Are Asset Taxes Useful in Reducing Consumption and Wealth Inequality?

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Abstract

I analyze asset tax reforms in an overlapping generations economy with preference heterogeneity and find that asset taxes have a limited impact on consumption and asset heterogeneity, even when the tax revenues are redistributed to the less wealthy. Higher asset taxes raise both the risk-free rate and the equity premium, largely offsetting their redistributive benefits. The equity premium increases not only because taxing risky assets reduces the wealthy's willingness to bear risk but also because these assets are reallocated to less wealthy individuals with a lower risk tolerance.

Calibrating the model to the US economy, I show that a 10 percent tax on risky assets increases the equity premium from 5.2 percent to 6.2 percent, reduces consumption Gini only from 0.33 to 0.31, and asset Gini from 0.55 to 0.54. The reduction in inequality is significantly larger if asset price changes are not taken into account.

J.E.L Codes: E6, H2, G1
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1 Introduction

Rising wealth inequality in recent decades has motivated numerous tax proposals targeting wealthy individuals, with the apparent intention of reducing wealth and consumption inequality. For example, Kamala Harris, a Democratic presidential candidate, has proposed a tax on unrealized capital gains for wealthy individuals with a net worth of more than 100 million dollars.[1] Other recent proposals had similarly featured a tax on unrealized capital gains that affects only a wealthy segment of the population. The tax proposals typically take the asset prices and returns as given and do not address how they will respond to the proposed tax changes.

This paper shows that the equilibrium response of the asset prices substantially reduces the ability of progressive asset taxes to decrease consumption and wealth inequality. Introducing a tax on unrealized capital gains for the wealthy redistributed to the remaining population will increase the equity premium and the risk-free rate that will largely mitigate the distributional effects. In the example studied, a 10 percent tax on risky assets on the top one percent of the population will reduce the consumption and asset inequality only negligibly: The Gini coefficient of consumption decreases from 0.33 before tax to 0.31 after tax, while the Gini coefficient for assets decreases from 0.55 to only 0.54. In contrast, if one counterfactually keeps the equilibrium prices unchanged, consumption and asset inequality are reduced substantially more, to 0.27 and 0.43, respectively.

The fact that asset prices respond to such a tax is perhaps not surprising, and a basic intuition can be inferred from a simple representative agent economy: a tax on risky asset returns effectively decreases the willingness of the agents to bear market risk, making them appear more risk-averse. This increases the equilibrium return on the risky assets in an off-setting manner. I show that if the agents are heterogeneous in their preferences, there is an additional powerful channel through which the asset prices respond. A tax on the wealthy, who have a low-risk aversion, reallocates the risky assets toward the less wealthy, who have a high-risk aversion and require a higher risk premium. This further pushes the equilibrium returns on the risky assets up to the extent that even the after-tax returns on the risky assets increase for the wealthy. The tax revenue is redistributed to the less wealthy, but the amount is quantitatively relatively minor, increasing the present value of incomes by only about 1.5 percent for the 10 percent tax on the wealthy mentioned before.

On a conceptual level, the paper builds upon the continuous time, heterogeneous preference, and overlapping generation endowment economy of Gârleanu and Panageas (2015) and incorporates nonlinear taxes on both risk-free and risky assets. I show that the taxes manifest themselves as three types of wedges. Two of them distort the stochastic

[1] See e.g. https://taxfoundation.org/blog/harris-unrealized-capital-gains-tax.

discount factor different agents face: a *risk-free wedge* and a *risk wedge*. The risk-free wedge is a wedge between the market return of the risk-free asset and the return for the agent; the risk wedge is a wedge between the equilibrium market price of risk and the market price of risk faced by the agent. The third one, a *transfer wedge*, modifies the present value of resources that agents face at birth. The mapping between the taxes and wedges is somewhat complex: a tax on risk-free assets affects both the risk-free and a risk wedge, while the tax on risky assets affects only the risk wedge, but nonlinearly. In addition, the mapping between the taxes and wedges is affected by the details of the government's transfer policies. Specifically, tax revenue from the risky assets exhibits instantaneous volatility, which must be borne by some of the agents in the form of transfers, further affecting their willingness to bear the risk and the risk wedge.

The mapping between taxes and wedges may be complex, but it also allows for a helpful separation of the whole problem into two subproblems. One can study how the equilibrium allocations respond to any given wedges independently of the problem of how the taxes are mapped into the wedges. The separation thus allows for an assessment of the strength of the three channels that any tax reform represents.

I calibrate the model to the US economy to match the risk-free rate of 1.52 percent, equity premium of 5.2 percent, and the Sharpe ratio of 0.285. The model is populated by 99 percent of individuals that exhibit high risk aversion and low intertemporal elasticity of substitution, and one percent of individuals who exhibit low risk aversion and high intertemporal elasticity of substitution, and accumulate most assets. I show that a positive risk wedge on the one percent of wealthy individuals increases the risk-free rate and the market price of risk. The market price of risk increases because the wealthy agents appear less willing to bear more risk. As a result, the low-risk aversion wealthy agents reduce their exposure to the risky asset, while the high-risk aversion agents increase it. Since the high-risk aversion agents also have low intertemporal elasticity of substitution, the risk-free rate also increases. I also show that the risk wedge generates a decrease in consumption inequality and a hump-shaped response in wealth inequality.

An increase in the risk-free wedge similarly increases the equilibrium risk-free rate by making the wealthy agents effectively appear to have a lower intertemporal elasticity of substitution. However, the market price of risk decreases as the wealthy agents increase their relative weight. The risk-free wedge has a secondary effect on consumption inequality but substantially increases asset inequality. Finally, the transfer wedge has, for realistic values, only a secondary effect on both prices and inequality measures. This is perhaps surprising, but it follows from the fact that asset and consumption inequality in the model is generated by differences in the portfolio choices of various agents, and changes in the present value of transfers do not change the portfolio dynamics very much.

I consider two reforms: a 10 percent tax on risky assets and a 10 percent tax on both risky and risk-free assets, which would be imposed on the one percent of the wealthy population. The two tax reforms deliver fairly similar outcomes, with the main difference being that the second reform increases the risk-free rate more and redistributes fewer resources to the 99 percent of non-wealthy. In both cases, the market price of risk increases substantially, from 0.285 to 0.320 and 0.314, respectively. Both reforms also increase the risk premium, from 5.2% to 6.2% and 6.4%. The increase is substantial and is a combination of both a lower effective risk aversion of the wealthy and a more weight attached to the high risk aversion of non-wealthy agents. I decompose the increase to the two channels and find that around 70% of the increase is due to the distributional change, and only 30% is due to the remaining factors. Consumption Gini is reduced only negligibly, from 0.33 to 0.31 and 0.31. Asset Gini even increases in the second reform from 0.55 to 0.56, while decreasing in the first reform, but only to 0.54. In partial equilibrium, consumption Gini is reduced more substantially, to 0.270 and 0.289; asset Gini is also reduced substantially.

1.1 Related Literature

The model framework is similar to the continuous-time complete-markets model of Gârleanu and Panageas (2015) that features Duffie-Epstein preferences, overlapping generations, and two types of agents with different values of risk aversion and intertemporal elasticity of substitution. The framework is useful for this paper's research question for two reasons. First, the existence of overlapping generations ensures that all types survive in the long run, and the model has a generic nondegenerate distribution of wealth.[2] Second, the model can quantitatively match key asset pricing moments, including the risk-free rate, equity premium, Sharpe ratio, and return volatility. This is important if one wants to investigate the asset pricing implications of various tax policies quantitatively.

The assumption that preference heterogeneity is modeled by having only two types

[2]With infinitely lived agents, long run survival happens only in special cases, see, e.g., Anderson (2005).

is standard in the literature. Guvenen (2009) and Gomes and Michaelides (2007) use this assumption in discrete time to address asset pricing puzzles with limited risk-sharing opportunities. Anderson (2005) and Colacito et al. (2019) study a discrete-time version of the model to characterize the conditions under which long-run stationary distribution exists. Borovička (2019) studies an alternative mechanism that generates a long-run survival in a two-type economy based on differences in agents' beliefs about the distribution of aggregate endowment. Backus et al. (2008) study equilibrium asset trades in economies with two types exhibiting extreme preferences similar to the examples in this paper.

Gomez (2024) offer an alternative mechanism that disproportionately exposes a subset of the population to risky assets: an equity constraint that prevents entrepreneurs from complete diversification and exposes them to idiosyncratic return fluctuations. The paper investigates the interaction of asset prices and inequality; However, it does not study tax-related questions addressed in this paper.

Aguiar et al. (2024) also study capital gains taxation with an explicit intention of modeling asset price changes. However, the objectives of both papers are quite different. While their objective is normative, to characterize the optimal capital gains tax, this paper is a purely positive exercise examining the channels through which a given tax reform affects the economy. At the same time, this paper presents a quantitative theory while Aguiar et al. (2024) investigate the principles of optimal capital gains tax. The theoretical framework is also different: their paper emphasizes income heterogeneity, while the insights of this paper are built on the heterogeneity of preferences.

2 The Model

I consider a continuous time endowment economy, where the aggregate endowment y follows a geometric Brownian motion

$$\frac{dY}{Y} = \mu_Y dt + \sigma_Y dB, \tag{1}$$

where B_t is a univariate Brownian motion, μ_Y is the mean consumption growth, and σ_Y is the volatility.

The economy is populated by a measure one of the finitely lived agents, whose die at any time at a constant death rate π, as in Yaari (1965) and Blanchard (1985). A new cohort of mass π is born per unit of time so that the total population size remains at measure one. The agents are of two types, A and B, with measures α and $1-\alpha$. The two types differ in their preferences and productivity. The expected lifetime utility of the agents is represented recursively by

$$V_t^i = V_0^i + \mathbb{E}_t \left[\int_{s=t}^{\infty} f^i(C_s^i, V_s^i)\, ds \right], \quad i = A, B, \tag{2}$$

where $f^i(C_t, V_t)$ is the recursive utility aggregator that aggregates current consumption C_t and the lifetime utility V_t of the agent. The aggregator f^i takes the Duffie-Epstein-Win form, as in Duffie and Epstein (1992a,b):

$$f^i(C,V) = \frac{\delta+\pi}{1-\rho^i} \left[(1-\gamma^i)V\right]^{\frac{\rho^i-\gamma^i}{1-\gamma^i}} \left[C^{1-\rho^i} - \left[(1-\gamma^i)V\right]^{\frac{1-\rho^i}{1-\gamma^i}} \right]. \tag{3}$$

The parameter γ^i represents the relative risk aversion of type i, while $1/\rho^i$ is the intertemporal elasticity of substitution in utilities. Both agents have a common discount rate δ. Expected utility is recovered if $\gamma^i = \rho^i$, in which case $f^i = \delta C^{1-\rho^i}/(1-\rho^i) - \delta V$.

