevant tasks:

$$K \equiv \int_0^a k(u)du,$$

$$L_L \equiv \int_a^1 l(u)du.$$

Clearly the production function described by Equation (11) shares some of the CES\CES characteristics that describe the technology with the capital skill complementarities as it is given in Equation (3). The factor demands can then be derived and will satisfy expressions very similar (given a) to the expressions given in Equations (4),(5) and (6):

$$L_H^d : \Omega L_H^{\rho_F-1}\left(L_H^{\rho_F} + \tilde{B}\, S_{\mathcal{G}}(K, L_L, a)^{\frac{\rho_F}{\rho_{\mathcal{G}}}}\right)^{\frac{1}{\rho_F}-1} = MP_{L_H} = w_H, \tag{12}$$

$$L_L^d : \Omega \tilde{B} L_L^{\rho_{\mathcal{G}}-1} S_{\mathcal{G}}(K, L_L, a)^{\frac{\rho_F}{\rho_{\mathcal{G}}}-1}\left(L_H^{\rho_F} + \tilde{B}\, S_{\mathcal{G}}(K, L_L, a)^{\frac{\rho_F}{\rho_{\mathcal{G}}}}\right)^{\frac{1}{\rho_F}-1} = MP_{L_L} = w_L, \tag{13}$$

[6] Section **??** shows how to calibrate the production described in Equation (11) to match the production function described in Equation (3) for some values $(Y, L_H, L_L, K, w_H, w_L, r)$

$$
\begin{aligned}
K^d :& \Omega \tilde{B}(1-a)^{\rho_{\mathcal{G}}-1}\Big(\int_0^a \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}} du\Big)^{1-\rho_{\mathcal{G}}} K^{\rho_{\mathcal{G}}-1} \\
&\left(L_H^{\rho_F} + \tilde{B}\, S_{\mathcal{G}}(K, L_L, a)^{\frac{\rho_F}{\rho_{\mathcal{G}}}}\right)^{\frac{1}{\rho_F}-1} S_{\mathcal{G}}(K, L_L, a)^{\frac{\rho_F}{\rho_{\mathcal{G}}}-1} - \delta = MP_K - \delta = r.
\end{aligned} \tag{14}
$$

3.2 Pension system

In line with the recommendations of the World Bank (World Bank, 1994), many economies operate multi-pillar PAYG pension systems that both mandate savings over the life cycle and redistribute income within generations.[7] Both of these features could hypothetically increase welfare in our model given that it includes both income heterogeneity and the presence of HTM workers.

Therefore, we model a stylized pension system that features both earnings-based (Bismarckian) and common-benefit (Beveridgean) pillars. Each worker type recognizes that any contributions to the Bismarckian pillar constitute forced savings (albeit in a technology return-dominated by capital investments) and thus accounts for it in their labor supply decision. Ignoring this link would overestimate the effective marginal tax rate.[8] However, any benefits from the Beveridgean pillar are treated as exogenous. Let $\kappa \in [0, 1]$ denote the fraction of contributions that goes toward the Bismarckian pillar (henceforth the "Bismarckian factor"). Reforming κ creates an equity-efficiency trade-off: the effective taxation of the Bismarckian scheme is lower, while the Beveridgean scheme offers an explicit mechanism for redistributing income intragenerationally.[9] Specifically, conditional on that the pension accounts must balance at any moment, pension benefits are equal to:

$$
b_j^i = \frac{\tau}{T - R_j^i}(\kappa w_j R_j^i + (1-\kappa)\Psi), \tag{15}
$$

[7]For studies that analyze the redistributive role of public pensions, see, e.g., Huggett and Ventura (1999), Fehr and Habermann (2008), Cremer and Pestieau (2011), Gustafsson (2023a), Gustafsson (2023b).

[8]See, e.g., Gustafsson (2023a) for a mathematical illustration.

[9]However, Sommacal (2006) and Gustafsson (2023a) show that this trade-off can collapse once the model allows for endogenous labor supply effects. Since this model includes endogenous retirement, the distributive effects of reforming this parameter are not trivial.

where:

$$\Psi = \Lambda_H^{RS} w_H R_H^{RS} + w_L[\Lambda_L^{RS} R_L^{RS} + \Lambda^{HTM} R^{HTM}], \tag{16}$$

measures the aggregate labor income in the economy at any moment in time.

3.3 Workers

Following a standard in the quantitative macroeconomic and public finance literature, we assume CRRA instantaneous utility functions over consumption for all workers: $\frac{c_j^i(t)^{1-\sigma}-1}{1-\sigma}$. For retirement leisure preferences, we convert the CRRA stock specification used in, e.g. Jacobs (2009), Heckman and Jacobs (2010), and Gustafsson (2023a), into a flow specification of the instantaneous utility of leisure while retired: $e^{\gamma t}(T-t)^{-\phi}$. In that way, the discounting of retirement leisure enters our model formulation explicitly, and we are able to turn discounting on or off in a simple fashion. This stock-to-flow conversion is described in Appendix A.2.

3.3.1 Rational savers

Upon entering the model, each worker draws a fixed education cost $\psi \geq 0$ from an exponential distribution. A worker will enroll in college education and become High-skilled H iff:

$$\psi \leq \int_0^T U(c_H^{RS*}(t), k_H^{RS*}(t), R_H^{RS*})e^{-\theta t}dt - \int_0^T U(c_L^{RS*}(t), k_L^{RS*}(t), R_L^{RS*})e^{-\theta t}dt \tag{17}$$

where the life-cycle utility maximization problem follows from a decision on consumption $c_j^{RS}(t)$ and retirement age R_j^{RS} to maximize:

$$U_j(c_j^{RS}(t), R_j^{RS}) = \int_0^T \frac{c_j^{RS}(t)^{1-\sigma}-1}{1-\sigma}e^{-\theta t}dt + \beta \int_{R_j^{RS}}^T e^{\gamma t}(T-t)^{-\phi}e^{-\theta t}dt, \tag{18}$$

subject to the intertemporal budget constraint:

$$w_j(1-\tau)\int_0^{R_j^{RS}} e^{-rt}dt + b_j^{RS}\int_{R_j^{RS}}^T e^{-rt}dt = \int_0^T c_j^{RS}(t)e^{-rt}dt, \tag{19}$$

and subject to the initial and terminal conditions $k(0) = k(T) = 0$. Here, θ is the discount rate, β is the utility weight attached to retirement leisure, σ is the inverse of the elasticity of intertemporal substitution, and ϕ is the inverse of the retirement elasticity with respect to the replacement income rate.

We write the corresponding Lagrangian as follows:

$$\begin{aligned}\mathcal{L} = &\int_0^T \frac{c_j^{RS}(t)^{1-\sigma} - 1}{1-\sigma} e^{-\theta t} + \beta \frac{(T - R_j^{RS})^{1-\phi} - 1}{1-\phi} + \\ &\mu_j \left\{ w_j(1-\tau) \int_0^{R_j^{RS}} e^{-rt} dt + b_j^{RS} \int_{R_j^{RS}}^T e^{-rt} dt - \int_0^T c_j^{RS}(t) e^{-rt} dt \right\}.\end{aligned} \tag{20}$$

The solution of the optimization problem provides an expression for the optimal consumption profile:

$$c_j^{RS*}(t) = \left[\frac{e^{(r-\theta)t}}{\mu_j} \right]^{\frac{1}{\sigma}}, \tag{21}$$

where μ_j is an unknown constant shadow-price term that satisfy equation 19. Consumption increases monotonically over the lifecycle if $r > \theta$.

Optimal retirement, R_j^{RS*} is the solution to:

$$\begin{aligned}\beta\big(T - R_j^{RS*}\big)^{-\phi} e^{(\gamma-\theta)R_j^{RS*}} = \mu_j \Bigg[&\big(w_j(1-\tau_l) - b_j^{RS*}\big) e^{-rR_j^{RS*}} + \\ &\frac{\tau_l\big(\kappa w_j T + (1-\kappa)\Psi\big)}{(R_j^{RS*} - T)^2} \int_{R_j^{RS*}}^T e^{-rt} dt \Bigg].\end{aligned} \tag{22}$$

The LHS of Equation 22 is the marginal utility cost of delaying retirement, which increases as R_j^{RS} approaches T. The first term inside the squared brackets on the RHS is the net-present value of the difference between the final earnings payment and the pension benefit level. The second term inside the squared brackets is the incremental pension benefits realized by delaying retirement.

Furthermore, from the expression given in Equation (21), we can analytically solve for the unknown constant term μ_j by exploiting the terminal condition that $k(T) = 0$:

$$\mu_j = \left\{ \frac{\sigma\big(e^{\frac{r(1-\sigma)-\theta}{\sigma}} - 1\big)}{\frac{r(1-\sigma)-\theta}{r}\big(w_j(1-\tau_l)(1 - e^{-rR_j^{RS}}) + b_j^{RS}(e^{-rR_j^{RS}} - e^{-rT})\big)} \right\}^{\sigma}. \tag{23}$$

By substituting the expression in Equation (23) into (22), the optimization problem for the rational savers is then reduced to identifying R_j^{RS*} that solves (22) and (23) simultaneously.

