Sources of consumer information *

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Abstract

A buyer can learn about a product, either through search or through the information disclosed by the seller. We analyze how this buyer-seller relationship is affected by lower search costs or an improvement in the seller's ability to fine-tune her disclosure of product information. Whereas a drop in search costs improves consumer surplus and decreases profit when the seller can resort to an optimal disclosure strategy, its impact is ambiguous if the seller is unable to provide information. When it is unlikely that the buyer's valuation is below marginal cost, the buyer does not benefit from optimal information disclosure by the seller if search costs are high. With such high search costs and no disclosure both parties can be better off than with lower search costs and optimal information disclosure. The seller then adopts a mass market strategy where she posts a low enough price so the buyer always purchases the product without search. By contrast, if it is sufficiently likely that the buyer's valuation is below marginal cost, then the buyer can benefit from sophisticated information disclosure for relatively low search costs. The corresponding outcome is better for both parties than an environment with higher search costs and no information disclosure. The optimal seller strategy targets a niche of high valuation buyers and prevents wasteful search by buyers with low valuations.

KEYWORDS: information disclosure, information acquisition, advertising, search.

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1 Introduction

Search environments can differ greatly with regard to the consumers' access to product information and how this access is shaped by the sellers or intermediaries such as online platforms. Think of a basic search engine that would rely solely on the content of the consumer's requests as opposed to search triggered by a targeted ad or that benefits from product steering on a market place or a vertical platform specialized in hotels, flights or real estate. The search engine is an effective way to gather a wealth of information about the product from different independent sources whereas the search directed by targeting or product steering is ideal for quickly finding a product which is reasonably suitable for the consumer's need. Whether the consumer benefits from undertaking his or her own search as opposed to relying on the information from the seller or a platform depends on a number of factors, including the level of search costs, the nature of the buyer's uncertainty about how much she or he likes the product but also the incentives of the seller in providing information and the concomitant choice of the seller's price. As another illustration, the progressive migration of trade from offline to online with the development of the digital economy has had significant implications, both for search and for the design of information transmitted from sellers to consumers. While information acquisition has become less costly, the ability of sellers to tailor information transmission optimally has been greatly enhanced by the use of big data and artificial intelligence as well as the availability of buyer personal data.

In this paper we study the impact on the market outcome of changes in the consumers' cost of information acquisition and in the sellers' ability to optimize their communication. We analyze the determination of price, quantity sold and how much consumers learn about the product before they buy it. We characterize which configurations of consumer search costs and seller information transmission capacity are more favorable to either side and to total welfare.

We consider a simple buyer seller setting where the buyer (he) has unit demand and his valuation is initially unknown to either party. The buyer can learn about the valuation through some product information provided by the seller (she) and can decide to learn more by incurring a fixed search cost, in which case he becomes perfectly informed.

We first characterize the seller's profit maximizing strategy assuming that she can perfectly control the informative signals that reach the buyer. We show that the seller optimally designs her product information to completely deter buyers from acquiring additional information. The optimal disclosure is characterized by a threshold on the match between the product and the buyer, such that the latter learns whether or not the match is above the threshold but no additional information. As the cost of information acquisition increases, the information disclosed to a consumer who purchases the product deteriorates (i.e., the threshold decreases). Consequently, the probability that the buyer purchases the product increases. This optimal information disclosure allows the seller to increase her price up to a point where she can extract the entire expected consumer surplus. If the information acquisition cost is high enough that the buyer has zero expected surplus, a further increase in that cost induces a decrease in price until information acquisition becomes so costly that the seller achieves a first-best outcome: the product is purchased if and only if the buyer's valuation exceeds marginal cost, and the entire surplus is extracted by the seller.

An increased cost of information for the buyer results in lower consumer welfare. This deterioration occurs because price increases whenever the buyer's surplus is strictly positive and the buyer, receiving poorer information from the seller, faces a larger expected loss from buying with a negative surplus. However, profit increases. Total surplus improves as well if information acquisition becomes more costly because the deterioration of the buyer's information (resulting from a lower threshold) also induces a higher purchase probability and it is always optimal for the seller to screen out all valuations below marginal cost.

Next we compare the outcome with optimal disclosure to the profit maximizing solution if the seller is unable to communicate product information and can only post a price. Without information disclosure, the seller chooses between a high price at which the buyer searches or a lower price inducing an immediate purchase without search. The latter strategy cannot be optimal if search costs are low because it would require posting a very low price whereas the buyer is willing to search even if price is at monopoly price or close to it. For higher search costs however, profit maximization can involve setting a price that deters search and the corresponding profit can even be above monopoly profit provided the expected buyer valuation is sufficiently larger than marginal cost. Contrary to what happens with optimal disclosure, profit needs not be monotonically increasing in search costs. If the expected buyer valuation is not large enough, it can be negatively impacted by a search cost increase for low search costs or even for all search costs. Similarly, buyer surplus is not necessarily decreasing in search costs. Whenever the seller finds it optimal to drop the price to induce an immediate purchase, the buyer's surplus discontinuously jumps up and can be larger than his full information monopoly surplus.

It is ambiguous whether the availability of seller provided product information can benefit the buyer or not. It partially prevents wasteful search as well as purchases with a very negative surplus by screening out buyers with very low valuations (below the optimal disclosure threshold). However, it enables the seller to sell at a price typically above the monopoly price a quantity which exceeds the monopoly quantity, thus capturing a large share or the entirety of total surplus. We show that, for large search costs with an expected buyer valuation above marginal cost, no product information disclosure is always preferable for the buyer. It is then possible that both sides are better off with large search costs and no information disclosure rather than with very low search costs and an elaborate communication of product information by the seller. With high search costs, the seller adopts a mass market strategy where she posts a low price at which all buyers buy immediately. We also show that, when it is sufficiently likely that buyer valuation is below marginal cost, then there is a range of relatively low search cost values at which the buyer benefits from the seller's communication. This suggests that, for such niche products, it might be harmful to limit the use of sophisticated communication based on user data for search engines where the cost of information acquisition is relatively small. Then the combination of low search costs and elaborate communication by the seller can be an improvement for both parties over the outcome that would prevail with larger search costs and no communication capacity by the seller, which could be viewed as a crude description of offline trade.

Related literature. This paper fits most directly in the literature on the interaction between consumer search and product information disclosure by firms. Early examples are Anderson and Renault (2006) and Bar-Isaac, Caruana, and Cuñat (2010), and more recent contributions include Lyu (2023) or Board and Lu (2018) who allow for competition. We are closest to Wang (2017), who studies a similar problem, although he restricts the seller's information disclosure to use the information transmission setup introduced by Lewis and Sappington (1994), which is quite different from optimal disclosure. This restriction on communication results in a profit maximizing strategy for the seller which shares more features with the no disclosure setting than with the optimal disclosure solution we characterize.[1]

Our work also contributes to the broader literature on information design applied to product information disclosure[2] (e.g., Saak, 2006, Smolin, 2023, or Hwang, Kim, and Boleslavsky, 2023 who consider oligopolistic competition). Information design applied to consumer information has also been investigated while considering providers of information other than the sellers themselves with an objective that can differ from profit maximization as in Armstrong and Zhou (2022), Roesler and Szentes (2017), and Terstiege and Wasser (2020, 2024).

Our finding that it is optimal to deter search completely is reminiscent of a result in Matyskova and Montes (2023) who introduce rational inattention to capture costly learning by the receiver in an otherwise standard information design framework as popularized by Kamenica and Gentzkow (2011). Finally, our analysis also uses the concepts of mass markets and niche markets along the lines of Johnson and Myatt (2006) which have been introduced in a search environment by Bar-Isaac, Caruana, and Cuñat (2012).

[1]He also does not allow for valuations below marginal cost.

[2]There is also a literature on product information disclosure that uses a signaling setting (see Celik and Drugov, 2024 for a recent contribution)

The rest of the paper unfolds as follows. The model is presented in the next section. Section 3 provides useful preliminary results and shows that an optimal disclosure strategy is a threshold strategy that deters information acquisition completely. Profit maximization disclosure and pricing strategies are studied in Section 4, which also provides some welfare analysis. The results obtained when the seller can resort to an optimal disclosure strategy are compared to those obtained when the seller can provide no information in Section 5.

2 Model

A profit-maximizing seller, with costs normalized to zero, wishes to sell an object to a buyer. The buyer's valuation is v, drawn from a distribution with support $[\underline{v}, \bar{v}]$, where $\underline{v} \leq 0 < \bar{v}$ and with a distribution function G which admits a continuously differentiable density g. The buyer is risk-neutral, and his utility from buying the product at price p is $v - p$ while his utility is 0 if he does not buy. The hazard rate, $\frac{g(v)}{1-G(v)}$, is strictly increasing on $[\underline{v}, \bar{v}]$, so profit $p[1 - G(p)]$ is strictly quasi-concave in p and is maximized at a unique monopoly price $p_M \in (0, \bar{v})$, which is the solution to

$$p_M = \frac{1 - G(p_M)}{g(p_M)}.$$

Let $\pi_M = \max_{p \geq 0} p[1 - G(p)] = p_M[1 - G(p_M)]$ be the monopoly profit. The prior expected valuation is $E(v) = \int_{\underline{v}}^{\bar{v}} v \, dG(v)$. The first-best *ex ante* total surplus is the prior expected value of v conditional on the valuation exceeding marginal cost 0, denoted by

$$\mu \equiv E(v \mid v \geq 0) = \frac{\int_0^{\bar{v}} v \, dG(v)}{1 - G(0)} \geq E(v).$$

The match realization v is initially not known by either party, and they share a common prior described by G. The seller can provide certified information about its "product type" to the buyer through advertising. Such information disclosure entails no cost for either side. Though the product type conveys no information about the match to the seller (reflected in a prior on v for the seller independent of the product type), product information can generate some informative signal about v for the buyer, who knows his tastes (his "buyer type") perfectly: in particular, complete revelation of the product type informs the buyer perfectly about v.

Anderson and Renault (2006) provide examples of such settings.[3] As illustrated by the analysis in Koessler and Renault (2012), the nature of the information that can be imparted to the buyer through the disclosure of product attributes critically depends on how the sets of product types and buyer types are mapped into buyer valuations. However, additional flexibility in fine-tuning information disclosure by the seller can be achieved if a platform can collect data pertaining to the buyer's taste and implement an appropriate product steering policy. The setting studied by De Corniere (2016) provides an example where the product space is a circle and a search engine, which learns consumer tastes perfectly, steers consumers towards products located within a certain distance of their bliss point, so they learn that their valuations for the products they are matched with are above some threshold. As long as the platform is committed to its product steering policy, which is known to both the buyer and the seller, it can convey information to the buyer regarding his valuation for a product he is targeted with on the platform.

We abstract from the details of how the disclosure of product information combined with the platform's product steering policy can generate relevant information for the buyer and merely assume that the seller can select any signal structure to achieve maximal profit, in line with standard information design (Myerson, 1982, Kamenica and Gentzkow, 2011, Taneva, 2019). Formally, the seller chooses a disclosure policy, defined as a measurable function $X : [\underline{v}, \bar{v}] \to \Delta(M)$ for some signal space M.

The seller also posts a price p, which is independent of v because she has no prior information about the match value. After observing the price and the product information disclosed by the seller, the buyer can choose to learn his match realization perfectly by incurring a search cost

[3]See also Koessler and Renault (2012), and Koessler and Skreta (2016, 2019) for further elaborations on product information disclosure in related frameworks.

$s \in (0, \bar{v})$ before deciding whether to purchase the product. The timing can be summarized as follows:

1. The seller chooses a price p and a disclosure policy X;
2. Nature determines the match v according to the prior G, and a signal realization $m \in M$ according to the distribution $X(v)$;
3. The buyer observes the price p, the disclosure policy X, and the realized signal m;
4. The buyer chooses $a \in \{drop, buy, search\}$;
5. If the buyer drops, he gets 0; if he buys, he gets $v - p$; if he searches, he pays s, observes v, then buys if $v \geq p$ and drops if $v < p$. The seller gets p if the buyer buys and gets 0 if the buyer drops.

We next provide a simple characterization of the firm's optimal disclosure strategies.

3 Information disclosure

We start by establishing an important result regarding the characterization of the optimal information disclosure strategy for the seller. We show that there is no loss of generality in restricting attention to a set of simple disclosure policies such that the buyer never chooses to search after observing the product information provided by the seller and he is merely informed whether his valuation exceeds some threshold value. The point of the next proposition is that, for any arbitrary disclosure policy and any price p, there exists such a simple threshold policy inducing no search, such that the probability that the buyer purchases the product at price p is the same as in the original policy.

