Inflation, Fiscal Rules and Cognitive Discounting*

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Abstract

Using a stylized, calibrated New Keynesian model, we provide a welfare ranking of passive monetary and active fiscal (PM/AF) rules when contractionary aggregate-demand shocks occasionally drive the nominal interest rate to the ELB. When the shocks are calibrated to match the U.S. frequency of ELB episodes under a traditional active monetary, passive fiscal (AM/PF) regime, a PM/AF regime in which debt increases trigger *increases* in government purchases and/or *cuts* in taxes—what we call super-active fiscal policies—can so reduce the frequency of the ELB that they outperform an AM/PF regime *if expectations are formed rationally.* This last condition is crucial. Welfare ranking of policy regimes depends critically on the way private sector agents form expectations; when agents display cognitive discounting, PM/AF regimes perform significantly worse. This result is robust to the government's long-run debt target and the presence of long-term debt. We also analyze fiscal rules calibrated to the U.S. response during the Great Recession and the COVID recession. Our paper is the first to analyze super-active fiscal policies and to evaluate the implications of cognitive discounting for the relative performance of AM/PF and PM/AF policy regimes.

Keywords: automatic stabilizers, cognitive discounting, fiscal and monetary interactions, government debt.

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1 Introduction

The 21st century has seen central banks in major industrialized economies facing multiple and often extended periods during which traditional interest rate policies have been constrained by the effective lower bound (ELB) on nominal interest rates. In response, a large literature has investigated the performance of different monetary-policy frameworks that might mitigate the effects of the ELB. These alternatives generally work by affecting expectations about future inflation and are therefore affected by how monetary policy behaves in the post-ELB environment. Other research has focused on the role of fiscal policy at the ELB. With monetary policy constrained, discretionary increases in government debt not accompanied by any commitment to raise future taxes or cut future expenditures can raise expected inflation when monetary policy cannot.

We study the role that active fiscal policy rules in the form of debt-financed fiscal expansions, rules that might appear to threaten the sustainability of the government's debt, can play in stabilizing inflation and output when the ELB can render monetary policy ineffective. Recent research on new monetary-policy frameworks to deal with the ELB assumes that central banks can commit to rules responding to inflation. In a similar vein, we incorporate simple fiscal rules governing purchases and taxes in response to the government debt level. Rather than focusing on the temporary adoption of fiscal rules when the economy encounters the ELB, we consider permanent policy regimes in which monetary policy responds weakly to inflation while fiscal policy raises spending or reduces taxes as debt rises.

Using a stylized, calibrated New Keynesian model, we find that, in the face of contractionary aggregate-demand shocks that occasionally drive the nominal interest rate to the ELB, a regime of passive monetary policy and super-active fiscal policy in which rising debt levels trigger *increases* in government purchases and/or *cuts* in taxes can reduce both the frequency of the ELB and the welfare costs of fluctuations in real economic activity and inflation. We show, however, that the welfare ranking of policy regimes depends critically on the way private sector agents form expectations.

The relevance of policies that reduce the primary surplus as debt levels rise is suggested by the policies adopted in the U.S. following the 2008-09 global financial crisis and the COVID induced recession of early 2020. During these recessions, large fiscal expansions occurred accompanied by increases in debt-to-GDP levels. As is well-known, the U.S. federal debt held by the public as a percent of GDP has been rising steadily since 2007; it was 35% at the start of the Great Recession (2007Q4) and had risen to 80% at the onset of the COVID recession (2019Q4).[1] Figure 1 illustrates the size and composition of the U.S. federal response during

[1] The peak to trough dates for the Great Recession in the U.S. are 2007Q4–2009Q2, while for the COVID recession the peak to trough dates are 2019Q4–2020Q2, according to the NBER's Business Cycle Dating Committee (`https://www.nber.org`). Because we interpret the budget constraint (5) as applying to the consolidated government sector, the relevant definition of debt in the model is government debt held outside

these recessions by showing the change in government *purchases* and *net taxes* as a share of GDP, divided by the change in debt as a share of GDP.[2] Also shown is the change in the *primary surplus* (net taxes minus purchases) relative to the change in debt. The bars on the left refer to the fiscal response during the Great Recession (2007Q4 to 2009Q2), while the bars on the right show the response in the COVID recession (2019Q4 to 2020Q2).

As the figure shows, most of the fiscal response in the U.S. during the Great Recession took the form of tax cuts and increases in transfers, with the fall in net taxes equaling 42% of the rise in debt. Government purchases (consumption plus investment) rose by only 6% of the change in debt, implying the primary surplus fell by 48% of the rise in debt. During the COVID recession, the fiscal response was *larger overall* measured relative to the debt level, with the primary surplus falling by 91% of the rise in debt *and* with the debt level at the end of 2019 much higher than during the Great Recession. Government purchases rose notably more during COVID (24% of the rise in debt) and cuts in net taxes were larger (67% of the rise in debt) compared to the previous recession.

Understanding the implications of fiscal policies that increase borrowing as debt levels rise motivates our paper, which makes four primary contributions. First, we evaluate the consequences of adopting simple fiscal rules that would normally be viewed as prescribing "irresponsible" fiscal behavior: debt-financed fiscal expansions unbacked by any promise of future tax increases or spending cuts. We refer to these policies as *super-active* fiscal policies. The resulting debt expansions generate expectations of higher future inflation. If combined with a weak monetary policy response to inflation, the rise in expected inflation lowers the real interest rate and boosts aggregate demand. To ensure a unique, stationary rational-expectations equilibrium, such fiscal rules must be combined with weak monetary policy responses to inflation that violate the Taylor principle.

Importantly, the ELB places no constraint on the government's ability to tax less and spend more to offset contractionary shocks to aggregate demand. In contrast, an active monetary policy seeking to stabilize current inflation will fail to raise inflation expectations if the ELB binds; active monetary policy becomes ineffective. Away from the ELB, however, active monetary policy can stabilize inflation and output without any fiscal stimulus. Thus, a welfare comparison of whether monetary policy or fiscal policy should be active will depend on their

the government sector, i.e. federal debt held by the public.

[2]The data source is the Federal Reserve Bank of St. Louis FRED database (`https://fred.stlouisfed.org`). The variables we use (and their FRED identifiers) are the following: U.S. federal government receipts (FGRECPT), expenditures (FGEXPND), interest payments (A091RC1Q027SBEA), transfer payments (W014RC1Q027SBEA), debt held by the public as a percent of GDP (FYGFGDQ188S), and GDP (GDP). *Net taxes* are equal to receipts minus transfers, while government *purchases* are equal to expenditures minus transfers minus interest payments. And the *primary surplus* is equal to net taxes minus government purchases. In Figure 1, for example, the bar shown for government purchases is given by (g(end) - g(start))/(debt(end) - debt(start)), where g and debt are expressed as a percent of GDP and where 'start' and 'end' refer to the quarter in which the recession begins and ends.

relative performances both at the ELB and away from it, and on the frequency of the ELB under each policy alternative.

Our second contribution is to carry out such a welfare comparison employing a model-consistent measure of the welfare costs of fluctuations. This has not previously been done when the ELB is considered. We show that super-active fiscal policies can dominate less active fiscal policies as well as active monetary policies, *if private sector agents form expectations rationally*. While it is well known that active fiscal policies *at the ELB* can lead to welfare improvements, our results demonstrate that these improvements can more than offset their poorer performance when the economy is not at the ELB. This remains the case when we increase the level of steady-state debt and when the government issues long-term debt. However, super-active fiscal policies are most effective with a high debt target and when debt is short-term. This normative assessment suggests that both the existing literature and policy discussions based on conventional wisdom have focused too exclusively on the role of active monetary policies in the face of the ELB.

Our third contribution is to provide an evaluation of such active monetary/passive fiscal (AM/PF) and passive monetary/active fiscal (PM/AF) policy regimes when expectations are not rational. The performance of policy rules depends heavily on their impact on future expectations, and this is particularly important at the ELB. An important question, therefore, is whether our findings are robust to deviations from rational expectations. To address this issue, we investigate the implications of bounded rationality in the form of cognitive discounting that causes less weight to be placed on expected future events. As far as we are aware, ours is the first analysis of alternative monetary and fiscal regimes under cognitive discounting. This form of deviating from rational expectations has significant implications for equilibrium determinacy. We show that some policies normally classified as regimes of passive monetary and active fiscal policies are inconsistent with a stable equilibrium. Cognitive discounting also leads to a large deterioration in the performance of active fiscal regimes.

Our fourth contribution is to calibrate the fiscal rules to reflect the size of the fiscal responses seen in the U.S. during the Great Recession and the COVID recession, and thus show that super-active fiscal policies are effective in stabilizing inflation and output in the face of the ELB, *if expectations are formed rationally*. Accounting for the ELB, the welfare costs under these policies are slightly lower than is achieved by active monetary policy. The fiscal response during the Great Recession was concentrated on tax cuts and transfer increases, while that in 2020 during the COVID pandemic represented a more balanced increase in purchases and cut in net taxes. In the model simulations, the COVID response reduces inflation volatility and results in a welfare improvement relative to the fiscal response during the Great Recession. Both examples of super-active fiscal policies are particularly effective if they are combined with a passive monetary policy that pegs the nominal interest rate to its steady state, eliminating the occurrence of the ELB. These results, however, are predicated on the public forming expectations rationally, and we find that this standard assumption about expectations is critical for the results. They are reversed under cognitive discounting.

Our paper is part of a large literature on designing policies to achieve macroeconomic stability in the face of the ELB. This literature has been prompted by the extended periods of low and even negative interest rates experienced in many countries, and by evidence of a declining natural rate of interest (see for example Holston, Laubach and Williams 2017) that increases the likelihood of future ELB episodes. The research on monetary policy frameworks to deal with the ELB has focused on both optimal monetary policy and on instrument rules. Work by Eggertsson and Woodford (2003; 2006), Adam and Billi (2006), Nakov (2008), and Billi, Galí, and Nakov (2024) shows that central banks should promise to keep their policy rate at the ELB for longer, not raising rates as soon as doing so might become feasible. If credible, such promises to make-up for past target misses induce expectations of higher future inflation, helping to mitigate the policy limitations arising from the ELB.[3] Other work has focused on altering the monetary authorities goals to include price level or average inflation objectives (e.g., Nessén and Vestin 2005, Budianto, Nakata, and Schmidt 2023) or including the price level or average inflation in the policy instrument rule to mimic the outcomes of optimal policy (e.g., Reifschneider and Williams 2000, Mertens and Williams 2019).

However, this work has ignored the role of fiscal policy. Nevertheless, monetary policy actions have implications for the government's budget, and the central bank's ability to achieve its inflation target depends on the behavior of the fiscal authority. Standard analyses of monetary policy assume, in the terminology of Leeper (1991), an *active monetary* (AM) policy aimed at controlling inflation, implicitly combined with a *passive fiscal* (PF) policy that ensures debt sustainability.[4] The Fed's policy framework review that led to the adoption of average inflation targeting in 2020, for example, took as given that any candidate framework would involve just such an AM/PF arrangement.[5]

Several authors have emphasized the role that active fiscal policy might play at the ELB. According to Sims (2016, p. 315), the key question is: "Can fiscal deficit finance replace

[3]Policy rules consistent with price-level targeting or average inflation targeting lead expectations to adjust in ways that mimic optimal policy at the ELB. The ELB and its implications for the choice of central-bank goals are investigated in Billi (2017; 2018; 2020), Billi, Söderström and Walsh (2023), among others.

[4]Leeper and Leith (2016) discuss the literature on interactions between monetary and fiscal policies and their role in determining macroeconomic outcomes, particularly the aggregate price level. Sablik (2019) provides a discussion of active and passive policies, budget deficits and inflation, linking active fiscal policies to the fiscal theory of the price level (FTPL) and to modern monetary theory (MMT). Sims (2024) offers a summary of post-WWII US inflation through the lens of the FTPL. For a detailed introduction to FTPL and its relation to New Keynesian analysis, see Cochrane (2023).

[5]As part of the FOMC's review of its policy framework, in June 2019 a research conference was held at the Federal Reserve Bank of Chicago. The papers from the conference were published in the *International Journal of Central Banking*, vol. 16(1), February 2020. However, none of the papers discussed the interactions between monetary and fiscal policies, or the role fiscal rules might play if monetary policy is limited in achieving its goals due to the ELB.

ineffective monetary policy" at the ELB? He concludes the answer is yes, but stresses that "fiscal expansion is not the same thing as deficit finance. It requires deficits aimed at, and conditioned on, generating inflation. The deficits must be seen as financed by future inflation, not future taxes or spending cuts." Hence, monetary policy that is ineffective at controlling inflation requires fiscal expansion that is not accompanied by any promise to generate future primary surpluses to finance those deficits; "budget balancing can become bad policy" (Sims 2000, p. 970). Similarly, Eggertsson (2006) calls for the government to "commit to being irresponsible" during periods at the ELB by creating money to fund a fiscal expansion, inducing expectations of higher future inflation. At the ELB, creating money and debt financing a fiscal expansion are equivalent, see Galí (2020).

Thus, an era with frequent periods at the ELB may require a more fundamental change in policy than simply maintaining an AM/PF policy regime while adopting a make-up rule based on a price-level target or average inflation. Passive monetary and active fiscal (PM/AF) policy regimes also need to be considered.[6] There is some evidence that switches between policy regimes in the U.S. have occurred in the past. Kim (2003) argued that VAR evidence from the U.S. on inflation and output responses is consistent with a PM/AF regime during the 1940s and 1950s. Davig and Leeper (2011) estimate a regime switching model and find the U.S. has alternated between active and passive regimes.[7] Bianchi and Melosi (2017) also find evidence of regime switches in an estimated DSGE model that incorporates exogenous episodes at the ELB.

Jacobson, Leeper and Preston (2023) offer an historical example of active fiscal policy in President Franklin Roosevelt's distinction between the *regular* budget, which was governed by conventional budget-balancing concerns, and the temporary *emergency* budget, for which there was no promise that future taxes would be raised to fund the deficit. Bianchi, Faccini and Melosi (2022) analyze temporary shock-specific policies (emergency budgets and temporary inflation targets) that can be interpreted as capturing Roosevelt's distinction between the regular and emergency budgets. The monetary authority does not respond to inflation generated by the emergency budget, essentially acting as if it were constrained by the ELB. Bianchi and Melosi (2019) investigate the consequences of a negative demand shock that leads to a monetary-fiscal policy conflict and the public is uncertain about how that conflict will be

[6] Liu, Miao and Su (2022) examine the relative performance of average inflation targeting (AIT) in AM/PF and PM/AF regimes. However, their focus, as is that of Beck-Friis and Willems (2017) and Hills and Nakata (2018), is on the implications for fiscal multipliers associated with exogenous fiscal shocks.

[7] Davig and Leeper (2011) use an estimated model to explore the impact of shocks to government purchases (such as the 2008 American Recovery and Reemployment Act) in different policy regimes, where the regimes are determined by the properties of policy rules for the nominal interest rate and lump-sum taxes net of transfers. Ascari, Florio and Gobbi (2020) also employ a regime-switching framework that allows for "timid" departures from regimes, ensuring determinacy as long as agents anticipate a future return to either an AM/PF or PM/AF regime. They define fiscal rules in terms of lump-sum taxes, and their focus is primarily on issues of determinacy and the effects of regimes on the impact of fiscal-spending shocks.

settled. Potential conflicts between the two policy authorities can lead to "dire" consequences, but allowing inflation to rise to reduce the recession-induced rise in real debt helps avoid the ELB and improves welfare if it is associated with a commitment to return to a passive fiscal policy, active monetary policy after the recession.

While emergency budgets represent *temporary* adoption of an active fiscal policy, we adopt an alternative perspective and consider the welfare implications of *permanently* adopting a PM/AF regime in an environment in which episodes at the ELB are frequent under a standard inflation-targeting AM/PF regime.[8] Our analysis is therefore consistent with the approach that assumes PF and investigates alternative monetary policy rules for dealing with the ELB. That is, regimes are analyzed as permanent choices among a variety of policy frameworks.[9] To rank alternatives, we use a model-consistent welfare measure of the costs of economic fluctuations. Thus, our measure of the performance of AM/PF and PM/AF policies will depend on their respective welfare costs at the ELB and away from it, as well as on the incidence of ELB episodes.

Our work is also related to that of Bianchi and Melosi (2017), Bianchi, Faccini and Melosi (2022), and Ascari, Florio and Gobbi (2023). The first of these papers finds that announcing a permanent shift to PM/AF *when at the ELB* significantly mitigates the negative effect of the binding ELB and potentially even avoids the ELB. In contrast, our focus in on the consequences of the permanent adoption of a PM/AF regime and, importantly, we provide a welfare-based metric for evaluating alternative regimes and show that the presence of cognitive discounting significantly alters the ranking of policy regimes.

Bianchi, Faccini and Melosi (2022) consider emergency budgets and temporary inflation targets and analyze a transfer shock based on the 2020 CARES Act with nominal interest rates positive throughout the experiment. Instead, we focus on the role of fiscal rules in contributing to macroeconomic stabilization in the face of aggregate demand shocks that can push the economy to the ELB, employing fiscal rules for both net taxes and government purchases in a stylized, calibrated New Keynesian model with a ELB constraint. Ascari, Florio and Gobbi (2023) examine the relative performance of inflation targeting (IT) and price-level targeting (PLT) in AM/PF and PM/AF regimes and employs a quadratic loss function to evaluate alternative outcomes with instrument rules for the nominal interest rate and for net taxes.[10] They find that AM/PF dominates PM/AF under PLT, while the comparison of AM/PF and PM/AF under IT depends on the size of the interest rate response to inflation. They show

[8] Bhattarai, Lee and Park (2014) provide an analytical characterization of inflation dynamics under AM/PF and PM/AF regimes when not constrained by the ELB.

[9] An exception is the proposal of *temporary* price-level targeting of Bernanke, Kiley and Roberts (2019).

[10] Their loss function differs from a quadratic welfare approximation in putting a larger weight on output gap volatility, and they define monetary policy in terms of an interest rate rule that responds to the price level or average inflation. Defining regimes in terms of what variables appear in an instrument rule contrasts with approaches that define regimes in terms of the objectives or goals adopted by the central bank. For a discussion of the choice between rules and goals in the context of monetary policy design, see Walsh (2015).

that with an active fiscal policy, in which the primary surplus is fixed, performance under IT is improved if monetary policy responds *negatively* to inflation. Our paper is complementary in that they consider a seemingly irresponsible IT regime (raising rates as inflation falls) under active fiscal policy, while our focus is on seemingly irresponsible, super-active, fiscal policies that reduce the primary surplus as debt levels rise.

While we follow much of the literature on active fiscal policy in assuming one-period debt, we also generalize the analysis to consider the role of long-term government debt. Caramp and Silva (2023) and Leeper (2021) emphasize how the presence of long-term debt implies revaluation effects on existing debt that affect the wealth channel of monetary policy under active fiscal policies. Leeper and Zhou (2021) investigate optimal commitment policies when debt is long-term, while Leeper, Leith and Liu (2021) consider discretionary policies, as does Harrison (2021), who highlights how debt sustainability becomes a constraint on the actions of the central bank when fiscal policy is active. Harrison's work is complementary to ours in that he uses, as we do, a model-consistent quadratic loss function to rank outcomes and specifies fiscal policy as setting an exogenous primary surplus, which as he notes, is one of the specifications we investigate. We consider a wider range of PM/AF policies allowing the primary surplus to vary endogenously, and we investigate the performance of those policies in the presence of cognitive discounting.

The performance of different policy rules depends importantly on the way a rule shapes future expectations. However, the literature evaluating PM/AF policies has adopted the assumption of rational expectations. A number of authors have analyzed the implications of deviations from rational expectations in the context of AM/PF policies. One approach is based on the cognitive discounting model due to Gabaix (2020). With cognitive discounting, agents place less weight on expectations of future events relative to the case of rational expectations. This dampens the role of future expectations and has been used to explore issues that arise at the ELB such as the forward guidance puzzle (e.g., Gabaix 2020) and the performance of monetary policy frameworks such as average inflation targeting (e.g., Budianto, Nakata, and Schmidt 2023).[11] Our paper is the first to employ cognitive discounting to analyze the relative performance of AM/PF and PM/AF regimes. We find that this alternative assumption about expectations has a major impact on the ranking of alternative policy regimes.

