Subjective Survival Beliefs, Cognitive Skills and Investments in Risky Assets

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Abstract: Financial resilience in old age is a major challenge in rapidly aging societies. In this paper, we study the role that subjective life expectancy plays in the decision to save and participate to the financial market among the elderly and how it interacts with financial literacy. We empirically show that the discrepancy between subjective survival beliefs and objective survival rates affects the decision to save and participate in the stock market and we document that this discrepancy is correlated with financial literacy. We then set up a quantitative life-cycle model to simulate alternative policy interventions. We find that survival literacy interventions informing individuals about their objective survival chances attenuate the longevity risk by encouraging wealth accumulation. Financial literacy policies lowering the costs of participating to the stock market incentivize investment in risky assets, but they benefit wealthy households more.

JEL: H31, G5, G11, E21

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1 Introduction

Financial resilience in old age is a major challenge in rapidly aging populations across developed countries. Economic theory suggests that agents can ensure adequate economic conditions post retirement by allocating their wealth to a well-diversified portfolio of financial assets during working life, while taking into account their expected life horizon. However, vast empirical evidence has shown that very few households participate to the stock market and that, on average, individuals underestimate their survival chances with respect to the national statistics survival rates, thus ending up outliving their financial resources.

In order to avoid scenarios of extreme poverty and low levels of well-being among an increasingly old population, policy makers can implement interventions aimed at increasing both financial and survival literacy of the working age adults who already completed formal education. These are two different types of human capital that are closely intertwined, hence public policies that focus on boosting cognitive skills in the form of financial literacy can potentially have an indirect effect on individuals' financial decisions through the modification of survival beliefs and vice versa. Ignoring the interactions between financial and survival literacy is likely to produce an underestimation of the effectiveness of these type of public interventions in changing individuals' economic decisions.

In this paper, we study the role that subjective life expectancy – and the discrepancy with respect to objective survival rates – plays in the decision to participate to the financial market among the elderly and how it interacts with individuals' level of financial literacy.

First, we use panel data from the English Longitudinal Study of Aging (ELSA) and conduct an empirical analysis to assess the impact of subjective survival beliefs on financial market participation decisions of the elderly in the UK, keeping constant cognitive skills as a proxy for the level of financial literacy. We show that the discrepancy between subjective survival beliefs and objective survival rates has a significant role in explaining the decision to participate in the stock market; in line with Spaenjers and Spira (2015), those who expect a shorter lifespan with respect to the relevant objective survival curve are significantly less likely to participate in the financial market. We further document that, the discrepancy between individuals' survival beliefs with respect to objective survival rates is strongly correlated with cognitive skills – our proxy of financial literacy – even after controlling for dispositional optimism and individuals' private information about their health conditions (captured by reported healthy/unhealthy habits). Namely, high cognitive individuals' subjective survival expectations are closer to objective survival rates than those of low cognitive individuals.

Since the literature has widely documented that cognitive skills, especially in the form of financial literacy, have a significant effect on the probability of investing in risky assets (see, for instance, Christelis, Jappelli, and Padula (2010)), then the question arises of what is the role of the interaction between financial literacy and survival literacy in determining stock market participation. In particular, what part of the well known effect of cognitive skills on participation is represented by a direct effect acting through increased levels of financial literacy and therefore lower participation costs and what part, instead, is mediated by an indirect effect of subjective beliefs on the expected life horizon.

In order to disentangle these direct and indirect mechanisms, in the second part of the paper, we set up and calibrate a life-cycle model of saving decisions with two assets (a risk free and a risky asset) and two types of agents (high cognitive and low cognitive skills). More specifically, households are ex ante heterogeneous depending on their level of cognitive skills which we assume to be constant, in the absence of financial literacy interventions, over the part of the life cycle that we model. Households belonging to different cognitive skills types have different income processes and different costs of participating to the financial market.

In our setting, agents make their consumption and investment decisions taking into account their subjective survival expectations, rather than the average objective survival rates from life tables as in standard life-cycle models. These expectations are individual-specific survival curves obtained from the answers given by a specific cohort of ELSA respondents – those born between 1956 and 1964 – to a set of questions about self-assessed probabilities of surviving up to a certain target age.

We use the calibrated model to perform a set of counterfactual experiments. We simulate our model under three alternative policy scenarios: first, a survival literacy intervention that informs low cognitive individuals about their actual survival rates from the life tables so to close the gap between subjective survival expectations and objective survival probabilities; second, a financial literacy intervention that provides information about the functioning of the financial market to low cognitive individuals and therefore lowers their participation (entry and per period) costs to the same level of those faced by high cognitive individuals; third, a combination of the previous two interventions.

We find that the survival literacy intervention has a positive effect on total wealth accumulation in and of itself, although smaller in magnitude than that of the financial literacy intervention.

Moreover, the effect of the combined policy on total wealth is stronger than the sum of the separate effects of the survival literacy and financial literacy policies. The financial literacy intervention, by lowering participation costs, is more effective than the survival literacy intervention in boosting entry in the financial market. In particular, higher entry rate is driven by households who are less liquidity constrained, i.e. have higher initial wealth.

In terms of welfare impact of the policies, both the survival literacy and the financial literacy policies improve households' welfare overall. However, the survival literacy intervention results in larger welfare gains (up to 6% of per period consumption) and it is more redistributive than the financial literacy intervention that, instead benefits more households at the top of the initial wealth distribution.

This paper contributes to two main strands of the literature. First, the paper relates to the empirical and quantitative studies about the role of subjective expectations, and of survival expectations in particular, on households' economic decisions. Manski (2004) seminal work shed light on the measurement of expectations in the form of subjective probabilities to understand agents' economic choices. More recently, Koşar and O'Dea (2023) provide a review of the growing literature using data on beliefs and expectations in the estimation of structural models of households' behavior. In particular, Gan et al. (2015) look at the impact of subjective mortality risk on consumption, saving, and bequest decisions in a structural life-cycle model. Spaenjers and Spira (2015) empirically analyze the link between subjective life horizon and equity portfolio shares and find a positive relationship. Two recent studies that are closest to ours are Heimer, Myrseth, and Schoenle (2019) and O'Dea and Sturrock (2021); they study the role of subjective survival beliefs in explaining the participation puzzle and the annuity puzzle, that is the fact that individuals participate too little in the financial market and purchase too few annuities with respect to what the theory would predict.

Second, this paper builds on the vast literature about cognitive skills, financial literacy, and household financial decisions at various stages of the life cycle. Van Rooij, Lusardi, and Alessie (2012) provide empirical evidence of a strong positive association between financial literacy and net worth. Lusardi, Michaud, and Mitchell (2017) use a life-cycle model to conclude that a relevant part of retirement wealth inequality is accounted for by differences in financial knowledge. Jappelli and Padula (2015) and Jappelli and Padula (2017) build life cycle models of consumption and saving and show, respectively, that there is a positive correlation between financial sophistication, on the one hand, and stock market participation and consumption growth, on the other hand. More recently, D'acunto et al. (2023) analyze administrative and survey micro data showing that high cognitive abilities (IQ) men have more accurate inflation expectations with relevant implications for their consumption dynamics over the life cycle.

To the best of our knowledge our paper is the first to look at the interaction between subjective life horizon and financial literacy in determining households' financial resilience in old age.

The rest of the paper is structured as follows. Section 2 presents the dataset and the descriptive statistics used for the empirical analysis, and Section 3 presents the empirical relation between households' survival beliefs and the stock market participation decision. Section 4 explores the interaction between survival beliefs and cognitive skills. Section 5 describes the life-cycle model and its calibration. Section 6 illustrates the counterfactual simulated experiments and the welfare analysis. Section 7 concludes.

2 Data

We use the English Longitudinal Study of Ageing data (hereafter ELSA) to study the relationship between subjective survival expectations and stock market participation of households. ELSA is a longitudinal survey that collects data from a representative sample of English people aged 50 years and above. It is a biennial survey (first wave in 2002) that aims to gather data to study the aspects of the ageing process, like social care, retirement, pension policies and social participation in England.

The original sample of ELSA (first wave) was selected from the Health Survey for England (HSE[1]) respondents in the period 1998-2001. After the first survey in 2002, younger age groups of ELSA were refreshed to balance the panel over time.

Sections 3 and 4 work with Wave 8 data of ELSA, while observations in Section 5 use information from all ELSA waves.

The variable of interest is household stock market participation, which is household-level financial information, thus, the variable is identical for each partner (when the household is a couple). We decide to select only the self-reported financial respondents of the household, as representative of the household characteristics. We identify the financial respondent as the individual who answered the Income&Asset interview module. If one household has more than one financial respondent, we assigned the role based on gender (males have precedence)

[1]More information about HSE at http://healthsurvey.hscic.gov.uk.

and age (the older win). For those households that change the financial respondent over time, the role is assigned to the individual who was the financial respondent more often and is still alive.

Table 1 shows the descriptive statistics of the sample used in Sections 3 and 4.

Table 1: Sample descriptive statistics

	All	Non-Stock Holders	Stock Holders
obs.	3,311	1,308	2,003
males (%)	54.9	51.1	68.1
age (median)	69	69	69
age (sd)	8.5	9.2	8.2
working (mean)	20.2	20.0	20.4
fin. wealth: median (thousands £)	33.0	5	74.1
fin. wealth: sd (thousands £)	208.3	106.7	258.8
tot. income: median (week £)	410.8	309.1	489.2
tot. income: sd (week £)	358.9	272.6	386.2
cognition	0.35	-0.02	0.58
cognition (sd)	1.37	1.41	1.29
financial lit.	3.58	3.13	3.87
financial lit. (sd)	1.16	1.21	1.02
discrepancy: % overestimating	38.8	35.7	40.9
discrepancy (mean)	0.84	1.01	0.72
discrepancy (mean, only underestimating)	1.1	1.23	0.98
smoker (%)	9.8	15.5	6.0
intense sport (%)	38.6	28.6	45.1
healthy eating (%)	57.4	52.1	60.7

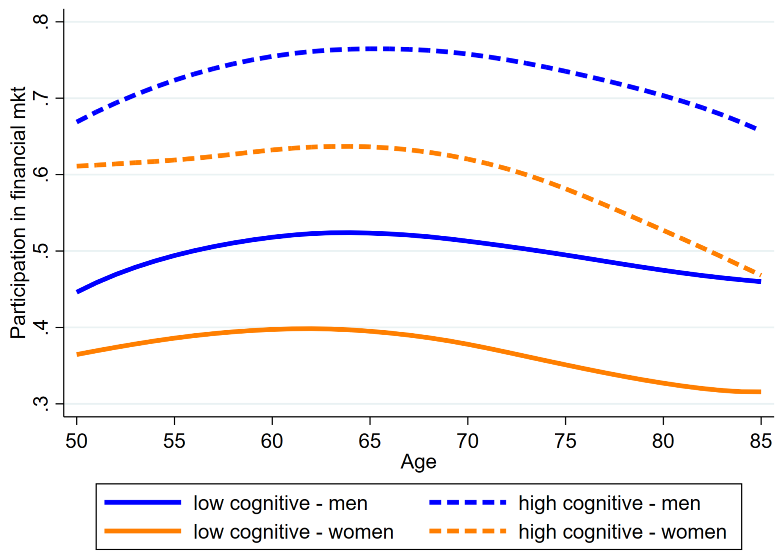
Note: Financial wealth does not include housing wealth. Thus, house value and mortgages are not taken into account.