To illustrate, consider the "truth-or-noise" disclosure policy *à la* Lewis and Sappington (1994), as popularized by Johnson and Myatt (2006): the set of signals is $M = [\underline{v}, \bar{v}]$ and for each v, the signal is $m = v$ with probability η, and is drawn from the distribution function G with probability $1-\eta$. A higher η results in more precise product information for the buyer, with full information achieved when $\eta = 1$ and no information transmitted when $\eta = 0$. Assuming the buyer's valuation is uniformly distributed on $[\underline{v}, \bar{v}] = [0, 1]$, we have $E(v) = \mu = \frac{1}{2}$ and the conditional expected match for the buyer, given that he observes the signal m, is $E(v \mid m) = \eta m + \frac{1}{2}(1-\eta)$. Assuming small enough search costs ($s \leq \frac{3-2\sqrt{2}}{4} < \frac{1}{8}$), the optimal truth-or-noise disclosure policy is $\eta = 1 - 8s$, and the optimal price is $p = p_M = \frac{1}{2}$ (see Wang, 2017). The equilibrium behavior of the buyer consistent with profit maximization is to buy immediately after observing $m \geq \frac{1}{2}$ and to search otherwise.[4] Total purchase probability is then $\frac{1}{2} + \frac{1}{4}(1-\eta) = \frac{1}{2} + 2s$.

A first step in simplifying the above disclosure policy is to apply the revelation principle (see, e.g., Myerson, 1982). The idea is that the same outcome (i.e., the same purchase and search probabilities for a buyer with valuation v, for all $v \in [\underline{v}, \bar{v}]$) can be achieved using a simpler "buy-or-search" information disclosure policy with only two signals, $M = \{buy, search\}$. For each v, the probability that the buyer receives the signal *buy* equals the probability that he buys immediately if he has valuation v under the original truth-or-noise policy: the signal $m = buy$ is sent with probability $\eta + \frac{1}{2}(1-\eta)$ if $v \geq \frac{1}{2}$, and with probability $\frac{1}{2}(1-\eta)$ if $v < \frac{1}{2}$. The key point is that the equilibrium conditions ensuring the buyer behaves as desired under the original disclosure policy imply that the new disclosure policy satisfies four *incentive compatibility constraints*: (BnS) and (BnD) ensure that a buyer who receives the signal *buy* has a higher expected surplus if he buys than if he searches or drops, respectively, and (SnB) and (SnD) ensure the buyer does not wish to buy immediately or drop out when observing $m = search$.

Because the seller ultimately cares about the total probability of a sale, she could bypass the search process by immediately disclosing to the buyer whether he would end up purchasing if

[4]The buyer is actually indifferent between buying and searching for $m \geq \frac{1}{2}$ and between searching and dropping out for $m < \frac{1}{2}$ but the seller could break those ties by lowering the price slightly.

he chose to search.[5] She can achieve this by resorting to the following "buy-or-drop" disclosure policy. In this alternative policy, $M = \{buy, drop\}$, and the buyer receives the signal $m = buy$ with probability 1 if $v \geq \frac{1}{2}$ (recall that in the original policy, a buyer with $v \geq \frac{1}{2}$ always buys, either because he receives a signal above $\frac{1}{2}$ or because, after searching, he finds out that his valuation exceeds the price $\frac{1}{2}$), and with probability $\frac{1}{2}(1-\eta)$ if $v < \frac{1}{2}$ (with $v < \frac{1}{2}$ in the truth-or-noise policy, a buyer buys only if he receives a noisy signal above $\frac{1}{2}$ and hence does not search). By construction, the purchase probability is the same as in the two disclosure policies described above: $\frac{1}{2} + \frac{1}{4}(1-\eta) = \frac{1}{2} + 2s$. However, we need to check that this policy with no search is incentive compatible, so a buyer observing a signal $m \in \{buy, drop\}$ complies with the recommendation. This is trivially the case for $m = drop$ because the buyer knows he can observe such a signal only if $v < \frac{1}{2}$ and hence will have a strictly negative expected surplus if he buys immediately or searches. For a buyer who is advised to buy, it is intuitive that this constitutes an even more favorable signal than what it was in the buy-or-search policy. Indeed, the only situations where he observes $m = buy$ whereas he would not observe it in the buy-or-search policy is when he would have searched and decided to buy in the end: in other words, in those cases he knows that $v > p$ so that buying immediately is optimal. Hence, if the incentive compatibility constraints (BnD) and (BnS) are satisfied with the buy-or-search policy, this should be all the more the case with the buy-or-drop policy.

In the above buy-or-drop disclosure policy, for $s > 0$, there is a positive probability that the buyer is recommended to buy no matter how low the actual valuation is. As in Anderson and Renault (2006) and Saak (2006), the posterior for a buyer receiving the signal *buy* can be made more favorable by concentrating all the *buy* signals on high realizations of v. Specifically, consider the threshold $\tilde{v} = \frac{1}{2} - 2s$ and assume the buyer receives $m = buy$ if and only if $v \geq \tilde{v}$. Then the purchase probability is $1 - \tilde{v} = \frac{1}{2} + 2s$, which is the same as with the previous policies we have considered. Again, a buyer who is advised to drop out will never want to buy or search

[5] A similar idea appears in Matyskova and Montes (2023), who show that an optimal information structure can be solved as a standard information design problem, subject to the constraint that the receiver does not have an incentive to acquire additional information.

at price $\frac{1}{2}$ because he is certain that $v < \frac{1}{2}$. For a buyer who should buy, concentrating the $m = buy$ signal on higher realizations of valuation v while keeping the overall probability of such a signal constant induces a more favorable posterior for the buyer who, as a result, will be more inclined to buy. So the corresponding incentive compatibility constraints (BnS) and (BnD) are not violated.

Applying analogous arguments to a generic disclosure policy, we prove the following proposition in the appendix.

Proposition 1 *For profit maximization, it is without loss of generality to consider a threshold disclosure policy. This policy recommends buying when $v \geq \tilde{v}$ and dropping when $v < \tilde{v}$, for some $\tilde{v} \in [\underline{v}, \bar{v}]$. Specifically, $M = \{drop, buy\}$, $X(buy \mid v) = 1$ if $v \geq \tilde{v}$, $X(drop \mid v) = 1$ if $v < \tilde{v}$, and the consumer follows the recommended action. Additionally, at the optimum for the firm, we have $p > \tilde{v}$.*

Proof. See Appendix A.1. ■

The above result identifies a relevant class of disclosure policies that the seller can resort to in order to achieve maximum profit. Each of these policies is fully characterized by the threshold value $\tilde{v}$, which should be weakly below the selected price p. The policy should also be incentive compatible so that those who learn that $v \geq \tilde{v}$ buy immediately and those who learn that $v < \tilde{v}$ drop out. The requirement that the threshold should be below the price ensures incentive compatibility for those who should drop out. For those who should buy immediately, the two incentive compatibility constraints are

$$E(v \mid buy) - p \geq 0, \tag{BnD}$$

so they do not drop out and

$$E(v \mid buy) - p \geq \mathbb{P}\{v \geq p \mid buy\} E(v - p \mid v \geq p, buy) - s,$$

or, equivalently,

$$E(p - v \mid v < p, buy)\mathbb{P}\{v < p \mid buy\} \leq s, \tag{BnS}$$

in order for them not to want to search.[6] For a threshold disclosure policy with threshold value $\tilde{v}$ these constraints are

$$\int_{\tilde{v}}^{\bar{v}} (v - p)g(v)\, dv \geq 0, \tag{Δ}$$

for (BnD) and

$$\frac{\int_{\tilde{v}}^{p} (p - v)g(v)\, dv}{1 - G(\tilde{v})} \leq s \tag{Σ}$$

for (BnS). For any $\tilde{v} \in [\underline{v}, \bar{v}]$, a price $p > \bar{v}$ would violate (Δ). Hence the seller's problem consists in selecting a price $p \in [0, \bar{v}]$ and a threshold value $\tilde{v} \in [\underline{v}, p]$ to maximize profit $p[1 - G(\tilde{v})]$ subject to constraints (Δ) and (Σ).

Proposition 1 tells us that, when using a threshold match policy with no search, the seller can achieve at least as much profit as with any alternative policy. There is no claim that profit could not be maximized by using some disclosure policy outside this specific class. However, eliminating search and resorting to threshold match disclosure typically enables the seller to achieve a higher profit than with an alternative disclosure strategy because she can sell the same quantity at a strictly higher price. We now return to the truth-or-noise example to briefly discuss why this is the case.

In the buy-or-search policy obtained by applying the revelation principle to the truth-or-noise policy, there are two sources of price elasticity in the buyer's demand at price $\frac{1}{2}$. One stems from the purchase decisions made by buyers who search and become fully informed. The second arises because at this price, incentive compatibility constraint (BnS) is actually binding so that, with a higher price, some buyers would switch from buying immediately to searching and then would possibly find out they don't want to buy. If the seller raises her price above $\frac{1}{2}$, she would lose some demand on both these fronts. Switching to the buy-or-drop policy

[6] The second formulation of (BnS) follows from rewriting $E(v \mid buy) - p$ as $\mathbb{P}\{v \geq p \mid buy\}E(v - p \mid v \geq p, buy) + \mathbb{P}\{v < p \mid buy\}E(v - p \mid v < p, buy)$.

clearly eliminates the first effect because there is no longer any search. But it also relaxes the two incentive constraints because, as explained earlier, the additional cases where the buyer receives $m = buy$ whereas he would not have received it in the buy-or-search policy involve valuations larger than the price. This makes buying more attractive as compared to searching or dropping out. Hence demand is perfectly inelastic at price $\frac{1}{2}$ and the seller can increase her price without losing any demand.

Now consider switching from the buy-or-drop policy where the signal $m = buy$ can be observed with a strictly positive probability for any $v \in [\underline{v}, \bar{v}]$ to a threshold match disclosure policy, while keeping unchanged the probability of observing the signal $m = buy$. In the threshold match policy, the signal $m = buy$ is associated with stochastically higher valuations in the sense that the posterior distribution of v conditional on $m = buy$ stochastically dominates the posterior distribution of v in the non threshold disclosure policy. It follows that $E(v \mid buy)$ is strictly higher and $\mathbb{P}\{v < p \mid buy\}E(p - v \mid, v < p, buy)$ is strictly lower so both constraints are further relaxed. This allows the seller to raise her price even higher without altering the buyer's purchase probability.

An alternative method for increasing profit when the incentive constraints are relaxed would be to decrease the threshold value in order to sell more rather than charging a higher price while keeping quantity constant or, indeed, moving both p and $\tilde{v}$ simultaneously. This price quantity tradeoff is at the heart of the general profit maximization problem we study in the next section.

4 Profit maximization

In this section, we characterize the seller's profit-maximizing strategy, both in terms of disclosure and pricing. From our analysis in Section 3 the seller faces a price quantity tradeoff resulting from two incentive compatibility constraints for those who buy: a no search constraint and a no drop constraint. Indeed, these constraints indicate that the product can be sold at a certain price provided that the information transmitted to the buyer is favorable enough, which in turn requires that only buyers with high enough valuations are induced to buy. Here we analyze how a change in the buyers' search cost impacts price and quantity as well as the resulting changes in profit, consumer welfare and total welfare.

Formally, from our discussion following Proposition 1, the seller chooses a price $p \in [0, \bar{v}]$ and a threshold $\tilde{v} \in [\underline{v}, p]$ that maximize $p(1 - G(\tilde{v}))$ subject to two incentive compatibility constraints: (Δ) and (Σ). The two constraints respectively ensure that, when the buyer is advised to buy, that is, when he learns that his valuation is above the threshold, he does not prefer to drop out and he does not prefer to search.

The *ex ante* first-best outcome is achieved by maximizing profit subject to (Δ) alone. It yields $p = \mu = E(v \mid v \geq 0)$ and $\tilde{v} = 0$ so the buyer buys only if his valuation exceeds the marginal cost. The resulting profit is $[1 - G(0)]\mu$ and the seller fully extracts total surplus, i.e., (Δ) binds. The seller achieves this first-best outcome if it satisfies constraint (Σ), i.e.,

$$s \geq \bar{s} := \frac{\int_0^{\mu} (\mu - v) g(v)\, dv}{1 - G(0)}. \tag{1}$$

If $s < \bar{s}$, then either both constraints (Δ) and (Σ) bind, or only constraint (Σ) binds.

When they bind, the no-drop and no-search constraints yield the two following equations, respectively labeled (Δ_b) and (Σ_b):

$$\int_{\tilde{v}}^{\bar{v}} (v - p) g(v)\, dv = 0, \tag{Δ_b}$$

$$\phi(\tilde{v}, p) := \frac{\int_{\tilde{v}}^{p} (p - v) g(v)\, dv}{1 - G(\tilde{v})} = s. \tag{Σ_b}$$

Note that $\phi(0, \mu) = \bar{s}$. The curves delineated by (Δ_b) and (Σ_b), denoted Δ_b and Σ_b, along with the associated incentive-compatible pricing and disclosure policies, are shown in Figure 1 for a match distribution uniform on $[0, 1]$, in which case (Σ_b) and (Δ_b) respectively yield $p = \tilde{v} + \sqrt{2s(\bar{v} - \tilde{v})}$ for $\tilde{v} \in [0, \bar{v} - 2s]$, and $p = \frac{\tilde{v}+1}{2}$, for $\tilde{v} \in [0, 1]$.