The rest of the paper is organized as follows. We employ a simple New Keynesian model for our analysis, and this is set out in Section 2, which also discusses the policy rules we consider, reviews the intuition for why active fiscal polices might improve outcomes facing the ELB, and discusses the calibration of the model. We use the model in Section 3 to investigate the effects of active fiscal polices in the face of a contractionary shock to aggregate demand. In Section 4 we investigate whether our results depend on the debt-to-GDP target and on the

[11] We review some of the macroeconomic evidence on cognitive discounting in online appendix A.2.1.

introduction of long-term debt. Section 5 evaluates the impact of a negative demand shock when the fiscal responses are calibrated to those seen in the U.S. during the Great Recession and COVID recession. Conclusions are summarized in Section 6.

2 The Model

We conduct our analysis using a canonical version of the New Keynesian model, augmented with a ELB constraint and with fiscal policy rules in which net taxes and purchases respond to the level of government debt. In contrast to the earlier literature, we assume expectations differ from rational expectations and are instead subject to cognitive discounting as in Gabaix (2020). In this section, we introduce the equations describing the model's equilibrium, consider how cognitive discounting and fiscal rules affect determinacy, discuss how the fiscal rules affect inflation stabilization in a regime of passive monetary policy and active fiscal policy, and then calibrate the model to recent U.S. data.

2.1 Private Sector

The behavior of the private sector is described by the equilibrium conditions that correspond to the closed-economy New Keynesian model with staggered price setting à la Calvo, flexible wages, and without capital accumulation. Government purchases are financed through lump-sum taxes and the issuance of debt. All equations are log-linearized around a steady state with no trend growth, zero price inflation, and with a subsidy that exactly offsets the steady-state distortions arising from price markups. The micro foundations of the model and the derivations of its reduced form are well known and can be found in the textbooks of Galí (2015, chapter 3) and Walsh (2017, chapter 8). Our notation generally follows Galí (2020). The expectations operator under cognitive discounting is denoted by E_t^{CD} whereas E_t indicates rational expectations.

Modifying the New Keynesian Phillips curve (NKPC) to reflect cognitive discounting is straightforward. Details can be found in Gabaix (2020) and the online appendix A.2.2. Letting $\bar{m} \in [0,1]$ be the micro-cognitive discounting factor, the New Keynesian Phillips curve with cognitive discounting is

$$\pi_t = \beta E_t^{CD}\{\pi_{t+1}\} + \kappa\tilde{y}_t = \beta M^f E_t\{\pi_{t+1}\} + \kappa\tilde{y}_t, \tag{1}$$

where π_t is the rate of price inflation between periods $t-1$ and t. The parameter $\beta \equiv 1/(1+\rho)$ denotes the household's discount factor, where ρ is the discount rate. $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$ denotes the output gap, where $\hat{y}_t \equiv \log(Y_t/Y)$ denotes (log) output in deviation from its steady state, and where $\hat{y}_t^n \equiv \log(Y_t^n/Y)$ represents the (log) deviation of the natural level of output, i.e.

output's equilibrium level in the absence of nominal rigidities, as a deviation around its steady state. The elasticity of inflation with respect to the output gap κ is unchanged from the basic model with rational expectations.

The parameter $M^f \in [0,1]$, given by

$$M^f \equiv \bar{m}\left[\theta + (1-\theta)\left(\frac{1-\beta\theta}{1-\beta\theta\bar{m}}\right)\right] \leq \bar{m} \leq 1,$$

captures the relevant discounting factor for future inflation in the adjustment of the aggregate price level. The parameter $\theta \in [0,1)$ denotes the Calvo index of price rigidity (the probability that a firm does not reset its price in a given period). Under rational expectations, $\bar{m} = 1$, implying $M^f = 1$, and (1) reduces to the standard NKPC.

The natural (flexible-price) level of output is given by $\hat{y}_t^n \equiv \Gamma \hat{g}_t$, where $\Gamma \equiv \frac{\bar{\sigma}(1-\alpha)}{\alpha+\varphi+\bar{\sigma}(1-\alpha)}$ and $\hat{g}_t \equiv (G_t - G)/Y$ denotes the deviation of (real) government purchases from its steady state as a share of steady-state output.[12] Note that $\bar{\sigma} \equiv \sigma(Y/C)$. The parameters α, σ and φ denote the degree of decreasing returns to labor in production, the household's coefficient of relative risk aversion and the curvature of labor disutility, respectively. The goods-market equilibrium condition is given by $Y_t = C_t + G_t$ or approximately $\hat{y}_t = (C/Y)\hat{c}_t + \hat{g}_t$, where $\hat{c}_t \equiv \log(C_t/C)$ denotes (log) private consumption expressed as a deviation from its steady state.

In addition, the slope of the Phillips curve is given by $\kappa \equiv \lambda\left(\bar{\sigma} + \frac{\alpha+\varphi}{1-\alpha}\right)$, where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\epsilon)}$. The elasticity of substitution among varieties of goods is denoted by $\epsilon > 1$. Flexible-price output satisfies the condition that the marginal rate of substitution between leisure and consumption and the marginal product of labor are equal. As neither of these depend on expectations, $\hat{y}_t^n$ is not affected by cognitive discounting.

The demand side of the economy is described by a dynamic IS equation

$$\tilde{y}_t = E_t^{CD}\{\tilde{y}_{t+1}\} - \frac{1}{\bar{\sigma}}\left(\hat{\imath}_t - E_t^{CD}\{\pi_{t+1}\} - \hat{r}_t^{CD}\right),$$

where $\hat{\imath}_t \equiv i_t - \rho$ denotes the short-term nominal interest rate expressed as a deviation from its steady state, and the latter corresponds to the discount rate $\rho \equiv 1/\beta - 1 > 0$. The short-term real interest rate is given by $\hat{r}_t \equiv \hat{\imath}_t - E_t^{CD}\{\pi_{t+1}\}$.

Cognitive discounting affects the IS equation by modifying the role of expected future output and inflation. Details can be found in the online appendix A.2.3.[13] In terms of rational

[12] We do not include a distortional markup shock in (1), so when expressed in terms of deviation from steady state, the flexible-price output level coincides with the efficient output level.

[13] See section XI.G.1, p. 11 of the online appendix to Gabaix (2015) for a version of his model with government purchases.

expectations, the IS equation becomes[14]

$$\tilde{y}_t = \bar{m} E_t \{\tilde{y}_{t+1}\} - \frac{1}{\bar{\sigma}} \left(\hat{\imath}_t - \bar{m} E_t \{\pi_{t+1}\} - \hat{r}_t^{CD}\right), \tag{2}$$

where $\hat{r}_t^{CD}$ is the real interest rate consistent with a zero output gap under cognitive discounting. With an active fiscal policy and rational expectations, Ricardian equivalence does not hold because agents do not expect the government to raise future taxes when, for example, current taxes are cut. With cognitive discounting, *even if future taxes are expected to be raised*, these future taxes are discounted by $\bar{m} < 1$. This generates an additional reason for Ricardian equivalence to fail.

Specifically, Gabaix (2020) argues that a debt-financed increase in transfer payments puts money in the pockets of households, but households then discount the future taxes implied by the higher government debt. On net, households feel wealthier; they spend more and work less, raising the natural interest rate. This channel implies $\hat{r}_t^{CD}$ depends directly on the stock of debt $\hat{b}_t$,

$$\hat{r}_t^{CD} \equiv \left(z_t - E_t^{CD}\{z_{t+1}\}\right) - \bar{\sigma}\,(1-\Gamma)\left(E_t^{CD}\{\hat{g}_{t+1}\} - \hat{g}_t\right) + \bar{\sigma} b_d \hat{b}_t.$$

where z_t is a preference shifter (aggregate-demand shock) which follows an exogenous $AR(1)$ process with autoregressive coefficient ρ_z and standard deviation σ_z.[15] The parameter b_d is given by

$$b_d \equiv (1-\bar{m})\,\beta\rho\left(\frac{C}{Y}\right)\left(\frac{\varphi}{\varphi + (1-\alpha)\bar{\sigma}}\right) \geq 0.$$

The stock of debt, however, has no wealth effect on households from the natural interest rate when $\bar{m} = 1$.[16] In terms of rational expectations, $\hat{r}_t^{CD}$ equals

$$\hat{r}_t^{CD} \equiv \left(z_t - \bar{m} E_t\{z_{t+1}\}\right) - \bar{\sigma}\,(1-\Gamma)\left(\bar{m} E_t\{\hat{g}_{t+1}\} - \hat{g}_t\right) + \bar{\sigma} b_d \hat{b}_t. \tag{3}$$

Under rational expectations, $\bar{m} = 1$, implying $M^f = 1$ and $b_d = 0$, so (2) and (3) reduce to the standard new Keynesian IS equation and definition of the natural interest rate.[17]

A key objective of our analysis is the evaluation of fiscal and monetary policy from a welfare

[14] Gabaix (2020, p. 2282) appeals to "a fringe of rational financial arbitrageurs with vanishing small consumption" to argue that inflation expectations in the definition of the real return are rational. We assume, instead, that households have access to nominal assets only and thus the relevant expected real return includes cognitive discounting of inflation in the equation for aggregate demand.

[15] This shock's innovation is an i.i.d. normally distributed process with zero mean and standard deviation given by $\sigma_{ez} = \sigma_z \sqrt{1-\rho_z^2}$. Furthermore, z_t is interpreted as a shock to the effective discount factor; it affects the household's marginal utility of consumption and marginal value of leisure, while leaving unaffected the marginal rate of substitution between consumption and leisure. Thus, z_t affects $\hat{r}_t^n$ but not $\hat{y}_t^n$ in the model.

[16] If debt has a wealth effect, $\bar{m} < 1$, households respond by increasing their consumption of goods and of leisure. The term $\varphi/\left(\varphi + (1-\alpha)\bar{\sigma}\right) < 1$ captures the net effect on goods consumption.

[17] Note that $\hat{r}_t^{CD}$ also depends on $\bar{m} E_t\{\hat{g}_{t+1}\}$. Given the fiscal policy rule (7) that we specify below, we have $E_t\{\hat{g}_{t+1}\} = \psi_g \hat{b}_t$. Thus $\hat{g}_{t+1}$ is known by agents at time t and therefore, once we impose our fiscal rules, is not affected by cognitive discounting.

perspective. For that purpose, we use as a welfare metric the second-order approximation of the average welfare loss experienced by the representative household as a consequence of fluctuations around an efficient steady state with zero inflation. We express this social welfare loss as a fraction of steady-state consumption

$$\mathbb{L} = \frac{1}{2}\left[\frac{\epsilon}{\lambda}var\left(\pi_t\right) + \frac{\kappa}{\lambda}var\left(\tilde{y}_t\right) + \frac{\gamma\kappa}{\lambda}var\left(\hat{g}_t\right)\right], \tag{4}$$

where $\gamma \equiv \Gamma\left(1 - \Gamma + \frac{\bar{\vartheta}}{\bar{\sigma}}\right)$, $\bar{\vartheta} \equiv \vartheta\left(Y/G\right)$, and ϑ denotes the curvature of utility from government purchases. The welfare loss has three components, respectively associated with the volatilities of inflation, the output gap, and government purchases. A discussion can be found in Woodford (2011).

The welfare loss given by (4) reflects the present discounted value of expected future output gaps and inflation. By employing (4) to rank equilibrium outcomes, we are implicitly assuming the policy authorities do not suffer from cognitive discounting. One might argue that a policymaker, knowing private sector agents form non-rational expectations, should also discard rational expectations in assessing the welfare consequences of alternative policies. In the related context of the literature on robust control, however, Sims (2001) argued "...the criteria for acceptable shortcuts in decision-making by a central bank should generally be much stricter than those applying to, say, a consumer buying a new washing machine. On the other hand, a 'representative agent' that summarizes the behavior of many individuals with disparate information sources, coordinated through many markets, may be well modeled as having fewer computational constraints than a monetary policy maker. In either case, the criteria for good descriptive modeling and good normative policy advice ought to be kept distinct." In contrast, Hansen and Sargent (2003) are characterized by Sims as "...recommending to policy-makers the same sub-rational behavior that they postulate in private agents."

We are sympathetic with Sims' position, and note that Gabaix (2020) employs the standard welfare loss appropriate under rational expectations in his analysis of optimal policy when private sector agents display cognitive discounting, as do Budianto, Nakata, and Schmidt (2023) in their analysis of average inflation targeting. We follow their approach, using (4), when we investigate the welfare implications of policy regimes.

2.2 Government Budget and Policy Regimes

The fiscal authority finances its spending through two sources: net taxes (*lump-sum taxes net of transfers*) and the issuance of nominally riskless one-period bonds with a nominal yield i_t. Section 4 extends the model to include long-term debt. After log-linearization around a steady state with no trend growth and zero inflation, the following difference equation describes the evolution of real government debt as a share of steady-state output, thereby representing the government's flow-budget constraint[18]

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} + \beta^{-1}b\left(\hat{\imath}_{t-1} - \pi_t\right) + \hat{g}_t - \hat{\tau}_t, \tag{5}$$

where $\hat{b}_t \equiv \left(B_t - B\right)/Y$ and $\hat{\tau}_t \equiv \left(T_t - T\right)/Y$ denote, respectively, deviations of (real) government debt and net taxes from their steady state, expressed as a fraction of steady-state output. B_t denotes the (real) market value of government debt. The parameter $b \equiv B/Y$ denotes the long-run debt target as a share of steady-state output. The debt accumulation equation given by (5) does not involve expectations and so is unaffected by cognitive discounting.

In (5) the government debt issuance for the current period, $\hat{b}_t$, is determined by three cost components expressed as deviations from their steady state. The first part is the cost to refinance (roll over) the outstanding debt, $\beta^{-1}\hat{b}_{t-1}$. The second part is the (real) interest cost to service the debt outstanding, $\beta^{-1}b\left(\hat{\imath}_{t-1} - \pi_t\right)$. And the third part captures the "*primary surplus*" (defined in official statistics as the fiscal balance net of any interest payments) which may be in surplus or deficit. We denote the primary surplus, $\hat{\tau}_t - \hat{g}_t$, by $\hat{s}_t$.

The fiscal authority controls the primary balance with rules that specify how both net taxes and government purchases respond to deviations of the outstanding stock of government debt from its steady state. When log-linearized, the fiscal rules we analyze take the form

$$\hat{\tau}_t = \psi_\tau \hat{b}_{t-1}, \tag{6}$$

$$\hat{g}_t = \psi_g \hat{b}_{t-1}. \tag{7}$$

These fiscal policy rules together imply a rule for the primary surplus in response to the debt level, $\hat{s}_t = \psi_s \hat{b}_{t-1}$, where $\psi_s \equiv \psi_\tau - \psi_g$.[19] Moreover, combining (5) through (7), we obtain

$$\hat{b}_t = \left(\beta^{-1} - \psi_s\right)\hat{b}_{t-1} + \beta^{-1}b\left(\hat{\imath}_{t-1} - \pi_t\right). \tag{8}$$

Hence, as this combined version of the government's flow budget shows, the choice of coefficients in the fiscal rules, ψ_τ and ψ_g, allows the fiscal authority to affect directly the accumulation of government debt through the response of the primary surplus $\psi_s \equiv \psi_\tau - \psi_g$. In particular, a lower ψ_τ and/or a higher ψ_g increase the outstanding debt from period $t-1$ that must be refinanced. All else equal, the lower ψ_s would lead to a higher debt issuance in period t to satisfy the government's budget. The government debt issuance, however, is not determined by fiscal policy alone; rather it is the result of the joint behavior of the fiscal authority and the central bank that will determine inflation and the nominal interest rate. To close the model economy, we need also a description of monetary policy.

[18] See, for example, Walsh (2017, chapter 4) and Galí (2020) for a discussion.

[19] In the model, $\hat{g}_t$ and $\hat{\tau}_t$ have different effects, which is why we do not characterize fiscal policy simply in terms of the primary surplus $\hat{s}_t$.

We adopt a standard rule for the nominal interest rate, namely a truncated Taylor-type rule that incorporates explicitly a ELB constraint ($i_t \geq 0$ implying $\hat{\imath}_t \geq -\rho$)

$$\hat{\imath}_t = \max\left[-\rho, \phi\pi_t\right]. \tag{9}$$

Consistent with the standard analysis of policy rules in the monetary policy literature, we assume that both the fiscal authority and the central bank are able to credibly commit to follow the policy rules (6), (7) and (9). At the same time, private sector agents in the economy form expectations knowing that the policy authorities will abide by those policy rules. A discussion of the feasibility of implementing such a set of policy rules is postponed until our concluding section.[20]

In our analysis we have two policy regimes, characterized by the configuration of the response coefficients ϕ, ψ_g and ψ_τ in the above monetary and fiscal rules. Namely, in the terminology of Leeper and Leith (2016), we study regimes M and F. Under **regime M**, active monetary (AM) policy aimed at controlling inflation is combined with a passive fiscal (PF) policy ensuring debt sustainability. With rational expectations, such an AM/PF regime is defined by $\phi > 1$ and $\psi_s \equiv \psi_\tau - \psi_g > \rho$. That is, in regime M, with $\phi > 1$ in rule (9), the nominal interest rate responds more than one-for-one to movements in inflation, and the Taylor principle is satisfied. With $\psi_s > \rho$, the coefficient on the lagged debt stock in (8) is smaller than unity, $\beta^{-1} - \psi_s = 1 + \rho - \psi_s < 1$, which ensures the debt level converges to its long-run target. Conversely, with rational expectations, under **regime F** active fiscal (AF) policy controls inflation and monetary policy responds passively (PM) to ensure debt sustainability when $\phi < 1$ and $\psi_s < \rho$.[21]

As Gabaix (2020) has shown, under cognitive discounting and a passive fiscal policy, there exists a $\phi^* < 1$ such that determinacy is obtained as long as $\phi > \phi^*$. That is, values of ϕ that fail the Taylor principle may still be consistent with a unique, stationary equilibrium. However, determinacy conditions in new Keynesian models with active fiscal policy rules and cognitive discounting have not previously been examined; we turn to this task next.

[20] One could add exogenous stochastic components to the policy rules (6), (7) and (9) without qualitatively affecting our results. We restrict attention to simple rules for both fiscal and monetary policy. Burgert and Schmidt (2014) show that government spending and debt rise under the optimal discretionary policy at the ELB, with the increase depending negatively on the initial debt level. Nakata (2017) studies the optimal commitment policy in a similar environment. He finds that the ability to commit to future policies implies a higher initial debt level leads to a larger spending response. In both these papers, fiscal and monetary instruments are optimally chosen by a single policymaker.

[21] The determinacy conditions for ϕ and ψ_τ under rational expectations are well known, see Leeper (1991). The previous literature has focused on tax policies, generally treating spending as an exogenous process. The online appendix A.1 discusses determinacy for the spending rule (7).

2.3 Equilibrium Determinacy Under Cognitive Discounting

Determinacy conditions under cognitive discounting have not previously been explored when both monetary and fiscal rules are governing policy. Gabaix (2020) shows that the presence of cognitive discounting affects the condition for equilibrium determinacy in a basic New Keynesian model with passive fiscal policy. Specifically, when $\bar{m} < 1$, the Taylor Principle is affected and a policy response that is "too weak" according to the normal Taylor Principle can still be consistent with determinacy. The intuition is straightforward. As $\bar{m}$ falls towards zero, the role of future expectations diminishes. In the limit, if $\bar{m} = 0$, the model would be completely static and a unique equilibrium exists for any response to inflation. Consistent with Gabaix, we find that, when $\bar{m} = 0.85$, the conditions on the monetary policy response to inflation consistent with active policy are affected significantly. In contrast, the conditions that determine whether fiscal policy is active or passive are much less affected.