Table 1 shows that 60.3% of households hold risky assets, defined as shares, bonds, stocks and shares ISAs or life insurance ISAs[2], following the definition of ELSA.

[2]ISA (Individual Saving Account) is a class of retail investment arrangement available to residents of the United Kingdom, with favourable tax conditions. They offer four types of accounts: cash ISA, stocks &

Stock-holding households are more likely to have a male financial respondent and show a median financial wealth that is more than ten times larger than non-stock-holding households. Also, the median weekly income of stock-holding households is higher than those who do not invest in risky assets, around 1.40 times larger. Figure 1 further shows the profile of the rate of participation in the financial market over the second half of the life-cycle in ELSA. It highlights the fact that participation levels are heterogeneous across gender and cognitive level of the financial respondent with male and high cognitive level respondents reporting higher participation rates than the female and low cognitive level counterparts.

Figure 1: Participation rate by financial respondent's gender and cognitive level

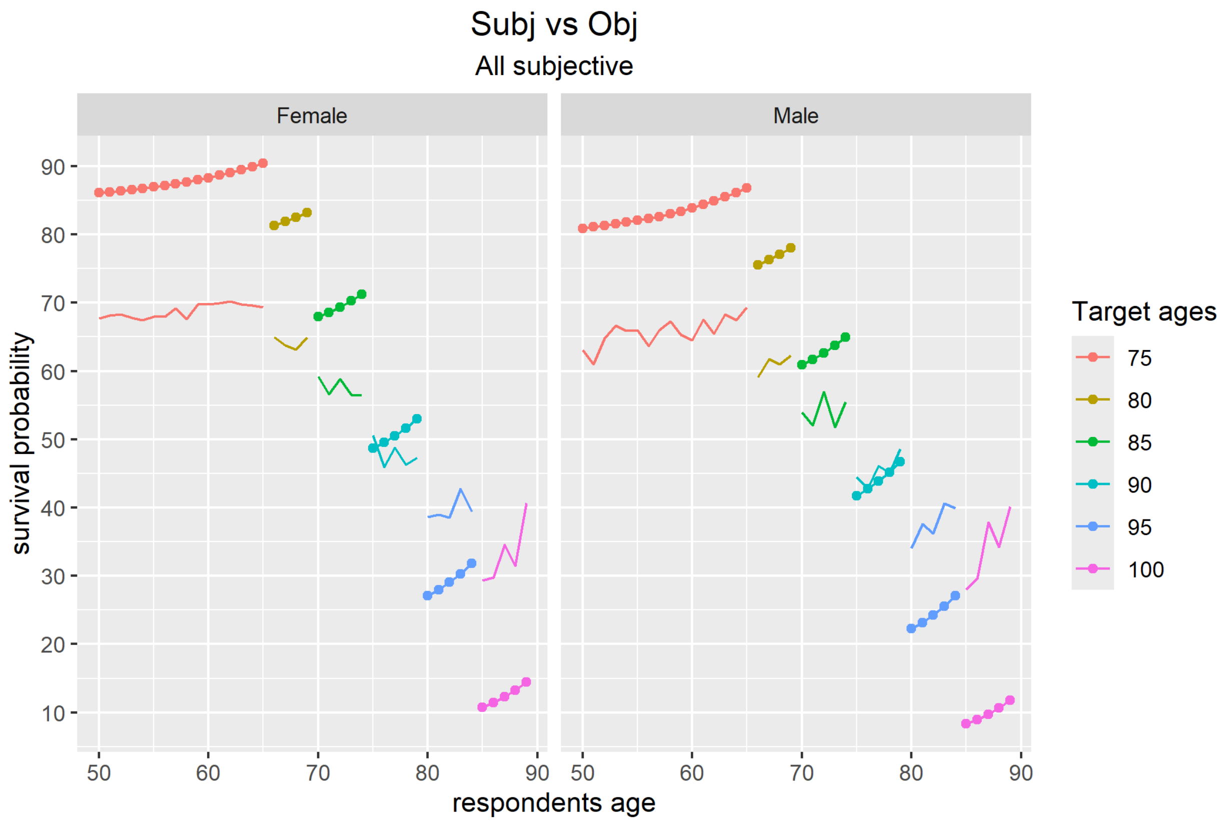


3 Survival Beliefs and Participation

We start by looking at the impact of survival beliefs on stock market participation decisions. We focus on underestimation of subjective survival as individuals on average underestimate their survival chances (at least up to age 80), see Figure 2. These findings are in line with the survival expectations literature (see Heimer, Myrseth, and Schoenle (2019) among others), where the young underestimate and the elderly overestimate their survival chances.

shares ISA, innovative finance ISA (IFISA) and lifetime ISA.

Figure 2: Subjective and objective survival probabilities by target age and gender. ELSA waves 6, 7 and 8 and ONS life tables 2012, 2014 and 2016.



We exploit data from the English Longitudinal Study of Ageing (ELSA) for a sample of individuals aged 50 or above interviewed in wave 8 (in 2016) to estimate the following baseline regression:

$$participation = \alpha_0 + \alpha_1 discrepancy + \alpha_2 cognitive\ skills + \beta' X + e \tag{1}$$

Our main outcome of interest, *participation*, takes value 1 if the respondent holds risky assets. Wealth information is collected at the household level, therefore we select the financial respondents of each household. As explained in Section 2, we do not include housing wealth in the analysis, therefore excluding also mortgages.

Our main regressor of interest is *discrepancy*, which takes value zero if the subjective survival probability is equal or above the life tables' probability for an individual with the same age and gender, and it takes positive values when the subjective probability is lower than the objective one. Higher values denote more inaccurate predictions (underestimation). We acknowledge that the respondent possesses more detailed information about their health and conditions compared to the researcher. However, the data consistently show a tendency to underestimate survival chances, even when controlling for a comprehensive set of individual characteristics collected in the survey. Therefore, we label our variable of interest as *discrepancy*. This variable is constructed according to the following equation:

$$accuracy = \begin{cases} \frac{subjective_i}{objective_i} & \text{if } \frac{subjective_i}{objective_i} < 1 \\ 1 & \text{if } \frac{subjective_i}{objective_i} \geq 1 \end{cases} \tag{2}$$

$$discrepancy = \frac{1 - accuracy}{\sigma_{accuracy}}$$

Table 2 reports estimation results. Column 1 shows that *discrepancy* has a negative and significant effect on *participation*, controlling for age, gender, family structure (having a partner and having children), income, health (limitations with ADL), education, and employment status. One standard deviation increase in discrepancy decreases participation by about 2.9 percentage points (average participation in the sample considered is 61%.).

This effect might be driven by potential confounding factors. In particular, Grevenbrock et al. (2021) identify cognitive skills and dispositional optimism as the main drivers of deviations of subjective survival probabilities from objective ones. Suppose low levels of cognitive skills imply a reduced ability to form expectations and express them in probability terms. In that case, we expect a larger discrepancy among those individuals with low cognitive abilities. Moreover, also individual general pessimistic attitudes might explain the underestimation of survival probability. We address these concerns by adding cognitive skills and dispositional optimism indicators among our regressors (see Columns 2 and 3 of Table 2). Our measure of cognitive skills is obtained by applying PCA to a set of indicators including memory tests and numeracy tests: larger values denote higher levels of cognition. Dispositional optimism is a binary indicator that takes value 1 if the respondent has optimistic attitudes towards life and 0 otherwise. We define *dispositional optimism* as a general optimistic attitude towards life and its events. We measure it following Steptoe and Wardle (2017) definition of optimism, based on two CASP-19 questions: "I feel that life is full of opportunities" and "I feel that the future looks good to me". After adding these controls, the effect of *discrepancy* slightly reduces in magnitude but remains significant. We also control for extreme or potentially uninformative values in self-reported probabilities, named *focal answers*. This variable takes the value 1

Table 2: Outcome: Participation (linear probability model).

	Dependent variable: Participation			
	(1)	(2)	(3)	(4)
discrepancy	-0.029***	-0.023***	-0.022**	-0.016*
	(0.009)	(0.009)	(0.009)	(0.009)
cognitive skills		0.032***	0.032***	0.030***
		(0.007)	(0.007)	(0.007)
fin. literacy		0.055***	0.055***	0.050***
		(0.008)	(0.008)	(0.008)
disposition optimism			0.010	0.005
			(0.018)	(0.018)
habits				*yes*
Controls	*yes*	*yes*	*yes*	*yes*
Observations	3,311	3,311	3,311	3,260

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
Additional controls: age, age squared, gender, having a partner, having children, income quartiles, economic status, number of ADLs, education, focal answers.

if self-reported survival is equal to 0, 50 or 100, following the definition of focal answers by Lillard and Willis (2001). The results show that these values do not drive the results of Table 2[3]. In column 4 of Table 2, we add control variables related to the respondent's lifestyle, such as smoking or doing regular sports activities. Controlling for such variables means including potential additional information about expected personal survival rates, unknown to the researcher.

The effects of *discrepancy* on stock market participation are consistently negative and significant, even if reduced in magnitude when adding controls.