The next lemma provides some general properties of Δ_b and Σ_b, none of which require an

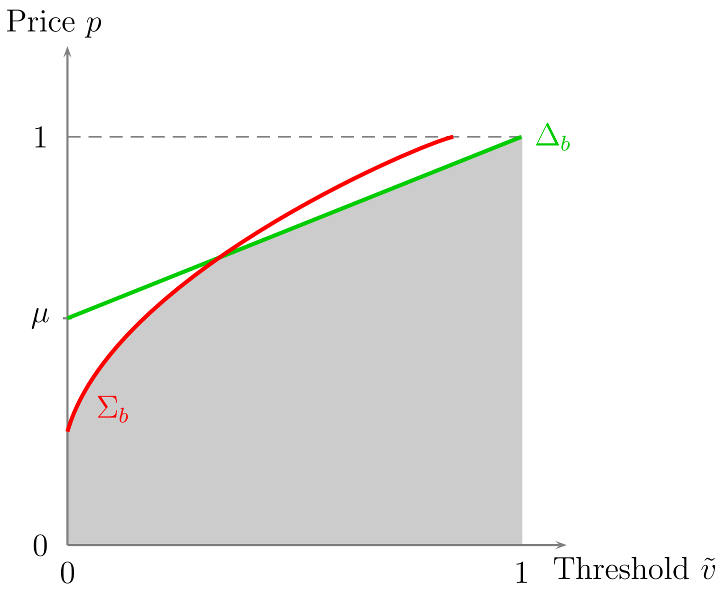

Figure 1: Pairs of disclosure thresholds and prices $(\tilde{v}, p)$ satisfying the no drop constraint (below the Δ_b curve) and the no search constraint (below the Σ_b curve) when the match distribution is uniform on $[0, 1]$.

increasing hazard rate for G.

Lemma 4.1

(i) Along the Δ_b curve, p is strictly increasing in $\tilde{v}$, with $p = \mu$ when $\tilde{v} = 0$ and $p = \bar{v}$ when $\tilde{v} = \bar{v}$.

(ii) Along the Σ_b curve, p is strictly increasing in $\tilde{v}$ and an increase in search cost s shifts the curve upwards (i.e, for any $\tilde{v}$, price is strictly increasing in s). Furthermore, if $s = \bar{s}$, $p = \mu$ when $\tilde{v} = 0$ and, if $s \leq \bar{s}$, then there exists $\tilde{v} \in (0, \bar{v})$ such that $p = \bar{v}$.

Proof. See Appendix A.2. ∎

Part (ii) of the lemma implies that if $s < \bar{s}$, then the Σ_b curve crosses the price axis at $p < \mu$, and then, the curves Δ_b and Σ_b cross at least once at some $\tilde{v} \in (0, \bar{v})$, with Σ_b crossing Δ_b from below at the first such crossing point.[7] Notice that any combination $(\tilde{v}, p)$ that satisfies both (Σ) and (Δ), which is to the right of this first intersection point necessarily yields a lower profit

[7] In Step 2 of the Proof of Lemma 4.2 we show that if G has an increasing hazard rate, then this is the unique crossing point.

than that first intersection point. Such points are dominated by a combination of the same $\tilde{v}$ with the price that binds (Δ). However the corresponding point, which is on Δ_b but specifies a higher threshold than the intersection point yields a lower profit than the intersection point: this is because, when (Δ) binds, the seller captures the full social surplus from the sale, which is decreasing in the threshold when it is above marginal cost, zero. It follows that when only (Σ) binds, the seller's solution is on Σ_b to the left of the first intersection point and, when both constraints bind, it is at the intersection point. Because Σ_b crosses Δ_b from below and an increase in s shifts Σ_b upwards it can easily be seen graphically that the intersection moves to the left on the increasing curve Δ_b. It follows that, at the first crossing point an increase in search cost results in a lower price and a lower information threshold (see Figure 2).

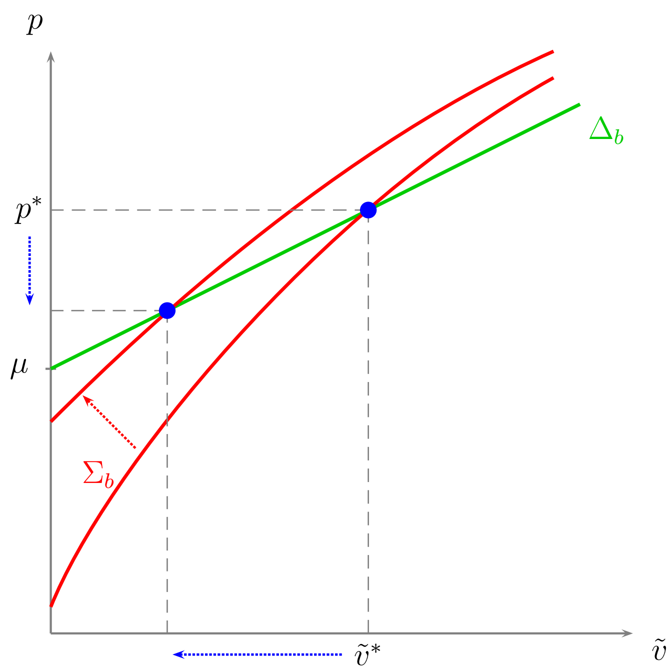

Figure 2: Impact of an increase in s when both (Δ) and (Σ) bind.

Assume now that $s \leq \bar{s}$ and consider the *relaxed program* where the seller chooses a price $p \in [0, \bar{v}]$ and a threshold $\tilde{v} \in [\underline{v}, p]$ to maximize profit $p(1 - G(\tilde{v}))$ subject to the no-search constraint (Σ) alone. Because profit is continuous and the set of valid combinations $(\tilde{v}, p)$ is compact, the relaxed program has a solution, and this solution necessarily binds (Σ).

We use a version of the first-order conditions that compares at a given point on Σ_b the slope of the iso-profit curve going through that point to the slope of Σ_b. Let us define $p'_\Sigma(\tilde{v})$ the derivative of price p with respect to threshold $\tilde{v}$ along the Σ_b curve. We also define, for any point $(\tilde{v}, p)$ on Σ_b, $p'_{ISO}(\tilde{v})$ as the derivative of price p with respect to threshold $\tilde{v}$ along the iso-profit curve going through that point: although this derivative depends on both p and $\tilde{v}$, p is a function of $\tilde{v}$ because of the requirement that Σ_b holds. The first-order conditions for a solution $(\tilde{v}^*, p^*)$ are:

- $p'_{ISO}(\tilde{v}^*) = p'_\Sigma(\tilde{v}^*)$ if $(\tilde{v}^*, p^*)$ is interior, i.e., $p^* < \bar{v}$ and $\tilde{v} > \underline{v}$;
- $p'_{ISO}(\tilde{v}^*) \geq p'_\Sigma(\tilde{v}^*)$ if $\tilde{v}^* = \underline{v}$;
- $p'_{ISO}(\tilde{v}^*) \leq p'_\Sigma(\tilde{v}^*)$ if $p^* = \bar{v}$.

We show below that the first-order conditions identify a unique solution to the relaxed program. Under the strictly increasing hazard rate property, $1 - G$ is log-concave, so profit $p(1-G(\tilde{v}))$ is log-concave and therefore quasi-concave in $(\tilde{v}, p)$. As a result, the isoprofit curves are strictly convex. Hence, the relaxed program has a unique solution if the no-search constraint (Σ) is convex (i.e., if ϕ is quasi-convex), as in Figure 1. The main challenge is that even with an increasing hazard rate, (Σ) is not necessarily convex. For example when the match distribution is $G(v) = v^2$, with support $[0, 1]$, the Σ_b curve is s-shaped as illustrated in Figure 3. In such a case, the first-order conditions may have multiple solutions; for instance, we cannot exclude the possibility of multiple tangent points between the iso-profit curves and the Σ_b curve.

Despite these non-convexities, we show in the next lemma that, with a strictly increasing hazard rate, the solution of the relaxed program is unique: the solution is either interior, with $p'_{ISO}(\tilde{v}) = p'_\Sigma(\tilde{v})$, or $p^* = \bar{v}$. This is established by showing that the function p'_{ISO} must cross the function p'_Σ from below. We then show that $\tilde{v}^* > 0$, so if $p^* < \bar{v}$ the solution is necessarily interior.

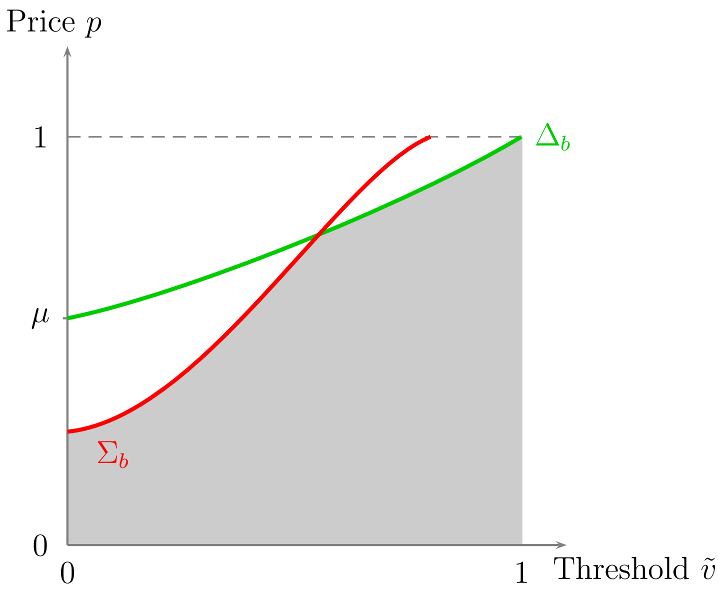

Figure 3: Pairs of disclosure thresholds and prices $(\tilde{v}, p)$ satisfying the no drop constraint (below the Δ_b curve) and the no search constraint (below the Σ_b curve) for a distribution of match values such that (Σ) is not convex.

Lemma 4.2 *Assume $s \leq \bar{s}$. In the relaxed profit-maximization program under (Σ), the optimum $(\tilde{v}^*, p^*)$ is unique and we have $\tilde{v}^* > 0$. If $p^* < \bar{v}$, then $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$, i.e.,*

$$p^* = \frac{\int_{\tilde{v}^*}^{p^*} 1 - G(v)\, dv}{G(p^*) - G(\tilde{v}^*)} \tag{2}$$

Proof. See Appendix A.2. ∎

Now consider an increase in the buyer's information acquisition cost, s. This relaxes constraint (Σ) because, as shown in Lemma 4.1, for any information threshold $\tilde{v}$, the constraint remains satisfied for higher price levels. Hence, sales could be maintained while raising the price but they could also be increased without having to drop the price. Lemma 4.3 below, shows that the seller does a bit of both by increasing price and decreasing the information threshold, as illustrated in Figure 4. The proof is a straightforward comparative statics argument using the increasing hazard rate property applied to the first order conditions for an interior solution, (2), so $p^* < \bar{v}$.[8]

[8]The argument also relies on the uniqueness of the solution to the first-order condition established in Lemma

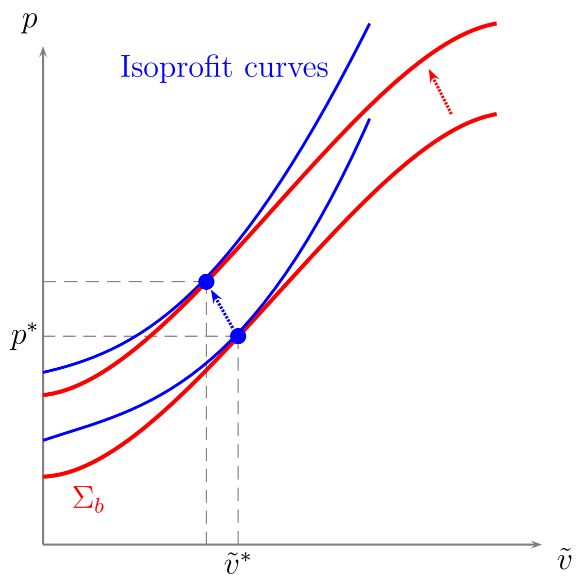

Figure 4: Impact of an increase in s when only (Σ) binds.

Lemma 4.3 *Assume $s \leq \bar{s}$. In the relaxed profit-maximization program under (Σ), if $p^* < \bar{v}$, then $\tilde{v}^*$ is strictly decreasing in s and p^* is strictly increasing in s.*

Proof. See Appendix A.2. ∎

The solution to the relaxed problem solves the seller's optimisation problem if and only if it satisfies constraint (Δ), in which case only constraint (Σ) binds. Else, both incentive compatibility constraints are binding. First, observe that if $s = 0$, then constraint (Σ) merely states that $p \leq \tilde{v}$, because at any higher price, the buyer would choose to search (which involves no cost in this case) to check that his valuation exceeds the price. Then, when the constraint binds, quantity is determined by price as in the standard monopoly problem so the solution of the relaxed program is $p^* = \tilde{v}^* = p_M$. The corresponding expected consumer surplus for buyers with valuation above p_M is strictly positive, so the no-drop constraint (Δ) holds strictly and this remains true for s close enough to 0. Next, consider $s = \bar{s}$. From Lemma 4.1(ii), the first-best outcome $(\tilde{v}, p) = (0, \mu)$ satisfies constraint (Σ), and yet from Lemma 4.2, the solution to the

4.2 so there cannot be any jump in the solution as s increases. It can be shown that for $p^* = \bar{v}$ an increase in s leaves price unchanged and decreases $\tilde{v}^*$ but this is irrelevant to the analysis below because the relaxed problem solution then violates constraint (Δ).

relaxed problem involves $\tilde{v}^* > 0$. Since the first-best outcome yields the highest profit that can be achieved under constraint (Δ), this constraint is necessarily violated by the solution to the relaxed problem. Hence, for s close to zero the seller's profit maximization solution coincides with that of the relaxed problem so only (Σ) binds, whereas for s close enough to $\bar{s}$, the profit maximizing solution binds both constraints.

