The Transmission of Foreign Shocks in a Networked Economy

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We develop a multi-country New Keynesian model with rich sectoral heterogeneity and both national and international production networks to capture the granular nature of recent supply-side shocks. Calibrating the model to the main Euro-Area countries and their trade partners, we analyze the transmission of international energy price shocks. Our results show that input-output linkages play a crucial role in shaping inflation dynamics, significantly amplifying the response and persistence of inflation through a feedback loop between selling prices and production costs. High trade integration across European economies allows inflationary pressures to propagate across borders via input-output linkages, with the interaction between national and international networks creating a larger inflationary impact than either in isolation. Moreover, heterogeneous production structures lead to diverse inflationary outcomes: countries with more integrated production structures experience prolonged inflationary pressures, while countries with more downstream-oriented industries see sharper but shorter-lived inflation spikes. We derive implications for monetary policy: a weaker systematic monetary policy response increases inflation volatility more when production networks are present, despite input-output linkages heightening the degree of monetary non-neutrality in response to monetary policy shocks. Finally, we find that production networks diminish (amplify) the differences in inflation (real GDP) responses between "leaning against the wind" and "looking-through" monetary policy following an energy price shock.

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1. Introduction

In recent years, the global economy has experienced a series of supply-side shocks that have significantly disrupted inflation dynamics and macroeconomic stability. Examples of these are energy price shocks, often triggered by geopolitical events, and supply chain disruptions. Despite their different underlying causes, these shocks share a key characteristic: they originate in specific sectors but quickly spread through complex production networks and international supply chains, ultimately affecting the whole economy. As a result, understanding how these shocks are transmitted through input-output (IO) linkages and spillover across countries and sectors has become a central focus of recent macroeconomic research.

In this paper, we investigate the transmission of supply-side shocks through production networks and their impact on inflation dynamics through the lens of a multi-country New Keynesian model of the global economy with rich sectoral heterogeneity and national and international production networks. Calibrating the model to the main Euro-Area countries and their trade partners, we use the model to analyze the macroeconomic effects of shocks to international energy prices and their transmission.

Our results show that production networks, by generating a feedback loop between increasing selling prices and rising production costs, are key in shaping inflation dynamics in response to the international energy price shock. First, we find that IO linkages significantly amplify the response and persistence of headline and core inflation. Namely, we find that the cumulative response of headline inflation would be up to 60% smaller and substantially shorter-lived if production networks were absent. Additionally, we show that high trade integration across European economies propagates inflationary pressures across borders via IO linkages, with the interaction between national and international networks generating a larger inflationary impact than either alone. Second, heterogeneity in production structures gives rise to differential inflation dynamics across countries: countries with more upstream industries and longer production chains (e.g., Germany) exhibit larger amplification and more persistence. In contrast, inflation is shorter-lived in countries with more downstream-oriented production structures and less complex production networks (e.g., Spain). Third, we explore the implications of our findings for the conduct of monetary policy. We show that a weaker systematic monetary policy response increases inflation volatility more with production networks conditional on these supply-side shocks, despite IO linkages and intermediate goods dampening the response of inflation to monetary policy shocks (Nakamura and Steinsson 2010; Rubbo 2023).

More in detail, we consider a model of the global economy with K countries and I sectors or industries within each country, and incomplete international financial markets. Depending on the monetary regime in place, countries may form part of currency unions or may have monetary autonomy. Firms in each sector use domestic labor alongside imported and domestically produced intermediate goods, leading to national and international production networks. At the sector level, we incorporate a high degree of heterogeneity to match observed data on labor shares, IO shares, and exposure to domestic and international markets through IO linkages. In addition, we also allow for nominal rigidities in nominal wages (Erceg *et al.* 2000) and staggered price setting à la Calvo (1983). We assume that nominal wage inflation is common across sectors, but we allow each sector's price-frequency probability to be heterogeneous across countries and industries.

We calibrate the model to 44 sectors per country and 6 regions: the four largest Euro-Area countries (Germany, France, Italy, and Spain), the rest of the Euro Area, and the rest of the world. We make sure that the model replicates observed trade flows between different sectors and countries using IO tables from the OECD and Eurostat. In addition, we use micro-level CPI data from Gautier *et al.* (2024) to calibrate the heterogeneous price-frequency adjustment probabilities across sectors and countries, allowing the model to capture the varying degrees of price rigidity observed in the data.

We next examine the effects and propagation of an increase in the international price of imported energy paid by European firms. We assume that this increase is driven by an exogenous wedge between the price charged by foreign exporting firms and the price paid by domestic importing firms (see, for example, Baqaee and Farhi 2024). This assumption aligns with the notion that, as seen in the recent energy crisis, fluctuations in international energy prices are often triggered by geopolitical rather than macroeconomic events.

In response to the increase in energy prices, we find that while Euro-Area headline inflation initially spikes due to the surge in energy prices, it declines very gradually over time, with core inflation becoming the primary driver of headline inflation, rather than energy prices. That is, we obtain a significant pass-through from headline to core inflation, which notably increases the persistence of inflationary pressures. Specifically, our results show that, on impact, core inflation increases by approximately 20% of the increase in headline inflation, consistent with previous empirical findings (Adolfsen *et al.* 2024). The increase in energy prices induces energy goods to become more expensive for households and energy production costs for firms to increase. As a result, firms respond by increasing the prices of their products. Therefore, through the production network, the costs of imported and domestically produced goods for firms increase further, leading to an additional increase in prices. This feedback between increasing selling prices and rising production costs results in a generalized increase in core and headline inflation.

The interaction between price rigidities and IO linkages adds more persistence to inflation dynamics. With staggered price-setting, the average selling price also incorporates the individual prices of those firms that have updated prices yet, in addition to those of updating firms. Next, in our model, a key component of firms' marginal costs is the price of the intermediate goods they purchase, and through the production network, also the costs faced by their suppliers. As a result, the stickiness and lagged adjustments in these prices are transmitted through firms' marginal costs and selling prices, ultimately amplifying and prolonging inflationary pressures.

We more formally quantify and isolate the role that production networks play through a series of counterfactuals. Namely, we consider three counterfactual economies where we sequentially turn off domestic, international, and national and international production networks.[1]

We find that without national and international production networks, cumulative headline inflation would be roughly up to 60% of our baseline, which includes a fully fledged production network structure. In particular, we find that despite headline inflation rising similarly on impact – driven by the rise of energy prices – it stabilizes and dies out much faster when production networks are absent, in line with the intuition provided above.

We further isolate the role played by national and international production networks separately. On the one hand, we find that the IO network contributes significantly to inflation persistence. Due to the high level of integration between industrial sectors across European economies, there are substantial spillovers from the shock, transmitted through cross-country links captured in the IO tables. On the other hand, it is crucial to account for both national and international production networks simultaneously. Specifically, the joint effect of these two dimensions is greater than the sum of their individual impacts in isolation. Intuitively, higher domestic inflation leads to increased export prices, which raises inflation abroad. In turn, increasing inflation abroad leads to higher import prices, further amplifying domestic inflation.

Second, we obtain that the increase in energy prices results in heterogeneous inflation developments across countries. For example, headline inflation increases sharply in Spain but it dissipates quickly. On the other hand, Germany shows the opposite dynamics. Namely, inflation in the German economy increases less than in Spain, but it displays more persistent dynamics. As a result, cumulative inflation in Germany surpasses that in Spain over time. The presence of heterogeneous production structures and consumption baskets can also rationalize this finding. That is, the energy share in the consumption baskets of Spanish households is greater than in Germany, which explains the initial heterogeneous inflation spike. However, the size of the IO network in Germany is significantly larger, meaning that the amplification effects previously described arise more strongly in the German economy, leading to a more persistent and over time larger response of inflation.

Finally, we analyze the implications of production networks and our findings for the con-

[1] In all these counterfactual economies we always keep energy as a production input for firms.

duct of monetary policy. Previous research has highlighted that the presence of intermediate goods and IO linkages leads to a higher degree of monetary non-neutrality in response to monetary policy shocks (Nakamura and Steinsson 2010; Basu 1995; Christiano 2016; Rubbo 2023). Namely, inflation tends to move less and output more in response to a monetary policy shock, as stickiness in the price of intermediate goods adds inertia to firms' marginal costs. Our model replicates these findings, with the presence of production networks dampening the response of inflation to a monetary policy shock.

We find that the systematic response of monetary policy is more relevant with IO linkages in the presence of energy price shocks, despite monetary policy shocks having a smaller effect on inflation. Namely, simulating the economy through a time series of shocks to the international price of energy, we find that a weaker systematic monetary policy response increases inflation volatility more when production networks are accounted for. Since international energy price shocks are amplified through the production network, a more passive monetary response allows the feedback between selling prices and production costs to build up further, resulting in a larger increase in inflation volatility. This finding shows that the implications of IO linkages for monetary policy are subtle: even though they may reduce the inflation effect of a given change in interest rates, such a monetary response becomes more relevant when it arrests the propagation of shocks that the production network severely amplifies.

Lastly, we study the impact of two monetary policy stances, "leaning against the wind" and "looking-through", on headline inflation and real GDP growth following an energy price shock. Comparing the inflation and output dynamics under each monetary stance, we find that the inflation (output) gap between the two stances is diminished (amplified) when production networks are present, highlighting the lessening (amplifying) role of IO linkages on inflation (output) due to the exacerbated monetary non-neutrality (Nakamura and Steinsson 2018; Rubbo 2023).

Related literature. Our paper contributes to several strands of the literature, at the intersection of the macroeconomic effects of production networks and the propagation of international macroeconomic shocks.

The seminal works of Acemoglu *et al.* (2012), Gabaix (2011), and Baqaee and Farhi (2019) study the propagation of granular shocks in production networks under flexible prices, abstracting from their inflationary effects. Building on these contributions, Pasten *et al.* (2020) and Rubbo (2023) incorporate IO linkages into frameworks with nominal rigidities. However, both papers focus on closed economies and thus cannot address international shocks, cross-country heterogeneity, and spillovers. In contrast, our paper analyzes these dynamics in an open economy, explicitly accounting for international trade and production network spillovers.

Earlier research examined the transmission of monetary policy in closed economies with simpler roundabout production structures. Nakamura and Steinsson (2010) develop a menu-cost model with heterogeneous sectoral nominal rigidities, showing that intermediate goods amplify monetary non-neutrality. While we replicate this finding, we also show that with supply-side shocks, the systematic component of monetary policy has a greater influence when IO linkages are present. Huang (2006) and Huang and Liu (2004) highlight that nominal rigidities and intermediate goods increase inflation persistence. Our model extends this by showing that a fully-fledged IO structure further amplifies persistence, beyond what would occur in a simpler roundabout economy.

Our paper also contributes to the growing literature that incorporates IO linkages in open economy models. Baqaee and Farhi (2024) study the propagation of shocks in an open economy model with production networks, but with a limited role for nominal price rigidities and monetary policy. Comin *et al.* (2023) develop a more tractable small open economy model with nominal rigidities, but focus on potentially binding capacity constraints. Ernst *et al.* (2023) consider a multi-country environmental model with flexible prices to study carbon taxes and climate clubs. Finally, Andrade *et al.* (2023) developed a 3-sector small open economy à la Gali and Monacelli (2005) to study the propagation of productivity shocks. Relative to this paper, and beyond focusing on energy price shocks, the quantitative nature of our model allows us, for example, to be able to uncover cross-country heterogeneity arising from diverse production structures.

Finally, we contribute to the literature exploring the transmission of energy shocks in macroeconomic models. An earlier contribution is Bodenstein *et al.* (2008), which studied optimal monetary policy in a closed-economy model with an energy sector. Gagliardone and Gertler (2023) explore the origins of the inflation surge in the US using a closed economy new Keynesian framework with oil, and find that oil price shocks were a key determinant. Auclert *et al.* (2023), Chan *et al.* (2024), and Bayer *et al.* (2023) explore the consequences of the recent energy crisis in open economy models with household heterogeneity, while the focus of the current paper is on the consequences of heterogeneity in the production sectors and across countries.

Roadmap. The paper proceeds as follows. Section 2 presents the international input-output New Keynesian framework. In Section 3 we describe the model calibration, and we derive the main results in section 4. Section 5 concludes the paper. Appendix A contains a detailed derivation of the linearized model.

2. Model

We consider a world economy composed of K asymmetric countries, indexed by k. The core of our model is a production structure characterized by national and international production networks through IO linkages. Namely, each country is comprised of I production sectors, heterogeneous within and between countries, that produce using labor and intermediate goods produced by other domestic and foreign sectors. A subset $I_E \in I$ of these sectors produces energy goods, while the complement set I_M produces only non-energy goods. Next to this, we also allow for staggered price setting and nominal wage rigidities.

2.1. Households

Each country k is inhabited by a large number of identical households. Each household is made up of an infinitely lived continuum of members, each specialized in a different labor service, indexed by $g \in [0, 1]$. Income is pooled within each household, acting as a risk-sharing mechanism. The representative household enjoys consumption ($C_{k,t}$) and dislikes labor ($\mathcal{N}_{gk,t}$). The agent's preferences per period are described by the CRRA utility function $U(C_{k,t}, \mathcal{N}_{gk,t}; Z_{k,t}) = \left(\frac{C_{k,t}^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_{gk,t}^{1+\varphi}}{1+\varphi}\, dg\right) Z_{k,t}$, where $Z_{k,t}$ is an exogenous preference shifter, σ and φ denote the inverse of the intertemporal elasticity of substitution and the inverse of the Frisch elasticity, respectively. Each household takes as given labor income since wages are set by labor unions and employment is decided by firms. Thus, the only decisions made by the household are the optimal allocation of consumption expenditures among different good varieties across different countries, and the optimal intertemporal allocation of consumption.

Consumption Baskets. There are I industries within each economy and the consumer has homothetic preferences over their products. Households have a consumption basket composed of goods and services produced by industries from different countries, which are aggregated under a nested CES function. Thus, households combine the different national varieties of each industry, subsequently they combine the goods and services produced by each of the different industries, and finally they combine the bundle of energy and non-energy goods.

In the top layer, households consume a bundle of energy and non-energy goods. The consumption aggregator $C_{k,t}$ is given by

$$C_{k,t} = \left[\widetilde{\beta}_k^{\frac{1}{\gamma}} C_{kE,t}^{\frac{\gamma-1}{\gamma}} + \left(1 - \widetilde{\beta}_k\right)^{\frac{1}{\gamma}} C_{kM,t}^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} \tag{1}$$

where $C_{kE,t}$ and $C_{kM,t}$ denote the consumption of energy and non-energy goods, respectively;

$\widetilde{\beta}_k$ is the initial share of expenditure on energy in country k, and $\gamma > 0$ measures the elasticity of substitution between energy and non-energy goods. A key advantage of this specification is that, given the relevance of energy products in economic fluctuations, particularly in the recent post–COVID period, it allows us to introduce a specific elasticity of substitution of the energy consumption that does not necessarily need to be equal to the elasticity of substitution between the rest of goods and services.[2]

In the middle layer, households combine the different energy sources and the different non-energy goods and services to produce, respectively, the energy ($C_{kE,t}$) and non-energy ($C_{kM,t}$) bundles,

$$C_{kE,t} = \left[\sum_{i \in I_E} \widetilde{\nu}_{ki}^{\frac{1}{\eta}} C_{ki,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad \text{and} \quad C_{kM,t} = \left[\sum_{i \in I_M} \widetilde{\upsilon}_{ki}^{\frac{1}{\iota}} C_{ki,t}^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}} \tag{2}$$

where $C_{ki,t}$ denote the consumption of goods from industry i, $\widetilde{\nu}_{ki}$ are the steady-state expenditure shares of each energy source $i \in I_E$ over the total energy consumption and $\widetilde{\upsilon}_{ki}$ the steady-state expenditures weight of each sector $i \in I_M$ of the total non-energy consumption. Within each of the two bundles, different products can be substituted with elasticities $\eta > 0$ and $\iota > 0$, respectively.

In the bottom layer, households combine the differentiated varieties of goods and services from each industry. Thus, the consumption bundle of industry i's good or service by the household in country k , $C_{ki,t}$, is given by

$$C_{ki,t} = \left[\sum_{l=1}^{K} \widetilde{\zeta}_{kl\,i}^{\frac{1}{\delta}} C_{kl\,i,t}^{\frac{\delta-1}{\delta}} \right]^{\frac{\delta}{\delta-1}} \tag{3}$$

where $C_{kl\,i,t}$ is the consumption of good i purchased from country l . Parameter $\widetilde{\zeta}_{kl\,i}$ represents the steady-state expenditure on the variety produced in country l over the total consumption of the industry i made by the household from country k, and δ represents the Armington trade elasticity of substitution between the different national producers of a particular good or service. Typically, the share of expenditure by households on its own country variety of an industry ($\widetilde{\zeta}_{kki}$) is larger than the on the rest of the countries' varieties, reflecting the presence of home bias in final consumption.

[2]Notice that, while we allow energy shares to be heterogeneous by countries, we assume that the elasticity of substitution between energy and non-energy goods is homogeneous across different economies.

Intertemporal Household Problem. International financial markets are incomplete, with households in each country only having access to two risk-free bonds. More precisely, the household in country k has access to a domestic bond, $B_{k,t}$, and an internationally traded bond, $B^K_{k,t}$, issued, without loss of generality, by country K and denominated in country K's currency. The agent maximizes the present discounted value of per-period utility flows, with discount factor β, subject to her budget constraint,

$$\begin{aligned} & P_{kC,t}C_{k,t} + B_{k,t} + B^K_{k,t}\left[1 - \Gamma(\text{NFA}^K_{k,t})\right]^{-1} \mathcal{E}_{kK,t} + \Xi_{k,t} \leq \\ & B_{k,t-1}(1 + i_{k,t-1}) + B^K_{k,t-1}\mathcal{E}_{kK,t}(1 + i_{K,t-1}) + \int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg + \Pi_{k,t} - T_{k,t} \end{aligned} \tag{4}$$

where $P_{kC,t}$ denotes the consumer price index in country k, derived formally in Appendix A, $\int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg$ is the nominal labor income received by the representative household, $\Pi_{k,t}$ denotes the profits made by the monopolistically competitive domestic firms and reimbursed to households, $\mathcal{E}_{kK,t}$ is the nominal exchange rate between the currency in country k and the currency in the country K. $\text{NFA}^K_{k,t} = B^K_{k,t}\mathcal{E}_{kK,t}$ is the net foreign asset position of households in the country k, and where $\Gamma(x) = \gamma_*\left(\exp\left\{x/\mathcal{Y}_{k,t}\right\} - 1\right)$ is an external financial intermediary premium that depends on the economy-wide net holdings of internationally traded foreign bonds as a ratio to the national nominal GDP $\mathcal{Y}_{k,t}$, with $\gamma_* > 0$.[3] The incurred intermediation premium is rebated to households in a lump-sum manner through the fiscal instrument $\Xi_{k,t}$. Finally, $T_{k,t}$ denotes government transfers, also rebated to households in lump sum.

The above program delivers two sets of different Euler conditions,

$$C^{-\sigma}_{k,t} = \mathbb{E}_t\beta C^{-\sigma}_{k,t+1}\frac{1 + i_{k,t}}{1 + \pi_{kC,t+1}}\frac{Z_{k,t+1}}{Z_{k,t}} \tag{5}$$

$$C^{-\sigma}_{k,t} = \mathbb{E}_t\beta C^{-\sigma}_{k,t+1}\frac{1 + i_{K,t}}{1 + \pi_{kC,t+1}}\left[1 - \Gamma(\text{NFA}^K_{k,t})\right]\frac{\mathcal{E}_{kK,t+1}}{\mathcal{E}_{kK,t}}\frac{Z_{k,t+1}}{Z_{k,t}} \quad \forall k \neq K \tag{6}$$

where we assume that the (log-)demand shock follows an AR(1) process:

$$z_{k,t} = \rho^z_k z_{k,t-1} + \varepsilon^z_{k,t} \tag{7}$$

where $z_{k,t} := \log Z_{k,t}$, and $\varepsilon^z_{k,t} \sim \mathcal{N}\left(0, \sigma^2_{kz}\right)$.

[3]The role of this intermediation premium is to stabilize the net foreign asset position in response to transitory shocks, a common practice in open economies with incomplete financial markets (Schmitt-Grohe and Uribe 2003). Furthermore, this specification guarantees that, in the non-stochastic steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero.

2.2. Firms

There are I industries in each economy, indexed by $i \in \{1, ..., I\}$, and within each industry there is a unit mass of firms. A unit mass of monopolistically competitive firms, indexed by $f \in (0, 1)$, produce differenciated varieties in each sector i of each country k. Firms employ labor, N_{fki}, and they use a bundle of intermediate inputs that they source from the rest of sectors and countries, X_{fki}, to produce their output Y_{fki}:

$$Y_{fki,t} = A_{ki,t} \left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{fki,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{fki,t}^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}\psi_{ki}} \tag{8}$$

Both factors are combined under a constant elasticity of substitution ψ; $\widetilde{\alpha}_{ki}$ can be interpreted as a measure of labor-biased demand in production in sector i and country k, $\widetilde{\vartheta}_{ki}$ can be interpreted as a measure of input-biased demand in production in sector i and country k, and ψ_{ki} modulates the degree of returns to scale in production. Notice that while we allow labor and intermediary shares to be heterogeneous by sectors and countries, we assume that the elasticity of substitution between labor and intermediary goods is homogeneous across different economies. We assume that the (log-)TFP shock follows an AR(1) process:

$$a_{ki,t} = \rho_{ki}^{a} a_{ki,t-1} + \varepsilon_{ki,t}^{a} \tag{9}$$

where $a_{ki,t} := \log A_{ki,t}$, and $\varepsilon_{ki,t}^{a} \sim \mathcal{N}\left(0, \sigma_{kia}^{2}\right)$.

Intermediate Input Baskets. Firms use an intermediate input bundle $X_{ki,t}$. Similarly to the households' consumption basket, they demand the output of the rest of firms in the economy and they aggregate those inputs under a nested CES aggregator. First, they combine all the existing varieties of a given sector, next they combine the inputs of the different sectors, and finally they combine the bundle of energy and non-energy sectors.

In the top layer, firms combine bundles of energy and non-energy inputs

$$X_{ki,t} = \left[\widetilde{\beta}_{ki}^{\frac{1}{\phi}} X_{kiE,t}^{\frac{\phi-1}{\phi}} + (1 - \widetilde{\beta}_{ki})^{\frac{1}{\phi}} X_{kiM,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{10}$$

where $X_{kiE,t}$ and $X_{kiM,t}$ denote the amount of intermediate inputs of energy and non-energy goods, respectively; $\widetilde{\beta}_{ki}$ can be interpreted as the energy intensity of sector i from country k, and $\phi > 0$ measures the substitutability between energy and non-energy intermediate inputs.[4]

[4]Notice that, while we allow energy shares to be heterogeneous by countries, we assume that the elasticity of

In the middle layer, both energy ($X_{kiE,t}$) and non-energy intermediate inputs ($X_{kiM,t}$) are themselves composite intermediate baskets:

$$X_{kiE,t} = \left[\sum_{j\in I_E} \tilde{\nu}_{kij}^{\frac{1}{\chi}} X_{kij,t}^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}} \quad \text{and} \quad X_{kiM,t} = \left[\sum_{j\in I_M} \tilde{\upsilon}_{kij}^{\frac{1}{\xi}} X_{kij,t}^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \tag{11}$$

where $X_{kij,t}$ denotes industry i's demand of intermediate use of goods from industry j; $\tilde{\nu}_{kij}$ is the relative importance of each energy source j over firm's i energy expenditure and $\tilde{\upsilon}_{kij}$ the weight of input from sector j on firms' i non-energy expenditure. The input bundles from different energy or non-energy sectors can be substituted with elasticities $\chi > 0$ and $\xi > 0$.

Finally, in the bottom layer, firms combine the different country varieties of each sector j to produce the composite intermediate input of that sector input, $X_{kij,t}$:

$$X_{kij,t} = \left[\sum_{j=1}^{I} \tilde{\zeta}_{kl\,ij}^{\frac{1}{\mu}} X_{kl\,ij,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{12}$$

where $X_{kl\,ij,t}$ is the intermediate input demand of sector i in country k, purchased from industry j in country l. The parameter $\zeta_{kl\,ij}$ reflects the importance of the country l in the supply of the input j to firms, and the different varieties of countries can be substituted with an Armington (1969) trade elasticity μ.

2.3. Price– and Wage–Setting

The price and wage rigidities are introduced in a way analogous to the Calvo (1983) framework. Firms (labor unions) specialized in any given good (labor) type can reset their price (nominal wage) only with probability $1-\theta_{ki}^{p}$ $(1-\theta_{k}^{w})$ each period, independently of the time elapsed since they last adjusted their price (wage). Notice that we allow nominal prices to be heterogeneously rigid, depending on their sectoral and geographical location. In addition, we adopt the local currency pricing paradigm (Devereux and Engel 2003), meaning that goods' prices are set in the currency of destination.