To further investigate the role of cognitive level in the relationship between *discrepancy* and *participation*, we estimate equation 1 for high and low cognitive individuals separately. In particular, we focus on the model specification in Column 2 of Table 2. Table 3 shows estimation results. *Low cognitive* individuals are those with a level of cognition below the

[3]See Table A.1 in the Appendix for the full set of parameter estimates, including *focal answers*. We also find that the probability of focal answers in subjective survival questions is not associated with a higher probability of focal answers in the other expectation questions of the ELSA questionnaire

first tercile of our cognition measure, *high cognitive* individuals are those with higher values instead[4].

Table 3: Outcome - Participation (linear probability model).

	Dependent variable:			
	Participation			
	low cognitive	*high cognitive*	*low cognitive*	*high cognitive*
	(1)	(2)	(3)	(4)
discrepancy	-0.013	-0.034***	-0.007	-0.026**
	(0.014)	(0.012)	(0.015)	(0.012)
habits			*yes*	*yes*
Controls	*yes*	*yes*	*yes*	*yes*
Observations	1,052	2,259	1,027	2,233

Note: *p<0.1; **p<0.05; ***p<0.01

Additional controls: age, age squared, gender, having a partner, having children, income quartiles, economic status, number of ADLs, education, focal answers.

The estimates show that the effects of *discrepancy* on stock market participation are mainly driven by *high cognitive* individuals, who are more likely to invest in risky assets.
We also test an additional model that controls for cognitive skills continuous variable in Table 3, even after splitting the sample by high and low cognitive. Cognitive abilities have a positive and significant effect on participation in both groups, with a slightly larger magnitude among *low cognitive* individuals.

4 Survival Beliefs and Cognitive Skills

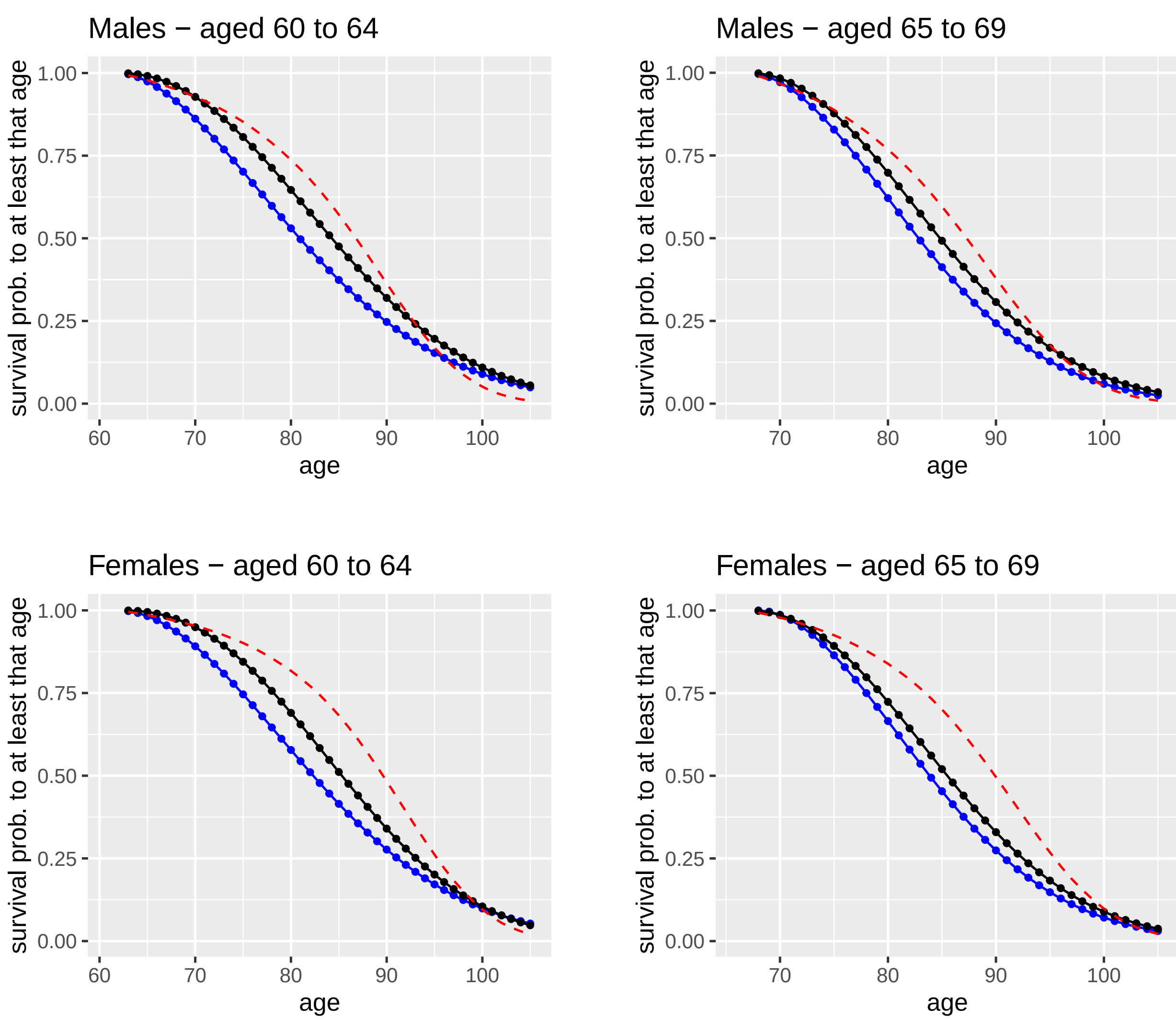
The estimates discussed in Section 3 suggest that underestimation of survival probabilities affects *participation* directly, reducing the probability of holding risky assets. However, we recognize that part of the effect is likely to be driven by heterogeneity in cognitive abilities, and potentially on individuals' attitudes towards life. In this section, we focus on these two

[4] To classify high and low cognitive individuals we rely on their position in the distribution of cognition by age-windows (50 to 55, 56 to 60 and so on) at the time of their first participation in ELSA. Thus, high and low cognition classification does not depend on current respondents' age.

main channels that might contribute to the formation of survival probabilities and show that cognitive skills is the more relevant channel in our context.

Figure 3 documents that individuals with low cognitive skills (blue curves) have subjective survival beliefs that are systematically more distant from the objective ones (red curves) with respect to high cognitive skills individuals (black curves). Figure 3 displays subjective survival curves constructed using self-reported probabilities of surviving to a set of given target ages, and assuming individual survival beliefs follow a Weibull distribution as in O'Dea and Sturrock (2021) (see Appendix B for details).

Figure 3: Survival curves: life tables (red), low cognitive skills (blue), high cognitive skills (black).



The correlation between cognitive skills and subjective survival probability, and in particular our measure of *discrepancy*, is statistically significant even after controlling for many potential confounding factors or removing the observations for which *focal answers* is equal to one, see Table 4. Column 1 presents OLS estimates controlling for a large set of individual characteristics, such as age, gender, having a partner, having children, income, economic status, health and education. In Column 2, we additionally control *habits* such as smoking behaviour or sports activities. The results show that cognition significantly reduces the underestimation of subjective survival expectations, while financial literacy does not play any role.

Table 4: Outcome - discrepancy of subjective survival expectations (OLS).

	Dependent variable:	
	discrepancy	
	baseline	habits
	(1)	(2)
cognitive skills	$-0.071***$	$-0.064***$
	(0.015)	(0.016)
fin. literacy	0.002	0.015
	(0.018)	(0.018)
dispositional optimism	$-0.325***$	$-0.298***$
	(0.033)	(0.033)
Controls	*yes*	*yes*
Observations	3,311	3,260

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$
Additional controls: age, age squared, gender, having a partner, having children, income quartiles, employment status, number of ADLs, education, focal answers.

Focusing only on the cognitive skills channel might be problematic if dispositional optimism is correlated with cognitive skills. Prima facie evidence suggests it might be the case: Figure 4 shows that individuals with high dispositional optimism scores are likely to have slightly higher cognitive skills. However, this correlation is not statistically significant once we control for the usual set of demographics, especially education, as shown in Table 5.

Figure 4: Distribution of cognitive skills for dispositional optimism equal to 0 or 1, by gender.

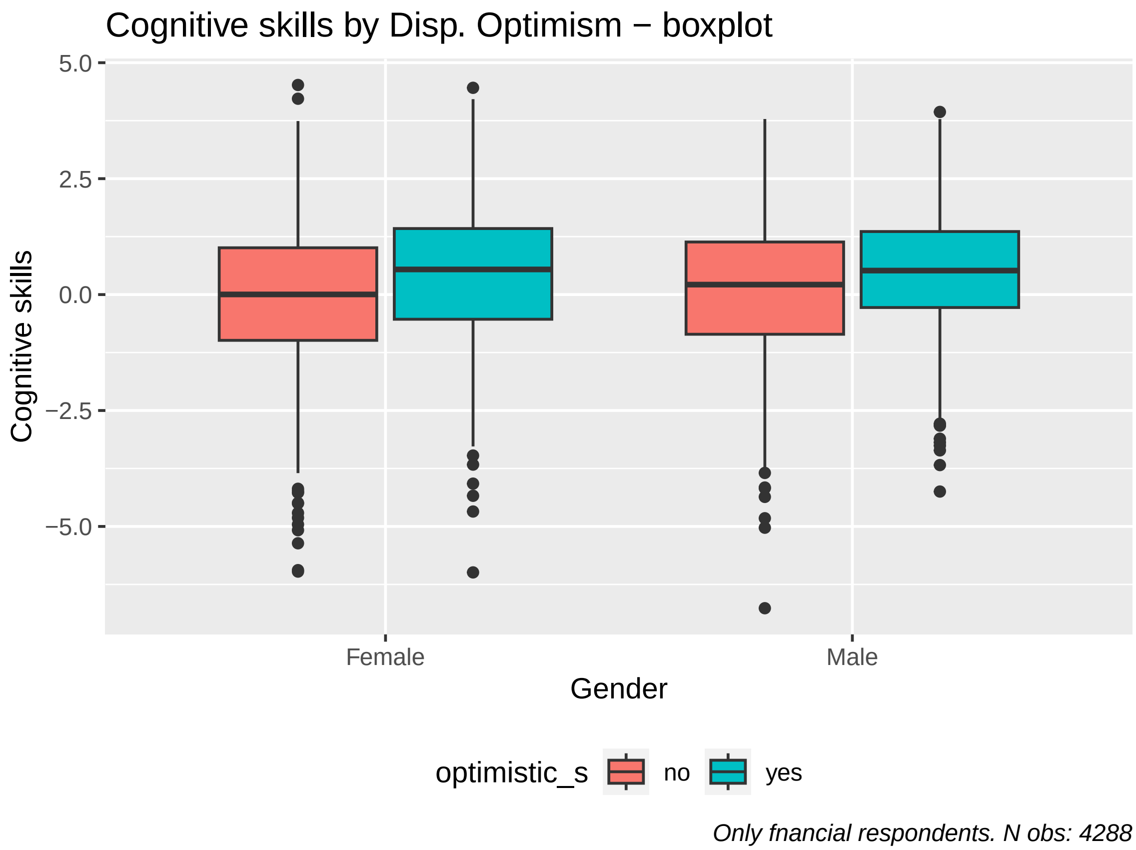


Table 5: Outcome - Cognitive skills (OLS) vs Individual dispositional Optimism and demographics.