We finally show that constraint (Δ) binds if and only if $s > s_1$, for some $s_1 \in (0, \bar{s})$. Lemma 4.3 shows that as s increases, the solution to the relaxed problem has price p^* increase while threshold $\tilde{v}^*$ decreases, unless $p^* = \bar{v}$. However, Δ_b is a strictly increasing relation between the information threshold and the price by Lemma 4.1(i) so that, if the solution of the relaxed program violates (Δ) for some s, then this remains the case for any higher search cost. Then the existence of a critical search cost $s_1 \in (0, \bar{s})$ follows from our discussion in the previous paragraph. From our discussion following Lemma 4.1, when both constraints bind (i.e. for $s \in (s_1, \bar{s})$), the seller's optimum must be at the first point of intersection of Σ_b with Δ_b, at which an increase in s causes price and information threshold to fall.

Collecting the above results, and now letting $(\tilde{v}^*, p^*)$ denote the solution to the full fledged profit maximization problem, we have the following proposition.

Proposition 2 *For all $s \geq 0$, the seller's profit is maximized at some unique combination of price and information threshold, $(p^*, \tilde{v}^*)$. Furthermore, there exists $s_1 \in (0, \bar{s})$ such that:*

- *If $s \leq s_1$, then only (Σ) is binding, $(p^*, \tilde{v}^*)$ is the unique solution to (2), $\tilde{v}^*$ is strictly decreasing in s, and p^* is strictly increasing in s;*
- *If $s \in (s_1, \bar{s})$, then both (Δ) and (Σ) are binding, $\tilde{v}^*$ and p^* are strictly decreasing in s.*
- *If $s \geq \bar{s}$, then only (Δ) is binding, and the seller achieves the first-best outcome, $\tilde{v}^* = 0$ and $p^* = \mu$.*

Figure 5 illustrates the seller's optimal strategies depending on the buyer's information acquisition cost s. In the first two figures, $s < \bar{s}$ and as seen in Lemma 4.1, Σ_b crosses Δ_b from below. In Figure (a), $s \in (0, s_1)$ and only (Σ) binds. In Figure (b), $s \in (s_1, \bar{s})$ and both (Δ) and (Σ) bind. In Figure (c), we have $s \geq \bar{s}$, only (Δ) binds, so the seller extracts all the surplus, which is maximized by the first-best solution $(\tilde{v}^*, p^*) = (0, \nu)$.

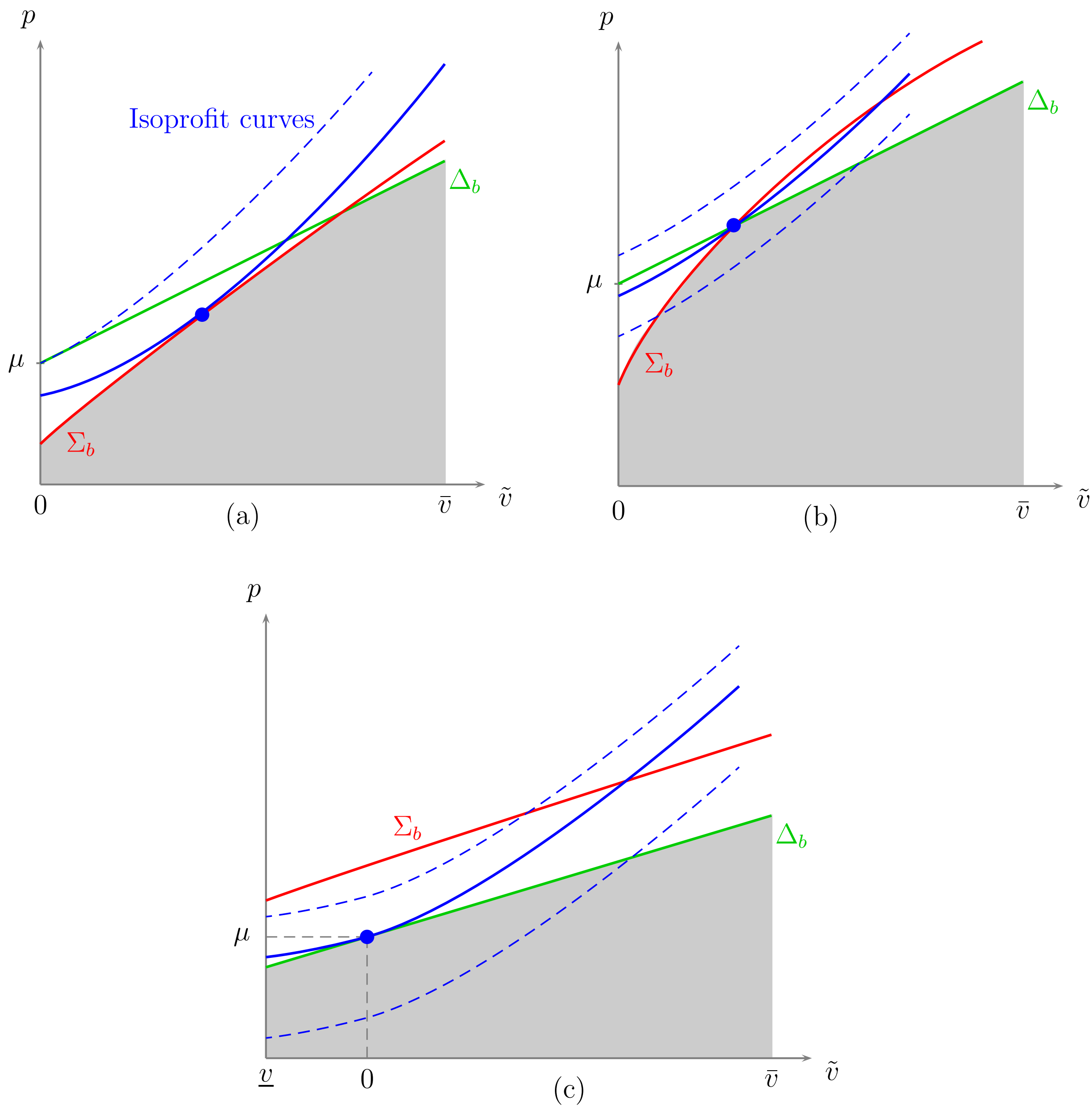

Figure 5: Optimal solution of the seller when (a) $s \in (0, s_1)$ and only (Σ) binds, (b) $s \in (s_1, \bar{s})$ and both (Δ) and (Σ) bind, (c) $s > \bar{s}$ and only (Δ) binds.

Proposition 2 provides a crisp characterization of the seller's optimal disclosure and pricing strategy. When information acquisition is costly for the buyer, $s > 0$, profit exceeds its full information monopoly level. Quantity is always larger than the monopoly quantity even though the seller typically prices above the monopoly price: the only instance when she does not is, for $\mu < p_M$, when search cost s is sufficiently large (close to or above $\bar{s}$). As information acquisition becomes more costly for the buyer, quantity increases. This is because it becomes easier to deteriorate the product information provided to the buyer while still preventing him from becoming informed through search. By lowering threshold $\tilde{v}^*$, the seller sells more to buyers who know less. As long as the buyer's rent is positive, (i.e., for $s < s_1$) she can do this while raising the price. When s rises above s_1, she captures the entire surplus so the no-drop constraint binds. However, for $s < \bar{s}$, she cannot achieve the first-best outcome $(\tilde{v}^*, p^*) = (0, \mu)$ which violates the no-search constraint (Σ). She can still increase her sales by lowering $\tilde{v}^*$, but she must lower the price to keep the no-drop constraint satisfied, so the buyer keeps purchasing the product despite a less favorable information. Price is therefore non monotonic in the search cost: it rises with the search cost for $s < s_1$ and then falls when s increases beyond s_1 until it reaches μ for $s = \bar{s}$.

Profit obviously rises as information acquisition becomes more costly thus loosening the no-search constraint until it no longer binds for $s \geq \bar{s}$. The implications for total welfare and consumer welfare are, somewhat surprisingly, just as straightforward. Threshold information transmission selects all valuations above the threshold and $\tilde{v}^*$ never falls below marginal cost. The drop in $\tilde{v}^*$ as s increases from 0 to $\bar{s}$ therefore unambiguously improves total welfare. It should be expected that the seller would never want to serve buyers with valuations below marginal cost if she captures the entire surplus (i.e. (Δ) binds) but we show in Lemma 4.2 that this is also the case when only (Σ) binds. Regarding consumer welfare, for $s < s_1$ consumer surplus is non-zero. Although the buyer purchases the product with a higher probability when the search cost increases, he pays a higher price. Furthermore, because $\tilde{v}^* < p^*$, all additional values of v for which he buys are below the price so that consumer surplus necessarily decreases. Hence consumer surplus is strictly decreasing in search cost for $s \in [0, s_1]$ and is zero thereafter.

The deterioration of consumer surplus as s rises from 0 to s_1 results from the seller's choice to take advantage of a relaxed no-search constraint (Σ) by increasing her sales and her price, as shown in Lemma 4.2. The following result summarizes the welfare properties of the seller's optimal strategy.

Proposition 3 *For $s \in [0, \bar{s}]$, an increase in the information acquisition cost s strictly increases profit and total welfare, strictly decreases consumer surplus if $s < s_1$, and leaves consumer surplus unchanged at 0 if $s \geq s_1$.*

From Proposition 3, if sellers can implement an optimal product information disclosure, a drop in search cost unambiguously benefits buyers and hurts sellers. We now contrast these result with those obtained in a situation where the seller can provide no information.

5 No product information disclosure

Here we analyze how the market outcome is affected if the seller cannot implement the optimal information design considered in the previous two sections. For the most part, we study the properties of the alternative extreme benchmark where the seller is unable to provide any product information to the buyer and can only post a price. We also discuss the situation where, if the seller wishes to provide any information, she must provide full information. We also consider the possibility that the seller has access to a specific, suboptimal, information transmission technology, drawing on the results in Wang (2017).

No information disclosure coincides with threshold information disclosure for $\tilde{v} = \underline{v}$. Hence, in line with our analysis in Sections 3 and 4, for a search cost level s, if the seller posts price p, the buyer prefers to buy if $\varphi(p) = \phi(\underline{v}, p) \leq s$ and $E(v) - p \geq 0$. The arguments underpinning Proposition 1 do not apply so the seller cannot rule out that profit maximization involves letting the buyer search because it would be too costly to prevent it when search costs are low. When he searches, the buyer anticipates that he will buy if and only if his valuation exceeds the price, so he will obtain the corresponding consumer surplus. Letting $\gamma(p)$ denote consumer surplus at price p, it is optimal for the buyer to search rather than drop if $\gamma(p) \geq s$.

Using integration by parts we have

$$\varphi(p) = \int_{\underline{v}}^{p} (p - v)g(v)dv = \int_{\underline{v}}^{p} G(v)dv, \tag{3}$$

and

$$\gamma(p) = \int_{p}^{\bar{v}} (v - p)g(v)dv = \int_{p}^{\bar{v}} 1 - G(v)dv, \tag{4}$$

so φ is strictly increasing (from $\varphi(\underline{v}) = 0$ to $\varphi(\bar{v}) = \bar{v} - E(v)$), while γ is strictly decreasing (from $\gamma(\underline{v}) = E(v) - \underline{v}$ to $\gamma(\bar{v}) = 0$).