Given R_j^{RS*} and μ_j, the optimal savings profile k_j^* is then described over the working life and the retirement period as:

$$k_j^*(t) = \begin{cases} e^{rt}\left[\frac{w_j(1-\tau_l)}{r}(1-e^{-rt}) - \frac{\sigma}{\mu_j^{\frac{1}{\sigma}}(r(1-\sigma)-\theta)}\left(e^{\frac{r(1-\sigma)-\theta}{\sigma}t}-1\right)\right], & \text{for } t \in [0, R_j^{RS*}); \\ e^{rt}\left[k_j^*(R_j^{RS*})e^{-rR_j^{RS*}} + \frac{b_j^{RS}}{r}(e^{-rR_j^{RS*}} - e^{-rt}) - \right. & \\ \left. \frac{\sigma}{\mu_j^{\frac{1}{\sigma}}(r(1-\sigma)-\theta)}\left(e^{\frac{r(1-\sigma)-\theta}{\sigma}t} - e^{\frac{r(1-\sigma)-\theta}{\sigma}R_j^{RS*}}\right)\right], & \text{for } t \in [R_j^{RS*}, T]. \end{cases} \tag{24}$$

3.3.2 Hand-to-mouth workers

While HTM workers do not save, we assume that they optimize over the retirement margin. Since their consumption $c^{HTM}(t)$ at any point is equal to their income in the same period:

$$c^{HTM}(t) = \begin{cases} w_L(1-\tau_l), & \text{for } t \in [0, R^{HTM}); \\ b^{HTM}, & \text{for } t \in [R^{HTM}, T], \end{cases} \tag{25}$$

their optimization problem reduces to identifying the retirement age R^{HTM*} that solves the following problem:

$$\max_{R^{HTM}} U^{HTM} = \int_0^{R^{HTM}} \frac{(w_L(1-\tau_l))^{1-\sigma}-1}{1-\sigma}e^{-\theta t}dt + \int_{R^{HTM}}^{T} \frac{(b^{HTM})^{1-\sigma}-1}{1-\sigma}e^{-\theta t}dt + \beta\int_{R_j^{HTM}}^{T} e^{\gamma t}(T-t)^{-\phi}e^{-\theta t}dt. \tag{26}$$

The condition for optimal retirement becomes:

$$R^{HTM*}: \frac{(w_L(1-\tau_l))^{1-\sigma}-1}{1-\sigma}e^{-\theta R^{HTM*}} - \frac{(b^{HTM})^{1-\sigma}-1}{1-\sigma}e^{-\theta R^{HTM*}} + \int_{R^{HTM*}}^{T}(b^{HTM})^{-\sigma}\frac{\tau_l(\kappa w_L T+(1-\kappa)\Psi)}{(R^{HTM*}-T)^2}e^{-\theta t}dt - \beta(T-R^{HTM*})^{-\phi}e^{(\gamma-\theta)R^{HTM*}} = 0. \tag{27}$$

This condition differs somewhat from the retirement condition for rational workers. Since HTM workers do not save, their consumption level will be subject to a discrete change at the timing of retirement. As long as $w_L(1-\tau_l) > b^{HTM}$, this discrete change will correspond to a consumption drop, which is consistent with the retirement-consumption puzzle. The difference between the first two terms in Equation (27) capture the tradeoff between the consumption level realized from labor income and pension income, respectively. The third term captures how the level of retirement consumption increases by delaying retirement, via returns to the Bismarckian pillar. The fourth term is identical to that of the rational worker, and captures the utility cost of delaying retirement in the form of foregone leisure.

3.4 Aggregation and closing the model

In general equilibrium, all markets clear so that demand equals supply in each market. The clearing conditions for the market for high-skilled labor:

$$L_H^{d*} = \Lambda_H^{RS*} R_H^{RS*}, \tag{28}$$

for low-skilled labor:

$$L_L^{d*} = \Lambda_L^{RS*} R_L^{RS*} + \Lambda^{HTM} R^{HTM*}, \tag{29}$$

and capital:

$$K^{d*} = \int_0^T \left[\Lambda_H^{RS*} k_H^*(t) + \Lambda_L^{RS*} k_L^*(t)\right] dt. \tag{30}$$

The solution also satisfies the aggregate resource constraint:

$$Y = C + I, \tag{31}$$

where

$$C = \int_0^T \left[\Lambda_H^{RS*} c_H^{RS*} + \Lambda_L^{RS*} c_L^{HS*} + \Lambda^{HTM} c^{HTM} \right] dt, \tag{32}$$

is the aggregate consumption, and

$$I = \delta \int_0^T \left[\Lambda_H^{RS*} k_H^*(t) + \Lambda_L^{RS*} k_L^*(t) \right] dt = \delta K, \tag{33}$$

aggregate investments, which is equal to depreciated capital, in the steady state.

4 Calibration

We continue and study the effects of increased automation on the provision and distribution of public pension benefits numerically, for each of the model specifications. Since we explore two different production function specifications, scale becomes an issue. To ensure that both model economies are comparable in terms of marginal returns to inputs and input/output ratios, we scale the task-based economy so that it matches the CSC model, not only in terms of the object described in Table 1, but also in terms of equilibrium output, capital, labor, and wage levels. The steps of this process are:

Step 1: We first calibrate the CSC specification. This involves both an external and an internal calibration. For the external calibration, we lift values for policy parameters from relevant documentation and data provided by `https://www.oecd.org` and `https://databank.worldbank.org`. We then consult the empirical literature for reasonable values for the elasticities governing both demand- and supply-side trade-offs. In particular, we fix the elasticity of substitution between capital and unskilled labor to $\rho_G = 0.4$, which implies substantial substitution between the two inputs.[10] Third, we discipline the remaining parameters for the model to match the capital-output ratio (K/Y), interest rate r, skill premium w_H/w_L, and the average retirement age $\bar{R}$ of an average OECD economy. We set $\sigma = 2$, which implies an elasticity of intertemporal substitution (EIS) equal to 0.5. This parameter describes the sensitivity of savings decisions to a change

[10] In particular, it is almost identical to the value Krusell et al. (2000) find for a different specification of the CSC technology.

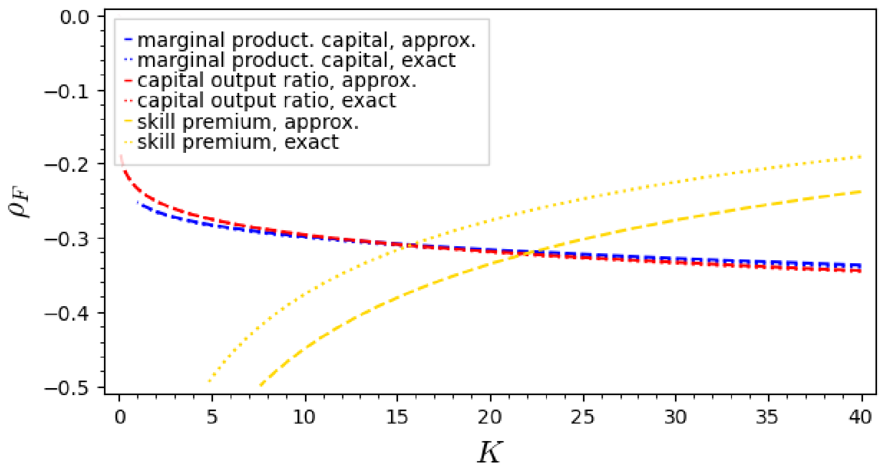
in the real interest rate, and thereby determines the resource cost of increasing the size of the public pension system given its capital crowding-out effect. Given its expected importance, we carry out a sensitivity analysis in which we set $\sigma = 1$, so that utility over consumption is logarithmic, which is equally standard in the quantitative macroeconomic literature. Furthermore, while a value for the EIS (far) below unity is consistent with empirical evidence, a value for $\sigma > 1$ results in a dominating income effect. This further motivates a sensitivity analysis with $\sigma = 1$ where the income and substitution effects perfectly offset each other. This analysis is carried out in Appendix B.1.

Figure 2 shows that the calibrated model is able to precisely fit the three targets determined by the form of the technology. Assuming $\Omega = 25$ and $\sigma_G = 1.66$, the figure compares the loci where the three targets are met exactly for two sets of values: first, for the approximate value for the quantity of input labor, where we weight the average amount of labor in the economy (the average retirement age minus twenty-five) by the share of skilled and unskilled workers in the economy. This sets $L_L = 25.4$ and $L_H = 12.1$. The second set of values for the input labor are the ones we obtain from the internal calibration process, in this case $L_L = 26.0$ and $L_H = 12.5$. The first two targets are consistent with a parameter value ρ_F around -0.3 over a wide range of values for the amount of capital in the economy (say between 10 and 30). The skill premium target appears to be the most sensitive to small differences in the input labor values. It is therefore this target that, in the end, determines precisely the calibrated value for the level of capital in the economy (in this case, $K = 18.0$ and $\rho_F = -0.31$, which corresponds to $\sigma_F = 0.763$, see Table 1).

Step 2: we adjust the baseline of the TB model to the calibrated CSC model to ensure that the economies are identical in equilibrium. To achieve this, we first observe that the quantities $(Y^*, L_H^*, L_L^*, K^*, w_H^*, w_L^*)$, and equilibrium target r^* are determined in the baseline CSC case such that:

$$Y^* = \Omega\left[L_H^{*\rho_F} + B\Big(AK^{*\rho_G} + L_L^{*\rho_G}\Big)^{\frac{\rho_F}{\rho_G}}\right]^{\frac{1}{\rho_F}}, \tag{34}$$

Figure 2: Sensitivity of calibration to K and ρ_F



Note: The figure shows the locii where each of the targets are met, in the (K,ρ_F) plane all else equal. The dotted lines show the evidence at the approximate values for L_L and L_H, such that $L_L = (1 - \Lambda_H)(63.5 - 25) = 25.4$ and $L_H = \Lambda_H(63.5 - 25) = 12.1$. The dashed lines present the same loci at the calibrated value when $\sigma = 2$.

and:

$$\begin{aligned} MP^*_{L_H} &= w^*_H, \\ MP^*_{L_L} &= w^*_L, \\ MP^*_K &= r^* - \delta, \end{aligned} \tag{35}$$

for the calibrated values of the constants A, B, ρ_F and ρ_G.