A share ω of the endowment y is distributed to the agents as the labor income. While all agents receive the same capital income, labor income depends on age and type. An agent of type i born at time s has labor earnings in period $t \geq s$

$$E_{t,s}^i = \omega \theta^i e_{t-s} Y_t,$$

where θ^A and θ^B are relative incomes of the two types, and e_a is an age-specific productivity profile common to both types for simplicity. We normalize the relative productivity of the two types by requiring that

$$\alpha\theta^L + (1-\alpha)\theta^H = 1,$$

and the life-cycle profile of earnings by assuming that

$$\pi \int_0^{\infty} e^{-\pi a} e_a\, da = 1. \tag{4}$$

The aggregate labor income of type i agents is $\omega\theta^i Y_t$, and aggregate labor income overall is ωY_t.

Assets. There are two tradeable assets: riskless bonds and risky stocks. The assets are continuously traded in frictionless markets. The riskless asset is in zero net supply and has a return of r. The risky asset, stock, pays dividends D representing a claim on a fraction $1-\omega$ of the aggregate output. The risky asset has a price P that evolves according to

$$dP = (\mu P - D)\,dt + \sigma P dB, \tag{5}$$

where μ and σ are drift and volatility parameters to be determined in equilibrium.

Taxes and Transfers. The bond and stock returns are subject to a tax. Both taxes are potentially time-varying and nonlinear, so the marginal tax rates the two types face can differ. We denote the tax rate on the bond returns faced by the type-i agent by $\tau^i_{B,t}$, and the tax rate on the stock returns faced by the type-i agent by $\tau^i_{P,t}$. To simplify the tax structure, it is assumed that dividends and capital gains are taxed at the same rate. It is also assumed that the government imposes the stock returns tax on unrealized capital gains on a continuous basis. Imposing the tax only on realized capital gains would introduce a nontrivial problem of choosing when to buy or sell stocks and would limit the tractability of the model. In addition to taxing assets, the government uses lump-sum transfers with a transfer flow $dTR^i_{t,s}$ to redistribute resources across types.

Since stock prices will exhibit instantaneous volatility and taxes collected will be proportional to stock prices, at least some transfers must exhibit instantaneous volatility as well.[3] I will specify the stochastic processes for the transfer flows more precisely when discussing the government budget constraint below.

Agents. Let $Z^i_{t,s}$ be the time-t financial assets of an agent of type i born at time $s \leq t$. The agents invest an amount $S^i_{t,s}$ of their financial wealth in the risky asset, while the remaining amount $Z^i_{t,s} - S^i_{t,s}$ into the risk-free asset. It is assumed that all agents enter annuities contracts. The contracts are actuarially fair, paying them $\pi Z^i_{t,s}$ while alive and collecting their remaining wealth upon death. The law of motion for the financial assets

[3] An alternative, not considered here is to assume that government consumption absorbs all instantaneous volatility and the transfers to households are risk-free.

is then

$$
\begin{aligned}
dZ^i_{t,s} = &\left[(1-\tau^i_{B,t})r_t\left(Z^i_{t,s}-S^i_{t,s}\right)+(1-\tau^i_{P,t})\mu_t S^i_{t,s}+\pi Z^i_{t,s}+E^i_{t,s}-C^i_{t,s}\right]dt+dTR^i_{t,s} \\
&+(1-\tau^i_{P,t})S^i_{t,s}\sigma_t dB_t.
\end{aligned} \tag{6}
$$

The agents start their life with zero initial financial wealth, that is, $Z_{t,t}=0$, and choose stochastic processes for consumption, financial assets, and stock holdings to maximize their lifetime utility (2) subject to the budget constraint (6) and the transversality condition, which is assumed to hold through the paper.

Government. The government does not consume resources and only redistributes resources across types. Given that the government has access to lump-sum taxes, it is without loss of generality to assume that the government continuously balances its budget constraint:

$$
\begin{aligned}
\alpha\, d\overline{TR}^A_t+(1-\alpha)\, d\overline{TR}^B_t = &\left[\alpha\tau^A_{B,t}\left(\overline{Z}^A_t-\overline{S}^A_t\right)+(1-\alpha)\tau^B_{B,t}\left(\overline{Z}^B_t-\overline{S}^B_t\right)\right]r_t\, dt \\
&+\left[\alpha\tau^A_{P,t}\overline{S}^A_t+(1-\alpha)\tau^B_{P,t}\overline{S}^B_t\right]\left(\mu_t\, dt+\sigma_t dB_t\right),
\end{aligned} \tag{7}
$$

where $\overline{Z}^i_t=\pi\int_{u=-\infty}^{t}e^{-\pi(t-u)}Z^i_{t,u}\, du$ is the aggregate financial wealth of type i and, similarly, $\overline{S}^i_t$ is the aggregate holding of stocks by type i and dTR^i_t are aggregate transfer flows to type i.

Many stochastic processes for the transfer flows satisfy the government's budget constraint (7). In what follows, I will somewhat limit the set of admissible processes by assuming that

$$
dTR^i_{t,s}=\mu^i_{TR,t,s}Y_t dt+\sigma^i_{TR,t}S^i_{t,s}dB_t, \tag{8}
$$

where $\mu^i_{TR,t,s}Y_t$ is the drift of the transfer flows, $\sigma^i_{TR,t}S^i_t$ is the volatility of the transfer flows. The equation (8) puts no restriction on the drift of the transfer process but limits the volatility to be proportional, for each type and age, to the risky asset holdings of the agents of the same type and age. The restriction means that government transfers will not effectively be able to redistribute risk among agents within a given type. It allows, however, to freely redistribute risk across types. The government budget constraint (7) restricts the transfer process's means and volatilities. The restriction on volatilities is, in particular, straightforward:

$$\alpha\sigma^A_{TR,t}\overline{S}^A_t + (1-\alpha)\sigma^B_{TR,t}\overline{S}^B_t = \left[\alpha\tau^A_{P,t}\overline{S}^A_t + (1-\alpha)\tau^B_{P,t}\overline{S}^B_t\right]\sigma_t.$$

One choice that does not redistribute risk across types at all sets $\sigma^i_{TR,t}$ equal to $\tau^i_{P,t}\sigma_t$. A second conceptual extreme sets $\sigma^i_{TR,t}$ to zero for one type and loads all the transfer risk to the other type.

$$\begin{aligned}\alpha\mu^A_{TR,t}Y_t + (1-\alpha)\mu^B_{TR,t}Y_t = {} & \left[\alpha\tau^A_{P,t}\overline{S}^A_t + (1-\alpha)\tau^B_{P,t}\overline{S}^B_t\right]\mu_t \\ & + \left[\alpha\tau^A_{B,t}\left(\overline{Z}^A_t - \overline{S}^A_t\right) + (1-\alpha)\tau^B_{B,t}\left(\overline{Z}^B_t - \overline{S}^B_t\right)\right]r_t\end{aligned}$$

Equilibrium. The equilibrium requires that the aggregate demand for stocks equals the stock value,

$$\alpha\overline{S}^A_t + (1-\alpha)\overline{S}^B_t = P_t, \tag{9}$$

and that the aggregate bond demand is zero or, equivalently,

$$\alpha\overline{Z}^A_t + (1-\alpha)\overline{Z}^B_t = P_t, \tag{10}$$

Finally, the consumption goods market clears when

$$\alpha\overline{C}^A_t + (1-\alpha)\overline{C}^B_t = Y_t, \tag{11}$$

with $\overline{C}^i_t$ again denoting aggregate consumption of a given type. For a given tax and transfer policy $\tau^i_{B,t}$, $\tau^i_{P,t}$ and $dTR^i_{t,s}$ satisfying the government budget constraint (7), the equilibrium consists, of stochastic processes for consumption $C^i_{t,s}$, financial assets $Z^i_{t,s}$ and stock holdings $S^i_{t,s}$, as well as the stochastic processes for the bond and stock prices r_t and P_t such that consumers choose $C^i_{t,s}$, $Z^i_{t,s}$ and $S^i_{t,s}$ to maximize (2) subject to (6) and the transversality condition and markets for stocks, bonds and consumption goods (9) - (11) clear.

2.1 A Representative Agent Economy

To understand the impact of taxes on the stochastic discount factor and asset prices, I start with an economy populated by infinitely lived representative agents. This special case is obviously limiting in that it does not allow one to think about how asset taxes affect inequality or the reallocation of resources. However, it permits a closed-form solution and determines a benchmark relationship between asset taxes and prices. The proof can be found in the Appendix.

Proposition 1. *Suppose that both types of agents have common preference parameters γ and ρ, identical productivity and that they are infinitely lived, with $\pi = 0$. Then, the equilibrium consists of the market price of risk and the risk-free rate*

$$\beta = \gamma \sigma_Y, \quad r = \frac{1}{1-\tau_B}\left(\delta + \rho \mu_Y - \gamma\left(1+\rho\right)\frac{\sigma_Y^2}{2}\right),$$

stock market volatility and expected stock return

$$\sigma = \sigma_Y, \quad \mu = \frac{1}{1-\tau_P}\left(\delta + \rho \mu_Y + \gamma\left(1-\rho\right)\frac{\sigma_Y^2}{2}\right),$$

and the stock price-to-output ratio $p = P/Y$ given by

$$p = \frac{(1-\tau_P)(1-\omega)}{g + \tau_P \mu_Y}.$$

There are several notable properties of the equilibrium. First, asset taxes do not affect the market price of risk or the pre-tax volatility of returns. The market price of risk does not change because, in an endowment economy, the tax policy cannot change the equilibrium volatility of consumption or the risk aversion of the representative agent. The tax policy does not affect the volatility of returns because of two effects. On the one hand, the tax reduces the after-tax volatility of returns, increasing the equilibrium pre-tax volatility. On the other hand, the agents receive transfers that are volatile by themselves, which reduces their willingness to bear stock market risk and the pre-tax volatility of returns. In equilibrium, both effects exactly cancel each other out, and the pre-tax volatility of returns is unchanged. Second, the risk-free rate and the expected stock market return increase proportionally with their respective taxes to keep the after-tax returns unchanged. It follows that the equity premium $\gamma\sigma_Y^2$ is also unaffected by the asset taxes.

Finally, as long as $\mu_Y > 0$, the stock price-to-output ratio decreases more than proportionally with the risky asset tax τ_P. This result is perhaps somewhat counterintuitive, but it is so because, in a growing economy, the expected tax on capital gains is positive since price increases are more common than price decreases. As a result, the tax from holding a risky asset amounts to more than just a tax on dividends.[4] It is straightforward to see that if $\mu_Y = 0$, and the economy is not growing, the stock price-to-output ratio decreases proportionally to the tax since the capital gains tax is expected to be zero, and only dividends are, in expectation, taxed.