We can therefore summarize the buyer's behavior for search cost s by describing his choice as a function of the seller's price p as follows:

$$\begin{cases} buy & \text{if } p \leq \min\{E(v), \varphi^{-1}(s)\} \\ search & \text{if } \varphi^{-1}(s) < p \leq \gamma^{-1}(s) \\ drop & \text{if } p > \max\{E(v), \gamma^{-1}(s)\}. \end{cases}$$

This means that the buyer drops for high prices, buys immediately for low prices, and searches for intermediate prices if the search cost is not too high. To see this it is useful to note that

$$\gamma(E(v)) = \varphi(E(v)). \tag{5}$$

As depicted in Figure 6, if $\varphi^{-1}(s) > \gamma^{-1}(s)$, i.e., $s > \gamma(E(v)) = \varphi(E(v))$, then the buyer never searches, regardless of the price: he buys for $p \leq E(v)$ and drops for $p > E(v)$. Otherwise, if $s < \varphi(E(v))$, the buyer searches for prices in the interval $(\varphi^{-1}(s), \gamma^{-1}(s)]$, drops for $p > \gamma^{-1}(s)$, and buys for $p \leq \varphi^{-1}(s)$. Figure 6 shows the regions of price and search cost combinations where the buyer chooses to buy, search or drop, assuming $E(v) \geq 0$.[9]

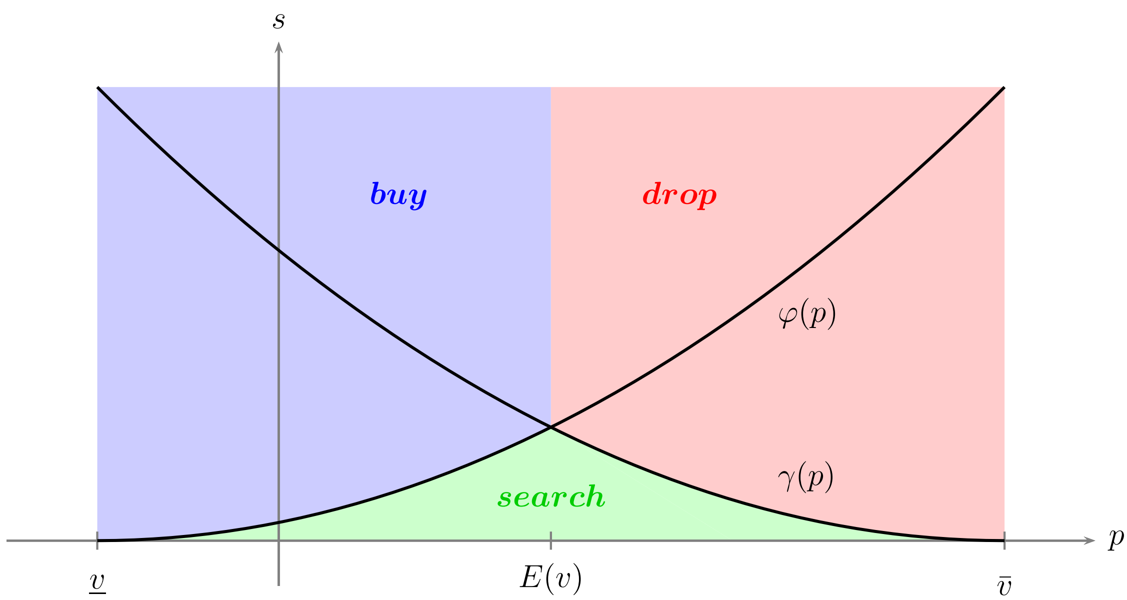

Figure 6: Functions $\varphi(p)$ and $\gamma(p)$, and optimal action of the buyer as a function of p and s under no disclosure.

Using the above characterization of the buyer's behavior, we can now determine the seller's optimal pricing. Suppose first that search costs are very large so that $s \geq \min\{\varphi(E(v)), \gamma(0)\}$. For $E(v) \geq 0$, in which case $s \geq \varphi(E(v))$, the buyer's choice at any positive price is either to buy immediately or to drop out. Hence the profit-maximizing price p^N is the largest price at which the buyer prefers to buy immediately, that is $p^N = E(v)$, and the profit is $E(v)$. If $E(v) \leq 0$ and hence $s \geq \gamma(0)$, the profit is zero because the buyer drops out for any positive price.

If, on the contrary, search costs are sufficiently low, i.e. $s \leq \min\{\varphi(\pi_M), \gamma(p_M)\}$, the seller can obtain monopoly profit π_M by charging monopoly price p_M, and the buyer searches because $\varphi^{-1}(s) \leq \pi_M < p_M \leq \gamma^{-1}(s)$. Her profit cannot exceed π_M because, in order to induce the buyer to purchase immediately, she must charge at most $\varphi^{-1}(s) \leq \pi_M$.

[9]Sliding the φ and γ curves sufficiently to the left shows that, if $E(v) \leq 0$, there is no strictly positive price at which the buyer buys immediately.

For intermediate search costs, $\min\{\varphi(\pi_M), \gamma(p_M)\} < s < \min\{\varphi(E(v)), \gamma(0)\}$, the seller can always secure a positive profit because $\bar{v} > 0$. She then chooses between charging the highest price such that the buyer purchases her product immediately $\varphi^{-1}(s)$ and the highest price at which the buyer searches, $\gamma^{-1}(s)$ (if that price is below p_M) or p_M. The latter strategy yields profit $\gamma^{-1}(s)[1-G(\gamma^{-1}(s))]$ or π_M to be compared to a profit of $\varphi^{-1}(s)$ if the product is bought immediately. First notice that for $E(v) \leq 0$, if $s \leq \varphi(E(v))$, then $\varphi^{-1}(s) \leq 0$ so the seller will always choose price $\gamma^{-1}(s)$ and have the buyer search, which yields a strictly positive profit because $s < \gamma(0)$. Hence we now assume $E(v) > 0$, which implies that $\min\{\varphi(E(v)), \gamma(0)\} = \varphi(E(v))$.

Suppose first that $\varphi(\pi_M) \leq \gamma(p_M)$. Then, for $s > \varphi(\pi_M)$, the seller can sell with certainty at price $\varphi^{-1}(s) > \pi_M$, since $\varphi^{-1}(s) \leq E(v)$. This is profit maximizing because the alternative strategy of charging $\gamma^{-1}(s)$ yields at most profit π_M. Now suppose $\gamma(p_M) < \varphi(\pi_M)$ so $s > \gamma(p_M)$ and hence $\gamma^{-1}(s) < p^M$. If price is $\gamma^{-1}(s)$, profit is $\gamma^{-1}(s)[1 - G(\gamma^{-1}(s))]$ which is π_M for $s = \gamma(p_M)$ and, because monopoly profit is strictly increasing on $[0, p_M]$ and γ^{-1} is strictly decreasing, it strictly decreases to $E(v)[1 - G(E(v))]$ as s increases from $\gamma(p_M)$ to $\varphi(E(v))$ (where we have used (5) to substitute $\varphi(E(v)) = \gamma(E(v))$). If price is $\varphi^{-1}(s)$, so the buyer purchases the product without search, the corresponding profit is $\varphi^{-1}(s)$ which strictly increases as s increases from $\gamma(p_M)$ to $\varphi(E(v))$. For s close to $\gamma(p_M)$, the corresponding profit is strictly less than π_M and hence, strictly less than the profit at price $\gamma^{-1}(s)$, because $\gamma(p_M) < \varphi(\pi_M)$. It is $E(v)$ for $s = \varphi(E(v))$ which is larger than profit at $\gamma^{-1}(\varphi(E(v))$. It follows that there exists a unique $\hat{s} \in [\gamma(p_M), \varphi(E(v))]$ at which the two profits cross, so $\hat{s}$ satisfies

$$\varphi^{-1}(\hat{s}) = \gamma^{-1}(\hat{s})(1 - G(\gamma^{-1}(\hat{s}))), \tag{6}$$

and profit is larger at $\gamma^{-1}(s)$ for $s \in (\gamma(p_M), \hat{s})$ and profit is larger at $\varphi^{-1}(s)$ for $s \in (\hat{s}, \varphi(E(v))]$.

The above arguments establish the following result.

Proposition 4 (Profit maximization under non disclosure) *Under no disclosure, the op-*

timal price p^N and profit π^N are described as follows as a function of the search cost s.

1. *If $s \leq \min\{\varphi(\pi_M), \gamma(p_M)\}$, then $p^N = p_M$, $\pi^N = \pi_M$ and the consumer searches;*

2. *If $\min\{\varphi(\pi_M), \gamma(p_M)\} < s < \min\{\varphi(E(v)), \gamma(0)\}$:*

   2a. *If $\varphi(\pi_M) < \gamma(p_M)$, then $p^N = \pi^N = \varphi^{-1}(s) > \pi_M$ and the consumer buys immediately;*

   2b. *If $\varphi(\pi_M) > \gamma(p_M)$ and*

   - *$E(v) \leq 0$, then $p^N = \gamma^{-1}(s) < p_M$, $\pi^N = \gamma^{-1}(s)(1 - G(\gamma^{-1}(s))) < \pi_M$ and the consumer searches;*
   - *$E(v) > 0$ and $s < \hat{s}$, then $p^N = \gamma^{-1}(s) < p_M$, $\pi^N = \gamma^{-1}(s)(1 - G(\gamma^{-1}(s))) < \pi_M$ and the consumer searches;*
   - *$E(v) > 0$ and $s > \hat{s}$, then $p^N = \pi = \varphi^{-1}(s)$ and the consumer buys immediately; in this range, $\pi^N = \varphi^{-1}(s) < \pi_M$ small s, and $\pi^N = \varphi^{-1}(s) > \pi_M$ for large s.*

3. *If $s \geq \min\{\varphi(E(v)), \gamma(0)\}$, then $p^N = \pi^N = \max\{0, E(v)\}$ and the consumer buys immediately if $E(v) \geq 0$, and drops otherwise.*

Notice that if $p_M \leq E(v)$, then $\pi_M \leq E(v)$, so $\varphi(\pi_M) < \gamma(p_M)$, and we are in case 2a of the proposition. If $\pi_M \geq E(v)$, then $\varphi(\pi_M) > \gamma(p_M)$, and we are in case 2b. If $\pi_M < E(v) < p_M$, we could be in either case 2a or 2b.

We now examine how the profit-maximizing strategy for the seller, with no product information disclosed, differs from the fully optimal solution with product information, using the following uniform example as an illustration. Assume the match value v is uniformly distributed with $\bar{v} - \underline{v} = 1$, $\bar{v} \in (0, 1]$, so $g(v) = 1$ and $G(v) = v - \underline{v}$ for every $v \in [\underline{v}, \bar{v}]$, $E(v) = \bar{v} - \frac{1}{2}$, and $\mu = \frac{\bar{v}}{2}$. A lower $\bar{v}$ implies that $G(0)$ is larger so the buyer's valuation is more likely to be below the zero marginal cost. This is equivalent to keeping the match value distribution unchanged and raising the marginal cost.

The monopoly price and profit are given by $p_M = \frac{\bar{v}}{2}$ and $\pi_M = \frac{\bar{v}^2}{4}$. The different regions for search costs in Proposition 4 are delineated by the following parameters:

$$\varphi(\pi_M) = \frac{(2-\bar{v})^4}{32}, \quad \gamma(p_M) = \frac{\bar{v}^2}{8}, \quad \varphi(E(v)) = \gamma(E(v)) = \frac{1}{8}, \quad \gamma(0) = \frac{(\bar{v})^2}{2},$$

$$\hat{s} = \frac{1}{8}\left((\bar{v} - 1 + \sqrt{(\bar{v}-1)(\bar{v}-5)})\right)^2.$$

Moving $\bar{v}$ from 1 to 0 allows for exploring the various possible configurations in Proposition 4. Much of the takeaway here applies more generally to sliding down the support $[\underline{v}, \bar{v}]$ while keeping the distribution of $v - \underline{v}$ unchanged.

First, the low search cost region corresponding to Proposition 4 (1), where the seller posts a monopoly price and the buyer searches, always exists. It involves search costs below

$$\min\{\varphi(\pi_M), \gamma(p_M)\} = \begin{cases} \gamma(p_M) & \text{if } \bar{v} \leq \hat{v} \in (\frac{1}{2}, 1) \\ \varphi(\pi_M) & \text{if } \bar{v} \geq \hat{v}. \end{cases}$$

where the crossing point at $\hat{v} \in (\frac{1}{2}, 1)$ indeed exists and is unique because $\varphi(\pi_M)$ is $\frac{1}{32}$ at $\bar{v} = 1$ increases to $\frac{81}{512}$ for $\bar{v} = \frac{1}{2}$, while $\gamma(p_M)$ decreases from $\frac{1}{8}$ to $\frac{1}{32}$ over the same range.

Beyond this point, two cases arise. For $\bar{v} \geq \hat{v}$, Proposition 4 (2a) applies for $s \in (\varphi(\pi_M), \varphi(E(v)))$ so the seller sells with probability 1 by charging $p^N = \varphi^{-1}(s)$ and the buyer buys immediately. As noted after Proposition 4, $\varphi(\pi_M) \leq \gamma(p_M)$ requires that $E(v) > 0$ so that for $s \geq \varphi(E(v))$ price is $E(v)$ and the buyer buys with certainty without search. Now for $\bar{v} < \hat{v}$, from Proposition 4 (2b), the seller's pricing behavior branches out in two directions depending on whether $E(v) > 0$ or not. If $E(v) \leq 0$, then for $\gamma(p_M) < s < \gamma(0)$, from the first bullet in 2b, price is $\gamma^{-1}(s)$ and the buyer searches, whereas, from item 3, the market breaks down for higher search costs (with a negative expected valuation, the buyer does not want to buy without search). If $E(v) > 0$, so the second bullet in 2b is relevant, then there is search for $\gamma(p_M) < s < \hat{s}$ and the buyer buys immediately for $s \geq \hat{s}$ and this remains true for $s \geq \varphi(E(v))$ as stated in item 3.