	Dependent variable: cognitive skills
dispositional optimism	-0.068
	(0.049)
Controls	*yes*
Observations	3,311

5 The Model

To better understand the mechanisms through which subjective survival beliefs affect investment choices and to uncover how they interact with public policies aimed at increasing cognitive skills in terms of financial literacy, we build and calibrate an heterogeneous agent life cycle model that incorporates all the relevant elements highlighted in the empirical analysis. In particular, our framework features two types of assets – risky and non risky – and two types of agents – high cognitive and low cognitive skills – as well as subjective survival probabilities, financial market participation costs, uncertainty in pre-retirement income, and borrowing and short sale constraints.

5.1 Household Problem

In each period t, with $t = 1, \ldots, T$, households maximize their expected lifetime utility of the form:

$$\max_{c_t, f_t} U(c_t) + E_t \left[\sum_{j=t+1}^{T+1} \beta^j \Pi^s(j-1, t)(\pi_j^s U(c_j) + (1 - \pi_j^s) b(w_j)) \right] \tag{3}$$

by choosing non-durable consumption, c_t, and the fraction invested in risky assets, f_t. They receive utility (U_t) from consumption and, when they die, they value bequest of wealth w_t according to a bequest function $b(w_t)$.

Let β be the discount factor, $\Pi^s(j,t)$ be the probability of living to age j conditional on being alive at age t and π_j^s be the probability of being alive at time t conditional on being alive at time $t-1$. The per-period utility is

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relative risk aversion.

The bequest function is specified as

$$b(w_t) = \phi_B \frac{(w_t + \phi_K)^{(1-\gamma)}}{1-\gamma}.$$

The parameter ϕ_K, which is positive, regulates the curvature of the bequest function and allows the utility of a zero bequest to be finite. The parameter ϕ_B represents the intensity of bequest motives.

The households' maximization problem is solved under the following budget constraint

$$s_t + a_t = (1 + r_t^s)s_{t-1} + (1 + r)a_{t-1} - c_t - k(I_t, I_{t-1}) + y_t \tag{4}$$

where s_t and a_t are the amounts of wealth invested in risky and risk-free asset respectively, with returns r_t^s (modelled as an IID process with excess return μ_s and variance σ_s^2) and r. $k(.)$ are participation costs, outlined below, and y_{t+1} is the income realization in period $t + 1$. Income is stochastic up to age 65 and it follows an AR(1) process with persistence ρ and variance on the innovation σ_y^2. After age 65 households start to receive pension income which is modeled as a fraction of the last income realization.

Participation costs are defined following Cocco (2005) and Alan (2012) as

$$k(I_t, I_{t-1}) = \begin{cases} 0 & \text{if } I_t = 0 \\ k^f & \text{if } I_t = 1 \text{ and } I_{t-1} = 0 \\ k^p & \text{if } I_t = 1 \text{ and } I_{t-1} = 1 \end{cases} \tag{5}$$

where $I_t = (s_t > 0)$ is an indicator function for investments in risky assets, k^f is an entry cost and k^p is a per-period participation cost.

Wealth at the beginning of period t is

$$w_t = (1 + r_t^s)s_{t-1} + (1 + r)a_{t-1}. \tag{6}$$

Households additionally face borrowing and short-sale constraints, $a_t \geq 0$ and $s_t \geq 0$.

Timing of the model At the beginning of the period, the household head observes the realization of the return of the risky investment r_t^s and the income realization y_t. Given the faction invested in risky assets in period $t-1$, f_{t-1}, she/he has an initial wealth of w_t. Having this information she/he decides how much to consume c_t and the fraction of wealth to invest in risky assets f_t.

Model solution The model is solved backwards. The continuous state variables (w_t, y_t) are discretized on a grid. Additionally, we have two states for the ownership of risky assets in the previous period (I_{t-1}). The value function is evaluated at each point of the state space and we take expectations with respect to shocks on income by converting the persistent income component into a discrete Markov chain.

The model is solved for approximately 600 individuals (i) and using individual specific subjective survival curves (their derivation is reported in Appendix B).

Initial conditions The model reproduces the behaviour of a specific cohort of individuals, those born between 1956 and 1964. We select household's financial head (that can be male or female) from ELSA data (waves 6 and 7). For each individual solution i (that uses a specific subjective survival curve), we simulate 50 histories of mortality[5] and income shocks and initialize wealth (w_0) and risky assets ownership (I_0) at their value in the data for individual i.

Model calibration Households are ex ante heterogeneous depending on their level of cognitive skills (high or low) which we assume to be constant, in the absence of financial literacy interventions, over the part of the life cycle that we model. Households belonging to different cognitive skills types have different participation costs and income processes.[6]

The model is calibrated separately for high- and low-cognitive individuals. To calibrate the parameters we target total wealth and participation in financial market. Model fit is reported in the next section. The calibrated parameters are reported in Table 6 for high and low cognitive types respectively. Participation costs are expressed as a fraction of total wealth.

To calibrate the bequest parameters ϕ_B and ϕ_k, we compute the marginal propensity to bequeath out of an extra pound and the consumption value of wealth at which the bequest motive becomes operative. We do it for a household head who starts period t with wealth x and dies next period with probability one.

The two bequest parameters are calibrated in such a way that the marginal propensity to bequeath is equal to 0.975 and the bequest motive becomes operative when x is above £10,000 for both low and high cognitive types.

[5]The mortality probabilities faced by individuals when simulating from the model are consistent with the life tables, but re-weighted to account for heterogeneity between low and high cognitive individuals. See Appendix B.4 for details on the computation of life table survival probabilities conditional on cognitive level.

[6]We define the cognitive level of the individual i by using cognitive tests at age 50 to 55. There are two levels of cognition: "low" if the individual is in the first tertile of the cognitive score distribution, and "high" if he is in the second or third tertile.

		Low cognitive	High cognitive
β	discount factor	0.975	0.975
γ	relative risk aversion	3	3
r	return of risk-free asset	0.02	0.02
μ^s	return of risky asset (mean)	0.04	0.04
σ_s	return of risky asset (sd)	0.2	0.2
k^f	entry costs	0.05	0.04
k^p	per-period costs	0.03	0.01

Table 6: Calibrated parameters.

5.2 Fit of the Model to the Data

Our calibrated model performs well in reproducing the empirical profiles of total wealth and participation to the stock market. In particular, Figure 5 shows that the simulated average total wealth fits the corresponding measure observed in the data over the life-cycle for high and low cognitive households aged 50 and above, with the exception of low cognitive households aged 61-70 for whom the model tends to over predict total wealth with respect to the data. Figure 6 shows how the model is in general successful in reproducing the decreasing pattern of participation over the life-cycle for both high and low cognitive individuals, but it tends to understate the rates of participation for high cognitive individuals aged 66 to 80.

Figure 5: Total Wealth

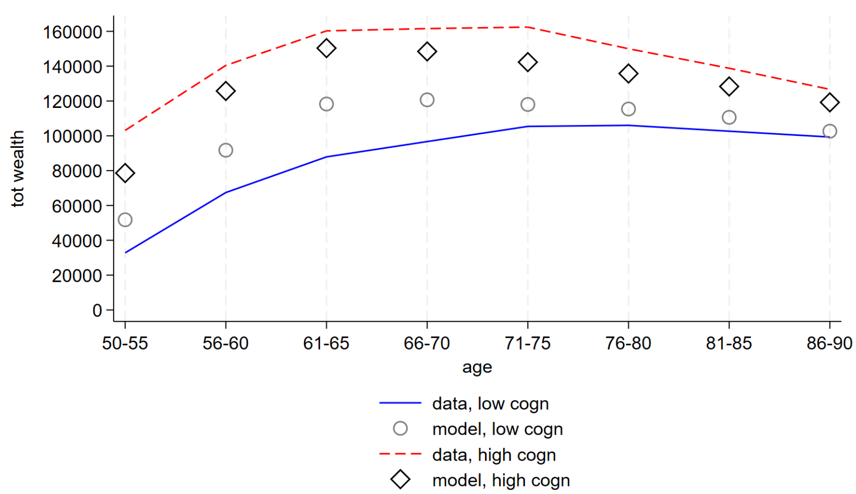
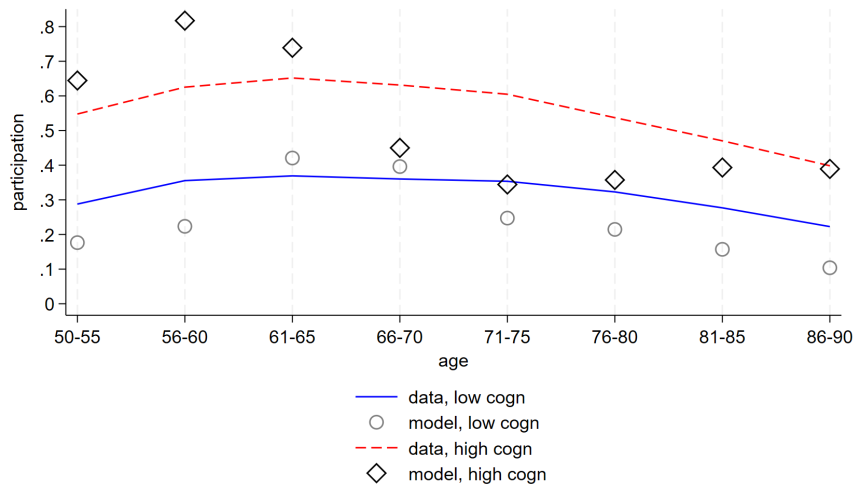


Figure 6: Participation



6 Counterfactual Experiments

We use our calibrated model to perform a set of counterfactual experiments. More specifically, we simulate our model under three alternative policy scenarios: i) a survival literacy intervention that informs low cognitive individuals about their actual survival rates from the life tables so to close the gap between subjective survival expectations and objective survival probabilities[7]; ii) a financial literacy intervention that provides information about the functioning of the financial market to low cognitive individuals and therefore lowers their participation (entry and per period) costs to the same level of those faced by high cognitive individuals; iii) a combination of the previous two interventions. We then study the effect of each policy on households' saving and investment choices with respect to the baseline scenario.