Given these constants and for a chosen value for the parameter η, we determine the parameter values $\breve{a}$, $\breve{\sigma}_{\mathcal{G}}$, $\breve{B}$ and $\breve{D}$ and such that:

$$\begin{aligned} &\psi(\breve{a}) = \frac{r^*}{w^*_L}, \\ &\breve{\sigma}_{\mathcal{G}} = \sigma_G, \\ &\breve{B} = B(1-\breve{a})^{\frac{\rho_F}{\breve{\rho}_{\mathcal{G}}}(\breve{\rho}_{\mathcal{G}}-1)}, \\ &A = \breve{D}^{\frac{\breve{\rho}_{\mathcal{G}}}{1-\breve{\rho}_{\mathcal{G}}}}(1-\breve{a})^{(\breve{\rho}_{\mathcal{G}}-1)}\left(\int_0^{\breve{a}} \psi(u)^{\frac{\breve{\rho}_{\mathcal{G}}}{1-\breve{\rho}_{\mathcal{G}}}} du\right)^{(1-\breve{\rho}_{\mathcal{G}})}. \end{aligned} \tag{36}$$

Since $\breve{a}$ corresponds by construction to the critical task a^* at the prices w^*_L and r^*, we set $\breve{a} = a^*$. Moreover, we observe that since the function $\psi()$ depends on D, the last equation is non-linear in D. If a solution of the previous equation system exists then the list $(Y^*, L_H^*, L_L^*, K^*, w_H^*, w_L^*, r^*)$ satisfies:

$$Y^* = \Omega\left[L_H^{*\rho_F} + \check{B}\left[(1-a^*)^{\check{\rho}_{\mathcal{G}}-1}\left(\int_0^{a^*}\psi(u)^{\frac{\check{\rho}_{\mathcal{G}}}{1-\check{\rho}_{\mathcal{G}}}}du\right)^{1-\check{\rho}_{\mathcal{G}}}K^{*\check{\rho}_{\mathcal{G}}} + L_L^{*\check{\rho}_{\mathcal{G}}}\right]^{\frac{\rho_F}{\check{\rho}_{\mathcal{G}}}}\right]^{\frac{1}{\rho_F}}, \tag{37}$$

as well as equations (12), (13), (14) and $\psi(a^*) = \frac{r^*}{w_L^*}$ which determine the marginal products and the optimal share a^* of tasks that are completed by automation. The parameter values for the external calibration, and their sources, are reported in Table 1, and the parameters obtained by the internal calibration are shown in Table 3. The model fit is documented in Table 2. Finally, Table 4 describes the equilibrium quantities and prices of the calibrated steady state. In addition to the targeted equilibrium objects, our baseline calibration produces a Gini coefficient equal to 0.338, which is comparable to the observed value of 0.34 for the OECD average.[11] The threshold value for the extensive margin of automation in the calibrated steady state is $a^* = 0.47863$.

Step 3: Our analysis proceeds by examining the consequences for either economy of growth-generating changes: on the one hand, changes that affect TFP and, on the other hand, changes that are specific to the marginal productivity of capital. Specifically, we ensure that all independent changes yield the same increase in total output[12]. Therefore, the change in output is identical in all scenarios, whether we compare the different economies or whether, given a particular technology, we compare alternative sources of change. The focus of our analysis is mostly on the interactions between the sources of growth and the distributional effects of the public pension system across the population.

[11] `https://www.statista.com/statistics/1461858/gini-index-oecd-countries/`

[12] In practice, we start with the CSC economy and increase the marginal product of capital by increasing the parameter A by 10%. We then determine the change to the TFP parameter Ω that produces the equivalent increase in total output. We then turn to the TB economy and determine the value of the parameter η that yields an equivalent change in total output, all else being equal. Finally, we determine the value of the TFP parameter in the TB economy in the same fashion.

Table 1: External calibration

	Parameter	Source/target
Population		
Time horizon	$T = 55$	Economic lifespan ages 25-80;
Hand-to-mouth population share	$\Lambda^{HTM} = 0.2$	20 % live hand to mouth;
Behavioral		
Inverse EIS	$\sigma = 2$	Standard;
Inverse retirement elasticity	$\phi = 1.5$	Ret. elasticity around 0.3;
Common production		
Total factor productivity	$\Omega = 25$	see discussion of Figure 2;
Elasticity of substitution L_h vs $G(K, L_l)$	$\sigma_F = 0.763$	see discussion of Figure 2;
Elasticity of substitution between L_L and K	$\sigma_G = \sigma_{\mathcal{G}} = 1.66$	Krusell et al. (2000);
Capital depreciation rate	$\delta = 0.08$	Standard;
Curvature capital productivity	$\eta = 0.48$	Kına (2024)
Policy		
Public pension contribution rate	$\tau^s = 0.154$	OECD data ;
Bismarckian factor	$\kappa = 0.67$	Bourlés and López-Cantor (2024).

Table 2: Targeted equilibrium objects and model values after internal calibration

	Object	Target	CSC	Source
1	Capital–Output ratio	3.00	3.00	Standard
2	Real interest rate (%)	4.00	4.00	Standard
3	Avg. Retirement age	63.50	63.50	OECD data
4	Skill premium	1.75	1.75	Standard
5	Proportion high-skilled	0.34	0.34	OECD (2023)

Note: The target column in the table describe the quantities that the internal calibration attempts to match. The CSC column describes the value of the model quantities at the calibrated parameters given in Tables 1 and 3.

5 Analysis of Simulations

With the baseline calibration as a reference point, we now proceed to numerically analyze the effects of automation-driven growth on the level and distribution of pension benefits. First, we perform a comparative static analysis on key equilibrium objects such as capital and labor shares, income measures, and prices, conditional on a 10% growth in the productivity of capital. We compare the consequences of this change in the productivity of capital to a change in the total factor productivity that yields an identical effect on total output. Second, we derive the optimal size of the public pension system to study whether automation-driven growth merits any reform to the contribution rate.

Table 3: Internal Calibration, parameters

Parameter		Specification CSC	TB
$\mathbb{E}[\psi]$	Mean of distribution of skill cost ψ	450.856	450.856
θ	Discount rate	0.027	0.027
β	Retirement utility parameter	204.232	204.232
B	Capital-low skilled productivity	5.563	4.105
A	Capital productivity	1.246	-
D	Scale capital productivity	-	1.344

Note: The Table shows the calibrated parameter values given the parameter values described in Table 1. The two economies are calibrated to the match the same equilibrium objects as described in Table 2. Entries with - indicate that the parameter is not defined in the given specification.

Table 4: Equilibrium objects

Y^*	w_H	w_L	b_H	b_L	K^*	L_H^*	L_L^*	r^*
5.209	0.123	0.070	0.031	0.027	15.600	12.000	26.500	0.040

Note: The Table contains the quantities and prices of the calibrated model. These output values are the same under both the CSC and the TB-specifications.

5.1 Comparative analysis

First, we study how increased automation affects labor supply, pension benefits, labor income, and lifetime income across the different worker types. The income and inequality measures are defined in Table 5 for a cross-section of the population at a given point in time

Table 5: Income and inequality measures

Earnings income (EI)	Pension income (PI)	Lifetime income (LI)
$w_j^i(1-\tau_l)R_j^i$	$b_j^i(T-R_j^i)$	EI+PI
Earnings inequality	Pension income inequality	Lifetime income inequality
$\frac{(1-\Lambda_H^{RS})EI_H^{RS}}{\Lambda_L^{RS}EI_L^{RS}+(1-\Lambda_H^{RS}-\Lambda_L^{RS})EI^{HTM}}$	$\frac{(1-\Lambda_H^{RS})PI_H^{RS}}{\Lambda_L^{RS}PI_L^{RS}+(1-\Lambda_H^{RS}-\Lambda_L^{RS})PI^{HTM}}$	$\frac{(1-\Lambda_H^{RS})LI_H^{RS}}{\Lambda_L^{RS}LI_L^{RS}+(1-\Lambda_H^{RS}-\Lambda_L^{RS})LI^{HTM}}$

Note: The Table contains the formulas for computing the various cross-sectional income and inequality measures used in the comparative analyses. The measures of inequality compare the average income among the high skill to the average among the low skill.

The effects of the different growth scenarios on factor shares, calculated as elasticities with respect to an increase in aggregate output, in response to an increase in total factor productivity or capital productivity, are presented in Table 6. These results are expected: Under both model specifications, automation leads to an increase in the capital–output ratio and, given complementarity, also to an increase in the high-skilled labor share of output. Meanwhile, the low-skilled labor share of output decreases. When comparing automation-driven growth scenarios between the CSC and TB specifications, we find that the increase in the capital share is substantially larger under the TB specification. This can be explained by the displacement effect present in the TB specification: as capital grows more productive, it not only becomes more productive in the tasks it is already performing, but it also gains a comparative advantage in an additional number tasks. This corresponds to an increase in a^*. This displacement effect generates a fall in the low-skilled labor share that is also substantially larger for the TB specification than for the CSC specification. For example, a one percent increase in output, resulting from improved automation, reduces the low-skilled labor share of national income by 0.411% under the CSC specification, while it falls by 0.571% under the TB specification. In the sensitivity analysis presented in Table B.3, when $\sigma = 1$, the higher value for the EIS, which implies that aggregate savings respond more to an increase in the interest rate, leads to an even higher increase in the capital–output ratio of 0.583 % for the CSC specifiction, and 0.901 % for the TB specification.