2.2 Complete Markets with Distortions

The two-asset structure is sufficient to dynamically complete the markets, given that the economy is subject to a univariate Brownian motions shock (Harrison and Kreps (1979)). It is then convenient to characterize equilibria with complete markets using the Martingale approach, as adapted for the stochastic differential utility by Schroder and Skiadas (1999). I will, however, extend the approach to allow for the distortions from the government's tax policies. The formulation will be recursive. As in Gârleanu and Panageas (2015), the consumption share of type-A consumers at time t will serve as an endogenous state variable. The consumption share of type-A consumers, x_t, is [5]

$$x_t = \alpha \frac{\overline{C}_t^A}{Y_t}.$$

I will, from now on, restrict the tax policies to depend on time only indirectly through the aggregate state x_t; that is, the tax on bonds is $\tau_{B,t}^i = \tau_B^i(x_t)$, the tax on stocks is $\tau_{P,t}^i = \tau_P^i(x_t)$. Similarly, the drift and diffusion of the transfer flow will be such that $\mu_{TR,t,s}^i = \mu_{TR,t-s}^i(x_t)$ and $\sigma_{TR,t}^i = \sigma_{TR}^i(x_t)$. The equilibrium consumption share x_t then follows a diffusion process

$$dx_t = \mu_x(x)dt + \sigma_x(x)dB_t$$

[4] The expected present value of transfers exactly compensates for the decreased asset value. The ratio of the transfer flow to output dTR/Y is not constant, however, but follows a diffusion with drift $\mu_{TR} = \tau_P p \mu$ and volatility $\sigma_{TR} = \tau_P p \sigma_Y$.

[5] It follows from (11) that the consumption share of type-B consumers must be $1 - x_t$.

for some drift and diffusion functions $\mu_x(x)$ and $\sigma_x(x)$ that will be determined in equilibrium.. The equilibrium stochastic discount factor that an agent i faces follows a diffusion process

$$\frac{d\zeta^i}{\zeta^i} = -\left(1 - \lambda_r^i(x)\right) r(x)dt - \left(1 - \lambda_\beta^i(x)\right) \beta(x)dB \tag{12}$$

for some functions $r(x)$ and $\beta(x)$ representing the risk-free rate and the market price of risk. The functions r and β are common to both types of agents. Distortive taxes drive a wedge between the stochastic discount factor the two agents face. Although the *risk-free wedge* λ_r^i equals directly to the tax on the bond returns, $\lambda_r^i = \tau_B^i$, we will keep the notation for the two separate to highlight the conceptual difference. The wedge on the market price of risk β, call *risk wedge* λ_β^i, is related to both the tax on bonds τ_B^i, tax on stocks τ_P^i as well as the volatility of transfers σ_{TR}^i in a complex way:

Proposition 2. *The risk wedge satisfies*

$$(1 - \lambda_\beta^i)\beta = \frac{\left(1 - \tau_P^i\right)\mu - \left(1 - \tau_B^i\right) r}{\sigma_{TR}^i + \left(1 - \tau_P^i\right)\sigma}. \tag{13}$$

The tax on capital gains τ_P^i reduces both the risk faced by the agents and the after-tax returns. On the net, it decreases the risk faced by the agents. Similarly, an increase in transfer volatility σ_{TR}^i reduces the risk wedge or the market price of risk by effectively increasing the instantaneous volatility of incomes.[6] The tax on the bonds also tends to reduce the risk wedge. If the marginal tax rates on stocks and bonds are equal, then the equity premium $\mu - r$ is reduced proportionally to the common rate, and the risk wedge is also equal to the common rate.

In what follows, I will treat the risk wedge λ_r^i and the risk-free wedge λ_β^i as given and characterize the equilibrium conditional on the wedge. Equation (13) can then be used to recover the underlying tax on the risky asset τ_P^i.

Agent's problem. The agent's problem will now be reformulated as a problem of maximizing the lifetime utility subject to the present value budget constraint, with future flows discounted by the stochastic discount factor (12). The agent's total wealth consists

[6]The assumption that the volatility of transfers is proportional to the stock holdings for each age and type is critical in deriving (13).

of the financial assets and the present value of earnings and transfers. Define $W^i_{t,s}$ to be the time-t total wealth of an agent born in s as

$$W^i_{t,s} = Z^i_{t,s} + Y_t h^i_{t,s}\left(1 + \lambda^i_{TR,t,s}\right) \tag{14}$$

where $Z^i_{t,s}$ are financial assets following the law of motion (6), $Y_t h^i_{t,s}$ is the present value of labor earnings, and $\lambda^i_{TR,t,s}$ is a *transfer wedge*, which measures the present value of transfer flows relative to the present value of the labor income:

$$h^i_{t,s} = \mathbb{E}_t \int_t^\infty e^{-\pi(u-t)} \frac{\zeta^i_u}{\zeta^i_t} \frac{Y_u}{Y_t} \omega \theta^i e_{u-s}\, du, \quad \lambda^i_{TR,t,s} = \frac{1}{h^i_{t,s}} \mathbb{E}_t \int_t^\infty e^{-\pi(u-t)} \frac{\zeta^i_u}{\zeta^i_t} \frac{Y_u}{Y_t} \mu^i_{TR,u-s}\, du.$$

The present value of the transfer flows takes into account the expectation of zero for its volatility component.

The budget constraint requires that the present value of consumption equals the present value of wealth:

$$\mathbb{E}_t \int_t^\infty e^{-\pi(u-t)} \frac{\zeta^i_u}{\zeta^i_t} C^i_{u,s} du = W^i_{t,s}, \tag{15}$$

and, by the martingale representation theorem, the total wealth follows the following stochastic differential equation:

$$dW^i_{t,s} = \left[((1-\lambda^i_{r,t})r_t + \pi)W^i_{t,s} - C^i_{t,s} + (1-\lambda^i_{\beta,t})\beta_t \sigma^i_{W,t,s} W^i_{t,s}\right] dt + \sigma^i_{W,t,s} W^i_{t,s} dB_t \tag{16}$$

for some wealth volatility process $\sigma^i_{W,t}$. The agent's is now to choose the consumption process C^i_t and the volatility process $\sigma^i_{W,t}$ that maximizes the lifetime utility (2) subject to the budget constraint (16), starting with zero financial wealth, $Z^i_{t,t} = 0$.

HJB equation. The agent's problem will now be characterized recursively. The problem is simplified because their choices are, conditionally, on total wealth W, independent of their age. Let $V^i(W, x)$ be type-i agent's value of having wealth W if the current state is

x. The value function satisfies the following HJB equation:

$$0 = \max_{C,\sigma_W} f^i\left(C, V^i\right) + \left[\left((1-\lambda_r^i)r + \pi\right)W - C + (1-\lambda_\beta^i)\beta\sigma_W W\right] V_W^i + \frac{1}{2}W^2\sigma_W^2 V_{WW}^i$$
$$+ \mu_x V_x^i + \frac{1}{2}\sigma_x^2 V_{xx}^i + W\sigma_W\sigma_x V_{Wx}^i, \tag{17}$$

where V_W^i, V_x^i, V_{WW}^i, V_{xx}^i and V_{Wx}^i are partial derivatives of the value function. The next proposition shows that the value function is homogeneous of degree $1-\gamma^i$ in the total wealth W, the consumption function is linear in W, and the volatility is independent of W:

Proposition 3. *The value function and the optimal policies of the agent satisfy*

$$V^i(W,x) = v^i(x)\frac{W^{1-\gamma^i}}{1-\gamma^i}, \quad C^i(W) = g^i(x)W$$

for some functions $v^i(x)$ and $g^i(x)$. In addition, the optimal wealth volatility $\sigma_W^i(W,x)$ is independent of W.

The proof is straightforward and is omitted. After canceling wealth and using the first-order conditions, the HJB equation (17) can be written in terms of the consumption-to-wealth ratio $g^i(x)$ and becomes a second-order differential equation in x:

$$0 = g^i + \frac{1-\rho^i}{\rho^i}\left((1-\lambda_r^i)r + \frac{(1-\lambda_\beta^i)^2\beta^2}{2\gamma^i}\right) - \frac{\delta}{\rho^i} - \pi - \left(\mu_x + (1-\lambda_\beta^i)\beta\frac{1-\gamma^i}{\gamma^i}\sigma_x\right)\frac{g_x^i}{g^i}$$
$$+ \frac{\sigma_x^2}{2}\left(1 + \frac{\rho^i}{\gamma^i}\frac{1-\gamma^i}{1-\rho^i}\right)\left(\frac{g_x^i}{g^i}\right)^2 - \frac{\sigma_x^2}{2}\frac{g_{xx}^i}{g^i}. \tag{18}$$

The equilibrium process for the aggregate consumption of each type can be determined from Ito's lemma and the budget constraint (16):

$$\frac{d\overline{C}_t^i}{\overline{C}_t^i} = \mu_{\overline{C}}^i dt + \sigma_{\overline{C}}^i dB_t, \tag{19}$$

with the drift and volatility parameters

$$\mu_{\overline{C}}^i = \frac{(1-\lambda_r^i)r - \delta}{\rho^i} + \Phi^i + \pi\left(\kappa^i - 1\right), \quad \sigma_{\overline{C}}^i = \frac{\gamma^i - \rho^i}{\gamma^i(1-\rho^i)}\frac{g_x^i}{g^i}\sigma_x + (1-\lambda_\beta^i)\frac{\beta}{\gamma^i},$$

where

$$\Phi^i = \frac{1+\rho^i}{\rho^i}\frac{(1-\lambda^i_\beta)^2\beta^2}{2\gamma^i} + (1-\lambda^i_\beta)\frac{\beta}{\gamma^i}\frac{\gamma^i-\rho^i}{1-\rho^i}\frac{g^i_x}{g^i}\sigma_x + \frac{1}{\gamma^i}\frac{\gamma^i-\rho^i}{1-\rho^i}\left(\frac{g^i_x}{g^i}\right)^2\frac{\sigma_x^2}{2}.$$

is a term that represents the precautionary savings motive of the agent and

$$\kappa^A = \frac{C^i_{t,t}}{\overline{C}^i_t} = \frac{\alpha}{x}g^A h^A_0(1+\lambda^A_{TR,0}), \quad \kappa^B = \frac{C^i_{t,t}}{\overline{C}^i_t} = \frac{1-\alpha}{1-x}g^B h^B_0(1+\lambda^B_{TR,0})$$

is the consumption of newborns relative to the aggregate consumption. The relative consumption depends on the newborns' present value of earnings and transfers $h^i_0(1+\lambda^i_{TR,0})$, where $h^i_0 = h^i_{t,t}$ is the newborns' present value of labor earnings, and $\lambda^i_{TR,0}$ is the newborns' transfer wedge.