Figure 7 shows that both survival literacy and financial literacy interventions have positive effects on total wealth accumulation over the life-cycle. When low cognitive households revise their survival expectation upwards and/or face lower participation costs, they save and invest more over the course of their lives, thus ending up with a larger stock of accumulated total wealth. In particular, the survival literacy intervention has a positive effect on wealth accumulation in and of itself – although it is roughly half of that of the financial literacy

[7] 97% of low cognitive individuals in our sample underestimate their actual survival probabilities with respect to the life tables.

intervention – and the effect of the third combined policy is stronger than the sum of the effects of the first two policies.

Figure 8 shows the rate of those who were not participating to the stock market in the baseline scenario and become participants in the counterfactual scenario (Entry). The financial literacy intervention leads up to 8% of non participants to become participants and has a relevant but decreasing impact over the life-cycle. The survival literacy intervention has the strongest impact on individuals aged 61-70 who were non participants, leading more than 3% of them to become participants when their survival expectations are revised upwards. Lastly, Figure 9 shows that, conditional on being participants in both baseline and counterfactual scenarios, neither a survival literacy intervention nor a financial literacy intervention has a relevant effect on the share invested in risky assets.

Figure 7: Total Wealth

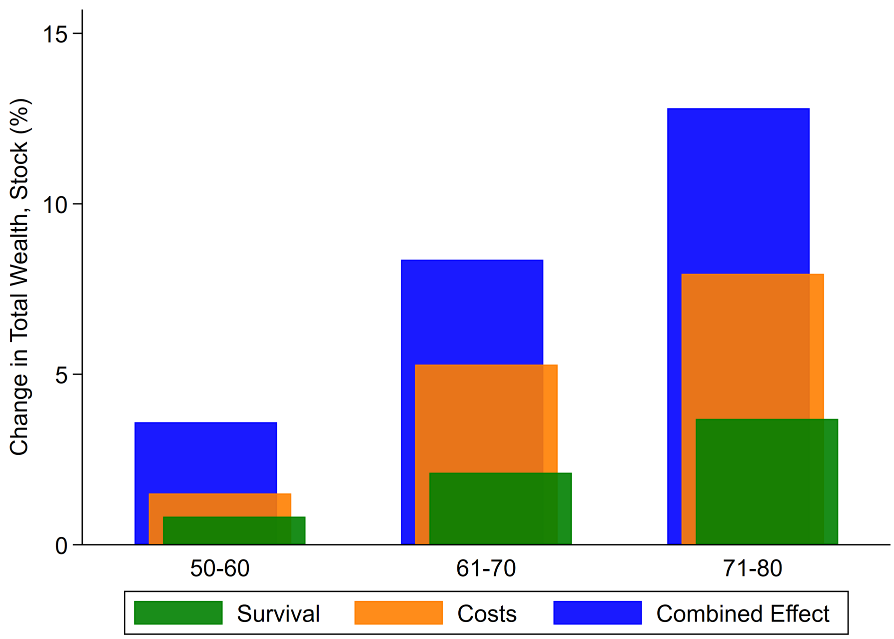


Figure 8: Entry

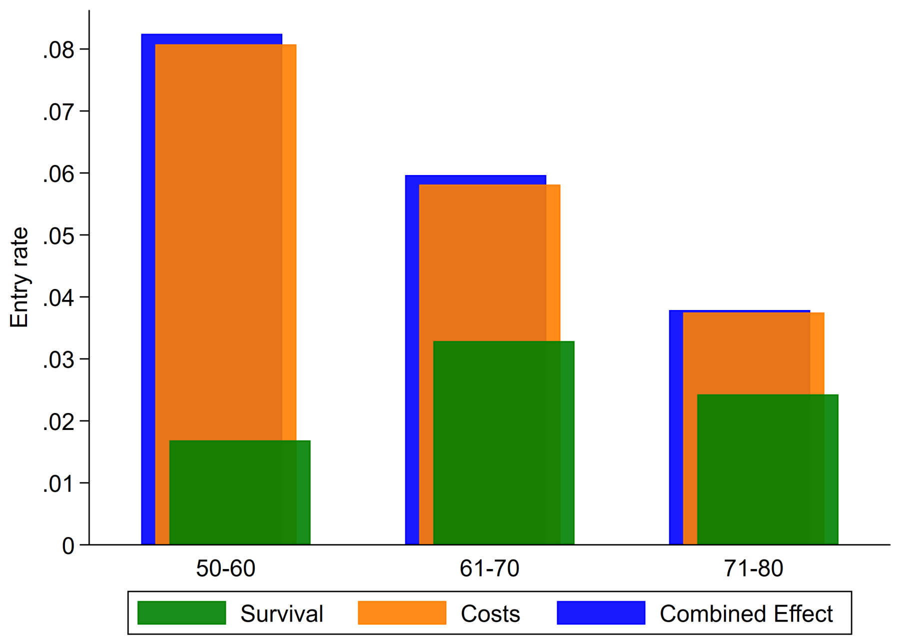
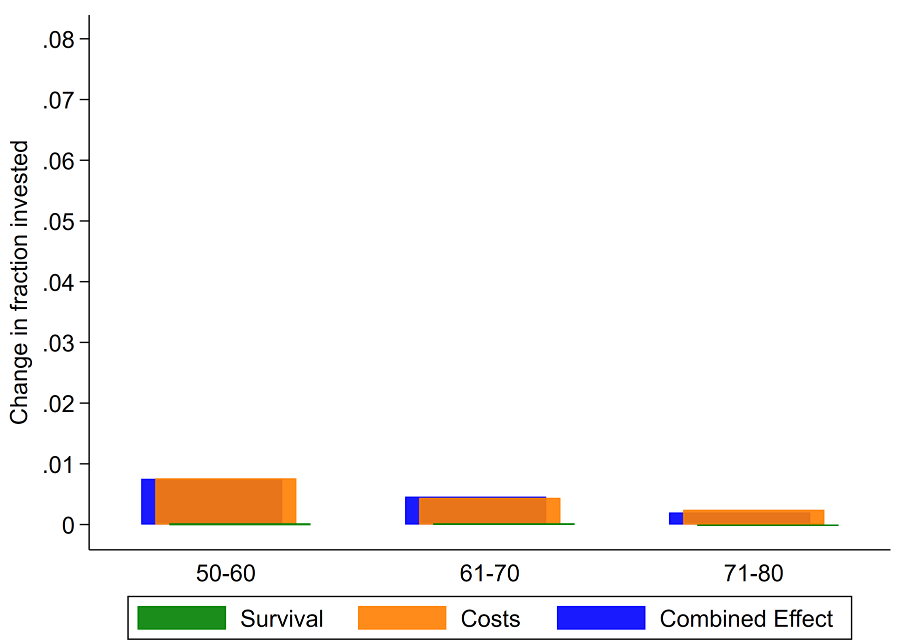


Figure 9: Fraction invested in risky assets



6.1 Heterogeneity

We then look at what subgroups of households drive the aggregate effects on entry rate presented above. Figures 10 and 11 suggest that households whose financial respondent is a man and those who have higher initial wealth tend to respond more strongly to both survival literacy and financial literacy interventions. These are the least financially constrained households who can afford to enter the stock market when they face higher survival chances and/or lower participation costs.

Figure 10: Entry by gender

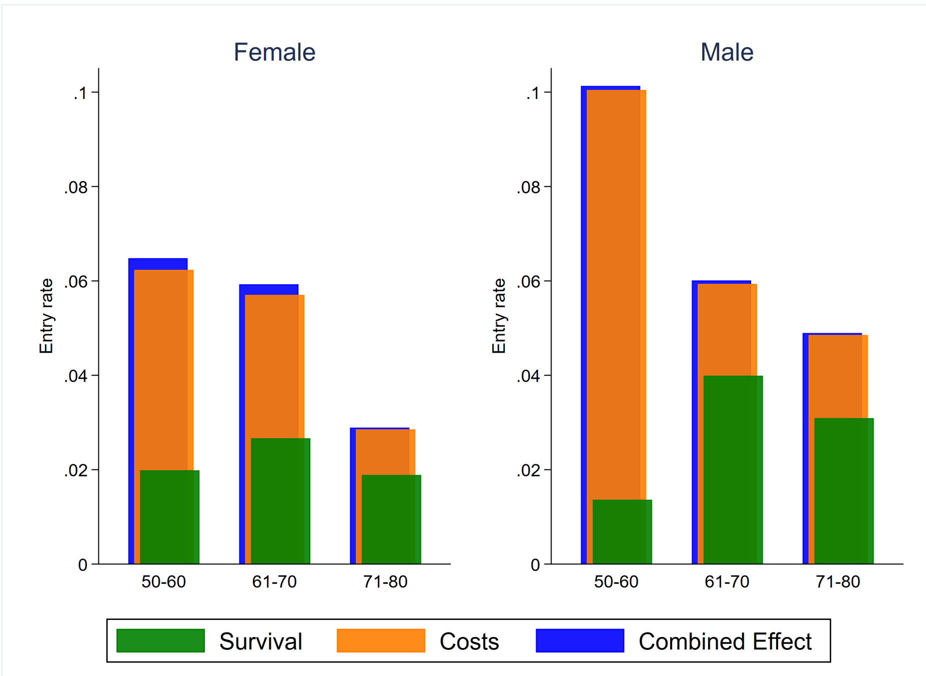
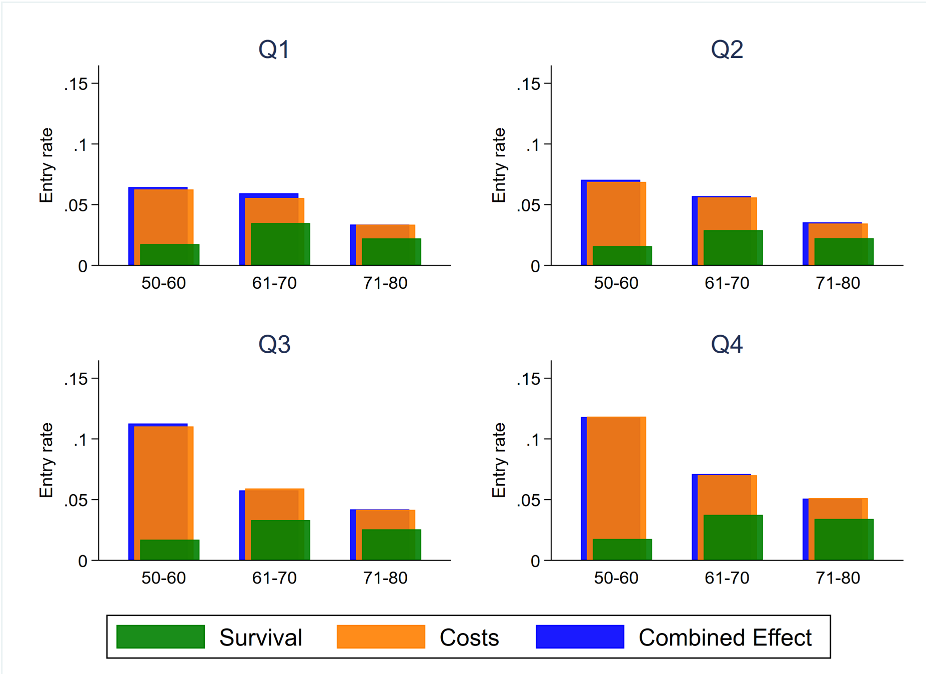


