Entry-encouraging vertical integration.*

Ramon Fauli-Oller (FAE. Universidad de Alicante)†
and Joel Sandonís (FAE. Universidad de Alicante)‡

January 17, 2025

Abstract

It is well known that a vertically integrated firm may have an incentive to raise the rivals' costs by increasing the wholesale prices it charges for the input. In this paper, we show that it is precisely this incentive to raise the rivals' costs that explains why vertical integration may have the additional effect of encouraging entry into a downstream industry, as the entrant will face more inefficient rivals. We consider a seting with an upstream firm that sets observable two-part tariff contracts to downstream firms. The downstream sector is made up of two incumbent firms and one potential entrant. Competition downstream is à la Cournot. Downstream firms can source the input also form an alternative, less efficient supply. We characterize the conditions under which entry-encouraging vertical integration is profitable and occurs in equilibrium in our setting, and show that it tends to be welfare enhancing. We also characterize a region in which vertical integration reduces the entrant's profits and can deter entry, and show that these mergers are always detrimental to welfare.

JEL Classification: L22;L40;L42

Keywords: entry-encouraging vertical integration, raising rivals' costs, two-part tariffs.

*Financial support from grant PID2022-142356NB-I00 financed by MICIU/AEI /10.13039/501100011033 and FEDER, UE and from Generalitat Valenciana (Research Project Groups 3/086, Prometeo/2021/073) is gratefully acknowledged. We thank Lluís Bru, Luis C. Corchón, Miguel González-Maestre, Ekaterina Kazakova, Gerard Llobet and Massimo Motta and the audiences at the 2024 PET Conference in Lyon, the 2024 EARIE Congress in Amsterdam, the 2024 Jornadas de Economía Industrial in Sevilla and the 2024 ASSET Meeting in Venice for helpful comments.

† Universidad de Alicante. Fundamentos del Análisis Económico (FAE). Campus de Sant Vicent del Raspeig, E-03071, Alicante, Spain. E-mail address: fauli@ua.es

‡Corresponding author: Universidad de Alicante. Fundamentos del Análisis Económico (FAE). Campus de Sant Vicent del Raspeig, E-03071, Alicante, Spain. E-mail address: sandonis@ua.es

1 Introduction

It is well known in the literature on vertical integration and market foreclosure, that a vertically integrated firm may have an incentive to raise the rivals' costs by increasing the wholesale prices it charges for the input, which has a negative impact on competition. In this paper, we show that it is precisely this incentive of the integrated firm to raise the rivals's costs which explains why vertical integration may have the additional effect of encouraging downstream entry, which enhances market competition. The intuition is straightforward: on the one hand, by raising the rivals' costs, the integrated firm allows the entrant to face more inefficient (independent) downstream competitors. On the other hand, vertical integration eliminates double marginalization, which reduces the integrated firm's marginal cost and forces the entrant to face a more efficient rival. We show in the paper that vertical integration increases the entrant's post-entry profits when the former effect dominates the latter, which would explain the existence of entry-encouraging vertical integration.

We consider a setting with an upstream firm that produces an input at no cost and offers observable take-it or leave-it two-part tariff supply contracts to downstream firms. The downstream sector is made up of two incumbent firms producing differentiated goods and one potential entrant. Competition downstream is à la Cournot.[1] Downstream firms have the possibility to source the input from a less efficient source of supply at cost c. Under take-it or leave-it two-part tariff contracts, a vertically integrated firm is able to bind the participation constraint of downstream firms and then, upon entry, the entrant's profits amount to the profits it can obtain when sourcing the input from the alternative supply (its outside option) which depend on (i) the cost of the alternative supply, which is exogenous and unaffected by vertical integration; (ii) the wholesale price faced by the independent incumbent firm, which increases with vertical integration, as the integrated firm aims to protect its market profits[2] and (iii) the cost of the integrated firm, which decreases with vertical integration due to the elimination of double marginalization. Therefore, the effect of vertical integration on the entrant's post-entry profits is ambiguous. We find a threshold value for c (the efficiency of the alternative supply) such that the entrant's post-entry profits increase for a sufficiently efficient alternative supply (low values of c). In this region, and for the appropriate range of entry costs, we find entry-encouraging vertical integration.

This result must be compared with the one found in the seminal paper by Hunold and Schad (2023). These authors are the first to study the theoretical connection between vertical integration and entry in a downstream industry. They obtain a clear-cut result: vertical integration never increases the entrant's profits.[3] The reason is that they consider a setting with only one incumbent

[1] We extend the model to Bertrand competition in Section 7.

[2] Note that the integrated firm obtains revenues not only from input sales, but also from selling the final good in the market. Therefore, it is interested in controlling the level of market competition and charges higher wholesale prices.

[3] This result holds under Bertrand downstream competition. Under Cournot competition,

firm and one potential entrant. In this scenario, the entrant's profits decrease with vertical integration because, upon entry, it faces a more aggressive rival, as vertical integration eliminates double marginalization. Therefore, when comparing the two settings, an important conclusion that we derive is that, when investigating the effect of vertical integration on entry, the number of downstream firms is a crucial aspect to take into consideration. In fact, we show that our result that vertical integration can increase the entrant's post-entry profits is robust to the general case of n-1 downstream firms plus one entrant.

Once we identify the region of parameters where vertical integration increases the entrant's post-entry profits, we are particularly interested in the case in which entry occurs under vertical integration and not under vertical separation, which we call entry-encouraging vertical integration. And this is the case only for an appropriate range of entry cost values. The next step in the analysis is then to study the profitability of entry-encouraging vertical integration, to asses when a vertical merger that induces entry does occur in equilibrium. We must compare the profits of the integrated firm with entry (so that we have three downstream firms) with the sum of their profits without integration, when there is no entry (and so we have only two downstream firms). We show that entry-encouraging vertical integration is profitable when the alternative supply is sufficiently inefficient and the goods are sufficiently differentiated. The intuition is the following: pushing entry through vertical integration can be profitable for the integrated firm when the positive effect of creating a new market (the market expansion effect), which increases with the degree of product differentiation, compensates for the negative competition effect (which decreases when the alternative supply is more inefficient). In order for entry-encouraging vertical integration to occur in equilibrium, the alternative supply must be sufficiently efficient to allow for vertical integration to increase the entrant's profits and sufficiently inefficient to make vertical integration profitable. And this interval is shown to be non-empty regardless of the degree of product differentiation. With respect to the welfare consequences of entry-encouraging vertical integration, for the admissible set of entry cost values, we find a large region of parameters in which it is welfare enhancing. There are two positive effects on welfare, namely, the market expansion effect of the introduction of a new differentiated good and the positive competition effect produced by entry; and two negative effects, the *raising rivals's cost* effect and the cost of entry. The market expansion effect is larger when the goods are more differentiated and the competition effect is larger when c is lower, that is, when the alternative supply is more efficient.

Besides analyzing the region in which vertical integration increases the entrant's profits, we also analyze the region in which it reduces the entrant's profits, which occurs when the alternative supply is sufficiently inefficient. In this case, we are interested in the region of entry cost values for which entry occurs only under vertical separation (and not under vertical integration). Studying

it holds as long as below-cost input pricing is not allowed, as we assume also in the present paper.

this region allows us to compare our results with those in Hunold and Schad (2023). In particular, we show that the region where entry-deterrent vertical integration occurs in equilibrium in our setting is much smaller than in Hunold and Schad (2023). The reason is two-fold: on the one hand, the region where vertical integration reduces the entrant's profits is smaller in our setting, because adding a second incumbent downstream adds a positive effect of vertical integration on the entrant's profits, which is absent in their setting. On the other hand, the region where entry-deterrent vertical integration is profitable is also smaller in our setting because, whereas in Hunold and Schad (2023) entry-deterrent vertical integration leads from a duopoly to a monopoly downstream, in our setting, it leads from a triopoly to a duopoly. Therefore, our result suggests that more intense downstream competition can alleviate the problem of market foreclosure (and its negative impact on social welfare) through entry-deterrent vertical integration.[4] In other words, as long as we have more competitive downstream markets, competition authorities should not be too concerned about the potential negative effect that vertical integration may have on entry. This idea is partially supported for example by Hortaçsu and Syverson (2007). In their empirical paper, the authors regress the entry rate with respect to the market share of vertically integrated firms in the cement and concrete (vertically related) markets. They obtain a positive but not significant coefficient and summarize their findings saying that: "There is little evidence that foreclosure is quantitatively important in these industries... and entry rates do not fall when markets become more (vertically) integrated".

The main result of the present paper about the possibility that vertical integration increases the entrant's profits and may induce entry in the downstream market, also holds when we extend the benchmark model to (i) vertical mergers involving more than one downstream firm (ii) secret contracts and (iii) Bertrand downstream competition. Theses extensions are analyzed in Sections 5, 6 and 7 respectively.

Concerning the related literature on vertical integration, since the single monopoly profit theory of the Chicago School there has been a long debate about the possible anticompetitive effects of vertical integration. The main ingredient of the discussion is whether the vertically integrated firm, trying to increase its profits, will exclude some of the downstream rivals from the market by denying them access to an essential input.

The Chicago School proponents defended that, in the absence of efficiency gains, vertical integration would be competitively neutral (e.g., Bork, 1978; Posner, 1979). The idea of neutrality of vertical integration can be supported by a benchmark model with an upstream monopolist supplying an essential input via take-it or leave-it observable two-part tariff supply contracts to two downstream

[4]Regarding the welfare consequences of entry-deterrent vertical integration, we obtain a clear-cut result (similar to Hunold and Schad (2023). For any admissible value of the entry-cost, the effect of entry-deterrent vertical integration on social welfare is negative. The entry cost saving is not enough to compensate for three negative effects, namely, a negative market expansion effect (elimination of one good), a negative competition effect (elimination of one firm) and the *raising rivals' costs* effect.

firms producing homogeneous goods. In this setting, both the integrated and unintegrated structures lead to full monopolization. More sophisticated (game theory based) models have studied some potential anticompetitive effects of vertical integration. For example, Sandonís and Fauli-Oller (2006) relax two of the assumptions of the benchmark model, namely, they introduce an alternative less efficient supply of the input upstream and product differentiation downstream. They obtain that the integrated firm does not foreclose the rival but sells the input to the latter firm at a higher price, following a *raising rivals' cost strategy.*[5] There exist a bunch of other theoretical papers that have also studied anticompetitive effects of vertical integration under other specific conditions, such as additional commitment power of the integrated firm (Ordover et al.,1990), secret contract offers (Hart and Tirole, 1990), exclusive dealing contracts (Chen and Riordan, 2007), upstream collusion (Normann, 2009) or the existence of a cost of switching suppliers (Chen, 2001).

Finally, our paper is very much related to a new line of research, opened very recently by Hunold and Schad (2023), that studies another potential anticompetitive effect of vertical integration that was neglected so far, namely, the potential negative effect of vertical integration on downstream entry.

The rest of the paper is organized as follows. The next section describes the model. The case with two incumbent downstream firms and one entrant is addressed in Section 3, Section 4 analyzes the case with a general number of downstream firms and Section 5 vertical mergers with two downstream firms. Section 6 extends the model to secret contracts. The Bertrand model is studied in Section 7. And a conclusions section puts the paper to an end. All the proofs are relegated to the Appendix.

2 The model

We have an upstream firm called Firm U producing an essential input at zero cost. There exist n downstream firms, denoted with a natural number from 1 to n, that transform this input into a final output on a one-to-one basis at no cost. The outputs sold by downstream firms are symmetrically differentiated. Firm i sells good i. The inverse demand function of good i is given by:

$$p_i = 1 - q_i - \gamma \sum_{j \neq i} q_j, i = 1, ...n,$$

where q_i is the quantity sold of good i and $0 \leq \gamma \leq 1$ is an inverse measure of product differentiation. The first $n-1$ firms are incumbents and firm n is a potential entrant. There also exists an alternative, less efficient supply of the input at cost c, with $0 < c < 1$.

[5] Other papers in the literature that have explored rising rivals' cost strategies associated to vertical integration are, for example, Salop and Scheffman (1983), Krattenmaker and Salop (1986), Salinger (1988), Hart and Tirole (1990), Ordover, Saloner and Salop (1990) and Chen (2001),

The timing of the game is like in Hunold and Schad (2023) and evolves as follows:

1. Firm U and firm 1 decide whether to merge or stay separated.
2. Firm n decides whether to enter the market at a fixed cost $\theta > 0$.
3. Firm U offers non-discriminatory[6] take-it or leave-it two-part tariff supply contracts to downstream firms including a fixed fee $F \geq 0$ and a wholesale price of $w \geq 0$.
4. Each independent downstream firm decides whether to accept or reject the proposed contract.
5. After observing the acceptance and rejection decisions, all active downstream firms compete à la Cournot.

A comment on the restrictions we set on the parameters conforming the two-part tariff supply contracts is warranted. Non-negative fees are assumed because negative fees would likely be considered anticompetitive. Non-negative wholesale prices (or more precisely a ban on below-cost pricing) is not imposed in Hunold and Schad (2023) but they point out "[p]rices below marginal cost may be considered anti-competitive and might be prohibited" (p.10). Apart from this antitrust argument, we introduce the assumption to better differentiate our results from the ones in Hunold and Schad (2023). With the non-negative wholesale price assumption, they would obtain that vertical integration never increases the entrant's post-entry profits. We obtain instead that even restricting ourselves to non-negative wholesale prices, there always exist a region of parameters where vertical integration increases the entrant's profits.

We solve the model by backward induction. In the last stage, firm i has constant marginal cost c_i, where $c_i = w$ if firm i has accepted the supply contract offered by U and $c_i = c$ if it has not accepted the contract offer and sources the input from the alternative supply. The (interior) equilibrium output and profit of firm i are given respectively by:

$$\begin{aligned} q(c_i, C_{-i}, n) &= \frac{2-\gamma+\gamma C_{-i} - c_i(2+\gamma(-2+n))}{(2-\gamma)(2+\gamma(n-1))} \\ \Pi(c_i, C_{-i}, n) &= [q_i(c_i, C_{-i}, n)]^2 \end{aligned} \tag{1}$$

where $C_{-i} = \sum_{j \neq i} c_j$. If firm n does not enter the market, we only have $n-1$ active firms, but (1) still applies.

We next proceed to solve the model for the case $n = 3$, to emphasize the important differences with respect to the case $n = 2$, solved in Hunold and Schad (2023), and will solve the general case with n firms in Section 4.

3 The case $n = 3$

We first solve the third stage, where the upstream firm decides the two-part tariff supply contract, taking into account that in the fourth stage, downstream

[6] In practice, we see that firms very often offer uniform contracts to their customers (see for example Lafontaine and Slade,1997, pp.15-16.).

firms will accept only contracts guaranteeing them no less than their outside option, namely, the profits they would get if they reject the contract and source the input from the alternative supply. We have to consider two different cases. On the one hand, an scenario in which the entrant (firm 3) has not entered the downstream market. This case has been already analyzed in Sandonis and Fauli-Oller (2006) and Hunold and Schad (2023). On the other hand, an scenario in which there has been entry, that we analyze below.