Figure 2 shows regions of determinacy, indeterminacy, and no stable equilibrium as functions of the parameters ϕ and ψ_τ in the top panels, and ϕ and ψ_g in the bottom panels.[22] The left column shows results when ψ_τ and ψ_g range from -0.4 to 0.4, while the right column narrows the range to -0.02 to 0.02 to highlight the boundaries between active and passive fiscal policies. Areas in white denote parameter regions with a unique, stationary equilibrium, in dark grey denote multiple equilibria, and in light grey denote no stationary equilibrium. Recall that under rational expectations, $\psi_s \equiv \psi_\tau - \psi_g > \rho$ and $\phi > 1$ define passive fiscal and active monetary policy respectively; the dashed lines in the figure use these standard values to divide the space into the traditional regions of determinacy, indeterminacy, and no stable equilibrium.

Consistent with Gabaix's result, monetary policy can be significantly less responsive to inflation than required by the Taylor principle and still be consistent with determinacy when fiscal policy is passive. There are now regions in which, when $\bar{m} = 1$, both monetary and fiscal policy would be viewed as passive, yet a unique equilibrium is obtained for $\bar{m} < 1$. For example, this is the case when $\psi_\tau = 0.02 > \rho$ or $\psi_g = -0.02 < -\rho$ and $\phi = 0.8$; a unique equilibrium occurs when taxes are increased or spending is reduced as the debt level rises to ensure debt sustainability, while monetary policy responds weakly to inflation. Some policies normally thought of as involving active fiscal and passive monetary policy, for example $\psi_\tau = 0 < \rho$ or $\psi_g = 0 > -\rho$ combined with, for example, $\phi = 0.8$ would, under the standard criterion, be classified as PM/AF, ensuring determinacy. However, under cognitive discounting, these policy combinations do not generate a stable equilibrium.

Compared to the case of rational expectations, it is primarily the dividing line between active and passive monetary policy that is affected. Under our baseline calibration, as fiscal policy becomes more active, weaker and weaker monetary policy responses are still "active."

[22] This figure employs the baseline calibration, discussed in Section 2.5, with $\bar{m} = 0.85$ and $\rho = 0.005$.

With super-active fiscal policy, the critical value of ϕ defining active monetary policy falls towards 0.6. The reduced impact of future expectations extends towards zero the range of monetary policy responses consistent with active monetary policy. It is the deviation from Ricardian equivalence due to cognitive discounting that accounts for policies normally viewed as PM/PF to support determinacy. If $b_d = 0$, then passive fiscal policy is defined by $\psi_s \equiv \psi_\tau - \psi_g > \rho$, just as in the standard analysis under rational expectations, and active monetary policy requires $\phi > 0.6$ under our calibration, regardless of the fiscal rule.[23] For example, consider a policy in which $\psi_s \equiv \psi_\tau - \psi_g = 0.006 > \rho$ and $\phi < 0.6$. Under rational expectations, this would represent a PM/PF regime and there would be multiple equilibria; under cognitive discounting, this policy supports a determinate equilibrium.

To understand why cognitive discounting affects the regions of determinacy, consider a positive shock to inflation expectations. With a weak monetary policy response, this results in a lower real interest rate and an expansion of aggregate demand that risks making the expectation of higher inflation self-fulfilling. However, the rise in inflation reduces the real value of outstanding debt. Under a fiscal policy in which the primary surplus responds by more than ρ to the debt level but only weakly so, the initial fall in the debt level lowers household wealth as Ricardian equivalence fails with cognitive discounting. This, in turn, reduces aggregate demand. This feedback of the debt level to demand can ensure determinacy even with a weak monetary policy response to inflation. This is also why the value of ϕ consistent with determinacy falls as the primary surplus responds more negatively to the debt level, strengthening the feedback from the debt level to demand, as seen in the right panels of Figure 2.

We next discuss how a regime of passive monetary policy and super-active fiscal policy affects the response of inflation to demand shocks, and how the inflation response depends critically on whether agents form expectations rationally or with cognitive discounting.

2.4 Inflation Stabilization in Alternative Policy Regimes

Because our focus is on the consequences of negative aggregate-demand shocks, we briefly review the key mechanism through which debt dynamics affect the inflation response to such shocks. This review is useful, as much of the literature comparing AM/PF with PM/AF regimes has focused on the implication for fiscal multipliers, while our focus is on systematic stabilization policies in the face of aggregate demand shocks and the ELB.

These dynamics are straightforward, and well known, under regime M (AM/PF). In principle, absent the ELB constraint, monetary policy can cut its policy rate to offset a negative demand shock, ensuring both inflation and the output gap are stabilized. The real interest rate falls, lowering debt servicing costs. With a passive fiscal policy, the coefficient on lagged

[23] See online appendix A.2.4 for a discussion of determinacy with cognitive discounting.

debt in the debt accumulation equation (8) is less than one, implying the debt variable $\hat{b}_t$ follows a stationary process that converges back to $\hat{b} = 0$. Under rational expectations, the debt variable does not appear in either the Phillips curve (1) or the IS equation (2) because $\bar{m} = 1$ and $b_d = 0$. In this case, under regime M these two equations are sufficient to determine inflation and output with no feedback from (8).

The equilibrium conditions for regime F (PM/AF) also consist of (1) and (2), together with the debt accumulation equation (8), but with $\psi_s < \rho$ and $\phi < 1$. As is well-known, $\phi < 1$ implies the two-equation system (1) and (2) in π_t and $\tilde{y}_t$ fails the Blanchard-Kahn conditions and is inconsistent with a unique, stationary equilibria. However, with $\psi_s < \rho$, (8) pins down the unique equilibrium level of inflation consistent with stationarity of the debt variable $\hat{b}_t$ by ensuring expected future inflation adjusts so that the discounted future deviation of debt from steady state converges to zero (see Leeper 1991).

This result is perhaps easiest to see when regime F involves $\psi_s = 0$ and $\phi = 0$ so that neither the primary surplus nor the nominal interest rate react.[24] Equation (8) in regime F then implies

$$\beta \hat{b}_t = \hat{b}_{t-1} - b\pi_t. \tag{10}$$

Because $\beta < 1$, this equation can be solved forward and, after imposing the condition that debt is stationary and taking expectations under cognitive discounting,[25] one obtains

$$\frac{\hat{b}_{t-1}}{b} = \sum_{i=0}^{\infty} \beta^i E_t^{CD} \pi_{t+i} = E_t \sum_{i=0}^{\infty} \left(\beta \bar{m}\right)^i \pi_{t+i}. \tag{11}$$

The right side of this expression is the present value of inflation, incorporating cognitive discounting. With the lagged stock of debt given, the present discounted value of inflation is fixed at $\hat{b}_{t-1}/b$. A fall in current inflation, $\pi_t < 0$, due to the negative demand shock, must therefore generate higher future inflation in regime F to ensure a stationary debt process. This reasoning is similar to the "unpleasant monetarist arithmetic" of Sargent and Wallace (1981), though they focused on the need for future seigniorage revenue to rise to balance the intertemporal government budget, while here debt sustainability is accomplished through the impact of inflation on the real value of the outstanding debt.[26] In the face of a negative demand shock, however, raising expectations of future inflation helps to stabilize the economy when the nominal interest rate fails to adjust. For this reason, a regime of active fiscal policy may be effective when monetary policy is unable to respond.

With cognitive discounting, as $\bar{m}$ declines below 1, the expected value of $\sum_{i=0}^{\infty} \left(\beta \bar{m}\right)^i \pi_{t+i}$

[24] This is equivalent to making fiscal policy exogenous and is the active fiscal policy considered by Ascari, Florio and Gobbi (2020) and by Harrison (2021). We relax this assumption in Section 3 when we report simulation results for active fiscal policy with $\psi_s < 0$ and passive monetary policy with $0 \leq \phi < 1$.

[25] This requires $\lim_{i\to\infty} \beta^{i+1} \left(\hat{b}_{t+i}/b\right) = 0$.

[26] A discussion can be found in Bhattarai, Lee and Park (2014).

in (11) also declines. Hence, the expected path of current and future inflation must rise to maintain the equality in (11).

In an active monetary policy regime, the rise in real debt holdings of households resulting from the fall in π_t is offset by the negative wealth effect as households anticipate a rise in the present value of future tax payments. Under active fiscal policy with $\psi_s = 0$, there is no negative wealth effect from future taxes offsetting the rise in the real value of household debt holdings. In addition, with cognitive discounting, the rise in $\hat{b}_t$ induces a further increase in demand and inflation.

By extension, a super-active fiscal policy with $\psi_s < 0$ implies the primary surplus falls as the debt level rises. In this case, (11) is modified to become

$$\frac{\hat{b}_{t-1}}{b} = E_t \sum_{i=0}^{\infty} \left(\frac{\beta \bar{m}}{1 - \beta \psi_s} \right)^i \pi_{t+i}. \tag{12}$$

Relative to the $\psi_s = 0$ case, if $\psi_s < 0$, a given fall in current inflation generates a larger rise in future inflation to ensure debt sustainability. This increase in expected future inflation acts to partially offset the initial fall in inflation, serving to help stabilize current inflation and the output gap in the face of contractionary aggregate-demand shocks. Thus, a super-active fiscal policy acts in a manner similar to a stronger cognitive discounting (a lower value of $\bar{m}$).

A rule-based, active fiscal policy—that is, a credible commitment to behave in ways that might appear to be irresponsible and shortsighted—can endogenously generate movement in expected inflation that serves to stabilize the economy. This is an advantage if, due to the ELB, monetary policy's response is limited. With cognitive discounting, the impact of future expectations is dampened relative to the situation in which expectations are formed rationally. What this implies for the effectiveness of active fiscal policy is, however, not obvious. Agents respond less to future expectations of inflation, but as implied by (12) future inflation must rise more as future expectations are discounted more heavily.

The economy is likely to experience periods when the ELB binds and periods when it doesn't. Even if the PM/AF policy of regime F performs best when at the ELB, it may perform much worse than AM/PF when the ELB is not a constraint on monetary policy. To assess under which policy regime social welfare is highest, we turn to a calibrated version of the model that can be used to conduct stochastic simulations.

2.5 Baseline Calibration

The simulations of the model reported in the next sections use the following baseline parameter values, which are summarized in Table 1. We set the discount factor $\beta = 0.995$ which implies a steady-state real interest rate ρ equal to 2% annual. We set $\sigma = 1$, $\varphi = 5$ and $\alpha = 0.25$. Setting $\vartheta = 1$ implies the utility of government purchases decreases at the same rate as the marginal utility of private consumption. Setting $\epsilon = 9$ implies a steady-state price markup of 12.5%. And we set $\theta = 0.75$ which is consistent with an average duration of price spells of one year (four periods in the model). As we normalize Y to 1, we set $C = 0.8$ and $G = 0.2$.

We set $b = 2.4$, which corresponds to a debt target equal to 60% of annual GDP. This value of b is chosen to be quantitatively comparable to the onset of the Great Recession and COVID recession, as we discuss later in Section 5. We calibrate the aggregate-demand shock in a benchmark AM/PF regime, by setting $\rho_z = 0.8$ to generate persistence and $\sigma_z = 0.028$ to obtain a frequency of the ELB near 25%. We set $\bar{m} = 0.85$ as in Gabaix (2020).[27] We summarize some of the empirical evidence on $\bar{m}$ in online appendix A.2.1.

The values for the parameters in the monetary and fiscal rules determine the policy regimes. For our baseline specification of an AM/PF regime, which we refer to as *regime M*, we set $\phi = 2$ for AM policy, and we set $\psi_\tau = 0.3$ and $\psi_g = 0$ for PF. This fiscal rule implies any increase in the debt-to-GDP ratio above its target is corrected about three quarters in one year by future taxes, in the absence of further deficits.[28] For the PM/AF regimes, we set $\phi = 0.4$, a value consistent with determinacy under cognitive discounting for all PM/AF regimes we consider. Table 2 summarizes the various combinations of ψ_τ and ψ_g we use in the simulations; these choices are discussed in Section 3.

We next present the outcomes of the model's stochastic simulations.

3 The Effects of Irresponsible Fiscal Stimulus Facing the ELB

The existing literature has shown that switching temporarily from active monetary policy to active fiscal policy is welfare improving whenever the ELB binds and expectation are rational. We use the stylized, calibrated New Keynesian model, given by (1)-(3) and (6)-(9), as a framework to study whether, as argued by Sims (2016), fiscal deficit finance can *replace* ineffective monetary policy when the economy faces frequent periods at the ELB, and whether Sims' argument holds in the presence of cognitive discounting. We examine the implications of permanently adopting a rule-based PM/AF regime that would appear to be "fiscally irresponsible" but serves as an automatic stabilizer that helps to offset aggregate-demand shocks when monetary policy is unable to respond.

We compare the model's outcomes, with and without the ELB constraint, under fiscal rules that cut net taxes as the debt level rises, raise government purchases as debt levels rise, or hold the primary surplus constant as debt levels rise. In the following section, we also

[27] See section E of Gabaix (2020, p. 2285) for a discussion of the empirical relevance of the value of $\bar{m} = 0.85$, which as he notes could be viewed as conservative.

[28] That is, given $\beta = 0.995$, $\psi_\tau = 0.3$, and $\psi_g = 0$, the debt condition (8) implies $\left(\beta^{-1} - \psi_\tau + \psi_g\right)^4 \approx 0.25$.

analyze whether the government's long-run debt target and the presence of long-term debt matter for the predictions of the model. Under each of these policy alternatives, we analyze the economy's adjustment to a contractionary shock that pushes down the natural rate of interest, aggregate demand and inflation. To rank the policy alternatives, we use (4), the model-consistent welfare measure of the costs of economic fluctuations arising from shocks buffeting the economy. We show that cognitive discounting leads to a large deterioration in the performance of PM/AF policy regimes.

3.1 Effects of Regime F and ELB without a Fiscal Response

The various policy scenarios we investigate are summarized in Table 2. For each scenario, the welfare costs of fluctuations and the frequency of the ELB are reported in Table 3, panel A with rational expectations ($\bar{m} = 1$) and panel B with cognitive discounting ($\bar{m} = 0.85$).

Scenario 1 is our benchmark representation of AM/PF regime 'M' (using the baseline calibration from Section 2.5). Monetary policy is active with a response to inflation given by $\phi = 2$. With $\psi_\tau = 0.3$ and $\psi_g = 0$, passive fiscal policy ensures net taxes adjust positively to movements in the level of debt. Given the discount factor β is 0.995, the coefficient on the lagged debt stock in the debt accumulation equation (8) is then smaller than unity, $\beta^{-1} - \psi_s \approx 0.7$, which ensures the debt level converges to the government's long-run debt target for any stationary inflation path. Scenario 2 ('no Tax or G') represents a PM/AF policy that holds both fiscal instruments constant, setting $\psi_\tau = \psi_g = 0$, implying $\psi_s = 0$. Hence, the coefficient on the lagged debt stock in the debt accumulation equation is larger than unity, $\beta^{-1} \approx 1.005$, implying an "irresponsible" debt outlook that will endogenously generate movement in expected inflation to ensure debt sustainability. We set $\phi = 0.4$ so monetary policy responds less than one-for-one to movement in inflation.[29]

To illustrate the implications of these policy scenarios, we report the dynamic responses under rational expectations with and without the ELB constraint when the economy experiences a negative three standard-deviation demand shock. Figure 3 shows the responses of key variables without the ELB constraint.[30] Scenario 1 (regime M) is denoted by red solid lines, while scenario 2 (no Tax or G) is denoted by blue dashed lines. Not surprisingly, regime M succeeds in stabilizing both inflation and the output gap much better than the PM/AF policy of scenario 2. This difference in the responses of those variables can be seen in the top row of the figure. The superior performance of regime M is reflected in the welfare loss from fluctuations, measured as a fraction of permanent consumption, as reported in panel A of Table 3. When the ELB constraint is ignored, the total welfare loss in regime M is about half that in the PM/AF scenario 2 (0.31% versus 0.64% of permanent consumption).

[29] This value of ϕ is low enough to guarantee determinacy under each of the PM/AF regimes we consider.

[30] In all the figures, variable are shown in quarterly rates (not annualized).

Of note under scenario 2, the lack of any fiscal response causes inflation to first become negative in the face of the contractionary demand shock; it then rises and turns positive before converging to zero (its steady state). The higher expected inflation this overshooting generates serves to ensure debt remains stationary. By contrast, in regime M inflation falls and then converges to zero from below. The inflation overshooting in scenario 2 means that expected future inflation will be higher than in regime M. Note also that regime M requires the nominal interest rate, expressed at a quarterly rate, to fall to -2% (-8% at an annual rate), clearly violating the ELB constraint.

Higher expected inflation is desirable at the ELB, because it helps to stabilize the economy. This suggests that the PM/AF scenario 2 may deliver improved performance relative to the regime M scenario 1 once the ELB is taken into account. As Figure 4 shows, when the ELB is accounted for, in the face of the negative demand shock both inflation and the output gap fall much *less* and inflation recovers more quickly in scenario 2 than in regime M. In the second row of the figure, the nominal interest rate barely touches the ELB under the PM/AF scenarios, but regime M remains at the ELB for 8 quarters. The real interest rate rises much less in scenario 2 than in regime M, serving to reduce the decline in the output gap seen under regime M.

Regarding the welfare implications of these policies when facing the ELB, as Table 3 reports, regime M now performs worse than scenario 2. Under regime M, the total welfare loss nearly triples when accounting for the ELB (jumping from 0.31% to 0.79% of permanent consumption). Instead in scenario 2, the total welfare loss hardly rises at all due to the ELB (it goes from 0.64% to 0.65%). A chief reason is that the ELB is encountered 25% of the time under regime M while less than 1% of the time under the PM/AF scenario 2. Thus, with the ELB and rational expectations, an active fiscal policy in which neither spending nor taxes react to the debt level yields a lower welfare loss than regime M.

But how sensitive is this conclusion to the assumption of rational expectations? Figure 5 repeats the scenarios from Figure 4 when rational expectations are replaced by cognitive discounting. Again, facing the ELB, the red solid line shows responses for scenario 1 (regime M) and the blue dashed line shows scenario 2 (no Tax or G). The response of the output gap is similar under scenarios 1 and 2 due to the ELB. Because cognitive discounting dampens the effects of future expectations, the total welfare loss rises slightly under regime M (from 0.79% to 0.81%), see Table 3. The loss due to inflation volatility in regime M actually falls, but this is more than offset by the increased loss from output gap volatility. With rational expectations, the decline in the output gap is partially offset by expected future increases in output and inflation; both these channels are muted under cognitive discounting.

While regime M is similar under rational expectations and cognitive discounting, this is not true for the fiscal regimes. As seen in Figure 5, with cognitive discounting the ELB now binds under the PM/AF scenarios, due to the reduced power of expectations of future higher inflation. However, a binding ELB is much less frequent than under regime M. The frequency of the ELB under the PM/AF scenario 2 rises significantly from 0.8% under rational expectations to 8.3% with cognitive discounting. This frequency is still much less than under regime M (27%). However, cognitive discounting leads to a tripling of the total welfare loss under the PM/AF scenario 2 (jumping from 0.65% to 1.99%). Hence, the apparent superiority of PM/AF policy under rational expectations disappears under cognitive discounting.

Policy scenario 2 held both fiscal instruments constant by setting $\psi_\tau = \psi_g = 0$. We next examine how the model outcomes are affected when these response coefficients are allowed to differ from zero in ways that would normally be considered "fiscally irresponsible," such as committing to cut net taxes and/or increase government purchases as the level of debt increases.

3.2 Seemingly Irresponsible Tax Cuts and Spending Hikes

Scenarios 3 to 5, labelled 'Tax', 'G' and 'G balanced', represent examples of active fiscal policies. Scenario 3 (Tax) holds government spending constant but cuts taxes as debt rises: $\psi_\tau = -0.3$, $\psi_g = 0$. Scenario 4 (G) involves increasing spending as debt rises while holding taxes constant: $\psi_\tau = 0$, $\psi_g = 0.3$. Both of these scenarios have the same effect on the primary surplus: $\psi_s = -0.3$. The values of ψ_τ and ψ_g are chosen to make them quantitatively comparable to the fiscal responses seen during the Great Recession and COVID recession, as we discuss later in Section 5. Instead scenario 5 (G balanced) increases both spending and taxes with the debt level to ensure the primary surplus remains constant: $\psi_\tau = \psi_g = 0.3$, $\psi_s = 0$. We pair all these active fiscal policies with a passive monetary policy ($\phi = 0.4$), the same as in the previous section.