Table 6: Comparative statics on input shares, elasticities w.r.t. a change in output Y

Model	K/Y	L_H/Y	L_L/Y
TFP (CSC)	0.059	0.170	−0.190
Automation (CSC)	0.505	0.288	−0.411
TFP (TB)	0.390	0.137	−0.291
Automation (TB)	0.954	0.252	−0.571

Note: The Table shows the percentage change for the factor shares of production, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

Table 7 documents the elasticity of prices with respect to output growth. For both model specifications, automation-driven growth increases the returns to capital and thereby also the interest rate. Importantly, the interest rate increases the most under automation-driven growth in the TB specification. As the interest rate increases, inequality between capital owners and hand-to-mouth individuals will also increase. This bolsters pension inequality as rational savers top up any forced savings in the illiquid pension system with assets that earn interest at a higher rate.

Any growth scenario is found to disproportionately benefit the returns to high-skilled labor. This is true whether the source of growth is specific to capital (i.e., automation) or general (i.e., TFP). The direct effect follows from the higher degree of complementarity between high-skilled workers and capital in both specifications. In the post-growth steady states, we observe an increase in the skill premium, which in turn carries over to a more unequal distribution of pension benefits as the pension system is partially earnings-based.

It is interesting to note that although automation-driven growth is unequally distributed, all workers experience a wage increase. At first glance, this suggests that the productivity effect of increased automation dominates the displacement effect among low-skilled workers in our preferred calibration. However, we find that this result can be explained by labor supply effects and mobility across skill groups via the decision on skill formation. In Table B.8 in the Appendix B.2, where we report the results under the assumption that the skill distribution is exogenous and fixed to the calibrated shares of high- and low-skilled rational workers, the low-skilled face marginally lower wages when automation drives growth in the TB specification. In the full model, the combination of higher wages among high-skilled and lower wages among low-skilled leads to an increased enrollment in tertiary education. This reduces the supply of low-skilled labor enough for the low-skilled wage level to increase. Ultimately, while the displacement effect of automation is seemingly larger than the productivity effect for low-skilled wages when skills are fixed, the mobility across skill reduces low-skilled labor supply to the extent that low-skilled wages increase relative to the baseline steady state.

Figure 3 shows how the proportion of high-skilled workers changes with a change in technology, the contribution rate to the public pension system, and the wage differential. In any scenario where growth leads to a disproportionally large increase in the wage of the high-skilled, ceteris paribus, more workers choose to upgrade their skill according to the decision in Equation (17). Since the pension system is partially redistributive within generations, a higher value of the contribution rate reduces the lifetime skill-premium and makes human capital investments less attractive.

Table 7: Comparative statics on prices, elasticities w.r.t. a change in output Y

Model	r	w_H	w_L	b_H^{RS}	b_L^{RS}	b^{HTM}
TFP (CSC)	−0.204	1.333	0.405	0.709	0.555	0.542
Automation (CSC)	0.068	1.211	0.253	0.504	0.305	0.371
TFP (TB)	0.007	1.234	0.332	0.551	0.381	0.453
Automation (TB)	0.359	1.061	0.138	0.271	0.046	0.231

Note: The Table shows the percentage change for various prices, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

Table 8 complements Table 7 by showing that increase in wage and pension benefit levels across the skill distribution carries over to the levels of earnings, pension, and lifetime income for all worker types. What appears clear is that, in the task-based specification, the income responses of both low-skilled worker types are substantially smaller in response to automation.

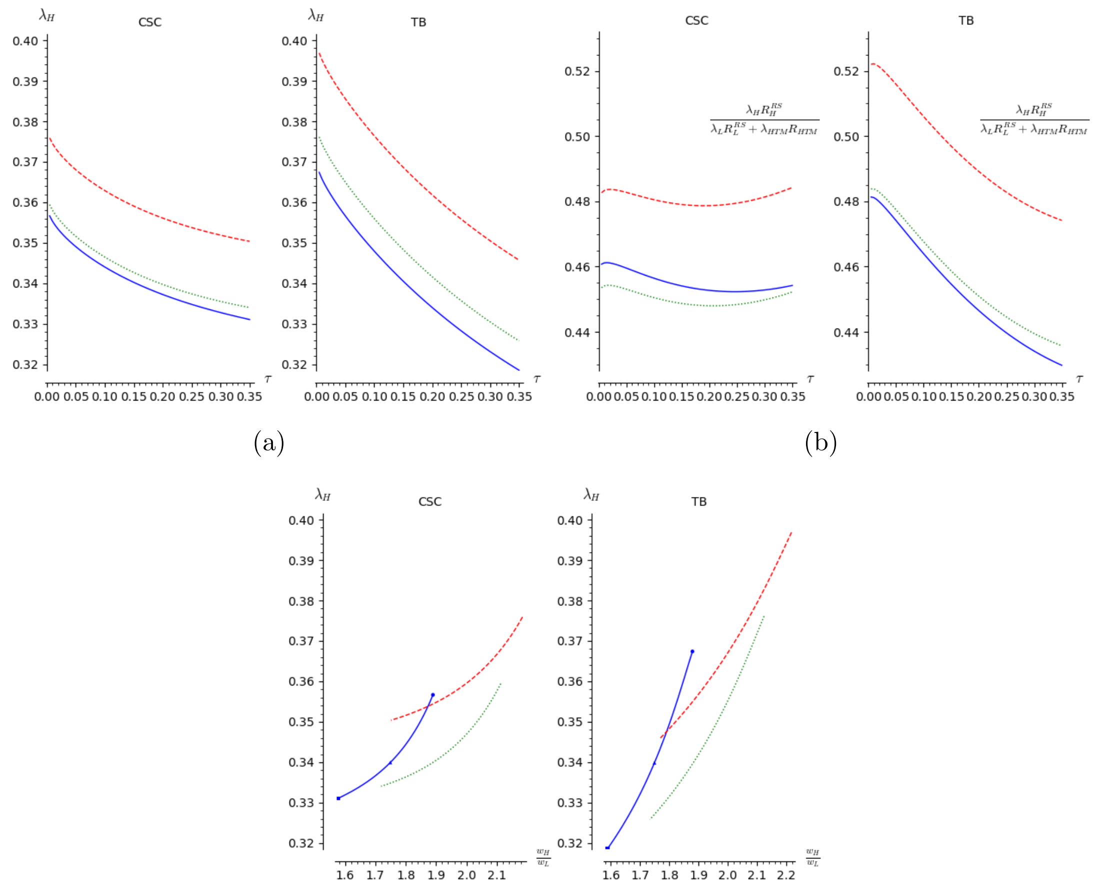
Table 8: Comparative statics on income measures, elasticities w.r.t. a change in output Y

Model	EI_H^{RS}	PI_H^{RS}	LI_H^{RS}	EI_L^{RS}	PI_L^{RS}	LI_L^{RS}	EI_L^{HTM}	PI_L^{HTM}	LI_L^{HTM}
TFP (CSC)	1.128	1.063	1.118	0.665	0.750	0.679	0.691	0.774	0.700
Automation (CSC)	0.975	0.908	0.965	0.382	0.515	0.404	0.430	0.470	0.434
TFP (TB)	1.005	0.943	0.996	0.515	0.613	0.531	0.566	0.568	0.566
Automation (TB)	0.792	0.728	0.783	0.147	0.304	0.173	0.230	0.161	0.223

Note: The Table shows the percentage change for various income measures, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

The inequality effects are presented in Table 9. Comparing the income inequality measures for the baseline scenarios, we find that earnings inequality is higher compared to pension income inequality. This mechanically follows from the fact that the pension system is partially redistributive, given $\kappa < 1$. Following automation-driven growth, under any specification, we find that both earnings inequality and pension inequality in-

Figure 3: Change in the proportion of high-skilled workers given changes in the contribution rate and the wage differential.



Note: The figures show the response of the share of high-skilled (panels a) and the ratio of high to low skill labour in panel (b) to a change in the public pension contribution rate. Each line corresponds to different technological assumptions: the continuous blue lines correspond to the baseline, the dashed red lines to a 10% increase to A in the CSC case, resp. to a change to η which generates an equivalent change to output in the TB case, and the dotted green lines to a 10% increase to Ω in the CSC case, resp. to a change to Ω which generates an equivalent change to output in the TB case. Panel (c) shows the association between the proportion of high skill workers and the wage differential as the contribution rate increases. In Panel (c), along each line when the contribution rate is low, $\tau = 0$, the proportion of high skill worker and the wage differential are the largest, while when the contribution is large, $\tau = 0.35$, both the differential and the share of high skill worker are the smallest.

crease, but earnings inequality remains larger throughout. For lifetime income inequality, we see that the pension system provides some insurance against automation-driven inequality, but this effect is modest since the Bismarckian pillar is substantially larger than the Beveridgean pillar. Overall, inequality effects are greater under the TB specification, as expected given the presence of displacement effects.

It is interesting to note that the baseline skill premium on earnings income (1.542) is lower than the calibrated skill premium on wages (1.75). This can be explained by the fact that high-skilled individuals retire at a younger age ($R_H^{RS} = 35.35$) than both rational low-skilled ($R_L^{RS} = 38.61$) and HTM workers ($R^{HTM} = 43.60$), following a dominating income effect when $\sigma > 1$. As shown in Table B.6 in Appendix B.1, when $\sigma = 1$, the skill-premium on earnings income (1.7) is closer to the wage skill-premium. This is because the distribution of retirement ages across different worker types is more concentrated around the mean, with $R_H^{RS} = 37.88$, $R^{RS_L} = 37.68$ and $R^{HTM} = 41.43$

Table 9: Inequality effects

Model	Earnings ineq.	Pension ineq.	Lifetime ineq.
Baseline	1.542	1.515	1.538
TFP (CSC)	1.668	1.598	1.658
Automation (CSC)	1.701	1.629	1.691
TFP (TB)	1.673	1.608	1.663
Automation (TB)	1.712	1.646	1.702

Note: The Table shows how earnings, pension and lifetime income inequality changes with either automation-driven or TFP-driven growth for both the CSC and TB specifications.