The firm (labor union) chooses the optimal price (wage) to maximize the discounted sum of profits (household welfare) subject to the good (labor) demand schedules. We document in Appendix A that such maximization programs yield the log-linearized wage, domestic price,

substitution between energy and non-energy intermediate goods is homogeneous across different economies.

and export price Phillips curves:

$$\pi_{wk,t} = \kappa_{wk}\left(\sigma\widehat{c}_{k,t} + \varphi\widehat{n}_{k,t} - \widehat{w}_{k,t}\right) + \beta\mathbb{E}_t\pi_{wk,t+1} + u^{w}_{ki,t} \tag{13}$$

$$\pi_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}_{ki,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^{p}_{ki,t} \tag{14}$$

$$\pi^{l}_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \widehat{p}^{l}_{ki,t} - \widehat{q}_{kl,t}\right) + \beta\mathbb{E}_t\pi^{l}_{ki,t+1} \tag{15}$$

where $\pi_{ki,t} = p_{ki,t} - p_{ki,t-1}$, $\pi_{wk,t} = w_{k,t} - w_{k,t-1}$ denote price and wage inflation, $\pi^{l}_{ki,t}$ export price inflation produced in country k and sold to country l; $\kappa_{ki} = \frac{(1-\theta^{p}_{ki})(1-\beta\theta^{p}_{ki})}{\theta^{p}_{ki}}\Theta_{ki}$ and $\Theta_{ki} \equiv \frac{\psi_{ki}}{\psi_{ki}+(1-\psi_{ki})\epsilon_{pki}}$. Both aggregate consumption $\widehat{c}_{k,t}$ and employment $\widehat{n}_{k,t}$ appear in log-deviations from their steady-state values, and the price variables (real marginal costs $\widehat{\mathrm{mc}}_{ki,t} = \widehat{\mathrm{mc}}^{n}_{ki,t} - p_{kC,t}$, the real price level $\widehat{p}_{ki,t} = p_{ki,t} - p_{kC,t}$, the real wage $\widehat{w}_{k,t} = w_{k,t} - p_{kC,t}$) appear in real terms so that they are stationary. Finally, $\widehat{q}_{kl,t}$ denotes the log-deviation of the real exchange rate between country k and country l:

$$\mathcal{Q}_{kl,t} = \frac{P_{lC,t}\mathcal{E}_{kl,t}}{P_{kC,t}}. \tag{16}$$

We additionally introduce an exogenous price wedge τ^{l}_{ki} between the price set by the exporting firm $\widetilde{P}^{l}_{ki,t}$ and the actual price paid by the importing firm $P^{l}_{ki,t}$:

$$P^{l}_{ki,t} = \left(1+\tau^{l}_{ki,t}\right)\widetilde{P}^{l}_{ki,t} \tag{17}$$

These price wedges are akin to the iceberg trade costs present in much of the trade literature (see, for example, Baqaee and Farhi 2024 for a similar specification). Our motivation for these is to have a source of exogenous movements in import prices (e.g. energy) that are not triggered by changes in economic activity of the exporting country. We allow for a richer specification for the (log-)price wedge, in the form of an AR(2) process:

$$\tau_{lkj,t} = \rho^{\tau}_{1,klj}\tau_{lkj,t-1} + \rho^{\tau}_{2,klj}\tau_{lkj,t-2} + \varepsilon^{\tau}_{lkj,t} \tag{18}$$

Finally, we assume that the price and wage cost-push shocks, micro-founded through markup shocks, follow two independent AR(1) processes:

$$u^{p}_{ki,t} = \rho^{p}_{ki}u^{p}_{ki,t-1} + \varepsilon^{p}_{ki,t} \tag{19}$$

$$u^{w}_{k,t} = \rho^{w}_{k}u^{w}_{k,t-1} + \varepsilon^{w}_{k,t} \tag{20}$$

In this open input-output (IO) economy, the price Phillips curve (14) depends on the international supply network through the real marginal costs faced by firm i in country k, $\widehat{\mathrm{mc}}_{ki,t}$. We show in Appendix A that (log-linearized) real marginal costs depend on its own output, its own productivity, and a weighted average of real wage expenses and intermediate input real prices,

$$\widehat{\mathrm{mc}}_{ki,t} = \frac{1-\psi_{ki}}{\psi_{ki}}\widehat{y}_{ki,t} - \psi_{ki}a_{ki,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\alpha_{ki}\widehat{w}_{k,t} + \sum_{l=1}^{K}\sum_{j=1}^{I}\frac{\mathcal{M}_{ki}}{\psi_{ki}}\omega_{kl\,ij}\widehat{p}_{kl\,ij,t} \tag{21}$$

where $\alpha_{ki} = \frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \frac{W_k N_{ki}}{\mathcal{M}_{ki}\mathrm{MC}_{ki}Y_{ki}}$ denotes the (steady-state) labor income share of total sales of firm i, $\omega_{kl\,ij} = \frac{P_{kl\,j}X_{kl\,ij}}{P_{ki}Y_{ki}} = \frac{P_{kl\,j}X_{kl\,ij}}{\mathcal{M}_{ki}\mathrm{MC}_{ki}Y_{ki}}$ denotes the (steady-state) IO expenditure share of total sales of firm i, and $\mathcal{M}_{ki}$ denotes the markup charged by firm i.

2.4. Monetary Authority

There is a monetary authority in each country $k \in K$. In terms of the monetary stance, we differentiate between those countries that belong to a monetary union and those countries that are member states of currency unions.

Non-members of Currency Unions. For those countries that are not part of the currency union, we assume that each central bank follows an inflation-targeting Taylor rule:

$$\begin{aligned} i_{k,t} &= \rho_{kr} i_{k,t-1} + \left(1-\rho_{kr}\right)\phi_{k\pi}\pi_{k\phi,t} + \left(1-\rho_{kr}\right)\phi_{ky}\widehat{y}_{k,t} \\ &\quad + \phi_{k\Delta\pi}(\pi_{k\phi,t} - \pi_{k,t-1}) + \phi_{k\Delta y}(\widehat{y}_{k,t} - \widehat{y}_{k,t-1}) + \varepsilon^{r}_{k,t} \end{aligned} \tag{22}$$

where the coefficients are allowed to vary by country; ρ_{kr} denotes the degree of interest rate smoothing in the monetary instrument, coefficients $\{\phi_{k\pi}, \phi_{ky}, \phi_{k\Delta\pi}, \phi_{k\Delta y}\}$ modulate the elasticity of the policy rate with respect to changes in aggregate inflation $\pi_{k\phi,t}$ and output $\widehat{y}_{k,t}$, measured as the log-deviation from its steady-state value, and their growth rate; acting as a stabilization tool. Furthermore, we allow the monetary authority to choose the particular inflation measure (CPI, PCE, PPI, GDP deflator, etc.) that they aim to stabilize

$$\pi_{k\phi,t} = \mathbf{\Phi}^{\top}\boldsymbol{\pi}_{k,t} = \sum_{l=1}^{K}\sum_{i=1}^{I}\phi_{kl\,i}\pi_{kl\,i,t}. \tag{23}$$

where $\sum_{l=1}^{K}\sum_{i=1}^{I}\phi_{kl\,i} = 1$, and $\pi_{kki,t} = \pi_{ki,t}$. For example, when $\phi_{kl\,i}$ is equal to the consumption share of sector i in the country k, then the central bank targets headline inflation

$\pi_{kC,t} = \sum_{l=1}^{K}\sum_{i=1}^{I}\beta_{kli}\pi_{kli,t}$.

Members of Currency Unions. We allow countries to be part of a currency union. Namely, suppose that a subset $K^{MU} \subset K$ of countries belongs to a monetary union. Without loss of generality, we assume that the central bank of a country $k^{MU} \in K^{MU}$ sets the nominal interest rate to stabilize the *union-wide* price inflation and output deviations, π_t^{MU} and $\widehat{y}_t^{MU}$,[5]

$$\begin{aligned} i_{k^{MU},t} = \rho^{\mathrm{MU}r}_{k\mathrm{MU}} i_{k^{MU},t-1} + \left(1-\rho^{\mathrm{MU}r}_{k\mathrm{MU}}\right)\phi_{\mathrm{MU}\pi}\pi_t^{\mathrm{MU}} + \left(1-\rho^{\mathrm{MU}r}_{k\mathrm{MU}}\right)\phi_{\mathrm{MU}\,y}\widehat{y}_t^{\mathrm{MU}} \\ + \phi_{\mathrm{MU}\Delta\pi}(\pi_t^{\mathrm{MU}} - \pi_{t-1}^{\mathrm{MU}}) + \phi_{k\Delta y}(\widehat{y}_t^{\mathrm{MU}} - \widehat{y}_{t-1}^{\mathrm{MU}}) + \varepsilon^r_{\mathrm{MU},t} \end{aligned} \tag{24}$$

where π_t^{MU} and $\widehat{y}_t^{MU}$ are defined as the GDP-weighted sum of member states' price inflation and output deviations,

$$\pi_t^{\mathrm{MU}} = \sum_{k=1}^{K}\phi_k^{\mathrm{MU}}\pi_{k\phi,t} \quad \text{and} \quad \widehat{y}_t^{\mathrm{MU}} = \sum_{k=1}^{K}\phi_k^{\mathrm{MU}}\widehat{y}_{k,t} \tag{25}$$

with $\phi_k^{\mathrm{MU}} = \frac{y_k}{\sum_{l=1}^{K^{\mathrm{MU}}} y_l}$ as the measure of the (steady-state) relative size of country k in the monetary union in terms of nominal GDP. The central banks in the rest of countries $l \neq k^{MU}$ that belong to the monetary union adopt a peg vis-a-vis the country k^{MU} that sets the monetary stance:

$$\mathcal{E}_{k,k^{MU},t} = \mathcal{E}_{k,k^{MU}} \quad \forall k \in K^{MU} \tag{26}$$

where $\mathcal{E}_{k,k^{MU}}$ is the bilateral nominal exchange rate in steady state.

2.5. Market Clearing, GDP, and Trade Balance

Market Clearing. Market clearing in the goods market requires that the quantity produced of each good matches the quantity demanded at home and abroad, either for direct consumption or intermediate use. That is,

$$Y_{ki,t} = \sum_{l=1}^{K} C_{lki,t} + \sum_{l=1}^{K}\sum_{j=1}^{I} X_{lkji,t} \tag{27}$$

Market clearing in the labor market requires that the aggregate labor supplied matches

[5]The specific location of the union-wide central bank is innocuous as long as it targets union-wide variables.

the sum of labor demand across sectors, for each country. That is,

$$N_{k,t} = \sum_{i=1}^{I} N_{ki,t} \tag{28}$$

Finally, the aggregate resource constraint of the economy requires that the net foreign position of country k equals its trade balance, defined as the nominal exports ($P_{k\text{EXP},t}\text{EXP}_{k,t}$) net of nominal imports ($P_{k\text{IMP},t}\text{IMP}_{k,t}$):

$$B^{K}_{k,t}\mathcal{E}_{kK,t} - (1 + i_{K,t-1})B^{K}_{k,t-1}\mathcal{E}_{kK,t} = P_{k\text{EXP},t}\text{EXP}_{k,t} - P_{k\text{IMP},t}\text{IMP}_{k,t} \tag{29}$$

Gross Domestic Product and Net Exports. Nominal GDP is defined as the sum of total household consumption and nominal net exports,

$$\mathcal{Y}_{k,t} = P_{kC,t}C_{k,t} + P_{k\text{EXP},t}\text{EXP}_{k,t} - P_{k\text{IMP},t}\text{IMP}_{k,t} \tag{30}$$

where $P_{k\text{EXP},t}$ and $P_{k\text{IMP},t}$ are the export and import price deflators.

The GDP deflator is defined as the ratio between nominal GDP measured using time–t prices and nominal GDP measured using steady-state prices:

$$P_{kY,t} = \frac{P_{kC,t}C_{k,t} + P_{k\text{EXP},t}\text{EXP}_{k,t} - P_{k\text{IMP},t}\text{IMP}_{k,t}}{P_{kC}C_{k,t} + P_{k\text{EXP}}\text{EXP}_{k,t} - P_{k\text{IMP}}\text{IMP}_{k,t}}$$

Real GDP is defined as nominal GDP deflated by the GDP deflator,

$$Y_{k,t} = \frac{\mathcal{Y}_{k,t}}{P_{kY,t}}. \tag{31}$$

Nominal exports are defined as

$$P_{k\text{EXP},t}\text{EXP}_{k,t} = \sum_{l\neq k}\sum_{i=1}^{I} P_{ki,t}C_{l\,ki,t} + \sum_{l\neq k}\sum_{i=1}^{I}\sum_{j=1}^{I} P_{ki,t}X_{l\,kji,t} \tag{32}$$

Nominal imports are defined as:

$$P_{k\text{IMP},t}\text{IMP}_{k,t} = \sum_{l\neq k}\sum_{i\in I} P_{kl\,i,t}C_{kl\,i,t} + \sum_{l\neq k}\sum_{i\in I}\sum_{j=1}^{I} P_{kl\,i,t}X_{kl\,ji,t} \tag{33}$$

3. Data and Calibration

We calibrate the model economy at a quarterly frequency to 6 countries: Spain, France, Italy, Germany, the Rest of the EA (REA), and the Rest of the World (ROW). The production structure within each country contains 44 sectors. We next discuss the calibration strategy and collect in Table 1 the main parameter values and the corresponding targets or sources.

Households. We set the household's discount factor β to 0.99, to target an annual real interest rate of 4.5%. The intertemporal elasticity of substitution σ is set to 1, a common value in the literature. The inverse of the Frisch elasticity φ is set to 1, in line with the estimates presented in Chetty *et al.* (2011). Households' borrowing premium γ_* is set to 0.001 so that the evolution of net foreign assets has only a small impact on the exchange rate and trade in the short run while guaranteeing that the net foreign asset position is stabilized at zero in the long run (Schmitt-Grohe and Uribe 2003).

The elasticity of substitution in consumption between energy and non-energy goods γ is set to 0.4 following Böhringer and Rivers (2021). The elasticity of substitution in consumption between energy sources η and between non-energy sectors ι is set to 0.9 following Atalay (2017). Household's trade elasticity δ is set to 1.[6]

To calibrate the quasi-consumption shares $\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kl\,i}\}$ we rely on the linearized model to target the respective consumption sectoral consumption shares in each country. More precisely, in Appendix A we show that once the model has been linearized, it is possible to read directly consumption shares from the data as long as we have as many quasi-consumption shares parameters as data targets. Implementing this strategy, we obtain consumption shares by country from Inter-country Input Output (ICIO) tables produced by the OECD, using 2019 as our baseline period.

Regarding wage rigidities, ECB (2009) report limited cross-sectoral heterogeneity in wage frequency adjustments for Euro-Area countries. Therefore, we fix the Calvo frequency wage adjustment probability θ_k^w to 0.75 for all countries, in line with the evidence presented in Christoffel *et al.* (2008) for the Euro Area.

Production. We start by setting the ψ_{ki} equal to one, as to have constant returns to scale in production. The elasticity of substitution in production between labor and intermediate inputs ψ is set to 0.5 (Atalay 2017). The elasticity of substitution in production between energy and non-energy goods ϕ is set to 0.4 (Böhringer and Rivers 2021). The elasticity of substitution

[6] A growing body of literature has estimated the value of these elasticities for different time horizons, finding that the values of trade elasticities are significantly greater than one in the long term but not in the short term, with values around 1 for horizons of up to two years (Boehm *et al.* (2023)). Given that the focus of our work is closer to a cyclical analysis rather than a long-term one, we choose the value of 1.

Parameter	Description	Value	Target / Source
Households			
β	Discount factor	0.99	R = 4.5% p.a.
σ	Inv. Intertemp. Elast. Subs.	1	Standard Value
φ	Inv. Frisch Elasticity	1	Chetty *et al.* (2011)
γ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
η	Elast. Subst. E	0.9	Atalay (2017)
ι	Elast. Subst. M	0.9	Atalay (2017)
δ	Trade Elasticity	1	Standard value
$\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kli}\}$	Quasi-shares consumption		ICIO tables (OECD)
θ^w_k	Calvo wage prob.	0.75	Christoffel *et al.* (2008)
Firms			
ψ_{ki}	Returns to scale	1	Constant returns to scale
ψ	Elast. Subst. N and X	0.5	Atalay (2017)
ϕ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
χ	Elast. Subst. M	0.2	Atalay (2017)
ξ	Elast. Subst. E	0.2	Atalay (2017)
μ	Trade Elasticity	1	Standard value
$\{\widetilde{\alpha}_{ki}, \widetilde{\vartheta}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$	Quasi-shares production		ICIO tables (OECD)
$\mathcal{M}_{ki}$	Markups		Labor shares (Eurostat)
θ^p_{ki}	Calvo price prob.		Gautier *et al.* (2024)
Monetary Policy			
$\rho_{k,r}$	Interest Rate Smoothing	0.7	Standard Value
$\phi_{k,\pi}$	Reaction to Inflation	1.5	Galí (2015)
$\phi_{k,y}$	Reaction to real GDP	0.125	Galí (2015)
Exogenous Shock Process			
$\rho^\tau_{1,kli}$	Persistence price wedge shock	1.17	Brent crude oil
$\rho^\tau_{2,kli}$	Persistence price wedge shock	-0.2	Brent crude oil
σ^τ_{kli}	Std. Dev. price wedge shock	1	Standard Value

Notes: List of calibrated parameters. See the main text for a discussion on targets, values, and data used.

TABLE 1. Calibration

in production between energy sectors χ and between non-energy sectors ξ is set to 0.2, following the estimates of Atalay (2017). Finally, as with households, we set the trade elasticity for firms μ, equal to one.

We follow the same strategy as with households to calibrate the quasi-shares in production $\{\widetilde{\alpha}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$. Namely, using the linearized model around the steady-state we directly read from the data shares in of each intermediate good in production as well as the shares of labor and production in total costs. Our data source here again is the 2019 ICIO tables from OECD.

We complement the ICIO tables with the Figaro database by Eurostat to calibrate the labor share of each industry. Namely, once the quasi-shares in production have been used, we calibrate the sector-specific markups $\mathcal{M}_{ki}$ to target the wage-bill-over-sales observed in the data.

Sectoral price rigidities are obtained from the PRISMA project conducted by the European Central Bank (Gautier *et al.* 2024). Using CPI micro-data from several EA countries, the authors report the frequency of price adjustment by COICOP categories for each country separately, and from the aggregate EA. Using the COICOP–to–NACE correspondence tables (Kouvavas *et al.* 2021), we compute the frequency of price adjustment by each NACE category in each country, and obtain the heterogeneous Calvo (1983) price rigidities θ^{p}_{ki} for Spain, France, Italy, Germany and REA. Finally, we assume that the ROW price rigidities coincide with the aggregate REA price rigidities.

A drawback of the evidence presented in Gautier *et al.* (2024) is that it does not contain consistent price adjustment frequency data on energy goods. Therefore, we complement this with the evidence presented in Dhyne *et al.* (2006) on price adjustments for energy goods for Euro-Area countries. In line with the data presented there, and not surprisingly, energy sectors in the model have the steepest price Phillips Curves, with nearly fully flexible prices.

Monetary Policy. All Taylor rule parameters are set to their standard values. The interest-rate smoothing coefficient ρ_{rk} is set to 0.7. The coefficients for inflation and output, $\phi_{\pi k}$ and ϕ_{yk}, are set to their standard values of 1.5 and 0.125, respectively. The other coefficients $\phi_{\pi\Delta k}$ and $\phi_{y\Delta k}$ are set to zero. Furthermore, we set $\phi_{kli} = \beta_{kli}$, so that central banks target headline inflation.

Exogenous Process. We shut down all shocks in the economy except for the price wedge shock. We fit the persistence coefficients to the time-series data of the Brent crude oil. The variance of the innovation is set to 1.

4. Results

A key determinant of the recent inflation surge in the Euro Area has been increasing energy prices (Arce *et al.* 2024). Motivated by this, we next use our model to explore the aggregate effects of a shock to the price of imported energy paid by European firms. We first analyze the dynamics of Euro-Area variables, with a focus on how these dynamics are shaped by IO linkages. Second, we formally analyze the contribution of production networks to inflation dynamics through a series of counterfactuals. Third, we explore how heterogeneity in production structures gives rise to differential inflation dynamics across countries. Finally, we derive implications for monetary policy.

The energy price shock we analyze is structured as follows. In both the model and the data, the energy mining sector of the rest of the world (ROW) extracts the main energy products.[7] These energy goods are then sold to Euro-Area firms that primarily belong to the energy sectors Coke and petrol refining and Electricity sector.[8] After being processed by these sectors, energy goods are then supplied to households as consumption goods, and to the remaining sectors of the economy as energy intermediate goods used in the production process.

In line with the previous reasoning, we consider a 10% increase in the price wedge τ_{klj} between the price charged by the energy mining sector located RoW and the price paid by Euro-Area firms. Formally, we set k = {ES,DE,FR,IT,REA}, l = ROW and j = energy mining.

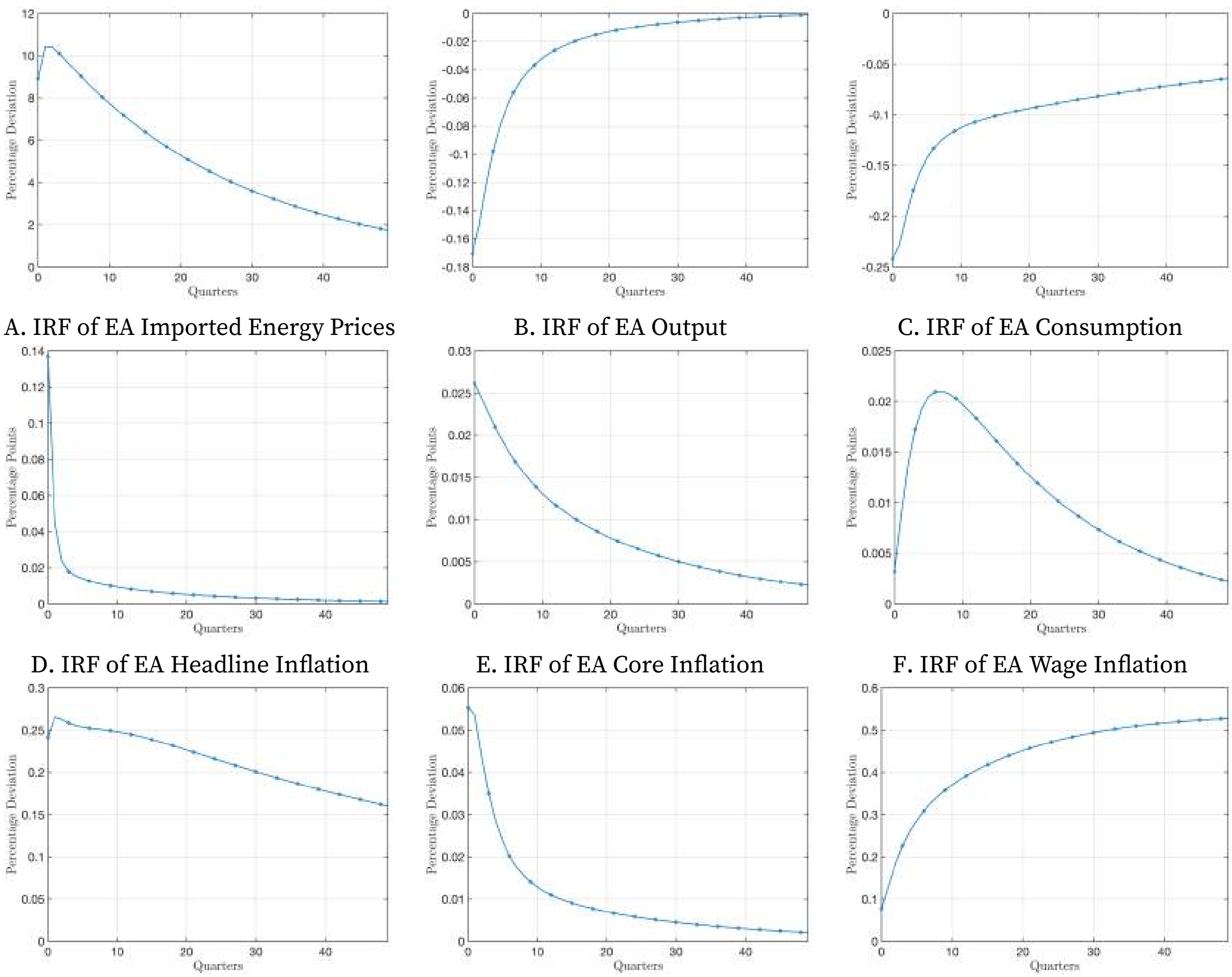
4.1. The Macroeconomic Effects of Rising International Energy Prices

Figure 1 shows the impulse response functions of Euro-Area GDP (panel 1B), real consumption (panel 1C), headline inflation (panel 1D), core inflation (panel 1E), services inflation (panel 1F), net exports (panel 1G), MU central bank policy rate (panel 1H), and the nominal exchange rate with respect to the ROW. The increase in the price of imported energy paid by Euro-Area firms is shown in panel 1A.