With vertical separation, taking into account that the upstream firm optimally sells the input to all downstream firms (see Fauli-Oller et al., 2013) and that the participation constraint of the downstream firms is binding, the upstream firm U maximizes:

$$\begin{aligned} & \underset{w}{Max}\{3\left(\Pi(w,2w,3)-\Pi(c,2w,3)+wq(w,2w,3)\right)\} \\ s.t.\ 0 & \leq \quad w \leq c \end{aligned}$$

The optimal two-part tariff contract[7] involves a wholesale price: $w_S^* = \begin{cases} \frac{\gamma((-2+\gamma)\gamma+2c(2+\gamma))}{4+\gamma(4+\gamma(-3+2\gamma))} \text{if } c > \frac{(2-\gamma)\gamma}{2(2+\gamma)} = c_S^C \\ 0 \quad \text{otherwise} \end{cases}$ and a fixed fee $F_S^* = \Pi(w_S^*, 2w_S^*, 3) - \Pi(c, 2w_S^*, 3)$. Observe that the outside option of the entrant is given by $\Pi(c, 2w_S^*, 3)$. In order to guarantee non-negative outputs, we need to impose the constraint $c \leq c_D^C = \frac{2-\gamma}{2+\gamma}$. This constraint guarantees that in the worst possible case for any firm (that is, facing two rivals with zero marginal costs when this firm has a marginal cost equal to c) its output is non-negative, namely, that $q(c,0,3) \geq 0$.[8]

With vertical integration, the integrated firm maximizes (taking into account that it optimally sells the input to all downstream firms (see Fauli-Oller et al., 2013) and that the participation constraint of the downstream firms is binding):

$$\begin{aligned} & \underset{w}{Max}\{\Pi(0,2w,3)+2(\Pi(w,w,3)-\Pi(c,w,3)+wq(w,w,3))\} \\ s.t.\ 0 & \leq \quad w \leq c \end{aligned}$$

We are implicitly assuming that the integrated firm cannot credibly commit to change its true marginal cost, which implies that it has to produce with a marginal cost equal to zero. This is a common assumption in the market foreclosure literature (Caprice, 2006, Chen, 2001, Hart and Tirole, 1990, Hunold and Stahl, 2016, Hunold and Schad, 2023, Normann, 2009, Reisinger and Tarantino, 2015, Rey and Tirole, 2007, Sandonis and Fauli-Oller, 2006).

The optimal two-part tariff contract involves a wholesale price: $w_I^* = \begin{cases} \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)} \text{if } c > \frac{(2-\gamma)\gamma}{4+6\gamma} = c_I^C \\ c \quad \text{otherwise} \end{cases}$ and a fixed fee $F_S^* = \Pi(w_I^*, w_I^*, 3) - \Pi(c, w_I^*, 3)$. Observe that the outside option of the entrant is given by $\Pi(c, w_I^*, 3)$.

[7] Notice that the upper bound of the constraint is due to the fact that we do not allow for negatives fees in the contract.

[8] This constraint also guarantees that outputs are never negative with vertical integration, which we analyze below.

In the second stage, the entrant will enter if it anticipates that its post-entry profits exceed the entry cost θ. These profits depend on whether vertical integration has occurred in the first stage of the game. Therefore, we have to compare the entrant's post-entry profits (its outside option) in both situations, which is formalized in the following proposition:

Proposition 1 *A threshold value $c_1^C > 0$ always exists such that a vertical merger increases the entrant's profits iff $c < \min\{c_1^C, c_D^C\}$, and decreases the entrant's profits otherwise, where c_1^C is defined in the Proof of Proposition 1 in the Appendix.*

Hunold and Schad (2023) obtain a qualitatively similar result in which vertical integration can improve the entrant's profits for sufficiently low values of c. However, their result holds only for *negative* wholesale prices (see their Lemma 4). In particular, when $n = 2$, the profits of the entrant amount to $\Pi(c, w_E^*, 2)$, whereas, with vertical integration, they are given by $\Pi(c, 0, 2)$. So the comparison of profits reduces to sign w_E^*. Then, under the assumption that the wholesale prices must be non-negative, $\Pi(c, w_E^*, 2) \geq \Pi(c, 0, 2)$ holds, with strict inequality if the wholesale price is positive.

Our contribution arises from the fact that, whereas in the case $n = 2$ vertical integration only has a negative effect on the entrant's profits (the fact that after vertical integration the entrant faces a more efficient rival), with $n = 3$ (and above), a new positive effect of vertical integration on the entrant's profits arises such that, even restricting ourselves to non-negative wholesale prices, we find a significant region of parameters where vertical integration increases the entrant's profits. To understand this new positive effect, note first that with $n = 3$, the entrant's equilibrium profits with vertical integration do depend (positively) on the wholesale price set by the integrated firm and, second, that this wholesale price is higher than the one set under vertical separation ($w_I^* > w_E^*$).[9] So the new (positive) effect is given by the fact that the entrant faces a more inefficient incumbent firm 2 with vertical integration than with vertical separation. And the size of this positive effect ($w_I^* - w_S^*$) increases as c decreases.[10] Joint with the fact that the size of the negative effect of vertical integration on the entrant‘s profits ($w_S^* - 0$) decreases as c decreases (in fact, for sufficiently low values of c, $w_S^* = 0$, and so the negative effect disappears), we obtain that for sufficiently low values of c (as Proposition 1 states), vertical integration increases the entrant's profits.

Comparing our threshold value c_1^C with $\widehat{c}$ (the counterpart for the case $n = 2$ in Hunold and Schad, 2023), we find that $\widehat{c} < c_1^C$, implying that the region in

[9] The reason is that the integrated firm directly participates in the final market and so it is interested in reducing competition to protect its markets profits. And the optimal way to do this is by raising the rivals' cost through a higher wholesale price.

[10] The reason is that even though both w_I^* and w_S^* decrease as c decreases (note that a lower c increases the outside option of downstream firms, which is optimally counteracted by the upstream firm via a lower wholesale price), the integrated firm reduces the wholesale price at a lower rate than the separated upstream firm as c decreases, because it needs to protect market profits, which calls for a higher w_I^*

which vertical integration reduces the entrant's profits shrinks when we move from two to three downstream firms. The reason is clear: with $n = 3$, the new positive effect that arises in the model counteracts the negative one, so that it is less likely that vertical integration reduces the entrant's profits. As a consequence, it seems that the possibility that vertical integration forecloses a rival by deterring entry is less of a concern when we increase the level of competition downstream. Note that vertical integration deters entry when its cost θ is higher than the entrant's profits with vertical integration and lower than its profits with vertical separation, namely, $\Pi(c, w_I^*, 3) < \theta < \Pi(c, 2w_S^*, 3)$. And the previous interval is non-empty, according to Proposition 1, only when $c > c_1^C$, that is, when vertical integration reduces the entrant's profits.

In what follows, we will divide the analysis into two separate subsections. In the first one, we will focus on the case in which a vertical merger reduces the entrant's profits ($c > c_1^C$) to determine the region in which entry-deterrent vertical integration is profitable and occurs in equilibrium, with the aim to compare the size of this region with the one in Hunold and Schad (2023). A welfare analysis of this kind of mergers closes this part. In the second subsection, we will study the case in which a vertical merger increases the entrant's profits ($c < c_1^C$) to determine also the region in which profitable entry-encouraging vertical integration occurs in equilibrium and its welfare consequences.

3.1 A vertical merger reduces the entrant's profits

The first step is to look at the profitability of entry-deterrent vertical integration, which is formalized in the following proposition.

Proposition 2 *Suppose that $\Pi(c, w_I^*, 3) < \theta < \Pi(c, 2w_S^*, 3)$, so that vertical integration deters entry. Then, vertical integration is profitable iff $c < \overline{c}^P$, where $\overline{c}^P$ is an increasing function of γ and is defined in the Proof of Proposition 2 in the Appendix.*

This result is akin to that in Proposition 4 in Hunold and Schad (2023). They also obtain that entry-deterrent vertical integration is profitable if c is sufficiently low ($c < \overline{c}_{Cournot}$). Furthermore, $\overline{c}_{Cournot}$ is also increasing in γ. And the intuition for the result is like the one they provide. Entry involves two opposite effects: a market expansion effect and a competition effect. The former reduces the profitability of entry-deterrent vertical integration and the latter increases it. Let us relate the size of both effects with the two relevant parameters of the model c and γ.

The lower is γ the greater the size of the market expansion effect induced by entry and the less the entrant will compete with the existing downstream firms. This becomes clear in the extreme cases of $\gamma = 0$ and $\gamma = 1$. In the first case, a completely new market is created through entry and no competition is generated, which reduces the incentives to deter entry. In the second case, entry does not produce any market expansion, but increases competition very much, because all downstream firms compete with a homogeneous good, which increases the incentives to deter entry.

Regarding parameter c, the lower is c, the more efficient the entrant would be when sourcing from the alternative supply and, therefore, the more the intensity of competition increases with entry. The higher is c, the lower the outside option of the entrant and the larger the fraction of the additional market profits produced by the introduction of a new good that the vertically integrated firm can appropriate, so that entry is less harmful for the integrated firm.

In order to add to the market foreclosure theory, we aim to identify cases where entry-deterrent vertical integration is an equilibrium outcome. This requires combining the results of the two previous propositions. Proposition 1 has informed that when c is sufficiently high, the entrant would obtain less profits under vertical integration than under vertical separation. So, in this case, there exist (intermediate) values of the entry cost such that entry only occurs under vertical separation and, therefore, vertical integration would result in market foreclosure. But this entry-deterrent vertical merger will only occur when it is profitable. Proposition 2 has shown that this is the case when c is sufficiently low. So entry-deterrent vertical integration will occur in equilibrium iff $c \in (c_1^C, \overline{c}^P]$, as the next proposition formally states:

Proposition 3 *Suppose that* $\Pi(c, w_I^*, 3) < \theta < \Pi(c, 2w_S^*, 3)$*, so that vertical integration deters entry. If* $\gamma \leq 0.81$*, vertical integration is unprofitable and never occurs in equilibrium. If* $\gamma \in (0.81, 0.93)$*, vertical integration is profitable and it occurs in equilibrium if* $c \in (c_1^C, \min\{\overline{c}^P, c_D^C\})]$.

Note first, that the interval $(c_1^C, \overline{c}^P]$ is non-empty only for sufficiently high values of γ: according to Proposition 3, for $\gamma \leq 0.81$, any vertical merger that reduces the entrant's profits is unprofitable. See Figure 1 below.

Place Figure 1 around here

Second, based on Propositions 4 and 5 in Hunold and Schad (2023), we can obtain an equivalence to Proposition 3 above for the case $n = 2$, where entry-deterrent vertical integration would occur (using Hunold and Schad's notation) if $c \in (\widehat{c}(\gamma), \overline{c}_{Cournot}(\gamma)]$, which is a non-empty set for all γ. The comparison of the results for $n = 2$ and $n = 3$ is clear-cut for $\gamma \leq 0.81$ because, as Proposition 3 shows, there will never be entry-deterrent vertical integration with $n = 3$. The same is true for $\gamma > 0.81$ because, in this case, $(c_1^C, \min\{\overline{c}^P, c_D^C\}] \subset (\widehat{c}(\gamma), \overline{c}_{Cournot}(\gamma)]$, which implies that for any value of c such that vertical integration deters entry for $n = 3$, it would do the same for $n = 2$. Figure 2 compares the regions where entry-deterrent vertical integration is profitable and takes place in equilibrium with $n = 2$ and $n = 3$. It makes clear that the region of profitable entry-deterrent vertical integration shrinks significantly as me move from $n = 2$ to $n = 3$.It is not only that the interval where entry-deterrent vertical integration occurs in equilibrium exists in our case only for very close substitutes ($\gamma > 0.81$). Furthermore, Figure 2 shows that our interval is also narrower than theirs regarding parameter c. On the one hand, (entry-deterrent) vertical integration is less profitable in our model, basically because it implies going from 3 to 2 firms in the downstream market, while in the case $n = 2$, it implies going from a duopoly to a monopoly. On the other hand, the region where vertical integration reduces the entrant's profits shrinks in the case $n = 3$ because, upon entry, the entrant faces a more inefficient downstream incumbent 2, which helps to increase its profits. Finally, note that for $\gamma > 0.93$, $c_D^C < c_1^C$ holds, which implies that any alternative supply satisfying the constraint $c < c_D^C$, prevents a vertical merger from reducing the entrant's profits.

Place Figure 2 around here

An important implication of the previous discussion is that the concern raised by Hunold and Schad (2023) on the potential market foreclosure consequences of a vertical merger is much less important when we add one more downstream incumbent.

We next turn to the analysis of the welfare consequences of entry-deterrent vertical integration. Given that production costs are zero, we can define the welfare function as:

$$W(q_1, q_2, q_3) = u(q_1, q_2, q_3) - \theta$$

where $u(q_1, q_2, q_3)$ is the utility function, separable in money, of a representative consumer:

$$u(q_1, q_2, q_3) = q_1 + q_2 + q_3 - \frac{q_1^2}{2} - \frac{q_2^2}{2} - \frac{q_3^2}{2} - \gamma q_1 q_2 - \gamma q_1 q_3 - \gamma q_2 q_3 + m$$

and θ is the entry cost. Note that the fact that we are analyzing the case of entry-deterrent vertical integration implies that we have to compare social welfare with vertical integration and two downstream firms and social welfare with vertical separation and three downstream firms. The welfare function for the case of two downstream firms can be obtained just by imposing $q_3 = 0$. The following proposition validates the result:

Proposition 4 *Social welfare under entry-deterrent vertical integration is always lower than under vertical separation and entry.*

Given that social welfare is measured by the utility of the representative consumer in the equilibrium quantities, a change in any element of the model that affects the outputs will change welfare, namely, the number of differentiated goods, the wholesale prices and the number of firms. Entry-deterrent vertical integration produces three negative effects on welfare: it reduces variety by eliminating one good; it eliminates one firm and, additionally, it increases the wholesale price charged to the independent downstream firm. The only positive effect of on welfare is the entry cost saving. However, this positive effect is of limited size given that the entry cost must satisfy $\Pi(c, w_I^*, 3) < \theta < \Pi(c, 2w_S^*, 3)$ and, as the above proposition states, it is never large enough to outweigh the three negative effects.[11]

[11] This result is akin to the one obtained in Hunold and Schad (2023) in the case $n = 2$.

In the following subsection, we proceed to study the region where a vertical merger increases the entrant's profits, that is, when $c < c_1^C$. This subsection contains the main contributions of our paper.

3.2 A vertical merger increases the entrant's profits

The first step is to study the profitability of entry-encouraging vertical integration. The following proposition formalizes the result.

Proposition 5 *Suppose* $\Pi(c, 2w_S^*, 3) < \theta < \Pi(c, w_I^*, 3)$ *holds, such that vertical integration encourages entry. Then, vertical integration is profitable iff* $c > \widetilde{c}^P$, *where* $\widetilde{c}^P$ *is an increasing function of* γ *and is defined in the Proof of Proposition 5 in the Appendix.*

Pushing entry through vertical integration tends to be profitable both when the positive market expansion effect of opening a new market is large (which occurs as the goods are more differentiated) and when the negative competition effect of entry is small (which occurs when the alternative supply is sufficiently inefficient). When c is high, the outside option of the independent downstream firms is low, which provides the integrated firm with additional incentives to increase the wholesale price with the aim to reduce market competition. So, ceteris paribus, higher values of c lead to higher profits of the integrated firm. As the goods become closer substitutes, entry increases more the intensity of competition and reduces the market expansion effect. Both effects lead to a less profitable vertical merger, which requires of a higher values of c that allow the integrated firm to increase more the wholesale price to compensate for the previous two negative effects.