In Table 10, we quantify the public pension income share of retirement consumption for both high- and low-skilled rational saving workers. Obviously, this share is one for HTM workers since they do not save. This share is also higher for low-skilled workers as the pension system is redistributive, so their replacement rate is higher than that of high-skilled workers. Since low-skilled workers face higher relative pension benefits, the Beveridgean pillar of the pension system causes larger disincentives for private savings among these workers. This illustrates the potential dilemma that the designers of public pensions face: if the system is designed to redistribute income, as via the Beveridgean component, it can mitigate some of the increased inequality caused by improved automation, specifically the inequality that carries over from the skill premium via the Bismarckian pillar. However, simultaneously, if the system becomes more redistributive, it would create larger disincentives for private savings among low-skilled workers. Such a reform therefore bolsters capital inequality, not only between rational savers and hand-to-mouth workers, but also between high-skilled and low-skilled rational savers.

Analyzing the sensitivity of these results to the economy with $\sigma = 1$, in Appendix B.1, Table B.7, we find that the differences in the public pension share of retirement consumption across skill-groups is smaller. This can be explained by the differences in when these worker types retire. When $\sigma = 1$, the high-skilled worker retires at a later age relative to the baseline calibration. The opposite is true for low-skilled workers. Consequently, high-skilled workers save less for retirement in the sensitivity analysis, while the low-skilled save more privately. By retiring later (earlier), the high-skilled (low-skilled) workers also contribute relatively more (less) to the pension system in the sensitivity analysis, which is realized as higher (lower) retirement benefits via the Bismarckian pillar. This further reduces (increases) incentives for private savings. However, the qualitative results that automation increases the difference in the public pension share of retirement consumption across skill-groups is robust.

Table 10: Public pension income share of retirement consumption

	CSC		TB	
Group	High Skill	Low Skill	High Skill	Low Skill
Baseline	0.288	0.414	0.288	0.414
TFP growth	0.270	0.413	0.262	0.401
Automation growth	0.259	0.402	0.248	0.387

Note: The Table shows how the ratio of public pension income to retirement consumption changes for the different growth scenarios under both the CSC and TB specifications. we do not present figures for the Hand to Mouth group, since in this case public pension income funds the entirety of consumption in retirement.

To summarize, the public pension system provides some insurance against automation-driven income inequality through the Beveridgean pillar. However, the same mechanism also reduces the incentives for private retirement savings among high- and low-skilled workers. The pension system can work to reduce inequality via the link between wage and retirement benefits, and at the same time increase inequality via the savings channel.

5.2 Pension policy and welfare

From the previous analysis, we conclude that automation can increase inequality in pension payments between high- and low-skilled workers, and between rational savers and hand-to-mouth consumers. Furthermore, since public pensions earn zero interest, hand-to-mouth consumers forego any compound interest from not saving in the capital market. In light of these sources of increased inequality, we now pose the question of whether automation-driven growth justifies any reform to the public pension size. To this end, we first quantify the optimal size of the public pension system, given by τ^*, in the baseline economy. We then analyze how the optimal value for this parameter changes if the economy is subject to growth, driven by either improvements to automation or to total factor productivity.

The central planner determine the size of the pension system τ so as to maximize social welfare which is:

$$\mathcal{W}(\tau) = \Gamma^{HTM}(1 - \Lambda_H^{RS} - \Lambda_L^{RS})U^{HTM} + \sum_{i \in \{L,H\}} \Gamma_j^{RS}\Lambda_j^{RS}U_j^{RS}, \tag{38}$$

where Γ^{HTM} and Γ_j^{RS} for $j \in \{L, H\}$ are welfare weights attached to the lifetime utility of hand-to-mouth and rational workers respectively. Assuming a standard Utilitarian central planner, these weights are normalized to unity $\Gamma^{HTM} = \Gamma_j^{RS} = 1$.

In turn, the analytical expressions for the optimal utility levels can be written as follows. For rational savers $j \in \{L, H\}$:

$$\begin{aligned} U_j^{RS} =& \frac{\mu_0^{-\frac{1-\sigma}{\sigma}}}{1-\sigma}\frac{\sigma}{r(1-\sigma)-\theta}\Big(\exp\big(\frac{r(1-\sigma)-\theta}{\sigma}T\big) - 1\Big) + \frac{1}{(1-\sigma)\theta}\Big(\exp(-\theta T) - 1\Big) + \\ & \beta\int_{R_j^{RS}}^{T} e^{\gamma t}(T-t)^{-\phi}e^{-\theta t}dt, \end{aligned} \tag{39}$$

and HTM workers:

$$U^{HTM} = \frac{(w_l(1-\tau_l))^{1-\sigma}-1}{(1-\sigma)\theta}\Big(1-\exp(-\theta R^{HTM})\Big)+ \frac{(b_l)^{1-\sigma}-1}{(1-\sigma)\theta}\Big(\exp(-\theta T)-\exp(-\theta R^{HTM})\Big)+\beta\int_{R^{HTM}}^{T} e^{\gamma t}(T-t)^{-\phi}e^{-\theta t}dt. \tag{40}$$

In this measure, θ essentially determines how the government discounts the welfare of older generations. If $\theta = 0$, the government attaches the same welfare weight to all generations alive.

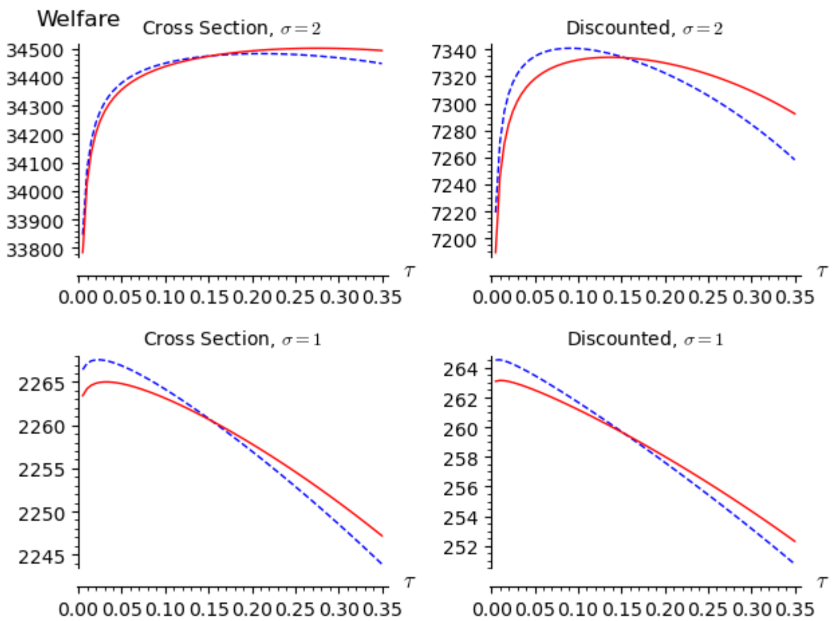
Ultimately, let τ_l^* denote the optimal size the pension system such that:

$$\{\tau_l^*\} = \arg\max_{\tau_l}\{\mathcal{W}(\tau_l)\}. \tag{41}$$

In Figure 4, we illustrate how welfare changes with the size of the pension system for the baseline calibration of both the CSC and TB specifications. Since the TB specification is calibrated to perfectly match the CSC economy, the welfare level is the same for both model economies at the calibrated value of $\tau = 0.154$. We compare two different discounting scenarios: one where the government does not discount the welfare of older generations (cross section), and one where the government applies the same discount rate as the workers (discounted). For the analyses carried out in this section, we have chosen to include the results of the sensitivity analysis where $\sigma = 1$. We find that the welfare effects of varying the size of the public pension system are very sensitive to different values for σ. However, importantly, the qualitative insights from the comparative analysis do not change.

Comparing the CSC and TB economies, we find that the latter supports a larger public pension system at the optimum than the former. More generally, increasing the size of the pension system beyond the calibrated value is less costly in terms of welfare under the TB specification compared to the CSC specification. This follows from the displacement effect of the TB specification: increasing the size of the pension system crowds out private savings, and therefore reduce the use robots. This leads to a higher marginal product of capital, and therefore an increased interest rate. As the interest rate

Figure 4: Optimal size of the pension system given the baseline calibrations.