Figure 11: Entry by initial wealth



6.2 Welfare Analysis with Subjective Survival Expectations

To compare the welfare of individuals under alternative scenarios of life expectancy, we have to modify the utility function of the model adding a positive shift, $\bar{b}$, so to make sure that individuals value their life, meaning that the expected value of being alive one additional period is greater than that of being dead (Hall and Jones (2007)).

Following De Nardi, Pashchenko, and Porapakkarm (2024), we calibrate the value of $\bar{b}$ so to set the statistical value of life (SVL) equal to 900,000£(in 2011). Where, the SVL represents the monetary value corresponding to the reduction in mortality risk that would prevent one statistical death and it is defined as the marginal rate of substitution between wealth and survival probability.

Then, we express welfare changes between a counterfactual scenario and our baseline scenario in terms of consumption equivalent variation (CEV), that is the proportion of consumption, π, that an individual is willing to pay to be indifferent between the counterfactual scenario (denoted with 2) and the baseline scenario (denoted with 1). In the context of our experiments, survival expectations in the counterfactual scenario, s_2, may differ from survival expectations in the baseline scenario, s_1, even if the actual number of periods lived by any individual is the same in both scenarios (experiments i) and iii)). Hence, we generalize the usual cev formula (Low, Meghir, and Pistaferri (2010)) by augmenting it with an additional term capturing this difference in expected survival expectations:

$$\pi = 1 - \left[\frac{E_0 U_1}{E_0 U_2|_{\pi=0}} + \bar{b} \frac{\left[\sum_t \beta^t s_{1,t} - \sum_t \beta^t s_{2,t}\right]}{E_0 U_2|_{\pi=0}} \right]^{\frac{1}{1-\gamma}} \tag{7}$$

Note that the formula above boils down to the usual $\pi = 1 - \left[\frac{E_0 U_1}{E_0 U_2|_{\pi=0}}\right]^{\frac{1}{1-\gamma}}$ when the baseline and counterfactual scenarios do not differ in terms of survival expectations (experiment ii)).

Figure 12 reports welfare implications of the survival literacy intervention – experiment i) –in terms of CEV by deciles of difference between subjective survival expectations and objective survival rates (life tables). If this difference is negative, the individual subjective survival expectations were below the life table survivals, if it is positive they were above. Most low cognitive individuals, who receive the survival literacy intervention, rate their survival far below the life table curves, so their survival expectations are revised upwards as a result of the treatment. As a consequence, they experience a large welfare gain (up to 7.8% of per period consumption) both because they value a longer life in itself and because this leads them to save and invest more when they realize they will live longer, as seen in the previous sections. Only individuals in the 10th decile, those who rate their survivals above the life table curves and whose survival expectations are revised downwards, experience a welfare loss with respect to the baseline.

Figure 13 presents the welfare effects of the financial literacy intervention – experiment ii) – in terms of CEV by deciles of initial wealth. All households gain from the financial literacy policy intervention and those who gain more – up to 0.27% of per period consumption – are the ones at the top of the initial wealth distribution. These are the least liquidity constrained households who have sufficient wealth to be able to enter the stock market when their participation costs become lower as a result of the policy.

Figure 12: Survival Literacy Intervention

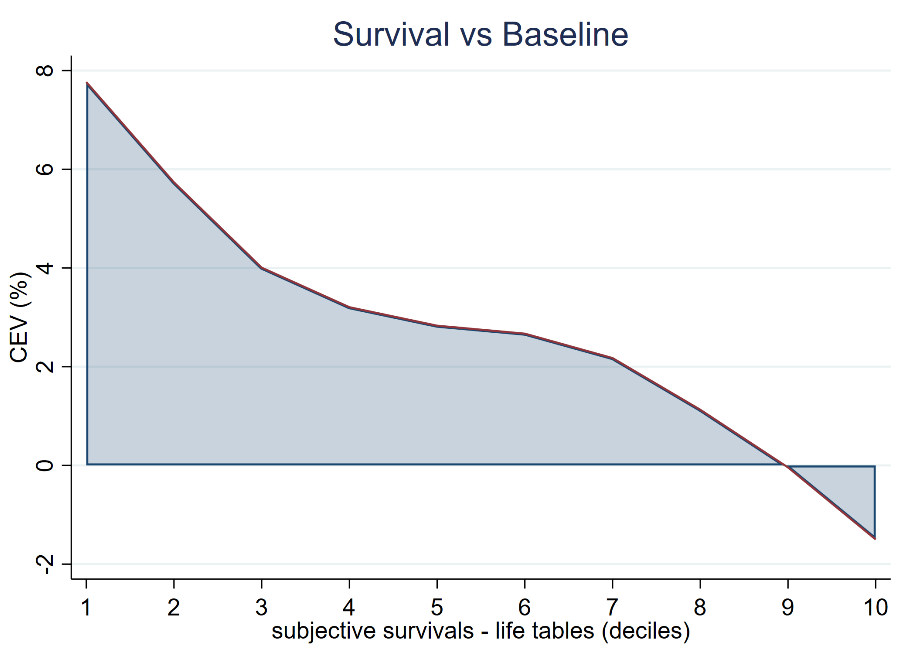
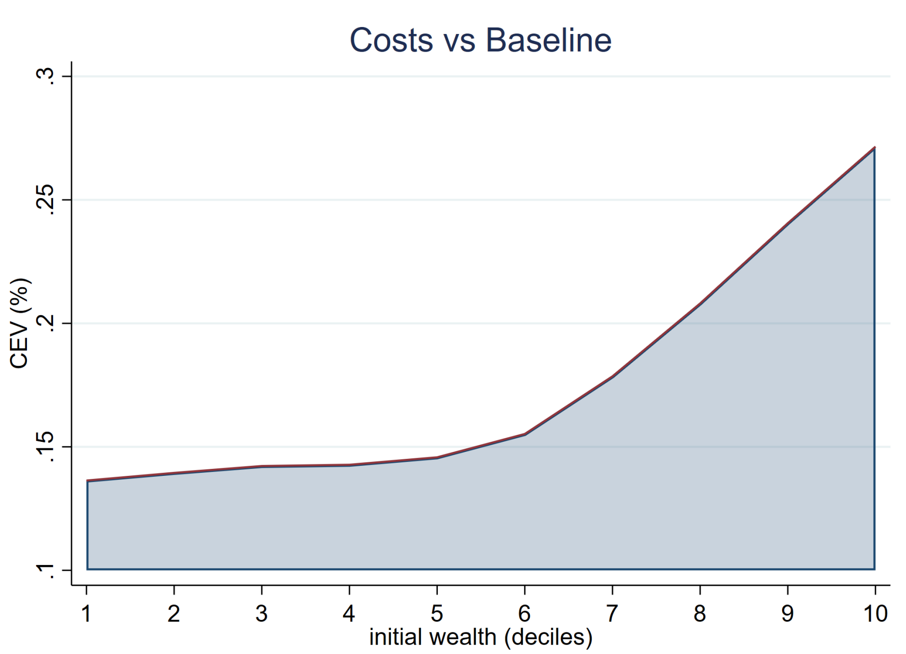


Figure 13: Financial Literacy Intervention



7 Conclusions

In this paper, we exploit panel data from the English Longitudinal Study of Aging (ELSA) and we empirically show that the discrepancy between subjective survival beliefs and objective survival rates has a significant role in explaining the decision to participate in the stock market. We further document that this discrepancy is strongly correlated with financial literacy as captured by cognitive skills.

We then set up a theoretical framework to investigate the role of the interaction between financial literacy and survival literacy in determining stock market participation. We calibrate a life-cycle model of saving decisions with risky and risk-free assets and high and low cognitive skills agents, who make consumption and investment decisions taking into account their subjective survival expectations.

We simulate our model under three alternative policy scenarios: a survival literacy intervention, a financial literacy intervention, and a combination of the two. We find that the survival literacy intervention has a positive effect on total wealth accumulation in and of itself, although smaller in magnitude than that of the financial literacy intervention. The financial literacy intervention, by lowering participation costs, is more effective than the survival literacy intervention in boosting entry in the financial market.

Both the survival literacy and the financial literacy policies are welfare improving overall. However, the financial literacy intervention results in smaller welfare gains than the survival literacy policy and it benefits more households at the top of the initial wealth distribution.

Our quantitative results suggest that survival literacy interventions are an important tool in the hands of policy makers who aim at attenuating the longevity risk by encouraging wealth accumulation. Financial literacy policies, instead, are the most effective type of intervention when the goal is to incentivize households to diversify their portfolio and invest in risky assets, but they tend to benefit wealthy households more.

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A Appendix

Figure A.1: Corresponds to Figure 2 in the main text.

Figure A.2: Corresponds to Figure 2 in the main text, excluding individuals who report a subjective survival chance equal to 50.