In contrast with the optimal disclosure solution characterized in Section 4, the seller is not guaranteed a profit above the full information monopoly level when the buyer's information acquisition is costly. This is the case only in scenario 2a in Proposition 4 where, if search costs are high enough, the seller switches from inducing search by posting the monopoly price to inducing an immediate purchase with a lower price and profit rises above the monopoly level after the switch: this arises for $\bar{v} > \hat{v}$ in the uniform example. For $\bar{v} < \hat{v}$, profit is actually non monotonic in search costs and it falls below its monopoly level for $\gamma(p_M) < s < \hat{s}$ if $1/2 < s < \hat{s}$ and for any s above $\gamma(p_M)$ if $\bar{v} \leq 1/2$. For $1/2 < \bar{v} < \hat{v}$, profit rises after $\hat{s}$ to reach $E(v)$, but it eventually exceeds monopoly profit only if $\bar{v}$ is sufficiently close to $\hat{v}$.

The impact on the buyer's welfare is also quite different from what we found with optimal information disclosure, where a drop in search costs unambiguously increases consumer surplus. In order for this to be the case with no information disclosure, we need $\bar{v} \leq 1/2$ so $E(v) \leq 0$, in which case consumer welfare is $\gamma(p_M) - s$, for $s \leq \gamma(p_M)$ and then zero for higher search costs. For $E(v) > 0$, consumer welfare is always discontinuous in s because it jumps up when the seller switches from inducing search to inducing immediate purchase, which happens at $s = \varphi(\pi_M)$ for $\bar{v} > \hat{v}$ or at $s = \hat{s}$ for $1/2 < \bar{v} < \hat{v}$. In both cases, consumer welfare can exceed the full information level $\gamma(p_M)$ right after the jump though it eventually falls to 0 as search costs increase. For search cost values below the jump, it is constant at $\gamma(p_M)$ when $\bar{v} > \hat{v}$ and it is weakly decreasing and reaches 0 before the jump if $v < \hat{v}$.

Keeping the cost of information acquisition fixed, profit is obviously higher with optimal disclosure than with no disclosure. Whether the buyer benefits from optimal disclosure by the seller is a more complicated matter. On the one hand, threshold information disclosure prevents him from searching or buying the product immediately when his valuation is too low. On the other hand, from our characterization of profit maximization with optimal disclosure in Proposition 2, whenever the buyer's surplus is strictly positive, price exceeds the monopoly price, and hence, the price with no information disclosure. We now show that there are configurations of information acquisition costs and likelihoods of the buyer's valuation being below marginal cost for which the consumer welfare comparison unambiguously favors either optimal disclosure or no disclosure.

First consider a large search cost but below $\varphi(E(v))$. Further assume that $\bar{v} > \frac{1}{2}$ so that $E(v) > 0$. From Proposition 4 (2), if s is sufficiently close to $\varphi(E(v))$, the seller charges price $p^N = \varphi^{-1}(s) < E(v)$ so the buyer purchases the product immediately and has a strictly positive surplus. On the other hand, with optimal disclosure, the buyer's surplus is zero if $s \geq s_1$ (see Proposition 3). The next result shows that $\bar{s} \leq \varphi(E(v))$, and since $s_1 < \bar{s}$, for s sufficiently close to $\varphi(E(v))$, the buyer's surplus is larger with no information disclosure than with optimal disclosure.

Proposition 5 *We have $\bar{s} \leq \varphi(E(v))$. Therefore, if $E(v) > 0$, the buyer is strictly better off with no information than with optimal disclosure by the seller for search costs s sufficiently close to and below $\varphi(E(v))$.*

Proof. See Appendix A.3. ■

The setting where Proposition 5 applies can be illustrated in the uniform example with $\bar{v} = 1$ or $\bar{v} = 0.7$. For $\bar{v} = 1$, we have $\varphi(\pi_M) = \frac{1}{32} < \gamma(p_M) = \frac{1}{8}$ so Proposition 4 (2a) applies and consumer welfare jumps up at $\frac{1}{32}$ and remains strictly positive until $\varphi(E(v)) = \frac{1}{8}$. Figure 7 shows that the buyer's surplus with no disclosure always dominates the buyer's surplus with optimal disclosure for all $s \in [\frac{1}{32}, \frac{1}{8})$, so search costs need not be very large.[10] In contrast, with $\bar{v} = 0.7$, profit maximization with no disclosure is described by Proposition 4 (2b) because $\varphi(\pi_M) > \gamma(p_M)$, and buyer surplus with no disclosure jumps up at $\hat{s}$ which is now larger than s_1. Then the buyer can be better off with no disclosure for large search costs only if s is sufficiently close to $\varphi(E(v)) = \frac{1}{8}$.

The uniform example with $\bar{v} = 1$ illustrates that both sides can be better off with relatively high search costs and no information disclosure than with low search costs and optimal

[10]Here this ranking holds for all values of s, but this is not a property that extends to any match distribution with only positive values in the support.

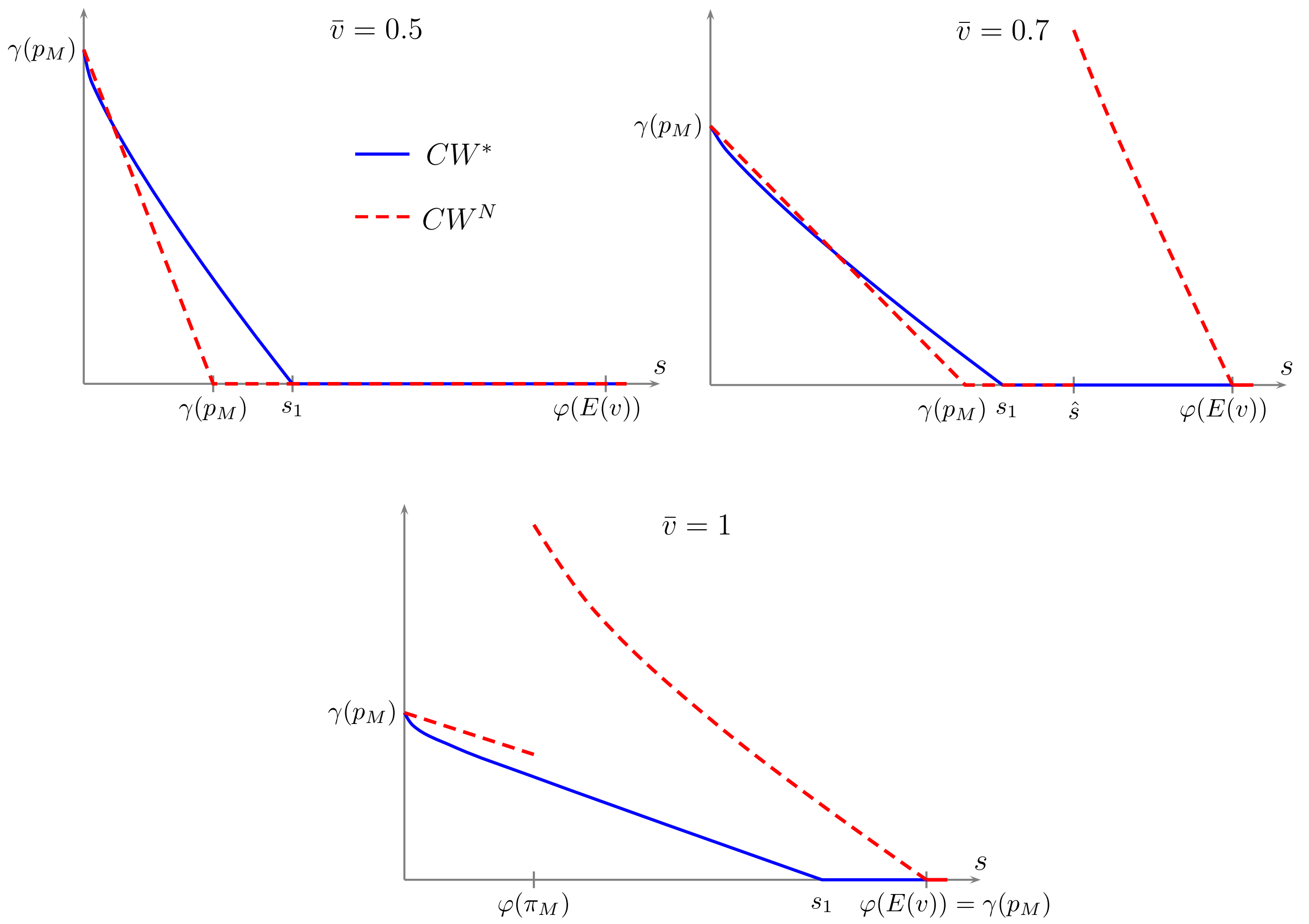

Figure 7: Comparison of consumer welfare under no disclosure (CW^N, in red dashed lines) and optimal disclosure (CW^*, in blue) when the match distribution is uniform on $[\underline{v}, \bar{v}]$, with $\underline{v} = \bar{v} - 1$, as a function of the search cost s for $\bar{v} = 0.5, 0.7, 1$.

information disclosure. At $s = \varphi(\pi_M) = \frac{1}{32}$, with no information disclosure, consumer surplus jumps up from $\frac{3}{32}$ to $\frac{1}{4}$ and the seller earns monopoly profit $\frac{1}{4}$. If search costs are increased above this level, profit rises above monopoly profit $\frac{1}{4}$ and there is some interval over which consumer surplus remains strictly larger than its monopoly level $\frac{1}{8}$. By contrast, if search costs drop substantially and become close to zero while the seller can switch to optimal information disclosure, then profit and consumer welfare are close to their monopoly levels. This illustrates that a drop in search costs concomitant with an increased sophistication in the seller's communication, that could result for instance from trading on digital platforms, can be detrimental to both parties.

A comparison of $\bar{v} = 1$ and $\bar{v} = 0.7$ in the uniform setting suggests that, if it is more likely that the buyer's willingness to pay is below marginal cost, the buyer can be better off with optimal disclosure at some relatively low search cost levels.[11] The next proposition shows that this holds more generally.

Proposition 6 *Consider a distribution function F with support $[0, \bar{w}]$ that admits a continuously differentiable density and has a strictly increasing hazard rate. Let $G(v) = F(v - \underline{v})$, with $\underline{v} \in [-\bar{w}, 0]$. If $\underline{v}$ is sufficiently low, then $\gamma(p_M) < s_1$, and there exists an interval of information acquisition costs, including $[\gamma(p_M), s_1)$, such that the buyer is strictly better off with optimal disclosure rather than no information provided by the seller.*

Proof. See Appendix A.3 ∎

The idea of the proposition is to slide down the support of the match value distribution while retaining its original shape, as we have done in the uniform example by moving $\bar{v}$. This is equivalent to keeping the match distribution unchanged and increasing the marginal cost.

The result can be illustrated in the uniform example by taking $\bar{v} = 0.7$ or $\bar{v} = 1$. In the latter case, the buyer's surplus with no information falls to 0 as s goes from 0 to $\gamma(p_M) = \frac{1}{32}$

[11] In the uniform case, the buyer always prefers no disclosure for s sufficiently close to zero, but this is not a general property.

and remains zero beyond that point. With optimal disclosure, the buyer's surplus only reaches 0 at $s = s_1 = \frac{\bar{v}}{10} > \frac{1}{32}$ and is larger than with no information except when s is very close to 0. Consider again a drop in search costs combined with an improvement in the seller's ability to communicate. As discussed above, for $E(v) \leq 0$, with no disclosure both sides benefit from the drop in search cost. Furthermore, for low search costs, the buyer can be better off with optimal disclosure than with no disclosure so that, the combined improvement in the search and communication technology can benefit both.

A Appendix

A.1 Proof of Proposition 1

First, according to the revelation principle (Myerson, 1982, Proposition 2), we can, without loss of generality, set $M = A$, making the information disclosure policy a direct recommendation system $X : [\underline{v}, \bar{v}] \to \Delta(A)$. Let $X(a \mid s)$ represent the probability that the consumer receives recommendation (signal) a given the match value v. Additionally, we can, again without loss of generality, require obedience from the consumer, meaning the consumer chooses action a upon receiving recommendation a. Thus, $X(a \mid s)$ denotes the probability that the consumer plays action a when the match value is v. Let $X(a)$ be the unconditional probability of signal a with policy X, $\mathbb{P}_X$ be the probability distribution over match values and signals induced by X, and $E_X(v \mid a)$ be the expectation of the match according to the posterior induced by X when the recommendation is a.

The incentive compatibility (obedience) constraints for consumers receiving the recommendation to *buy* are:

$$E_X(v - p \mid buy) \geq 0, \tag{BnD}$$

so they don't drop, and

$$s \geq \mathbb{P}_X(v < p \mid buy) E_X(p - v \mid buy, v < p), \tag{BnS}$$

so they don't search. Similarly, incentive compatibility constraints for consumers receiving the recommendation to *drop* are:

$$E_X(v - p \mid drop) \leq 0, \tag{DnB}$$

so they do not buy, and

$$s \geq \mathbb{P}_X(v \geq p \mid drop) E_X(v - p \mid drop, v \geq p), \tag{DnS}$$

so they do not search. Finally, incentive compatibility constraints for consumers receiving the recommendation to *search* are:

$$\mathbb{P}_X(v \geq p \mid search)E_X(v - p \mid search, v \geq p) - s \geq 0, \tag{SnD}$$

so they do not drop, and

$$\mathbb{P}_X(v \geq p \mid search)E_X(v - p \mid search, v \geq p) - s \geq E_X(v - p \mid search), \tag{SnB}$$

so they do not buy.