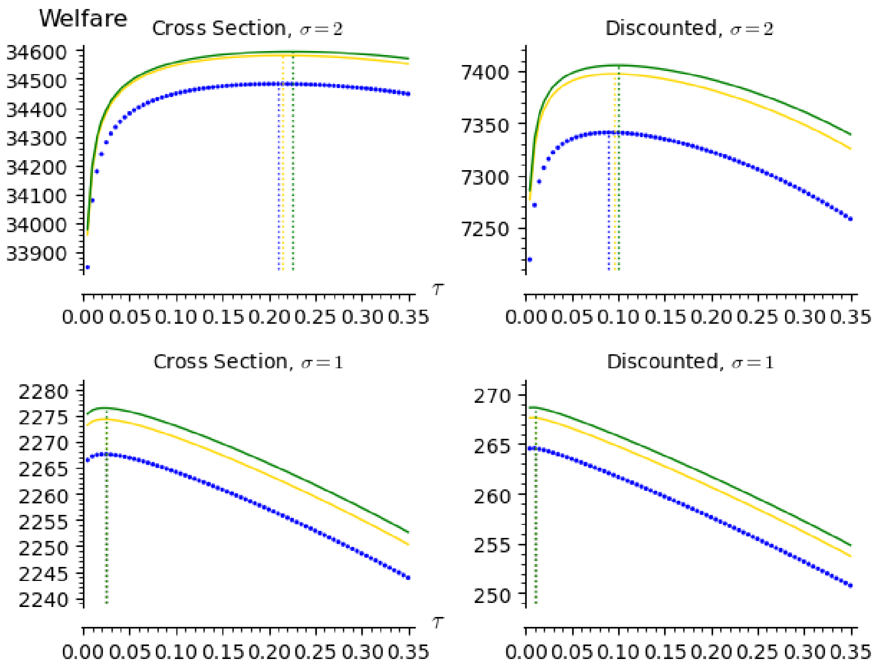
Note: The figures show the behavior of overall welfare in the economy under two specifications for the preferences: $\sigma = 2$, top row, and $\sigma = 1$, bottom row. The left-most column shows how the instantaneous welfare (i.e. when $\theta = 0$ in the economy , as defined in equation (38), varies with the size of the pension system, τ. The right-most provides the same illustration when the discount rate is set to its calibrated value $\theta = 0.032$. Blue dotted lines corresond to the CSC specification, and red solid lines to the TB specification.

increases, robots close to the extensive margin of automation become relatively costlier to operate compared to low-skilled labor. Consequently, the increased demand for low-skilled labor results in higher equilibrium wages for low-skilled workers relative to the CSC specification.

For both technology specifications, we also note that the optimal contribution rate τ^* is higher for a higher value of σ, which in turn translates to a lower value of the elasticity of intertemporal substitution. This is reasonable since if savings are more inelastic, the capital crowding-out effect of increasing the size of the pension system is smaller. We find that τ^* exceeds the calibrated value by a sizable amount only when we evaluate welfare for the cross section and when $\sigma = 2$.

Relative to the baseline calibration, the effects of automation-driven growth on the optimal size of the public pension system are not trivial. On the one hand, automation-driven growth leads to an increased skill premium and thus increases earnings inequality. Given the Utilitarian welfare measure, this should warrant an increase in the contribution

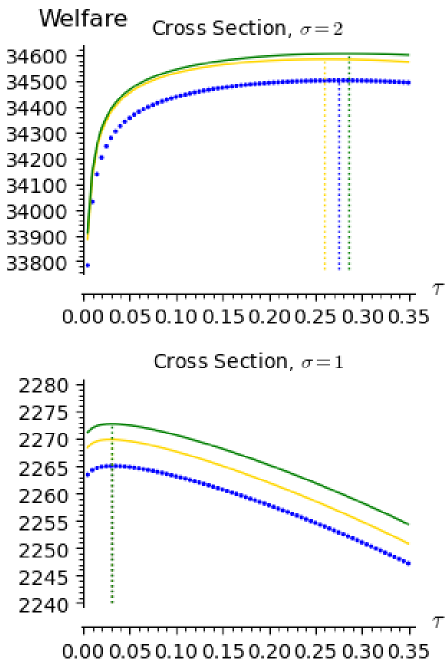
Figure 5: Optimal size of the pension system under CSC-technology.



Note: The figures show the response of overall welfare in the economy assuming the CSC specification in response to alternative changes to the technology. The blue dotted lines correspond to the baseline calibration; the yellow solid lines describe the welfare response to the automation-driven growth scenario, and the green solid lines show the welfare response to the TFP growth scenario. The top row shows the calibration for the preferences such that $\sigma = 2$, and $\sigma = 1$ in the bottom row. The left-most column shows how the instantaneous welfare (i.e. when $\theta = 0$ in the economy , as defined in equation (38), varies with the size of the pension system, τ. The right-most provides the same illustration when the discount rate is set to its calibrated value $\theta = 0.032$.

rate as the pension system provides intragenerational redistribution via the Beveridgean pillar. On the other hand, automation-driven growth increases the interest rate, which in turn translates to a higher opportunity cost of contributing to the public pension system. This should have a negative effect on the optimal size of the public pension system. As illustrated in Figures 5 and 6, a somewhat surprising finding is that most growth scenarios we consider do not merit any change in the size of the optimal public pension system, suggesting that the two effects largely offset each other.

Figure 6: Optimal size of the pension system under TB-technology.



Note: The figures show the response of overall welfare in the economy assuming the TB specification in response to alternative changes to the technology. The blue dotted lines correspond to the baseline calibration; the yellow solid lines describe the welfare response to the automation-driven growth scenario, and the green solid lines show the welfare response to the TFP growth scenario. The top row shows the calibration for the preferences such that $\sigma = 2$, and $\sigma = 1$ in the bottom row. The left-most column shows how the instantaneous welfare (i.e. when $\theta = 0$ in the economy , as defined in equation (38), varies with the size of the pension system, τ. The right-most provides the same illustration when the discount rate is set to its calibrated value $\theta = 0.032$.

6 Conclusions

Public pension programs are typically funded through payroll taxes or labor income-indexed social contributions. Since automation has been shown to reduce labor shares and possibly equilibrium wages in exposed sectors, the present paper explores the implications of improved automation on a number of variables related to public pensions, such as the size and distribution of pension benefits, as well as the opportunity cost of the system. In a second stage, we quantify the optimal size and design of public pensions and study how these characteristics vary when economic growth follows from improvements in automation. These analyses are carried out in an overlapping generations model with indivisible labor and heterogenous workers. Rational workers make decisions on skill formation, consumption/savings, and when to retire. Meanwhile, hand-to-mouth workers only decide when to retire.

We consider two different production function specifications to model automation. The first exhibits capital-skill complementarity, where automated capital is more complementary to high-skilled workers than to the low-skilled. The other is a task-based specification in which improved automation displaces low-skilled workers.

Calibrating the model to average OECD macro-regularities, we find that improved automation can increase pension inequality through the earnings-related pillar of the pension system, and that the redistributive pillar provides some insurance against this. However, the redistributive pillar also generates greater disincentives for private savings among low-skilled households. As such, this increases inequality in private pension savings. Importantly, we find that, when the skill-distribution is exogenously determined, automation reduces the wage level of low-skilled workers in the task-based specification. When skill formation is endogenous, automation increases the wages across the skill-distribution following as the increased skill-premium increases the number of workers who upgrade their skills, thus reducing the labor supply of low-skilled workers. This emphasizes the potential long-run benefits of public policy that promote effective skill-enhancing activities.

From the welfare analysis, we find that the model with task-based production merits a larger public pension system. This follows from the capital crowding-out effects of increasing the size of the pension system being smaller under such a system, since capital displaces low-skilled labor. If capital is crowded out and returns to capital increase, low-skilled labor will gain a comparative advantage in the production of more tasks. This increase in labor demand then results in higher equilibrium wages for these workers. Interestingly, we do not find that increased automation has any important effect on the size of the optimal public pension system. While increasing the pension system can reduce some of the wage-driven income inequality, the same reform increases the interest rate and therefore also the opportunity cost of contributing to the public pension system. Our findings suggest that these effect largely offset each other.

Ultimately, our results highlight important interaction effects between automation and the performance of public pensions. While public pensions are often designed to reduce intra-generational inequality, this pillar can bolster inequality in capital income as a higher replacement rate for low-skilled workers creates larger disincentives for these individuals to save privately for retirement. More generally, we show that the way technology is specified also matters for the quantitative assessments of the optimal size of public pensions. For example, how much capital displaces human labor affects the resource cost of expanding public pension systems, particularly in terms of crowding out private savings in the economy.

There exist several interesting paths for future work. The focus of this paper is on automation, and specifically capital that can substitute for low-skilled workers. With the recent emergence of AI technologies, it is natural to ask how our current results may change if one includes capital that substitutes for high-skilled workers. If the productivity growth in AI is more rapid than that of automation, it may reduce economic inequality. At the same time, if AI also contributes to the reduction of the labor share in the economy, it could exacerbate the funding issues already faced by public pension systems.

Furthermore, our analysis is restricted to unfunded pay-as-you-go pension systems. While this is by far the most common way of financing public pensions, a major question in the field of pension economics is whether public pensions should or could be privatized, and how such a reform could be made without violating the Pareto criterion. Given that our findings suggest increased inequality between workers who save privately for retirement and those who only rely on public pensions, automation should increase the long-term gains of capitalizing public pension systems.

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A Analytical steps

A.1 Calculation of μ_j given R_j^{RS}, equations (19) and (21)

Given a value for R_j^{RS} and keeping all prices as given, we can substitute the expression for the optimal consumption (21) into the intertemporal budget constraint (19) to obtain a general expression for μ_j:

$$\mu_j|_R = \Big(\frac{r}{f}(e^{fT}-1)\Big)^{\sigma}\Big(\tilde{w}(1-e^{-rR}) + b_j(e^{-rR}-e^{-rT})\Big)^{-\sigma}, \tag{A1}$$

where $f \equiv \frac{r(1-\sigma)-\theta}{\sigma}$. In the absence of a pension system, that is if $\tau = 0$, the expression simplifies to:

$$\mu_j|_R = \Big(\frac{r}{f}(e^{fT}-1)\Big)^{\sigma}\Big(w(1-e^{-rR})\Big)^{-\sigma} = \Big(\frac{r}{wf}\Big)^{\sigma}\Big(\frac{e^{fT}-1}{1-e^{-rR}}\Big)^{\sigma}. \tag{A2}$$

A.2 Specifying utility for retirement.