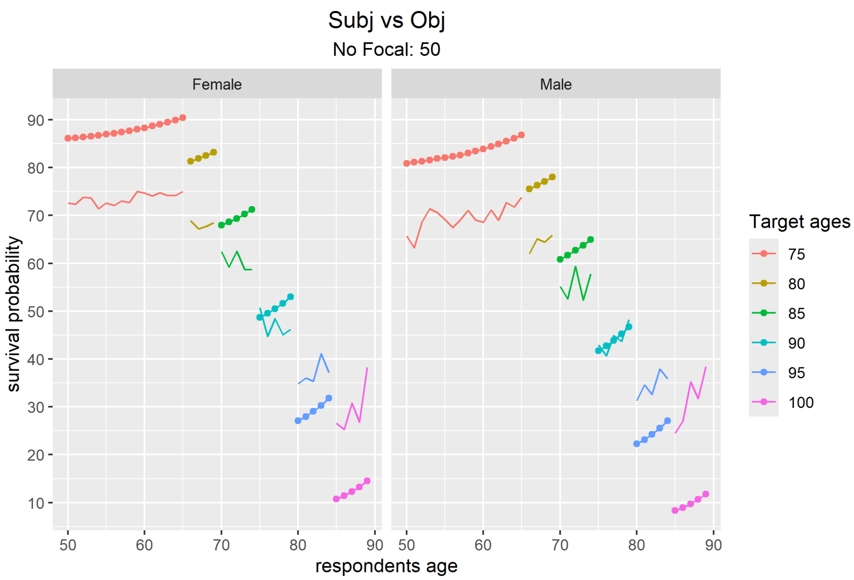

Figure A.3: Corresponds to Figure 2 in the main text, excluding individuals who report a subjective survival chance equal to 50, 0 and 100.

Table A.1: full parameter estimates of Table 2.
Outcome: Participation (linear probability model). Additional controls: age, age squared, gender, having a partner, having children, income quantiles, economic status, number of ADLs, education, focal answers. Habits (Col. (4)): smoker (dummy), mild and intense sport (more than once a week, at least once a week respectively), healthy eating (at least 4 portions of fruit/vegetables per day).

	Dependent variable:			
	Participation			
	(1)	(2)	(3)	(4)
discrepancy	$-0.029***$	$-0.023***$	$-0.022**$	$-0.016*$
	(0.009)	(0.009)	(0.009)	(0.009)
underestimation (dummy)	-0.019	-0.027	-0.026	-0.028
	(0.018)	(0.018)	(0.018)	(0.018)
cognitive skills		$0.032***$	$0.032***$	$0.030***$
		(0.007)	(0.007)	(0.007)
fin. literacy		$0.055***$	$0.055***$	$0.050***$
		(0.008)	(0.008)	(0.008)

dispositional optimism			0.010	0.005
			(0.018)	(0.018)
focal answers	0.012	0.021	0.021	0.022
	(0.017)	(0.017)	(0.017)	(0.017)
age	0.016	0.015	0.015	0.016
	(0.014)	(0.013)	(0.013)	(0.014)
age^2	-0.000	-0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)
male	$0.056***$	$0.038**$	$0.038**$	$0.044**$
	(0.017)	(0.018)	(0.018)	(0.018)
adl	$0.090***$	$0.072***$	$0.071***$	$0.067***$
	(0.020)	(0.020)	(0.020)	(0.020)
partner	$0.067***$	$0.067***$	$0.067***$	$0.064***$
	(0.022)	(0.021)	(0.021)	(0.022)
child	$-0.100***$	$-0.099***$	$-0.099***$	$-0.099***$
	(0.021)	(0.021)	(0.021)	(0.021)
income qrt 2^{nd}	$0.062***$	$0.054**$	$0.053**$	$0.046*$
	(0.024)	(0.024)	(0.024)	(0.024)
income qrt 3^{rd}	$0.168***$	$0.145***$	$0.145***$	$0.138***$
	(0.027)	(0.026)	(0.026)	(0.026)
income qrt 4^{th}	$0.222***$	$0.191***$	$0.190***$	$0.178***$
	(0.029)	(0.028)	(0.028)	(0.028)
education: mid	$0.198***$	$0.148***$	$0.148***$	$0.144***$
	(0.019)	(0.020)	(0.020)	(0.020)
education: high	$0.259***$	$0.188***$	$0.187***$	$0.182***$
	(0.022)	(0.023)	(0.023)	(0.023)
Job (excluded: Employee)				
Seeking work	0.037	0.028	0.028	0.028
	(0.034)	(0.034)	(0.034)	(0.034)
Self-Employed	$-0.311***$	$-0.293***$	$-0.293***$	$-0.279***$
	(0.051)	(0.054)	(0.054)	(0.055)
Sick and not seeking	$-0.201***$	$-0.170***$	$-0.169***$	$-0.153***$
	(0.039)	(0.039)	(0.039)	(0.040)
Retired	$0.068***$	$0.061**$	$0.061**$	$0.060**$

	(0.025)	(0.024)	(0.024)	(0.024)
Unoccupied	−0.005	0.003	0.003	−0.004
	(0.052)	(0.049)	(0.049)	(0.049)
smoker				−0.094∗∗∗
				(0.027)
mild sport				0.001
				(0.025)
intense sport				0.025
				(0.017)
healthy eating				0.031∗
				(0.016)
Constant	−0.345	−0.509	−0.505	−0.529
	(0.466)	(0.460)	(0.460)	(0.469)
Observations	3,311	3,311	3,311	3,260

Note:*p<0.1; **p<0.05; ***p<0.01

Table A.2: full parameter estimates of Table 3.
Outcome: Participation (linear probability model). Additional controls: age, age squared, gender, having a partner, having children, income quantiles, economic status, number of ADLs, education. Habits (Col. (4)): smoker (dummy), mild and intense sport (more than once a week, at least once a week respectively), healthy eating (at least 4 portions of fruit/vegetables per day).

	Dependent variable:			
	Participation			
Cognition:	*low*	*high*	*low*	*high*
	(1)	(2)	(3)	(4)
discrepancy	−0.013	−0.034∗∗∗	−0.007	−0.026∗∗
	(0.014)	(0.012)	(0.015)	(0.012)
underestimation (dummy)	−0.024	−0.022	−0.031	−0.023
	(0.033)	(0.022)	(0.033)	(0.023)
dispositional optimism	0.018	0.006	0.017	−0.001

	(0.035)	(0.021)	(0.035)	(0.021)
focal answers	0.034	−0.005	0.040	−0.005
	(0.027)	(0.018)	(0.028)	(0.018)
smoker			−0.069	−0.076***
			(0.042)	(0.029)
mild sport			0.034	−0.038
			(0.039)	(0.030)
intense sport			0.042	0.042**
			(0.034)	(0.017)
healthy eating			0.031	0.032**
			(0.028)	(0.016)
age	0.040*	0.020	0.035	0.020
	(0.024)	(0.014)	(0.025)	(0.014)
age^2	−0.000	−0.000	−0.000	−0.000
	(0.000)	(0.000)	(0.000)	(0.000)
male	0.062**	0.037**	0.068**	0.037**
	(0.028)	(0.017)	(0.029)	(0.017)
adl	−0.012*	−0.025***	−0.007	−0.025***
	(0.007)	(0.007)	(0.007)	(0.007)
partner	0.021	0.075***	0.030	0.072***
	(0.034)	(0.021)	(0.035)	(0.021)
child	−0.103**	−0.098***	−0.101**	−0.099***
	(0.041)	(0.022)	(0.042)	(0.022)
2nd income quartile	0.156***	0.075***	0.163***	0.073***
	(0.035)	(0.024)	(0.036)	(0.024)
3rd income quartile	0.207***	0.138***	0.199***	0.138***
	(0.043)	(0.025)	(0.044)	(0.025)
4th income quartile	0.263***	0.210***	0.236***	0.203***
	(0.056)	(0.028)	(0.057)	(0.028)
mid education	0.171***	0.115***	0.166***	0.113***
	(0.030)	(0.019)	(0.031)	(0.019)
high education	0.265***	0.157***	0.264***	0.149***
	(0.049)	(0.022)	(0.050)	(0.023)
Job: excluded Employee				

Seeking work	0.064	0.012	0.073	0.007
	(0.078)	(0.031)	(0.080)	(0.031)
Self-Employed	−0.178	−0.267***	−0.187	−0.245**
	(0.134)	(0.097)	(0.134)	(0.097)
Sick and not seeking	−0.174**	−0.130**	−0.167**	−0.109*
	(0.078)	(0.063)	(0.080)	(0.063)
Retired	0.021	0.069***	0.017	0.068***
	(0.052)	(0.024)	(0.053)	(0.024)
Unoccupied	−0.021	0.065	−0.041	0.070
	(0.080)	(0.051)	(0.082)	(0.051)
Constant	−1.213	−0.388	−1.094	−0.349
	(0.858)	(0.480)	(0.884)	(0.481)
Observations	1,052	2,259	1,027	2,233

Note:* p<0.1; ** p<0.05; *** p<0.01

Table A.3: Outcome - Inaccuracy of expectations (OLS).