While $\hat{s}_t$ follows the same fiscal rule in scenarios 3 and 4, they can have different effects on dynamics and welfare because $\hat{g}_t$ and $\hat{\tau}_t$ enter the model differently, as government purchases but not taxes affect the natural interest rate.[31] We find the effects of scenario 3 (Tax) and scenario 4 (G) on inflation and the output gap are very similar. To make the figures easier to read, therefore, we do not show dynamic responses for scenario 3; however, we do report welfare outcomes for scenario 3 in Table 3.

Figure 4 shows the responses of scenario 4 (green, dot-dashed lines) and scenario 5 (black dotted lines) under rational expectations with the ELB. Scenario 4 (G) results in the primary surplus falling as the debt level rises, the opposite of conventional wisdom that seeks to stabilize the level of debt by increasing the primary surplus if debt increases. When debt levels rise, Scenario 4 (G) corresponds, therefore, to using further debt issuance to finance the endogenous increase in spending, while in scenario 5 (G balanced) taxes are raised to finance spending.

[31] For our calibration, $\bar{\sigma} = 1.25$ and $\Gamma = 0.1515$, which implies $\Delta\hat{r}_t^{CD}/E_t\{\Delta\hat{g}_{t+1}\} = -\bar{\sigma}(1-\Gamma) = -1.0606$.

By keeping the real interest rate low, the G policy helps to stabilize both inflation and the output gap when the economy experiences a large, contractionary demand shock. As a result, despite a modest decline in the primary surplus of 0.6% necessary to fuel the debt-financed fiscal expansion, the G policy actually helps to *stabilize* the level of debt. In the bottom row of the figure, the debt-to-GDP ratio rises only 2.5% under the G policy, while it rises above 11% in regime M.

With rational expectations and the ELB, as panel A of Table 3 reports, the total welfare loss falls from 0.65% to just 0.28% in going from scenario 2 (no Tax or G) to scenario 4 (G) in which spending rises with the debt level. Both inflation volatility and output gap volatility are reduced by the G policy. Importantly, the ELB constraint never binds under the G policy and the weak response of monetary policy ($\phi = 0.4$).[32] Scenario 5 (G balanced) is less expansionary than G policy as it involves tax-financed changes in spending to maintain a constant primary surplus. Under this G balanced policy, there is a very small but positive frequency of a binding ELB. This occurs only 0.1% of the time. The G balanced policy generates a larger total welfare loss than G policy (0.44% versus 0.28%), but this loss is still less than that produced by regime M. Thus, adopting the standard assumption of rational expectations and with the ELB, the various scenarios ranked from best (lowest loss) to worst (highest loss) are "G, Tax, G balanced, and M."

The superior performance of the PM/AF polices is due to the positive inflation that they generate after the initial drop in inflation as a result of the negative demand shock. This contrasts sharply with regime M in which inflation remains negative throughout the convergence back to steady state. The credible commitment to a policy that generates positive inflation in the future has a large impact under rational expectations, the assumption underlying Figure 4 and panel A of Table 3. How sensitive are these rankings to deviations from rational expectations? The answer to this question turns out to be very sensitive.

As was seen comparing scenarios 1 and 2, results are quite different under cognitive discounting. The dampened impact of expectations affects both the comparison of PM/AF policies with regime M and the relative comparison among PM/AF policies. Figure 5 shows the effects of these scenarios under cognitive discounting with the ELB and can be compared to the rational expectations cases shown in Figure 4. With cognitive discounting, the G policy boosts inflation relative to regime M and relative to scenario 2 (no Tax or G). With expected future inflation also higher under the PM/AF policies, the real interest rate rises less (it actually falls under the G policy), and this leads to smaller drops in the output gap. When future expectations are discounted, fiscal measures (debt-to-GDP, purchases, net taxes, and primary surplus) under the PM/AF policies all display greater volatility than with rational

[32] If this PM/AF policy had involved a stronger monetary policy response to inflation, for example letting $\phi = 0.8$, the frequency of the ELB would be 10%, still significantly lower than under the traditional AM/PF regime in which the ELB frequency is 25%.

expectations. Because the higher debt level generates a positive wealth effect on aggregate demand, the output gap under the G policy eventually turns positive before converging back to steady state. This contrasts with the effects under rational expectations where the output gap converges without overshooting.

The welfare ranking of policies is dramatically altered if expectations are characterized by cognitive discounting, see Table 3. Rather than outperforming regime M, all the PM/AF policies do worse than regime M when facing the ELB constraint; moreover, the relative performance among the PM/AF policies is flipped.[33] With the ELB, the ranking of scenarios from best to worst changes from "G, Tax, G balanced, and M" under rational expectations to "M, G balanced, G, and Tax" with cognitive discounting. The PM/AF policies lead to significant increases in the welfare loss due to greater inflation volatility and greater output gap volatility compared to the outcomes under rational expectations. This is the case even though all the PM/AF policies significantly reduce the frequency of the ELB. For example, while this frequency is 27% under regime M, it is less than 9% under each of the PM/AF policies, and falls to as low as 4% under the G balanced policy.

Employing lower values of $\bar{m}$, leading to an additional weakening of the effects of future policies, would further exacerbate our finding that cognitive discounting causes a large deterioration in the performance of PM/AF policies, even though the frequency of a binding ELB is greatly reduced relative to an AM/PF policy. Modern models of monetary policy emphasize both rational expectations and the importance of expectations for the effectiveness of monetary policy. An important lesson from our analysis is that varying ones assumptions about expectations formation can reverse the ranking of alternative policy regimes.

4 The Role of the Debt Level and Its Duration

In this section, we focus on the G policy (scenario 4) in which spending rises with the debt level and evaluate the robustness of our results to the long-run debt target, and we extend the model to include long-term debt. We find that, in the face of contractionary aggregate-demand shocks that occasionally drive the nominal interest rate to the ELB, a regime of active fiscal and passive monetary policy can become more effective when the government adopts a high debt target and issues debt of short duration. We find this result holds under rational expectations, but with cognitive discounting the G policy becomes more effective with a high debt target and long-term debt.

[33] These results on active fiscal policies are consistent with the findings of Budianto, Nakata, and Schmidt (2023) that cognitive discounting can lead price-level and average inflation targeting to perform worse than inflation targeting under passive fiscal policies.

4.1 Does the Debt Target Matter?

We next analyze whether the government's long-run debt target affects the predictions of the model. Recall, from the debt accumulation equation, the effect of the real interest rate on the debt process depends on the long-run debt target, b. As (10) showed, when the nominal interest rate does not react, the impact of inflation on debt, conditional on the lagged debt stock, is increasing in b. This observation suggests that, under an active fiscal and passive monetary policy, the fluctuations in the inflation rate necessary to ensure debt remains stationary may be smaller when the debt target is higher.

Figure 6 compares the responses to a negative demand shock with the ELB for scenarios 4 (G) and 6 (G high b) which differ only in the calibrated value of b. Responses for the baseline value of b are shown by green lines (solid for CD, dashed for RE), and are the same as in the previous figures. Responses for the high debt target are shown by heavy blue lines. In scenario 4 the baseline value $b = 60\%$ as a share of annual GDP, whereas scenario 6 sets $b = 200\%$. In both cases, the policy regime sets $\psi_\tau = 0$ and $\psi_g = 0.3$ for AF, combined with $\phi = 0.4$ for PM. The responses of inflation and the output gap under rational expectations are little affected by the debt target. The fact that there is any effect only arises because expected spending affects the equilibrium real interest rate and the spending is debt financed ($\psi_\tau = 0$). With cognitive discounting, in addition, debt-financed spending has a wealth effect, see (3).

With the much-higher debt target in scenario 6 (G high b), the output gap responds similarly as for the baseline value for b under rational expectations. However, the responses of inflation differ slightly, with inflation overshooting less when b is higher. With a higher b, the debt-to-GDP ratio is more volatile and, as a consequence, so is the primary surplus. The impact of the higher b is visibly more pronounced under cognitive discounting. Cognitive discounting leads to a much larger initial drop in inflation followed by a larger overshooting. Still, the higher b dampens both the initial decline and the overshooting of inflation. The primary surplus falls the most, and the debt correspondingly rises the most, with cognitive discounting and a high debt target.

From a welfare perspective, the higher debt target *improves* stabilization policy under rational expectations and under cognitive discounting, due mainly to the lower inflation volatility, as Table 3 reports. Under cognitive discounting and the ELB the total welfare loss falls from 1.92% to 1.4% of permanent consumption with the higher debt target, while under rational expectations the loss reduction is from 0.28% to 0.23%. In both cases, outcomes are better with the high debt target. Moreover, because the high debt target renders the G policy more effective in stabilizing inflation, the frequency of the ELB is *reduced* (from 7.6% to 3.6%) with cognitive discounting.

Overall, not surprisingly, a higher debt target increases debt volatility. However, the implications for inflation stability are quite the opposite of conventional wisdom that seeks to stabilize the level of debt by increasing the primary surplus if debt increases. Under a credible commitment to an "irresponsible" fiscal rule that raises government purchases when debt levels rise, a higher debt target helps to stabilize inflation and to improve welfare when the ELB is a concern. This is true under either assumption about expectations.

4.2 Does the Presence of Long-Term Debt Matter?

The previous analysis was based on the assumption that all government debt took the form of one-period discount bonds. We next investigate whether the conclusions are affected if the government issues long-term debt. Harrison (2021), for example, finds that the presence of long-term debt can lessen the recessionary impact of a negative demand shock. Following Woodford (2001), we assume government bonds are perpetuities whose coupon declines at rate $\eta \in [0, 1]$ in each period.[34] We denote the price of the bond by Q_t.

In addition to these perpetuities, assume there are one-period bonds in zero net supply. The representative household's first-order conditions for the two types of bond imply

$$1 = \beta E_t^{CD} \left(\frac{1+i_t}{1+\pi_{t+1}} \right) \left(\frac{MUC_{t+1}}{MUC_t} \right) = \beta E_t^{CD} \left(\frac{1+\eta Q_{t+1}}{Q_t} \right) \left(\frac{1}{1+\pi_{t+1}} \right) \left(\frac{MUC_{t+1}}{MUC_t} \right), \quad (13)$$

where MUC_t denotes the marginal utility of consumption at time t. When these two Euler conditions are linearized around a zero inflation steady state, the relationship between the deviation of the one-period rate from its steady state, $\hat{\imath}_t$, and the deviation from steady-state of the one-period holding return on the perpetuity is given by

$$\hat{\imath}_t = \eta \beta E_t^{CD} \left\{ \hat{Q}_{t+1} \right\} - \hat{Q}_t = \eta \beta \bar{m} E_t \left\{ \hat{Q}_{t+1} \right\} - \hat{Q}_t, \quad (14)$$

where $\hat{Q}_t \equiv (Q_t - Q)/Q$ denotes the percent deviation of the bond price from its steady state, and the steady-state bond price is given by $Q \equiv \beta/(1-\eta\beta)$. The average duration of the bond is $1/(1-\eta\beta)$, which is increasing in η.

The recent literature typically assumes the average duration of U.S. government debt is 5 years. Given that our model is calibrated at a quarterly frequency and $\beta = 0.995$, this implies a value for η of 0.955. Relative to rational expectations, cognitive discounting implies a change in the one-period rate will cause, given the current price of the long-term bond $\hat{Q}_t$, a larger change in the expected future bond price to maintain equality between the expected holding period returns on the two financial assets.

In addition, the flow budget constraint for the government, when log-linearized around a

[34] Declining perpetuities of this form have been used by, for example, Chen, Cúrdia and Ferrero (2012), Bianchi, Faccini and Melosi (2022), Caramp and Silva (2023), and Harrison (2021), among others.

steady state with no trend growth and zero inflation, takes the more general form

$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} + \beta^{-1}b\left(\eta\beta\hat{Q}_t - \hat{Q}_{t-1} - \pi_t\right) + \hat{g}_t - \hat{\tau}_t. \tag{15}$$

When all debt is one-period, as in the previous analysis, $\eta = 0$ and $\hat{\imath}_{t-1} = -\hat{Q}_{t-1}$, implying (15) reduces to (5).

When $\eta > 0$, there are two key difference that distinguish (15) from (5). First, the (real) interest cost to service the debt outstanding is *increasing* in η. Namely, while a rise in the past bond price reflects a fall in the nominal yield, a rise in the current bond price results in a revaluation of long-term debt that increases the government's liabilities. Second, long-term debt also *amplifies* the impact of monetary policy on the bond price itself. Solving (14) forward gives

$$\hat{Q}_t = -E_t\sum_{i=0}^{\infty}(\eta\beta\bar{m})^i\,\hat{\imath}_{t+i}, \tag{16}$$

which shows that the current bond price depends negatively on the future path of the short-term nominal interest rate. But this relationship reduces to $\hat{Q}_t = -\hat{\imath}_t$ when $\eta = 0$.[35]

Figure 7 compares the responses to a negative demand shock with the ELB for scenarios 4 (G) and 7 (G long debt) which differ only in the debt duration. In scenario 7 the debt duration is set to 5 years ($\eta = 0.955$). Conditional on the assumption made about expectations, the presence of long-term debt has little impact. For either RE or CD, the inflation and output gap responses for short-term debt and long-term debt are virtually identical. With short-term debt, the nominal interest rate initially falls but then overshoots before converging to steady state. The bond price $\hat{Q}_t$ mirrors (with opposite sign) the path of the nominal interest rate. With long-term debt, the bond price falls more to reflect the discounted value of the entire path of the nominal interest rate, see (16).

The lower bond price generates a revaluation effect that acts to reduce the value of outstanding debt. However, this effect is more than offset by the fall in inflation which raises the real value of debt. The resulting rise in the debt-to-GDP ratio, in turn, implies more fiscal spending and further boosts the debt-to-GDP ratio. These effects are amplified under cognitive discounting; the nominal interest rate, the bond price, inflation and the output gap are all more volatile. Because the role of expectations about future equilibrium outcomes play a smaller role with CD, current endogenous variables must adjust more in equilibrium.

Regarding the welfare consequences of long-term debt, as Table 3 reports, under rational expectations scenario 7 (G long debt) performs marginally *worse* than scenario 4 (G), with the

[35] Define $\hat{Q}_t^{RE} = -E_t^{RE}\sum_{i=0}^{\infty}(\eta\beta)^i\,\hat{\imath}_{t+i}$ as the long-term bond price under rational expectations. Then under cognitive discounting, $\hat{Q}_t = -E_t^{RE}\sum_{i=0}^{\infty}(\eta\beta\bar{m})^i\,\hat{\imath}_{t+i} \geq -E_t^{RE}\sum_{i=0}^{\infty}(\eta\beta)^i\,\hat{\imath}_{t+i} = Q_t^{RE}$, so for a given path of the one-period rate, the price of a long-term bond is higher (the long-term rate is lower) under cognitive discounting than under rational expectations.

total welfare loss rising from 0.28% to 0.32% of permanent consumption if debt is long-term. However, scenario 7 with long-term debt still performs better than regime M, the AM/PF regime (which generates a welfare loss of 0.79%), *if expectations are formed rationally.*[36]

Welfare costs are much larger with cognitive discounting. Regime M dominates scenario 4 (G), as discussed previously. Scenario 7 (G long debt) also does significantly worse than regime M, more than doubling the welfare costs (from 0.81% to 1.71%). However, scenario 7 with long-term debt does perform slightly better than scenario 4 with short-term debt (1.71% versus 1.92%). This improvement is primarily due to the less frequent episodes at the ELB in scenario 7, leading to lower inflation volatility. The ELB frequency falls from 7.6% in scenario 4 to 6.6% in scenario 7.

Under either assumption about expectations, long-term debt can increase the welfare costs of output gap volatility in a PM/AF regime; although this adverse effect is more than offset by the lower welfare costs of inflation volatility in the presence of cognitive discounting (see Table 3). This outcome is the opposite of conventional wisdom that seeks to stabilize the government's burden of debt repayments by issuing debt instruments with longer maturity. Long-term debt can render the unfunded fiscal expansion less effective, because long-term debt dampens the wealth effects generated by rising debt levels. In welfare terms, however, the way expectations are formed determines whether in a PM/AF regime with long-term debt the greater costs of fluctuations in the output gap are more than offset by the greater stability of inflation and by the reduced frequency of the ELB.

5 Irresponsible Policy Responses in Recent Recessions

We use the data underlying Figure 1 to calibrate the fiscal rule parameters ψ_τ and ψ_g to study the consequences of "irresponsible" fiscal policies during the two past recessions through the lens of our model of Section 4.2 with long-term debt. The Great Recession (GR) scenario sets $\psi_\tau = -0.42$ and $\psi_g = 0.06$, combined with a passive monetary policy $\phi = 0.4$. The COVID scenario sets $\psi_\tau = -0.67$ and $\psi_g = 0.24$, maintaining $\phi = 0.4$.[37] We calibrate the debt target b to the debt-to-GDP level in the data at the start of each recession; this implies $b = 35\%$ for GR and $b = 80\%$ for COVID, as a share of annual GDP. In both scenarios, the debt duration is set to five years ($\eta = 0.955$). Table 4 summarizes the experiments, and Table 5 reports the outcomes with RE (panel A) and with CD (panel B). We discuss the results under rational

[36] Harrison (2021) finds that with an active fiscal policy setting an exogenous primary surplus ($\psi_g = \psi_\tau = 0$) and assuming prices are extremely rigid (average duration of price spells near two years), long-term debt leads to a welfare improvement regardless of the ELB.

[37] The Coronavirus Aid, Relief, and Economic Security (CARES) Act of 2020 was enacted during the COVID recession that ended in 2020Q2, while the American Rescue Plan (ARP) of 2021 was enacted after the recession ended. If the COVID period is extended to 2021Q2 to include the ARP, the calibration of the COVID scenario would be $\psi_\tau = -0.38$ and $\psi_g = 0.15$, implying $\psi_s = -0.53$, which is overall quite similar to the GR scenario.

expectations and then under cognitive discounting.

Figure 8 compares the responses to a negative demand shock for the GR scenario (heavy blue lines) and COVID scenario (green lines). Measured by the fall in the primary surplus as debt rises, the GR policy, with $\psi_s = -0.48$, might be expected to be less expansionary than the COVID policy, with $\psi_s = -0.91$. However, the two policies do a very similar job in stabilizing inflation and the output gap. Differences appear in the behavior of the fiscal variables themselves. Under the COVID policy, debt and the primary surplus move more. The reason is that a higher debt target increases the volatility in debt and government purchases, which serves to stabilize inflation in a PM/AF regime (see Section 4.1). Rows 1 and 2 of panel A in Table 5 report the welfare outcomes under rational expectations for the GR and COVID policies when $\phi = 0.4$. As expected, the higher debt target of the COVID policy serves to reduce a little inflation volatility, and thus improves welfare regardless of the ELB.