Now, consider an arbitrary direct disclosure policy X_0 that satisfies all the incentive compatible conditions above. We show that there exists an incentive compatible disclosure policy Y such that $Y(search) = 0$ yielding the same ultimate probabilities of buying and dropping as X_0. Take Y defined as follows:

$$Y(buy \mid v) = X_0(buy \mid v) + X_0(search \mid v) \text{ and } Y(drop \mid v) = X_0(drop \mid v), \text{ if } v \geq p,$$

$$Y(buy \mid v) = X_0(buy \mid v) \text{ and } Y(drop \mid v) = X_0(drop \mid v) + X_0(search \mid v), \text{ if } v < p.$$

Then we have

$$E_Y(v \mid buy) = \frac{X_0(buy)}{Y(buy)}E_{X_0}(v \mid buy) + \frac{X_0(search \mid v \geq p)[1 - G(p)]}{Y(buy)}E_{X_0}(v \mid search, v \geq p).$$

The first expectation is at least p from the incentive compatibility condition (BnD) of X_0, and the second is at least p by construction, so a buyer receiving signal *buy* from the disclosure policy X_0 does not deviate to *drop*, i.e., the incentive compatibility condition (BnD) is also satisfied for Y. He will not search either because his benefit from searching (which only arises when $v < p$) is smaller than with disclosure policy X_0. Formally, we have

$$\mathbb{P}_{X_0}(v < p \mid buy) = \frac{\mathbb{P}_{X_0}(v < p, buy)}{X_0(buy)} \geq \frac{\mathbb{P}_Y(v < p, buy)}{Y(buy)} = \mathbb{P}_Y(v < p \mid buy),$$

because $\mathbb{P}_{X_0}(v < p, buy) = \mathbb{P}_Y(v < p, buy)$ and $Y(buy) \geq X_0(buy)$, and we have

$$E_{X_0}(p - v \mid buy, v < p) = E_Y(p - v \mid buy, v < p),$$

so the RHS of (BnS) is lower for $X = Y$ than for $X = X_0$.

The arguments for showing that a buyer who receives the *drop* signal will not want to deviate are similar. For a buyer receiving signal *drop*, $E_Y(v \mid drop)$ is a convex combination of $E_{X_0}(v \mid drop)$ and $E_{X_0}(v \mid search, v < p)$, which are both lower than p, so (DnB) is also satisfied for $X = Y$. Finally, consider the buyer's incentive to search instead of dropping when he receives the *drop* signal, i.e., condition (DnS). We have

$$\mathbb{P}_{X_0}(v \geq p \mid drop) E_{X_0}(v - p \mid drop, v \geq p) \geq \mathbb{P}_Y(v \geq p \mid drop) E_Y(v - p \mid drop, v \geq p),$$

because $\mathbb{P}_{X_0}(v \geq p \mid drop) \geq \mathbb{P}_Y(v \geq p \mid drop)$ and $E_{X_0}(v - p \mid drop, v \geq p) = E_Y(v - p \mid drop, v \geq p)$, so the RHS of (DnS) is lower for $X = Y$ than for $X = X_0$.

Next, consider some incentive compatible disclosure policy Y such that $Y(search) = 0$. Let $\tilde{v}$ be the unique solution to $1 - G(\tilde{v}) = Y(buy)$, and define a threshold disclosure policy Z such that $Z(buy \mid v) = 1$ if $v \geq \tilde{v}$ and $Z(drop \mid v) = 1$ if $v < \tilde{v}$. There is no information acquisition with either Y or Z, and they both yield the same probability of purchase.

Clearly, we have $E_Y(v \mid buy) \leq E_Z(v \mid buy)$ and $E_Y(v \mid drop) \geq E_Z(v \mid drop)$, so the incentive constraints (BnD) and (DnB) are satisfied for $X = Z$. Consider next the constraint (BnS). It is obviously satisfied if $p \leq \tilde{v}$, so let $p > \tilde{v}$. We have

$$E_Z(v \mid buy, v < p) \geq E_Y(v \mid buy, v < p),$$

and

$$\mathbb{P}_Z(v < p \mid buy) \leq \mathbb{P}_Y(v < p \mid buy),$$

so the RHS of (BnS) is smaller with $X = Z$ than with $X = Y$. Similarly, consider the constraint (DnS). It is obviously satisfied if $p \geq \tilde{v}$, so let $p < \tilde{v}$. We have

$$E_Z(v - p \mid drop, v \geq p) \leq E_Y(v - p \mid drop, v \geq p),$$

and

$$\mathbb{P}_Z(v \geq p \mid drop) \leq \mathbb{P}_Y(v \geq p \mid drop),$$

so the RHS of (DnS) is smaller with $X = Z$ than with $X = Y$.

Given the disclosure policy Z above, a firm charging $p \leq \tilde{v}$ could increase p slightly above $\tilde{v}$ without violating any incentive compatibility constraint and hence make more profit. So we must have $p > \tilde{v}$.

A.2 Additional results and proofs for Section 4

Proof of Lemma 4.1. (i) Along the Δ_b curve p is strictly increasing in $\tilde{v}$ because the LHS of (Δ_b) is $E(v \mid v \geq \tilde{v}) - p$, which strictly decreasing in p and strictly increasing in $\tilde{v}$. The fact that $p = \mu$ for $\tilde{v} = 0$ and $p = \bar{v}$ for $\tilde{v} = \bar{v}$ directly follows from (Δ_b).

(ii) To show the properties of the Σ_b curve we first show that $\phi(\tilde{v}, p) = \frac{\int_{\tilde{v}}^{p} (p-v)g(v)\, dv}{1-G(\tilde{v})}$ is strictly increasing in p and strictly decreasing in $\tilde{v}$ for $p \in [0, \bar{v}]$ and $\tilde{v} \leq p$. By integrating by parts, we have

$$\phi(\tilde{v}, p) = \frac{\int_{\tilde{v}}^{p} G(v) - G(\tilde{v})\, dv}{1 - G(\tilde{v})}.$$

Hence,

$$\frac{\partial \phi}{\partial p} = \frac{G(p) - G(\tilde{v})}{1 - G(\tilde{v})} > 0, \tag{7}$$

$$\frac{\partial \phi}{\partial \tilde{v}} = -\frac{g(\tilde{v})}{(1 - G(\tilde{v}))^2} \int_{\tilde{v}}^{p} 1 - G(v)\, dv < 0. \tag{8}$$

Hence, along the Σ_b curve, p is strictly increasing in $\tilde{v}$, and Σ_b shifts upward if s increases.

The fact that $p = \mu$ for $\tilde{v} = 0$ if $s = \bar{s}$ directly follows from the definition of $\bar{s}$.

Finally, for $p = \bar{v}$, (Σ_b) is $\frac{\int_{\tilde{v}}^{\bar{v}} (\bar{v}-v) g(v)\, dv}{1-G(\tilde{v})} = s$, i.e.,

$$E(v \mid v \geq \tilde{v}) = \bar{v} - s.$$

As $\tilde{v}$ increases from 0 to $\bar{v}$, the left hand side increases strictly and continuously from μ to $\bar{v}$. For $s \in (0, \bar{s}]$, the right hand side is in $(\mu, \bar{v})$ because, from Equation (1) and using $\mu = \frac{\int_0^{\mu} v dG(v)}{1-G(0)}$, we have $\bar{s} = \frac{\mathbb{P}\{v \geq \mu\}}{\mathbb{P}\{v \geq 0\}} E(v - \mu \mid v \geq \mu) < \bar{v} - \mu$. Hence, the equality holds for some unique $\tilde{v} \in (0, \bar{v})$.

■

Before proving the next lemmas we prove the following result.

Claim 1 If at $(\tilde{v}^*, p^*)$ we have $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$, then $p_M < p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$.

Proof of Claim 1. The slope of the iso-profit curve at some $(\tilde{v}, p)$ is given by

$$p'_{ISO}(\tilde{v}) = \frac{g(\tilde{v}) p}{1 - G(\tilde{v})}. \tag{9}$$

Using (7) and (8) in the proof of Lemma 4.1, the slope of the Σ_b curve at $(\tilde{v}, p)$ is given by

$$p'_{\Sigma}(\tilde{v}) = -\frac{\frac{\partial \phi}{\partial \tilde{v}}}{\frac{\partial \phi}{\partial p}} = \frac{g(\tilde{v})}{1 - G(\tilde{v})} \frac{\int_{\tilde{v}}^{p} 1 - G(v)\, dv}{G(p) - G(\tilde{v})}. \tag{10}$$

If $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$, then

$$p^* = \frac{\int_{\tilde{v}^*}^{p^*} 1 - G(v)\, dv}{G(p^*) - G(\tilde{v}^*)}, \tag{11}$$

which can be rewritten as

$$\int_{\tilde{v}^*}^{p^*} 1 - G(v) - p^* g(v)\, dv = \int_{\tilde{v}^*}^{p^*} \left(\frac{1 - G(v)}{g(v)} - p^* \right) g(v)\, dv = 0. \tag{12}$$

Because $\frac{g(\cdot)}{1-G(\cdot)}$ is increasing and $p^* > \tilde{v}^*$, we have $\frac{1-G(p^*)}{g(p^*)} < \frac{1-G(v)}{g(v)}$ for $v \in [\tilde{v}^*, p^*]$, so

$$\int_{\tilde{v}^*}^{p^*} \left(\frac{1-G(p^*)}{g(p^*)} - p^* \right) g(v)\, dv < \int_{\tilde{v}^*}^{p^*} \left(\frac{1-G(v)}{g(v)} - p^* \right) g(v)\, dv = 0.$$

This inequality implies

$$\frac{1-G(p^*)}{g(p^*)} < p^*, \tag{13}$$

and therefore, from the increasing hazard rate, $p^* > p_M = \frac{1-G(p_M)}{g(p_M)}$.

To prove that $p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$, we use (11) and the fact that $\frac{1-G(v)}{g(v)}$ is decreasing in v:

$$p^* = \frac{\int_{\tilde{v}^*}^{p^*} \frac{(1-G(v))}{g(v)} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} < \frac{\int_{\tilde{v}^*}^{p^*} \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} = \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)} \frac{\int_{\tilde{v}^*}^{p^*} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} = \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}. \tag{14}$$

■

Proof of Lemma 4.2. The proof proceeds in two steps. First we show that first-order conditions have only one solution. Second, we show that $\tilde{v}^* > 0$ so that, if there is a corner solution, then we have $p^* = \bar{v}$.

Step 1. To show that there is only one solution to first-order conditions we first show that whenever $p'_{ISO}(\tilde{v}) = p'_{\Sigma}(\tilde{v})$ then the difference $p'_{ISO} - p'_{\Sigma}$ is strictly increasing in the neighborhood of $\tilde{v}$ along the Σ_b curve. This in turn implies that $p'_{ISO} - p'_{\Sigma}$ can only switch sign from being negative to being positive and hence switches sign only once. This ensures that there is a unique solution which is either interior, in which case $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$, at $\tilde{v}^* = \underline{v}$ if $p'_{ISO}(\underline{v}) \geq p'_{\Sigma}(\underline{v})$ or $\tilde{v}^*$ is such that $p^* = \bar{v}$, which would require $p'_{ISO}(\tilde{v}^*) \leq p'_{\Sigma}(\tilde{v}^*)$. Therefore there is a unique point $(\tilde{v}^*, p^*)$ satisfying the FOC.

From Equations (9) and (10) we have

$$p'_{ISO}(\tilde{v}) - p'_{\Sigma}(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})} \left(p - \frac{\int_{\tilde{v}}^{p} 1-G(v)\, dv}{G(p) - G(\tilde{v})} \right).$$

Because $\frac{g(\tilde{v})}{1-G(\tilde{v})}$ is positive and strictly increasing, it suffices to show that $f(\tilde{v}) := p - \frac{\int_{\tilde{v}}^{p} 1-G(v)\, dv}{G(p)-G(\tilde{v})}$

is strictly increasing in $\tilde{v}$ at whenever $f'(\tilde{v}) = 0$. We have

$$f'(\tilde{v}) = p'_{\Sigma}(\tilde{v}) - \frac{d}{d\tilde{v}} \frac{\int_{\tilde{v}}^{p} 1 - G(v)\, dv}{G(p) - G(\tilde{v})},$$

so, using (10), we get:

$$f'(\tilde{v}) = \frac{g(\tilde{v})}{1 - G(\tilde{v})} \frac{\int_{\tilde{v}}^{p} 1 - G(v)\, dv}{G(p) - G(\tilde{v})} - \frac{d}{d\tilde{v}} \frac{\int_{\tilde{v}}^{p} 1 - G(v)\, dv}{G(p) - G(\tilde{v})}.$$

$$= \frac{g(\tilde{v})}{1 - G(\tilde{v})} \frac{N}{D} - \frac{N'D - ND'}{D^2},$$

where $N = \int_{\tilde{v}}^{p} 1 - G(v)\, dv$, $D = G(p) - G(\tilde{v})$, $N' = p'_{\Sigma}(\tilde{v})(1 - G(p)) - (1 - G(\tilde{v}))$ and $D' = p'_{\Sigma}(\tilde{v})g(p) - g(\tilde{v})$.