Equilibrium. We will now characterize the equilibrium of the economy in a closed form. By definition, the consumption share, output, and aggregate consumption of a given type are related as follows:

$$x_t Y_t = \alpha\overline{C}^A_t \tag{20}$$

$$(1-x_t)\,Y_t = (1-\alpha)\overline{C}^B_t. \tag{21}$$

Equating volatilities on both sides, we solve for the volatility of the consumption share σ_x and the market price of risk β as

$$\sigma_x = \frac{\frac{\gamma^B}{1-\lambda^B_\beta} - \frac{\gamma^A}{1-\lambda^A_\beta}}{\frac{\gamma^A}{1-\lambda^A_\beta}\frac{1}{x} + \frac{\gamma^B}{1-\lambda^B_\beta}\frac{1}{1-x} + \frac{\rho^A-\gamma^A}{(1-\lambda^A_\beta)(1-\rho^A)}\frac{g^A_x}{g^A} - \frac{\rho^B-\gamma^B}{(1-\lambda^B_\beta)(1-\rho^B)}\frac{g^B_x}{g^B}}\sigma_Y \tag{22}$$

$$\beta = \bar{\gamma}\left(\sigma_Y + x\frac{\rho^A-\gamma^A}{\gamma^A(1-\rho^A)}\frac{g^A_x}{g^A}\sigma_x + (1-x)\frac{\rho^B-\gamma^B}{\gamma^B(1-\rho^B)}\frac{g^B_x}{g^B}\sigma_x\right), \tag{23}$$

where $\bar{\gamma}$ is the harmonic mean of the tax-adjusted risk aversion parameters of the two types:

$$\bar{\gamma} = \left(x\frac{1-\lambda^A_\beta}{\gamma^A} + (1-x)\,\frac{1-\lambda^B_\beta}{\gamma^B}\right)^{-1}.$$

The risk wedge λ^i_β determines both σ_x and β by effectively augmenting the coefficients of relative risk aversion for the two types. Agents facing a positive risk wedge behave as if they were more risk averse: they require a higher excess return to compensate for taking on the risk as if their risk aversion coefficient was $\gamma^i/(1-\lambda^i_\beta)$. The market price of risk then depends on the harmonic mean of the effective risk aversion coefficients of the two types $\bar{\gamma}$.

As shown by equation (22), the volatility of consumption share σ_x depends critically on the difference between the augmented risk aversion of the two types. As long as $\gamma^B/(1-\lambda^B) < \gamma^A/(1-\lambda^A)$ and type B agents are effectively less risk averse, they are relatively more exposed to the return fluctuations and a positive income shock increases their consumption and wealth share. An increase in the risk wedge on the type-B agents will dampen the dynamics of consumption and wealth since it closes the gap between the effective risk aversions of both types.

Equating the drifts of both sides of (20) and (21) yields expressions for the drift of the consumption share μ_x and the risk-free rate r:

$$r = \zeta\delta + \bar{\rho}\left[\mu_Y + \pi\left(1 - x\kappa^A - (1-x)\kappa^B\right) - x\Phi^A - (1-x)\,\Phi^B\right] \tag{24}$$

$$\mu_x = (1-x)\left(\mu_Y + \pi - \frac{(1-\lambda^B_r)r - \delta}{\rho^B} - \Phi^B\right) - \pi(1-x)\kappa^B - \sigma_x\sigma_Y, \tag{25}$$

where

$$\bar{\rho} = \left(x\frac{1-\lambda^A_r}{\rho^A} + (1-x)\,\frac{1-\lambda^B_r}{\rho^B}\right)^{-1}, \quad \zeta = \bar{\rho}\left(\frac{x}{\rho^A} + \frac{1-x}{\rho^B}\right).$$

The risk-free rate is determined by the typical three factors: compensation for the instantaneous discounting δ, compensation for the willingness to accept a given expected growth rate of consumption, and the precautionary savings motive. However, each type's expected consumption growth rate is not equal to the output drift μ_Y. With finite lifetimes, there are two additional factors: due to annuities contracts, the consumption growth of the survivors increases by π. On the other hand, deceased agents are replaced by newborns with lower consumption, reducing the aggregate consumption growth rate for each type. The relative consumption of the newborns, averaged over the two types, is represented by the term $x\kappa^A + (1-x)\kappa^B$. Changes in the transfer policies that will change the transfer wedge $\lambda^i_{TR,0}$ directly impact the equilibrium risk-free rate through κ^i. Finally, the term $x\Phi^A + (1-x)\,\Phi^B$ in (24) represents the precautionary savings motive, whose strength, in turn, depends on the risk wedge, both directly through the term $(1-\lambda^i_\beta)\beta$, and indirectly through the volatility of the consumption share σ_x.

The risk-free wedge matters for the risk-free rate by augmenting the respective coefficients of intertemporal substitution. Analogously to the relationship between the risk wedge and risk aversion, a higher risk-free wedge increases the risk-free rate that the agents require to be compensated for a given consumption growth, effectively lowering the intertemporal elasticity of substitution. The risk-free rate is then determined by the harmonic mean of the effective intertemporal elasticities of substitution $(1-\lambda^i_r)/\rho^i$, as represented by $\bar{\rho}$.

Equilibrium. For given wedges $\{\lambda^i_r, \lambda^i_\beta, \lambda^i_{TR,0}\}$, the equilibrium is given by functions g^A, g^B, σ_x, β, μ_x, r that satisfy the HJB equations (18) for each type, the volatility restrictions (22), (23), the drift restrictions (24) and (25), and the government budget constraint (7). The present values of labor incomes h^i_0 can be computed easily once the functional form for e_u is specified.

Normalization. As one can see from examining the equilibrium equations, one can always normalize one value of the risk wedge λ^i_β to zero. The equilibrium value of β will then scale to keep the after-wedge value $(1-\lambda^i_\beta)\beta$ unchanged, and the equilibrium values of σ_x, μ_x and g^i will be unchanged as well. The result follows from one degree of freedom in (13) and holds for any tax and transfer policy. In the remainder of the text, I will keep the general notation but normalize the risk wedge on type A, λ^A_β, to be zero whenever possible.

Asset Prices. Define the ratio of stock price to output by $p_t = P_t/Y_t$. The market clearing condition (10) can be rewritten with the help of the equilibrium relationships (20) and (21) to express the stock price to output ratio as

$$p_t = \frac{x_t}{g^A_t} + \frac{1-x_t}{g^B_t} - \alpha \overline{h}^A_t - (1-\alpha)\overline{h}^B_t,$$

where $\overline{h}^i_t = \pi \int_{-\infty}^t e^{-\pi(t-s)} h^i_{t,s}(1+\lambda^i_{TR,t,s})\, ds$ is the cross-sectional aggregate present value of earnings. The volatility of returns σ equals to

$$\sigma = \frac{p_x}{p}\sigma_x + \sigma_Y.$$

2.3 Linear Wedges

The first part of Proposition (1) for the representative agent can be understood as a neutrality result, where the tax policies do not affect the two values that matter from the agent's perspective: their equilibrium stochastic discount factor and their wealth. The general model offers a related result: tax policies that generate linear wedges accompanied by transfers with a zero present value at birth do not affect the stochastic discount factor and allocation of resources.[7]

Proposition 4. *Suppose that the tax and transfer policy is such that $\lambda_\beta^A = \lambda_\beta^B$, $\lambda_r^A = \lambda_r^B$, and that $\lambda_{TR,0}^A = \lambda_{TR,0}^B = 0$. Let λ_β and λ_r be the common values of the risk and risk-free wedge.*

If $\{g^A, g^B, \beta, r, \sigma_x, \mu_x\}$ constitute the economy's equilibrium, then $\{g^A, g^B, (1-\lambda_\beta)\beta, (1-\lambda_r)r, \sigma_x, \mu_x\}$ constitute the equilibrium in an economy with no taxes and transfers.

Proof. It follows from the discussion of normalization that an equilibrium with a linear risk wedge is equivalent to an equilibrium with no risk wedge. Similarly, if equations (18), (24) and (25) hold for interest rate r and a common risk-free wedge $1-\lambda_r$, then they hold for a risk-free rate $(1-\lambda_r)r$ and a zero risk-free wedge. Finally, if the newborns' present value of transfers is zero, then the value of h_0^i is identical to its respective value in equilibrium with no taxes and transfers. As a result, the equilibrium set of equations (18), (22), (23), (24) and (25) continues to hold. Since the government budget constraint (7) trivially holds in an economy with no taxes or transfers, $(1-\lambda_\beta)\beta$, $(1-\lambda_r)r$, σ_x and μ_x constitute the equilibrium in an economy with no taxes and transfers. □

Three comments clarify the scope of the proposition. First, the assumption that the transfer wedge $\lambda_{TR,0}^i$ is always zero is not inconsistent with the requirement that the government budget constraint (7) holds. The government budget constraint requires that the cross-sectional average of transfers, not the present value for newborns, equals the taxes collected. Transfer schemes that backload the transfers will tend to have a cross-sectional average larger than the present discounted value at birth.

Second, the proposition is silent about what values of the tax on risky assets τ_P^i support the linear risk wedge. The tax on risky assets may or may not be itself linear. It will be linear only if the transfer volatility is identical for both types, as seen from (13).

[7]The assumption of zero expected transfers at birth has no counterpart in the infinitely lived representative-agent economy. The representative agent analog is that changes in transfers leave the total wealth unchanged.

The proposition implies that any tax on risky assets consistent with the linear tax wedge will not affect the allocation of consumption across individuals.

Third, the proposition is silent about what happens to the asset prices. As in the second part of Proposition (1), the asset price will respond to changes in τ_P^i, even if the equilibrium allocation will not. Unlike Proposition (1), however, the volatility of asset returns σ will generally respond to τ_P^i as well.

Proposition 4 is helpful in that it clearly shows that there are only three ways in which the real allocation in the economy can be affected relative to an economy with no taxes and transfers. The allocations can change i) because of nonlinearity in the risk-free wedge (tax on the risk-free asset), ii) nonlinearity in the risk wedge, or iii) redistribution through the transfers, as expressed by the present values of transfers at birth. Since any tax policy will manifest itself in one of those three wedges, one can decompose any tax reform into the respective three channels to assess their strength and the impact of the change on the equilibrium quantities.

3 Calibration

The model is calibrated to the US economy, with one unit of time corresponding to a year. The parameters are summarized in Table 1.

To a large extent, the calibration of the externally set parameters follows Gârleanu and Panageas (2015). The parameters σ_Y and μ_Y of the endowment process (1) are set to match the first two moments of annual consumption growth, with $\mu_Y = 0.02$ and $\sigma_Y = 0.041$. The share of labor income ω is set to $\omega = 0.92$ to match the fact that dividend and net interest payments to households are 0.08 of personal income in NIPA. The fraction of type A agents in the population is set to $\alpha = 0.99$.

The life-cycle earnings profile takes the form used in Gârleanu and Panageas (2015),

$$e_u = A_1 e^{-\eta_1 u} + A_2 e^{-\eta_1 u},$$

where A_1 and A_2 are constrained to satisfy the normalization (4). I follow their parameterization by setting $A_1 = 30.72$, $A_2 = -30.29$, $\eta_1 = 0.0525$ and $\eta_2 = 0.0611$. The parameters imply that the earnings profile is hump-shaped, peaking after approximately 16 years of working, and slowly decays after.