Macroeconomic outcomes depend on the fiscal rules *and* on the monetary policy rule, that is, on the value of ϕ, the response of the nominal interest rate to inflation. The GR and COVID scenarios considered so far set $\phi = 0.4$, implying that when the economy is away from the ELB, the nominal interest rate adjusts with inflation but by much less than one-for-one. While this behavior constitutes a passive monetary policy, it will have consequences for the economy when at the ELB. The expectation of a future recovery from the ELB will also generate expectations of a rise in the nominal interest rate, which, in turn, will *weaken* the current expansionary impact of higher expected inflation. Therefore, we also consider scenarios that set $\phi = 0$ to investigate the consequences of the central bank pegging the nominal interest rate to its steady state.[38]

Figure 9 shows the responses to a negative demand shock for the COVID policy with $\phi = 0.4$ (green lines) and with $\phi = 0$ (heavy blue lines). The latter scenario is labeled "COVID no MP" to indicate no response of monetary policy. The impact of the nominal rate peg on inflation is large. While inflation falls 1% when $\phi = 0.4$, it declines only 0.3% and returns to zero more quickly when $\phi = 0$. Under the nominal rate peg, the output gap also falls less and returns more quickly to zero. Because the nominal interest rate cannot adjust under the peg, the bond price remains at its steady state—that is, the duration of government debt plays no role under the peg. The resulting larger (real) interest expense to service the debt causes a larger rise in the debt-to-GDP ratio. With a larger rise in debt, government purchases rise more and net taxes fall more under the peg. Rows 3 and 4 of panel A in Table 5 report the welfare outcomes under rational expectations for the GR and COVID policies when $\phi = 0$. In both cases, pegging the nominal interest rate results in a large welfare improvement,

[38] With $\phi = 0$, (16) pins the bond price to its steady state value, $\hat{Q}_t = \hat{\imath}_t = 0$ for all t, so the debt condition (15) reduces to (5), which means that the duration of government debt is irrelevant for the outcomes under the nominal rate peg. Ascari, Florio and Gobbi (2023) find that with an active fiscal policy in which $\psi_g = \psi_\tau = 0$, setting $\phi < 0$ can lead to a welfare improvement.

due to the lower inflation volatility. As a result, the GR and COVID policies achieve the same welfare when the nominal rate is pegged. The nominal rate peg, by definition, also eliminates the occurrence of the ELB.

These results lead us to a final question that brings us back to our benchmark policy. Would an active fiscal policy combined with a peg on the nominal interest rate perform better than a standard regime of active monetary policy and passive fiscal policy? As we discussed earlier in this paper, scenario 1 is a standard representation of AM/PF policy, or regime M. The welfare results of that policy were reported in Table 3. In regime M, the total welfare loss from fluctuations was 0.79% and 0.31% respectively with and without the ELB. The loss is reduced to only 0.17% under both the GR and COVID policies when the nominal rate is pegged (Table 5 panel A). Hence, even if the ELB is ignored in the analysis, an active fiscal policy combined with a peg results in a large welfare improvement relative to regime M. The welfare gain achieved by an active fiscal policy and a peg is *several times larger* once the consequences of the ELB are taken into account. Furthermore, while the ELB inexorably binds frequently under an AM/PF regime, an active fiscal policy combined with a peg on the nominal interest rate would rule out episodes at the ELB.

These results are, however, significantly affected if rational expectations are replaced by cognitive discounting. Even when the ELB is ignored in the analysis, the GR and COVID policies under cognitive discounting lead to significant increases in the welfare loss due to inflation volatility and to output gap volatility compared to the outcomes under rational expectations (see Table 5). The loss under the GR policy increases from 0.33% to 2.33% (from 0.31% to 2.57% under the COVID policy) when moving from rational expectations to cognitive discounting. Losses are lower when accompanied by an interest rate peg, but are again much larger with cognitive discounting. Thus, as the welfare implications of a comparison of Table 3 and Table 5 suggests, super-active fiscal policies are much more dependent on the assumption of rational expectation than is a standard representation of AM/PF policy, or regime M.

6 Concluding Remarks

The challenges facing central banks in a low interest rate environment, when episodes at the ELB may be frequent and long lasting, are well known. Much is also known about the relative performance of alternatives to inflation targeting such as price-level targeting and average inflation targeting. The research on alternative policy frameworks has typically assumed the central bank can credibly commit to a policy rule and has *almost always* assumed that the broad framework of policy is one of active monetary policy and passive fiscal policy (AM/PF). It is this last assumption that we question. We show that, in the face of aggregate demand shocks and the ELB, a *credible commitment* to active fiscal policy and passive monetary policy

(AF/PM) can yield welfare gains. The superior performance of such a policy regime when monetary policy is constrained by the ELB outweighs the advantages of active monetary policy when the ELB is not a threat. In fact, we find that the incidence of the ELB is reduced to zero *and* the welfare costs of economic fluctuations are the lowest under an active fiscal policy and a nominal interest rate peg among the scenarios we consider, *if private sector agents form expectations rationally.*

The model we employ and the policy rules we analyze are stylized, but we think the results call into question the exclusive focus on monetary policy as the means of achieving inflation targets and maintaining macroeconomic stability. The fiscal rules we study involve seemingly irresponsible fiscal actions, that is, raising spending or cutting taxes as debt levels rise. Such actions generate expectations of the higher inflation necessary to ensure the government's real debt level remains stationary. Higher expected inflation helps offset a negative demand shock by reducing the real interest rate. At the ELB, monetary policy is limited in its ability to generate higher expected inflation; central banks can talk, but they cannot backup their statements if their primary policy instrument cannot be reduced. In contrast, the fiscal authority can always act because its instruments are not constrained by the ELB.

These results were robust to whether the government's long-run debt target was calibrated to equal 60% of annual GDP or to the much higher level of 200% of annual GDP. They were also robust to whether government debt was assumed to be of one-period duration or calibrated to match an average duration of 5 years. The advantage of super-active fiscal policy over active monetary policy was greatest when the debt target was high and the maturity of debt was short, *if expectations are formed rationally.*

The results are not, however, robust to the assumption of rational expectations. Rational expectations are key to why seemingly irresponsible fiscal actions may generate stabilizing movement in inflation expectations; they are also key to the performance of active monetary policy under "make-up" strategies such as price-level targeting or average inflation targeting. Replacing rational expectations with a model of cognitive discounting, which reduced the role of future expectations and introduced a new channel through which Ricardian equivalence fails to hold even with passive fiscal policy, had a significant effect on the results. Under cognitive discounting, the combination of PM/AF policies produce a much larger welfare loss than the AM/PF policy we examined. This outcome highlights the crucial role expectations play in evaluating alternative monetary and fiscal policies and emphasizes the importance of investigating the impact of deviations from rational expectations when assessing policy frameworks.

As is common in the literature, we assume for both AM/PF and PM/AF regimes that policymakers can commit to simple, implementable rules. While this assumption is widely accepted for independent central banks, a host of political issues arise in the case of fiscal policy. Changes in taxes and spending raise the issue of which taxes and which spending will be adjusted. The resulting choices have distributional implications whose political consequences may limit the ability to precommit to future fiscal actions. The debates over debt limits in the Euro area and in the U.S. are well known. Cutting taxes or raising spending when debt levels rise in a recession may gain more political popularity than implementing austerity as debt levels rise. However, the fiscal rules we analyze would call for tax increases and spending cuts when debt levels fall. Such policies might be feasible if debt levels are falling during economic booms. Regardless, one can question the ability of fiscal authorities to credibly commit to the types of policies we find would perform well in an environment of low interest rates, frequent episodes at the ELB, and rational expectations.

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Table 1: Baseline calibration.

Parameter	Description	Value
β	Discount factor	0.995
σ	Curvature of consumption utility	1
ϑ	Curvature of government purchases utility	1
φ	Curvature of labor disutility	5
ϵ	Elasticity of substitution of goods	9
α	Index of decreasing returns to labor	0.25
θ	Calvo index of price rigidities	0.75
G	Government purchases share of output	0.2
ϕ	{ 'Active' monetary policy response to inflation	2
	or 'passive' monetary policy response to inflation	0.4
ψ_τ	Fiscal policy, net taxes response to debt	0.3
ψ_g	Fiscal policy, purchases response to debt	0
b	Debt-to-GDP target	2.4
η	Bond coupon decay rate	0
ρ_z	Persistence of aggregate-demand shock	0.8
σ_z	Std. deviation of aggregate-demand shock	0.028
$\bar{m}$	Degree of cognitive discounting	0.85

Notes: Values are shown in quarterly rates. Shock volatility σ_z chosen to obtain under regime M a frequency of the ELB near 25%.

Table 2: Policy scenarios under regimes M and F.

Scenario	Policy coefficients ϕ	ψ_τ	ψ_g	b	η	Regime	Implications for a rise in debt
1. M	2	0.3	0	2.4	0	M	$\hat{\tau}_t$ hike
2. No tax or G	0.4	0	0	2.4	0	F	No fiscal response
3. Tax	0.4	−0.3	0	2.4	0	F	$\hat{\tau}_t$ cut, debt financed
4. G	0.4	0	0.3	2.4	0	F	$\hat{g}_t$ hike, debt financed
5. G balanced	0.4	0.3	0.3	2.4	0	F	$\hat{g}_t$ and $\hat{\tau}_t$ hike, balanced budget
6. G high b	0.4	0	0.3	8.0	0	F	$\hat{g}_t$ hike, debt financed
7. G long debt	0.4	0	0.3	2.4	0.955	F	$\hat{g}_t$ hike, debt financed

Notes: In regime F, $\psi_s \equiv \psi_\tau - \psi_g \leq 0$, i.e. super-active fiscal. The debt duration is one quarter if $\eta = 0$ and 5 years if $\eta = 0.955$.

Table 3: Welfare loss under regimes M and F.

Scenario	$\mathbb{L}$(%) no ELB Tot.	π_t	$\tilde{y}_t$	$\hat{g}_t$	$\mathbb{L}$(%) with ELB Tot.	π_t	$\tilde{y}_t$	$\hat{g}_t$	ELB freq. (%)
A. Rational expectations ($\bar{m} = 1$)									
1. M	0.31	0.30	0.01	0.00	0.79	0.74	0.05	0.00	25.0
2. No tax or G	0.64	0.57	0.07	0.00	0.65	0.58	0.07	0.00	0.8
3. Tax	0.31	0.28	0.03	0.00	0.31	0.28	0.03	0.00	0.0
4. G	0.28	0.24	0.04	0.00	0.28	0.24	0.04	0.00	0.0
5. G balanced	0.43	0.35	0.07	0.01	0.44	0.36	0.07	0.01	0.1
6. G high b	0.23	0.17	0.04	0.03	0.23	0.17	0.04	0.03	0.0
7. G long debt	0.31	0.27	0.05	0.00	0.32	0.27	0.05	0.00	0.0
B. Cognitive discounting ($\bar{m} = 0.85$)									
1. M	0.39	0.36	0.03	0.00	0.81	0.72	0.09	0.00	27.0
2. No tax or G	1.96	1.76	0.21	0.00	1.99	1.77	0.21	0.00	8.3
3. Tax	2.39	2.25	0.13	0.00	2.07	1.94	0.12	0.00	8.6
4. G	2.13	1.94	0.14	0.04	1.92	1.74	0.14	0.04	7.6
5. G balanced	1.33	1.07	0.19	0.08	1.35	1.08	0.19	0.08	4.0
6. G high b	1.43	1.07	0.15	0.22	1.40	1.04	0.15	0.21	3.6
7. G long debt	1.76	1.55	0.15	0.06	1.71	1.50	0.15	0.06	6.6

Notes: $\mathbb{L}$ is the permanent consumption loss from fluctuations. The total loss may differ from the sum of its components due to rounding.

Table 4: Great Recession and COVID, regime F scenarios.

Scenario	Policy coefficients ϕ	ψ_τ	ψ_g	b	η	Regime	Implications for a rise in debt
1) GR	0.4	-0.42	0.06	1.4	0.955	F	$\hat{\tau}_t$ cut and $\hat{g}_t$ hike, debt financed
2) COVID	0.4	-0.67	0.24	3.2	0.955	F	Idem
3) GR no MP	0	-0.42	0.06	1.4	0.955	F	Idem
4) COVID no MP	0	-0.67	0.24	3.2	0.955	F	Idem

Notes: In regime F, $\psi_s \equiv \psi_\tau - \psi_g \leq 0$, i.e. super-active fiscal. The debt duration is one quarter if $\eta = 0$ and 5 years if $\eta = 0.955$.

Table 5: Welfare loss under Great Recession and COVID, regime F scenarios.

Scenario	$\mathbb{L}$(%) no ELB Tot.	π_t	$\tilde{y}_t$	$\hat{g}_t$	$\mathbb{L}$(%) with ELB Tot.	π_t	$\tilde{y}_t$	$\hat{g}_t$	ELB freq. (%)
A. Rational expectations ($\bar{m} = 1$)									
1) GR	0.33	0.29	0.04	0.00	0.33	0.29	0.04	0.00	0.0
2) COVID	0.31	0.27	0.04	0.00	0.31	0.27	0.04	0.00	0.0
3) GR no MP	0.17	0.15	0.02	0.00	0.17	0.15	0.02	0.00	0
4) COVID no MP	0.17	0.15	0.02	0.00	0.17	0.15	0.02	0.00	0
B. Cognitive discounting ($\bar{m} = 0.85$)									
1) GR	2.33	2.20	0.13	0.00	2.13	2.00	0.13	0.00	9.4
2) COVID	2.57	2.41	0.14	0.02	2.35	2.19	0.14	0.02	10.4
3) GR no MP	0.87	0.80	0.07	0.00	0.87	0.80	0.07	0.00	0
4) COVID no MP	1.00	0.93	0.07	0.01	1.00	0.93	0.07	0.01	0

Notes: $\mathbb{L}$ is the permanent consumption loss from fluctuations. The total loss may differ from the sum of its components due to rounding. If no MP, $\phi = 0$ implies the policy rate is pegged to its steady state and therefore the ELB has no effects.

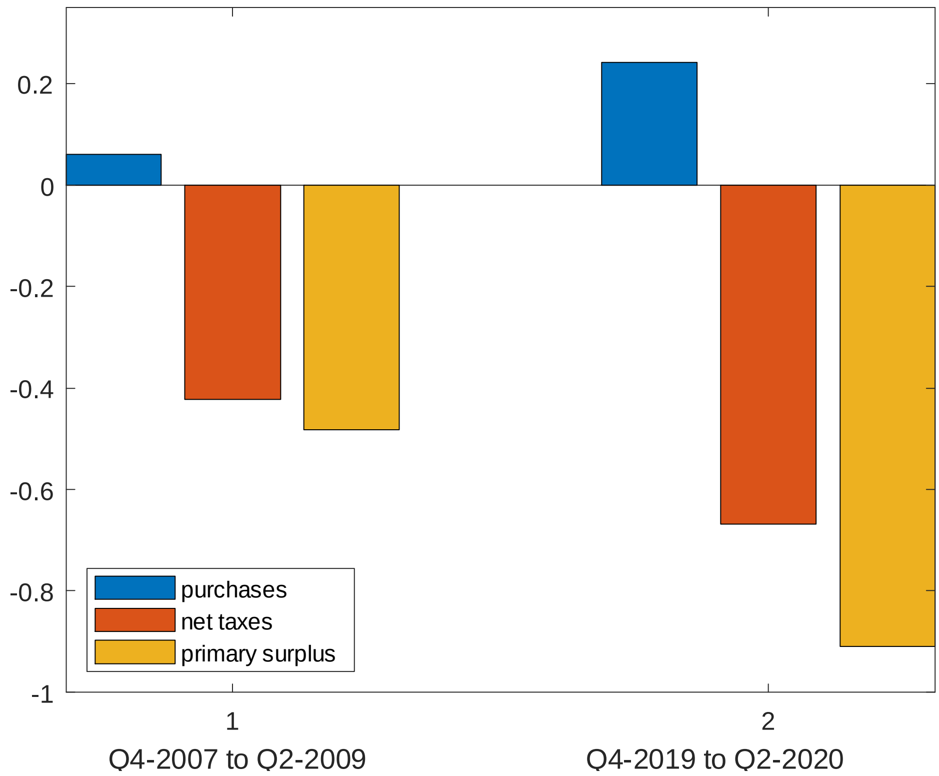

Figure 1: Composition of U.S. federal responses during the Great Recession (1) and COVID induced recession (2). Each bar is the change in category divided by change in debt held by the public. Data source FRED.

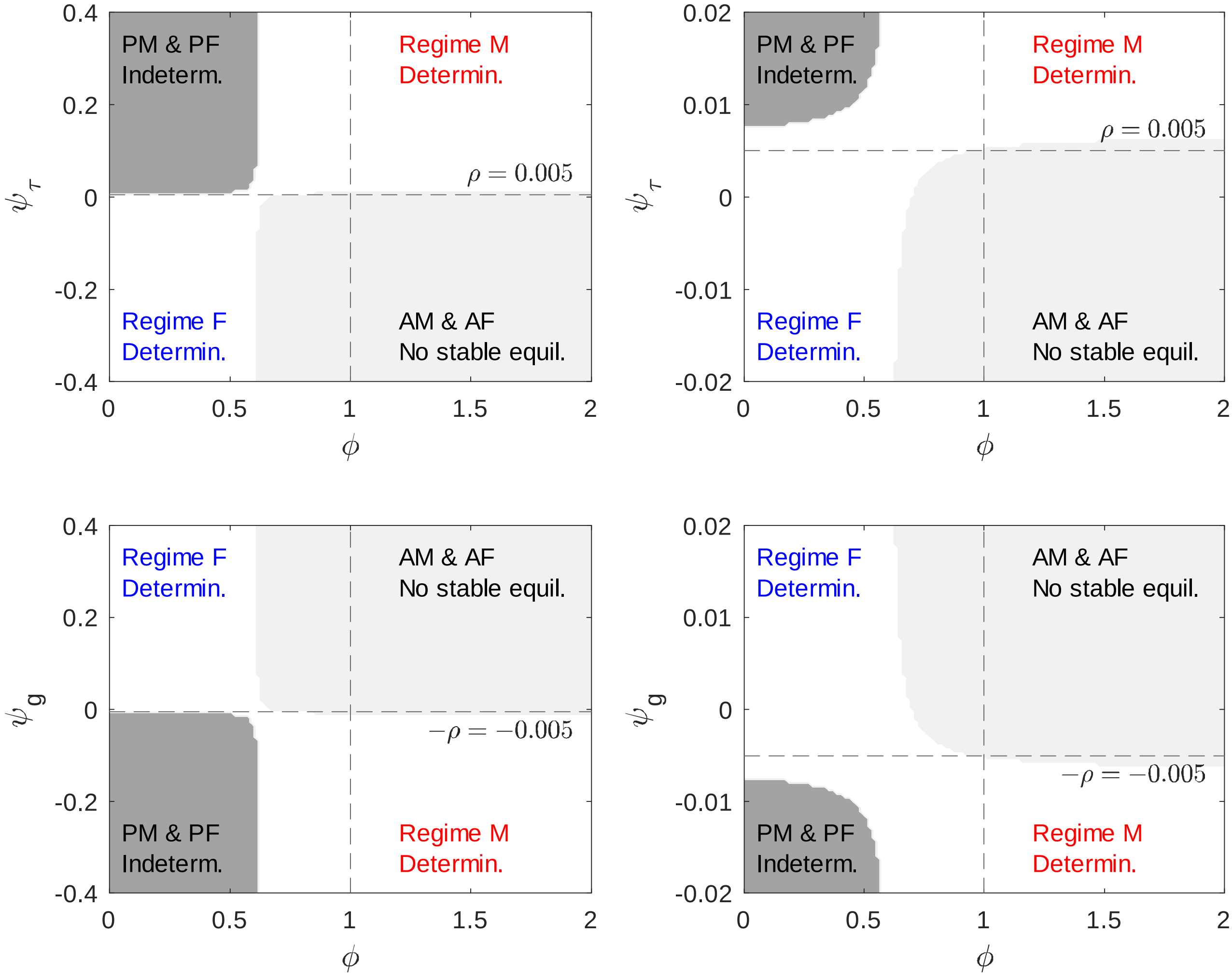

Figure 2: Equilibrium determinacy in regimes M and F with cognitive discounting ($\bar{m} = 0.85$). In the top row $\psi_g = 0$, while in the bottom row $\psi_\tau = 0$. The right column provides a close-up of the left column (note the change in the range for the fiscal parameter).