The increase of production costs for Euro-Area firms induces them to decrease labor demand and hence production, with value-added (real GDP) falling. In addition, the increase in international energy prices means a negative wealth shock for households, reducing their demand for domestic goods. Overall, we obtain a fall in real consumption larger than the fall in real GDP. Both imports and exports fall, with net exports increasing, due to the relative larger increase in the price of imported goods compared to exported goods. The systematic monetary policy stance of the central bank of the monetary union reacts by increasing the

[7] In our data, this corresponds with the Mining and quarrying of energy products sectors, which accounts for sections B.5 and B.6 in the ISIC, Rev.4 classification.

[8] In our data, these correspond with sections C.19 and D.35 in the ISIC, Rev.4 , classification respectively.



A. IRF of EA Imported Energy Prices
B. IRF of EA Output
C. IRF of EA Consumption
D. IRF of EA Headline Inflation
E. IRF of EA Core Inflation
F. IRF of EA Wage Inflation
G. IRF of EA Net Exports
H. IRF of EA Policy Rate
I. IRF of EA Nominal Exchange Rate vs. ROW

FIGURE 1. Effects of an International Energy Price Shock on Euro-Area Variables

Notes: Impulse response functions (IRF) of import energy prices (panel 1A), Euro-Area real GDP (panel 1B), Euro-Area real consumption (panel 1C), Euro-Area headline inflation (panel 1D), Euro-Area core inflation (panel 1E), Euro-Area wage inflation (panel 1F), Euro-Area net exports (panel 1G), Euro-Area nominal interest rate (panel 1H), and Euro-Area nominal exchange rate vs. Rest of the World (panel 1I) to a 10% peak increase in imported energy prices.

policy rate to control inflation. As a result, the currency of the monetary union appreciates, generating a rise in the nominal exchange rate with respect to the ROW.

Headline inflation responds immediately and sharply, reflecting the high price flexibility of energy sectors in the model and the non-negligible share of energy goods in households' consumption basket. The inflationary spike is followed by a more persistent rise in core inflation, which remains elevated long after the initial shock, contributing to the persistent increase of headline inflation. On impact, the pass-through of headline to core inflation is significant, amounting to roughly 20%, in line with the empirical evidence presented in Adolfsen *et al.* (2024). Intuitively, the increase in energy prices energy induces production costs for firms to increase. As a result, firms respond by increasing the prices of their products. Therefore, through the IO linkages, the costs of imported and domestically produced goods for firms further increase, leading to an additional rise in prices. This feedback between increasing selling prices and rising production costs results in a generalized increase in core and headline inflation. Finally, we obtain a remarkably persistent and hump-shaped response of wage inflation, which contributes to the persistent rise in core inflation.

The interaction between price rigidities and production networks is responsible for the persistent increase in core and headline inflation. We outline here the intuition for this result and show it quantitatively in the next section through counterfactuals. Consider for simplicity the case under constant returns to scale, $\psi_{ki} = 1$, domestic currency pricing, fixed nominal exchange rate, $\tau_{kl\,i,t} = \tau_{l\,i,t}$ follows an AR(1) process and with no further exogenous shocks in the economy. We show in Appendix A.6 that one can write the Phillips curves (14) in price level terms and in period $t + h$, as

(34)

$$\boldsymbol{p}_{t+h} = (\boldsymbol{I} - \boldsymbol{\Delta}\overline{\boldsymbol{\Omega}})^{-1} \left[\boldsymbol{\Delta}\overline{\boldsymbol{\Omega}} \left(\boldsymbol{R}^h \boldsymbol{\tau}_t + \sum_{s=0}^{h-1} \boldsymbol{e}^{\tau}_{t+s} \right) + (\boldsymbol{I} - \Delta) \sum_{s=1}^{h} \boldsymbol{\pi}_{t+h-s} + \boldsymbol{\Delta}\overline{\boldsymbol{\alpha}} \sum_{s=0}^{h} \boldsymbol{\pi}^w_{t+h-s} + \boldsymbol{\Delta}\boldsymbol{K}^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+h+1} \right]$$

where $\boldsymbol{\pi}_t = \boldsymbol{p}_t - \boldsymbol{p}_{t-1}$ and $\boldsymbol{\pi}^w_t = \boldsymbol{w}_t - \boldsymbol{w}_{t-1}$ with $\boldsymbol{p}_t = \begin{bmatrix} p_{11,t} & \dots & p_{1I,t} & p_{21,t} & \dots & p_{KI,t} \end{bmatrix}^\intercal$ and $\boldsymbol{w}_t = \begin{bmatrix} w_{1,t} & \dots & w_{1,t} & w_{2,t} & \dots & w_{2,t} & w_{K,t} & \dots & w_{K,t} \end{bmatrix}^\intercal$, and we have introduced nominal marginal costs (21), $\widehat{\mathbf{mc}}^n_t = \overline{\boldsymbol{\alpha}}\boldsymbol{w}_t + \overline{\boldsymbol{\Omega}} \left(\boldsymbol{\tau}_t + \boldsymbol{p}_t \right)$, where $\widehat{\mathbf{mc}}^n_t = \begin{bmatrix} \widehat{mc}^n_{11,t} & \dots & \widehat{mc}^n_{1I,t} & \widehat{mc}^n_{21,t} & \dots & \widehat{mc}^n_{KI,t} \end{bmatrix}^\intercal$,

$\overline{\boldsymbol{\alpha}} = \text{diag}\begin{pmatrix}\overline{\alpha}_{11} & \overline{\alpha}_{12} & \dots & \overline{\alpha}_{KI}\end{pmatrix}$ with $\overline{\alpha}_{ki} = \mathcal{M}_{ki}\alpha_{ki}$, and

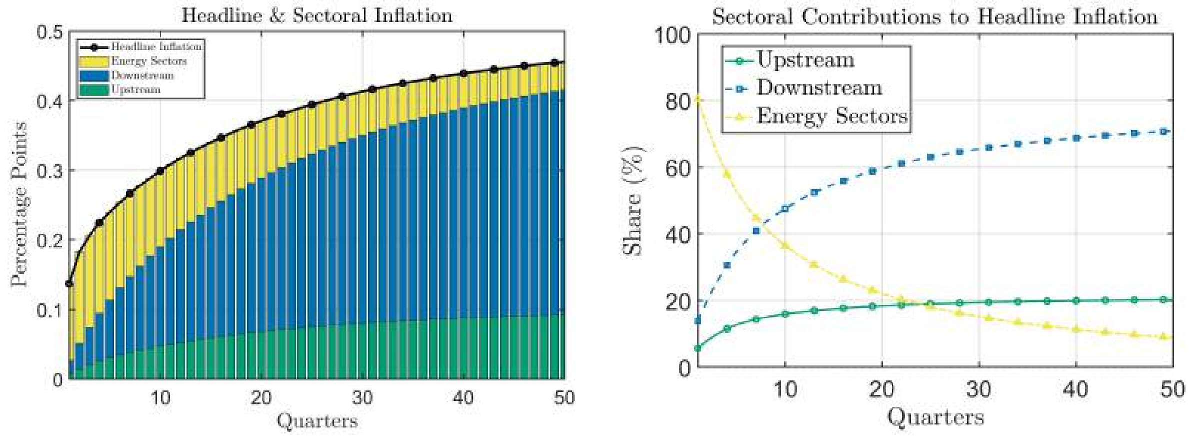
$$\overline{\boldsymbol{\Omega}} = \begin{bmatrix} \overline{\Omega}_{11} & \overline{\Omega}_{12} & \dots & \overline{\Omega}_{1K} \\ \overline{\Omega}_{21} & \overline{\Omega}_{22} & \dots & \overline{\Omega}_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\Omega}_{K1} & \overline{\Omega}_{K2} & \dots & \overline{\Omega}_{KK} \end{bmatrix}, \quad \overline{\Omega}_{kl} = \begin{bmatrix} \overline{\omega}_{kl\,11} & \overline{\omega}_{kl\,12} & \dots & \overline{\omega}_{kl\,1I} \\ \overline{\omega}_{kl\,21} & \overline{\omega}_{kl\,22} & \dots & \overline{\omega}_{kl\,2I} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\omega}_{kl\,I1} & \overline{\omega}_{kl\,I2} & \dots & \overline{\omega}_{kl\,II} \end{bmatrix}$$

with $\overline{\omega}_{kl\,ij} = \mathcal{M}_{ki}\omega_{kl\,ij}$, $\boldsymbol{K} = \text{diag}\begin{pmatrix}\kappa_{11} & \kappa_{12} & \dots & \kappa_{KI}\end{pmatrix}$ and $\boldsymbol{\Delta} = (\boldsymbol{I} - \boldsymbol{K})^{-1}\boldsymbol{K}$, and $\boldsymbol{\tau}_t = \boldsymbol{R}\boldsymbol{\tau}_{t-1} + \boldsymbol{e}_t^{\tau}$ where $\boldsymbol{\tau}_t = \begin{bmatrix}\tau_{11,t} & \dots & \tau_{1I,t} & \tau_{21,t} & \dots & \tau_{KI,t}\end{bmatrix}^{\top}$, and $\boldsymbol{I}$ denotes the identity matrix.

Equation (34) reveals how the IO structure amplifies the persistence induced by the nominal price and wage rigidities. Any increase in the price-wedge $\boldsymbol{\tau}_t$ is passed-through to sectoral prices, with the intensity of the pass-through measured by the position of the shocked sector in the IO network and its price rigidities (direct effect). This direct effect is amplified through the ridigity-adjusted Leontief inverse $(\boldsymbol{I} - \boldsymbol{\Delta}\overline{\boldsymbol{\Omega}})^{-1}$, which additionally considers the transmission through the IO network of the direct intermediate purchases. In addition to the impact effect of the price wedge, the persistence of nominal prices and wages, coming from the Calvo (1983) rigidities, is amplified through the ridigity-adjusted Leontief inverse. Intuitively, the resulting sectoral price changes after the granular shock affect sectors in the economy differently, depending on their exposure through the IO network and the limited pass-through induced through price and wage rigidities, which builds up over time. This feedback between the persistence of selling prices and wages (marginal costs) builds up through the entire production network, resulting in the slow-decaying pattern of headline and core inflation displayed in Figure 1.

We follow the methodology proposed in Antràs *et al.* (2012) to rank sectors according to their relative proximity to the final consumer. According to their measure, a sector is more downstream (i.e. closer to the final consumer) when a larger share of its output is used as final consumption. Conversely, more upstream sectors are the ones that sell a larger fraction its output as intermediate input for other sectors.[9] Implementing their methodology, we find that the three most downstream sectors are *Health and Education Services* (NACE P-Q), *Public Administration* (NACE O), and *Accommodation and food services* (NACE I); whilst the three most upstream sectors (apart from energy-related sectors) are *Basic metals* (NACE C.24), *Chemical products* (NACE C.20) and *Warehousing* (NACE H.52-H.53). In order to study

[9]More precisely, the upstreamness measure of an industry i, U_i derives from the solution of the system $U_{ki} = 1 + \sum_{l\in K}\sum_{j\in I}\frac{X_{kl\,ji}}{Y_{ki}}U_{kj}$, where $\frac{X_{kl\,ji}}{Y_{ki}}$ denotes the share of industry i output sold to industry j in country l and U_j is the upstreamness measure of industry j. Therefore, this measure take into account recursively also the upstreamness of the sectors to which industry i supplies intermediate inputs.



A. CIRF of Headline Inflation

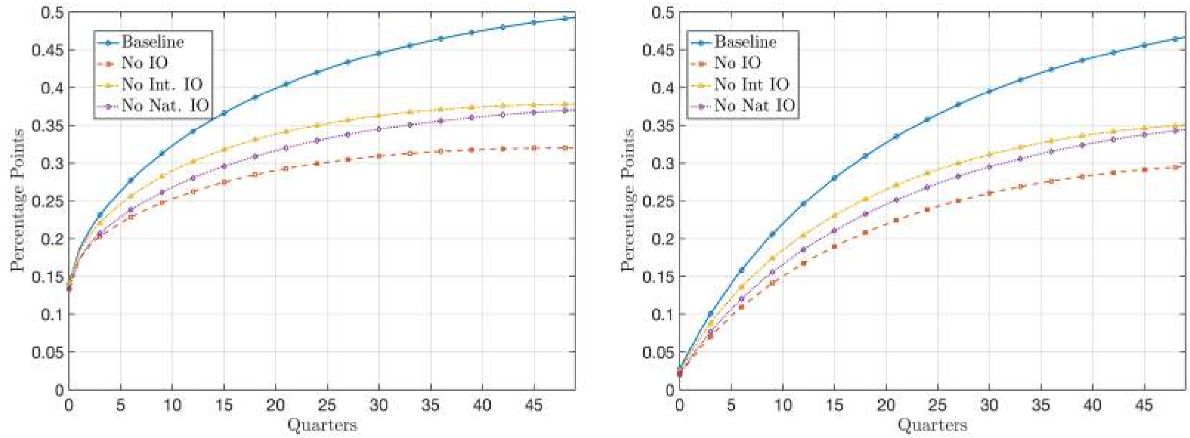
B. Contributions of Upstream, Downstream and Energy sectors

FIGURE 2. Inflation Dynamics and its Sectoral Decomposition

Notes: Panel 2A: CIRF of headline inflation and contributions of upstream and downstream sectors (Antràs *et al.* 2012). Panel 2B: contributions of upstream and downstream sectors as a percent of total headline inflation.

the sectoral composition of EA headline inflation, we catalog each of the 44 sectors in the economy into three categories: energy, upstream or downstream. We consider *Mining* (NACE B), *Coke and refined petroleoum* (NACE C.19), and *Electricity* (NACE d.35) as energy sectors. Out of the remanining non-energy sectors, we label a sector as upstream if its Antràs *et al.* (2012) upstreamness measure is above the median, and downstream otherwise.

We plot in figure 2 the decomposition of the cumulative EA headline inflation dynamics after the energy price shock (reported in panel 1C). In panel 2A we document that the initial increase in headline inflation is entirely driven by the increase in energy sectors, directly transmitted to consumption prices. Over time, the energy price falls, reverting to its initial level; and upstream and downstream sectors start contributing to headline inflation. We find that the share of inflation coming from upstream sectors stabilizes, whilst the share coming from downstream sectors is more persistent, increasing steadily over time (panel 2B reports each component's share in total headline inflation). This finding is explained through the intuition developed in the price dynamics equation (34): upstream sectors are less affected by the energy price increase since their intermediate input share in production is small, while downstream sectors depend directly on the intermediate input purchases on other sectors, including energy, and indirectly through their customers' IO network. Given that the pass-through is limited through price rigidities along the supply chain, this further increases the persistence of headline inflation.



A. CIRF of Headline Inflation

B. CIRF of Core Inflation

FIGURE 3. Inflation Dynamics and Production Networks

Notes: Cumulative IRF of Euro-Area headline (panel 3A) and core (panel 3B) inflation for the baseline and turning off the full, international, or national IO structure. When turning off the IO structure, we always keep the use of energy as an intermediate input.

4.2. Dissecting the Role of Production Networks

We next assess the role played by both national and international production networks. Toward this end, we consider a series of counterfactual economies where we turn off the production network entirely ($\omega_{kl\,ij} = 0 \quad \forall k, l, i, j$), only the domestic production network ($\omega_{kl\,ij} = 0 \quad \forall l = k$), or only the international network ($\omega_{kl\,ij} = 0 \quad \forall l \neq k$). In all these counterfactual economies we always maintain energy as both a consumption good and production input for firms.

Figure 3 shows the cumulative impulse response functions (CIRF) of headline inflation (panel 3A) and core inflation (panel 3B) for our baseline calibration and for each of the three counterfactual economies.

We focus first on the dashed red lines, which represent our counterfactual economy with national and international IO links removed altogether. On impact, headline inflation increases roughly by the same amount as in baseline calibration (solid blue lines). This is a consequence of headline inflation being driven initially by the rise of international energy prices, which is common across counterfactuals.

However, the presence of IO linkages is key in explaining inflation dynamics beyond impact. When we conduct our first counterfactual by turning off the production networks, cumulative inflation increases only 60% of our baseline at the end of the simulation horizon (compare the blue and red lines). On the one hand, this is a consequence of the smaller increase in core inflation (panel 3B). Without IO links, the feedback loop between increasing selling prices and rising production costs is absent, significantly dampening the increase in core inflation. On the other hand, inflation shows less persistence, dying out substantially quicker than in our baseline. More precisely, in our baseline simulation, inflation continues to rise steadily throughout the entire simulation horizon, whereas in the absence of production networks it stabilizes much earlier. This finding formalizes the intuition provided earlier, whereby the presence of intermediate goods in the marginal costs of firms in interaction with IO linkages leads to more persistent inflation dynamics.

The next two counterfactuals dissect the contribution of national (dotted purple line) and international (dashed yellow line) production networks. We find that the international IO network is a key factor that adds significant persistence on headline and core inflation. Given the upstream position of the energy within production chains, the shock propagates particularly strongly to other productive sectors. Consequently, due to the high level of integration between industrial sectors across European economies, there are significant spillovers from the effects of the shock through the cross-country links captured in the input-output tables. This finding highlights the quantitative relevance of the multi-country dimension to account for international spillover effects, which explain around 20% of the overall response of headline inflation, and would be underestimated in a simpler small open economy framework.

Importantly, we also observe that national and international production networks interact with each other. Namely, note that without IO linkages (red line) cumulative inflation increases by 0.32 percentage points (p.p.), while with only national or international production networks, it increases by roughly 0.37 p.p.. In other words, the "marginal effect" of each of them is approximately 0.05 p.p., and adding those to the counterfactual without IO we get to 0.42 p.p. This falls short of the 0.49 p.p. increase of our baseline, representing 85% of it.

In other words, it is crucial to account for both the national and international dimensions of production networks simultaneously. Intuitively, higher domestic inflation leads to increased export prices, which contributes to higher inflation abroad. In turn, rising inflation abroad translates into higher import prices, feeding back into domestic inflation. This interaction between national and international production networks amplifies the inflationary episode, resulting in a larger impact than if these dimensions were considered in isolation.

Finally, we report the effect of alternative foreign shocks on real output, inflation and the nominal policy rate in Figure A1 (Appendix B). We consider positive innovations in the ROW economy to the demand shock, monetary policy, sectoral TFP, sectoral price cost-push shocks, and wage cost-push shocks. For the case of sectoral disturbances, we consider a disturbance to the most downstream sector according to Antràs *et al.* (2012), *Health and Education Services*, and to the most upstream sector, *Basic metals*. We find that production networks dampen (amplify) the transmission of foreign demand shocks on output (inflation), while their effect on the policy rate is less significant. We find that production networks amplify the transmission of foreign monetary policy shocks on output, while their effect on inflation or the policy rate is less significant. We find that upstream foreign TFP shocks do not have a significant effect on real output, inflation or the policy rate, unless the production network is considered. For the case of downstream foreign TFP shocks, production networks amplify their effect on the three macroeconomic variables considered. The same lesson can be taken away from the upstream and downstream foreign price cost-push shocks, with upstream sectors not having any impact on macroeconomic dynamics, and downstream sectors generating an amplification through the production network. Lastly, production networks dampen the transmission of foreign price cost-push shocks on the three macroeconomic variables. We delegate the deeper analysis to Appendix B.

4.3. Heterogeneous Production Structures and Cross-country Heterogeneity

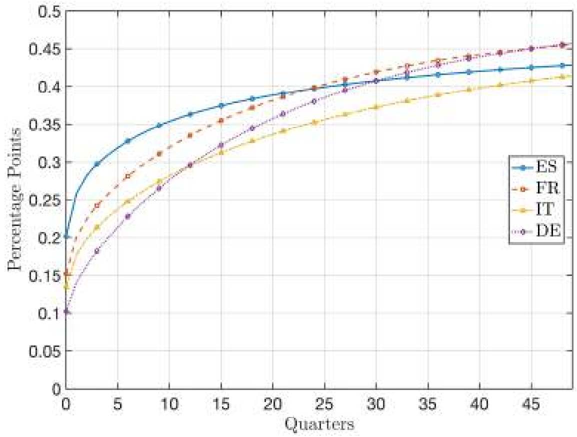
The previous sections have analyzed the effects of an international energy price shock on Euro-Area variables and the role played by production networks in its transmission. In this section, we instead show that such a common shock propagates differently across countries that differ in their production structures.

Figure 4 shows the cumulative impulse response functions of headline (panel 4A) and core inflation (panel 4B) for the main Euro-Area countries: Spain (blue line), Germany (purple line), Italy (yellow line), and France (red line).

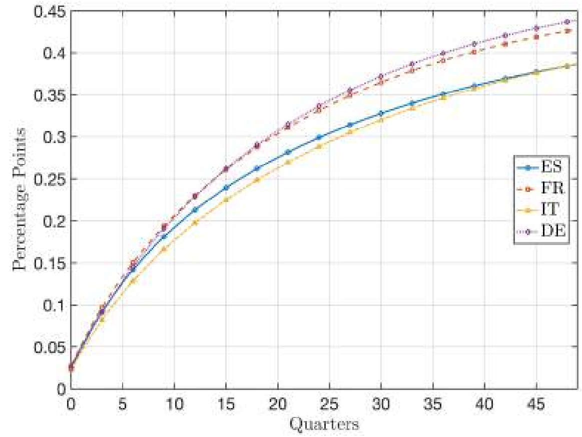
We find that the shock results in significantly heterogeneous inflation developments across countries, despite all European countries facing the same increase in imported energy prices. More precisely, we see that Spain suffers the largest spike in headline inflation in the first periods. However, note that this is also the country where inflation also stabilizes the fastest. In contrast, we observe the inflation dynamics in Germany. In response to the increase in energy prices, headline inflation increases the least in the German economy. In sharp contrast to Spain, headline inflation in Germany shows substantial persistence, increasing steadily over time. The dynamics of France and Italy sit somewhere between these two extremes.

The dynamics of headline inflation can be better understood by looking at core inflation, shown in panel 4B. Germany's core inflation rises gradually and remains elevated for a prolonged period. Spain, on the other hand, experiences a more transient rise in core inflation. This differential evolution in core inflation rates helps explain the varying persistence of headline inflation dynamics between the two countries, since most of the headline inflation at longer horizons is explained by core inflation dynamics.

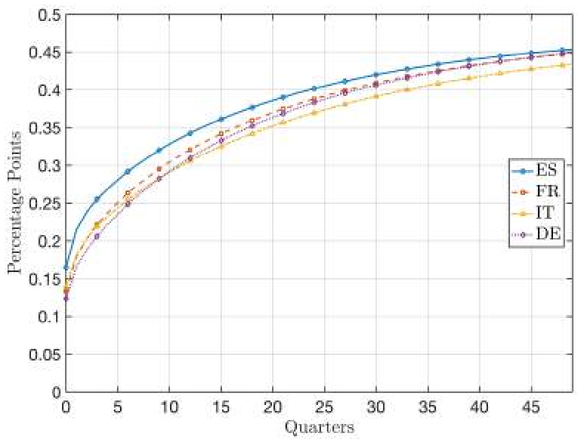
Heterogeneity in production structures and households' consumption baskets across countries can rationalize these differential inflation dynamics. In the model, as well as in



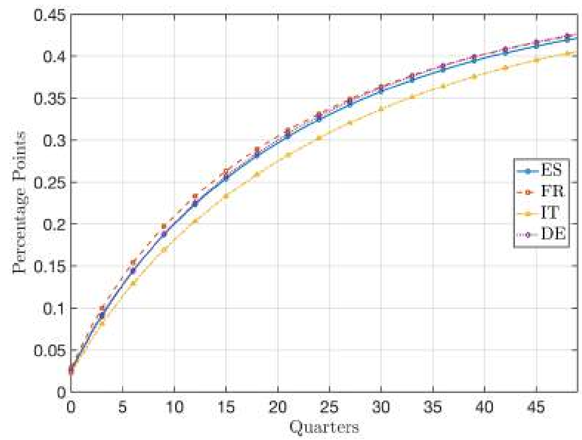
A. CIRF of Headline Inflation



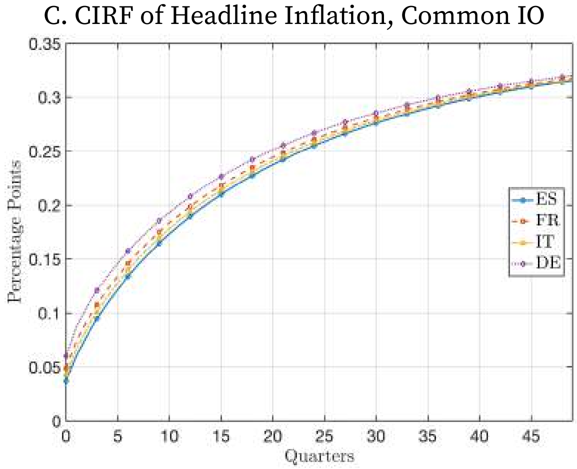
B. CIRF of Core Inflation



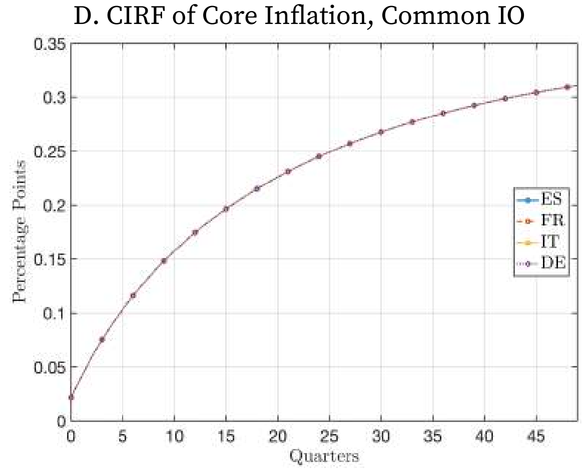
C. CIRF of Headline Inflation, Common IO



D. CIRF of Core Inflation, Common IO



E. CIRF of Headline Inflation, Common IO and Consumption Shares



F. CIRF of Core Inflation, Common IO and Consumption Shares

FIGURE 4. Cross-country Heterogeneity

Notes: Cumulative IRF of headline (panel 4A) and core (panel 4B) inflation for Spain (ES), France (FR), Italy (IT), and Germany (DE). Panels 4C and 4D reproduce the analysis with an homogeneous IO network, and 4E and 4F reproduce the analysis under both homogeneous IO production network and consumption shares.

the data, the consumption share of energy goods in Spain is the largest (4.49%). Therefore, headline inflation in Spain is the one that is most directly affected by the rise in energy prices. In contrast, Germany has a smaller share of energy consumption (4.09%), naturally leading to a smaller response of headline inflation on impact. However, Germany's production structure is characterized by a stronger industry exposure to the use of energy goods and long production chains. The longer production network structure of the German economy explains the persistent rise of inflation, as the feedback loops described in the previous sections apply more strongly here. On the other side, Spain has a more downstream-oriented production structure, with less complex IO linkages, resulting in lower amplification associated with production networks in this case.[10]

To isolate the role of production networks, we consider the case in which the IO matrix is homogeneous across countries.[11] Panels 4C-4D present the resulting cumulative IRFs of headline and core inflation. We find that equalization of the production network between countries affects the inflation gap between the different countries. Under no IO heterogeneity, the persistence induced by the network is the same across countries, so that no cumulative IRF crosses the others. Spain, which has a higher energy share in the CPI basket, reacts more initially and ends up with the highest cumulative inflation. Finally, to eliminate the gap coming from the heterogeneous consumption shares, we consider the case in which the consumption share matrix is homogeneous across countries.[12] Panels 4E-4F present the resulting cumulative IRFs of headline and core inflation. We find that equalizing the consumption shares across countries, on top of production network, reduces the gap in inflation between the different countries. The remaining distance between the curves can be explained by the heterogeneous price rigidities: the average price duration in Spain is 4.18 quarters, having relatively flexible prices, whereas in Germany prices are more rigid, lasting for 4.50 quarters on average.[13]

[10] Although the values of the upstreamness measure do not have a straightforward quantitative interpretation, the sales-weighted average of the sectors in the Spanish and German economies shows that, on average, Spanish sectors are 6.4% closer to final demand than German sectors.