Given that we are in the region in which a vertical merger increases the entrant's profits ($c < c_1^C$), in order to determine when entry-encouraging vertical integration occurs in equilibrium, we need to combine Propositions 1 and 5, which is formalized in the following proposition.

Proposition 6 *Suppose* $\Pi(c, 2w_S^*, 3) < \theta < \Pi(c, w_I^*, 3)$ *holds, such that vertical integration encourages entry. Then, vertical integration is profitable and occurs in equilibrium iff* $\widetilde{c}^P < c < \min\{c_1^C, c_D^C\}$.

The above proposition states that entry-encouraging vertical integration occurs in equilibrium when c is sufficiently high to guarantee that it is profitable and sufficiently low to guarantee that vertical integration increases the entrant's profits. Figure 3 plots the region defined in Proposition 6. As we can see in Figure 3, the interval $[\widetilde{c}^P, c_1^C]$ is non-empty regardless of the degree of product differentiation and the region where entry-encouraging vertical integration occurs in equilibrium is significantly large. This is the main contribution of our paper. In our setting with two downstream incumbents and one entrant, besides finding that the region where a vertical merger can deter entry is very small, we find a large region of parameters for which a vertical merger induces entry in the market. And, as the following discussion will show, in most cases, those entry-encouraging vertical mergers that occur in equilibrium are welfare enhancing.

Place Figure 3 around here

Figure 4 plots the social welfare comparison for two particular values of the entry cost, namely, $\theta = 0$ and $\theta > 0$.

We compare social welfare with vertical integration and three firms and social welfare with vertical separation and two firms. Figure 4 shows that even for large (feasible) entry costs $\theta < \Pi(c, w_I^*, 3)$, we still find a significantly large region of parameters in which entry-encouraging vertical integration enhances social welfare. This is the case for low values of c and γ (specifically, when $c < c^W(\gamma)$ in Figure 4). Note that low values of c prevent the integrated firm from charging too high wholesale prices and also imply a large competition effect of entry, whereas low values of γ increase the market expansion effect of the introduction of a new good. On the other hand, as the entry cost θ decreases, the threshold function $c^W(\gamma)$ shifts upwards, increasing the region where a vertical merger enhances welfare. Interestingly, Figure 4 shows that even with a null cost of entry ($\theta = 0$), we still find a (relatively small) region where a vertical merger can be detrimental to welfare. This is the case for high values of c and γ (specifically when $c > c^W(\gamma)$ in Figure 4).[12]

Place Figure 4 around here

4 The case with n firms

In this section, we extend the benchmark model to study if our result that vertical integration can increase the entrant's profits is robust to a general setting with $(n-1)$ downstream incumbents and 1 potential entrant. We will compare the outside option of the entrant in the cases of vertical integration and vertical separation. For the case of vertical separation, the upstream firm maximizes (taking into account that it optimally sells the input to all downstream firms, see Fauli-Oller et al., 2013):

$$n\Pi\left(r,(n-1)r,n\right)+nrq\left(r,(n-1)r,n\right)-n(\Pi(c,(n-1)r,n)$$

The optimal two-part tariff contract (see Proposition 1 in Fauli-Oller et al., 2013), leads to the following outside option of any of the independent downstream firms (which equals its equilibrium profits).[13]

[12] If $\theta = 0$, we know that a vertical merger would never deter entry. Specifically, there would be entry both with vertical integration and vertical separation. But in the previous discusssion and just for the sake of the argument, we are assuming that we have no entry with vertical separation.

[13] Note that for $c \leq c_{1N}^E$, $r_E^* = 0$.

$$\Pi(c,(n-1)r_E^*,n)=\begin{cases} \frac{(-2+\gamma+c(2+\gamma(-2+n)))^2}{(-2+\gamma)^2(2+\gamma(-1+n))^2} & \text{if } 0<c<c_{1N}^E \\ \frac{\begin{array}{c}(-2+\gamma)^2(4-2c(2+\gamma(n-2))\\(1+\gamma(n-1))+\gamma(-6+\\+\gamma(n-3)(n-1)+4n))^2\end{array}}{\begin{array}{c}4(2+\gamma(n-1))^2(4+\gamma(4(n-2)+\\+\gamma(6+\gamma(n-1)+n(n-6))))^2\end{array}} & \text{if } c_{1N}^E<c<c_{2N}^E \\ 0 & \text{otherwise} \end{cases}$$

where $c_{1N}^E=\frac{(2-\gamma)\gamma}{4+2\gamma(-2+n)}$ and $c_{2N}^E=\frac{4+\gamma(-6+\gamma(-3+n)(-1+n)+4n)}{2(2+\gamma(-2+n))(1+\gamma(-1+n))}$.

For the case of vertical integration, the integrated firm maximizes:

$$\Pi(0,(n-1)r,n)+(n-1)rq(r,(n-2)r,n)+(n-1)(\Pi(r,(n-2)r,n)-\Pi(c,(n-2)r,n))$$

The optimal two-part tariff contract (see Proposition 2 in Fauli-Oller et al., 2013) leads to the following outside option of any of the independent downstream firms (which equals its equilibrium profits):

If $\gamma<\frac{2}{n-1}$,

$$\Pi(c,(n-2)r_I^*,n)=\begin{cases} \frac{(-2+2c+\gamma)^2}{(-2+\gamma)^2(2+\gamma(-1+n))^2} & \text{if } 0<c<c_{1N}^I \\ \frac{\left(-2+\gamma-\frac{\begin{array}{c}2c(-4+\gamma(8+3\gamma(-1+\\+n)-4n))(2+\gamma(-2+\\+n))+\gamma^2(-2+\gamma)(-2+\\+\gamma(-1+n))(-2+n)\end{array}}{2(4+4\gamma(-2+n)+\gamma^2(7+(-7+n)n))}\right)^2}{(-2+\gamma)^2(2+\gamma(n-1))^2} & \text{if } c_{1N}^I\leq c<c_{2N}^I \\ 0 & \text{otherwise} \end{cases}$$

If $\gamma\geq\frac{2}{n-1}$,

$$\Pi(c,(n-2)r_I^*,n)=\begin{cases} \frac{(-2+\gamma+c(2+\gamma(-2+n)))^2}{(-2+\gamma)^2(2+\gamma(-1+n))^2} & \text{if } 0<c<c_{0N}^I \\ \frac{\left(-2+\gamma-\frac{\begin{array}{c}2c(-4+\gamma(8+3\gamma(-1+\\+n)-4n))(2+\gamma(-2+\\+n))+\gamma^2(-2+\gamma)(-2+\\+\gamma(-1+n))(-2+n)\end{array}}{2(4+4\gamma(-2+n)+\gamma^2(7+(-7+n)n))}\right)^2}{(-2+\gamma)^2(2+\gamma(n-1))^2} & \text{if } c_{0N}^I<c<c_{2N}^I \\ 0 & \text{otherwise} \end{cases}$$

where[14] $c_{0N}^I=\frac{(2-\gamma)(-2+\gamma(-1+n))}{2(2+\gamma(-2+n))(-2+n)}$, $c_{1N}^I=-\frac{\gamma(-2+\gamma)(-2+\gamma(-1+n))}{-8+2\gamma(4+3\gamma(-1+n)-2n)}$ and $c_{2N}^I=\frac{(-2+\gamma)(8+\gamma(8(-2+n)-\gamma(-2(-5+n)+\gamma(-2+n))(-1+n)))}{2(-4+\gamma(8+3\gamma(-1+n)-4n))(2+\gamma(-2+n))}$.

Directly comparing the previous expressions, we obtain the following result:

Proposition 7 *Vertical integration increases the entrant's post-entry profits if $c<c(n)$ and decreases the entrant's profits otherwise, where $c(n)$ is defined in the Proof of Proposition 7 in the Appendix.*

It is direct to check that $c(2)=\frac{1}{4}\gamma(2-\gamma)$, which is the threshold value identified in Proposition 4 in Hunold and Schad (2023), and that $c(3)=c_1^C$, which is

[14] Note that if $c<c_{0N}^I$, $r_I^*=0$.

the threshold value identified in Proposition 1 above. We have already noted after Proposition 1, that $c(2) < c(3)$ but, in general, $c(n)$ is non-monotonic with respect to n.

5 Merging with the two incumbent downstream firms

In this section, we extend the benchmark model to allow the upstream firm to merge with either one or the two downstream incumbents. In the case of a vertical merger with the two downstream incumbents, at first sight we could think that we are back to the original model in Hunold and Schad (2023), where the outside option of the entrant does not depend on the wholesale price charged by the merged firm, which leads to the result that, in their setting, a vertical merger never encourages entry. However, things are not that simple in our setting, where a vertical merger with the two incumbents reduces horizontal market competition, providing additional incentives for entry in the market. We show below that, indeed, we can find profitable entry-encouraging vertical mergers with the two incumbents in equilibrium.

We will assume that a two-firms vertical merger arises in equilibrium when the merger participants jointly obtain more profits than with either one-firm vertical merger or no merger. The ultimate goal is to find first, whether vertically merging with the two incumbents can be entry-encouraging and, second, whether this merger is profitable and arises in equilibrium.

Let us start by analyzing the equilibrium when the upstream firm merges with the two downstream incumbents. In the last stage, we have the merged firm producing two differentiated goods with zero marginal cost and one potential entrant. Upon entry, the last stage equilibrium profits of the merged firm and the entrant are given respectively by: $\Pi(w) = \frac{(1+\gamma)(2+\gamma(-1+w)))^2}{2(-2+(-2+\gamma)\gamma)^2}$ and $\Pi^E(w) = q^E(w)^2$, where $q^E(w) = \frac{-1+w+\gamma w}{-2+(-2+\gamma)\gamma}$is the entrant's equilibrium output. The outside option of the entrant is $\Pi^E(c)$. In the third stage, the merged firm chooses the contract to maximize $\Pi(w) + \left(\Pi^E(w) - \Pi^E(c)\right) + wq^E(w)$. Notice that the merged firms maximize total market profits, given that the outside option of the entrant does not depend on the wholesale price. The optimal contract is given by $w_E^* = \frac{\gamma(-2+\gamma^2)}{-2+\gamma(-4+\gamma+3\gamma^2)}$ and $F_E^* = \Pi^E(w_E^*) - \Pi^E(c)$.

Next, we compare the entrant's profits in the case of a merger with the two downstream firms, in the case of a merger with only one downstream firm and in the case of no merger. The following proposition states the result.

Proposition 8 *If $c < \min\{s_1, c^D\}$, the entrant's profits when the upstream merges with the two incumbent firms is higher than when it merges with only one incumbent or there is no merger.*

The previous lemma identifies cases where a vertical merger with the two incumbent downstream firms increases the entrant's profits as compared with no merger and a vertical merger with only one incumbent. The following proposition identifies when an entry-encouraging two-firms vertical merger materializes as an equilibrium outcome. We need both that the vertical merger increases the entrant's profits and that it is profitable. The following proposition validates the result.

Proposition 9 *Suppose that* $\max\{\Pi(c, w_I^*, 3), \Pi(c, 2w_S^*, 3)\} < \theta < \Pi^E(c)$. *Then, the upstream firm will merge with the two incumbent downstream firms in equilibrium (and there will be entry) if* $\gamma < 0.9$ *and* $r(\gamma) < c < \min\{s_1, c_D^C\}$, *where* $r(\gamma)$ *is an increasing function of* γ *and is defined in the Proof of Proposition 9 in the Appendix.*

The intuition for the previous result is akin to the one in Proposition 5. Entry-encouraging two-firms vertical mergers tend to be profitable when c is high and γ is low. In the former case the competition effect of entry is small and in the latter case, the market expansion effect of entry is large.

Concerning the effect of an entry-encouraging two-firms vertical merger on social welfare, it is direct to show that for the particular case of a null entry cost, it would be always welfare-enhancing. The reason is that the sum of the market expansion effect plus the competition effect of entry outweighs the negative effect in the form of a higher wholesale price charged to the entrant. Obviously, as we increase the entry cost, we can find a region where social welfare decreases with the two-firms vertical merger.

6 Secret contracts

In this section, we show that our result that vertical integration can increase the entrant's profits for a sufficiently efficient alternative supply is robust to the case of secret contracts (and passive beliefs).[15] Hunold and Schad (2023) also extend their model to study the case of secret contracts and obtain the same result than under observable contracts, namely, that in a Cournot setting, vertical integration always reduces the entrant's profits. In their model with only one incumbent downstream and one entrant, the vertically integrated firm can observe (and optimally react) to a possible deviation of the entrant to source the input from the alternative supply, so the setting is equivalent to the case of observable contracts. Adding a second downstream incumbent, however, enriches the model because even though the integrated firm can still observe and react to a deviation by the entrant, firm 2 cannot observe the entrant's behavior and so it does not react to any deviation. This adds an interesting trade-off that leads, as we show below, to a (large) region of parameters where vertical integration increases the entrant's profits. This is the case when the alternative supply is sufficiently efficient, as it occurs under observable contracts too.

With vertical separation, the equilibrium wholesale prices are the ones that maximize the bilateral profits of the upstream firm with each downstream firm,

[15] See Rey and Tirole (2007).

taking into account that, as the other competitors do not observe the other firms' contracts, they will stick to their equilibrium outputs. Then, the upstream firm faces a situation akin to the one of a bilateral monopoly whose joint profit maximization requires a wholesale price equal to marginal cost.[16] Then, in equilibrium, $w_i = 0$ and each downstream firm produces $q(0,0,3)$ (see equation,1). Next, we calculate the outside option of downstream firms, that coincide with the post-entry profits of the entrant, given that the upstream firm offers take-it or leave-it supply contracts. The outside option amounts to the profits a downstream firm can obtain by using the alternative supply, taking into account that the other competitors do not observe the acceptance/rejection decisions and, therefore, they cannot condition their strategy on this event and stick to the equilibrium outputs. Let $P(q_i, Q_{-i}) = p_i = 1 - q_i - \gamma Q_{-i}$, where $Q_{-i} = \sum_{j \neq i} q_j$ be the inverse demand function of downstream firm i as it was defined in Section 2. Then, the outside option amounts to:

$$\underset{q}{Max}\left(P(q, 2q(0,0,3)) - c\right) q \;= \frac{(-1 + c(1+\gamma))^2}{4(1+\gamma)^2}. \tag{2}$$

The case of vertical integration is more involved. As before, the independent downstream firms do not observe either the other firm's contract nor its acceptance/rejection decision. However, the integrated firm do observe all of them. It is common knowledge that the integrated firm operates at zero marginal cost. The equilibrium wholesale prices must again maximize the bilateral profits of the integrated firm and each independent downstream firm, given that the remaining firm sticks to the equilibrium output. But in this case, the integrated firm reacts optimally to changes in the contract. Suppose that firm 3 produces in equilibrium q_3. Then, if firm 2 accepts a contract with a wholesale price w, the outputs of the integrated firm and firm 2 (for a given q_3) are the following:

$q_1(w, q_3) = \frac{2+\gamma(-1+(-2+\gamma)q_3+w)}{4-\gamma^2}$ and $q_2(w, q_3) = \frac{(-2+\gamma)(-1+\gamma q_3)-2w}{4-\gamma^2}$.

The next step is to find the contract that maximizes the bilateral profits of the integrated firm and firm 2 as follows:

$\underset{w}{Max}[(1 - q_1(w,q_3) - \gamma q_2(w,q_3) - \gamma q_3)q_1(w,q_3) + (1 - q_2(w,q_3) - \gamma q_1(w,q_3) - \gamma q_3)q_2(w,q_3)]$.