	Dependent Variable			
	discrepancy			
	Baseline	*Habits*	*NoFocal Baseline*	*NoFocal Habits*
	(1)	(2)	(3)	(4)
cognitive skills	−0.071 * **	−0.064 * **	−0.076 * **	−0.066 * **
	(0.015)	(0.016)	(0.016)	(0.016)
fin. literacy	0.002	0.015	−0.026	−0.018
	(0.018)	(0.018)	(0.019)	(0.019)
dispositional optimism	−0.325 * **	−0.298 * **	−0.205 * **	−0.177 * **
	(0.033)	(0.033)	(0.040)	(0.040)
focal answers	0.320 * **	0.307 * **	0.436 * **	0.428 * **
	(0.038)	(0.038)	(0.039)	(0.039)
age	0.020	0.019	0.093***	0.094***
	(0.025)	(0.025)	(0.028)	(0.028)

age2	−0.000	−0.000	−0.001***	−0.001***
	(0.000)	(0.000)	(0.000)	(0.000)
male	−0.016	−0.021	0.019	0.019
	(0.031)	(0.031)	(0.034)	(0.034)
adl	0.081***	0.067***	0.062***	0.054***
	(0.010)	(0.011)	(0.012)	(0.013)
partner	−0.084**	−0.082**	−0.101**	−0.096**
	(0.038)	(0.038)	(0.042)	(0.042)
child	0.085**	0.087**	0.092**	0.089*
	(0.042)	(0.042)	(0.046)	(0.045)
2nd income quartile	0.002	0.009	−0.030	−0.024
	(0.042)	(0.042)	(0.047)	(0.047)
3rd income quartile	0.038	0.048	0.003	0.017
	(0.047)	(0.047)	(0.052)	(0.051)
4th income quartile	−0.029	−0.008	−0.076	−0.051
	(0.053)	(0.052)	(0.057)	(0.057)
mid education	−0.048	−0.034	−0.062	−0.045
	(0.034)	(0.034)	(0.038)	(0.038)
high education	−0.012	0.011	−0.028	−0.007
	(0.043)	(0.043)	(0.047)	(0.046)
Job (excluded: Employee)				
Seeking work	0.068	0.083	0.061	0.079
	(0.063)	(0.063)	(0.068)	(0.067)
Self-Employed	0.332**	0.298*	0.323*	0.269
	(0.168)	(0.167)	(0.188)	(0.187)
Sick and not seeking	0.260**	0.205**	0.332***	0.258**
	(0.102)	(0.102)	(0.112)	(0.112)
Retired	0.038	0.045	0.081	0.085*
	(0.047)	(0.047)	(0.050)	(0.050)
Unoccupied	0.065	0.062	0.087	0.072
	(0.092)	(0.091)	(0.101)	(0.100)
smoker		0.235***		0.280***
		(0.051)		(0.058)
mild_sport		−0.159***		−0.048

		(0.050)		(0.057)
inte_sport		-0.129^{***}		-0.133^{***}
		(0.032)		(0.035)
Constant	0.116	0.311	-2.260^{**}	-2.252^{**}
	(0.869)	(0.866)	(0.957)	(0.951)
Observations	3,311	3,311	2,165	2,165

Note:* p<0.1; ** p<0.05; *** p<0.01

B Appendix - Subjective Survival Curves

B.1 Life table: components

m_x**:** is known as the central rate of mortality. That is the average number of deaths each year at age x last birthday in the relevant three-year period, divided by the average population at that age over the same period.

q_x**:** is the mortality rate between age x and *(x+1)*. That is, the probability that a person aged x exactly will die before reaching age *(x+1)*.

l_x**:** is the number of males or females surviving to exact age x of 100,000 live births who are assumed to be subject throughout their lives to the mortality rates experienced in the specified three-year period.

d_x**:** is the number of males or females dying between exact age x and $x + 1$ described similarly to l_x, that is, $d_x = l_x - l_{x+1}$.

e_x**:** is the average period life expectancy at exact age x, which is the average number of further years that those aged x exactly will live based on the mortality rates experienced in the specified three-year period.

B.1.1 Number of survivors and probability of surviving

As stated above, l_x represents the number of people alive at exact age x. Generally, the l_x represents a hypothetical population and not a precise population estimate, therefore the l_0 (initial population) is an arbitrary number that the ONS sets to 100,000.

The l_x value is of particular interest for the purpose of this study because it can be used to calculate the survival probability from age x to age $x + n$ as follows:

$$s_{x,x+n} = \frac{survivors\ at\ age\ x+n}{survivors\ at\ age\ x} * 100 = \frac{l_{x+n}}{l_x} * 100$$

where $s_{x,x+n}$ is the expected survival probability of an agent of age x to age $x + n$, at a given year y. Note that I used period life-tables, where the l_x at year y represents the number of surviving at exact age x years in the specific year y, under the projected assumptions for mortality rates in year y for ages up to age x.

An example The following equation is used to calculate the probability of a female aged 40 years in 2018 surviving to age 75 years:

$$\frac{l_{75}}{l_{40}} * 100 = \frac{80,277}{98,595} * 100 = 81.4\%$$

That is, a female aged 40 years in 2018 has a 81.4% chance of surviving to age 75 years.

I will refer to the survival probability computed as described in the example as *objective* survival probability, and to the answers to the ELSA questions as *subjective* survival probability.

B.2 ELSA subjective survival questions

ELSA respondents answer the following - subjective survival believes - questions:

What are the chances that you will live to be ..X.. or more?

where X is a specific *target age* that depends on respondents current age as follow

Table B.1: Respondents age and Target age.

Age	Target Age (X)
≤ 65	75
66 - 69	80
70 - 74	85
75 - 79	90
80 - 84	95
85 - 90	100

if the respondent's age is less than 70, then a follow-up question is asked:

What are the chances that you will live to be 85 or more?

Therefore for each respondent, ELSA provides at least one subjective survival point. If the respondent is below 70, ELSA has two survival points.

We follow O'Dea and Sturrock (2021) to construct individual subjective survival curves assuming a Weibull distribution for subjective beliefs. We estimate individual Weibull parameters using Non-Linear Least Squares (NLS) and then we use them to draw individual survival curves (see 3). We then compute year-to-year expected survival probability and year-to-year mortality rates using the subjective survival curves.

B.3 Weibull distribution and parameters

The Weibull distribution has two parameters λ_i and k_i, the scale and the shape parameter, respectively. This distribution allows to compute the survival probability $S_i(\alpha)$ of an individual i of age z to the target age α as:

$$S(\alpha) = exp\Big[-\Big(\frac{\alpha - z}{\lambda_i}\Big)^{k_i}\Big] \quad : \quad \lambda_i, k_i > 0 \tag{1}$$

In other words, Equation 1 represents the probability of survival to at least age α of an agent i who is z years old.

To estimate the two Weibull parameters λ_i, k_i of each individual i, O'Dea and Sturrock (2021) make the weak additional assumption that individuals are almost certain not to live beyond age 110 years, assuming that the agents' survival probability at age 110 years is the one provided by ONS life tables[8]. This assumption implies that individuals aged 70 years or more have two survival points, while those aged 69 years or less have three survival points. This is because the former group answers to only one survival expectation question, while the latter answers to two survival expectation questions.

I follow O'Dea and Sturrock (2021), which estimate λ_i and k_i Weibull parameters with nonlinear least squares. In particular, the estimation procedure minimizes:

$$(\hat{\lambda}_i, \hat{k}_i) = arg\ min_{\lambda_i, k_i} \sum_{\alpha \in A_i} \Bigg(S_i^{subjective}(\alpha) - exp\Big[-\Big(\frac{\alpha - z}{\lambda_i}\Big)^{k_i}\Big]\Bigg)^2 \tag{2}$$

where $S_i^{subjective}(\alpha)$ is the subjective survival probability and $A_i = [75, 85, 110]$ or $A_i = [X, 110]$ is the vector composed by the three or two target ages (depending on the current age of the respondent, see Table B.1).

B.4 Year-To-Year subjective survival and mortality rates

Using l_x, $s_{x,x+t}$ (see Section B.1) and the Weibull distribution, we have (for $s_{50,51}$ and $s_{50,52}$):

$$\begin{cases} s_i(50, 51) = exp\Big[-\Big(\frac{51-50}{\lambda_i}\Big)^{k_i}\Big] & = \frac{l^i_{51}}{l^i_{50}} \\ s_i(50, 52) = exp\Big[-\Big(\frac{52-50}{\lambda_i}\Big)^{k_i}\Big] & = \frac{l^i_{52}}{l^i_{50}} \end{cases} \tag{3}$$

[8] $S_{i,110}^{objective} \in [0.001, 0.003]$.

conditioning on survivorship to age 50, we can fix $l_{50} = 100000$. Then:

$$\begin{cases} l^i_{51} \;=\; l^i_{50} \cdot s_i(50,51) \;=\; 100000 \cdot s_i(50,51) \\ l^i_{52} \;=\; l^i_{50} \cdot s_i(50,52) \;=\; 100000 \cdot s_i(50,52) \end{cases} \tag{4}$$

from which we obtain:

$$l^i_{51,52} \;=\; \frac{l^i_{52}}{l^i_{51}} \;=\; \frac{s_i(50,52)}{s_i(50,51)} \tag{5}$$

where $l^i_{51,52}$ represents individual subjective *year-to-year survival probability* between age 51 and age 52, given that today agent i is 50 years old.
We then compute individual specific *subjective year-to-year mortality* as $m^i_{x,x+1} = 1 - l^i_{x,x+1}$.

A Appendix - Life table survival probabilities by cognitive level

We use objective mortality rates conditional on the level of cognition (high or low) for both men and women. To derive them, we use ELSA data linked to administrative death records which allow to know the exact year of death of any individual (including attriters) up until February 2013 (this information is available in the public data release of ELSA).

Data are biennial but, given the linkage with administrative death records, we construct a dummy variable taking value one if the individual dies by next year and zero otherwise. This allows us to estimate the probability of dying by $t+1$ conditional on being alive in t, for each age and cognitive level in t. We do it estimating a fixed effect regression and controlling for a four-grade polynomial in age interacted with a dummy for having a low cognitive level. Details on the procedure used to derive mortality rates are provided below.

1. We estimate the probability of being of cognition level i ($\hat{Pr}(Cogn\_Skill_t = i)$) and of dying by $t+1$ conditional on cognition level i ($\hat{Pr}(death^D_{t+1}|Cogn\_Skill_t = i)$) using all observations. To control for cohort effects, we estimate these probabilities using fixed-effect regressions. When we predict from the estimated regressions, we set the fixed effect equal to the average fixed effect for those born between 1956 and 1964 (the cohort of interest);

2. the probability of dying by $t+1$ at each age t is given by:

$$\hat{Pr}(death^D_{t+1}) = \sum_{i=1}^{4} \hat{Pr}(Cogn\_Skill_t = i) * \hat{Pr}(death^D_{t+1}|Cogn\_Skill_t = i); \qquad (1)$$

3. We compare the estimated probability with the life tables for each age t:

$$\frac{\hat{Pr}(death^{LT}_{t+1})}{\hat{Pr}(death^D_{t+1})} = \alpha_t$$

4. We rescale each conditional probability in such a way that the unconditional probability matches the life tables:

$$\hat{Pr}(death^{LT}_{t+1}) = \sum_{i=1}^{4} \hat{Pr}(Cogn\_Skill_t = i) * \hat{Pr}(death^C_{t+1}|Cogn\_Skill_t = i)$$

with $\hat{Pr}(death^C_{t+1}|Cogn\_Skill_t = i) = \alpha_t * \hat{Pr}(death^D_{t+1}|Cogn\_Skill_t = i)$.