[11] This matrix assumes that, within Euro Area economies, all sectors have the same productive structure. Therefore, within a given sector i, the weight of any sector j (υ_{kij} in our notation) as well as the different national varieties l ($\zeta_{kl\,ij}$ in our notation) is the same for all firms in the euro area. Within each sector, these values are calibrated as the average of all Euro Area countries. For consistency, this implies that the weight of a given sector in the GDP of its country is the same in all euro area countries.

[12] In this case we set the different consumption shares by sector as well as by national varieties to be the equal to the mean for all households across the Euro Area.

[13] These differences in sectoral price flexibility are particularly pronounced in energy and food sectors. However, the differences vanish when we consider only core CPI sectors (the average price duration is 6.45 quarters in Spain vs. 6.54 quarters in Germany), which explains the overlapping of core CPI inflation dynamics in the four countries in panel 4F.

4.4. Production Networks and Monetary Policy

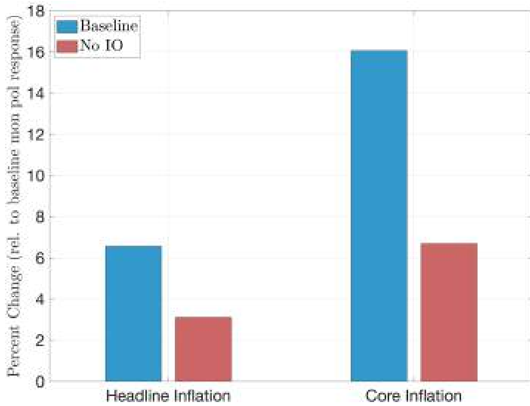
Our previous findings indicate that IO links play a central role in explaining the inflationary effects and cross-country propagation of international energy price shocks. Next, we investigate the implications of these findings for monetary policy.

Previous research has shown that the presence of intermediate goods and IO links tends to increase the degree of monetary policy (see, for example, Nakamura and Steinsson 2010; Rubbo 2023). That is, upon a monetary policy shock, inflation tends to respond less and output more than in a counterfactual where these features are absent. This finding is also present in our framework. To see this, in panel 5B of Figure 5 we show the cumulative impulse response functions of headline inflation upon a monetary shock that increases Euro-Area interest rates. We observe there that monetary tightening leads to a larger drop in inflation when the IO structure is absent (red line), relative to our baseline calibration (blue line). Intuitively, the presence of intermediate goods with sticky prices reduces the volatility of marginal costs and reduces the pass-through of wages into prices (see expression 34), leading to a muted inflation response.

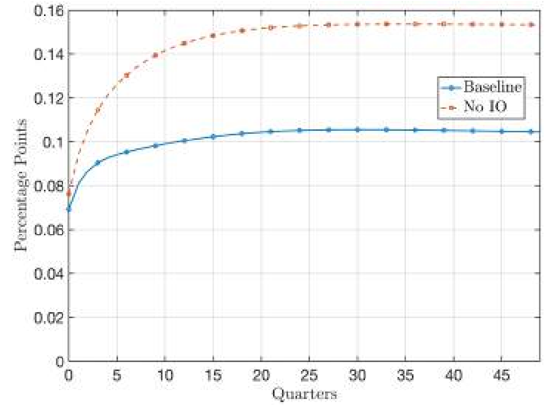
The panel 5A of Figure 5 considers the following exercise. We draw a series of shocks to the international price of imported energy faced by European firms. We then simulate the model with and without production networks subject to those shocks and compute the volatility of headline and core inflation. When doing so, we consider two different inflation coefficients in the Taylor rule (24): the first with our baseline calibration $\phi_{MU_\pi} = 1.5$, and the second considers a weaker systematic response $\phi_{MU_\pi} = 1.1$. Figure 5 shows the increase in inflation volatility when we move from $\phi_{MU_\pi} = 1.5$ to $\phi_{MU_\pi} = 1.1$ in the economy with production networks. The red bars show the same statistic in the economy without IO links.

First, we observe that monetary policy has a greater impact on core inflation than on headline inflation, both with and without IO links. Specifically, the increase in inflation volatility from a weaker monetary policy response is more than twice as large for core inflation compared to headline inflation. This difference arises from cross-sector heterogeneity in price flexibility and its interaction with import intensities. Sectors contributing to core inflation, such as services and manufacturing, have stickier prices compared to energy- and food-producing sectors in our dataset, which exhibit a higher pass-through from marginal costs to selling prices. In addition, the domestic energy and food sectors are heavily dependent on imported goods as key production inputs. Since domestic monetary policy has limited influence over the international prices of these imported goods, which strongly affect domestic prices, the monetary policy rule has a smaller impact on headline inflation than on core inflation.

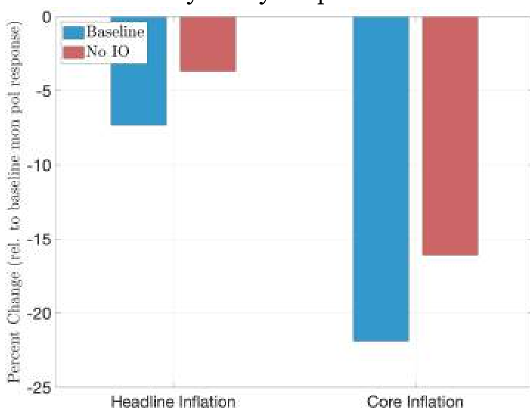
Second, the results of this simulation show that the systematic response of monetary



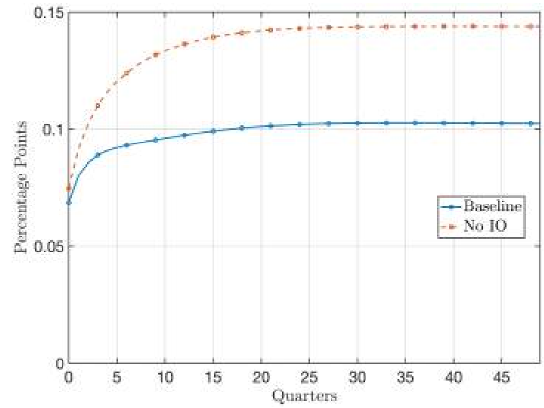
A. Percent Change in Inflation Volatility under Weaker Monetary Policy Response



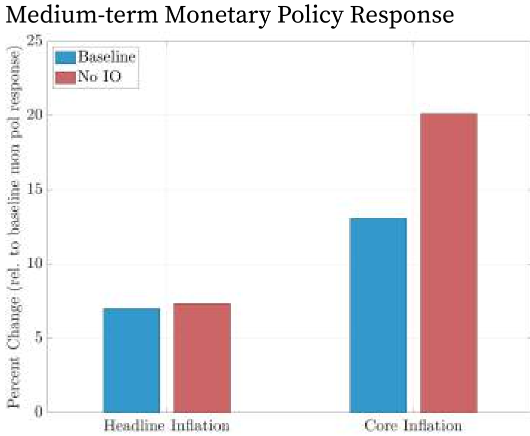
B. CIRF of Headline Inflation to a Monetary Policy Shock



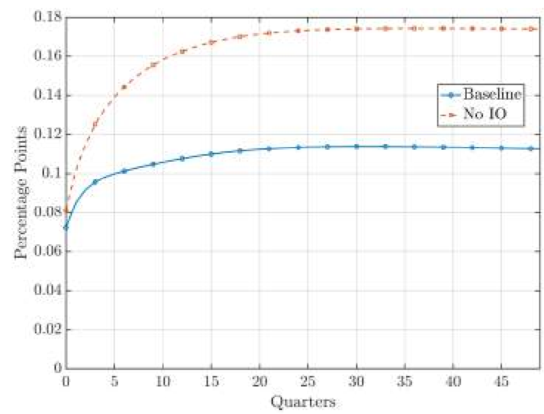
C. Percent Change in Inflation Volatility adding Medium-term Monetary Policy Response



D. CIRF of Headline Inflation to a Monetary Policy Shock



E. Percent Change in Inflation Volatility under Medium-term Monetary Policy Response



F. CIRF of Headline Inflation to a Monetary Policy Shock

FIGURE 5. Monetary Policy and Production Networks

Notes: Panel 5A: percent change in inflation volatility (conditional on energy price shocks) with a lower coefficient on inflation in the Taylor rule. Panel 5B: CIRF of headline inflation to a monetary policy shock (easing) in the baseline and without IO. Panel 5C: percent change in inflation volatility (conditional on energy price shocks) with an additional coefficient on 1-year-ahead inflation expectations in the Taylor rule. Panel 5D: CIRF of headline inflation to a monetary policy shock (easing) under a medium-term Taylor rule with IO, and without IO. Panel 5E: percent change in inflation volatility (conditional on energy price shocks) with only medium-term inflation expectations in the Taylor rule. Panel 5F: CIRF of headline inflation to a monetary policy shock (easing) under a medium-term Taylor rule with IO, and without IO.

policy becomes more significant in the presence of IO linkages, despite the higher degree of monetary non-neutrality following monetary shocks. Specifically, by comparing the red and blue bars, we observe that inflation volatility increases by more than twice as much when production networks are included. This finding aligns with our earlier results: IO linkages amplify the inflationary response to international energy price shocks. Consequently, even though a given change in interest rates has a smaller direct effect (as shown in the right panel), a monetary policy response that fails to contain the propagation of such shocks and allows the IO amplification to build up will result in greater inflation volatility.

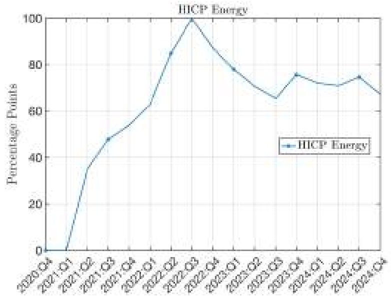
The panel 5C of Figure 5 considers a similar exercise to the previous one. In this case, instead of reducing the sensitivity of the policy rate towards inflation, we enlarge the systematic component in the Taylor rule of the monetary union (24) by additionally considering a reaction towards 4-quarters-ahead inflation expectations, with the same sensitivity as the inflation coefficient $\phi_{MU_\pi} = 1.5$ for both contemporaneous and expected inflation. Panel 5C shows the fall in inflation volatility when we move from the benchmark to a combination of short-term and medium-term reaction in the economy with production networks. The red bars show the same statistic in the economy without IO links. In panel 5D of Figure 5 we show the cumulative impulse response functions of headline inflation upon a monetary shock that increases Euro-Area interest rates. As in the previous case, a monetary easing leads to a larger increase in inflation when the IO structure is absent (red line), relative to our baseline calibration (blue line).

First, we find that volatility is reduced in both cases, with and without IO linkages. A stronger policy rate response towards inflation, whether realized or expected, tames down inflation dynamics. Second, we find that the fall in volatility is larger in the IO baseline economy. Since the IO network adds persistence to inflation dynamics, which then feeds into inflation expectations, a larger policy rate movement is necessary in the IO economy. As a result, volatility falls by more in the IO framework.

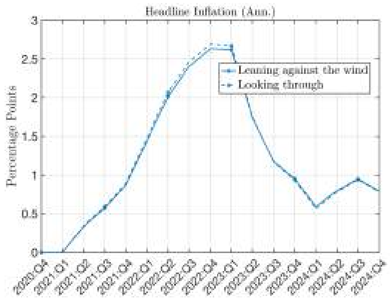
Lastly, we consider the case in which the central bank only reacts to medium-term inflation expectations. This case naturally speaks to the discussion on the benefits and costs of "looking-through" monetary policy in periods under energy price disturbances. Proponents of the "looking-through" approach argue that, since monetary policy affects the economy with a lag and energy price disturbances are short-lived, central banks should not immediately hike interest rates in the presence of energy price increases. Theoretically, we model the "looking-through" behavior by replacing inflation in the Taylor rule of the monetary union (24) for medium-term (4-quarters-ahead) inflation expectations. The panel 5E of Figure 5 presents the percent increase in inflation volatility under a "looking-through" Taylor rule, compared to our baseline case (24). We find that inflation volatility increases because the systematic component of monetary policy is partially muted. Interestingly, and compared to the two previous cases studied above, we now find that the increase in volatility is larger in the no IO counterfactual, both for headline and core inflation. This finding can be explained from the additional persistence induced by IO networks: since the policy rate only reacts to medium-term inflation expectations, and the pass-through of the energy price shock to inflation expectations depends on the persistence that such shock generates in inflation dynamics, the central bank policy rate is more sensitive to disturbances in the IO economy. As a result, the increase in inflation volatility is less in the baseline IO economy. In panel 5F of Figure 5 we show the cumulative impulse response functions of headline inflation upon a monetary shock that increases Euro-Area interest rates. As in the previous cases, a monetary easing leads to a larger increase in inflation when the IO structure is absent (red line), relative to our baseline calibration (blue line), with more pronounced effects on inflation due to the muted reaction of the systematic component of monetary policy.

To complement our analysis on the discussion on "leaning against the wind" vs. "looking-through" monetary policy, we use our theoretical framework to simulate the effect of a permanent increase of 10% in the price of imported energy on annualized headline inflation, annualized core inflation, and quarter-on-quarter real GDP growth, depending on the monetary stance in the Euro Area. We frame the "leaning against the wind" and "looking-through" monetary stances as more extreme cases than the one discussed in panels 5E-5F. In this case, under "leaning against the wind" monetary policy the central bank only reacts to inflation changes, setting $\phi_{\mathrm{MU}\,y} = 0$ in the Taylor rule (24). Alternatively, under the "looking-through" scenario, the central bank maintains the policy rate unchanged by fitting in monetary policy shocks. The panels 6B-6D in figure 6 show the dynamics of Euro Area headline inflation, core inflation, and real GDP growth. After the permanent increase in the price of oil, annualized headline inflation increases gradually up to four quarters and falls thereafter in a persistent manner. Annualized inflation follows a similar pattern, exhibiting greater persistence, and quarterly real GDP growth falls on impact, recovering thereafter. Unsurprisingly, we find that both inflation measures increase by more in the "looking-through" monetary policy, since the systematic component becomes inactive, eliminating the stabilization role of monetary policy. In turn, "leaning against the wind" monetary policy causes a larger drop in economic activity.

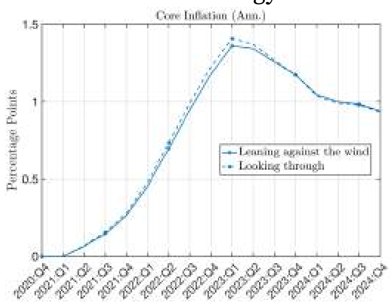
We now explore the role on the IO structure in the economy, and how does its presence or absence affect the previous results. The panels 6E-6G in figure 6 show the dynamics of Euro Area headline inflation, core inflation, and real GDP growth shutting down the IO structure as discussed in section 4.2, maintaining the sequence of energy price shocks used in panels 6B-6D. Qualitatively, the dynamics are similar to those of the baseline with production networks. The solid line in panel 6H represents the difference between "looking-through" and "leaning against the wind" headline inflation (the difference between the two



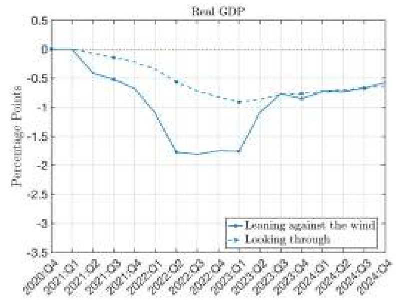
A. Euro area Energy HICP



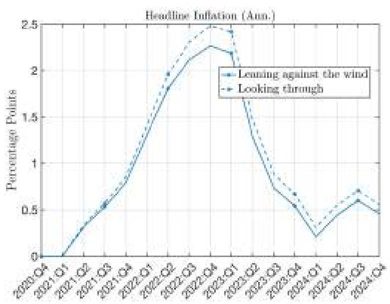
B. Annual Headline Inflation



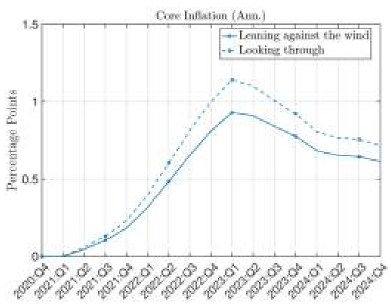
C. Annual Core Inflation



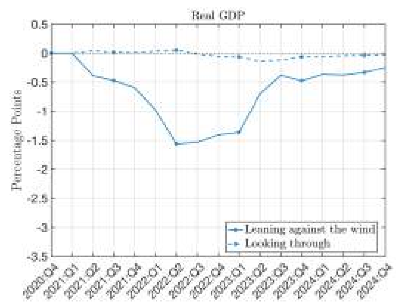
D. Quarter-on-Quarter Real GDP Growth



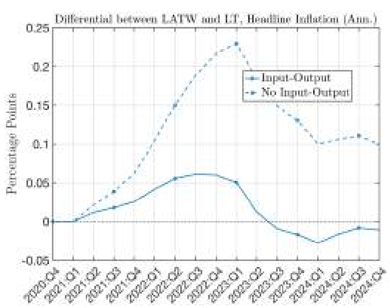
E. Annual Headline Inflation, no IO



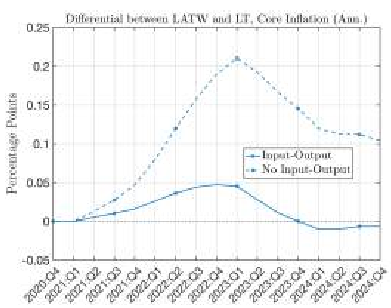
F. Annual Core Inflation, no IO



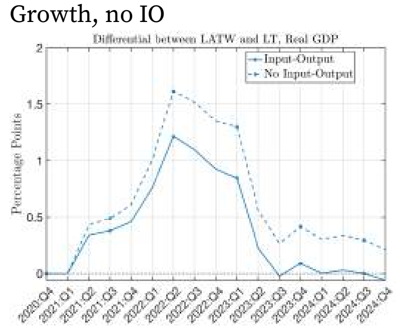
G. Quarter-on-Quarter Real GDP Growth, no IO



H. Annual Headline Inflation difference between "looking-through" vs. "leaning against the wind", with and without IO.



I. Annual Core Inflation difference between "looking-through" vs. "leaning against the wind", with and without IO.



J. Quarter-on-Quarter Real GDP Growth difference between "looking-through" vs. "leaning against the wind", with and without IO.

FIGURE 6. Leaning Against the Wind, Looking-Through, and Production Networks

Notes: Macroeconomic dynamics in the Euro Area after a permanent 10% increase in the price of imported energy. Panel 6B: annualized headline inflation dynamics under "leaning against the wind" and "looking-through" monetary policy. Panel 6C: annualized core inflation dynamics under "leaning against the wind" and "looking-through" monetary policy. Panel 6D: quarter-on-quarter real GDP growth dynamics under "leaning against the wind" and "looking-through" monetary policy. Panel 6E: annualized headline inflation dynamics under "leaning against the wind" and "looking-through" monetary policy without IO. Panel 6F: annualized core inflation dynamics under "leaning against the wind" and "looking-through" monetary policy without IO. Panel 6G: quarter-on-quarter real GDP growth dynamics under "leaning against the wind" and "looking-through" monetary policy without IO. Panel 6H: difference between "leaning against the wind" and "looking-through" headline inflation, with IO and without IO. Panel 6I: difference between "leaning against the wind" and "looking-through" core inflation, with IO and without IO. Panel 6J: difference between "leaning against the wind" and lines in panel 6B). This measure can be interpreted as a gap or excess inflation arising from "looking-through" monetary policy. Equivalently, the dashed line in panel 6H represents the difference between "looking-through" and "leaning against the wind" headline inflation dynamics shutting down the IO structure (the difference between the two lines in panel 6E). We find that the presence of production networks alleviates the inflation differential between both monetary policy stances, both for headline and core inflation (see panel 6I). These findings can be rationalized through the exacerbated monetary non-neutrality induced by production networks (Nakamura and Steinsson 2018; Rubbo 2023), since the difference between the "leaning against the wind" monetary policy and "looking-through" is due to a sequence of expansionary monetary policy shocks in the latter case, with lessened effects on headline inflation under IO. The presence of production networks alleviates the differential in real GDP growth on impact (see panel 6G), although its cumulative sum is affected by the additional persistence induced by production networks in the "looking-through" case, in which the effects of the energy price shock discussed in the previous sections are active, compared to "leaning against the wind", in which its consequences are partially muted due to systematic monetary policy. This causes a larger effect on cumulative real GDP growth under production networks.

In summary, the presence of the IO network diminishes the differences in headline and core inflation dynamics between "leaning against the wind" and "looking-through" monetary policy following an energy price shock, while production networks amplify the differential in the real GDP growth responses.

5. Conclusions

This paper highlights the critical role of production networks in shaping the transmission and persistence of inflation in response to international energy price shocks. Using a multi-country New Keynesian model with rich sectoral heterogeneity and input-output linkages, we show that production networks significantly amplify inflation through a feedback loop between rising selling prices and production costs. The interaction between national and international networks intensifies inflationary pressures, with cross-border spillovers further magnifying the persistence of inflation. This effect is particularly pronounced in countries with more integrated production structures, such as Germany, where inflation lasts longer, while countries with less complex networks, like Spain, experience shorter-lived inflation spikes. Our findings highlight the importance of an active monetary policy response to supply-side shocks, as weaker stabilization policies lead to greater inflation volatility when production networks are present. Furthermore, we find that production networks alleviate the tensions on inflation between "leaning against the wind" and "looking-through" monetary policy after an energy price shock, while these are exacerbated on real GDP.

Our framework is well-suited for analyzing the inflationary impact of macroeconomic shocks and policies characterized by a high degree of sectoral granularity. Examples include trade policies like tariffs, environmental measures such as carbon pricing, supply-side bottlenecks, and disruptions in global value chains. On the modeling side, further progress can be made by incorporating the complexity of investment input-output networks (vom Lehn and Winberry 2021; Quintana 2024). These represent promising avenues for future research, offering potential insights into how capital investments and production structures interact to shape the transmission of shocks and policies across sectors and countries.

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Appendix

Appendix A. Model Derivation and Log-linearization

In this section we derive model equilibrium conditions, outlining the final set of log-linearized equations.