We obtain the following optimal wholesale price:

$w(q_3) = \frac{\gamma(-2+\gamma)^2(-1+\gamma q_3)}{-8+6\gamma^2}$. Given that the contract will be symmetric in equilibrium, the independent downstream firms will produce the same output and, therefore, we can write: $q_2(w(q_3), q_3) = q_3$, leading to $q_2^* = q_3^* = \frac{2(-1+\gamma)}{-4+\gamma(-2+5\gamma)}$. To obtain q_1^* we proceed by computing $q_1(w(q_3^*), q_3^*) = \frac{-4+\gamma(2+\gamma)}{-8+2\gamma(-2+5\gamma)}$. Note that $w(q_3^*) = \frac{\gamma(-2+\gamma)^2}{8-2\gamma(-2+5\gamma)}$, is the optimal wholesale price (and the previous expressions represent the equilibrium outputs) only when it is lower than c. Then, we can write the equilibrium wholesale price as:

$$w^* = \begin{cases} \frac{\gamma(-2+\gamma)^2}{8-2\gamma(-2+5\gamma)} \text{if } \frac{\gamma(-2+\gamma)^2}{8-2\gamma(-2+5\gamma)} < c \\ c \text{ otherwise} \end{cases}.$$

[16] This result is standard (Rey and Tirole, 2007).

Let us study first the unrestricted case $w^* = \frac{\gamma(-2+\gamma)^2}{8-2\gamma(-2+5\gamma)}$. We need to compute the outside option of downstream firm 2, the profits it can obtain, deviating from the equilibrium and sourcing the input from the alternative supply. Firm 3 does not observe this decisions so it will stick to its equilibrium output. However, the integrated firm does observe it and will react optimally. Then, the output of the integrated firm will be $q_1(c, q_3^*)$ and the outside option of firm 2 will amount to:

$$\underset{q}{Max}\ (P(q, q_1(c, q_3^*) + q_3^*) - c)\, q \ = \frac{((-2+\gamma)(-4+3\gamma^2) + 2c(-4+\gamma(-2+5\gamma)))^2}{(-2+\gamma)^2(2+\gamma)^2(-4+\gamma(-2+5\gamma))^2}. \tag{3}$$

In the restricted case, $w^* = c$, so that the profits of firm 2 can be written (to make them comparable with the previous profit expressions):

$$\underset{q}{Max}\ (P(q, q(0, 2c, 3) + q_3(c, c, 3) - c)\, q \ = \left(\frac{2-\gamma-2c}{2(2-\gamma)(1+\gamma)}\right)^2 = \Pi(c, c, 3) \tag{4}$$

To see whether vertical integration increases the entrant's post-entry profits we need to compare (3) with (2) for $\frac{\gamma(-2+\gamma)^2}{8-2\gamma(-2+5\gamma)} < c$ and (4) with (2) for lower values of c. From the way profits are written, it is easy to see that we have just to compare the total output sold by the two downstream incumbents in both the scenarios of vertical integration and vertical separation in the event that the entrant sources the input from the alternative supply. The higher this joint output, the lower the entrant's profits. With vertical separation, it amounts to $2q(0, 0, 3) = \frac{1}{1+\gamma}$. With vertical integration, it depends on whether the integrated firm sets the unrestricted wholesale price or is restricted to set $w^* = c$. In the former case, the sum of the outputs produced by competitors amount to $q_1(c, q_3^*) + q_3^* = \frac{16+4(-3+c)+2(-4+c)\gamma^2-5(-1+c)\gamma^3}{(-4+\gamma^2)(-4+\gamma(-2+5\gamma))}$ and, in the latter case, to $q(0, 2c, 3) + q_3(c, c, 3) = \frac{-2+c+\gamma(1-c)}{(-2+\gamma)(1+\gamma)}$. The following proposition formalizes the result of the comparison.

Proposition 10 *Vertical integration increases the entrant's post-entry profits under secret contracts (and passive beliefs) if* $c < \min\{\frac{(-2+\gamma)^2}{(1+\gamma)(4-\gamma(-2+5\gamma))}, c_D^C\}$ *and reduces the entrant's profits otherwise.*

Proposition 10 shows that the effect of vertical integration on the entrant's post-entry profits under secret contracts is qualitatively similar to the one under observable contracts. Nevertheless, the intuition that explains the result is different. The result is based on the comparison between the sum of the outputs of firms 1 and 2 under vertical integration and vertical separation when the entrant sources the input from the alternative supply. The comparison is more clear in the case in which the wholesale price in the integrated case is restricted to be $w^* = c$. Suppose we start from the limit case $c = 0$, when the incumbent firms' outputs are the same in both scenarios. As we increase c, however, firm 1 produces more under vertical integration than under vertical separation, due to an indirect effect: it faces more inefficient firms 2 and 3 (the latter firms have zero cost under vertical separation and cost c under vertical integration). Regarding firm 2, its output decreases under vertical integration due to a direct effect: the integrated firm optimally raises firm 2's cost through the wholesale price it charges to this firm. Proposition 10 shows that the direct effect always dominates the indirect one, leading to the result. In the unrestricted case we have that, even though firm 2's output is lower under vertical integration, firm 1's output is increasing in c, as this firm observes and optimally reacts to the entrant's deviation towards the alternative supply. This explains that for sufficiently high values of c, the sign of the comparison is reversed and vertical integration reduces the entrant's post-entry profits.

7 The case of price competition

In this section, we extend the benchmark model to study whether the results obtained in a Cournot setting are robust to the case of downstream price competition. We will start by analyzing whether vertical integration can increase the entrant's profits in the new setting. Under vertical separation, in the last stage of the game, each downstream firm maximizes:

$$\underset{p_i}{Max}\Pi_i(C_i, C_j, C_k) = \{(p_i - C_i)x_i(p_i, p_j, p_k)\}$$

where C_i denotes the marginal cost of firm i and can be $C_i = w$ if firm i accepts the supply contract and $C_i = c$ if it rejects the supply contract and sources the input from the alternative supply. The term $x_i(p_i, p_j, p_k)$ denotes the direct demand functions and they are given by:

$$\begin{aligned} x_i(p_i, p_j, p_k) &= \frac{1}{1+2\gamma} - \frac{(1+\gamma)p_i}{(1-\gamma)(1+2\gamma)} - \frac{\gamma(p_j + p_k)}{(1-\gamma)(1+2\gamma)} \\ i, j, k &= 1, 2, 3,\ i \neq j \neq k. \end{aligned}$$

Solving for the system of the three first order conditions $\frac{\partial \Pi_i(C_i)}{\partial p_i} = 0$, we obtain the equilibrium prices:

$$p_i^*(C_i.C_j, C_k) = \frac{2(1+C_i) + \gamma^2(-3 + \sum_{i=1}^{3} C_i) + \gamma(1 + 3C_i + Cj + C_k)}{4+6\gamma}.$$

The equilibrium profits are given by:

$$\Pi_i^*(C_i.C_j, C_k) = \frac{(1+\gamma)(2 - C_i + \gamma(1 + C_j + C_k - 3C_i) + \gamma^2(-3 + \sum_{i=1}^{3} C_i))^2}{4(1-\gamma)(1+2\gamma)(2+3\gamma)^2}.$$

In the third stage of the game, the upstream firm chooses the contract (w, F) to maximize (taking into account that the upstream firm finds optimal to sell the input to the three independent downstream firms[17] and that the participation constraint is binding):

$$\begin{aligned} & \underset{w}{Max}\{3(wx_i(w,w,w)+\Pi_i^*(w,w,w)-\Pi_i^*(c,w,w))\}, \\ s.t.\ 0 \quad \leq & \quad w \leq c. \end{aligned}$$

This leads to the following optimal contract[18]:

$$\begin{aligned} w_S^* &= \begin{cases} \frac{\gamma((-1+\gamma)\gamma(2+3\gamma)+2c(1+\gamma)(-2+(-3+\gamma)\gamma))}{(1+\gamma)(-4+\gamma(-8+\gamma(-1+5\gamma)))} & \text{if } c \geq \frac{\gamma^2}{2+3\gamma+\gamma^2} \\ c & \text{otherwise} \end{cases}; \\ F^* &= \Pi_i^*(w_S^*,w_S^*,w_S^*)-\Pi_i^*(c,w_S^*,w_S^*). \end{aligned}$$

Notice that the outside option of the entrant is given by $\Pi_3^*(c,w_S^*,w_S^*)$. In order to guarantee non-negative outputs, we need to impose the constraint that $c \leq c_D^B = \frac{(-1+\gamma)(2+3\gamma)}{-2+(-3+\gamma)\gamma}$. This constraint guarantees that in the worst possible case for any firm (specifically, facing two rivals with zero marginal costs when it has a marginal cost equal to c) its output is non-negative, for example, that $x_3(p_3^*(c,0,0),p_1^*(0,0,c),p_2^*(0,0,c) \geq 0$.[19]

Under vertical integration, each independent downstream firm maximizes in the last stage of the game:

$$\underset{p_j}{Max}\{(p_j-C_j)x_j(p_j,p_i,p_k)\},\ j,k=2,3,\ j\neq k,$$

where C_i denotes the marginal cost of firm i and can be $C_i = w$ if firm i accepts the supply contract and $C_i = c$ if it rejects the supply contract and sources the input from the alternative supply. The integrated firm maximizes its market profits plus its input revenues. We must distinguish two cases:

(i) When both independent downstream firms accept the contract, the integrated firm maximizes:

$$\underset{p_1}{Max}\{p_1x_1(p_1,p_2,p_3)+wx_2(p_2,p_1,p_3)\ +wx_3(p_3,p_1,p_2).$$

In this case, solving for the system of the three first order conditions, we obtain the equilibrium prices:

$$\begin{aligned} p_{1I}^*(w) &= \frac{2+\gamma+6\gamma w+\gamma^2(-3+4w)}{4+6\gamma}; \\ p_{2I}^*(w) &= p_{3I}^*(w)=\frac{2(1+w)+\gamma(1-3\gamma+4(1+\gamma)w)}{4+6\gamma}. \end{aligned}$$

[17]It is tedious but straightforward to show that it is always optimal for the upstream firm to sell the input to all downstream firms.

[18]Note that $x_i(w,w,w) = x_i(p_i^*(w,w,w),p_j^*(w,w,w),p_k^*(w,w,w))$.

[19]Note that the same constraint also guarantees that outputs are non-negative under vertical integration, which we analyze below.

The equilibrium profits are given by:

$$\begin{aligned}\Pi_1^*(w) &= \frac{1-\gamma^2}{4+8\gamma}+\frac{(2+\gamma(2+\gamma))w}{2+3\gamma}-\frac{(4+\gamma(2+\gamma)(5+2\gamma))w^2}{((2+3\gamma)^2};\\ \Pi_2^*(w) &= \Pi_3^*(w)=\frac{(1-\gamma^2)(2+3\gamma-2w-4\gamma w)^2}{4(1+2\gamma)(2+3\gamma)^2}.\end{aligned}$$

(ii) When one downstream firms accepts the contract (let's say firm 2) and the other firm rejects it, the integrated firm maximizes:

$$\underset{p_1}{Max}\{p_1x_1(p_1,p_2,p_3)+wx_2(p_2,p_1,p_3)\}.$$

In this case, solving for the system of the three first order conditions, we obtain the equilibrium prices:

$$\begin{aligned}p_{1N}^* &= \frac{2+\gamma^2(-3+c+2w)+\gamma(1+c-3w)}{4+6\gamma};\\ p_{2N}^* &= \frac{2(1+w)+\gamma^2(-3+c+2w)+\gamma(1+c-3w)}{4+6\gamma};\\ p_{3N}^* &= \frac{2+c(1+\gamma)(2+\gamma)+\gamma(1+w+\gamma(-3+2w))}{4+6\gamma}.\end{aligned}$$

And the profits obtained by the firm rejecting the contract (firm 3) are given by:

$$\Pi_{3N}^*(c,w)=\frac{(1+\gamma)(2+c(-2+(-3+\gamma)\gamma)+\gamma(1+w+\gamma(-3+2w)))^2}{4(1-\gamma)(1+2\gamma)(2+3\gamma)^2}.$$

In the third stage, the upstream firm chooses the contract (w,F) to maximize (taking into account that it sells the input to the two independent downstream firms[20] and that the participation constraint is binding):

$$\begin{aligned}&\underset{w}{Max}\{\Pi_1^*(w)+wx_2(w)+wx_3(w)+\\ &+(\Pi_2^*(w)-\Pi_{2N}^*(c,w))+(\Pi_3^*(w)-\Pi_{3N}^*(c,w))\}\\ s.t.\ 0 &\leq w\leq c\end{aligned}$$

With some abuse of notation[21], we have used $\Pi_{2N}^*(c,w)$ in the above maximization program, which has not been explicitly defined. However, it is intuitive

[20] It is tedious but straightforward to show that it is always optimal for the integrated firm to sell the input to all downstream firms.

[21] Note that $x_1(w)=x_i(p_i(w),p_j(w),p_k(w))$.

that $\Pi_{2N}^*(c,w) = \Pi_{3N}^*(c,w)$ by symmetry. This leads to the following optimal contract:

$$\begin{aligned} w_I^* &= \begin{cases} \frac{\gamma((-1+\gamma)(1+2\gamma)(2+3\gamma)+c(1+\gamma)(-2+(-3+\gamma)\gamma))}{-4+\gamma(1+\gamma)(-8+\gamma(-3+10\gamma))} & \text{if } c \geq \frac{(-1+\gamma)\gamma(1+2\gamma)(2+3\gamma)}{-4+3\gamma(1+\gamma)(-2+3\gamma^2)} \\ c & \text{otherwise} \end{cases} \\ F^* &= \Pi_3^*(w_I^*) - \Pi_{3N}^*(c, w_I^*) \end{aligned}$$

The outside option of the entrant is given by $\Pi_{3N}^*(c, w_I^*)$. It can be seen that it is always positive.

The next step is to compare the entrant's profits with both vertical separation and vertical integration, namely, to sign $\Pi_3^*(c, w_S^*, w_S^*) - \Pi_{3N}^*(c, w_I^*)$. The following proposition formalizes the result. Both the fact that we have $w_I^* \geq w_S^*$ (with strict inequality if the separated firm is unconstrained) and the collusive effect that arises with vertical integration explain that the previous expression may be negative. This possibility is validated in the following proposition.

Proposition 11 *Vertical integration increases the entrant's post-entry profits iff* $\gamma < 0.66$ *and* $c \in (c_2^B, c_4^B)$*,where* c_2^B *and* c_4^B *are defined in the Proof of Proposition 11 in the Appendix.*

As we can see in Proposition 11, vertical integration increases the entrant's profits only for intermediate values of c. Interestingly, this is different to what we obtained under Cournot competition, where vertical integration increases the entrant's profits for low values of c. The reason is that in the Bertrand model, competition is so intense that, even with vertical separation, the upstream firm has incentives to set large wholesale prices. This implies that for low values of c, with both vertical integration and vertical separation, the upstream is constrained to set a wholesale price equal to c. In this case, with vertical separation the entrant faces two rivals with costs c, whereas with vertical integration, it faces a rival with cost c and the integrated firm with cost zero. Therefore, for low values of c, vertical integration reduces the entrant's profits under Bertrand competition.