The death rate is set to $\pi = 0.02$, implying an expected working lifespan of 50 years.

Table 1: Model Parameters

parameter	value	target
μ_Y	0.02	mean US consumption growth
σ_Y	0.041	standard deviation of log US consumption growth
π	0.02	average US death rate
ω	0.92	Gârleanu and Panageas (2015)
ρ	0.001	exogenously set
α	0.99	exogenously set
ρ^A	20.45	equity premium of 5.2%
γ^A	10	exogenously set
ρ^B	1.544	annual risk-free rate of 1.52%
γ^B	1.619	Sharpe ratio of 0.285

The instantaneous discount rate is set to $\rho = 0.001$. The risk aversion of the A type is set exogenously to $\gamma^A = 10$. This value is a standard upper bound used in the literature; see Mehra and Prescott (1985). The remaining three parameters, ρ^A, γ^B, and ρ^B, are calibrated to match three targets in an economy with no taxes and transfers. The equity premium of 5.2%, the annual risk-free rate of 1.2%, and the Sharpe ratio 0f 0.285. This yields $\rho^A = 20.45$, implying the elasticity of intertemporal substitution of type A of 0.049, $\gamma^B = 1.619$, and $\rho^B = 1.544$, implying the elasticity of intertemporal substitution of type B of 0.648. Although the calibration procedure is slightly different, the values are close to the ones obtained by Gârleanu and Panageas (2015). In computing the equity premium, I follow Barro (2006) by reporting the results for levered equity, that is in zero net supply, as a redundant asset does not change prices and allocations and has the expected excess returns equal $1 + lr$ times the expected excess returns of unlevered equity, where lr is the leverage ratio, set to 0.5.

To compute the equilibrium, I discretize the state space $X = [0, 1]$ with 1000 equispaced gridpoints. I compute the HJB equation (18) as a nonlinear equation for given functions β and r; I and use the upwind finite difference scheme as in Achdou et al. (2021) to compute the partial derivatives.[8] I then update the pricing functions β and r, and iterate until convergence. The results regarding consumption and wealth inequality are computed using numerical simulations.

[8]There is a second pair of differential equations used to compute the present values of earnings and transfers, that are computed similarly.

Table 2: Model Moments

statistics	aggregate	type A	type B
risk-free rate r*	1.52%		
equity premium*	5.17		
market price of risk*	0.285		
volatility of returns	17.3%		
mean consumption	1	0.90	8.93
mean assets	8.82	6.54	235.05
consumption Gini	0.33	0.28	0.72
assets Gini	0.55	0.42	0.80
consumption to output volatility	1	0.64	4.35

*Targeted moments.

Table 2 shows the aggregate statistics for the benchmark economy. The calibrated economy matches the equity premium, the risk-free rate, and the market price of risk in the data by construction. The equilibrium return volatility is 17.3 %, close to the empirical value of 18.2 %.

The calibrated economy exhibits a substantial overall inequality in consumption and assets. The Gini coefficient is 0.33 for consumption, comparable to the empirical values of around 0.3 (Fisher et al. (2015)). The Gini coefficient in assets of 0.55 for assets is somewhat lower than the empirical counterpart of 0.85 (in 2010, see Kuhn and Rios-Rull (2016)). There is also a significant heterogeneity across the two types. Type-B agents accumulate substantially more assets, about 46 times more than type-A agents. This is because their portfolios are strongly skewed toward the risky asset: in fact, type A agents hold negative positions in the risky asset on average because a negative position provides a hedge against their labor market risk. Not unexpectedly, the consumption of type-B agents is more than eleven times higher than that of type-A agents but also more than six times as volatile. The model also delivers significant heterogeneity within each type of agent due to age differences. Predictably, the within-group heterogeneity is higher among the type-B agents with steeper consumption profiles who accumulate relatively more assets over their lifespan. However, even for type-A agents, the life-cycle components produce a consumption Gini of 0.28, which is not too far below the overall average.

4 Understanding the Role of Wedges

This section will explore how exogenous changes in the three wedges affect the prices and allocations in the calibrated economy, with a special emphasis on the effects on inequality between the two types, within the two types, and overall. Although the model allows the wedges to depend on x, I will only consider constant wedges to make the exercise as simple as possible.

Risk wedge. Figure 1 shows how the risk wedge on type B agents λ_β^B affects asset prices and inequality in consumption and wealth. As λ_β^B increases from zero to 50 percent, type B agents, who now behave as if they were more risk averse, push the average equilibrium market price of risk β up from 0.285 to 0.36. The market price of risk increases by less than in the infinitely lived representative agent economy because any increase in β increases the demand for the risky asset by type A agents. As a result, the equilibrium after-wedge value for type B agents $(1 - \lambda_\beta^B)\beta$ decreases, type B agents reduce exposure to the risky asset, and type A agents increase it.

The risk-free rate increases along the market price of risk, from 1.5 % to 4%. The two changes are directly related because an increase in the risk wedge changes the steepness of the consumption profiles for the two types, increasing it for type-A agents and decreasing it for type-B agents. This change for type-A agents dominates and requires a higher risk-free rate to support their profile, as shown in the equation (24).

The overall consumption Gini is shown in the third panel of Figure 1. The decrease is driven primarily by changes in within-group consumption inequality of type-A agents. The between-group inequality and the within-group inequality of type-B agents are hump-shaped and contribute to the concavity of the relationship between λ_β^B and consumption Gini. In particular, they moderate the initial decline in the overall inequality.

The between-group inequality is hump-shaped because changes in β and r have two opposing effects on the consumption profiles: an increase in either of the two increases the consumption growth rate but also reduces the present value of labor income. This is precisely what happens to type A agents. For type-B agents, the present value of labor income decreases by less, and the consumption growth rate decreases. For low values of λ_β^B, the present value effect, which affects type-A agents more, dominates, and consumption inequality between the two groups increases. For higher values of λ_β, the effect on the expected consumption growth rate dominates, and consumption inequality

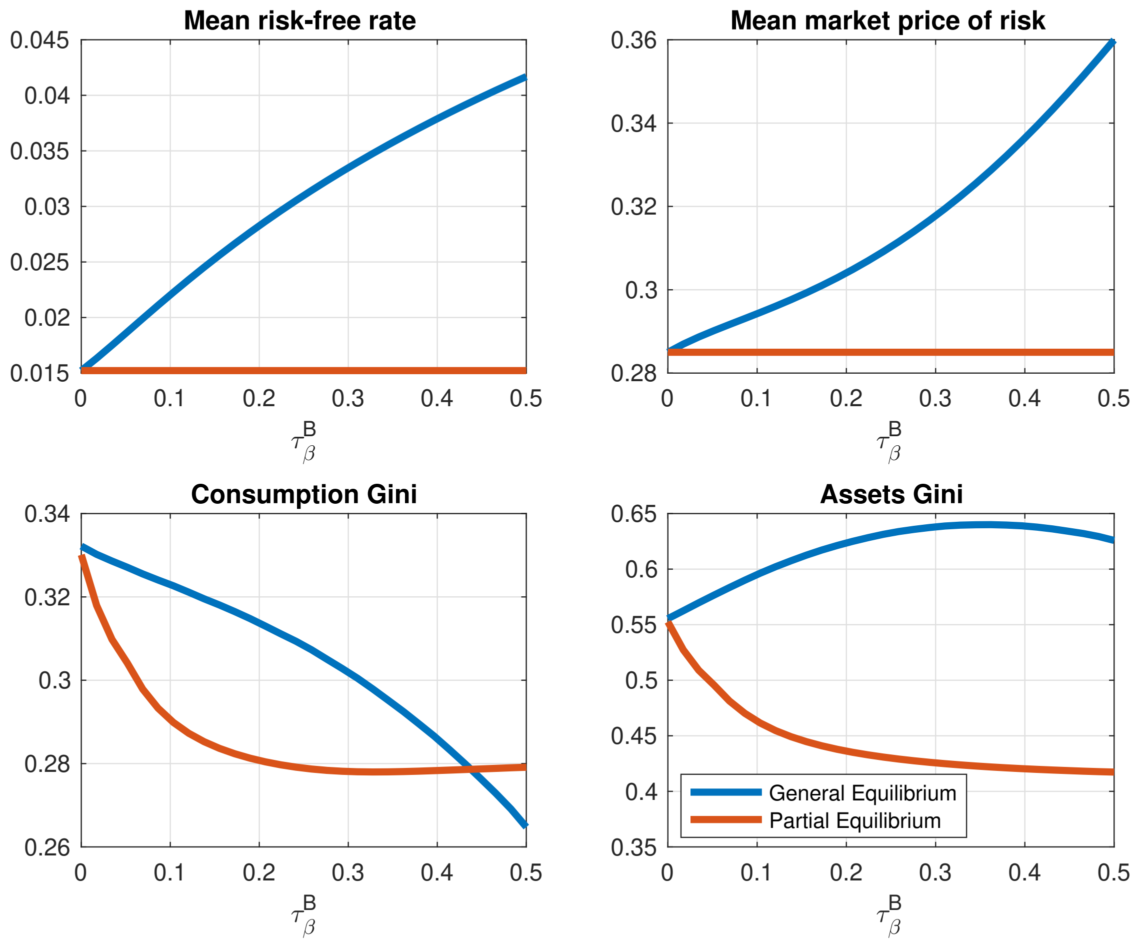

Figure 1: Risk wedge λ_β^B and the equilibrium risk-free rate r, market price of risk β, consumption and assets Gini. Partial equilibrium keeps r, β, σ_x and μ_x fixed.

between the two groups decreases.

Within-group consumption inequality of type A agents decreases with λ_β^B for the following reason. Type-A agents have a low consumption growth rate, low enough for their consumption-to-output ratio to decrease over the life cycle. Higher λ_β^B, by increasing their consumption growth rate, "flattens" their consumption profile and reduces consumption inequality within the type. Consumption inequality of type-B agents, on the other hand, first increases and then decreases because $(1-\lambda_\beta^B)\beta$ and r move in opposite directions.

The fourth panel of Figure 1 shows that, unlike consumption inequality, asset inequality is not monotonically decreasing in λ_β^B. It increases until $\lambda_\beta^B \approx 0.35$ and decreases after. The increase in the interest rate reduces the mean value of the assets for both types, but the decline is initially more pronounced for the type-A agents and later for the type-B agents, leading to the hump-shaped profile of the asset inequality between the two groups. The hump-shaped profile is also present in the within-type asset inequality for both types and the overall inequality. Despite the later decline, the asset Gini at $\lambda_\beta^B = 0.5$ is 0.63, still higher than at $\lambda_\beta^B = 0.0$, however.