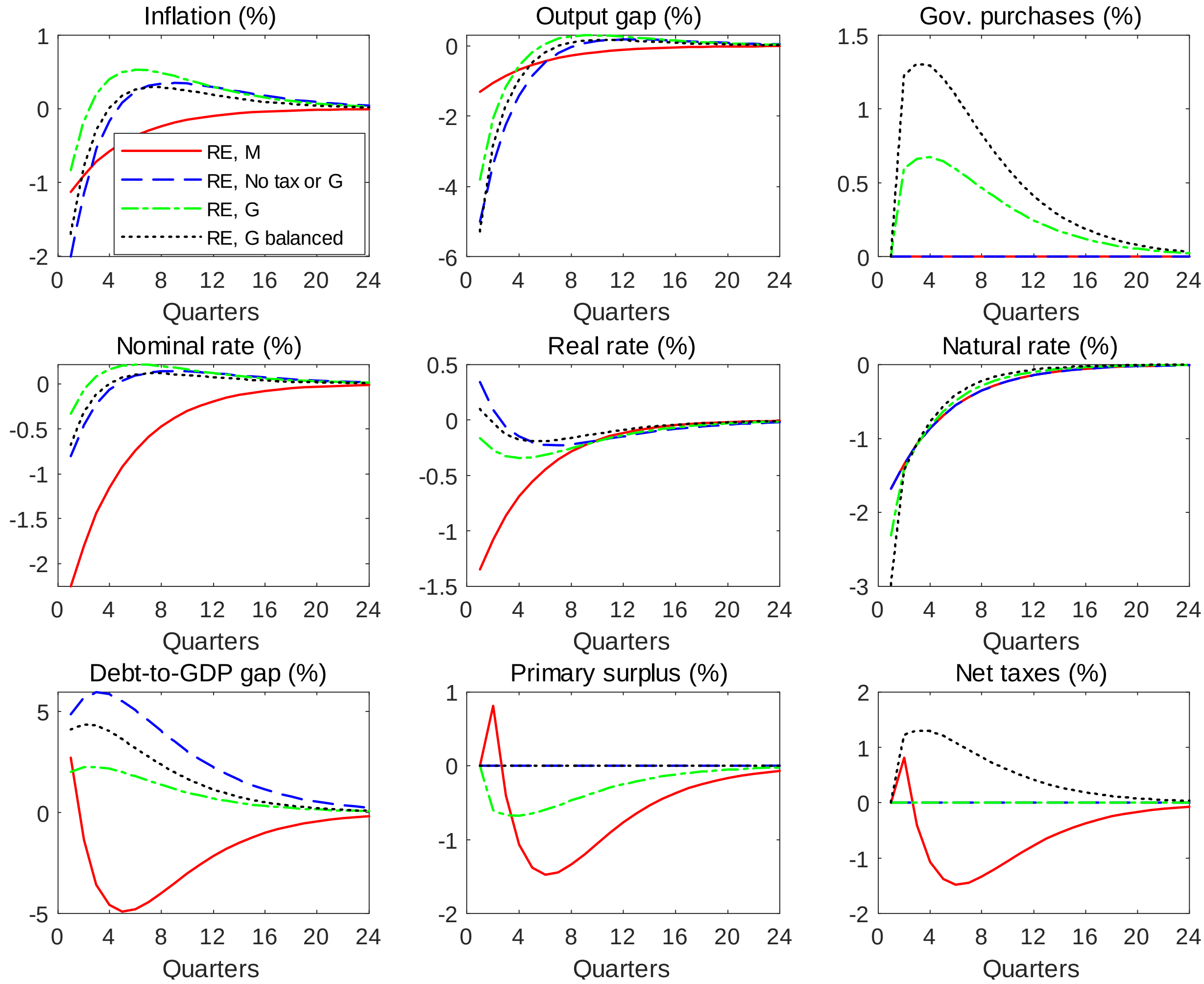

Figure 3: Dynamic effects of regimes M and F without ELB, under rational expectations (RE). Deviation from steady state in response to $-3sd$ demand shock.

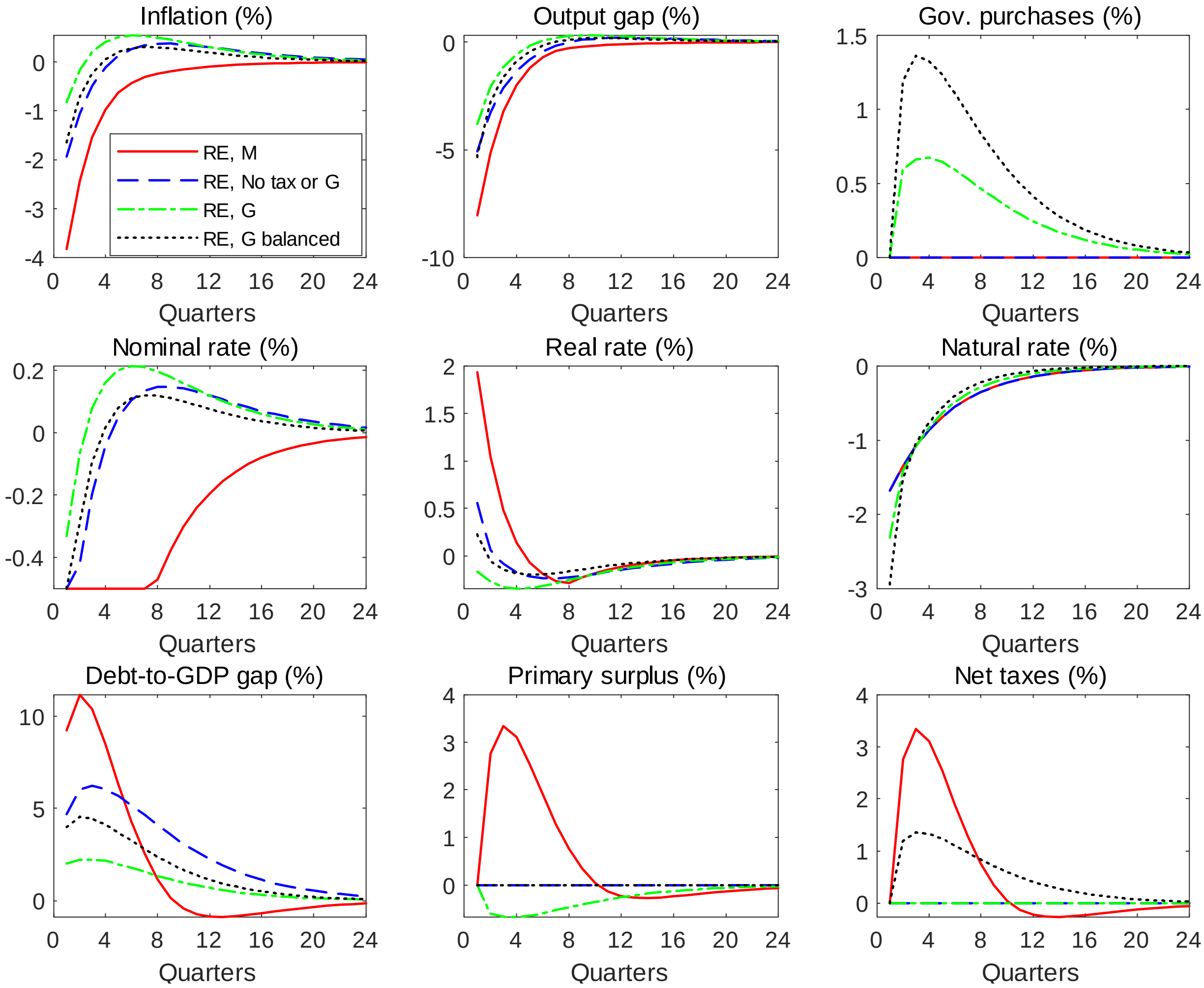

Figure 4: Dynamic effects of regimes M and F with ELB, under rational expectations (RE). Deviation from steady state in response to $-3sd$ demand shock.

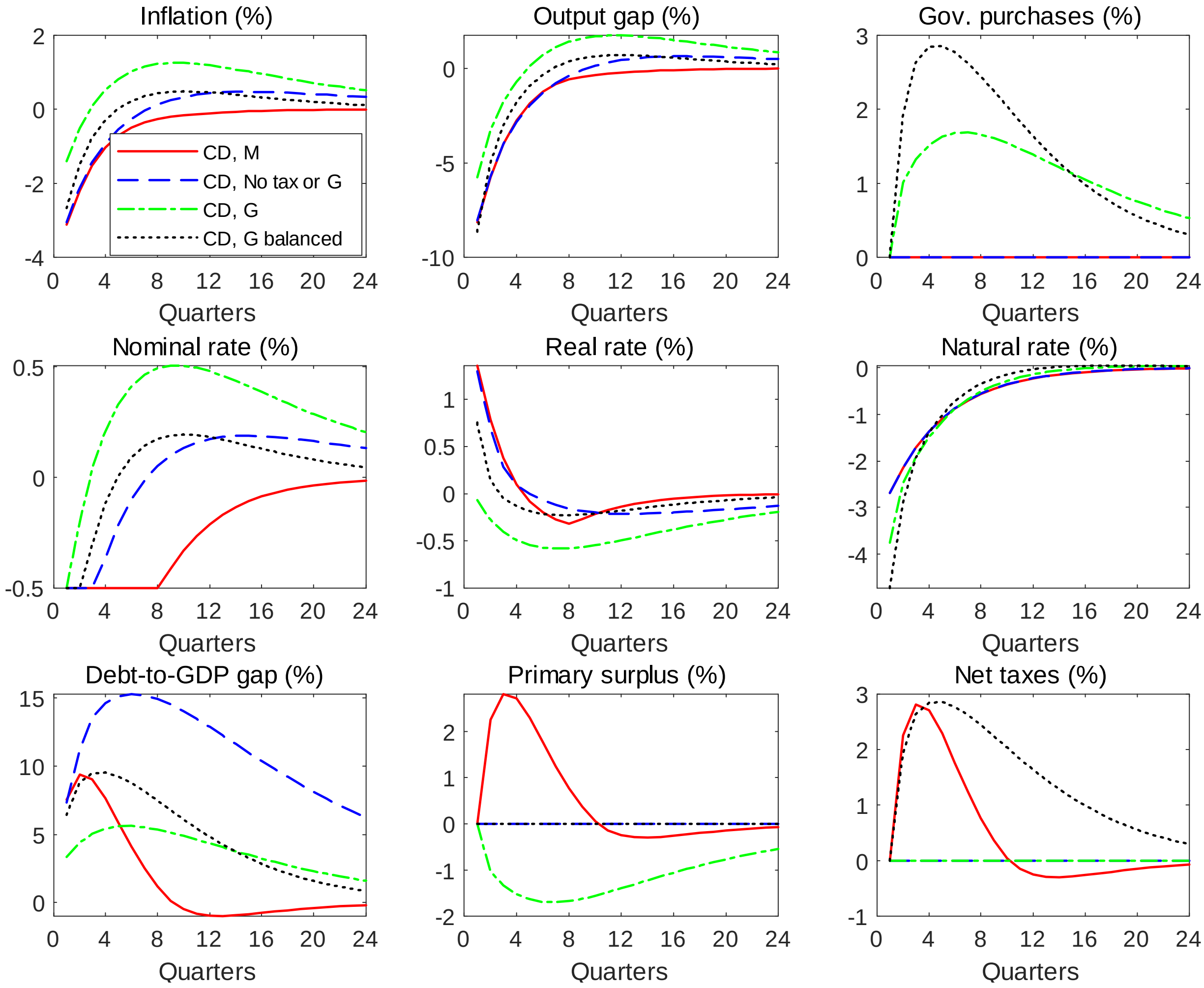

Figure 5: Dynamic effects of regimes M and F with ELB, under cognitive discounting (CD). Deviation from steady state in response to $-3sd$ demand shock.

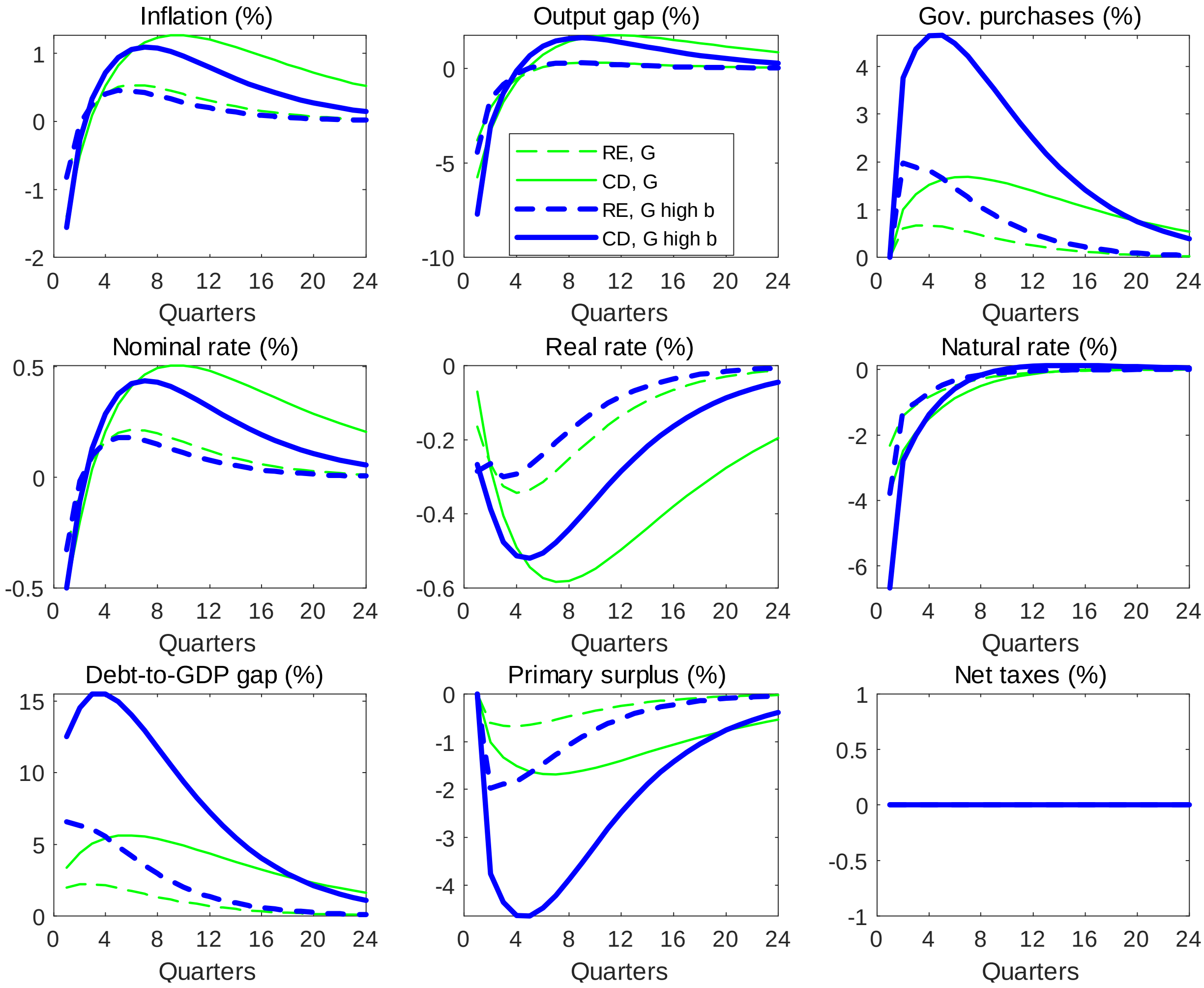

Figure 6: Dynamic effects of G and higher debt target ($b = 60\%$ versus 200% annual) with ELB, under rational expectations (RE) and cognitive discounting (CD). Deviation from steady state in response to $-3sd$ demand shock.

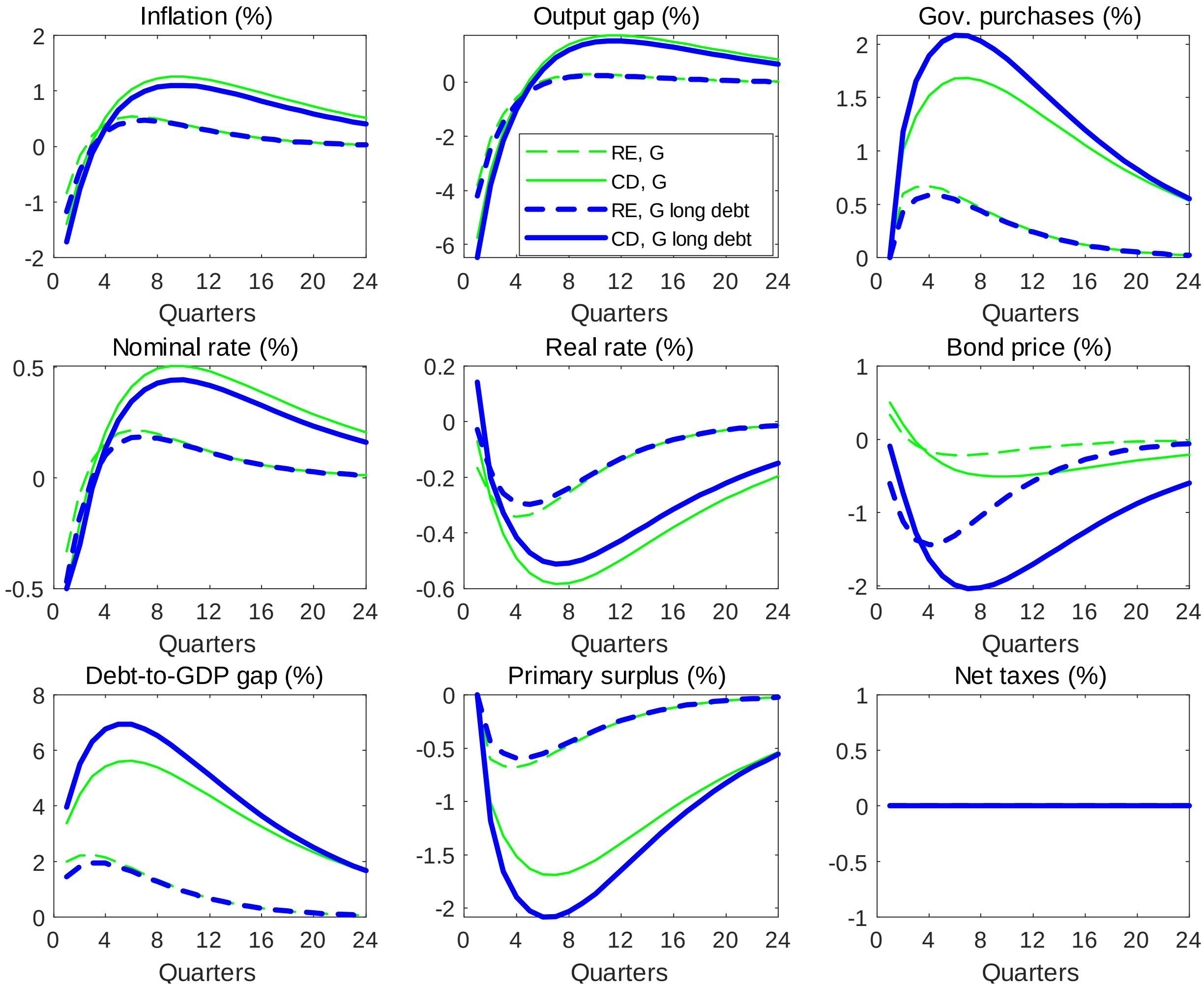

Figure 7: Dynamic effects of G and long-term debt (one-quarter versus five-year duration) with ELB, under rational expectations (RE) and cognitive discounting (CD). Deviation from steady state in response to $-3sd$ demand shock.