For larger values of c, the separated firm is unconstrained, and the integrated firm is still constrained so, in this case, the integrated firm charges a higher wholesale price. For even larger values of c, both firms are unconstrained, but it is still true that the wholesale price set by the integrated firm is higher. This anticompetitive effect of vertical integration (that will increase the entrant's profit) should be contrasted with the competitive effect (that tends to reduce the entrant's profits) of vertical integration coming from the fact the integrated firm produces at zero cost. The above proposition states that the former effect dominates the latter for intermediate values of c.

In what follows, we divide the analysis into two separate subsections as we did in the Cournot model. In the first one, we will focus on the case in which a vertical merger reduces the entrant's profits with the aim to determine the region in which entry-deterrent vertical integration is profitable and occurs in equilibrium. A welfare analysis will close this subsection. In the second, we will study the case in which a vertical merger increases the entrant's profits to determine the region in which profitable entry-encouraging vertical integration occurs in equilibrium as well as its welfare consequences.

7.1 A vertical merger reduces the entrant's profits

The first step in the analysis is to look at the profitability of entry-deterrent vertical integration, which is formalized in the following proposition.

Proposition 12 *Suppose that* $\Pi^*_{3N}(c, w^*_I) < \theta < \Pi^*_3(c, w^*_S, w^*_S)$*, so that vertical integration deters entry. Then, vertical integration is profitable if either* $0 < \gamma < 0.66$ *and* $c < \widehat{c}^P$ *or* $\gamma > 0.66$*, where* $\widehat{c}^P = \left\{ \begin{smallmatrix} \widehat{c}^P_1 \text{ when } \gamma \leq 0.65 \\ \widehat{c}^P_2 \text{ when } 0.65 < \gamma \leq 0.66 \end{smallmatrix} \right.$

It is intuitive that entry-deterrent VI tends to be profitable when (i) the market expansion effect of entry would be small (which occurs when γ is high) and so the integrated firm does not lose too much by deterring entry and (ii) the competition effect of entry would be large (which occurs when c is low, so that the entrant obtains a relatively large part of the overall rents) and when γ is high, so that the entrant would face intense competition upon entry. The above proposition formalizes this intuition.

We aim to identify cases where entry-deterrent vertical integration is an equilibrium outcome. This requires combining the results of the two previous Propositions. Proposition 11 shows that when c is either sufficiently high or sufficiently low, the entrant would obtain less profits with vertical integration than with vertical separation. So, in those cases, there exist (intermediate) values of the entry cost, specifically $\Pi^*_{3N}(c, w^*_I) < \theta < \Pi^*_3(c, w^*_S, w^*_S)$, such that entry only occurs with vertical separation and, therefore, vertical integration would result in market foreclosure. Proposition 12 tells us that entry-deterrent vertical integration is profitable when c is sufficiently low. The following proposition formalizes the previous discussion.

Proposition 13 *If* $\gamma \in (0, 0.6]$ *and* $c \in (0, c^B_2]$ *or if* $\gamma \in (0.6, 0.66]$ *and* $c \in (0, c^B_2]$ *or* $c \in (c^B_4, \min\{\widehat{c}^P, c^B_D\})$ *and if* $\gamma \in (0.66, 1)$ *and* $c \in (0, c^B_D)$*, entry-deterrent vertical integration is profitable and will occur in equilibrium.*

For $\gamma \leq 0,6$, entry deterrent vertical integration occurs in equilibrium only for low enough values of c ($c \leq c^B_2$) because, in that interval of γ, we have $c^B_2 < \widehat{c}^P < c^B_4$ and then, for high values of c ($c > c^B_4$), the vertical merger reduces the entrant's profits but it is unprofitable. When $\gamma \in (0.60, 0.66]$, entry-deterrent vertical integration occurs both for sufficiently low values of c ($c \leq c^B_2$) and for sufficiently high values of c ($c^B_4 < c < \min\{\widehat{c}^P, c^B_D\}$), because $c^B_2 < c^B_4 < \min\{\widehat{c}^P, c^B_D\}$ holds. Finally, when $\gamma > 0,66$, all mergers satisfying the constraint $c \leq c^B_D$ reduce the entrant's profits and are profitable.

Regarding the welfare implications of entry-deterrent vertical integration under Bertrand competition, we must compare social welfare with vertical integration and two firms and social welfare with vertical separation and three firms.

Assume for a moment that the entry cost is equal to zero and suppose that we have entry with vertical integration and no entry with vertical separation, just for the sake of the argument. In this case, social welfare should be higher under vertical separation. Note that, in this case, vertical integration has no positive effect on welfare: it eliminates one good plus one firm and the integrated firm sets higher wholesale prices than with vertical separation.

When the entry cost is positive, however, deterring entry has at least the positive effect of saving on the entry cost, which could reverse the sign of the welfare comparison. For sufficiently high values of entry costs (among the feasible ones), entry-deterrent vertical integration becomes welfare enhancing. This occurs in the region of high values of γ and low values of c, where the negative effects of entry-deterrent vertical integration are mitigated. Specifically, high values of γ reduce the negative market expansion effect of deterring entry and low values of c restrict the capacity of the integrated firm to set high wholesale prices.

7.2 A vertical merger increases the entrant's profits

First, we study profitability of entry-encouraging vertical integration, which is validated in the following proposition.

Proposition 14 *Suppose that* $\Pi_3^*(c, w_S^*, w_S^*) < \theta < \Pi_{3N}^*(c, w_I^*)$ *hold, such that vertical integration encourages entry. Then, vertical integration is profitable iff* $\gamma < 0.86$ *and* $c \in [\widetilde{c}, c_D^B]$, *where* $\widetilde{c} = \begin{cases} l_2 & \text{if } 0<\gamma<0.8250 \\ h_2 & \text{if } 0.8250<\gamma<0.8664 \end{cases}$

Inducing entry through vertical integration tends to be profitable when (i) the positive market expansion effect of opening a new market is large (which occurs as the goods are more differentiated) and (ii) the negative competition effect of entry is small (which occurs when the alternative supply is sufficiently inefficient. When c is high, the outside option of the independent downstream firms is low, which provides the integrated firm with additional incentives to increase the wholesale price with the aim to reduce market competition. So, ceteris paribus, higher values of c lead to higher profits of the integrated firm. As the goods become closer substitutes, entry increases more the level of competition and reduces the market expansion effect. Both effects lead to a less profitable vertical merger.

Combining Propositions 11 and 14 we identify, in the following proposition, the conditions under which entry-encouraging vertical integration is an equilibrium outcome of the game.

Proposition 15 *Entry-encouraging vertical integration is profitable and will occur in equilibrium if* $\gamma < 0.46$ *and* $c \in [l_2, c_4^B]$ *or if* $0.46 < \gamma < 0.66$ *and* $c \in [c_2^B, \min\{c_4^B, c_D^B\}]$.

Proposition 15 shows that for sufficiently high values of γ all vertical mergers that increase the entrant's profits are profitable. However, for low values of γ, we may have that some vertical mergers that increase the entrant's profits are not profitable. They are profitable only for sufficiently high values of c.

Regarding the social welfare consequences of entry-encouraging vertical integration, the intuition is very similar to the one in the Cournot setting.

We have to compare social welfare with vertical integration and three firms and social welfare with vertical separation and two firms. It can be seen that even for large (feasible) entry costs $\theta < \Pi^*_{3N}(c, w^*_I)$, we still find a significantly large region of parameters in which entry-encouraging vertical integration enhances social welfare. This occurs for low values of c and γ. Note that low values of c prevent the integrated firm from charging too high wholesale prices and also imply a large competition effect of entry, while low values of γ increase the market expansion effect of the introduction of a new good. On the other hand, as the entry cost θ reduces, the region where a vertical merger enhances welfare increases.

8 Conclusions

Hunold and Schad (2023) show that in the particular case of two downstream firms (one incumbent and one entrant) vertical integration reduces the entrant's post-entry profits. This is intuitive because with vertical integration the entrant faces an (integrated) rival with zero marginal cost, whereas under vertical separation, it faces a rival with a positive marginal cost (equal to the optimal wholesale price charged by the independent upstream firm, assuming that below-cost pricing is not allowed by the antitrust authorities).

In the present paper, we show that if we increase the number of downstream incumbents to at least two, vertical integration may increase the entrant's post-entry profits, which opens the door to the existence of entry-encouraging vertical integration. The key difference is that with more than one incumbent, the entrant's post-entry profits under vertical integration do depend on the wholesale price set by the integrated firm. This increases the entrant's profits because the optimal wholesale price set by the integrated firm is larger than the one set by the independent upstream firm. Notice that the integrated firm obtains revenues not only from input sales, but also from selling the final good in the market. Therefore, it is more interested in controlling the level of market competition and follows a *raising rivals's cost* strategy. We find a large region of parameters where entry-encouraging vertical integration occurs in equilibrium and show that most of the time it is welfare enhancing. We also analyze the region of parameters where vertical integration is entry-deterrent. We check that the region where we have profitable entry-deterrent vertically integration in equilibrium is significantly smaller than in Hunold and Schad (2023). We may then conclude that increasing the number of downstream competitors should reduce the concern of the antitrust authorities about the foreclosure effect of vertical integration as an entry-deterrent mechanism.

We find also that our main result that vertical integration can induce entry in a downstream market is robust to a number of extensions. In particular, we have extended the result to the case of a general number of incumbents and one entrant, to the case in which the upstream firm may choose whether to acquire one or the two incumbent downstream firms, to secret contracts and, finally, to the case of Bertrand downstream competition.

9 Appendix

Proof of Proposition 1

Proof. For $0 < c \leq c_I^C$ we have $w_S^* = 0$ and $w_I^* = c$, where $c_I^C = \frac{(2-\gamma)\gamma}{4+6\gamma}$. In this case, the comparison between the entrant's profits under vertical separation and under vertical integration reduces to sign $\Pi(c,0,3) - \Pi(c,c,3)$, which is negative.

For $c_I^C < c \leq \min\{c_S^C, c_D^C\}$ we have $w_S^* = 0$ and $w_I^* = \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)}$, where $c_S^C = \frac{(2-\gamma)\gamma}{2(2+\gamma)}$. In this case, the comparison reduces to sign $\Pi(c,0,3) - \Pi(c, \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)}, 3)$, which is negative, given that $\frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)} > 0$.

For $c_S^C < c \leq c_D^C$, we have $w_S^* = \frac{\gamma((-2+\gamma)\gamma+2c(2+\gamma))}{4+\gamma(4+\gamma(-3+2\gamma))}$ and $w_I^* = \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)}$. In this case, the comparison reduces to sign $\Pi(c, 2(\frac{\gamma((-2+\gamma)\gamma+2c(2+\gamma))}{4+\gamma(4+\gamma(-3+2\gamma))}), 3) - \Pi(c, \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)}, 3)$, which is a concave function of c, with two roots, c_1^C and c_2^C. For $\gamma \in (0, 0.9339)$, we have $c_S^C < c_1^C < c_D^C < c_2^C$. This implies that vertical integration increases the entrant's profits iff $c \in (c_S^C, c_1^C]$, where $c_1^C = \frac{(-2+\gamma)(1+\gamma)(-2+\gamma(-1+2\gamma))}{12-\gamma(-12+\gamma(17+2\gamma))}$. For $\gamma \in (0.9339, 1)$, we have $c_S^C < c_D^C < c_1^C < c_2^C$, which implies that the difference is negative in the entire interval (c_S^C, c_D^C).

Proposition 1 results from a straightforward exercise of merging the three regions we have just described. ■

Proof of Proposition 2:

Proof. $$\bar{c}^P = \frac{-(((-2+\gamma)(64-128H+\gamma(192-576H+\gamma(192-800H+\gamma(112-208H+\gamma(4(5+42H+\gamma(-12(7+13H)+\gamma(-2(17+75H)+\gamma(-9+73H+\gamma(-22+24H+\gamma(-19+4\gamma(1+\gamma))H))))))))))}{(128+\gamma(512+\gamma(576+\gamma(64+\gamma(-264+\gamma(-240+\gamma(-80+\gamma(-56+\gamma(-41+4\gamma(4+\gamma)))))))))}$$

with $H = \sqrt{\frac{-256+\gamma^2(384+\gamma^2(48+\gamma(-288+\gamma(96+\gamma(72+\gamma(-132+\gamma(80+\gamma(-13+\gamma+2\gamma^2))))))))}{(-4+3\gamma^2)(16+32\gamma-12\gamma^3+7\gamma^4+\gamma^5 2\gamma^6)^2}}$.

For $0 < c \leq c_S^C$, $w_S^* = 0$ and $w_{I2}^* = c$ (notice that w_{I2}^* denotes the optimal wholesale under *entry deterrent* vertical integration, namely, we only have 2 active downstream firms). The difference between the profits of integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 under vertical separation can be written:
$(\Pi(0,c,2) + cq(c,0,2))) - 3\Pi(0,0,3) - 2\Pi(c,0,3) =$
$= \frac{(-2+\gamma)^2\gamma(4-3\gamma)+2c^2\gamma^2(14+\gamma(20+7\gamma))+4c(-2+\gamma)(4+\gamma(2+\gamma(-1+\gamma+\gamma^2)))}{4(-4-4\gamma+\gamma^2)^2} > 0$. This result is based on the fact that the previous expression is a convex function of c with two roots, and the lowest of the roots is higher than c_S^C.

For $c_S^C < c \leq c_{I2}^C$, $w_S^* = \frac{\gamma((-2+\gamma)\gamma+2c(2+\gamma))}{4+\gamma(4+\gamma(-3+2\gamma))}$ and $w_{I2}^* = c$, where $c_{I2}^C = \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$. The difference between the profits of integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 under vertical separation can be written:

$$(\Pi(0,c,2)+cq(c,0,2)))-$$
$$-\ 3\left(\Pi(w_S^*,2w_S^*,3)-\Pi(c,2w_S^*,3)+w_S^*q(w_S^*,2w_S^*,3)\right)-\Pi(c,2w_S^*,3)=$$
$$=\frac{\begin{array}{c}-(-2+\gamma)^2\gamma(-64+\gamma(-176+\gamma(-128+\gamma(-24+\gamma(-12+\gamma(13+\\+\gamma(16+\gamma(-1+6\gamma))))))))+2c^2\gamma^2(96+\gamma(384+\gamma(160+\gamma(-432+\\+\gamma(-114+\gamma(156+\gamma(-43+4\gamma(-2+7\gamma))))))))+4c(-2+\gamma)(64+\gamma(160+\\+\gamma(80+\gamma(32+\gamma(76+\gamma(-78+\gamma(-93+\gamma(11+\gamma(-9+4(-3+\gamma)\gamma)))))))))\end{array}}{4(16+32\gamma-12\gamma^3+7\gamma^4+\gamma^5-2\gamma^6)^2}>$$

0. This result is based on the fact that the difference is a convex function of c with two roots and the lowest of the roots is higher than c_{I2}^C.

For $c_{I2}^C<c\leq c_D^C$, $w_S^*=\frac{\gamma((-2+\gamma)\gamma+2c(2+\gamma))}{4+\gamma(4+\gamma(-3+2\gamma))}$ and $w_{I2}^*=\frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$.