The general equilibrium effects are important in propagating consumption and asset inequality. The red lines in Figure 1 show that, in the absence of general equilibrium effects (with the functions r, β, σ_x and μ_x being kept at their benchmark values), the risk wedge is, at least in its lower range, substantially more powerful in reducing consumption inequality. This is a consequence of several factors. First and foremost, the between-group inequality is reduced very strongly rather than being hump-shaped in λ_β^B. Without price changes, the average consumption of type-A agents is unchanged, and only the type-B agents reduce their average consumption. The within-group inequality also decreases substantially for type-B agents. Although the within-group inequality of type-A agents does not change by construction, the first two effects dominate for lower values of λ_β^B, and the consumption inequality decreases substantially faster. Asset inequality also always decreases with λ_β^B, in sharp contrast to an increase in the asset inequality to 0.63 in general equilibrium.

Risk-free wedge. Consider now the role of the risk-free wedge on type B agents λ_r^B, as shown in Figure 2. An increase in λ_r^B from zero to 50 percent increases the risk-free rate substantially, from 1.5 percent to 5.5 percent. The direction of the change is expected - type B agents now effectively have a lower intertemporal elasticity of substitution and push the increase rate up. However, the strength of this channel is perhaps surprising. The response of the risk-free rate is so strong that even the after-wedge rate $(1 - \lambda_r^B)r$ increases. This is because finite lifetimes provide an "accelerating" mechanism in the model: an increase in r reduces the present value of earnings at birth, in turn increasing the slope of the consumption profile, pushing the equilibrium risk-free rate further up.

As λ_r^B increases, type A agents reduce their investment in the risky asset relative to type-B agents. This increases the weight of the type-B anges and reduces the market price of risk. The response of the market price is not as strong as a response to the risk wedge, and the market price of risk is reduced from 0.285 to 0.25.

Changes in λ_r^B have almost no effect on consumption inequality. The result is a consequence of several factors pulling inequality in opposite directions and balancing

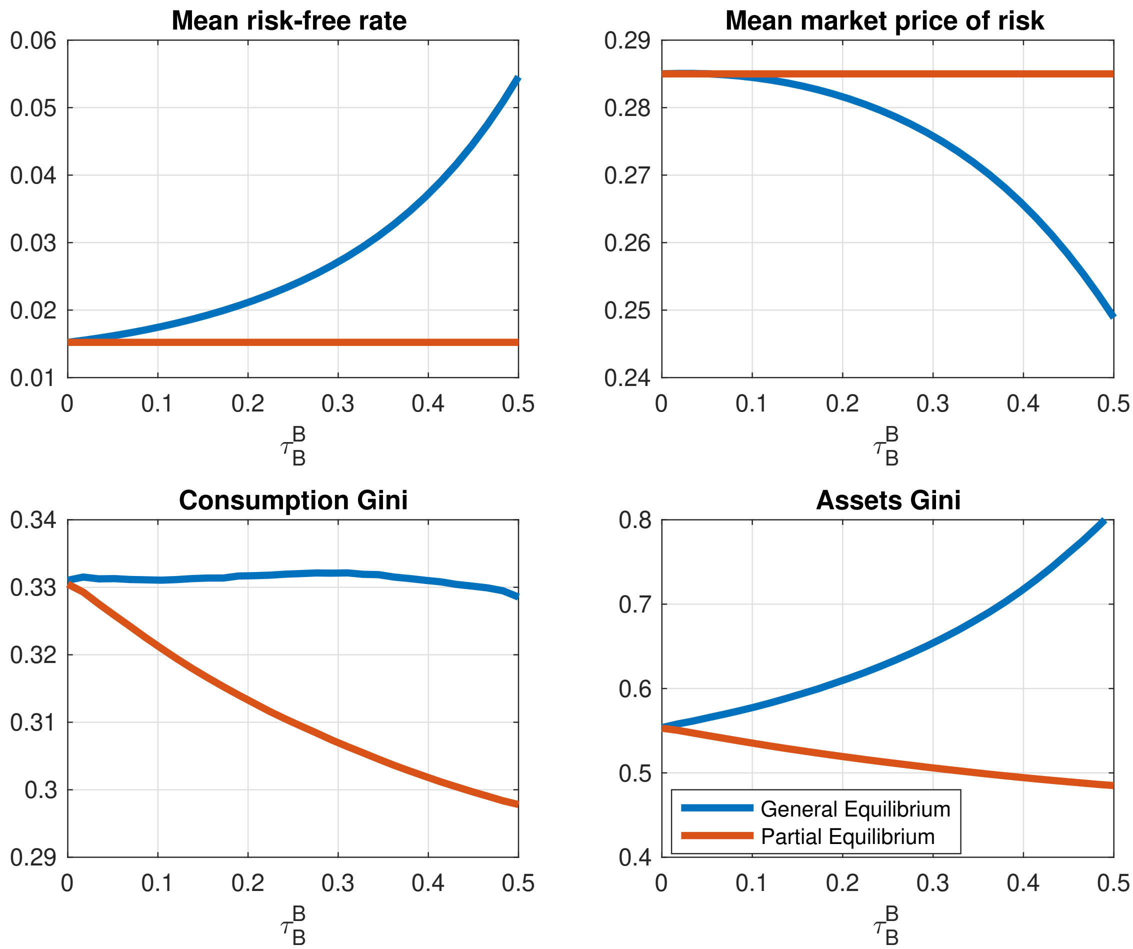

Figure 2: Risk-free wedge λ_r^B and the equilibrium risk-free rate r, market price of risk β, consumption and assets Gini. Partial equilibrium keeps r, β, σ_x and μ_x fixed.

each other. The between-group consumption inequality increases because type-A agents reduce their investment in risky assets relative to type-B agents. Withing-group inequalities move in opposite directions for the two types. For type-A agents, the increase in the interest rate dominates, and the inequality is reduced, similarly to an increase in the risk wedge. For type B agents, on the other hand, the within-group inequality decreases. On the other hand, asset inequality increases substantially from 0.56 to 0.8, with contributions from both between-group and within-group inequalities.

Equilibrium price and distribution changes again play a crucial role in the economy's response to the wedge. The red lines in Figure 2 show that consumption and asset inequality decrease in the absence of general equilibrium effects. Unlike in general equilibrium, the between-group consumption inequality is reduced because only type-B

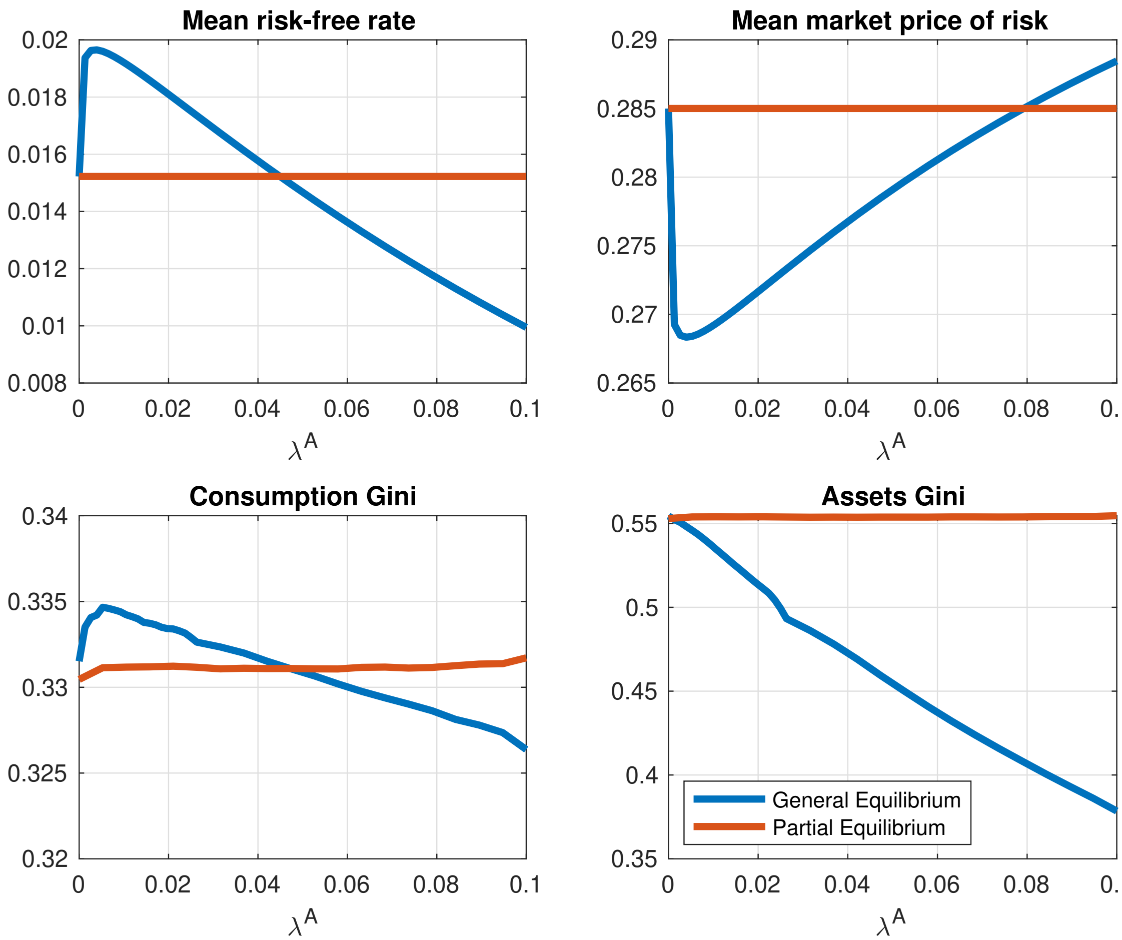

Figure 3: Relative present value of transfers $\lambda^A_{TR,0}$ and the equilibrium risk-free rate r, market price of risk β, consumption and assets Gini. Partial equilibrium keeps r, β, σ_x and μ_x fixed.

agents are now affected by the higher wedge. The consumption inequality of type-A agents is constant, while the consumption inequality of type-B agents now decreases as well. Asset inequality follows a similar pattern and decreases from 0.56 to 0.49 overall.

Transfer wedge. Changes in the present value of transfers to type A agents, as shown in Figure 3, have a relatively limited impact on the equilibrium prices and inequality. The figure considers an increase in the present value of transfers to type-A agents from zero to ten percent, which is a rather sizeable value, given that type-A agents comprise 99 percent of the population. At $\lambda^A_{TR,0}$ of 10 percent, the risk-free rate decreases from 1.52% to 1%, although the dynamics are not monotone. Changes in the market price of risk are also not monotone, and for $\lambda^A_{TR,0}$ of 10 percent, the market price of risk increases from 0.285 to 0.288.

The consumption inequality is likewise affected marginally, with the Gini coefficient decreasing to only 0.327. On the other hand, asset inequality decreases substantially. This is driven primarily by the decrease of between-group inequality, with type-A agents increasing consumption due to the higher present value of transfers.