We start from a stock-specification for retirement utility $v(R)$ that builds on the assumption that an individual full-time retires at age R and remains so throughout the rest of its lifetime. Assuming a CRRA functional form, this specification can be written as:

$$v(R) = \frac{(T-R)^{1-\phi}}{1-\phi}, \tag{A3}$$

as in, e.g., Jacobs (2009), Gustafsson (2023a) and Gustafsson (2023b). From this stock-specification, we specify a constant flow utility function $\Gamma(T-R)$ over the period from R to T such that:

$$\frac{(T-R)^{1-\phi}}{1-\phi} \approx \int_R^T \Gamma(T-R)e^{-\theta t}dt, \tag{A4}$$

so that the flow utility takes the form:

$$\Gamma(T-R) \approx e^{\gamma t}(T-t)^{-\phi}. \tag{A5}$$

This allows us to include discounting in utility and welfare measures explicitly, rather than assuming any discounting to be implicit to the weight of retirement preferences β.

A.3 Expression for the Aggregate Production Function with Task-Based choice in production

To simplify the discussion we focus on the analysis of the aggregate part of the model which is affected by automation as described in Equation (10). For any task $u \in [0, \alpha^*)$ which uses capital it must be the case that at the margin:

$$p_{\mathcal{G}}\psi(u)^{\rho_{\mathcal{G}}}k(u)^{\rho_{\mathcal{G}}-1}\mathcal{G}^{1-\rho_{\mathcal{G}}} = r+\delta, \tag{A6}$$

where $p_{\mathcal{G}}$ measures the value of the aggregate $\mathcal{G}$ (we could carry instead the analysis from first principle detailing how $p_{\mathcal{G}}$ depends on the functions F and $\mathcal{G}$ and the derivative $\frac{\partial F}{\partial \mathcal{G}}$).

Furthermore, for any task $u \in (\alpha^*, 1]$ which uses labour in quantities $l(u)$ it must be the case that:

$$p_{\mathcal{G}} l(u)^{\rho_{\mathcal{G}}-1} \mathcal{G}^{1-\rho_{\mathcal{G}}} = w. \tag{A7}$$

The following observations will be useful in what follows:

$$\begin{aligned} \mathcal{G} &= S^{\frac{1}{\rho_{\mathcal{G}}}}, \\ \mathcal{G}^{1-\rho_{\mathcal{G}}} &= S^{\frac{1}{\rho_{\mathcal{G}}}-1}, \end{aligned} \tag{A8}$$

where we define $S \equiv \int_0^1 y(u)^{\rho_{\mathcal{G}}} du$.
The first order condition expressed in equation (A6) can be rewritten into two expressions:

$$\begin{aligned} k(u) &= (\frac{r+\delta}{p_{\mathcal{G}}})^{\frac{1}{\rho_{\mathcal{G}}}-1} \mathcal{G}\, \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}}, \\ \psi(u)^{\rho_{\mathcal{G}}} k(u)^{\rho_{\mathcal{G}}} &= \frac{r+\delta}{p_{\mathcal{G}}} \mathcal{G}^{\rho_{\mathcal{G}}-1} k(u). \end{aligned} \tag{A9}$$

From the first of these two expressions we obtain an additional expression for $\psi(u)^{\rho_{\mathcal{G}}} k(u)^{\rho_{\mathcal{G}}}$:

$$\psi(u)^{\rho_{\mathcal{G}}} k(u)^{\rho_{\mathcal{G}}} = \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}} \Big[\frac{r+\delta}{p_{\mathcal{G}}} \mathcal{G}^{\rho_{\mathcal{G}}-1}\Big]^{\frac{\rho_{\mathcal{G}}}{\rho_{\mathcal{G}}-1}}, \tag{A10}$$

and we observe that the expression within the squared brackets is independent of u. This observation and the two expressions in (A9) yields two expressions for $\int_0^{\alpha^*} \psi(u)^{\rho_{\mathcal{G}}} k(u)^{\rho_{\mathcal{G}}} du$. We find:

$$\begin{aligned} \mathcal{I}(\alpha^*) &\equiv \int_0^{\alpha^*} \psi(u)^{\rho_{\mathcal{G}}} k(u)^{\rho_{\mathcal{G}}} du = \Big[\frac{r+\delta}{p_{\mathcal{G}}} \mathcal{G}^{\rho_{\mathcal{G}}-1}\Big]^{\frac{\rho_{\mathcal{G}}}{\rho_{\mathcal{G}}-1}} \int_0^{\alpha^*} \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}} du, \\ \mathcal{I}(\alpha^*) &\equiv \int_0^{\alpha^*} \psi(u)^{\rho_{\mathcal{G}}} k(u)^{\rho_{\mathcal{G}}} du = \Big[\frac{r+\delta}{p_{\mathcal{G}}} \mathcal{G}^{\rho_{\mathcal{G}}-1}\Big] \int_0^{\alpha^*} k(u) du. \end{aligned} \tag{A11}$$

Denote K^* the aggregate capital across the tasks that use it, so that $K^* \equiv \int_0^{\alpha^*} k(u)du$. The second of the expressions above therefore imply that:

$$\frac{r+\delta}{p_{\mathcal{G}}} \mathcal{G}^{\rho_{\mathcal{G}}-1} = \frac{\mathcal{I}(\alpha^*)}{K^*} \tag{A12}$$

The first expression can therefore be rewritten as:

$$\mathcal{I}(\alpha^*) = \Big[\frac{\mathcal{I}(\alpha^*)}{K^*}\Big]^{\frac{\rho_{\mathcal{G}}}{\rho_{\mathcal{G}}-1}} \int_0^{\alpha^*} \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}} du, \tag{A13}$$

which we can solve for $\mathcal{I}(\alpha^*)$ to find:

$$\mathcal{I}(\alpha^*) = \Psi(a^*) K^{*\rho_{\mathcal{G}}}, \tag{A14}$$

where we set

$$\Psi(a^*) \equiv \Big(\int_0^{a^*} \psi(u)^{\frac{\rho_{\mathcal{G}}}{1-\rho_{\mathcal{G}}}} du\Big)^{1-\rho_{\mathcal{G}}}.$$

Similarly we can define:

$$\mathcal{J}(\alpha^*) \equiv \int_{\alpha^*}^{1} l(u)^{\rho_\mathcal{G}} du, \tag{A15}$$

and we can solve for $\mathcal{J}(\alpha^*)$ in a similar fashion, and we find:

$$\mathcal{J}(\alpha^*) = (1-\alpha^*)^{1-\rho_\mathcal{G}} L^{*\rho_\mathcal{G}}, \tag{A16}$$

where $L^{*\rho_\mathcal{G}} \equiv \int_{\alpha^*}^{1} l(u)du$ is the aggregate labour used to produce $\mathcal{G}$. Putting all these expression together we find (assuming optimal choices for all tasks):

$$\begin{aligned} \mathcal{G} &= \left(\int_0^1 y(u)^{\rho_\mathcal{G}} du\right)^{\frac{1}{\rho_\mathcal{G}}} \\ &= (1-a^*)^{\frac{1-\rho_\mathcal{G}}{\rho_\mathcal{G}}} \left[(1-a^*)^{\rho_\mathcal{G}-1}\Psi(a^*)K^{\rho_\mathcal{G}} + L_L^{\rho_\mathcal{G}}\right]^{\frac{1}{\rho_\mathcal{G}}} \equiv \mathcal{G}(K, L, a^*) \end{aligned} \tag{A17}$$

which is the expression given in Equation (11).

We can furthermore describe how the function $\mathcal{G}(k, 1, a)$, i.e. the production per unit of low skilled labour, behaves when a is determined optimally, so that $\psi(a^*) = \frac{r+\delta}{w_l}$. First we observe that the ratio of prices can be expressed in terms of the production function itself:

$$\frac{r+\delta}{w_l} = \frac{\mathcal{G}'_k(k,1,a)}{\mathcal{G}(k,1,a) - k\mathcal{G}'_k(k,1,a)}, \tag{A18}$$

and the RHS of the expression is:

$$\frac{\mathcal{G}'_k(k,1,a)}{\mathcal{G}(k,1,a) - k\mathcal{G}'_k(k,1,a)} = \Psi(a^*)(1-a^*)^{\rho_\mathcal{G}-1}k^{\rho_\mathcal{G}-1}, \tag{A19}$$

and therefore, given some value k, a^* solves :

$$\psi(a^*) = \Psi(a^*)(1-a^*)^{\rho_\mathcal{G}-1}k^{\rho_\mathcal{G}-1}. \tag{A20}$$

From the definition of a^* we can determine how a^* responds to k, we find:

$$\frac{da^*}{dk} = \frac{1}{k}\frac{\rho_\mathcal{G}-1}{\frac{\psi'(a^*)}{\psi(a^*)} - \frac{\Psi'(a^*)}{\Psi(a^*)} + \frac{\rho_\mathcal{G}-1}{1-a^*}}, \tag{A21}$$

which, given our assumptions concerning $\psi(a)$, $\Psi(a^*)$, $\rho_\mathcal{G}$ and a, is positive. Finally, the definition of a^* in equation (A20) allows us to write the output $\mathcal{G}(k, 1, a^*)$ as:

$$\begin{aligned} \mathcal{G}(k,1,a^*) =& \left[\Psi(a^*)k^{\rho_\mathcal{G}} + (1-a^*)^{1-\rho_\mathcal{G}}\right]^{1/\rho_\mathcal{G}} \\ =& \left[\Psi(a^*)k^{\rho_\mathcal{G}} + \frac{\Psi(a^*)}{\psi(a^*)}k^{\rho_\mathcal{G}-1}\right]^{1/\rho_\mathcal{G}} \\ =& k\Psi(a^*)^{1/\rho_\mathcal{G}}\left[1 + \frac{1}{\psi(a^*)k}\right]^{1/\rho_\mathcal{G}}. \end{aligned} \tag{A22}$$

For cases where k is large enough for the second term in the previous expression to reach a limiting value, output per head $\mathcal{G}(k, 1, a^*)$ is therefore almost proportional to k .