The difference between the profits of integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 under vertical separation can be written:

$$(\Pi(0,w_{I2}^*,2)+\Pi(w_{I2}^*,0,2)-\Pi(c,0,2)+w_{I2}^*q(w_{I2}^*,0,2)))-$$
$$-\ 3\left(\Pi(w_S^*,2w_S^*,3)-\Pi(c,2w_S^*,3)+w_S^*q(w_S^*,2w_S^*,3)\right)-\Pi(c,2w_S^*,3)=$$
$$=\frac{\begin{array}{c}-2c(-2+\gamma)(-4+3\gamma^2)(-64+\gamma(-192+\gamma(-192+\gamma(-112+\gamma(-20+\gamma(84+\\+\gamma(34+\gamma(9+22\gamma))))))))--(2+\gamma)^2\gamma(128+\gamma(384+\gamma(256+\gamma(-192+\\+\gamma(-264+\gamma(-40+\gamma(12+\gamma(-28+\gamma(1+2\gamma)(9+(-1+\gamma)\gamma)))))))))+\\+c^2(-4+3\gamma^2(128+\gamma(512+\gamma(576+\gamma(64+\gamma(-264+\\+\gamma(-240+\gamma(-80+\gamma(-56+\gamma(-41+4\gamma(4+\gamma))))))))))\end{array}}{2(-4+3\gamma^2)(16+32\gamma-12\gamma^3+7\gamma^4+\gamma^5-2\gamma^6)^2}.$$

For $\gamma>0.8893$, this difference is positive in the entire interval. For $\gamma<0.8893$, the difference is positive (negative) if $c<(>)\ \overline{c}^P$. This result is based on the fact that the difference is a convex function of c with two roots, the lowest of the roots ($\overline{c}^P$) belongs to the interval (c_{I2}^C,c_D^C) and the highest of the roots is higher than c_D^C. ■

Proof of Proposition 4

Proof. We have to study the welfare consequences of entry-deterrent vertical integration only in the region where it occurs in equilibrium, as Proposition 3 describes. Notice that vertical integration is entry-deterrent when $\Pi(c,w_I^*,3)<\theta<\Pi(c,2w_S^*,3)$. In order to prove that welfare is always higher under vertical separation we have to compute as the entry cost the highest possible value in the previous interval, that is $\Pi(c,2w_S^*,3)$. If social welfare is higher for this value of the entry cost, we know that it will be also higher for any smaller value. In this region, we have that both under vertical integration and vertical separation, the upstream firms sets the unrestricted royalty. This means that we have to sign the following expression:

$$u(q(0,w_{I2}^*,2),x(w_{I2}^*,0,2))-(u(q(w_S^*,2w_S^*,3),q(w_S^*,2w_S^*,3),q(w_S^*,2w_S^*,3))-$$
$$\Pi(c,2w_S^*,3))=$$
$$=\frac{\begin{array}{c}(32+2c(2+\gamma)(-4+3\gamma^2)(-8+\gamma(2+\gamma(-2+\gamma))(-4+3\gamma+6\gamma^2))+\\+c^2(2+\gamma)^2(1+2\gamma)(-4+3\gamma^2)(4+\gamma(4+\gamma(-1+2\gamma)))+\gamma(32+\gamma(-16+\\+\gamma(80+\gamma(2+\gamma(-130+\gamma(56+\gamma(41+\gamma(-45+\gamma(11+2\gamma))))))))))\end{array}}{4(-4+3\gamma^2)(4+\gamma(8+\gamma-\gamma^2+2\gamma^3))^2}<$$

0. For $0.81<\gamma<0.90$, the result is based on the fact that this a convex function

of c with two roots y_1 and y_2, such that $y_1 < c_1^C < c_D^C < y_2$ which implies that the function is negative in the entire interval. For $0.90 < \gamma < 0.93$, the result is based on the fact that this a convex function of c with two roots y_1 and y_2, such that $y_1 < w_{I2}^* < c_D^C < y_2$ which implies that the function is negative in the entire interval. ■

Proof of Proposition 5

Proof. We have to compare the profits of the integrated firm when there are 3 firms downstream (entry has taken place) with the sum of the profits of the upstream firm and downstream firm 1, when there are only two downstream firms in the market. For $0 < c < c_I^C$, we have that the integrated firm is constrained to set a wholesale price equal to c and the unintegrated upstream firms is constrained to set a wholesale price equal to 0. The difference between the profits of the integrated firms and the sum of the profits of the upstream firm and downstream firm 1 can be written as: $\Pi(0,2c,3)+2cq(c,c,3)-2(\Pi(0,0,2)-\Pi(c,0,2))-\Pi(c,0,2)=$

$$= \frac{(-(-2+\gamma)^2\gamma(4-3\gamma)+4c(8+\gamma(12+\gamma(-4-6\gamma+\gamma^3)))+4c^2(-12+\gamma(-16+\gamma(4+\gamma(10+3\gamma)))))}{4(-4-4\gamma+\gamma^2+\gamma^3)^2}.$$

This is a concave function of c with two roots t_1 and t_2 such that $c_I^C < t_1 < t_2$, which implies that the function is negative in the entire interval $(0, c_I^C]$. For $c_I^C < c < c_P^C$, we have that the integrated firm is unconstrained and sets a wholesale price equal to $w_I^* = \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)}$ and the unintegrated upstream firms is constrained to set a wholesale price equal to 0. The difference between the profits of the integrated firms and the sum of the profits of the upstream firm and downstream firm 1 can be written as: $\Pi(0,2w_I^*,3)+2(w_I^*q(c,2w_I^*,3)+\Pi(w_I^*,w_I^*,3)-\Pi(c,w_I^*,3))-2(\Pi(0,0,2)-\Pi(c,0,2))-\Pi(c,0,2)=$

$$= \frac{\begin{array}{c}-(((-2+\gamma)^2\gamma(-16+\gamma(-20+\gamma^2(9+2\gamma(2+\gamma))))+4c^2(4+\\ +\gamma(4+3\gamma))(-4+\gamma(-8+\gamma(2+\gamma(6+\gamma))))+4c(-2+\gamma)\\ (-16+\gamma(-32+\gamma(2+\gamma(2+\gamma))(-6+\gamma(8+\gamma)))))\end{array}}{(4(-2+\gamma)^2(1+\gamma)^2(2+\gamma)^2(-4+\gamma(-4+5\gamma))))}.$$

This is a concave function of c with two roots b_1 and $\widetilde{c}^P$, such that $c_I^C < \widetilde{c}^P < c_P^C < b_1$, which implies the function is negative for $c_I^C < c < \widetilde{c}^P$ and positive for $\widetilde{c}^P < c < c_P^C$, where

$$\widetilde{c}^P = \frac{\begin{array}{c}-(((-2+\gamma)(-16+32L+\gamma(16(-2+7L)+\gamma(-12+96L+\\ +\gamma(4-48L+\gamma(12-90L+\gamma(10+\gamma-15L+16\gamma L+5\gamma^2L)))))))\end{array}}{2(4+\gamma(4+3\gamma))(-4+\gamma(-8+\gamma(2+\gamma(6+\gamma)))))}$$

and $L = \sqrt{-\frac{64+\gamma^2(-32+\gamma(-32+\gamma(-4+\gamma(24+\gamma(6+\gamma(6+\gamma))))))}{(2+\gamma)^4(2+\gamma-\gamma^2)^2(-4+\gamma(-4+5\gamma))}}$. For $c_P^C < c < c_D^B$, we have that the integrated firm is unconstrained and sets a wholesale price equal to $w_I^* = \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)}$ and the unintegrated upstream firms is unconstrained and sets a wholesale price equal $w_{S2}^* = \frac{\gamma(4c+\gamma(-2+\gamma))}{2(4-2\gamma^2+\gamma^3)}$. The difference between the profits of the integrated firms and the the sum of the profits of the upstream firm and downstream firm 1 can be written as: $\Pi(0,2w_I^*,3)+2(w_I^*q(c,2w_I^*,3)+\Pi(w_I^*,w_I^*,3)-\Pi(c,w_I^*,3))-2(\Pi(w_{S2}^*,w_{S2}^*,2)-\Pi(c,w_{S2}^*,2))-\Pi(c,w_{S2}^*,2)=$

$$
= \frac{\begin{array}{c} -(((-2+\gamma)^2\gamma(-256+\gamma(-320+\gamma(320+\gamma(496+ \\ +\gamma(-176+\gamma(-348+\gamma(60+\gamma(129+2\gamma(-10+ \\ +\gamma(-11+\gamma(3+\gamma))))))))))) + 4c^2(-256+\gamma(-768+ \\ +\gamma(-320+\gamma(768+\gamma(656+\gamma(-240+ \\ \gamma(-428+\gamma(72+\gamma(216+\gamma(-8+3\gamma(2+ \\ +\gamma(6+\gamma))))))))))) + 4c(-2+\gamma)(256+\gamma(-512+ \\ +\gamma(128+\gamma(608+\gamma(48+\gamma(-288+\gamma(-104+ \\ +\gamma(74+\gamma(74+\gamma(-14+\gamma(-14+\gamma(6+\gamma))))))))))))) \end{array}}{(4(-2+\gamma)^2(1+\gamma)^2(2+\gamma)^2(4+(-2+\gamma)\gamma^2)^2(-4+\gamma(-4+5\gamma))))}
$$

. This is a concave function of c with two roots m_1 and m_2, such that $0 < m_2 < c_P^C < c_D^B < m_1$, which that the function is positive in the entire interval (c_P^C, c_D^B). ■

Proof of Proposition 7

Proof.

$$
c(n) = \frac{\begin{array}{c} -(((-2+\gamma)(-8(-2+n) - 2\gamma^2(-2+n)^3 + \gamma^4(-2+n)(-1+n)^2+ \\ +\gamma^3(-1+n)(-1+2n) - 4\gamma(7+n(-7+2n)))) \end{array}}{\begin{array}{c} (2(2+\gamma(-2+n))(12+\gamma^3(-2+n)^2(-1+n) - 8n- \\ -4\gamma(-2+n)(-3+2n) - \gamma^2(-17+(-9+n)(-3+n)n)))) \end{array}}
$$

If $c \geq c_{2N}^E$, the profits of the entrant are zero both with vertical integration and vertical separation. If $c_{2N}^I \leq c < c_{2N}^E$, the profits of the entrant are positive with vertical separation and zero with vertical integration. If $c_{1N}^E \leq c < c_{2N}^I$, the optimal wholesale prices with both vertical integration and vertical separation are the unrestricted ones (namely, they are lower than c) and the corresponding profit comparison between the entrant's profits under vertical separation and under vertical integration leads to:

$$
\frac{(-2+\gamma)^2(4-2c(2+\gamma(n-2))(1+\gamma(n-1))+\gamma(-6+\gamma(n-3)(n-1)+4n))^2}{4(2+\gamma(n-1))^2(4+\gamma(4(n-2)+\gamma(6+\gamma(n-1)+n(n-6))))^2} > (<)
$$

$$
> (<) \frac{(-2+\gamma-\frac{2c(-4+\gamma(8+3\gamma(-1+n)-4n))(2+\gamma(-2+n))+\gamma^2(-2+\gamma)(-2+\gamma(-1+n))(-2+n)}{2(4+4\gamma(-2+n)+\gamma^2(7+(-7+n)n))})^2}{(-2+\gamma)^2(2+\gamma(n-1))^2} \text{if } c >
$$

$(<)\ c(n)$.

If $\gamma < \frac{2}{n-1}$ and $c_{1N}^I \leq c < c_{1N}^E$ the optimal royalty with vertical integration is the unrestricted one while it is zero with vertical separation. It is easy to conclude that, in this region, the profits of the entrant are higher with vertical integration than with vertical separation. The reason is that whereas with both vertical separation and vertical integration the entrant faces an equally efficient downstream firm 1 with a marginal cost equal to 0, with vertical integration the entrant faces a more efficient n-2 independent downstream firms than under vertical separation. Specifically, with vertical integration, the independent downstream firms have a marginal cost equal to $r_I^* > 0$ while it has cost 0 with vertical separation. If $\gamma < \frac{2}{n-1}$ and $0 \leq c < c_{1N}^I$ the optimal royalty with vertical integration is equal to c while it is zero with vertical separation. This implies that the entrant's profits are higher under vertical integration than under vertical separation.

If $\gamma > \frac{2}{n-1}$ and $c_{0N}^I \leq c < c_{1N}^E$, the optimal royalty with vertical integration is the unrestricted one (and positive) while it is zero with vertical separation. It is direct to see that, in this region, the profits of the entrant are higher with vertical integration than with vertical separation. If $\gamma > \frac{2}{n-1}$ and $0 \leq c < c^{I}_{0N}$, the wholesale prices with both vertical integration and separation are equal to zero. In this case, both profits coincide. ■

Proof of Proposition 8

Proof. We start by comparing the entrant's profits when the upstream firm merges with the two incumbent downstream firms and when it merges with only one of them. This means to sign $\Pi^E(c) - \Pi(c, w_I^*, 3)$,

where $w_I^* = \begin{cases} \frac{\gamma(2-3\gamma+\gamma^2+c(2+\gamma))}{4-\gamma(-4+5\gamma)} \text{if } c > \frac{(2-\gamma)\gamma}{4+6\gamma} = c_I^C \\ c \quad \text{otherwise} \end{cases}$.

In both the cases of restricted and unrestricted optimal wholesale price, we have that the difference is a convex function with two roots, which are higher than c^D. This implies that $\Pi^E(c) - \Pi(c, w_I^*, 3) > 0$, for $c \in [0, c_D^C]$.

Next we compare the entrant's profits when the upstream firm merges with the two incumbent downstream firms and when there is no merger. This means to sign $\Pi^E(c) - \Pi(c, 2w_S^*, 3)$, where $w_S^* = \begin{cases} \frac{\gamma((-2+\gamma)\gamma+2c(2+\gamma))}{4+\gamma(4+\gamma(-3+2\gamma))} \text{if } c > \frac{(2-\gamma)\gamma}{2(2+\gamma)} = c_S^C \\ 0 \quad \text{otherwise} \end{cases}$.