5 Two Tax Reforms

This section studies tax reforms and their implications for equilibrium prices and quantities. It builds on the insights from the previous section, but rather than starting with wedges as primitives, it characterizes the equilibrium mapping from taxes to wedges. Since the mapping between taxes and wedges is nonlinear, even simple tax reforms will often manifest themselves through nonlinear equilibrium wedges, distorting the allocations with different intensities through the state space.

I consider two tax reforms. Both tax reforms tax only the type-B agents and redistribute the proceeds to type-A agents. In addition, both tax reforms shield the type A agents from increased transfer volatility by setting the volatility of transfers to type-A agents to zero, $\sigma^A_{TR} = 0$. On the other hand, type-B agents receive zero expected transfers, $\mu^B_{TR} = 0$, and all the expected transfers go to type-A agents to balance the budget constraint (7). The two reforms differ in which assets are taxed. In the first reform, the government introduces a 10 percent tax on risky asset holdings of type-B agents, $\tau^B_P = 0.1$, but risk-free assets are not taxed, $\tau^B_B = 0$. The second tax reform taxes all assets of type-B agents, risky and riskless, at 10 percent. The two tax reforms are characterized by the set of parameters in Table 3.

Table 3: Two Tax Reforms: Parameters

	τ^A_p	τ^B_p	τ^A_B	τ^B_B	σ^A_{TR}	σ^B_{TR}	μ^A_{TR}	μ^B_{TR}
tax on risky assets	0	0.1	0	0	0	$\tau^B_p \sigma$	g.b.c.	0
tax on all assets	0	0.1	0	0.1	0	$\tau^B_p \sigma$	g.b.c.	0

g.b.c means that the parameter is determined from the government budget constraint.

Both tax reforms are arbitrary, both in design and magnitude. However, they broadly represent popular motivations: taxing the wealthy segment of the population that holds

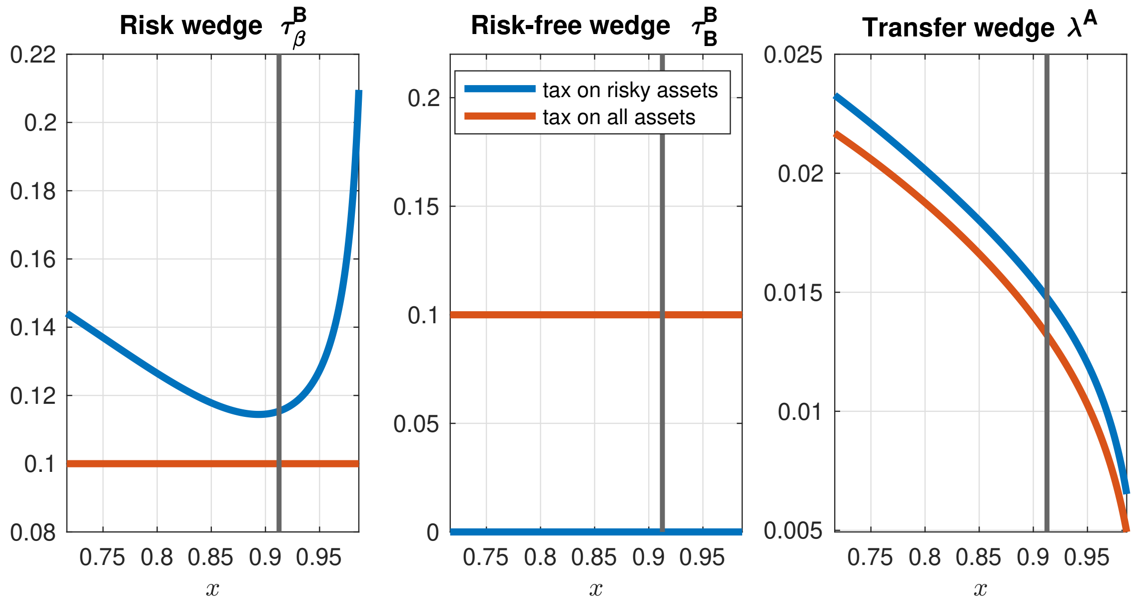

Figure 4: Risk wedge, risk-free wedge, and transfer wedge for the two tax reforms as a function of the consumption share x. The vertical line represents the average value of x in the benchmark.

most of the economy's assets and transferring the proceeds to the less wealthy segment.

Figure 4 shows the equilibrium distribution of the three wedges produced by the tax reforms. The first tax reform generates strictly positive risk and transfer wedges; the second reform adds a strictly positive risk-free wedge. The risk wedge in the first reform is higher than the tax rate because the tax rate reduces, relative to type-A agent, stock returns, but not the returns of the risk-free asset (see equation 13). The risk wedge is also highly nonlinear, rising substantially for both low and high values of the distribution of the consumption share x. High values of x correspond to times when the type-A agents consume relatively more, and the economy exhibits lower consumption and wealth inequality. A tax on risky assets is thus particularly distortive in those periods. The second tax reform, on the other hand, not only reduces the risk wedge but also keeps it uniform across the distribution at the common rate of 10 percent.

The transfer wedge is relatively small in magnitude. In the first reform, the transfers increase the earnings of type-A agents somewhere between 0.6% and 2.3% of their labor earnings, with 1.5% around the mean of the distribution. The second reform reduces the transfers because type-B agents predominantly hold negative positions in the risk-free asset. The reduction is relatively small, however. In both cases, the transfer wedge monotonically decreases in x and is higher at times of higher inequality.

Table 4: Two Tax Reforms: Results

Statistics	Benchmark	Tax on risky assets G.E.	Tax on risky assets P.E.	Tax on all assets G.E.	Tax on all assets P.E.
Market price of risk β	0.285	0.320	0.285	0.314	0.285
Risk-free rate r	1.52%	1.67%	1.52%	1.81%	1.52%
Expected equity return μ	6.69%	7.89%	6.79%	8.18%	6.80%
Equity premium $\mu - r$	5.17%	6.22%	5.27%	6.36%	5.28%
mean consumption B to A	9.89	8.27	2.73	8.59	3.01
mean assets B to A	35.97	32.61	8.79	35.77	9.90
consumption Gini	0.33	0.31	0.27	0.32	0.29
assets Gini	0.55	0.54	0.43	0.56	0.43

G.E. and P.E. denote general and partial equilibrium.

As table 4 shows, the pricing implications of the two reforms are broadly similar. The risk-free rate increases to 1.67% in the first reform and to 1.81% in the second reform, where both the risk wedge and the risk-free wedge contribute to its increase. Both reforms increase the market price of risk by about the same amount, from 0.285 to 0.320 in the first reform and to 0.314 in the second reform. This is mainly due to the increase in the risk wedge; the risk-free wedge in the second reform has only negligible impact, as seen from the top right panel Figure 2. The positive transfer wedge works in the opposite direction, but its impact is also negligible. The (pre-tax) equity premium increases by 1.05 percent in the first reform and even more by 1.19 percent in the second reform. The equity premium increases since the expected equity returns increase even more than the risk-free rate. This finding contrasts with the results from the representative agent economy in Proposition 1, where the equity premium didn't change.

The reaction of asset prices means that the two reforms have only a negligible effect on the redistribution of resources. The ratio of mean consumption of the two groups, a measure of between-group inequality, decreases from 9.89 to only 8.27 and 8.59 in the two reforms. The overall consumption of Gini has also been reduced slightly, from 0.33 to 0.31 and 0.32, respectively. The overall asset Gini even slightly increased in the second reform, from 0.55 to 0.56.

The limited ability of progressive tax reforms to redistribute resources and affect inequality is mainly due to the response of asset prices. The third and fifth columns show the inequality measures where asset prices are fixed at the pre-reform level. Without as-

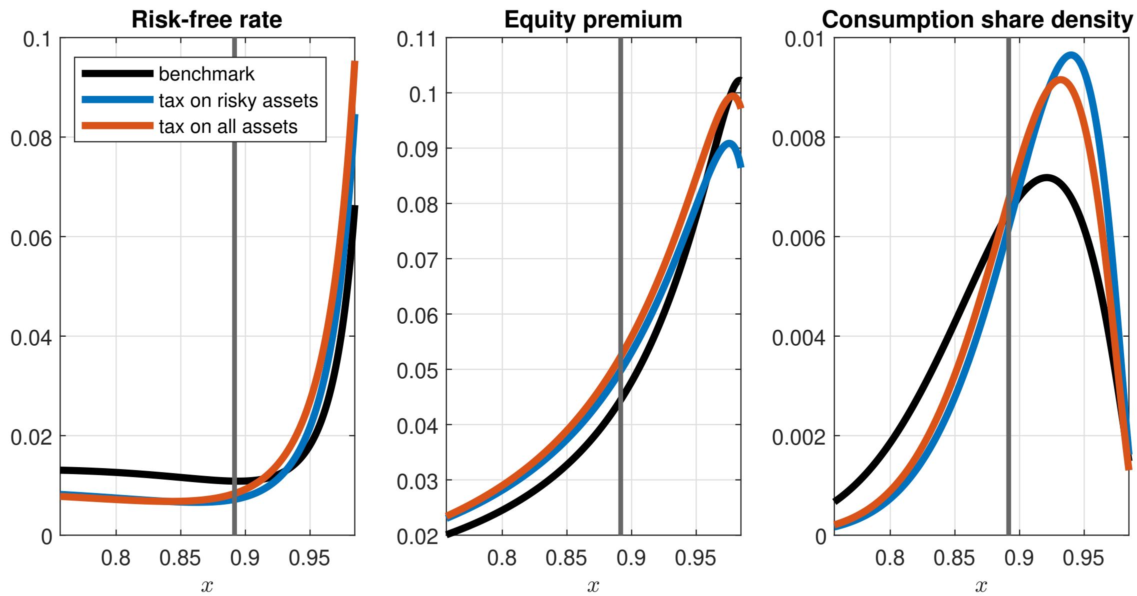

Figure 5: Risk-free rate, equity premium, and the density of the consumption share as a function of the consumption share x. The vertical line represents the average value of x in the benchmark.

set price responses, the between-group inequality (the ratio of the average consumption of the two groups) declines drastically to around 3 in both reforms. Both consumption and asset inequality also decline: consumption Gini decreases to 0.27 and 0.29, while asset Gini decreases to 0.43.

Looking under the hood of the aggregate statistics shows several changes in the distribution of prices and quantities. The left and middle panels of Figure 5 show the distribution of the risk-free rate and the equity premium across a range of consumption shares. Recall that higher values of x are associated with less consumption inequality between the two groups and x increases after negative endowment shocks ("bad times"). They are also associated with higher returns on risk-free assets and higher equity premiums. Both tax reforms increase the volatility of the risk-free rate by decreasing it in good times and increasing it in bad times. The effect on the equity premium is not monotone. The middle panel of Figure 5 shows that the equity premium increases for most values of the consumption share, except for around the top five percent of the share range. This is especially prominent in the first tax reform, where the risk wedge rises at those values, depressing return volatility and, in turn, the equity premium.