A.1. Households

Consumption Demand Curves. The allocation optimal allocation between energy and non-energy goods is the result of a cost minimization programme min $P_{kE,t}C_{kE,t} + P_{kM,t}C_{kM,t}$ subject to (1). Similarly, the optimal allocation between energy (non-energy) consumption is the result of a cost minization programme min $\sum_{i\in I_E} P_{kiC,t}C_{ki,t}$ (min $\sum_{i\in I_M} P_{kiC,t}C_{ki,t}$) subject to (2). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme min $\sum_{l=1}^{K} P_{kl\,i,t}C_{kl\,i,t}$ subject to (3). The implied demand curves are given by

(A.1)
$$P_{kE,t} = P_{kC,t}\left(\frac{\widetilde{\beta}_k C_{k,t}}{C_{kE,t}}\right)^{\frac{1}{\gamma}} \quad \text{and} \quad P_{kM,t} = P_{kC,t}\left(\frac{(1-\widetilde{\beta}_k)C_{k,t}}{C_{kM,t}}\right)^{\frac{1}{\gamma}}$$

(A.2)
$$P_{kiC,t} = P_{kE,t}\left(\frac{\widetilde{\nu}_{ki}C_{kE,t}}{C_{ki,t}}\right)^{\frac{1}{\eta}} \quad \forall \quad i \in I_M \quad \text{and} \quad P_{kiC,t} = P_{kM,t}\left(\frac{\widetilde{\upsilon}_{ki}C_{kM,t}}{C_{ki,t}}\right)^{\frac{1}{\iota}} \quad \forall \quad i \in I_E$$

(A.3)
$$P_{kl\,i,t} = P_{kiC,t}\left(\frac{\widetilde{\zeta}_{kl\,i}C_{ki,t}}{C_{kl\,i,t}}\right)^{\frac{1}{\delta}} \quad \forall \quad l \in K.$$

The log-linearized versions of the consumption demand curves (A.1)-(A.3) are given by

$$\widehat{p}_{kE,t} = \frac{1}{\gamma}(\widehat{c}_{k,t} - \widehat{c}_{kE,t}) \quad \text{and} \quad \widehat{p}_{kM,t} = \frac{1}{\gamma}(\widehat{c}_{k,t} - \widehat{c}_{kM,t}) \tag{A.4}$$

$$\widehat{p}_{kiC,t} - \widehat{p}_{kE,t} = \frac{1}{\eta}(\widehat{c}_{kE,t} - \widehat{c}_{ki,t}) \quad \text{and} \quad \widehat{p}_{kiC,t} - \widehat{p}_{kM,t} = \frac{1}{\iota}(\widehat{c}_{kM,t} - \widehat{c}_{ki,t}) \tag{A.5}$$

$$\widehat{p}_{kl\,i,t} - \widehat{p}_{kiC,t} = \frac{1}{\delta}\left(\widehat{c}_{ki,t} - \widehat{c}_{kl\,i,t}\right) \tag{A.6}$$

where $\widehat{p}_{kE,t} = p_{kE,t} - p_{kC,t}$, $\widehat{p}_{kM,t} = p_{kM,t} - p_{kC,t}$, $\widehat{p}_{kiC,t} = p_{kiC,t} - p_{kC,t}$, and $\widehat{p}_{kl\,i,t} = p_{kl\,i,t} - p_{kC,t}$ are well-defined as a ratio of prices.[14]

Consumption Baskets. The log-linearized consumption aggregator (1) is given by

$$\widehat{c}_{k,t} = \beta_k \widehat{c}_{kE,t} + (1-\beta_k)\widehat{c}_{kM,t} \tag{A.7}$$

where $\beta_k = \frac{P_{kE}C_{kE}}{P_{kC}C_k} = \widetilde{\beta}_k^{\frac{1}{\gamma}} \left(\frac{C_{kE}}{C_k}\right)^{\frac{\gamma-1}{\gamma}}$ and $(1-\beta_k) = \frac{P_{kM}C_{kM}}{P_{kC}C_k} = (1-\widetilde{\beta}_k)^{\frac{1}{\gamma}} \left(\frac{C_{kM}}{C_k}\right)^{\frac{\gamma-1}{\gamma}}$ can be verified using the steady-state consumption aggregator (1) and the demand curves (A.1).

The log-linearized versions of the energy and non-energy consumption aggregators (2) are given by

$$\widehat{c}_{kE,t} = \sum_{i \in I_E} \nu_{ki}\widehat{c}_{ki,t} \quad \text{and} \quad \widehat{c}_{kM,t} = \sum_{i \in I_M} \upsilon_{ki}\widehat{c}_{ki,t} \tag{A.8}$$

where $\nu_{ki} = \frac{P_{kiC}C_{ki}}{P_{kE}C_{kE}} = \widetilde{\nu}_{ki}^{\frac{1}{\eta}} \left(\frac{C_{ki}}{C_{kE}}\right)^{\frac{\eta-1}{\eta}}$ and $\upsilon_{ki} = \frac{P_{kiC}C_{ki}}{P_{kM}C_{kM}} = \widetilde{\upsilon}_{ki}^{\frac{1}{\iota}} \left(\frac{C_{ki}}{C_{kM}}\right)^{\frac{\iota-1}{\iota}}$ can be verified using the steady-state energy and non-energy consumption aggregators (2) and the demand curves (A.2).

The log-linearized version of final layer of the consumption aggregator, (3), is given by

$$\widehat{c}_{ki,t} = \sum_{l=1}^{K} \zeta_{kl\,i}\widehat{c}_{kl\,i,t} \tag{A.9}$$

where $\zeta_{kl\,i} = \frac{P_{kl\,i}C_{kl\,i}}{P_{kiC}C_{ki}} = \widetilde{\zeta}_{kl\,i}^{\frac{1}{\delta}} \left(\frac{C_{kl\,i}}{C_{ki}}\right)^{\frac{\delta-1}{\delta}}$ can be verified using the steady-state international consumption aggregator (3) and the consumption demand curves (A.3).

Price Indices. The different price indices can be derived by combining the consumption demand curves previously derived with the different consumption aggregators. The con-

[14]The individual price levels are not well-defined in steady state, but their ratio is.

sumption price index, the energy and non-energy price index, and the consumption import price index are given by $P_{kC,t} = \left[\widetilde{\beta}_k P_{kE,t}^{1-\gamma} + (1-\widetilde{\beta}_k) P_{kM,t}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, $P_{kE,t} = \left[\sum_{i \in I_E} \widetilde{\nu}_{ki} P_{kiC,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, $P_{kM,t} = \left[\sum_{i \in I_M} \widetilde{\upsilon}_{ki} P_{kiC,t}^{1-\iota}\right]^{\frac{1}{1-\iota}}$, and $P_{kiC,t} = \left[\sum_{l=1}^{K} \widetilde{\zeta}_{kl\,i} P_{kl\,i,t}^{1-\delta}\right]^{\frac{1}{1-\delta}}$. Their log-linearized counterparts are given by

$$0 = \beta_k \widehat{p}_{kE,t} + (1-\beta_k)\widehat{p}_{kM,t} \tag{A.10}$$

$$\widehat{p}_{kE,t} = \sum_{i \in I_E} \nu_{kj} \widehat{p}_{kiC,t} \quad \text{and} \quad \widehat{p}_{kM,t} = \sum_{i \in I_M} \upsilon_{ki} \widehat{p}_{kiC,t} \tag{A.11}$$

$$\widehat{p}_{kiC,t} = \sum_{l=1}^{K} \zeta_{kl\,i} \widehat{p}_{kl\,i,t} \tag{A.12}$$

Intertemporal Household Problem. The log-linearized version of the household's first-order conditions (5)-(6) are given by

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{k,t} - \mathbb{E}_t \pi_{kc,t+1}) + \mathbb{E}_t \widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z) z_{k,t} \tag{A.13}$$

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{K,t} - \mathbb{E}_t \pi_{kC,t+1}) + \mathbb{E}_t \widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z) z_{k,t} - \frac{1}{\sigma}\mathbb{E}_t \Delta e_{kK,t+1} - \frac{1}{\sigma}\gamma_* \text{nfa}_{k,t}^K \quad \forall k \neq K \tag{A.14}$$

where we define the different log-linear NFA positions as $\text{nfa}_{k,t}^K = B_{k,t}^K \mathcal{E}_{kK,t}/\mathcal{Y}_k$ and $\text{nfa}_{k,t}^{\text{MU}} = B_{k,t}^{\text{MU}} \mathcal{E}_{k\text{MU},t}/\mathcal{Y}_k$ since $B_{k,t}^K = 0$ and $B_{k,t}^{\text{MU}} = 0$ in steady-state.

Combining the log-linearized first-order conditions for the holdings of domestic and internationally traded bonds (A.13)-(A.14), yields a risk-adjusted Uncovered Interest Parity (UIP) condition $i_{k,t} - i_{K,t}^* = \mathbb{E}_t \Delta e_{kK,t+1} + \gamma_* \text{nfa}_{k,t}^K$.

A.2. Firms

Intermediate Input Demand Curves. The optimal allocation between labor and intermediate inputs is the result of a cost minimization programme $\min W_{k,t} N_{fki,t} + P_{kiX,t} X_{fki,t}$ subject to (8). The optimal allocation between energy and non-energy intermediate goods is the result of a cost minimization programme $\min P_{kiXE,t} X_{kiE,t} + P_{kiXM,t} X_{kiM,t}$ subject to (10). Similarly, the optimal allocation between energy (non-energy) intermediate inputs is the result of a cost minization programme $\min \sum_{j \in I_E} P_{kijX,t} X_{kij,t}$ ($\min \sum_{j \in I_M} P_{kijX,t} X_{kij,t}$) subject to (11). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme $\min \sum_{l=1}^{K} P_{kl\,j,t} X_{kl\,ij,t}$ subject to (12). The implied intermediate input demand curves are given by

(A.15) $$W_{k,t} = \mathrm{MC}_{fki,t} A_{ki,t}^{\frac{\psi-1}{\psi\psi_{ki}}} \psi_{ki} Y_{fki,t}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}} \left(\frac{\widetilde{\alpha}_{ki} Y_{fki,t}}{N_{fki,t}}\right)^{\frac{1}{\psi}}$$

(A.16) $$P_{kiX,t} = \mathrm{MC}_{ki,t} A_{ki,t}^{\frac{\psi-1}{\psi\psi_{ki}}} \psi_{ki} Y_{fki,t}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}} \left(\frac{\widetilde{\vartheta}_{ki} Y_{fki,t}}{X_{fki,t}}\right)^{\frac{1}{\psi}}$$

(A.17) $$P_{kiXE,t} = P_{kiX,t} \left(\frac{\widetilde{\beta}_{ki} X_{ki,t}}{X_{kiE,t}}\right)^{\frac{1}{\phi}} \quad \text{and} \quad P_{kiXM,t} = P_{kiX,t} \left(\frac{(1-\widetilde{\beta}_{ki}) X_{ki,t}}{X_{kiM,t}}\right)^{\frac{1}{\phi}}$$

(A.18)

$$P_{kijX,t} = P_{kiXE,t} \left(\frac{\widetilde{\nu}_{kij} X_{kiE,t}}{X_{kij,t}}\right)^{\frac{1}{\chi}} \quad \forall \quad j \in I_E \quad \text{and} \quad P_{kijX,t} = P_{kiXM,t} \left(\frac{\widetilde{\upsilon}_{kij} X_{kiM,t}}{X_{kij,t}}\right)^{\frac{1}{\xi}} \quad \forall \quad j \in I_M$$

(A.19) $$P_{klj,t} = P_{kijX,t} \left(\frac{\zeta_{kl\,ij} X_{kij,t}}{X_{kl\,ij,t}}\right)^{\frac{1}{\mu}} \quad \forall \quad l \in K$$

The log-linearized versions of the labor and intermediate inputs demand curves (A.15)-(A.19) are given by

(A.20) $$\widehat{w}_{k,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} + \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} \widehat{y}_{fki,t} + \frac{1}{\psi}\left(\widehat{y}_{fki,t} - \widehat{n}_{fki,t}\right)$$

(A.21) $$\widehat{p}_{kiX,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} + \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} \widehat{y}_{fki,t} + \frac{1}{\psi}\left(\widehat{y}_{fki,t} - \widehat{x}_{fki,t}\right)$$

(A.22)

$$\widehat{p}_{kiXE,t} - \widehat{p}_{kiX,t} = \frac{1}{\phi}\left(\widehat{x}_{ki,t} - \widehat{x}_{kiE,t}\right) \quad \text{and} \quad \widehat{p}_{kiXM,t} - \widehat{p}_{kiX,t} = \frac{1}{\phi}\left(\widehat{x}_{ki,t} - \widehat{x}_{kiM,t}\right)$$

(A.23)

$$\widehat{p}_{kijX,t} - \widehat{p}_{kiXE,t} = \frac{1}{\chi}\left(\widehat{x}_{kiE,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j \in I_E \quad \text{and} \quad \widehat{p}_{kijX,t} - \widehat{p}_{kiXM,t} = \frac{1}{\xi}\left(\widehat{x}_{kiM,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j \in I_M$$

(A.24) $$\widehat{p}_{klj,t} - \widehat{p}_{kijX,t} = \frac{1}{\mu}\left(\widehat{x}_{kij,t} - \widehat{x}_{kl\,ij,t}\right) \quad \forall \quad l \in K$$

where $\widehat{w}_{k,t} = w_{k,t} - p_{kC,t}$, $\widehat{\mathrm{mc}}_{ki,t} = \widehat{\mathrm{mc}}^{n}_{ki,t} - p_{kC,t}$, $\widehat{p}_{kiX,t} = p_{kiX,t} - p_{kC,t}$, $\widehat{p}_{kiXE,t} = p_{kiXE,t} - p_{kC,t}$, $\widehat{p}_{kiXM,t} = p_{kiXM,t} - p_{kC,t}$, $\widehat{p}_{kijX,t} = p_{kijX,t} - p_{kC,t}$, and $\widehat{p}_{klj,t} = p_{klj,t} - p_{kC,t}$ are well-defined as a ratio of prices.

Intermediate Inputs Baskets. The log-linearized intermediary input aggregator (10) is given by

$$\widehat{x}_{ki,t} = \beta_{ki}\widehat{x}_{kiE,t} + (1-\beta_{ki})\widehat{x}_{kiM,t} \tag{A.25}$$

where $\beta_{ki} = \frac{P_{kiXE}X_{kiE}}{P_{kiX}X_{ki}} = \widetilde{\beta}_{ki}^{\frac{1}{\phi}}\left(\frac{X_{kiE}}{X_{ki}}\right)^{\frac{\phi-1}{\phi}}$ and $(1-\beta_{ki}) = \frac{P_{kiXM}X_{kiM}}{P_{kiM}X_{ki}} = (1-\widetilde{\beta}_{ki})^{\frac{1}{\phi}}\left(\frac{X_{kiM}}{X_{ki}}\right)^{\frac{\phi-1}{\phi}}$ can be verified using the steady-state intermediate input aggregator (10) and the input demand curves (A.17).

The log-linearized versions of the energy and non-energy intermediate input aggregators (11) are given by

$$\widehat{x}_{kiE,t} = \sum_{j\in I_E}\nu_{kij}\widehat{x}_{kij,t} \qquad \text{and} \qquad \widehat{x}_{kiM,t} = \sum_{j\in I_M}\upsilon_{kij}\widehat{x}_{kij,t} \tag{A.26}$$

where $\nu_{kij} = \frac{P_{kijX}X_{kij}}{P_{kiXE}X_{kiE}} = \widetilde{\nu}_{kij}^{\frac{1}{\chi}}\left(\frac{X_{kij}}{X_{kiE}}\right)^{\frac{\chi-1}{\chi}}$ and $\upsilon_{kij} = \frac{P_{kijX}X_{kij}}{P_{kiXM}X_{kiM}} = \widetilde{\upsilon}_{kij}^{\frac{1}{\xi}}\left(\frac{X_{kij}}{X_{kiM}}\right)^{\frac{\xi-1}{\xi}}$ can be verified using the steady-state energy and non-energy intermediate input aggregators (11) and the demand curves (A.18).

The log-linearized version of the final layer of the intermediate input aggregator, (12), is given by

$$\widehat{x}_{kij,t} = \sum_{l=1}^{K}\zeta_{kl\,ij}\widehat{x}_{kl\,ij,t} \tag{A.27}$$

where $\zeta_{kl\,ij} = \frac{P_{kl\,j}X_{kl\,ij}}{P_{kijX}X_{kij}} = \widetilde{\zeta}_{kl\,ij}^{\frac{1}{\mu}}\left(\frac{X_{kl\,ij}}{X_{kij}}\right)^{\frac{\mu-1}{\mu}}$ can be verified using the steady-state international intermediate input aggregators (12) and the demand curve (A.19).

Price Indices. The different price indices can be derived by combining the intermediate input demand curves previously derived with the different intermediate input aggregators. The marginal cost of production, the intermediate input price index, the energy and non-energy input price index, and the input import price index are given by $MC_{ki,t} = \frac{Y_{ki,t}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{A_{ki,t}^{\psi_{ki}}\psi_{ki}}\left[\widetilde{\alpha}_{ki}W_{k,t}^{1-\psi} + \widetilde{\vartheta}_{ki}P_{kiX,t}^{1-\psi}\right]^{\frac{1}{1-\psi}}$, $P_{kiX,t} = \left[\widetilde{\beta}_{ki}P_{kiXE,t}^{1-\phi} + (1-\widetilde{\beta}_{ki})P_{kiXM,t}^{1-\phi}\right]^{\frac{1}{1-\phi}}$, $P_{kiXE,t} = \left[\sum_{j\in I_E}\widetilde{\nu}_{kij}P_{kijX,t}^{1-\chi}\right]^{\frac{1}{1-\chi}}$, $P_{kiXM,t} = \left[\sum_{j\in I_M}\widetilde{\upsilon}_{kij}P_{kijX,t}^{1-\xi}\right]^{\frac{1}{1-\chi}}$, and $P_{kijX,t} = \left[\sum_{l=1}^{K}\widetilde{\zeta}_{kl\,ij}P_{kl\,j,t}^{1-\mu}\right]^{\frac{1}{\mu}}$. Their log-linearized counterparts are given by

$$\widehat{\text{mc}}_{ki,t} = \frac{1-\psi_{ki}}{\psi_{ki}}\widehat{y}_{ki,t} - \psi_{ki}a_{ki,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{W_k N_{ki}}{P_{ki}Y_{ki}}\widehat{w}_{k,t} + \frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}}\widehat{p}_{kiX,t} \tag{A.28}$$

$$\widehat{p}_{kiX,t} = \widetilde{\beta}_{ki}\widehat{p}_{kiXE,t} + (1-\widetilde{\beta}_{ki})\widehat{p}_{kiXM,t} \tag{A.29}$$

$$\widehat{p}_{kiXE,t} = \sum_{j\in I_E} \nu_{kij}\widehat{p}_{kijX,t} \quad \text{and} \quad \widehat{p}_{kiXM,t} = \sum_{j\in I_M} \upsilon_{kij}\widehat{p}_{kijX,t} \tag{A.30}$$

$$\widehat{p}_{kijX,t} = \sum_{l=1}^{K} \zeta_{klij}\widehat{p}_{klj,t} \tag{A.31}$$

where $\frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \left(\frac{W_k Y_{ki}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{\text{MC}_{ki}\psi_{ki}}\right)^{1-\psi} \widetilde{\alpha}_{ki}$ and $\frac{\mathcal{M}_{ki}}{\psi_{ki}}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}} = \left(\frac{P_{kiX} Y_{ki}^{\frac{1-\psi_{ki}}{\psi_{ki}}}}{\text{MC}_{ki}\psi_{ki}}\right)^{1-\psi} \widetilde{\vartheta}_{ki}$ can be derived using (A.15)-(A.16) in steady-state, and $\mathcal{M}_{ki} = \frac{\epsilon_{pk}}{\epsilon_{pk}-1}$.

Production Structure. The log-linearized version of the production function (8) is given by

$$\widehat{y}_{fki,t} = a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\widehat{n}_{fki,t} + \mathcal{M}_{ki}\vartheta_{ki}\widehat{x}_{fki,t} \tag{A.32}$$

where the identities $\mathcal{M}_{ki}\alpha_{ki} = \mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \frac{N_{ki}}{Y_{ki}}\psi_{ki}Y_{ki}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\widetilde{\alpha}_{ki}Y_{ki}}{N_{ki}}\right)^{\frac{1}{\psi}}$ and $\mathcal{M}_{ki}\vartheta_{ki} = \mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}} = \frac{X_{ki}}{Y_{ki}}\psi_{ki}Y_{ki}^{\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}}\left(\frac{\widetilde{\vartheta}_{ki}Y_{ki}}{X_{ki}}\right)^{\frac{1}{\psi}}$ can be verified using the first-order conditions from the firms' problem (A.15) and (A.16) in steady-state and the standard monopolistic competition pricing condition in steady-state, $P_{ki} = \mathcal{M}_{ki}\text{MC}^n_{ki}$. Using the previous identities, together with the production function (8) in steady-state, one can verify that

$$\mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}} = \frac{W_k N_{ki}}{\text{MC}^n_{ki}Y_{ki}} + \frac{P_{kiX}X_{ki}}{\text{MC}^n_{ki}Y_{ki}} = \psi_{ki}. \tag{A.33}$$

A.3. Price– and Wage–Setting

Price–Setting. Following Galí (2015), we assume that each firm $f \in [0,1]$ produces a continuum of differentiated goods, using technology given by the production function (8). In particular, $Y_{ki,t}$ is an index of production, and defined by

$$Y_{ki,t} = \left(\int_0^1 Y_{fki,t}^{\frac{\epsilon_{pk,t}-1}{\epsilon_{pk,t}}} df\right)^{\frac{\epsilon_{pk,t}}{\epsilon_{pk,t}-1}}, \tag{A.34}$$

where $Y_{fki,t}$ denotes the quantity of type-f good produced by firm f in period t. Note that $\epsilon_{pk,t}$ represents the elasticity of substitution among good varieties. The implied sectoral price index is

$$P_{ki,t} = \left(\int_0^1 P_{fki,t}^{1-\epsilon_{pki,t}} df \right)^{\frac{1}{1-\epsilon_{pki,t}}} . \tag{A.35}$$

Producers of each differentiated variety face the demand function

$$Y_{ik,t+l|t} = \left(\frac{P_{fki,t}}{P_{ki,t+l}} \right)^{-\epsilon_{pki,t}} Y_{ki,t+l} \tag{A.36}$$

Firms set prices à la Calvo, which implies that the aggregate price dynamics are described by the equation

$$\Pi_{ki,t}^{1-\epsilon_{pki,t}} = \theta_{ki}^p + (1-\theta_{ki}^p) \left(\frac{P_{ki,t}^*}{P_{ki,t-1}} \right)^{1-\epsilon_{pki,t}} \tag{A.37}$$