When $c < c_S^C$, the difference is a concave function of c, with two roots, one is negative and the other is positive and higher than c_S^C, which implies that $\Pi^E(c) - \Pi(c, 2w_S^*, 3) > 0$ for $c \in [0, c_S^C]$. When $c > c_S^C$, the difference is a convex function of c, with two positive roots. If $\gamma > 0.69$, we have that the two roots are higher than c^D, which implies that $\Pi^E(c) - \Pi(c, 2w_S^*, 3) > 0$ for $c \in [c_S^C, c^D]$. If $\gamma < 0.69$, the lowest root s_1 is lower than c^D and the highest root higher than c^D, where $s_1 = \frac{4+\gamma(8+\gamma)}{8+\gamma(14+\gamma(5+2\gamma))}$. This implies that $\Pi^E(c) - \Pi(c, 2w_S^*, 3) > 0$ for $c \in [c_S^C, s_1]$ and $\Pi^E(c) - \Pi(c, 2w_S^*, 3) < 0$ for $c \in [s_1, c_D^C]$. ■

Proof of Proposition 9

Proof. We start by comparing the profitability of a merger with the two incumbents that induces entry and the merger with only one incumbent without entry. The relevant cut-off for this case is $r(\gamma) = \begin{cases} s_2(\gamma) \text{ if } \gamma < 0.76 \\ t_2(\gamma) \quad \text{otherwise} \end{cases}$, where $s_2(\gamma)$ and $t_2(\gamma)$ are very cumbersome expressions of γ and are available upon request. For $0 < c \leq c_{I2}^C$, $w_{I2}^* = w_E^* = c$. The difference between the profits of the integrated firm under the merger with the two incumbents and the sum of the profits of the integrated firm under the merger with only one incumbent plus the profits of the independent incumbent in the latter case can be written: $\Pi(c) + cq^E(c) - (\Pi(0, c, 2) + cq(c, 0, 2)) - \Pi(c, 0, 2) < 0$. This result is based on the fact that the previous expression is a concave function of c with two roots both of which are higher than c_{I2}^C. For $c_{I2}^C < c \leq w_E^*$, $w_{I2}^* = \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$ and $w_E^* = c$. The difference between the profits of the integrated firm under the merger with the two incumbents and the sum of the profits of the integrated firm under the merger with only one incumbent plus the profits of the independent incumbent in the latter case can be written: $\Pi(c)+cq^E(c)-(\Pi(0, w_{I2}^*, 2)+w_{I2}^*q(w_{I2}^*, 0, 2)+\Pi(w_{I2}^*, 0, 2)-\Pi(c, 0, 2))-\Pi(c, 0, 2)$. For $\gamma < 0.76$, the difference is negative in the entire interval and for $\gamma > 0.76$ the difference is positive (negative) if $c >$ ($<$) $t_2(\gamma)$. The result is based on the fact that the difference is a concave function of c with two roots and the lowest of the roots $(t_2(\gamma)))$ is higher (lower) than w_E^* if γ is lower (higher) than 0.76 and the highest of the roots is always above w_E^*. For $w_E^* < c \leq \min\{s_1,c^D\}$, $w_{I2}^* = \frac{\gamma(2-\gamma)^2}{2(4-3\gamma^2)}$ and $w_E^* = \frac{\gamma(-2+\gamma^2)}{-2+\gamma(-4+\gamma+3\gamma^2)}$. The difference between the profits of the integrated firm under the merger with the two incumbents and the sum of the profits of the integrated firm under the merger with only one incumbent plus the profits of the independent incumbent in the latter case can be written: $\Pi(w_E^*) + \left(\Pi^E(w_E^*) - \Pi^E(c)\right) + w_E^* q^E(w_E^*) - (\Pi(0, w_{I2}^*, 2) + w_{I2}^* q(w_{I2}^*, 0, 2) + \Pi(w_{I2}^*, 0, 2) - \Pi(c, 0, 2)) - \Pi(c, 0, 2)$. For $\gamma > 0.76$, the difference is positive in the entire interval. For $\gamma < 0.76$ the difference is positive (negative) if c is higher (lower) than $s_2(\gamma)$. The reason is based on the fact that the difference is a concave function of c with two roots. The lowest of the roots $(s_2(\gamma))$ is higher (lower) than w_E^* if γ is lower (higher) than 0.76. For $\gamma > 0.76$, the lowest of the roots is lower than w_E^* and the highest of the roots is above $\min\{s_1,c_D^C\}$. The cut-off $r(\gamma)$ results from a straightforward exercise of merging the three regions we have just described.

We continue by comparing the profitability of a merger with the two incumbents that induces entry given that with no merger there is no entry. The relevant cut-off for this case is $r'(\gamma) = \begin{cases} n_2(\gamma) & \text{if } \gamma < 0.59 \\ k_2(\gamma) & \text{otherwise} \end{cases}$, where $n_2(\gamma)$ and $k_2(\gamma)$ are very cumbersome expressions of γ and are available upon request. For $0 < c \leq c_P^C$, $w_{S2}^* = 0$ and $w_E^* = c$. The difference between the profits of the integrated firm under the merger with the two incumbents and the sum of the profits of the upstream firms and the two independent downstream firms can be written: $\Pi(c) + cq^E(c) - 2(\Pi(0, 0, 2) - \Pi(c, 0, 2)) - 2\Pi(c, 0, 2) < 0$. This result is based on the fact that the previous expression is a concave function of c with two roots both of which are higher than c_P^C. For $c_P^C < c \leq w_E^*$, $w_{S2}^* = \frac{\gamma(4c--\gamma(2-\gamma))}{2(4-2\gamma^2+\gamma^3)}$ and $w_E^* = c$. The difference between the profits of the integrated firm under the merger with the two incumbents and the sum of the profits of the upstream firms and the two independent downstream firms can be written: $\Pi(c) + cq^E(c) - 2\left(\Pi(w_{S2}^*, w_{S2}^*, 2) - \Pi(c, w_{S2}^*, 2) + w_{S2}^* q(w_{I2}^*, w_{S2}^*, 2)\right) - 2\Pi(c, w_{S2}^*, 2)$. For $\gamma < 0.59$, the difference is negative in the entire interval and for $\gamma > 0.59$ the difference is positive (negative) if $c > (<)\ k_2(\gamma)$. The result is based on the fact that the difference is a concave function of c with two roots and the lowest of the roots $(k_2(\gamma)))$ is higher (lower) than w_E^* if γ is lower (higher) than 0.59 and the highest of the roots is always above w_E^*. For $w_E^* < c \leq \min\{s_1,c_D^C\}$, $w_{S2}^* = \frac{\gamma(4c--\gamma(2-\gamma))}{2(4-2\gamma^2+\gamma^3)}$ and $w_E^* = \frac{\gamma(-2+\gamma^2)}{-2+\gamma(-4+\gamma+3\gamma^2)}$. The difference between the profits of the integrated firm under the merger with the two incumbents and the sum of the profits of the upstream firms and the two independent downstream firms can be written: $\Pi(w_E^*) + \left(\Pi^E(w_E^*) - \Pi^E(c)\right) + w_E^* q^E(w_E^*) - 2\left(\Pi(w_{S2}^*, w_{S2}^*, 2) - \Pi(c, w_{S2}^*, 2) + w_{S2}^* q(w_{I2}^*, w_{S2}^*, 2)\right) - 2\Pi(c, w_{S2}^*, 2)$. For $\gamma > 0.59$, the difference is positive in the entire interval. For $\gamma < 0.59$ the difference is positive (negative) if c is higher (lower) than $n_2(\gamma)$. The reason is based on the fact that the difference is a concave function of c with two roots. The lowest of the roots $(n_2(\gamma))$ is higher (lower) than w_E^* if γ is lower (higher) than

0.59. For $\gamma > 0.59$, the lowest of the roots is lower than w_E^* and the highest of the roots is above $\min\{s_1, c_D^C\}$. The cut-off $r'(\gamma)$ results from a straightforward exercise of merging the three regions we have just described.

The result in proposition results from the fact that $r(\gamma) > r'(\gamma)$ for $\gamma \in [0, 1]$. ■

Proof of Proposition 10

Proof. In the restricted case, we check that $\frac{-2+c+\gamma(1-c)}{(-2+\gamma)(1+\gamma)} - \frac{1}{1+\gamma} = \frac{c(1-\gamma)}{(-2+\gamma)(1+\gamma)} < 0$, which implies that, in this region, vertical integration increases the entrant's profits. In the unrestricted case, we have to sign $\frac{16+4(-3+c)+2(-4+c)\gamma^2-5(-1+c)\gamma^3}{(-4+\gamma^2)(-4+\gamma(-2+5\gamma))} - \frac{1}{1+\gamma}$. It is direct to see that the difference is negative if $c < \min\{\frac{(-2+\gamma)^2}{(1+\gamma)(4-\gamma(-2+5\gamma))}, c_D^C\}$ and positive otherwise. ■

Proof of Proposition 11

Proof. First, we study the region where c is so low that with both vertical separation and vertical integration the upstream firm is constrained to set $w = c$, namely, $c \in (0, \min\{c_S^B, c_D^B\}]$, where $c_S^B = \frac{\gamma^2}{2+3\gamma+\gamma^2}$. In this region, we have to sign $\Pi_i^*(c, c, c) - \Pi_{3N}^*(c, c)$. It is direct to see that this is a concave function with two roots, the lowest one is equal to zero and the highest one, $c_1^B = \frac{2(-2-\gamma+3\gamma^2}{-4-3\gamma+6\gamma^2}$.Given that $c_1^B > \min\{c_S^B, c_D^B\}$, we can conclude that vertical integration reduces the entrant's profits in the entire region.

Second, we study the region where the upstream firm sets the unconstrained wholesale price under vertical separation whereas it sets $w = c$ under vertical integration. This region is defined by the values of c such that when $\gamma < 0.69$, $c \in [c_S^B, c_I^B]$ and when $0.69 < \gamma < 0.88$ $c \in [c_S^B, c_D^B]$, where $c_I^B = \frac{(-1+\gamma)\gamma(1+2\gamma)(2+3\gamma)}{-4+3\gamma(1+\gamma)(-2+3\gamma^2)}$. When $\gamma > 0.880$, we have that $c_D^B < c_S^B$ and so both firms are restricted to set $w = c$, the region analyzed in the previous paragraph. Observe that the optimal wholesale price with vertical integration is higher than the one with vertical separation. In this region, we have to sign $\Pi_i^*(c, w_S^*, w_S^*) - \Pi_{3N}^*(c, c)$. As long as $\gamma < 0.81$, this is a convex function of c with two roots c_2^B and c_3^B where $c_2^B = \frac{2(-1+\gamma)\gamma^2(2+3\gamma)}{-4+\gamma(-8+\gamma(3+2\gamma)(1+3\gamma))}$ and $c_3^B = \frac{2(-1+\gamma)(2+3\gamma)(-4+\gamma(-8+\gamma(-1+4\gamma)))}{16+\gamma(52+\gamma(20+\gamma(-67+\gamma(-37+24\gamma))))}$. If $\gamma < 0.66$, we have $c_S^B < c_2^B < c_I^B < c_3^B$. This implies that vertical integration reduces the entrant's profits when $c_S^B < c < c_2^B$ and increases the entrant's profits when $c_2^B < c < c_I^B$. If $0.66 < \gamma < 0.69$, we have $c_S^B < c_I^B < c_2^B < c_3^B$. This implies that vertical integration reduces the entrant's profits in the entire interval $(c_S^B, c_I^B]$. If $0.69 < \gamma < 0.74$, we have $c_S^B < c_D^B < c_2^B < c_3^B$. This implies that vertical integration reduces the entrant's profits in the entire interval $(c_S^B, c_D^B]$. If $0.74 < \gamma < 0.81$, we have $c_S^B < c_D^B < c_3^B < c_2^B$. This implies that vertical integration reduces the entrant's profits in the entire interval $(c_S^B, c_D^B]$. If $0.81 < \gamma < 0.88$, $\Pi_i^*(c, w_S^*, w_S^*) - \Pi_{3N}^*(c, c)$ is a concave function of c with two roots c_2^B and c_3^B and we have $c_2^B < 0 < c_S^B < c_D^B < c_3^B$. This implies that vertical integration reduces the entrant's profits in the entire interval $(c_S, c_D^B]$.

Lastly, we study the region where the upstream firm sets the unconstrained wholesale price with both vertical separation and vertical integration. This region is defined by the values of $c \in (c_I^B, c_D^B)$, where $c_I^B < c_D^B$ only if $\gamma < 0.69$. In this region, we have to sign $\Pi_i^*(c, w_S^*, w_S^*) - \Pi_{3N}^*(c, w_I^*)$, which is a concave function of c with two roots c_4^B and c_5^B, where $c_4^B = \frac{(2+\gamma)(2+3\gamma)(-2+(-7+\gamma)\gamma(1+2\gamma))}{(1+\gamma)(-2+(-3+\gamma)\gamma)(12+\gamma(28+5\gamma(11+6\gamma)))}$ and $c_5^B = \frac{(2+3\gamma)(32+\gamma(128+\gamma(228+\gamma(128+\gamma(-185+\gamma(-231+20\gamma(1+3\gamma)))))))}{(-2+(-3+\gamma)\gamma)(-32+\gamma(-160+\gamma(-364+\gamma(-392+\gamma(-33+\gamma(281+150\gamma))))))}$. If $\gamma < 0.66$, we have $c_I^B < c_4^B < c_D^B < c_5^B$. This implies that vertical integration increases the entrant's profits when $c \in (c_I^B, c_4^B]$. If $0.66 < \gamma < 0.69$, we have $c_4^B < c_I^B < c_D^B < c_5^B$. This implies that vertical integration reduces the entrant's profits in the entire region.

Proposition 11 results from a straightforward exercise of merging the three regions we have just described. ■

Proof of Proposition 12:

Proof. We start by solving the case of vertical integration for the case of two downstream firms. The direct demand functions are given by: $x(p_i, p_j) = \frac{1}{1+\gamma} - \frac{p_i}{1-\gamma^2} + \gamma\frac{p_j}{1-\gamma^2} \quad i, j = 1, 2, i \neq j$.In the market stage the integrated firm maximizes $p_1 x(p_1, p_2) + w x(p_2, p_1)$ and firm 2 maximizes $(p_2 - w)\ x(p_2, p_1)$. This leads to the following equilibrium prices and profits: $p_1^*(w) = \frac{2-\gamma-\gamma^2+3\gamma w}{4-\gamma^2}$, $p_2^*(w) = \frac{2-\gamma-\gamma^2+(2+\gamma^2)w}{4-\gamma^2}$, $\pi_1(w) = p_1^*(w)x(p_1^*(w), p_2^*(w))$ and $\pi_2(w) = (p_2^*(w) - w)x(p_2^*(w), p_1^*(w))$. Given that the participation constraint is binding and that the integrated firm always sells the input to firm 2, this firm solves: $\max_r \{\pi_1(w) + wx(p_2^*(w), p_1^*(w)) + \pi_2(w) - \Pi_2(c)\}$, where $\Pi_2(c)$ represents the outside option of firm 2. The solution to this program results in $w^* = \min\{w_{I2}^*, c\}$, where $w_{I2}^* = \frac{\gamma(2+\gamma)^2}{2(4+5\gamma^2)}$ and $f^* = \pi_2(w^*) - \Pi_2(c)$. Next, we define the cut-off value $\widehat{c}$ that appears in Proposition 12:

$\widehat{c}^P = \begin{cases} \widehat{c}_1^P & \text{when } \gamma \leq 0.6519 \\ \widehat{c}_2^P & \text{when } 0.6519 < \gamma \leq 0.6609 \end{cases}$ where

$\widehat{c}_1^P$and $\widehat{c}_2^P$are complex functions of γ and are available upon request. For all $\gamma \in (0, 1)$ and $0 < c < \min\{c_S^B, c_D^B\}$, we have that $w_S^* = w_{I2}^* = c$ and the difference between the profits of the integrated firm and the sum of the profits of the independent upstream firm plus the profits of downstream firm 1 under vertical separation can be written:

$\pi_1(c) + cx(p_2^*(c), p_1^*(c)) + \pi_2(c) - \Pi_2(c) - 3\,(cx(p^*(c,c,c), p^*(c,c,c), p^*(c,c,c))) - \Pi_1^*(c,c,c) =$

$= \frac{1}{4(1+2\gamma)(-4+\gamma^2)^2(-1+\gamma^2)}(\gamma(-2+\gamma+\gamma^2)^2(-4+(-3+\gamma)\gamma(1+\gamma)) - 2c(-1+\gamma)(2+\gamma)(8+\gamma(12+$
$+\gamma(-2+\gamma(2+\gamma)(-11+\gamma^2)))) + c^2(-48+\gamma(-32+\gamma(92+\gamma(-1+\gamma)(-120+\gamma(-83+\gamma(3+\gamma(7+\gamma))))))) > 0$. This result is based on the fact that this is a convex function of c. For $0 < \gamma < 0.58$ and for $0.72 < \gamma < 0.76$ it has two roots $s_1$and s_2, such that $0 < c < c_S^B < s_1 < s_2$. which implies that the difference is positive in the entire interval $(0, \min\{c_S^B, c_D^B\})$. For $0.58 < \gamma < 0.72$, the function has no roots which implies that it is positive. For $0.7639 < \gamma < 1$, the difference is a concave function with two roots one of the is negative and the positive one is higher than the $\max\{c_S^B, c_D^B\}$, which implies that the difference is positive in the entire interval $(0, \min\{c_S^B, c_D^B\})$.