If $f(\tilde{v}) = 0$, then $p = \frac{N}{D}$ and $p'_{\Sigma}(\tilde{v}) = p'_{ISO}(\tilde{v}) = \frac{g(\tilde{v})p}{1 - G(\tilde{v})}$ so $f'(\tilde{v})$ can be rewritten as follows:

$$f'(\tilde{v}) = \frac{g(\tilde{v})}{1 - G(\tilde{v})} p - \frac{N' - pD'}{D}$$

$$= \frac{g(\tilde{v})}{1 - G(\tilde{v})} p - \frac{p'_{\Sigma}(\tilde{v})(1 - G(p)) - (1 - G(\tilde{v})) - p(p'_{\Sigma}(\tilde{v})g(p) - g(\tilde{v}))}{G(p) - G(\tilde{v})}$$

$$= \frac{g(\tilde{v})}{1 - G(\tilde{v})} p - \frac{\frac{g(\tilde{v})p}{1 - G(\tilde{v})}(1 - G(p)) - (1 - G(\tilde{v})) - p(\frac{g(\tilde{v})p}{1 - G(\tilde{v})} g(p) - g(\tilde{v}))}{G(p) - G(\tilde{v})}$$

$$= \frac{1}{G(p) - G(\tilde{v})} \left(\frac{g(\tilde{v})p(G(p) - G(\tilde{v}))}{1 - G(\tilde{v})} - \frac{g(\tilde{v})p(1 - G(p))}{1 - G(\tilde{v})} + (1 - G(\tilde{v})) + \frac{pg(\tilde{v})pg(p)}{1 - G(\tilde{v})} - pg(\tilde{v}) \right)$$

$$= \frac{1}{G(p) - G(\tilde{v})} \left(\frac{pg(\tilde{v})}{1 - G(\tilde{v})} \Big[G(p) - G(\tilde{v}) - (1 - G(p)) + pg(p) - (1 - G(\tilde{v})) \Big] + (1 - G(\tilde{v})) \right)$$

$$= \frac{1}{G(p) - G(\tilde{v})} \left(\frac{pg(\tilde{v})g(p)}{1 - G(\tilde{v})} \Big[p - 2\frac{1 - G(p)}{g(p)} \Big] + (1 - G(\tilde{v})) \right).$$

Using the properties that $G(p) - G(\tilde{v}) \geq 0$ (because $p \geq \tilde{v}$) and $p \geq \frac{1 - G(p)}{g(p)}$ (because $p \geq p_M$) a sufficient condition for $f'(\tilde{v}) > 0$ is

$$p < \frac{1 - G(\tilde{v})}{g(\tilde{v})} \frac{1 - G(\tilde{v})}{1 - G(p)}.$$

$p \geq \tilde{v}$ also implies $\frac{1-G(\tilde{v})}{1-G(p)} \geq 1$ so a sufficient condition for $f'(\tilde{v}) > 0$ is $p < \frac{1-G(\tilde{v})}{g(\tilde{v})}$, which follows from Claim 1.

Step 2. We show that $\tilde{v}^* > 0$. From our analysis above, it suffices to show that for $s \leq \bar{s}$, $p'_{ISO}(0) < p'_{\Sigma}(0)$. Indeed this implies that there is no $\tilde{v} \leq 0$ at which $p'_{ISO}(\tilde{v}) = p'_{\Sigma}(\tilde{v})$, which in turn implies that we cannot have $p'_{ISO}(\tilde{v}) \geq p'_{\Sigma}(\tilde{v})$ at $\tilde{v} = \underline{v}$, so first-order conditions never hold for $\tilde{v} \leq 0$.

Let $p'_{\Delta}(\tilde{v})$ be the slope of the Δ_b curve at $(\tilde{v}, p)$. We can write $p'_{\Delta}(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})} h(\bar{v})$ and $p'_{\Sigma}(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})} h(p)$, where

$$h(x) = \frac{\int_{\tilde{v}}^{x} 1 - G(v)dv}{G(x) - G(\tilde{v})},$$

and

$$h'(x) = \frac{[1 - G(x)][G(x) - G(\tilde{v})] - g(x) \int_{\tilde{v}}^{x} 1 - G(v)dv}{[G(x) - G(\tilde{v})]^2}.$$

Standard arguments show that the increasing hazard rate property of G implies that the integral is bounded below by $\frac{[1-G(x)][G(x)-G(\tilde{v})]}{g(x)}$ so $h'(x) < 0$. It follows that at any crossing point between Σ_b and Δ_b, $p'_{\Delta}(\tilde{v}) < p'_{\Sigma}(\tilde{v})$. Now consider $\tilde{v} = 0$. The two curves cross at $\tilde{v} = 0$ and $p = \mu$. Furthermore, at that point, profit is maximized subject to the constraint Δ alone so we have $p'_{ISO}(\tilde{v}) = p'_{\Delta}(\tilde{v}) < p'_{\Sigma}(\tilde{v})$, which proves the result for $s = \bar{s}$. Now because h is decreasing, $p'_{\Sigma}(\tilde{v})$ becomes larger for prices $p < \mu$ whereas $p'_{ISO}(\tilde{v}) = \frac{pg(0)}{1-G(0)}$ is lower. Furthermore, for $s < \bar{s}$, (Σ) binds at $p < \mu$ when $\tilde{v} = 0$, so we have $p'_{ISO}(\tilde{v}) < p'_{\Sigma}(\tilde{v})$ when $\tilde{v} = 0$. ■

Proof of Lemma 4.3. From the proof of Claim 1, the first-order condition $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$ can be rewritten as (12). Differentiating with respect to s yields, after simplification:

$$\frac{dp^*}{ds} = \frac{1 - G(\tilde{v}^*) - p^* g(\tilde{v}^*)}{1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*)} \frac{d\tilde{v}^*}{ds}. \tag{15}$$

Note that the denominator is negative,

$$1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*) < 0, \tag{16}$$

because $1-2G(p^*)+G(\tilde{v}^*)-p^*g(p^*) = 1-G(p^*)-p^*g(p^*)+G(\tilde{v}^*)-G(p^*)$, $1-G(p^*)-p^*g(p^*) < 0$ and $G(\tilde{v}^*) - G(p^*) < 0$.

Next, we differentiate the binding constraint (Σ_b), which gives

$$\frac{d\phi}{ds} = \frac{d\tilde{v}^*}{ds}\frac{\partial\phi}{\partial\tilde{v}^*} + \frac{dp^*}{ds}\frac{\partial\phi}{\partial p^*} = 1.$$

Using (7) and (8) we get:

$$\frac{dp^*}{ds} = \frac{1-G(\tilde{v}^*)}{G(p^*)-G(\tilde{v}^*)} + \frac{g(\tilde{v}^*)\int_{\tilde{v}^*}^{p^*}(1-G(v))\,dv}{(G(p^*)-G(\tilde{v}^*))(1-G(\tilde{v}^*))}\frac{d\tilde{v}^*}{ds}. \tag{17}$$

Combining (15) and (17) we get:

$$\frac{d\tilde{v}^*}{ds}\left(\frac{1-G(\tilde{v}^*)-p^*g(\tilde{v}^*)}{1-2G(p^*)+G(\tilde{v}^*)-p^*g(p^*)} - \frac{g(\tilde{v}^*)\int_{\tilde{v}^*}^{p^*}(1-G(v))\,dv}{(G(p^*)-G(\tilde{v}^*))(1-G(\tilde{v}^*))}\right) = \frac{1-G(\tilde{v}^*)}{G(p^*)-G(\tilde{v}^*)}. \tag{18}$$

Observe that the RHS of (18) is positive, the second fraction of the LHS is positive, and the denominator of the first fraction of the LHS is negative as observed in (16). To show that $\frac{d\tilde{v}^*}{ds} < 0$ it remains to show that the numerator of the first fraction of the LHS of (18) is positive, which follows from $p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$ (Claim 1). We conclude that $\frac{d\tilde{v}^*}{ds} < 0$, and hence from Equations (15) and (16) that $\frac{dp^*}{ds} > 0$. ■

A.3 Proofs for Section 5

Proof of Proposition 5. To show that $\bar{s} \leq \varphi(E(v))$, for every $x \in [\underline{v}, \bar{v}]$ define $\nu(x) = E(v \mid v > x)$. Consider now $V(x) = \frac{\int_x^{\nu(x)}(\nu(x)-v)g(v)dv}{1-G(x)}$ so $\bar{s} = V(0)$ and $\varphi(Ev) = V(\underline{v})$. We show that $V' < 0$. It has the sign of

$$\big[G(\nu(x)) - G(x)]\nu'(x) - (\nu(x)-x)g(x)\big][1-G(x)] + g(x)\int_x^{\nu(x)}(\nu(x)-v)g(v)dv.$$

Substituting for $\nu'(x)$ and integrating by parts, V' has the sign of

$$\left(\frac{G(\nu(x)) - G(x)}{1 - G(x)} - 1\right) \int_x^{\nu(x)} 1 - G(v)dv + \frac{G(\nu(x)) - G(x)}{1 - G(x)} \int_{\nu(x)}^{\bar{v}} 1 - G(v)dv.$$

Increasing hazard rate for G allows for constructing an upper bound for the above term which is zero. The second part of the proposition follows from Proposition 4. ∎

Proof of Proposition 6. Clearly, for $\underline{v}$ low enough, we have $E(v) \leq 0$, so $\varphi(\pi_M) > \gamma(p_M)$, and Proposition 4 implies that under no disclosure consumer welfare is 0 for every $s \geq \gamma(p_M)$. Proposition 2 implies that under optimal disclosure consumer welfare is strictly positive for every $s < s_1$. Therefore, if $\gamma(p_M) < s_1$, consumer welfare is strictly higher under optimal disclosure for every s in $[\gamma(p_M), s_1)$ and for every s slightly below $\gamma(p_M)$.

To show that $\gamma(p_M) < s_1$ for $\underline{v}$ sufficiently low, let us first consider the price y such that (Δ) is binding for the disclosure threshold $\tilde{v} = p_M$. Then,

$$\int_{p_M}^{\bar{v}} (v - y)\, dv = 0 \iff \int_{p_M}^{y} (v - y)\, dv + \int_{y}^{\bar{v}} (v - y)\, dv = 0$$

$$\iff \int_{p_M}^{y} (y - v)\, dv = \int_{y}^{\bar{v}} (v - y)\, dv \iff \phi(p_M, y) = \frac{\gamma(y)}{1 - G(p_M)}.$$

Let $(\tilde{v}^*, p^*)$ be the seller's optimum for search cost s_1. Because (Σ) is binding at s_1, we have $s_1 = \phi(\tilde{v}^*, p^*)$. At s_1, we also have (see Claim 1) $\frac{1-G(p_M)}{g(p_M)} = p_M < p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$. By the increasing hazard rate property, we have $p_M > \tilde{v}^*$. This implies $y > p^*$ because $(\tilde{v}^*, p^*)$ and (p_M, y) belong to Δ_b, and Δ_b is increasing. Hence, because $\phi(\tilde{v}, p)$ is decreasing in $\tilde{v}$ and increasing in p (see the proof of Lemma 4.1), we get

$$s_1 = \phi(\tilde{v}^*, p^*) > \phi(p_M, y) = \frac{\gamma(y)}{1 - G(p_M)}.$$

To show that $s_1 > \gamma(p_M)$, it suffices to show that $\frac{\gamma(y)}{1-G(p_M)} > \gamma(p_M)$, i.e.,

$$\frac{\gamma(y)}{\gamma(p_M)} \frac{1}{1-G(p_M)} > 1. \tag{19}$$

Next, we show that

$$\frac{d}{dp_M} \frac{\gamma(y)}{\gamma(p_M)} > 0, \tag{20}$$

which is equivalent to $\frac{dy}{dp_M} < \frac{\frac{\gamma'(p_M)}{\gamma(p_M)}}{\frac{\gamma'(y)}{\gamma(y)}}$. Standard arguments show that $\frac{\gamma'(p)}{\gamma(p)}$ is decreasing in p. Hence, because $y = E(v \mid v \geq p_M) > p_M$, we have $\frac{\frac{\gamma'(p_M)}{\gamma(p_M)}}{\frac{\gamma'(y)}{\gamma(y)}} > 1$. Finally, we show $\frac{dy}{dp_M} < 1$. Because (p_M, y) binds (Δ), we have $\int_{p_M}^{\bar{v}} (v-y)\,dv = 0$. Integrating by parts and simplifying yields

$$y = p_M - \frac{\gamma(p_M)}{\gamma'(p_M)}.$$

Because $-\frac{\gamma(p_M)}{\gamma'(p_M)}$ is decreasing, we get $\frac{dy}{dp_M} < 1$, and therefore (19).

To conclude the proof of the proposition, note that decreasing $\underline{v}$ is equivalent to increasing the marginal cost, i.e., increasing p_M, while keeping the distribution G fixed so that $\frac{1}{1-G(p_M)}$ becomes arbitrarily large. Therefore, by (20), $\frac{\gamma(y)}{\gamma(p_M)} \frac{1}{1-G(p_M)}$ becomes larger than 1, proving (19) and therefore $s_1 > \gamma(p_M)$. ■

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