Log-linearized: $\pi_{ki,t} = (1-\theta_{ki}^p)(p_{ki,t}^* - p_{ki,t-1})$. A firm that resets its price at time t faces the following problem $\max_{P_{ki,t}^*} \sum_{l=0}^{\infty} \theta_{ki}^{pl} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+l}}{P_{ki,t+l}} [P_{ki,t}^* Y_{ki,t+l|t} - \mathcal{C}_{ki,t+l}(Y_{ki,t+l|t})] \right\}$ subject to the sequence of demand constraints (A.36). The optimality condition associated with the problem takes the form

$$\sum_{l=0}^{\infty} \theta_{ki}^{pl} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+l} Y_{ki,t+l|t}}{P_{ki,t+l}} [P_{ki,t}^* - \mathcal{M}_{ki,t} \mathrm{MC}_{ki,t+l|t}^n] \right\} = 0 \tag{A.38}$$

where $\mathrm{MC}_{ki,t+l|t}^n$ denotes the nominal marginal cost in period $t+l$ for a firm which last reset its price in period t, and $\mathcal{M}_{ki,t} = \frac{\epsilon_{pki,t}}{\epsilon_{pki,t}-1}$. A first-order Taylor expansion of (A.38) around the zero inflation steady state yields

$$p_{ki,t}^* = (1-\beta\theta_{ki}^p) \sum_{l=0}^{\infty} (\beta\theta_{ki}^p) \mathbb{E}_t \left(\mathrm{mc}_{ki,t+l|t}^n + \mu_{ki,t}^n \right) \tag{A.39}$$

where $\mathrm{mc}_{ki,t+l|t}^n \equiv \log \mathrm{MC}_{ki,t+l|t}^n$ is the log marginal cost, and $\mu_{ki,t}^n := \log \mathcal{M}_{ki,t}$ is the log of the desired gross markup. The log marginal cost for an individual firm that last set its price in period t is given by $\mathrm{mc}_{ki,t+l|t}^n = w_{k,t+l} - \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t+l} - \log\psi_{ki} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} y_{ki,t+l|t} - \frac{1}{\psi}\left[\log\tilde{\alpha}_{ki} + y_{ki,t+l|t} - n_{ki,t+l|t}\right]$ where $n_{ki,t+l|t}$ denotes the log employment in period $t+l$ for a firm that last reset its price in period t, and where we have made use of (A.20). Letting $\text{mc}^n_{ki,t} = \int_0^1 \text{mc}^n_{fki,t}\, df$ represent the log average marginal cost, it follows that $\text{mc}^n_{ki,t} = (1-\theta^p_{ki}) \sum_{l=0}^{\infty} \theta^{pl}_{ki}\, \text{mc}^n_{ki,t|t-l} = w_{k,t} - \frac{\psi-1}{\psi\psi_{ki}} a_{ki,t} - \log \psi_{ki} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} y_{ki,t} - \frac{1}{\psi}\left[\log \widetilde{\alpha}_{ki} + y_{ki,t} - n_{ki,t}\right]$. Thus, the following relation holds between firm-specific and economy-wide marginal costs $\text{mc}^n_{ki,t+l|t} = \text{mc}^n_{ki,t+l} - \frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}}\left(y_{ki,t+l|t} - y_{ki,t+l}\right) - \frac{1}{\psi}\left[\left(y_{ki,t+l|t} - y_{ki,t+l}\right) - \left(n_{ki,t+l|t} - n_{ki,t+l}\right)\right]$. Notice that, making use of both marginal cost expressions (A.15)-(A.16), the identity $x_{ki,t+l|t} - x_{ki,t+l} = n_{ki,t+l|t} - n_{ki,t+l}$ is satisfied. Hence, we can write $y_{ki,t+l|t} - y_{ki,t+l} = \left[\mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}}\right](n_{ki,t+l|t} - n_{ki,t+l}) = \psi_{ki}(n_{ki,t+l|t} - n_{ki,t+l})$, where we have used the identity (A.33), and where we have used the linearized production function (A.32). Hence, we can finally write the relation between marginal costs as $\text{mc}^n_{ki,t+l|t} = \text{mc}^n_{ki,t+l} - \left[\frac{(1-\psi_{ki})(1-\psi)}{\psi\psi_{ki}} + \frac{1}{\psi}\left(1 - \frac{1}{\psi_{ki}}\right)\right]\left(y_{ki,t+l|t} - y_{ki,t+l}\right) = \text{mc}^n_{ki,t+l} + \frac{1-\psi_{ki}}{\psi_{ki}}\left(y_{ki,t+l|t} - y_{ki,t+l}\right) = \text{mc}^n_{ki,t+l} - \frac{(1-\psi_{ki})\epsilon_{pki}}{\psi_{ki}}\left(p^*_{ki,t} - p_{ki,t+l}\right)$. Introducing this last expression into the log-linearized firms' FOC (A.39), we can write $p^*_{ki,t} = (1-\beta\theta^p_{ki})\sum_{l=0}^{\infty}(\beta\theta^p_{ki})\mathbb{E}_t\left[p_{ki,t+l} - \Theta_k \right.$ where $\Theta_{ki} \equiv \frac{\psi_{ki}}{\psi_{ki}+(1-\psi_{ki})\epsilon_{pki}}$, $\widehat{\mu}_{ki,t} \equiv \mu_{ki,t} - \mu_{ki}$ is the deviation between the average and desired markups, with $\mu_{ki,t} = p_{ki,t} - \text{mc}^n_{ki,t}$, and $\widehat{\mu}^n_{ki,t} \equiv \mu^n_{ki,t} - \mu_{ki}$. Combining the (linearized) inflation dynamics (A.37) with the above expression, we can write (14), where $\kappa_{ki} = \frac{(1-\theta^p_{ki})(1-\beta\theta^p_{ki})}{\theta^p_{ki}}\Theta_{ki}$, $u^p_{ki,t} = \kappa_{ki}\widehat{\mu}^n_{ki,t}$, $\widehat{p}_{ki,t} = p_{ki,t} - p_{kC,t}$. Inserting the marginal cost equation (A.28), and the intermediate inputs price indices A.29 where $\kappa_{ki} = \frac{(1-\theta^p_{ki})(1-\beta\theta^p_{ki})}{\theta^p_{ki}}\Theta_{ki}$, $u^p_{ki,t} = \kappa_{ki}\widehat{\mu}^n_{ki,t}$, $\widehat{p}_{ki,t} = p_{ki,t} - p_{kC,t}$. Inserting the marginal cost equation (A.28), and the intermediate inputs price indices A.29 where $\kappa_{ki} = \frac{(1-\theta^p_{ki})(1-\beta\theta^p_{ki})}{\theta^p_{ki}}\Theta_{ki}$, $u^p_{ki,t} = \kappa_{ki}\widehat{\mu}^n_{ki,t}$, $\widehat{p}_{ki,t} = p_{ki,t} - p_{kC,t}$. Combining the log-linearized intermediate input prices indices (A.28)-(A.31) we obtain the marginal cost equation (21).

Note that sectoral-inflation rates and sectoral-level real prices ($\widehat{p}_{ki,t}$) are related through:

$$\pi_{ki,t} = \widehat{p}_{ki,t} - \widehat{p}_{ki,t-1} + \pi_{kC,t}. \tag{A.40}$$

Writing (A.10)-(A.12) in first-differences, we can obtain consumer price inflation,

$$\pi_{kC,t} = \sum_{i=1}^{I}\sum_{l=1}^{K}\beta_{kli}\pi_{kli,t} \tag{A.41}$$

where $\beta_{kli} = \frac{P_{kli}C_{kli}}{P_{kC}C_k} = \zeta_{kli}\left[\nu_{ki}\beta_k\mathbb{1}_{\{i\in I_E\}} + \upsilon_{ki}(1-\beta_k)\left(1 - \mathbb{1}_{\{i\in I_E\}}\right)\right]$.

Wage–Setting. Following Erceg *et al.* (2000), wage stickiness is introduced in a way analogous to price stickiness. We assume that firms employ a continuum of differentiated labor services.

In particular, $N_{fki,t}$ is an index of labor input used by firm f, and defined by

$$N_{fki,t} = \left(\int_0^1 N_{fgki,t}^{\frac{\epsilon_{wk}-1}{\epsilon_{wk}}} dg \right)^{\frac{\epsilon_{wk}}{\epsilon_{wk}-1}}, \tag{A.42}$$

where $N_{fgki,t}$ denotes the quantity of type-g labor employed by firm f in period t. Note that $\epsilon_{wk,t}$ represents the elasticity of substitution among labor varieties. Note also the assumption of a continuum of labor types, indexed by $g \in [0,1]$.

Let $W_{gk,t}$ denote nominal wage for type-g labor prevaling in period t. Nominal wages are set by workers of each type (or a union representing them) and taken as given by firms. Given the wages effective at any point in time for the different types of labor services, cost minimization yields a corresponding set of demand schedules for each firm f and labor type g, given the firm's total employment $N_{fk,t}$,

$$N_{fgki,t} = \left(\frac{W_{gk,t}}{W_{k,t}} \right)^{-\epsilon_{wk,t}} N_{fki,t}, \tag{A.43}$$

where

$$W_{k,t} \equiv \left(\int_0^1 W_{gk,t}^{1-\epsilon_{wk,t}} dg \right)^{\frac{1}{1-\epsilon_{wk,t}}} \tag{A.44}$$

is an aggregate wage index. Combining the previous conditions, one can obtain a convenient aggregation result, $\int_0^1 W_{gk,t} N_{fgki,t} dg = W_{k,t} N_{fki,t}$. That is, the wage bill of any given firm can be expressed as the product of the wage index and the firm's employment index.

Consider a union resetting its members' wage in period t, and let $W^*_{k,t}$ denote the newly set wage. The union chooses $W^*_{k,t}$ in a way consistent with utility maximization of its members' households, taking as given the decisions of other unions as well as all the path of aggregate consumption and prices. Specifically, the union seeks to maximize $\max_{W^*_{k,t}} \mathbb{E}_t \sum_{l=0}^{\infty} (\beta\theta^w_k)^l \left(\frac{C_{k,t+l} W^*_{k,t} N_{k,t+l|t}}{P^c_{t+l}} \cdot \frac{N^{1+\varphi}_{k,t+l|t}}{1+\varphi} \right)$ subject to the sequence of labor demand schedules $N_{k,t+l|t} = \left(\frac{W^*_{k,t}}{W_{k,t+l}} \right)^{-\epsilon_{wk,t}} \int_0^1 N_{gk,t} dg$, where $N_{k,t+l|t}$ denotes the level of employment in period $t+l$ among workers that last reset their wage in period t. The first-order condition is given by $\sum_{l=0}^{\infty} (\beta\theta^w_k)^l \, \mathbb{E}_t \left[N_{k,t+l|t} C^{-\sigma}_{t+l} \left(\frac{W^*_{k,t}}{P^c_{t+l}} - \mathcal{M}_{wk,t} \mathrm{MRS}^{\varphi}_{k,\ldots} \right. \right.$ 0, where $\mathcal{M}_{wk,t} = \frac{\epsilon_{wk,t}}{\epsilon_{wk,t}-1}$, and $\mathrm{MRS}_{k,t+l|t} = C^{-\sigma}_{t+l} N^{\varphi}_{k,t+l|t}$ denotes the marginal rate of substitution between household consumption and employment in period $t+l$ relevant to the workers resetting their wage in period t. Log-linearizing the above expression around a zero inflation steady-state yields the wage setting rule

$$w^*_{k,t} = (1-\beta\theta^w_k)\sum_{l=0}^{\infty}(\beta\theta^w_k)^l\,\mathbb{E}_t\left(\mathrm{mrs}_{t+l|t} + \mu^n_{wk,t} + p_{kc,t+l}\right) \tag{A.45}$$

where $\mu^n_{wk,t} = \log \mathcal{M}_{wk,t}$ and $\mathrm{mrs}_{t+l|t} = \sigma c_{k,t+l} + \varphi n_{k,t+l|t}$. Letting $\mathrm{mrs}_{t+l} = \sigma c_{k,t+l} + \varphi n_{k,t+l}$ define the economy's average marginal rate of substitution, where $n_{k,t+l} = \log \int_0^1 \int_0^1 N_{fgk} df dg$ denotes the log aggregate employment. Up to a first-order approximation, $\mathrm{mrs}_{t+l|t} = \mathrm{mrs}_{t+l} + \varphi(n_{k,t+l} - n_{k,t+l|t}) = \mathrm{mrs}_{t+l} - \epsilon_{wk}\varphi(w^*_{k,t} - w_{k,t+l})$. Hence, we can write (A.45) as

$$w^*_{k,t} = \frac{1-\beta\theta^w_k}{1+\epsilon_{wk}\varphi}\sum_{l=0}^{\infty}(\beta\theta^w_k)^l\,\mathbb{E}_t\left[(1+\epsilon_{wk}\varphi)w_{k,t+l} - \left(\widehat{\mu}_{wk,t+l} - \widehat{\mu}^n_{wk,t+l}\right)\right] \tag{A.46}$$

where $\widehat{\mu}_{wk,t+l} = \mu_{wk,t+l} - \mu_{wk}$ denotes the deviations of the economy's log average wage markup $\mu_{wk,t+l} = w_{k,t+k} - p_{kc,t+l} - \mathrm{mrs}_{k,t+l}$ from its steady-state level, and $\widehat{\mu}^n_{wk,t+l} = \mu^n_{wk,t+l} - \mu_{wk}$.

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by $W_{k,t} = \left(\theta^w_k W^{1-\epsilon_{wk}}_{k,t-1} + (1-\theta^w_k)(W^*_{k,t})^{1-\epsilon_{wk}}\right)^{\frac{1}{1-\epsilon_{wk}}}$. Log-linearized, $w_{k,t} = \theta^w_k w_{k,t-1} + (1-\theta^w_k)w^*_{k,t}$. Combing the last expression with (A.46), and letting $\pi_{wk,t} = w_{k,t} - w_{k,t-1}$, we obtain the baseline wage inflation equation (13), where $\kappa_{wk} = \frac{(1-\theta^w_k)(1-\beta\theta^w_k)}{\theta_w(1+\epsilon_{wk}\varphi)}$, $\widehat{\mu}_{wk,t} = \widehat{w}_{k,t} - \sigma\widehat{c}_{k,t} - \varphi\widehat{n}_{k,t}$, and $u^w_{k,t} = \kappa_{wk}\widehat{\mu}^n_{wk,t}$. Note that nominal wage inflation is related to real wages through:

$$\pi^w_{k,t} = \widehat{w}_{k,t} - \widehat{w}_{k,t-1} + \pi_{Ck,t}. \tag{A.47}$$

A.4. Monetary Authority

The log-linearized bilateral nominal exchange rate (26) is given by $e_{k,k^{MU},t} = e_{k,k^{MU}}, \forall k \in K^{MU}$. In stationary terms, taking first differences, this can be written as

$$\Delta e_{k,k^{MU},t} = 0 \quad \forall k \in K^{MU} \tag{A.48}$$

Log-linearizing the expression for the real exchange rate (16) and first-differencing, we obtain

$$\Delta q_{kl,t} = \Delta e_{kl,t} + \pi_{l,t} - \pi_{k,t}. \tag{A.49}$$

Similarly, log-linearizing and first-differencing the symmetry of nominal exchange rates condition $\mathcal{E}_{kl,t} = \mathcal{E}^{-1}_{lk,t}$ yields

$$\Delta e_{kl,t} = -\Delta e_{lk,t} \tag{A.50}$$

A.5. Market Clearing, GDP, and Trade Balance

Market Clearing. We first consider the goods market clearing condition (27). Pre-multiplying by $\frac{P_{ki,t}}{P_{k,t}C_{k,t}} = \frac{P_{ki,t}}{E_{k,t}}$, and making use of (A.8),

$$\begin{aligned}
\frac{P_{ki,t}Y_{ki,t}}{E_{k,t}} &= \sum_{l=1}^{K} \frac{P_{ki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}X_{lkji,t}}{E_{k,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{k,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lki,t}C_{lki,t}}{E_{l,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{l,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \quad \forall i \in I
\end{aligned} \tag{A.51}$$

which we can write in steady-state as

$$\lambda_{ki} = \sum_{l=1}^{K} y_{lk}\beta_{lki} + \sum_{l=1}^{K}\sum_{j=1}^{I} y_{lk}\lambda_{lj}\omega_{lkji} \quad \forall i \in I$$

where the Domar weight for sector i in country k is $\lambda_{ki} = \frac{P_{ki}Y_{ki}}{y_k}$, the nominal GDP ratio between countries l and k is defined as $y_{lk} = \frac{P_{lC}C_l}{P_{kC}C_k}$, and the IO share is given by $\omega_{lkji} = \frac{P_{lki}X_{lkji}}{P_{lj}Y_{lj}} = \frac{P_{ki}X_{lkji}}{P_{lj}Y_{lj}} = \zeta_{lkji}\vartheta_{lj}\left[\nu_{lji}\beta_{lj}\mathbb{1}_{\{i\in I_E\}} + \upsilon_{lji}(1-\beta_{lj})\left(1-\mathbb{1}_{\{i\in I_E\}}\right)\right]$, where we have made use of the law of one price in steady-state, $P_{klj} = P_{lj}$. Notice that β_{lki} and ω_{lkji} can be extracted directly from the data.

Hence, we can write the log-linearized version of the goods market clearing condition (27),

(A.52)

$$Y_{ki}\hat{y}_{ki,t} = \sum_{l=1}^{K}\left(C_{lki}\hat{c}_{lki,t} + \sum_{j=1}^{I} X_{lkji}\hat{x}_{lkji}\right) \implies \lambda_{ki}\hat{y}_{ki,t} = \sum_{l=1}^{K} y_{lk}\left(\beta_{lki}\hat{c}_{lki,t} + \sum_{j=1}^{I} \lambda_{lj}\omega_{lkji}\hat{x}_{lkji}\right)$$

where we have pre-multiplied the first expression by $\frac{P_{ki}}{E_k}$.

The log-linearized version of the labor market clearing condition (28) is given by

$$\widehat{n}_{k,t} = \sum_{i=1}^{I} \delta_{ki}\widehat{n}_{ki,t} \tag{A.53}$$

where $\delta_{ki} = \frac{\alpha_{ki}\lambda_{ki}}{1-\sum_{j=1}^{I}\left(1-\frac{\psi_{kj}}{\mathcal{M}_{kj}}\right)\lambda_{kj}} = \frac{W_kN_{ki}}{P_{ki}Y_{ki}}\frac{P_{ki}Y_{ki}}{P_{kC}C_k}\frac{P_{kC}C_k}{W_kN_k} = \frac{W_kN_{ki}}{W_kN_k} = \frac{N_{ki}}{N_k}$ can be derived using where we have made use of (A.58).

The NFA from the "global" country K (29) can be log-linearized to

$$\frac{1}{\beta}\sum_{k=1}^{K-1} \mathrm{nfa}_{k,t-1}^{K} - \sum_{k=1}^{K-1} \mathrm{nfa}_{k,t}^{K} = \Upsilon_K \left(\widehat{\exp}_{K,t} - \widehat{\mathrm{imp}}_{K,t} + \widehat{p}_{K\mathrm{EXP},t} - \widehat{p}_{K\mathrm{IMP},t}\right), \tag{A.54}$$

the NFA from country $k \neq K$ $k \notin \mathrm{MU}$, (29) can be log-linearized to

$$\mathrm{nfa}_{k,t}^{K} - \frac{1}{\beta}\mathrm{nfa}_{k,t-1}^{K} = \Upsilon_k \left(\widehat{\exp}_{k,t} - \widehat{\mathrm{imp}}_{k,t} + \widehat{p}_{k\mathrm{EXP},t} - \widehat{p}_{k\mathrm{IMP},t}\right) \tag{A.55}$$

where the linearized export and import price deflators are given by:

(A.56)

$$\widehat{p}_{k\mathrm{IMP},t} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\frac{P_{kli}C_{kli} + \sum_{j=1}^{I} P_{kli}X_{klji}}{P_{k,IMP}IMP_k}\widehat{p}_{kli,t}\right] = \sum_{l\neq k}\sum_{i=1}^{I}\Upsilon_k^{-1}\left(\beta_{kli} + \sum_{j=1}^{I}\lambda_{kj}\omega_{klji}\right)\widehat{p}_{kli,t}$$

$$\widehat{p}_{k\mathrm{EXP},t} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\frac{P_{ki}C_{lki} + \sum_{j=1}^{I} P_{ki}X_{lkji}}{P_{k,EXP}EXP_k}(\widehat{p}_{ki,t} + \tau_{kli,t})\right]$$

$$= \sum_{l\neq k}\sum_{i=1}^{I}\frac{\mathcal{Y}_{lk}}{\Upsilon_k}\left(\beta_{lki} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\right)(\widehat{p}_{ki,t} + \tau_{kli,t}) \tag{A.57}$$

Gross Domestic Product and Net Exports. Let us now move to nominal GDP (30). In steady state, assuming zero net exports, $P_{k\mathrm{EXP}}\mathrm{EXP}_k - P_{k\mathrm{IMP}}\mathrm{IMP}_k = 0$, we can write $\mathcal{Y}_k = P_{kC}C_k$. Using the household's budget constraint (4) in steady state, we can write

$$\mathcal{Y}_k = P_{kC}C_k = W_kN_k + \Pi_k = W_kN_k + \sum_{i=1}^{I}\left(1 - \frac{\psi_{ki}}{\mathcal{M}_{ki}}\right)P_{ki}Y_{ki} \tag{A.58}$$

where the last equality makes use of (A.33).

Log-linearizing the real GDP (31) definition,

$$\widehat{y}_{k,t} = \frac{P_{kC} C_k}{y_k} \widehat{c}_{k,t} + \frac{P_{k\mathrm{EXP}} \mathrm{EXP}_k}{y_k} \widehat{\mathrm{exp}}_{k,t} - \frac{P_{k\mathrm{IMP}} \mathrm{IMP}_k}{y_k} \widehat{\mathrm{imp}}_{k,t} = \widehat{c}_{k,t} + \Upsilon_k \left(\widehat{\mathrm{exp}}_{k,t} - \widehat{\mathrm{imp}}_{k,t} \right)$$

where second equality uses that nominal consumption expenditures will be equal nominal GDP in steady state, and $\Upsilon_k = \frac{P_{k\mathrm{EXP}} \mathrm{EXP}_k}{y_k} = \frac{P_{k\mathrm{IMP}} \mathrm{IMP}_k}{y_k}$ is the export (or import) share of nominal GDP.

The nominal exports expression (32) can be log-linearized to:

$$\begin{aligned} \widehat{\mathrm{exp}}_{k,t} &= \sum_{l \neq k} \sum_{i \in I} \left(\frac{P_{ki} C_{lki}}{P_{k\mathrm{EXP}} \mathrm{EXP}_k} \widehat{c}_{lki,t} + \sum_{j=1}^{I} \frac{P_{ki} X_{lki}}{P_{k\mathrm{EXP}} \mathrm{EXP}_k} \widehat{x}_{lkji,t} \right) \\ &= \sum_{l \neq k} \sum_{i \in I} \frac{y_{lk}}{\Upsilon_k} \left(\beta_{lki} \widehat{c}_{lki,t} + \sum_{j=1}^{I} \lambda_{lj} \omega_{lkji} \widehat{x}_{lkji,t} \right) \end{aligned} \tag{A.59}$$

where the export share of nominal GDP is given by

(A.60)

$$\Upsilon_k = \frac{P_{k\mathrm{EXP}} \mathrm{EXP}_k}{y_k} = \sum_{l \neq k} \sum_{i=1}^{I} y_{lk} \left[\beta_{lki} + \sum_{j=1}^{I} \lambda_{lj} \omega_{lkji} \right] = \left(\sum_{i=1}^{I} \lambda_{ki} \right) - \left(\beta_{kki} + \sum_{j=1}^{I} \lambda_{kj} \omega_{kkji} \right)$$

(A.61)

$$= \frac{P_{k\mathrm{IMP}} \mathrm{IMP}_k}{y_k} = \sum_{l \neq k} \sum_{i=1}^{I} \left[\beta_{kli} + \sum_{j=1}^{I} \lambda_{kj} \omega_{klji} \right]$$

Similarly, the nominal imports expression (33) can be log-linearized to

$$\begin{aligned} \widehat{\mathrm{imp}}_{k,t} &= \sum_{l \neq k} \sum_{i \in I} \left(\frac{P_{kli} C_{kli}}{P_{k\mathrm{IMP}} \mathrm{IMP}_k} \widehat{c}_{kli,t} + \sum_{j=1}^{I} \frac{P_{kli} X_{klji}}{P_{k\mathrm{IMP}} \mathrm{IMP}_k} \widehat{x}_{klji,t} \right) \\ &= \sum_{l \neq k} \sum_{i \in I} \Upsilon_k^{-1} \left(\beta_{kli} \widehat{c}_{kli,t} + \sum_{j=1}^{I} \lambda_{kj} \omega_{klji} \widehat{x}_{klji,t} \right) \end{aligned} \tag{A.62}$$

Now we can combine the linearized expression for real gdp, que the expressions for real imports and exports:

$$\widehat{y}_{k,t} = \widehat{c}_{k,t} + \sum_{l \neq k} \sum_{i \in I} \left(\frac{P_{ki} C_{lki}}{y_k} \widehat{c}_{lki,t} + \sum_{j=1}^{I} \frac{P_{ki} X_{lkji}}{y_k} \widehat{x}_{lkji,t} - \frac{P_{kli} C_{kli}}{y_k} \widehat{c}_{kli,t} - \sum_{j=1}^{I} \frac{P_{kli} X_{klji}}{y_k} \widehat{x}_{klji,t} \right)$$

$$= \widehat{c}_{k,t} + \sum_{l \neq k} \sum_{i \in I} \left(y_{lk} \beta_{lki} \widehat{c}_{lki,t} + \sum_{j=1}^{I} y_{lk} \lambda_{lj} \omega_{lkji} \widehat{x}_{lkji,t} - \beta_{kli} \widehat{c}_{kli,t} - \sum_{j=1}^{I} \lambda_{kj} \omega_{klji} \widehat{x}_{klji,t} \right)$$