B Sensitivity analysis

B.1 Sensitivity analysis with $\sigma = 1$

Our preferred calibration relies on a value of $\sigma = 2$, which implies an elasticity of intertemporal substitution (EIS) equal to 0.5. While a value for the EIS (far) below unity is consistent with empirical evidence, a value for $\sigma > 1$ results in a dominating income effect. We therefore test the sensitivity of our results by running a parallel calibration where we set $\sigma = 1$. In this environment, the income and substitution effects perfectly offset each other.

Table B.1 contains the internal calibration, and Table B.2 the equilibrium objects of the calibrated steady state. Table B.3 contains the comparative statics on input shares, which can be compared to Table 6 in the main paper. Table B.4 contains the comparative statics on prices, which can be compared to Table 7 in the main paper. Table B.5 contains the comparative statics on income measures, which can be compared to Table 8 in the main paper. Table B.6 contains the effects on the different inequality measures, which can be compared to Table 9 in the main paper. Last, Table B.7 contains the effects on the public pension income share of retirement consumption across skill-groups, which can be compared to Table 10 in the main paper. Figure B.1 shows the change in the proportion of high-skilled workers given changes in the contribution rate and the wage differential, which can be compared to Figure 3 in the main paper.

Table B.1: Internal Calibration

		Specification	
Parameter		CSC	TB
$\mathbb{E}[\psi]$	Mean of distribution of skill cost ψ	25.509	25.509
θ	Discount rate	0.032	0.032
β	Retirement utility parameter	14.431	14.431
B	Capital-low skilled productivity	5.074	3.714
A	Capital productivity	1.167	-
D	Scale capital productivity	-	1.096

Note: The Table shows the calibrated parameter values given the parameter values described in Table 1 with $\sigma = 1$. The two economies are calibrated to the match the same equilibrium objects as described in Table 2. Entries with - indicate that the specific parameter does not exist in the given specification.

Table B.2: Equilibrium objects

Y^*	w_H	w_L	b_H	b_L	K^*	L_H^*	L_L^*	r^*
6.676	0.155	0.089	0.048	0.032	20.000	12.900	25.600	0.040

Note: The Table contains the quantities and prices of the calibrated model. These output values are the same under both the CSC and the TB-specifications.

Table B.3: Comparative statics on input shares, elasticities w.r.t. a change in output Y

Model	K/Y	L_H/Y	L_L/Y
TFP (CSC)	0.275	0.089	-0.173
Automation (CSC)	0.583	0.203	-0.389
TFP (TB)	0.484	0.068	-0.149
Automation (TB)	0.901	0.176	-0.544

Note: The Table shows the percentage change for the factor shares of production, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

Table B.4: Comparative statics on prices, elasticities w.r.t. a change in output Y

Model	r	w_H	w_L	b_H^{RS}	b_L^{RS}	b^{HTM}
TFP (CSC)	0.108	0.933	0.451	0.864	0.775	0.820
Automation (CSC)	0.232	0.853	0.300	0.704	0.496	0.592
TFP (TB)	0.195	0.865	0.383	0.745	0.623	0.709
Automation (TB)	0.369	0.741	0.183	0.507	0.237	0.398

Note: The Table shows the percentage change for various prices, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

Table B.5: Comparative statics on income measures, elasticities w.r.t. a change in output Y

Model	EI_H^{RS}	PI_H^{RS}	LI_H^{RS}	EI_L^{RS}	PI_L^{RS}	LI_L^{RS}	EI_L^{HTM}	PI_L^{HTM}	LI_L^{HTM}
TFP (CSC)	0.914	0.906	0.913	0.779	0.819	0.786	0.797	0.783	0.795
Automation (CSC)	0.812	0.794	0.809	0.496	0.593	0.513	0.535	0.516	0.533
TFP (TB)	0.831	0.820	0.829	0.646	0.702	0.655	0.679	0.633	0.673
Automation (TB)	0.674	0.653	0.671	0.262	0.391	0.284	0.326	0.263	0.318

Note: The Table shows the percentage change for various income measures, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.

Table B.6: Inequality effects

Model	Earnings ineq.	Pension ineq.	Lifetime ineq.
Baseline	1.708	1.569	1.687
TFP (CSC)	1.751	1.601	1.728
Automation (CSC)	1.809	1.643	1.783
TFP (TB)	1.765	1.614	1.742
Automation (TB)	1.839	1.668	1.813

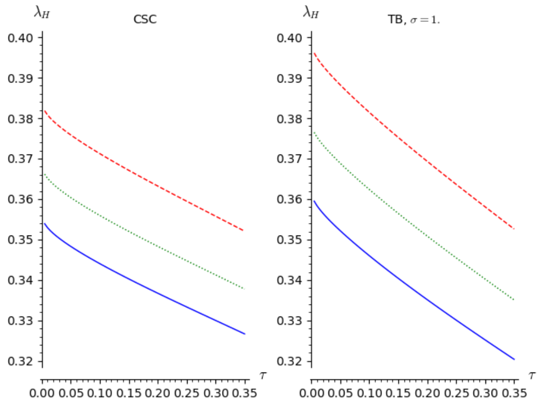
Note: The Table shows how earnings, pension and lifetime income inequality changes with either automation-driven or TFP-driven growth for both the CSC and TB specifications.

Table B.7: Public pension income share of retirement consumption

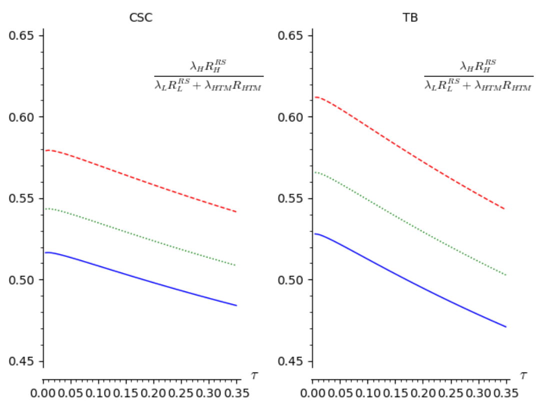
	CSC		TB	
Group	High Skill	Low Skill	High Skill	Low Skill
Baseline	0.325	0.380	0.325	0.380
TFP growth	0.313	0.370	0.304	0.361
Automation growth	0.300	0.359	0.286	0.345

Note: The Table shows how the ratio of public pension income to retirement consumption changes for the different growth scenarios under both the CSC and TB specifications. we do not present figures for the Hand to Mouth group, since in this case public pension income funds the entirety of consumption in retirement.

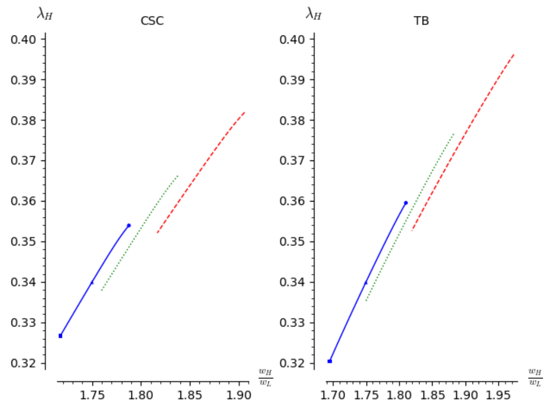
Figure B.1: Change in the proportion of high-skilled workers given changes in the contribution rate and the wage differential. $\sigma = 1$.



(a)



(b)



(c)

Note: The figures show the response of the share of high-skilled (panels a) and the ratio of high to low skill labour in panel (b) to a change in the public pension contribution rate. Each line corresponds to different technological assumptions: the continuous blue lines correspond to the baseline, the dashed red lines to a 10% increase to A in the CSC case, resp. to a change to η which generates an equivalent change to output in the TB case, and the dotted green lines to a 10% increase to Ω in the CSC case, resp. to a change to Ω which generates an equivalent change to output in the TB case. Panel (c) shows the association between the proportion of high skill workers and the wage differential as the contribution rate increases. In Panel (c), along each line when the contribution rate is low, $\tau = 0$, the proportion of high skill worker and the wage differential are the largest, while when the contribution is large, $\tau = 0.35$, both the differential and the share of high skill worker are the smallest.

B.2 Comparative statics when the skill distribution is exogenous

Table B.8: Comparative statics on prices, elasticities w.r.t. a change in output Y, Distribution of skill fixed, $\Lambda_H = 0.36, \Lambda_L = 0.44$.

Model	r	w_H	w_L	b_H^{RS}	b_L^{RS}	b^{HTM}
$\sigma = 1.0$						
TFP (CSC)	0.076	1.141	0.394	1.046	0.746	0.763
Automation (CSC)	0.174	1.344	0.153	1.121	0.382	0.417
TFP (TB)	0.149	1.148	0.313	0.994	0.590	0.638
Automation (TB)	0.315	1.377	−0.018	1.034	0.042	0.130
$\sigma = 2.0$						
TFP (CSC)	−0.226	1.393	0.391	0.739	0.563	0.530
Automation (CSC)	−0.096	1.689	0.121	0.738	0.360	0.257
TFP (TB)	−0.086	1.425	0.255	0.669	0.423	0.411
Automation (TB)	0.031	1.733	−0.046	0.688	0.198	0.062

Note: The Table shows the percentage change for various prices, given a percentage increase in output Y - conditional on either automation-driven or TFP-driven growth for both the CSC and TB specifications.