The last panel of Figure 5 shows that the tax reforms also generate a change in the distribution of the consumption share density. The average value of x, shown for the benchmark distribution, increases slightly in the two reforms, more in the first than in the second. As a result, the two tax reforms tend to increase the risk-free rate and the equity premium not only by increasing its average for a given distribution of the consumption share but also by shifting the distribution toward higher values of the equity premium and the risk-free rate.

Table 5 makes the last conclusion more precise by computing the prices and allocations for a fixed state distribution and comparing them with the equilibrium distribution. I consider only the first tax reform for simplicity. Keeping the distribution unchanged, the risk-free rate declines from 1.52% to 1.41%, and the equity premium and the equity premium increase, relative to the benchmark, and the equity premium increases only slightly from 5.17% to 5.42%. The shift in the equilibrium distribution is thus the dominant force behind the price changes: in the case of the equity premium, the increase for a fixed distribution is responsible for only $(5.49 - 5.17)/(6.22 - 5.17) = 30\%$ of the total change. The remaining 70%

A related decomposition, shown in the last column of Table 5, shows the equilibrium in the first tax reform when the transfer wedge is set to zero, and only the transfer wedge remains. The risk-free rate increases substantially more, to 2.35%, along with the expected return on the risky asset. That leaves the equity premium almost the same as under the fixed distribution, at 5.43 %. The risk wedge by itself reduces the between-group inequality only minimally, with the ratio of the average consumption of the two types at 9.60, down from 9.89 in the benchmark. The Gini coefficient of consumption of 0.32 lies between the benchmark value of 0.33 and the after-tax-reform value of 0.31, with a larger inequality both between the types and within type-B agents. Asset Gini increases to 0.61. Even though the risk wedge is now nonlinear, those findings are in line with the findings of Section 4.

6 Conclusions

The paper aims to characterize the general equilibrium of asset taxation, emphasizing the implication for the equilibrium prices of risk-free and risky assets. Asset taxes have only a moderate impact on the reduction of consumption and asset inequality. This is mainly due to the response of the asset prices: a tax on risky assets increases both the

Table 5: Tax on Risky Assets: A decomposition

Statistics	Full reform benchmark distribution	Full reform equilibrium distribution	Only risk wedge
Market price of risk β	0.300	0.320	0.296
Risk-free rate r	1.41%	1.67%	2.35%
Expected equity return μ	6.90%	7.89%	7.78%
Equity premium $\mu - r$	5.49%	6.22%	5.43%
mean consumption B to A	—	8.27	9.60
mean assets B to A	—	32.61	42.94
consumption Gini	—	0.31	0.32
assets Gini	—	0.54	0.61

Column "benchmark distribution" fixes the distribution of x at the pre-tax level.

risk-free rate and the equity premium, mitigating the effects of the taxes. I show that the asset price responses are responsible for the tepid reduction in consumption and asset inequality. In their absence, both consumption and asset inequality are reduced to a much larger extent.

The findings rest on several assumptions. Most importantly, the taxes are type-specific rather than a function of assets or wealth. That substantially simplifies the analysis because one does not have to track the individual asset holdings within a given type. It is likely that this assumption does not affect the quantitative results because the accumulated assets of type-B agents are substantially and distinctly higher than the assets of type-A. A tax with an appropriately chosen minimum wealth threshold somewhere between would deliver similar results. The paper also assumes complete markets, which, among other things, implies that the returns on investment are equalized across individuals. Relaxing this assumption, as in Guvenen et al. (2023) or Gomez (2024), opens up new channels through which asset of wealth taxation could impact equilibrium prices and quantities.

The paper offers a positive analysis of given tax policies and intentionally steers away from normative findings. However, it should be noted that, in the current calibration, all the inequality in wealth and consumption comes from differences in preferences, not from differences in endowments. It is thus unclear whether a utilitarian government with full commitment would want to redistribute resources toward type-A agents. Putting more weight on the utility of type-A agents or adding heterogeneity in endowments would resolve this tension. Such considerations are left for future research.

Appendix

Proof of Proposition 1. It follows from equation (18) that, in an infinite-horizon, representative agent economy, $C_t = gW_t$, where the constant consumption-wealth ratio g is

$$g = \frac{\delta}{\rho} + \left(1 - \frac{1}{\rho}\right)\left((1-\tau_B)r + \frac{\beta^2}{2\gamma}\right).$$

The stochastic process for total wealth and consumption (19) then reduces to:

$$\frac{dC}{C} = \frac{dW}{W} = \frac{1}{\rho}\left[(1-\tau_B)r - \delta + (1+\rho)\frac{\beta^2}{2\gamma}\right]dt + \frac{\beta}{\gamma}dB.$$

In equilibrium, $C_t = Y_t$. Equating drifts and volatilities of both stochastic processes yields the equilibrium values of β and r in the proposition.

Total wealth satisfies the budget constraint (14). If $p = P/Y$ is the constant price-output ratio, the risky price equation (5) immediately implies $\sigma = \sigma_Y$. The transfer process (8) yields $\sigma_{TR} = \tau_P\sigma_Y$. Inverting (13) to express μ gives the value of the expected return μ in the proposition.

It can be established that $h_t = (\omega + \mu\tau_P p)/g$. Using the market clearing conditions $Y_t = C_t = gW_t$ and $Z_t = P_t$ to rewrite (14), cancelling Y_t and rearranging yields the expression for p given in the proposition.

□

Proof of Proposition 2. The relationship (13) is obtained from differentiating (14), equating drift and volatility terms, and simplifying. Differentiate both sides of (14):

$$dW^i_{t,s} = dZ^i_{t,s} + dh^i_{t,s}Y_t + h^i_{t,s}dY_t + dh^i_{t,s}dY_t \tag{26}$$

The present value of earnings $h^i_{t,s}$ has a law of motion

$$dh^i_{t,s} = \mu_{h,t,s}dt + \sigma^i_{h,t,s}h^i_{t,s}dB_t, \tag{27}$$

for some volatility process $\sigma^i_{h,t,s}$ and for the drift

$$\mu^i_{h,t,s} = -\omega\theta^i e_{t-s} - \mu^i_{tr,t-s} + \left[(1-\tau^i_{B,t})r_t - \mu_Y + \pi + (1-\lambda^i_{\beta,t})\beta\sigma_Y\right] h^i_{t,s} + \left[(1-\lambda^i_{\beta,t})\beta - \sigma_Y\right] \sigma^i_{h,t,s} h^i_{t,s}.$$

Using the laws of motion (6), (16) and (27) for $Z^i_{t,s}$ and $W^i_{t,s}$ and $h^i_{t,s}$, rewrite (26), and equate drifts and volatilities. Rearranging the terms by using (14) yields a drift and volatility restriction

$$\begin{aligned}
(1-\lambda^i_{\beta,t})\beta\sigma^i_{W,t}W^i_t &= \left[(1-\tau^i_{P,t})\mu_t - (1-\tau^i_{B,t})r_t\right] S^i_{t,s} \\
&\quad + \left[\omega\theta^i e_{t-s} + \mu^i_{tr,t-s} + \mu^i_{h,t,s} + \left(\mu_Y - (1-\tau^i_{B,t})r_t - \pi\right) h^i_{t,s} + \sigma^i_{h,t,s} h^i_{t,s}\sigma_Y\right] Y_t \\
\sigma^i_{W,t}W^i_t &= \sigma^i_{tr,t}S^i_{t,s} + (1-\tau^i_{P,t})S^i_{t,s}\sigma_t + \left(\sigma^i_{h,t,s} + \sigma_Y\right) h^i_{t,s} Y_t
\end{aligned}$$

Finally, use the expression for $\mu^i_{h,t,s}$, cancel terms, subtract the volatility equation multiplied by $(1-\lambda^i_{\beta,t})\beta$ and cancel terms to obtain (13).

□

References

Achdou, Y., J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll (2021, 04). Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach. *The Review of Economic Studies 89*(1), 45–86. 20

Aguiar, M., B. Moll, and F. Scheuer (2024). Putting the "finance" into "public finance": A theory of capital gains taxation. Technical report, LSE. 5

Anderson, E. W. (2005). The dynamics of risk-sensitive allocations. *Journal of Economic Theory 125*(2), 93–150. 4, 5

Backus, D. K., B. R. Routledge, and S. E. Zin (2008). Who holds risky assets? Technical report, New York University. 5

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *The Quarterly Journal of Economics 121*(3), 823–866. 20

Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of Political Economy 93*(2), 223–247. 5

Borovička, J. (2019). Survival and long-run dynamics with heterogeneous beliefs under recursive preferences. *Journal of Political Economy 128*(1), 206–251. 5

Colacito, R., M. M. Croce, and Z. Liu (2019). Recursive allocations and wealth distribution with multiple goods: Existence, survivorship, and dynamics. *Quantitative Economics 10*(1), 311–351. 5

Duffie, D. and L. G. Epstein (1992a, 05). Asset Pricing with Stochastic Differential Utility. *The Review of Financial Studies 5*(3), 411–436. 6

Duffie, D. and L. G. Epstein (1992b, 02). Stochastic differential utility. *Econometrica 60*(2), 353–394. 6

Fisher, J., D. S. Johnson, and T. M. Smeeding (2015). Inequality of income and consumption in the u.s.: Measuring the trends in inequality from 1984 to 2011 for the same individuals. *Review of Income and Wealth 61*(4), 630–650. 21

Gârleanu, N. and S. Panageas (2015). Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing. *Journal of Political Economy 123*(3), 670–685. 2, 4, 11, 19, 20

Gomes, F. and A. Michaelides (2007, 11). Asset Pricing with Limited Risk Sharing and Heterogeneous Agents. *The Review of Financial Studies 21*(1), 415–448. 5

Gomez, M. (2024). Wealth inequality and asset prices. Technical report, Columbia University. 5, 32

Guvenen, F. (2009). A parsimonious macroeconomic model for asset pricing. *Econometrica 77*(6), 1711–1750. 5

Guvenen, F., G. Kambourov, B. Kuruscu, S. Ocampo, and D. Chen (2023, 01). Use it or lose it: Efficiency and redistributional effects of wealth taxation*. *The Quarterly Journal of Economics 138*(2), 835–894. 32

Harrison, J. and D. M. Kreps (1979). Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory 20*(3), 381–408. 11

Kuhn, M. and V. Rios-Rull (2016). 2013 update on the u.s. earnings, income, and wealth distributional facts: A view from macroeconomics. *Quarterly Review 37*(1). 21

Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of Monetary Economics 15*, 335–359. 20

Schroder, M. and C. Skiadas (1999). Optimal consumption and portfolio selection with stochastic differential utility. *Journal of Economic Theory 89*(1), 68–126. 11

Yaari, M. E. (1965). Uncertain lifetime, life insurance, and the theory of the consumer. *Review of Economic Studies 32*(2), 137–150. 5