A.6. Transmission Mechanism in the Phillips Curve

Let us consider the simplifying case of $\tau_{kl\,ij} = \tau_{lj}$, producer currency pricing and no additional shocks. Starting from a general Phillips curve, where $\pi_{ki,t} = p_{ki,t} - p_{ki,t-1}$ denotes the inflation rate of a good in sector i in country k, $\pi_{ki,t} = \kappa_{ki} \left(\widehat{\mathrm{mc}}^{n}_{ki,t} - p_{ki,t} \right) + \beta \mathbb{E}_t \pi_{ki,t+1}$. We now seek to obtain the Phillips curve in price terms. We can write the previous Phillips curve in matrix form,

$$\begin{aligned}
\boldsymbol{\pi}_t &= \boldsymbol{K} \left(\widehat{\mathbf{mc}}^{n}_t - \boldsymbol{p}_t \right) + \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} \\
&= \boldsymbol{K} \left[\widehat{\mathbf{mc}}^{n}_t - \left(\boldsymbol{\pi}_t + \boldsymbol{p}_{t-1} \right) \right] + \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} \\
&= (\boldsymbol{I} - \boldsymbol{K})^{-1} \boldsymbol{K} \left(\widehat{\mathbf{mc}}^{n}_t - \boldsymbol{p}_{t-1} \right) + (\boldsymbol{I} - \boldsymbol{K})^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} \\
&= \boldsymbol{\Delta} \left[\overline{\boldsymbol{\alpha}} \boldsymbol{w}_t + \overline{\boldsymbol{\Omega}} (\boldsymbol{\tau}_t + \boldsymbol{p}_t) - \boldsymbol{p}_{t-1} \right] + \boldsymbol{\Delta} \boldsymbol{K}^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} \\
&= \boldsymbol{\Delta} \left(\overline{\boldsymbol{\alpha}} \boldsymbol{w}_t + \overline{\boldsymbol{\Omega}} \boldsymbol{\tau}_t + \overline{\boldsymbol{\Omega}}\, \boldsymbol{p}_t - \boldsymbol{p}_{t-1} \right) + \boldsymbol{\Delta} \boldsymbol{K}^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} \\
&= \boldsymbol{\Delta} \left(\overline{\boldsymbol{\alpha}} \boldsymbol{w}_t + \overline{\boldsymbol{\Omega}} \boldsymbol{\tau}_t + \overline{\boldsymbol{\Omega}} \boldsymbol{\pi}_t + \overline{\boldsymbol{\Omega}}\, \boldsymbol{p}_{t-1} - \boldsymbol{p}_{t-1} \right) + \boldsymbol{\Delta} \boldsymbol{K}^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} \\
&= (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} \boldsymbol{\Delta} \left[\overline{\boldsymbol{\alpha}} \boldsymbol{w}_t + \overline{\boldsymbol{\Omega}} \boldsymbol{\tau}_t - (\boldsymbol{I} - \overline{\boldsymbol{\Omega}})\, \boldsymbol{p}_{t-1} \right] + (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} \boldsymbol{\Delta} \boldsymbol{K}^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1} \qquad \text{(A.63)}
\end{aligned}$$

where we have introduced nominal marginal costs (under CRS), $\widehat{\mathbf{mc}}^{n}_t = \overline{\boldsymbol{\alpha}} \boldsymbol{w}_t + \overline{\boldsymbol{\Omega}} \left(\boldsymbol{\tau}_t + \boldsymbol{p}_t \right)$, and the different objects are defined in section 4.1. We can also rewrite (A.63) in terms of the price level as

$$\boldsymbol{p}_t = (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} (\boldsymbol{I} - \boldsymbol{\Delta})\, \boldsymbol{p}_{t-1} + (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} \boldsymbol{\Delta} \left(\overline{\boldsymbol{\alpha}} \boldsymbol{w}_t + \overline{\boldsymbol{\Omega}} \boldsymbol{\tau}_t \right) (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} \boldsymbol{\Delta} \boldsymbol{K}^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+1}$$

Iterating forward, we can write

$$\begin{aligned}
\boldsymbol{p}_{t+h} = {} & (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} (\boldsymbol{I} - \boldsymbol{\Delta}) \sum_{s=1}^{h} \boldsymbol{\pi}_{t+h-s} + (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} \boldsymbol{\Delta} \overline{\boldsymbol{\alpha}} \sum_{s=0}^{h} \boldsymbol{\pi}^{w}_{t+h-s} + (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}} \left[\boldsymbol{R}^{h} \boldsymbol{\tau}_t + \sum_{s=0}^{h-1} \boldsymbol{e}^{\tau}_{t+s} \right] \\
& + (\boldsymbol{I} - \boldsymbol{\Delta} \overline{\boldsymbol{\Omega}})^{-1} \boldsymbol{\Delta} \boldsymbol{K}^{-1} \beta \mathbb{E}_t \boldsymbol{\pi}_{t+h+1}
\end{aligned}$$

where we have made the simplifying assumption that the price-wedge shock follows an AR(1) process $\boldsymbol{\tau}_t = \boldsymbol{R} \boldsymbol{\tau}_{t-1} + \boldsymbol{e}^{\tau}_t$. Similarly, the wage Phillips curve can be written as

$$\boldsymbol{\pi}^{w}_t = \boldsymbol{K}_w \left(\sigma \boldsymbol{c}_t + \varphi \boldsymbol{n}_t - \boldsymbol{w}_t + \boldsymbol{\beta}\, \boldsymbol{p}_t \right) + \beta \mathbb{E}_t \boldsymbol{\pi}^{w}_{t+1}$$

where $p_{C,t} = \boldsymbol{\beta}\,\boldsymbol{p}_t$ with

$$\boldsymbol{c}_t = \begin{bmatrix} c_{1,t} \\ c_{2,t} \\ \vdots \\ c_{K,t} \end{bmatrix}, \quad \boldsymbol{n}_t = \begin{bmatrix} n_{1,t} \\ n_{2,t} \\ \vdots \\ n_{K,t} \end{bmatrix}, \quad \boldsymbol{K}_w = \begin{bmatrix} \kappa_{w1} & 0 & \dots & 0 \\ 0 & \kappa_{w2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \kappa_{wK} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_{111} & \beta_{112} & \dots & \beta_{11I} \\ \beta_{121} & \beta_{122} & \dots & \beta_{12I} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{KK1} & \beta_{KK2} & \dots & \beta_{KKI} \end{bmatrix}$$

Or, in wage levels,

$$\boldsymbol{w}_t = \boldsymbol{w}_{t-1} + \boldsymbol{K}_w \beta\, \boldsymbol{p}_t + \boldsymbol{K}_w[\sigma \boldsymbol{c}_t + \varphi \boldsymbol{n}_t] + \beta \mathbb{E}_t \boldsymbol{\pi}^w_{t+1}$$

Iterating forward h periods,

$$\boldsymbol{w}_{t+h} = \sum_{s=0}^{h-1} \boldsymbol{\pi}^w_{t+s} + \boldsymbol{K}_w \beta \sum_{s=0}^{h} \boldsymbol{\pi}_{t+s} + \boldsymbol{K}_w[\sigma \boldsymbol{c}_{t+h} + \varphi \boldsymbol{n}_{t+h}] + \beta \mathbb{E}_t \boldsymbol{\pi}^w_{t+h+1}$$

Appendix B. The Transimission of (Other) Foreign Shocks in a Networked Economy

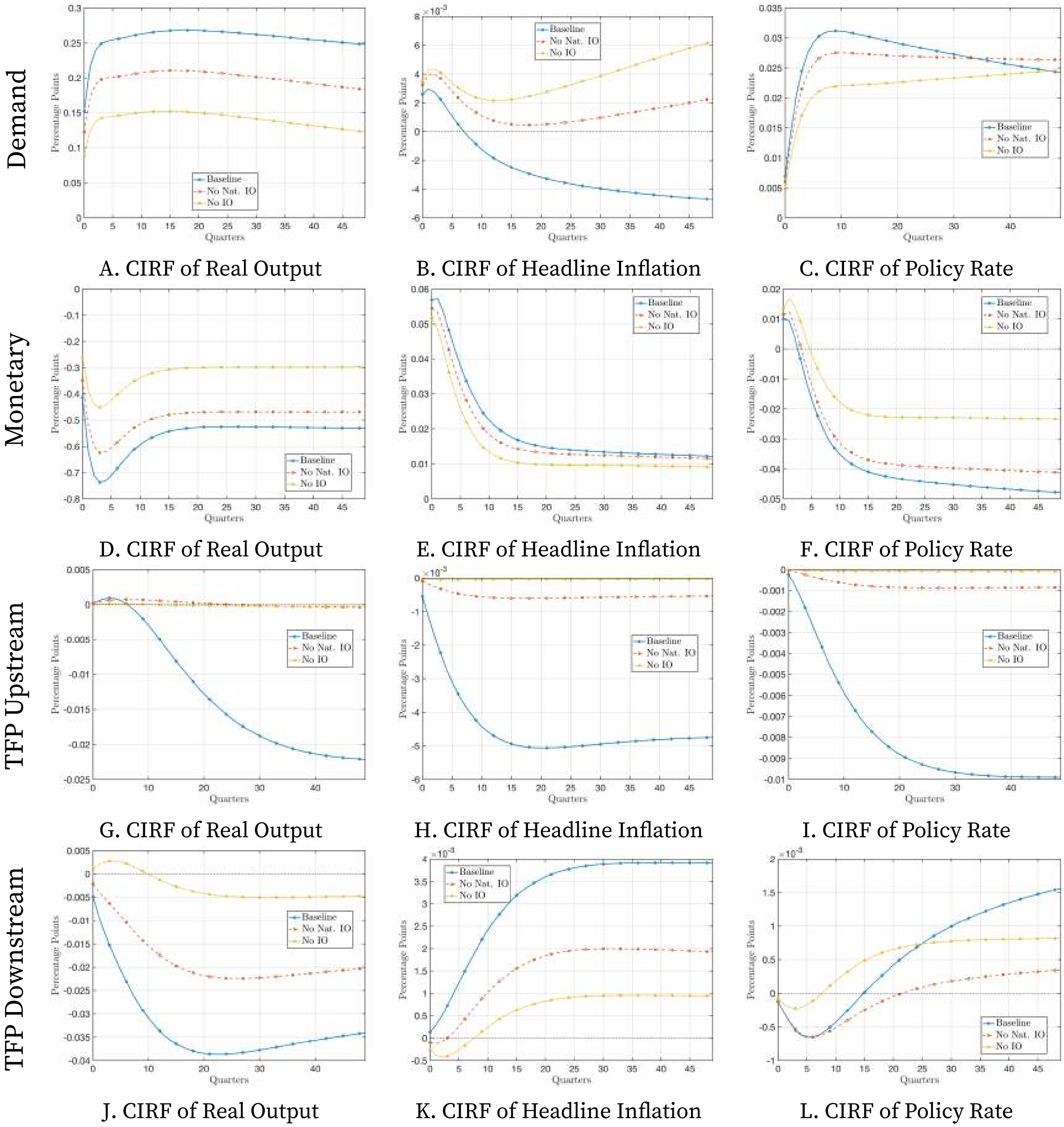
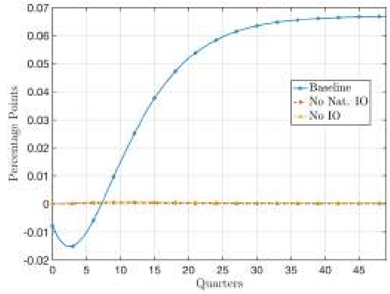


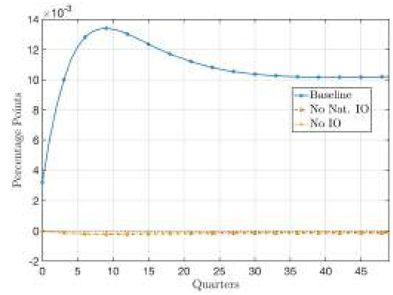
FIGURE A1. Macroeconomic Dynamics after Foreign Shocks

Notes: CIRFs of real output (first column), headline inflation (second column), and policy rate (third column) in the Euro Area after a foreign demand shock (first row), foreign monetary policy shock (second row), foreign TFP shock in upstream sector (third row), foreign TFP shock in downstream sector (fourth row), foreign price cost-push shock in upstream sector (first row, next page), foreign price cost-push shock in downstream sector (second row, next page), and foreign wage cost-push shock (third row, next page).

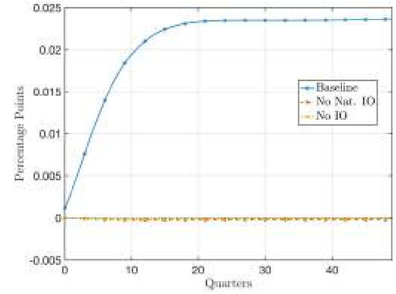
Price Upstream



M. CIRF of Real Output

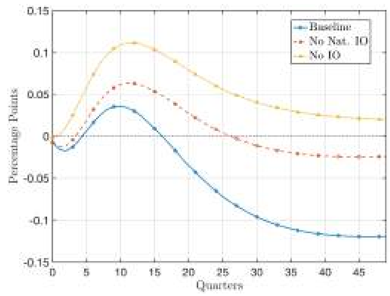


N. CIRF of Headline Inflation

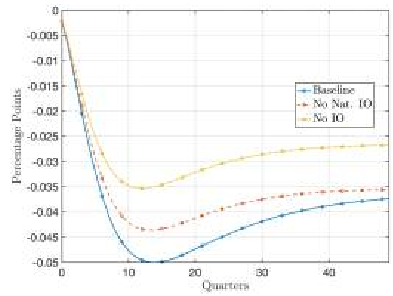


O. CIRF of Policy Rate

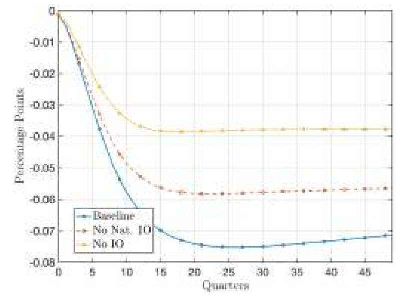
Price Downstream



P. CIRF of Real Output

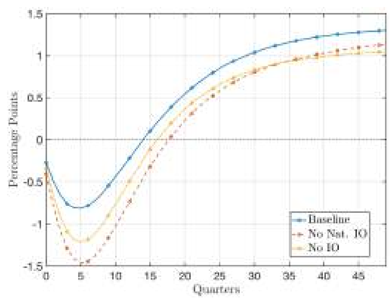


Q. CIRF of Headline Inflation

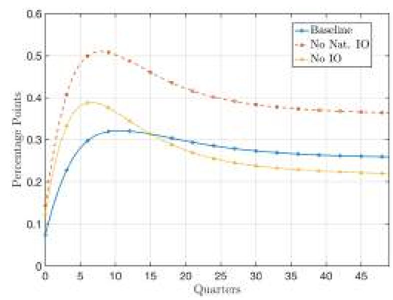


R. CIRF of Policy Rate

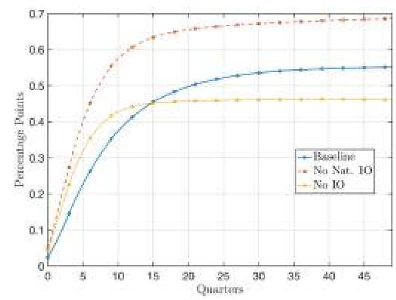
Wage



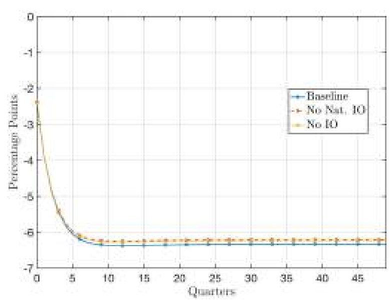
S. CIRF of Real Output



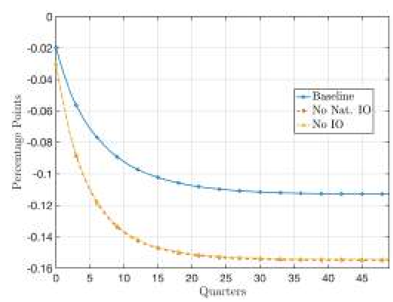
T. CIRF of Headline Inflation



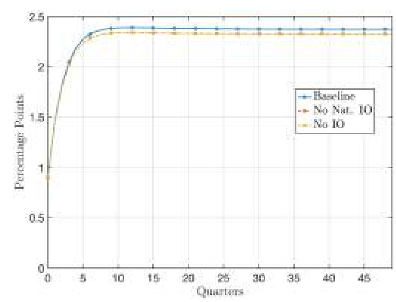
U. CIRF of Policy Rate



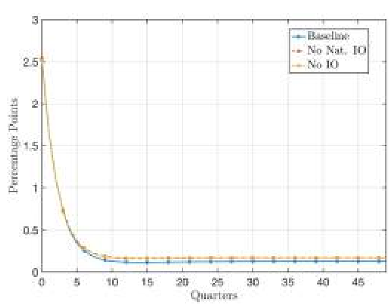
V. CIRF of Real Output



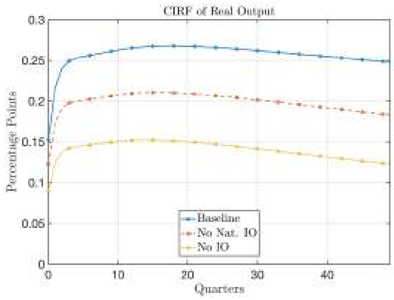
W. CIRF of Headline Inflation



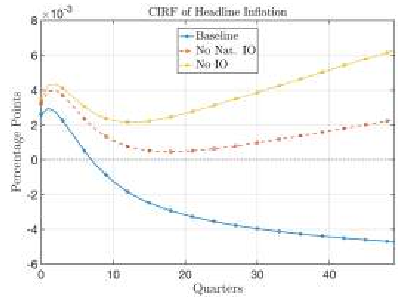
X. CIRF of Policy Rate



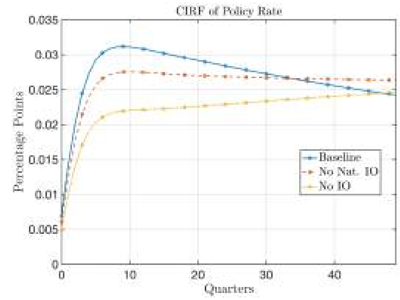
Y. CIRF of Real Output



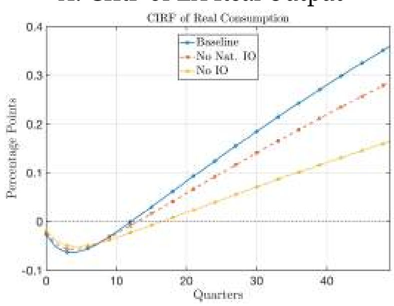
A. CIRF of EA Real Output



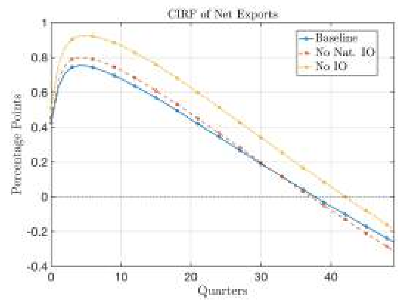
B. CIRF of EA Headline Inflation



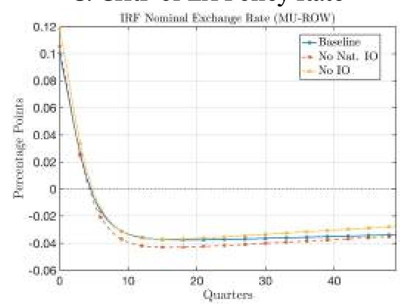
C. CIRF of EA Policy Rate



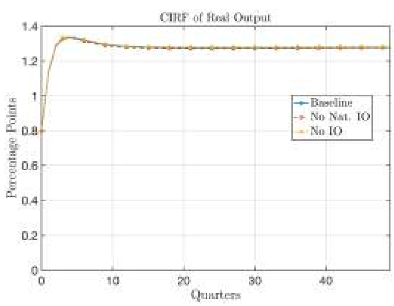
D. CIRF of EA Consumption



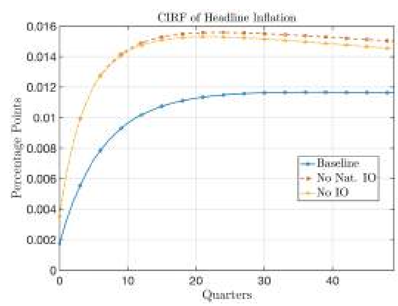
E. IRF of EA Net Exports



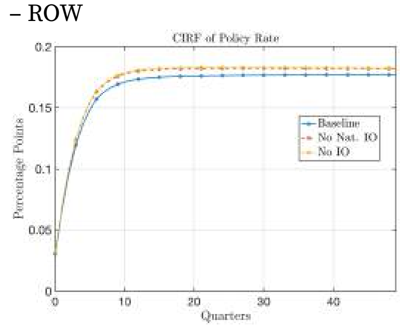
F. IRF of Nominal Exchange Rate EA – ROW



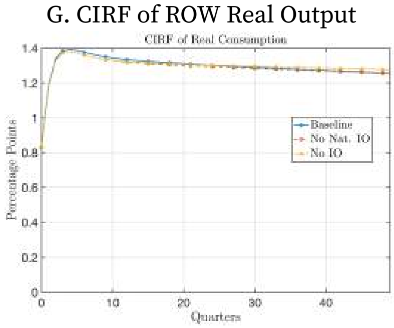
G. CIRF of ROW Real Output



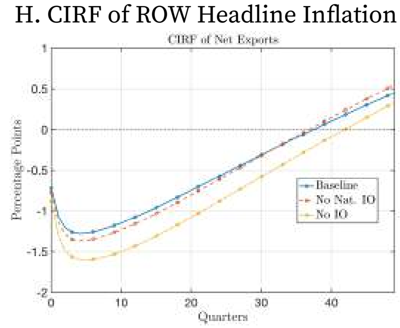
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



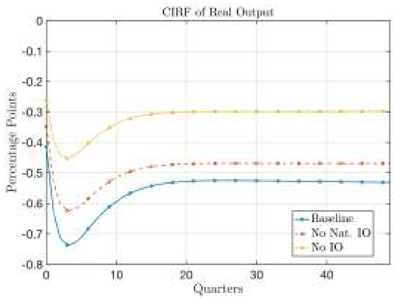
J. CIRF of ROW Consumption



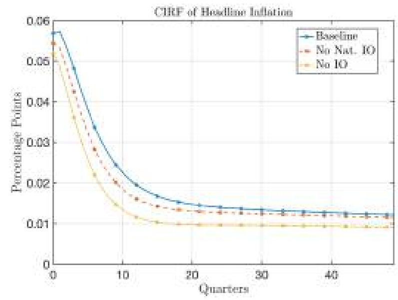
K. CIRF of ROW Net Exports

FIGURE A2. Effects of a Foreign Demand Shock on Euro-Area Variables

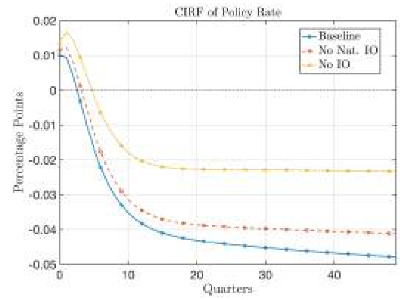
Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive demand shock in ROW.



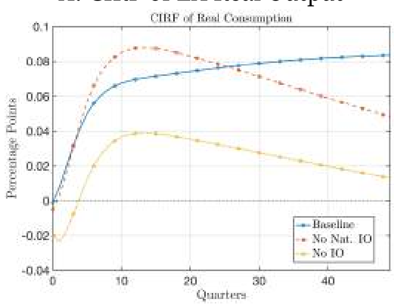
A. CIRF of EA Real Output



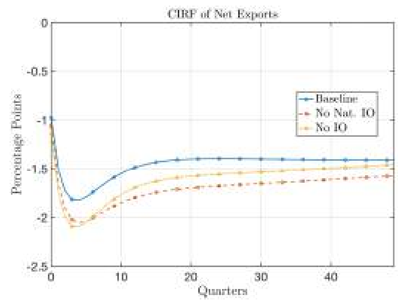
B. CIRF of EA Headline Inflation



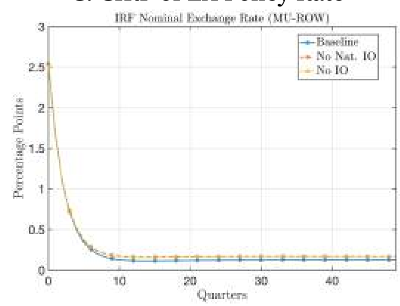
C. CIRF of EA Policy Rate



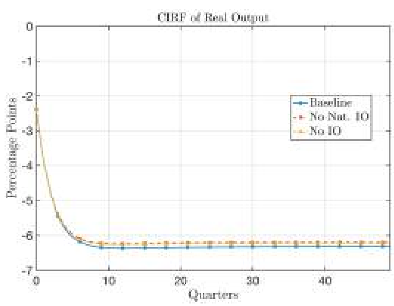
D. CIRF of EA Consumption



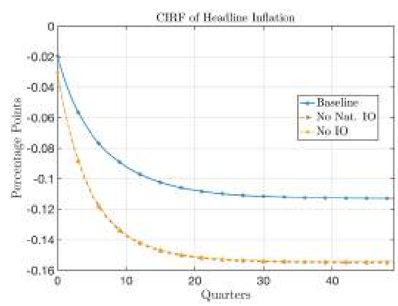
E. IRF of EA Net Exports



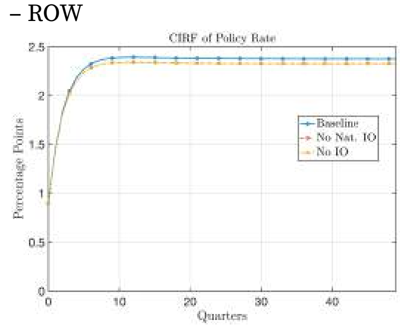
F. IRF of Nominal Exchange Rate EA – ROW



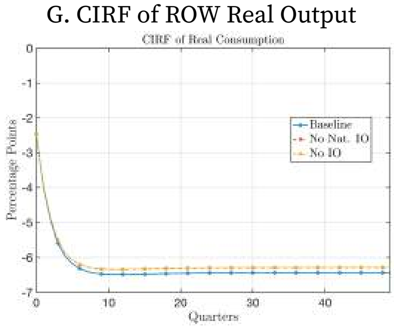
G. CIRF of ROW Real Output



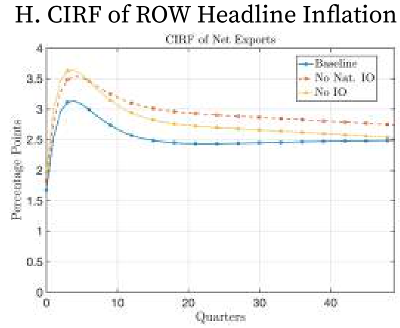
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



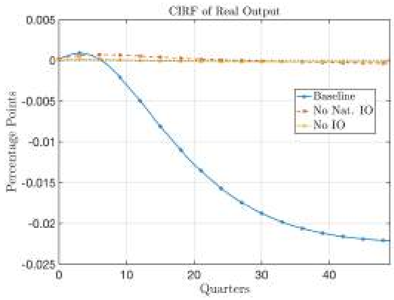
J. CIRF of ROW Consumption



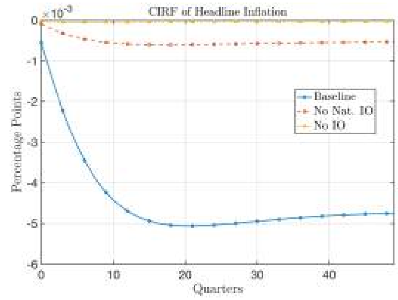
K. CIRF of ROW Net Exports

FIGURE A3. Effects of a Foreign Monetary Policy Shock on Euro-Area Variables

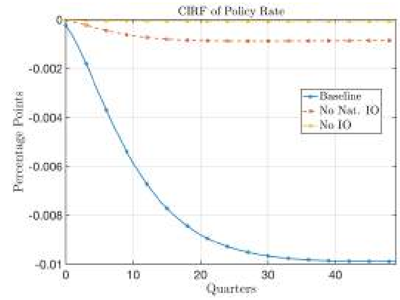
Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a contractionary monetary policy shock in ROW.