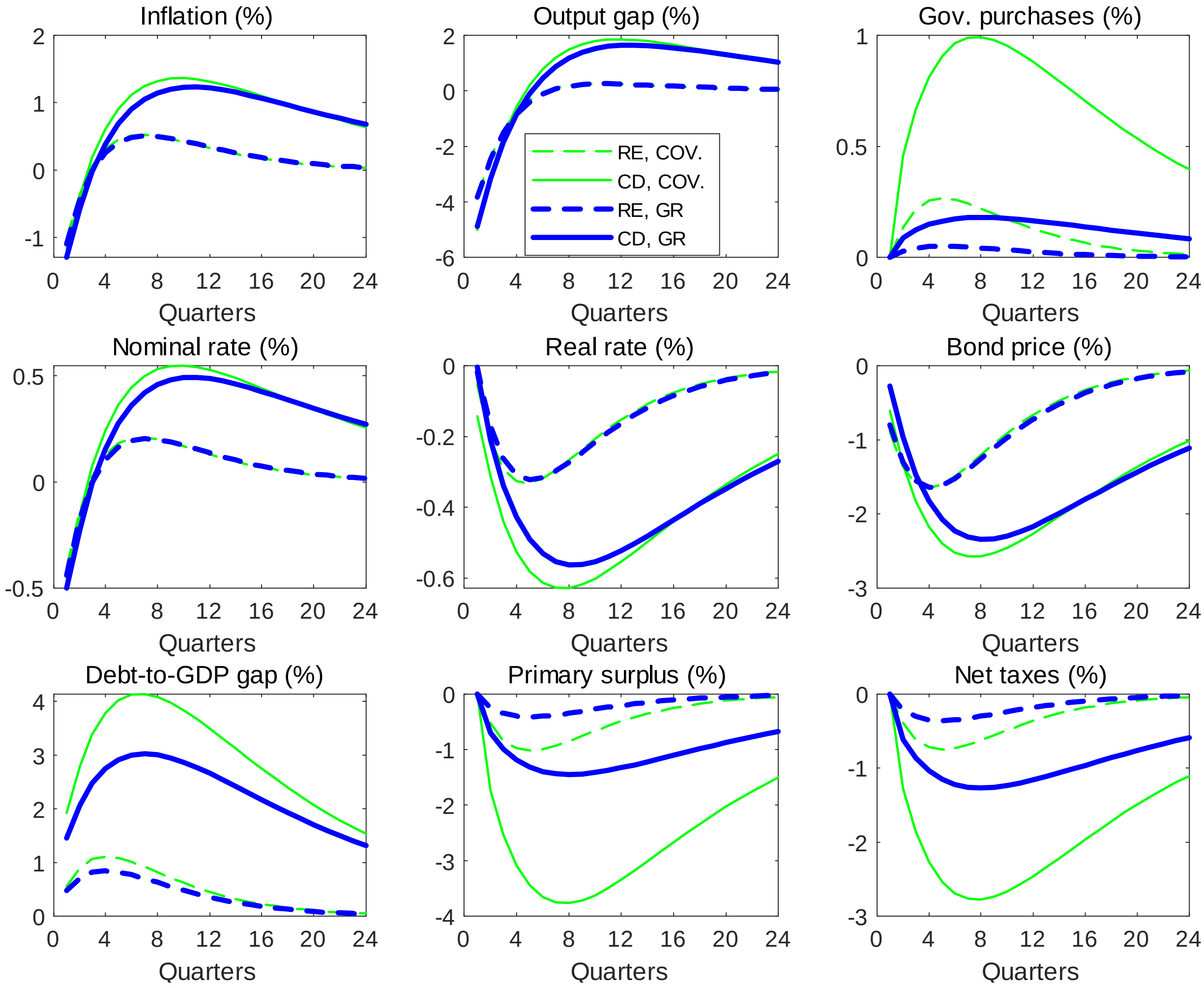

Figure 8: Dynamic effects of irresponsible fiscal stimulus as during Great Recession and COVID facing the ELB, under rational expectations (RE) and cognitive discounting (CD). Deviation from steady state in response to $-3sd$ demand shock.

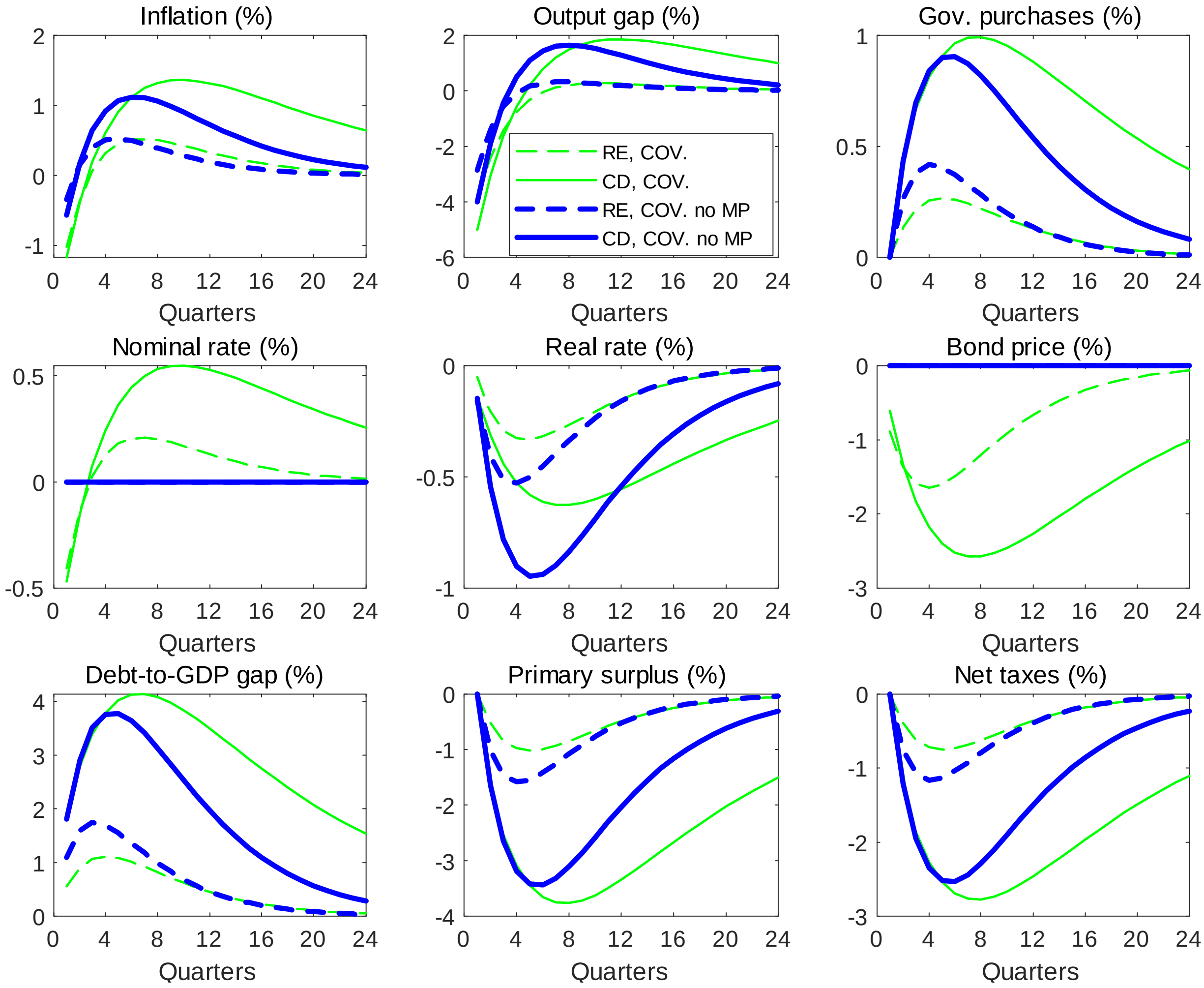

Figure 9: Dynamic effects of irresponsible fiscal stimulus as during COVID facing the ELB, with and without a monetary policy response, under rational expectations (RE) and cognitive discounting (CD). Deviation from steady state in response to $-3sd$ demand shock.

A Online Appendix to "Inflation, Fiscal Rules and Cognitive Discounting"; Not For Publication

In this appendix, we discuss equilibrium determinacy with a fiscal spending rule under rational expectations in the basic New Keynesian model of Section 2 with one-period debt, and then introduce cognitive discounting into the model and review evidence on the value of the cognitive discounting parameter $\bar{m}$. After modifying the New Keynesian Phillips curve (NKPC) and the IS equation, we discuss the effects of cognitive discounting on the model's determinacy conditions.

A.1 Determinacy with a Fiscal Spending Rule

The literature on active fiscal policy has focused on rules for lump-sum taxes. Conditions for determinacy in this case are well-known from Leeper (1991). Spending rules of the form (7) are less commonly analyzed. Because $\hat{g}_t$ appears in the definition of the natural rate of interest and therefore directly affects aggregate demand, (2), the determinacy conditions for spending rules potentially differ from those for tax rules.

When the definition of the natural interest rate and the policy rules for $\hat{\imath}_t$, $\hat{g}_t$ and $\hat{\tau}_t$ are substituted into (1), (2) and (5), the model under rational expectations ($\bar{m} = 1$) takes the form

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t,$$

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \frac{1}{\bar{\sigma}} \left(\phi \pi_t - E_t \pi_{t+1} \right) + \frac{1}{\bar{\sigma}} \left(1 - \rho_z \right) z_t - \left(1 - \Gamma \right) \psi_g \left(\hat{b}_t - \hat{b}_{t-1} \right),$$

and

$$\beta \hat{b}_t = \hat{b}_{t-1} + b \left(\phi \pi_{t-1} - \pi_t \right) + \beta \left(\psi_g - \psi_\tau \right) \hat{b}_{t-1}.$$

To assess the restrictions on the policy parameters that ensure a unique, stationary rational-expectations equilibrium, i.e. equilibrium determinacy, set the exogenous preference shock z_t to zero and write the model as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & \frac{1}{\sigma} & 1 & -\left(1-\Gamma\right)\psi_g \\ 0 & 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \pi_t \\ E_t \pi_{t+1} \\ E_t \tilde{y}_{t+1} \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & -\kappa & 0 \\ 0 & \frac{\phi}{\sigma} & 1 & -\left(1-\Gamma\right)\psi_g \\ b\phi & -b & 0 & 1+\beta\left(\psi_g - \psi_\tau\right) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \pi_t \\ \tilde{y}_t \\ \hat{b}_{t-1} \end{bmatrix}, \tag{17}$$

or

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \\ E_t\tilde{y}_{t+1} \\ \hat{b}_t \end{bmatrix} = C \begin{bmatrix} \pi_{t-1} \\ \pi_t \\ \tilde{y}_t \\ \hat{b}_{t-1} \end{bmatrix},$$

where

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\ \frac{b}{\beta}\phi(1-\Gamma)\psi_g & \frac{1}{\sigma}\phi - \frac{1}{\sigma\beta} - \frac{b}{\beta}(1-\Gamma)\psi_g & \frac{\kappa}{\sigma\beta}+1 & \sigma(1-\Gamma)\psi_g\left(\psi_g - \psi_\tau + \frac{1}{\beta} - 1\right) \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 & \psi_g - \psi_\tau + \frac{1}{\beta} \end{bmatrix}.$$

With two forward-looking variables, the Blanchard-Kahn conditions for determinacy require that two eigenvalues of C lie outside the unit circle.

The standard analysis of monetary and fiscal interactions assumes fiscal spending is exogenous and taxes follow a simple rule such as (6). Setting $\psi_g = 0$, the characteristic equation for the system is obtained by solving

$$\det \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 0 & \frac{1}{\beta} - \lambda & -\frac{\kappa}{\beta} & 0 \\ 0 & \frac{1}{\sigma}\left(\phi - \frac{1}{\beta}\right) & \frac{\kappa}{\sigma\beta} + 1 - \lambda & 0 \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 & -\psi_\tau + \frac{1}{\beta} - \lambda \end{bmatrix} = 0.$$

This determinant can be written as

$$-\lambda\left(-\psi_\tau + \frac{1}{\beta} - \lambda\right) \det \begin{bmatrix} \frac{1}{\beta} - \lambda & -\frac{\kappa}{\beta} \\ \frac{1}{\sigma}\left(\phi - \frac{1}{\beta}\right) & \frac{\kappa}{\sigma\beta} + 1 - \lambda \end{bmatrix} = 0. \tag{18}$$

One eigenvalue is $\lambda = 0$, another is $\lambda = 1/\beta - \psi_\tau$. The other two are determined by the determinant of the 2×2 matrix in (18), which is exactly that obtained in the basic New Keynesian model when debt is ignored; it has one eigenvalue outside the unit circle if $\phi < 1$ and two if $\phi > 1$. The condition for active fiscal policy is $|1/\beta - \psi_\tau| > 1$, or $\psi_\tau < \rho$.[39] Importantly, the conditions on ϕ for active and passive monetary policy do not depend on the fiscal policy parameter ψ_τ, and the conditions on fiscal policy do not depend on the monetary policy parameter ϕ.

The situation is different when $\psi_\tau = 0$ and $\psi_g \neq 0$. Now, the characteristic equation is

[39] Note that because it is the absolute value of the eigenvalues that matters, $1/\beta - \psi_\tau < -1$, or $\psi_\tau > 1/\beta + 1 = 2 + \rho$ is also consistent with an active fiscal policy. We rule out such a large positive tax response to debt and focus on $\psi_\tau < 2 + \rho$.

obtained from

$$\det\begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 0 & \frac{1}{\beta}-\lambda & -\frac{\kappa}{\beta} & 0 \\ \frac{b}{\beta}\phi\left(1-\Gamma\right)\psi_g & \frac{1}{\sigma}\phi-\frac{1}{\sigma\beta}-\frac{b}{\beta}\left(1-\Gamma\right)\psi_g & \frac{\kappa}{\sigma\beta}+1-\lambda & \sigma\left(1-\Gamma\right)\psi_g\left(\psi_g+\frac{1}{\beta}-1\right) \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 & \psi_g+\frac{1}{\beta}-\lambda \end{bmatrix}=0. \tag{19}$$

The term $\left(1-\Gamma\right)\psi_g$ reflects the presence of $\left(\hat{b}_t-\hat{b}_{t-1}\right)$ in the aggregate demand equation. However, the determinant on the left side of (19) equals

$$\left(\psi_g+\frac{1}{\beta}-\lambda\right)\det\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & \frac{1}{\beta}-\lambda & -\frac{\kappa}{\beta} \\ \frac{b}{\beta}\phi\left(1-\Gamma\right)\psi_g & \frac{1}{\sigma}\phi-\frac{1}{\sigma\beta}-\frac{b}{\beta}\left(1-\Gamma\right)\psi_g & \frac{\kappa}{\sigma\beta}+1-\lambda \end{bmatrix}$$
$$-\left[\sigma\left(1-\Gamma\right)\psi_g\left(\psi_g+\frac{1}{\beta}-1\right)\right]\det\begin{bmatrix} -\lambda & 1 & 0 \\ 0 & \frac{1}{\beta}-\lambda & -\frac{\kappa}{\beta} \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 \end{bmatrix}. \tag{20}$$

One can order the four eigenvalues from smallest (λ_1) to largest (λ_4) in absolute value; $|\lambda_1|<1$ and $|\lambda_4|>1$. Note that when $\psi_g+1/\beta=1$, $\lambda=1$ ensures the value of (20) is zero. If λ is increasing in ψ_g, then one eigenvalue would exceed 1 if $\psi_g+1/\beta>1$. Figure 10 shows $|\lambda_2|$ and $|\lambda_3|$ as a function of ψ_g. When $\phi=0.4$, monetary policy is passive and $|\lambda_2|$ is always less than 1 (blue dotted line). Determinacy then requires $|\lambda_3|>1$ which occurs for $\psi_g+1/\beta>1$ (heavy blue dotted line). Thus, one has determinacy with passive monetary policy when fiscal policy is active with $\psi_g>1-(1/\beta)=-\rho$. When monetary policy is active ($\phi=2.0$, shown in red), determinacy requires $\psi_g<-\rho$ (so that $|\lambda_2|<1$, $|\lambda_3|>1$), while no stationary equilibrium exists when $\psi_g>-\rho$ (so that $|\lambda_2|>1$, $|\lambda_3|>1$).

A.2 Introducing Cognitive Discounting

This section provides details of the model under cognitive discounting.

A.2.1 Macroeconomic Evidence on Cognitive Discounting in NK Models

We briefly review here some of the macroeconomic evidence on the value of the cognitive discounting parameter $\bar{m}$. Gabaix (2020, p. 2285) bases his calibration of $\bar{m}$ to 0.85 with reference to matching the inertia in the Phillips curve and IS curve found in Galí and Gertler (1999), Lindé (2005), and Fuhrer and Rudebusch (2004).

Those papers do not estimate $\bar{m}$ directly, but there have been attempts to estimate di-

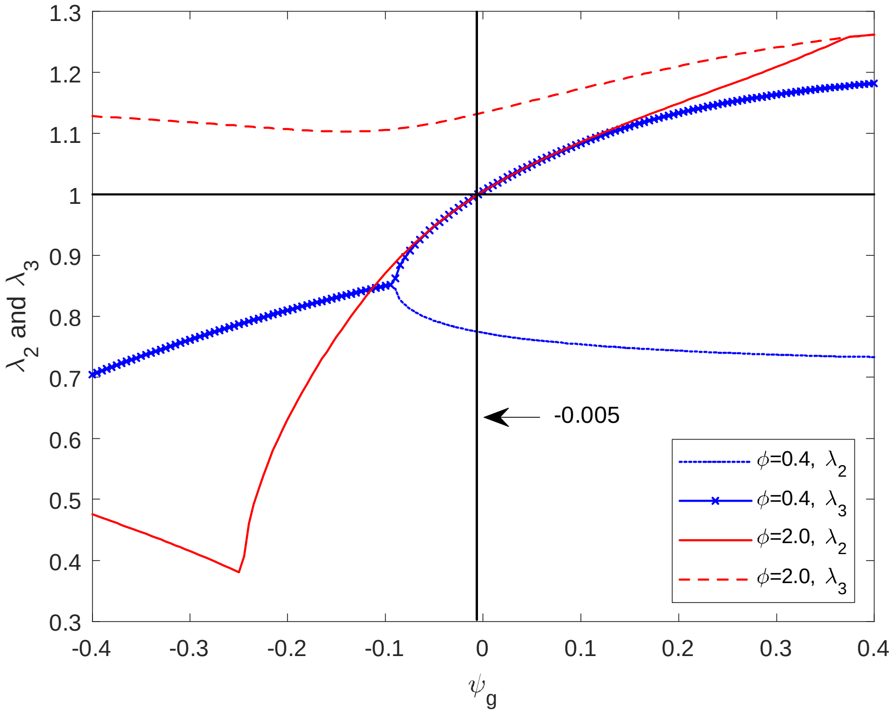

Figure 10: Eigenvalues λ_2 and λ_3 as a function of ψ_g for $\phi = 0.4$ (blue) and $\phi = 2.0$ (red). The vertical line is at $\psi_g = -\rho = -0.005$.

rectly the degree of cognitive discounting in New Keynesian models. For example, Andrade, Coredeiro, and Lambais (2019) provide maximum likelihood estimates for Gabaix's behavioral New Keynesian model and obtain a point estimate for $\bar{m}$ of 0.68 (standard deviation 0.07). This is significantly less than 1. They also employ estimation methods that are robust to weak identification. They find that $\bar{m}$ has an upper bound of 0.84 that rises to 0.95 when the wider confidence intervals associated with a robust estimator are used. Ilabaca, Meggiorini, and Milani (2020) estimate the behavioral New Keynesian model using U.S. macro data. Their estimates for the period 1982–2007 imply $\bar{m} = 0.71$ and $M^f = 0.41$ ($\bar{m} = 0.85$ and $M^f = 0.60$ for their pre-1979 sample).

Perhaps most relevant for the calibration of our model, given our focus on the ELB, is the work of Hirose et al. (2024). They incorporate the ELB and estimate the resulting non-linear New Keynesian model under the assumption of cognitive discounting. Their Bayesian estimation yields posterior means for $\bar{m}$ of 0.856 and 0.861 depending on the way the monetary policy rule is specified.[40]

These results are broadly consistent with the literature on approaches used to solve the forward guidance puzzle reviewed in Nakata et al. (2019).

[40] They assume a policy rule of the form $R_t = \max[R_t^*, 1]$ for the gross nominal rate where R^* is described by a Taylor rule with inertia. When the lagged nominal rate in the rule is R_{t-1}^*, they estimate $\bar{m}$ to be 0.856; when R_{t-1} is in the rule, the estimate of $\bar{m}$ is 0.861.