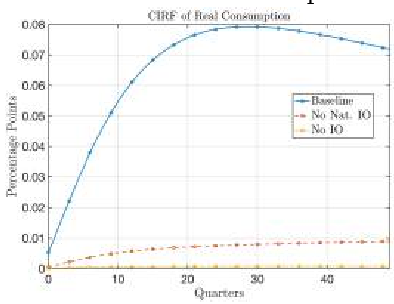
A. CIRF of EA Real Output



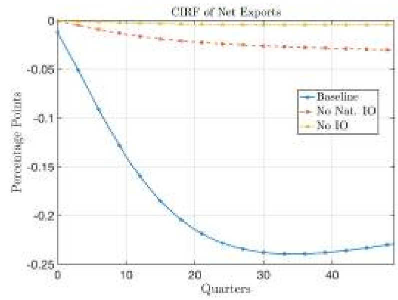
B. CIRF of EA Headline Inflation



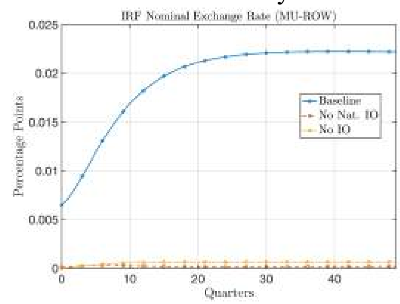
C. CIRF of EA Policy Rate



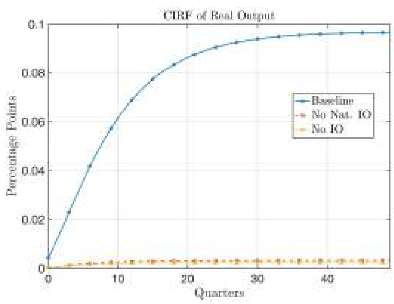
D. CIRF of EA Consumption



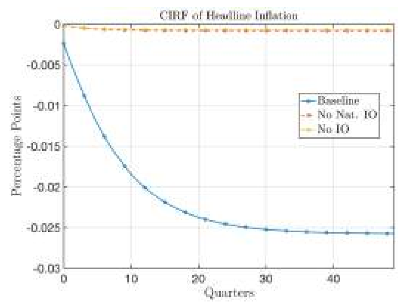
E. IRF of EA Net Exports



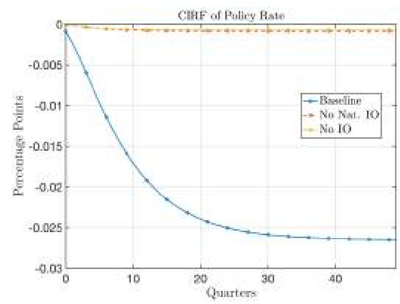
F. IRF of Nominal Exchange Rate EA – ROW



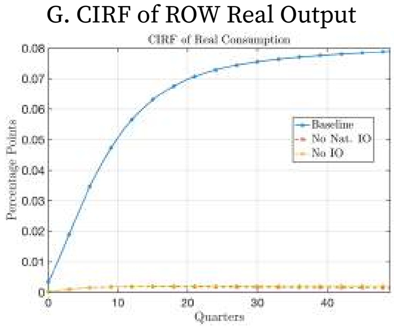
G. CIRF of ROW Real Output



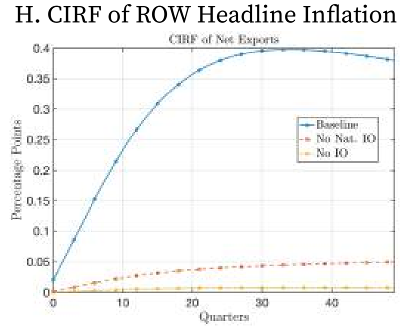
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



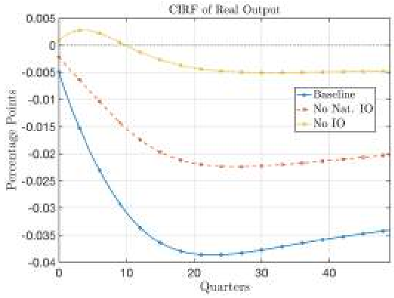
J. CIRF of ROW Consumption



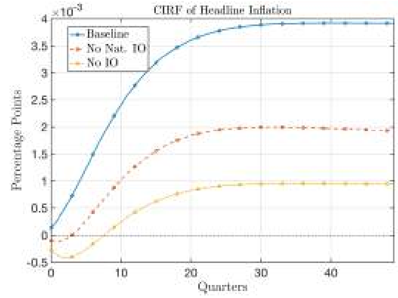
K. CIRF of ROW Net Exports

FIGURE A4. Effects of a Foreign TFP Shock (Upstream) on Euro-Area Variables

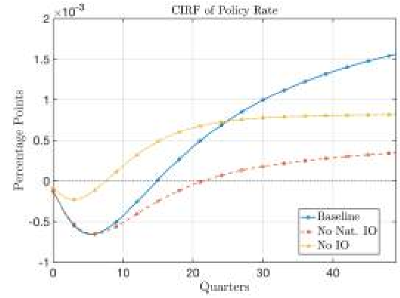
Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive TFP shock (most upstream sector) in ROW.



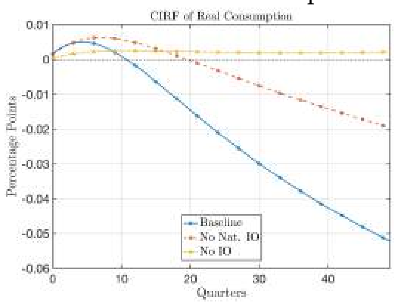
A. CIRF of EA Real Output



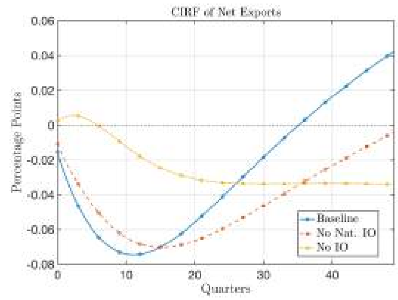
B. CIRF of EA Headline Inflation



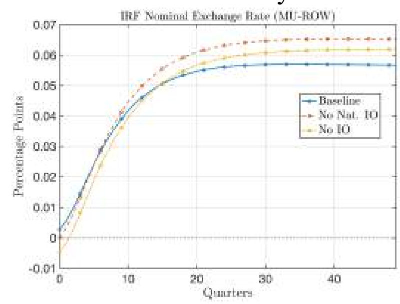
C. CIRF of EA Policy Rate



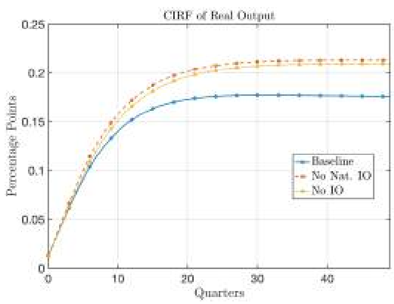
D. CIRF of EA Consumption



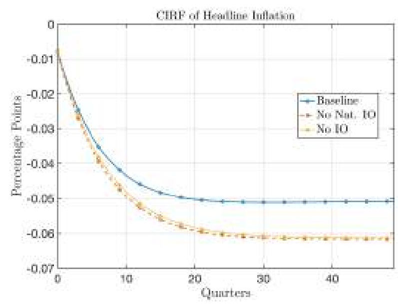
E. IRF of EA Net Exports



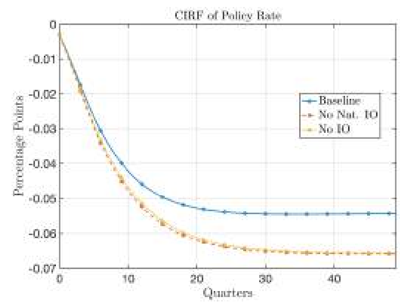
F. IRF of Nominal Exchange Rate EA – ROW



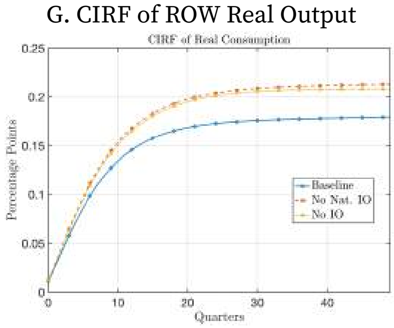
G. CIRF of ROW Real Output



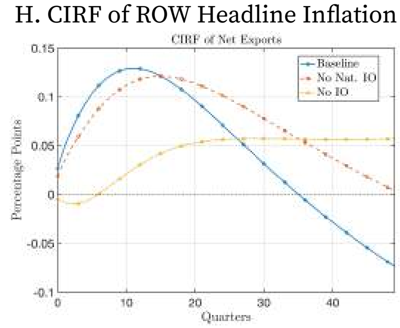
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



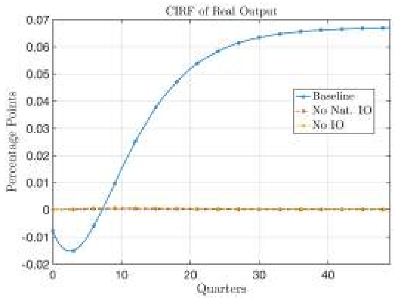
J. CIRF of ROW Consumption



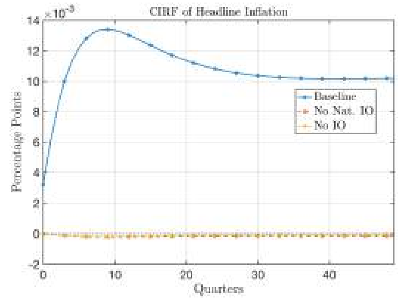
K. CIRF of ROW Net Exports

FIGURE A5. Effects of a Foreign TFP Shock (Downstream) on Euro-Area Variables

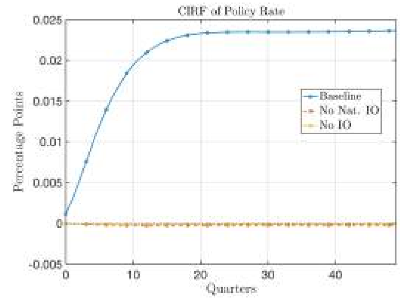
Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive TFP shock (most downstream sector) in ROW.



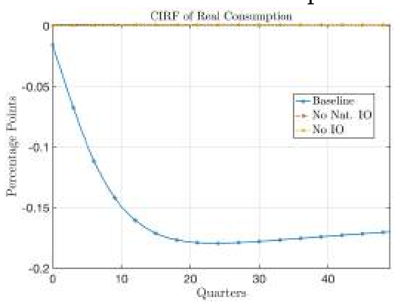
A. CIRF of EA Real Output



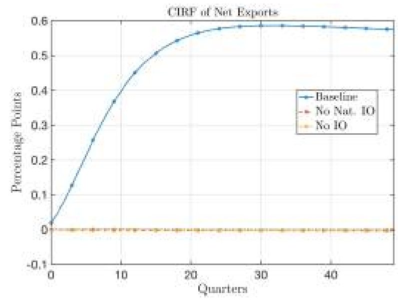
B. CIRF of EA Headline Inflation



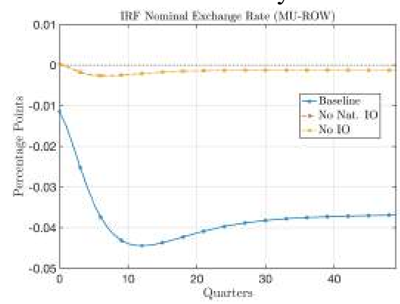
C. CIRF of EA Policy Rate



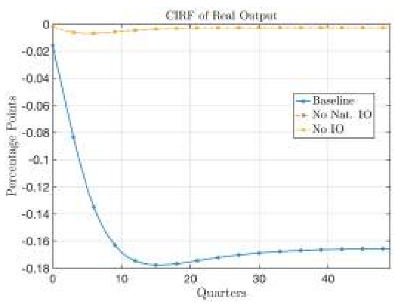
D. CIRF of EA Consumption



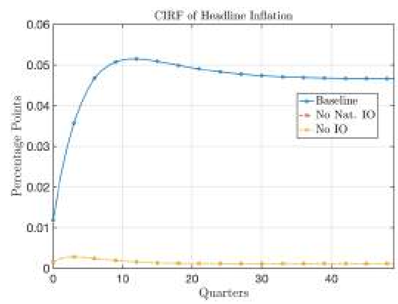
E. IRF of EA Net Exports



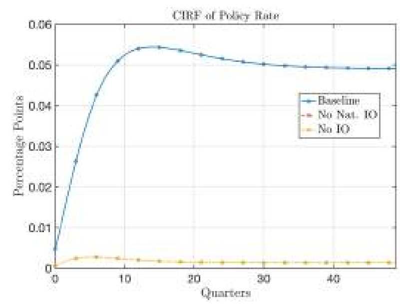
F. IRF of Nominal Exchange Rate EA – ROW



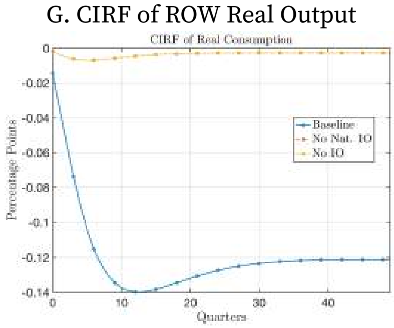
G. CIRF of ROW Real Output



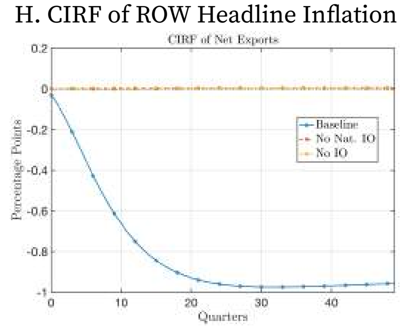
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



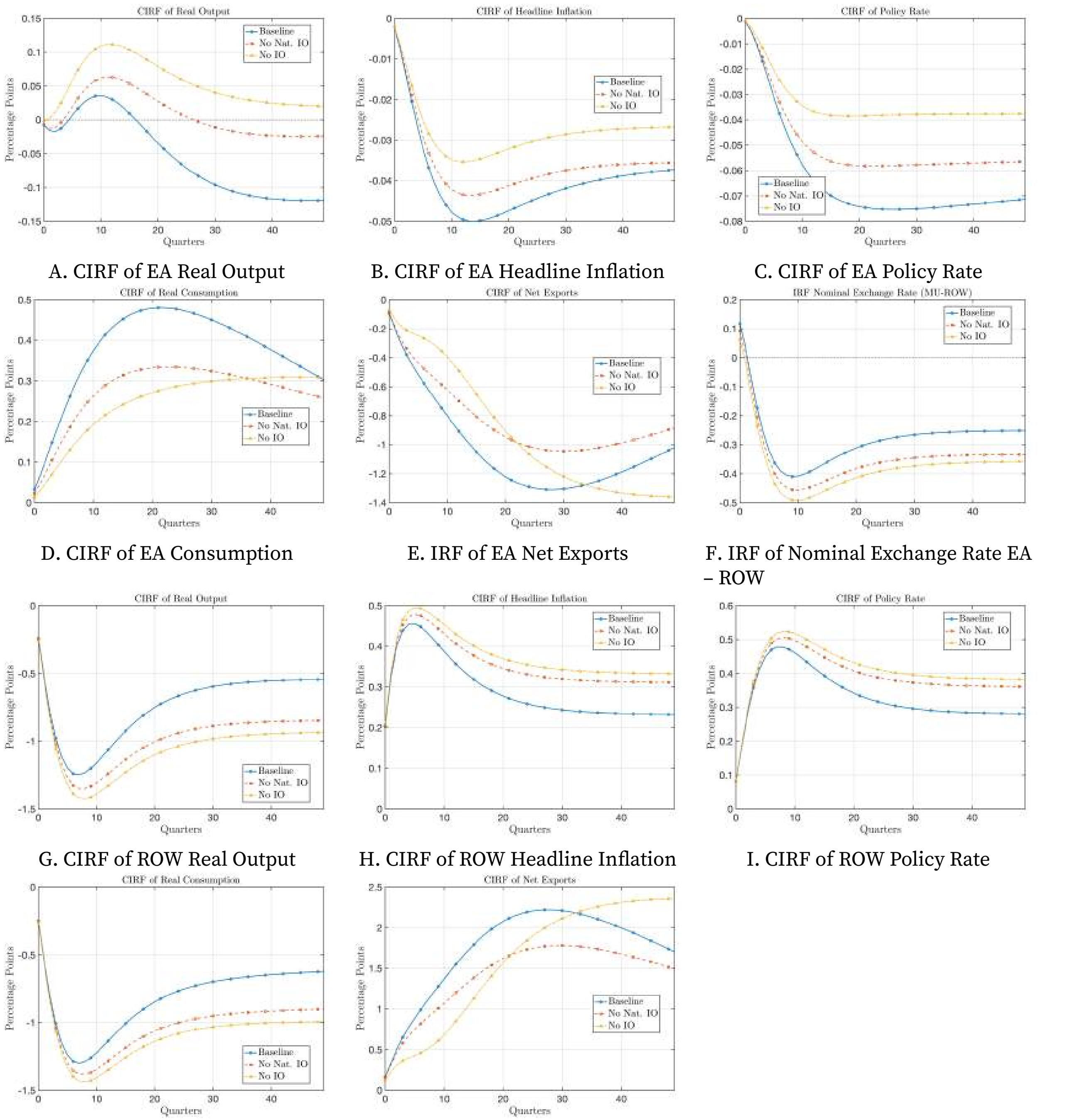
J. CIRF of ROW Consumption



K. CIRF of ROW Net Exports

FIGURE A6. Effects of a Foreign Price Cost-Push Shock (Upstream) on Euro-Area Variables

Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive price cost-push shock (most upstream sector) in ROW.



A. CIRF of EA Real Output

B. CIRF of EA Headline Inflation

C. CIRF of EA Policy Rate

D. CIRF of EA Consumption

E. IRF of EA Net Exports

F. IRF of Nominal Exchange Rate EA – ROW

G. CIRF of ROW Real Output

H. CIRF of ROW Headline Inflation

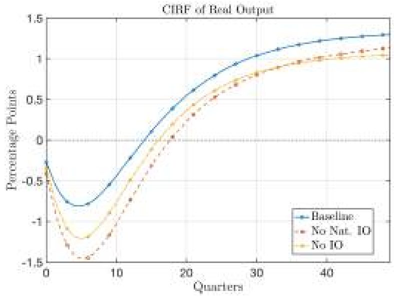
I. CIRF of ROW Policy Rate

J. CIRF of ROW Consumption

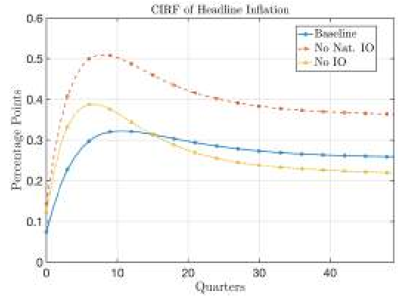
K. CIRF of ROW Net Exports

FIGURE A7. Effects of a Foreign Price Cost-Push Shock (Downstream) on Euro-Area Variables

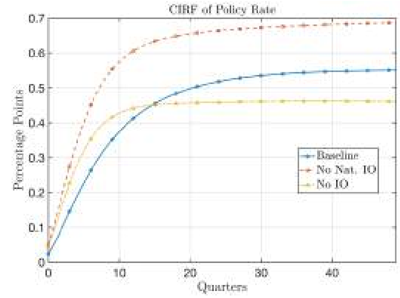
Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive price cost-push shock (most downstream sector) in ROW.



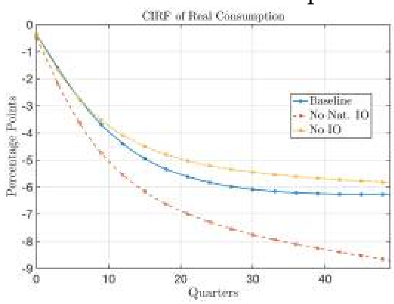
A. CIRF of EA Real Output



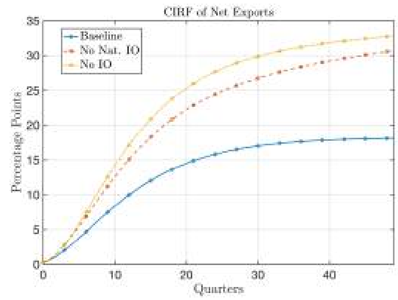
B. CIRF of EA Headline Inflation



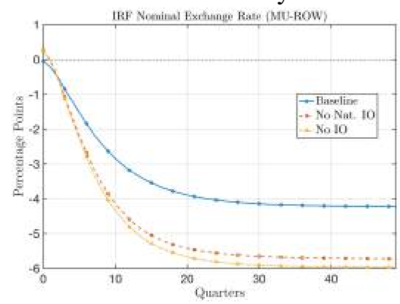
C. CIRF of EA Policy Rate



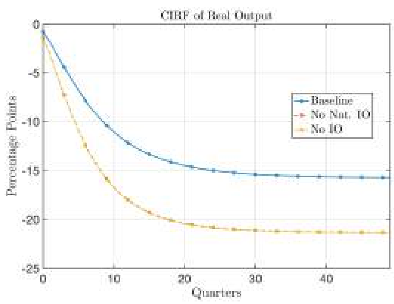
D. CIRF of EA Consumption



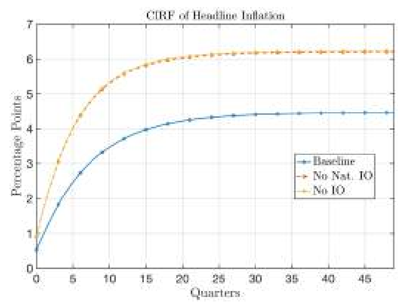
E. IRF of EA Net Exports



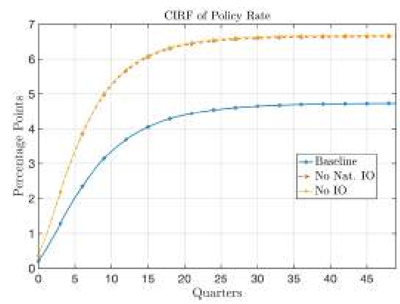
F. IRF of Nominal Exchange Rate EA – ROW



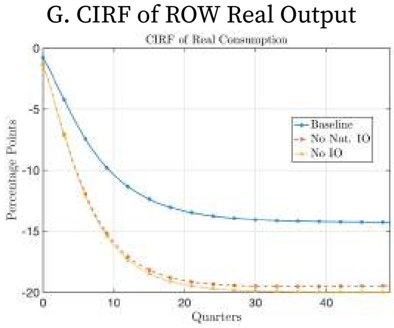
G. CIRF of ROW Real Output



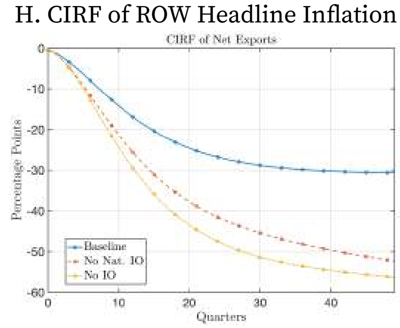
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



J. CIRF of ROW Consumption



K. CIRF of ROW Net Exports

FIGURE A8. Effects of a Foreign Wage Cost-Push Shock on Euro-Area Variables

Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive wage cost-push shock in ROW.