A.2.2 Derivation of the NKPC with Cognitive Discounting

The New Keynesian Phillips curve under cognitive discounting is derived by Gabaix (2020) for the case of constant returns to scale ($\alpha = 0$). We adapt the proofs of Lemma 2 and Proposition 2 in appendix X.B of Gabaix (2020, p. 2322) to deal with the case of decreasing returns to scale ($\alpha < 1$). This generalization only affects the mapping from real marginal cost to the output gap.

With rational expectations, Galí (2015) shows that the first-order condition for a price-setting firm under Calvo pricing takes the form

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[\Theta\mu_{t+k} + (p_{t+k} - p_{t-1})\right], \tag{21}$$

where μ is real marginal cost and $\Theta \equiv (1 - \alpha) / (1 - \alpha + \alpha\epsilon)$. Note that p_{t-1} has a coefficient of 1 on both sides, so add p_{t-1} and subtract p_t from each side to obtain

$$p_t^* - p_t = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[\Theta\mu_{t+k} + (p_{t+k} - p_t)\right]. \tag{22}$$

In addition, $p_t = \theta p_{t-1} + (1 - \theta) p_t^*$, implying $\pi_t \equiv p_t - p_{t-1} = (1 - \theta)(p_t^* - p_{t-1})$. Rearranging, $\pi_t = \left(\frac{1-\theta}{\theta}\right)(p_t^* - p_t)$.

Let $\bar{m}$ be the cognitive discounting factor, and replace rational expectations with the behavioral expectations operator E_t^{CD} (i.e. $E_t^{CD} x_{t+k} \equiv \bar{m}^k E_t x_{t+k}$ for any variable x_t) in (22) to obtain

$$\begin{aligned} p_t^* - p_t &= (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t^{CD} \left[\Theta\mu_{t+k} + (p_{t+k} - p_t)\right] \\ &= (1 - \beta\theta)\,\Theta \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k E_t\mu_{t+k} + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta\bar{m})^k E_t (p_{t+k} - p_t). \end{aligned} \tag{23}$$

Note that

$$p_{t+k} - p_t = \pi_{t+k} + \pi_{t+k-1} + \ldots + \pi_{t+1.}$$

Thus, the second summation in (23) can be written as

$$\begin{aligned}\sum_{k=0}^{\infty}(\beta\theta\bar{m})^{k}(p_{t+k}-p_{t}) &= 0+\beta\theta\bar{m}(\pi_{t+1})+(\beta\theta\bar{m})^{2}(\pi_{t+2}+\pi_{t+1})+(\beta\theta\bar{m})^{3}(\pi_{t+3}+\pi_{t+2}+\pi_{t+1})+\ldots\\ &= \left(\frac{1}{1-\beta\theta\bar{m}}\right)\left[\beta\theta\bar{m}\pi_{t+1}+(\beta\theta\bar{m})^{2}\pi_{t+2}+(\beta\theta\bar{m})^{3}\pi_{t+3}+\ldots\right]\\ &= \left(\frac{1}{1-\beta\theta\bar{m}}\right)\sum_{k=1}^{\infty}(\beta\theta\bar{m})^{k}\pi_{t+k}.\end{aligned}$$

Therefore,

$$\begin{aligned}\pi_{t} &= \left(\frac{1-\theta}{\theta}\right)(p_{t}^{*}-p_{t})\\ &= \left(\frac{1-\theta}{\theta}\right)(1-\beta\theta)\,\Theta\sum_{k=0}^{\infty}(\beta\theta\bar{m})^{k}E_{t}\mu_{t+k}+\left(\frac{1-\theta}{\theta}\right)\left(\frac{1-\beta\theta}{1-\beta\theta\bar{m}}\right)\sum_{k=1}^{\infty}(\beta\theta\bar{m})^{k}\pi_{t+k}.\end{aligned}$$

Using the forward operator F (i.e. $Fx_t \equiv x_{t+1}$), the first summation on the right of the equal sign is

$$\begin{aligned}E_{t}\sum_{k=0}^{\infty}(\beta\theta\bar{m})^{k}\mu_{t+k} &= E_{t}\sum_{k=0}^{\infty}(\beta\theta\bar{m}F)^{k}\mu_{t}\\ &= \left(\frac{1}{1-\beta\theta\bar{m}F}\right)\mu_{t},\end{aligned}$$

while the second summation is

$$\begin{aligned}\sum_{k=1}^{\infty}(\beta\theta\bar{m})^{k}\pi_{t+k} &= \sum_{k=0}^{\infty}(\beta\theta\bar{m}F)^{k}\beta\theta\bar{m}F\pi_{t}\\ &= \left(\frac{1}{1-\beta\theta\bar{m}F}\right)\beta\theta\bar{m}F\pi_{t}.\end{aligned}$$

Thus,

$$\pi_{t}=\left(\frac{1-\theta}{\theta}\right)(1-\beta\theta)\,\Theta\left(\frac{1}{1-\beta\theta\bar{m}F}\right)\mu_{t}+\left(\frac{1-\theta}{\theta}\right)\left(\frac{1-\beta\theta}{1-\beta\theta\bar{m}}\right)\left(\frac{1}{1-\beta\theta\bar{m}F}\right)\beta\theta\bar{m}F\pi_{t}.$$

Multiplying both sides by $\theta(1-\beta\theta\bar{m}F)$ yields

$$\theta(1-\beta\theta\bar{m}F)\,\pi_{t}=(1-\theta)(1-\beta\theta)\,\Theta\mu_{t}+(1-\theta)\left(\frac{1-\beta\theta}{1-\beta\theta\bar{m}}\right)\beta\theta\bar{m}F\pi_{t}.$$

Collecting terms, this yields the New Keynesian Phillips curve with cognitive discounting as

$$\pi_t = \beta M^f E_t \pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta \mu_t, \tag{24}$$

where

$$M^f \equiv \bar{m} \left[\theta + (1-\theta) \left(\frac{1-\beta\theta}{1-\beta\theta\bar{m}} \right) \right],$$

and where real marginal cost is proportional to the output gap, $\mu_t = \left(\bar{\sigma} + \frac{\alpha+\varphi}{1-\alpha}\right) \tilde{y}_t$, and the factor of proportionality depends on the returns to scale parameter α.

A.2.3 Derivation of the IS equation with Cognitive Discounting

To incorporate cognitive discounting into the IS equation, we initially simplify by ignoring the preference shock z_t that appeared in $\hat{r}_t^{CD}$ below (2). We use the results of Proposition 18, equation (133) on p. 5 of the online appendix to Gabaix (2020) and express current consumption as a function of current assets b_{t-1} and the expected future path of the real interest rate, consumption and transfers $\mathcal{T}_{t+k}$, to obtain

$$\hat{c}_t = \beta\rho\chi b_{t-1} + \beta E_t^{CD} \sum_{k=0}^{\infty} \beta^k \left[-\frac{1}{\sigma} \left(\hat{\imath}_{t+k} - E_t \pi_{t+k+1} \right) + \rho \hat{c}_{t+k} + \rho\chi \mathcal{T}_{t+k} \right], \tag{25}$$

where $\chi \equiv \varphi / (\varphi + \tilde{\sigma})$ with $\tilde{\sigma} \equiv \sigma (\omega h / c)$ arises from the endogenous response of labor hours to a transfer (which depends on the importance of wage income relative to total consumption) and $\beta^{-1} = R = 1 + \rho$.[41]

We consider the case in which debt reverts to a steady-state level b. Assume households correctly foresee the future taxes needed to service the interest cost of the steady-state level of debt b, so that

$$\mathcal{T}_{t+k} = -\beta\rho b + \hat{\mathcal{T}}_{t+k},$$

where $\hat{\mathcal{T}}_t^{CD}$ represents transfers that may depend on deviations of debt from its steady-state value. In this case, terms that involve b in (25) are

$$\beta\rho\chi \left[b - b\beta\rho \sum_{k=0}^{\infty} \beta^k \right] = \beta\rho\chi \left(1 - \frac{\beta\rho}{1-\beta} \right) b = 0,$$

and the steady-state debt level does not appear in the IS equation.

[41] Gabaix has the real interest rate multiplied by $-1/\sigma R^2$ as he approximates βR_t as $\beta R (1 + \hat{r}_t / R) = 1 + \hat{r}_t / R$ after using the fact that $\beta R = 1$. We instead use $\beta R_t = \beta (1 + r_t) = (1 + r_t) / (1 + r) \approx 1 + \hat{r}_t$. Thus, he obtains $-(1/\sigma R)(\hat{r}_t / R) = -\left(1/\sigma R^2\right) \hat{r}_t$ while we have $-(1/\sigma R) \hat{r}_t$.

Applying cognitive discounting, (25) can now be written in terms of $\hat{b}_t = b_t - b$ as[42]

$$\hat{c}_t = \beta\rho\chi\hat{b}_{t-1} + \beta E_t \sum_{k=0}^{\infty} \bar{m}^k \beta^k \left[-\frac{1}{\sigma} \left(\hat{\imath}_{t+k} - \bar{m}\pi_{t+k+1} \right) + \rho\hat{c}_{t+k} + \rho\chi\hat{\mathcal{T}}_{t+k} \right].$$

Using the forward operator F, this can be written as

$$\hat{c}_t = \beta\rho\chi\hat{b}_{t-1} + \beta E_t \left(1 - \beta\bar{m}F\right)^{-1} \left(-\frac{1}{\sigma} \left(\hat{\imath}_t - \bar{m}\pi_{t+1} \right) + \rho\hat{c}_t + \rho\chi\hat{\mathcal{T}}_t \right).$$

Premultiplying through by $1 - \beta\bar{m}F$, collecting terms involving $\hat{c}_t$, rearranging after noting that $1 - \beta\rho = 1 - \rho/\left(1+\rho\right) = 1/(1+\rho) = \beta$, and finally dividing by β gives

$$\hat{c}_t = \bar{m}E_t\hat{c}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \bar{m}E_t\pi_{t+1} \right) + \rho\chi\left(\hat{b}_{t-1} - \beta\bar{m}\hat{b}_t + \hat{\mathcal{T}}_t \right). \tag{26}$$

Let the lump-sum net transfer to households equal

$$\hat{\mathcal{T}}_t = b\left(\hat{\imath}_{t-1} - \pi_t \right) - \beta\hat{s}_t,$$

implying that the debt evolution equation (5) becomes

$$\hat{b}_t = \beta^{-1}\left[\hat{b}_{t-1} + b\left(\hat{\imath}_{t-1} - \pi_t \right) - \beta\hat{s}_t \right] = \beta^{-1}\left(\hat{b}_{t-1} + \hat{\mathcal{T}}_t \right).$$

Using this equation for $\hat{b}_t$ in (26), one obtains

$$\hat{c}_t = \bar{m}E_t\hat{c}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \bar{m}E_t\pi_{t+1} \right) + \rho\chi\left(1 - \bar{m}\right)\left(\hat{b}_{t-1} + \hat{\mathcal{T}}_t \right).$$

or

$$\hat{c}_t = \bar{m}E_t\hat{c}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \bar{m}E_t\pi_{t+1} \right) + \beta\rho\chi\left(1 - \bar{m}\right)\hat{b}_t. \tag{27}$$

The final step is to re-express (27) in terms of the output gap $\tilde{y}_t \equiv \hat{y}_t - \Gamma\hat{g}_t$. To do so, replace $\hat{c}_t$ with $(Y/C)\left(\hat{y}_t - (1-\Gamma)\,\hat{g}_t\right)$ to obtain

$$\left(\frac{Y}{C}\right)\left[\tilde{y}_t - (1-\Gamma)\,\hat{g}_t\right] = \bar{m}E_t\left(\frac{Y}{C}\right)\left[\tilde{y}_{t+1} - (1-\Gamma)\,\hat{g}_{t+1}\right] - \frac{1}{\sigma}\left(\hat{\imath}_t - \bar{m}E_t\pi_{t+1} \right) + \beta\rho\chi\left(1 - \bar{m}\right)\hat{b}_t,$$

which can be written as

$$\tilde{y}_t = \bar{m}E_t\tilde{y}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \bar{m}E_t\pi_{t+1} - \hat{r}_t^{CD} \right), \tag{28}$$

[42] Note that for $k = 0$, $E_t^{BR}\pi_{t+1} = \bar{m}E_t\pi_{t+1}$ which accounts for the $\bar{m}\pi_{t+k+1}$ term in brackets.

where, after adding back the preference shock z_t that appears in $\hat{r}_t^{CD}$,

$$\hat{r}_t^{CD} \equiv (z_t - \bar{m} E_t z_{t+1}) + \bar{\sigma}(1-\Gamma)(\hat{g}_t - \bar{m} E_t \hat{g}_{t+1}) + \bar{\sigma} b_d \hat{b}_t, \tag{29}$$

and, using $\chi = \varphi / (\varphi + \tilde{\sigma}) = \varphi / (\varphi + \bar{\sigma}(1-\alpha))$,[43]

$$b_d \equiv (1-\bar{m}) \beta \rho \left(\frac{C}{Y}\right)\left(\frac{\varphi}{\varphi + \bar{\sigma}(1-\alpha)}\right).$$

A.2.4 Equilibrium Determinacy with Cognitive Discounting

Under cognitive discounting, the model now consists of equation (24), (28), the debt equation (5), and the policy rules for the nominal interest rate, taxes, and government spending. Substituting out the policy variables and using the definition of $\hat{r}_t^{CD}$, yields the following three equations

$$\pi_t = \beta M^f E_t \pi_{t+1} + \kappa \tilde{y}_t,$$

$$\tilde{y}_t = \bar{m} E_t \tilde{y}_{t+1} - \frac{1}{\bar{\sigma}}(\phi \pi_t - \bar{m} E_t \pi_{t+1}) + \frac{1}{\bar{\sigma}}(1 - \bar{m}\rho_z) z_t + (1-\Gamma)\psi_g \left(\hat{b}_{t-1} - \bar{m}\hat{b}_t\right) + b_d \hat{b}_t,$$

$$\hat{b}_t = \left(\beta^{-1} - \psi_\tau + \psi_g\right)\hat{b}_{t-1} + \beta^{-1} b (\phi \pi_{t-1} - \pi_t).$$

Ignoring shocks, this system can be written as

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \beta M^f & 0 & 0 \\ 0 & \frac{1}{\bar{\sigma}}\bar{m} & \bar{m} & b_d - (1-\Gamma)\psi_g \bar{m} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ E_t \pi_{t+1} \\ E_t \tilde{y}_{t+1} \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & -\kappa & 0 \\ 0 & \frac{1}{\bar{\sigma}}\phi & 1 & -(1-\Gamma)\psi_g \\ \beta^{-1} b \phi & -\beta^{-1} b & 0 & \beta^{-1} - \psi_\tau + \psi_g \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \pi_t \\ \tilde{y}_t \\ \hat{b}_{t-1} \end{bmatrix}.$$

Under rational expectations ($\bar{m} = M^f = 1$ and $b_d = 0$). If $\psi_g = 0$ as in the usual analysis of lump-sum taxes only, the model becomes

$$\begin{bmatrix} \pi_t \\ E_t \pi_{t+1} \\ E_t \tilde{y}_{t+1} \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{1}{\beta} & -\frac{\kappa}{\beta} & 0 \\ 0 & \frac{1}{\sigma}\phi - \frac{1}{\sigma\beta} & \frac{\kappa}{\sigma\beta} + 1 & 0 \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 & \frac{1}{\beta} - \psi_\tau \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \pi_t \\ \tilde{y}_t \\ \hat{b}_{t-1} \end{bmatrix}.$$

This yields the standard result that one root is $\beta^{-1} - \psi_\tau$ which is stable if $\beta^{-1} - \psi_\tau < 1$ (i.e. $\psi_\tau > \rho$) and unstable if $\psi_\tau < \rho$.

When $\bar{m} < 1$ and $\psi_g \neq 0$, define $\Phi \equiv \left[b_d - (1-\Gamma)\psi_g \bar{m}\right]$ and $\psi_s = \psi_\tau - \psi_g$. The system

[43] Note that $\omega h / c = \left((1-\alpha) h^{1-\sigma}\right)/c = (1-\sigma)(Y/C)$ so $\tilde{\sigma} = (1-\sigma)\bar{\sigma}$.

becomes

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \\ E_t\tilde{y}_{t+1} \\ \hat{b}_t \end{bmatrix} = C \begin{bmatrix} \pi_{t-1} \\ \pi_t \\ \tilde{y}_t \\ \hat{b}_{t-1} \end{bmatrix},$$

where

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{1}{M^f\beta} & -\frac{1}{M^f}\frac{\kappa}{\beta} & 0 \\ -\frac{b}{\bar{m}\beta}\phi\Phi & \frac{\beta\phi-1}{\bar{m}\beta\sigma} + \frac{b}{\bar{m}\beta}\Phi & \frac{1}{\bar{m}} + \frac{1}{\bar{m}}\frac{\kappa}{\sigma\beta} & \frac{1}{\bar{m}}\psi_g(\Gamma-1) + \frac{1}{\bar{m}}\Phi\left(\psi_s - \frac{1}{\beta}\right) \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 & \frac{1}{\beta} - \psi_s \end{bmatrix}.$$

When $\psi_g = 0$, $\Phi = b_d$ and

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{1}{M^f\beta} & -\frac{1}{M^f}\frac{\kappa}{\beta} & 0 \\ -\frac{b}{\bar{m}\beta}\phi b_d & \frac{\beta\phi-1}{\bar{m}\beta\sigma} + \frac{b}{\bar{m}\beta}b_d & \frac{1}{\bar{m}} + \frac{1}{\bar{m}}\frac{\kappa}{\sigma\beta} & \frac{1}{\bar{m}}b_d\left(\psi_\tau - \frac{1}{\beta}\right) \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 & \frac{1}{\beta} - \psi_\tau \end{bmatrix}.$$

The characteristic equation is obtained from

$$\det \begin{bmatrix} -\lambda & 1 & 0 & 0 \\ 0 & \frac{1}{M^f\beta} - \lambda & -\frac{1}{M^f}\frac{\kappa}{\beta} & 0 \\ -\frac{b}{\bar{m}\beta}\phi b_d & \frac{\beta\phi-1}{\bar{m}\beta\sigma} + \frac{b}{\bar{m}\beta}b_d & \frac{1}{\bar{m}} + \frac{1}{\bar{m}}\frac{\kappa}{\sigma\beta} - \lambda & \frac{1}{\bar{m}}b_d\left(\psi_\tau - \frac{1}{\beta}\right) \\ \frac{b}{\beta}\phi & -\frac{b}{\beta} & 0 & \frac{1}{\beta} - \psi_\tau - \lambda \end{bmatrix} = 0.$$

The determinant on the left is equal to

$$\left(\frac{1}{\beta} - \psi_\tau - \lambda\right) \det \begin{bmatrix} -\lambda & 1 & 0 \\ 0 & \frac{1}{M^f\beta} - \lambda & -\frac{1}{M^f}\frac{\kappa}{\beta} \\ -\frac{b}{\bar{m}\beta}\phi b_d & \frac{\beta\phi-1}{\bar{m}\beta\sigma} + \frac{b}{\bar{m}\beta}b_d & \frac{1}{\bar{m}} + \frac{1}{\bar{m}}\frac{\kappa}{\sigma\beta} - \lambda \end{bmatrix}$$
$$+\frac{1}{\bar{m}}b_d\left(\psi_\tau - \frac{1}{\beta}\right)\left(\frac{1}{M^f}\frac{\kappa}{\beta}\right)\frac{b}{\beta}(\lambda - \phi).$$

Thus, even when only lump-sum taxes are employed as the fiscal policy instrument, the deviation from Ricardian equivalence resulting from cognitive discounting, as reflected in $b_d \neq 0$, implies there is no longer a clear separation in which the condition for active and passive fiscal (monetary) policy is independent of the monetary (fiscal) policy parameter. See Figure 2 of the paper for an illustration.