For $0 < \gamma < 0.8807$ and $c_S^B < c < \min\{c_{I2}^B, c_D^B\}$, we have that $w_S^* = \frac{\gamma((-1+\gamma)\gamma(2+3\gamma)+2c(1+\gamma)(-2+(-3+\gamma)\gamma))}{(1+\gamma)(-4+\gamma(-8+\gamma(-1+5\gamma)))}$ and $w_{I2}^* = c$ and the difference between the profits of the integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 under vertical separation can be written:

$$\pi_1(c) + cx(p_2^*(c), p_1^*(c)) + \pi_2(c) - \Pi_2(c) -$$
$$- 3\left(w_S^* x(p^*(w_S^*, w_S^*, w_S^*), p^*(w_S^*, w_S^*, w_S^*), p^*(w_S^*, w_S^*, w_S^*)) + \right.$$
$$\left. + \Pi_1(w_S^*, w_S^*, w_S^*) - \Pi_1(c, w_S^*, w_S^*)\right) - \Pi_1(c, w_S^*, w_S^*) =$$
$$= \frac{\begin{array}{c} -\gamma(-2+\gamma+\gamma^2)^2(64+\gamma(304+\gamma(448+\gamma(-8+ \\ +\gamma(-500+\gamma(-197+\gamma(180+\gamma(47+6\gamma(-4+ \\ +\gamma)))))))))+2c^2\gamma^2(160+\gamma(960+\gamma(1888+\gamma(432+ \\ +\gamma(-3150+\gamma(-3016+\gamma(1371+\gamma(2454+\gamma(-101+ \\ +\gamma(-652+\gamma(23-3\gamma(-6+\gamma)))))))))))+4c(-1+\gamma) \\ (2+\gamma)(-64+\gamma(-288+\gamma(-304+\gamma(464+\gamma(1204+ \\ +\gamma(526+\gamma(-689+\gamma(-700+\gamma(8+ \\ +\gamma(209+3\gamma(10+\gamma(-5+\gamma)))))))))))) \end{array}}{4(1+2\gamma)(-4+\gamma^2)^2(-1+\gamma^2)(-4+\gamma(-8+\gamma(-1+5\gamma)))^2}$$

This is a concave function of c with two roots. On of them is negative and the other one, $\widehat{c}_1^P$, is positive. For $0 < \gamma < 0.6519$, we have that $0 < c_S^B < \widehat{c}_1^P < c_{I2}^B$, which implies that the difference is positive for $c_S^B < c < \widehat{c}_1^P$ and it is negative for $\widehat{c}_1^P < c < c_{I2}^B$. For $0.6519 < \gamma < 0.8807$, we have that $\widehat{c}_1^P \min\{c_{I2}^B, c_D^B\}$, which implies that the difference is positive in the entire interval $(c_S^B, \min\{c_{I2}^B, c_D^B\})$.Note that for $\gamma > 0.8807$ the previous interval is empty.

For $0 < \gamma < 0.6626$ and for $c_{I2}^B < c < c_D^B$, we have that $w_S^* = \frac{\gamma((-1+\gamma)\gamma(2+3\gamma)+2c(1+\gamma)(-2+(-3+\gamma)\gamma))}{(1+\gamma)(-4+\gamma(-8+\gamma(-1+5\gamma)))}$ and $w_{I2}^* = \dfrac{\gamma(2+\gamma)^2}{2(4+5\gamma^2)}$ and the difference between the profits of the integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 under vertical separation can be written:

$$\pi_1(w_{I2}^*) + w_{I2}^* x_2(p_2^*(w_{I2}^*, 0), p_1^*(0, w_{I2}^*)) + \pi_2(w_{I2}^*) - \Pi_2(c) -$$
$$-3\left(w_S^* x(p^*(w_S^*, w_S^*, w_S^*), p^*(w_S^*, w_S^*, w_S^*), p^*(w_S^*, w_S^*, w_S^*))\right) - \Pi_1(c, w_S^*, w_S^*) =$$
$$= \frac{\begin{array}{c} ((-8c(-1+\gamma)(2+\gamma)(-2+\gamma^2)(4+5\gamma^2)+4c^2(-2+\gamma^2)^2(4+5\gamma^2)+ \\ +(-1+\gamma)(2+\gamma)^2(16+\gamma(-16+\gamma(24+\gamma(-12+\gamma(-5+\gamma)))))) \end{array}}{4(-4+\gamma^2)^2(-1+\gamma^2)(4+5\gamma^2)} -$$
$$- \frac{\begin{array}{c} 16+64\gamma-4c(-2+\gamma)(1+\gamma)^2(-2+(\gamma(-3+\gamma))(-4+\gamma(-8+ \\ +3(-1+\gamma)\gamma))+2c^2(-2+(-3+\gamma)\gamma)^2(-4+ \\ +\gamma(-12-11\gamma+3\gamma^3))+\gamma^2(56+\gamma(-72+\gamma(-107+ \\ +\gamma(14+\gamma(35+6\gamma(-2+\gamma)))))) \end{array}}{4(1+2\gamma)(4+\gamma(8+\gamma-5\gamma^2))^2}.$$

This is a convex function of c with two roots, $\widehat{c}_2^P$ and v_2. For $0 < \gamma < 0.6519$, we have $\widehat{c}_2^P < c_{I2}^B < c_D^B < v_2$, which implies that the difference is negative in the entire interval (c_{I2}^B, c_D^B). For $0.6519 < \gamma < 0.6609$, we have $c_{I2}^B < \widehat{c}_2^P < c_D^B < v_2$, which implies that the difference is negative for $\widehat{c}_2^P < c < c_D^B$, and it is positive for $c_{I2}^B < c < \widehat{c}_2^P$. For $0,6609 < \gamma < 0.6626$, we have $c_{I2}^B < c_D^B < \widehat{c}_2^P < v_2$, which implies that the difference is positive in the entire interval (c_{I2}^B, c_D^B). For $\gamma > 0.6626$, the previous interval is empty. Proposition 12 results from a straightforward exercise of merging the three regions we have just described. ■

Proof of Proposition 14

Proof. We start by solving the case of vertical separation for the case of two downstream firms. In the market stage, if the two downstream have accepted the supply contract, firm i (i=1,2) maximizes $(p_i - w)x(p_i, p_j)$. This leads to the following equilibrium prices and profits $p_i^*(w,w) = p_j^*(w,w) = \frac{-1+\gamma-w}{-2+\gamma}$, $\pi_i(w,w) = (p_i^*(w,w) - w)x(p_i^*(w,w), p_j^*(w,w)) \quad i,j = 1,2 \ i \neq j$. If only one downstream firm (say firm 1) have accepted the supply contract, firm 1 maximizes $(p_1 - w)x(p_1, p_2)$ and firms 2 maximizes $(p_2 - c)x(p_2, p_1)$. This leads to the following equilibrium prices and profits $p_1^*(w,c) = \frac{\gamma - c\gamma + \gamma^2 - 2(1+w)}{-4+\gamma^2}$, $p_2^*(w,c) = \frac{-2-2c+\gamma+\gamma^2-\gamma w}{-4+\gamma^2}$, $\Pi_1(w,c) = (p_1^*(w,c) - w)x(p_1^*(w,c), p_2^*(w,c))$ and $\Pi_2(w,c) = (p_2^*(w,c) - c)x(p_2^*(w,c), p_1^*(w,c))$. Given that the participation constraint is binding and that the upstream firm always sells the input to both firms, it solves: $\max\limits_{r}\{2(wx(p_1^*(w,w), p_2^*(w,w)) + \pi_2(w,w) - \Pi_2(w,c)\}$. The solution to this program results in $w^* = \min\{w_{S2}^*, c\}$, where $w_{S2}^* = \frac{\gamma(2c(-2+\gamma^2)+\gamma(-2+\gamma+\gamma^2))}{2(-4+\gamma^2(2+\gamma))}$ and $f^* = \pi_2(w^*, w^*) - \Pi_2(w^*, c)$.

Suppose that $0 < c < \min\{c_E^B, c_D^B\}$. In this region, both with vertical integration (and three downstream firms) and vertical separation (and two downstream firms), the upstream firm is constrained to set a restricted wholesale price equal to c. The difference between the profits of the integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 with vertical separation can be written:

$$\Pi_1^*(c) + 2cx(p_2^*(0,c,c), p_1^*(0,c,c), p_3^*(0,c,c)) + 2(\Pi_3^*(c) - \Pi_{3N}^*(c,c)) - 2\left(cx(p_1^*(c,c), p_2^*(c,c))\right) - \Pi_1(c,c) =$$

$$\frac{\begin{array}{l}(8+\gamma(20+\gamma(-10+4c(-1+\gamma)(2+3\gamma)(-4+3\gamma(-2 \\ +\gamma+\gamma^2))+\gamma(-49+\gamma(-4+\gamma(38+6\gamma-9\gamma^2)))- \\ -2c^2(20+\gamma(38+\gamma(-34+5\gamma(-11+\gamma(4+3\gamma))))))))\end{array}}{(4(-2+\gamma)(-1+\gamma)(1+\gamma)(1+2\gamma)(2+3\gamma)^2)}$$

.This is a concave function of c with two roots, h_1 and h_2. For $0 < \gamma < 0.825$, we have $0 < c_E^B < h_2 < h_1$, which implies that this function is negative in the entire interval $(0, c_E^B)$. For $0.825 < \gamma < 0.8664$, we have $0 < h_2 < \min\{c_E^B, c_D^B\} < h_1$. which implies that the function is negative in the interval $(0, h_2)$ and positive in the interval $(h_2, \min\{c_E^B, c_D^B\})$. For $0.8664 < \gamma < 1$, we have $0 < c_D^B < h_2 < h_1$, which implies that the function is negative in the entire interval.

Suppose now that $c_E^B < c < \min\{c_I^B, c_D^B\}$. In this region, the integrated firm is constrained to set a restricted wholesale price equal to c with vertical integration, whereas with vertical separation the upstream firm sets the unconstrained royalty w_{S2}^*. The difference between the profits of the integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 with vertical separation can be written:

$$\Pi_1^*(c) + 2cx(p_2^*(0,c,c), p_1^*(0,c,c), p_3^*(0,c,c)) + 2(\Pi_3^*(c) - \Pi_{3N}^*(c,c)) - 2\left(w_{S2}^* x(p_1^*(w_{S2}^*, w_{S2}^*), p_2^*(w_{S2}^*, w_{S2}^*)) + + \Pi_1(w_{S2}^*, w_{S2}^*) - \Pi_1(w_{S2}^*, c)\right) - \Pi_1(w_{S2}^*, c) =$$

$$= \frac{\begin{array}{c} -(((2+\gamma-3+\gamma^2)^2(-16+\gamma(-16+\gamma(20+\gamma(20+ \\ +\gamma(-4-3\gamma+\gamma^3)))))-4c(-1+\gamma)+\gamma 2(2+3\gamma) \\ (-40+\gamma(-52+\gamma(69+\gamma(60+\gamma(-28+3\gamma(-6+\gamma+\gamma^2))))))+ \\ +2c^2(64+\gamma(192+\gamma(112+\gamma(-224+\gamma(-552+\gamma(-168+ \\ +\gamma(536+\gamma(298+\gamma(-174+5\gamma(-23+\gamma(4+3\gamma)))))))))))) \end{array}}{(4(-2+\gamma)^2(-1+\gamma)(1+\gamma)(1+2\gamma)(2+3\gamma)^2(-4+\gamma^2(2+\gamma))))}$$

.This is a concave function with two positive roots l_1 and l_2. When $\gamma \in (0, 0.8250)$, we have $0 < c_E^B < l_2 < \min\{c_I^B, c_D^B\} < l_1$, which implies that the function is negative for $0 < c < l_2$ and positive for $l_2 < c < \min\{c_I^B, c_D^B\}$. When $\gamma \in (0.8250, 0.8485)$ we have $l_2 < c_E^B < c_D^B < l_1$, which implies that the function is positive in the entire interval $c_E^B < c < c_D^B$.

Suppose now that $c_I^B < c < c^D$. In this region, both with vertical separation and vertical integration, the upstream firm sets the unconstrained royalties w_{S2}^* and w_I^*. The difference between the profits of the integrated firm and the sum of the profits of the independent upstream firms plus the profits of downstream firm 1 with vertical separation can be written:

$$\Pi_1^*(w_I^*) + 2w_I^* x(p_2^*(0, w_I^*, w_I^*), p_1^*(0, w_I^*, w_I^*), p_3^*(0, w_I^*, w_I^*))$$
$$+2(\Pi_3^*(w_I^*) - \Pi_{3N}^*(w_I^*, c)) - 2\,(w_{S2}^* x(p_1^*(w_{S2}^*, w_{S2}^*), p_2^*(w_{S2}^*, w_{S2}^*)) +$$
$$+\Pi_1(w_{S2}^*, w_{S2}^*) - \Pi_1(w_{S2}^*, c)) - \Pi_1(w_{S2}^*, c) =$$

$$= \frac{\begin{array}{c} (-1024+\gamma(-5120+4c^2\gamma(-4+\gamma^2(2+\gamma))(-32+ \\ +\gamma(-128+\gamma(-208+\gamma(-176+\gamma(138+\gamma(652+\gamma(657+ \\ +\gamma(251+\gamma(-6+\gamma(-126+\gamma(-109+\gamma(-19+6\gamma))))))))))) \\ -4c(2+3\gamma)(-256+\gamma(-768+\gamma(448+\gamma(1+\gamma)(1536+ \\ +\gamma(496+\gamma(-2976+\gamma(-180+\gamma(2452+\gamma(-310+\gamma(-790+ \\ \gamma(363+\gamma(79+3\gamma(-39+\gamma(-1+2\gamma)))))))))))))+ \\ +\gamma(-8704+\gamma(256+\gamma(22016+\gamma(23296+\gamma(-16096+ \\ +\gamma(-43104+\gamma(-15212+\gamma(23540+\gamma(27488+\gamma(4721+ \\ \gamma(-10947+\gamma(-6316+\gamma(490+9\gamma(97+ \\ +3\gamma(7+2\gamma))))))))))))))))) \end{array}}{\begin{array}{c} (4(1+2\gamma)(2+3\gamma)^2(-4+\gamma^2)^2(-1+\gamma^2)(-4+\gamma^2(2+\gamma)) \\ (-4+\gamma(1+\gamma)(-8+\gamma(-3+10\gamma)))) \end{array}}$$

For $\gamma \in (0, 0.6936)$ this is a concave function with two roots, one is negative and the other one is positive (n_1). We have that $0 < c < c_I^B < c_D^B < n_1$, which implies that the function is positive in the entire interval (c_I^B, c_D^B). Proposition 14 results from a straightforward exercise of merging the three regions we have just described. ■

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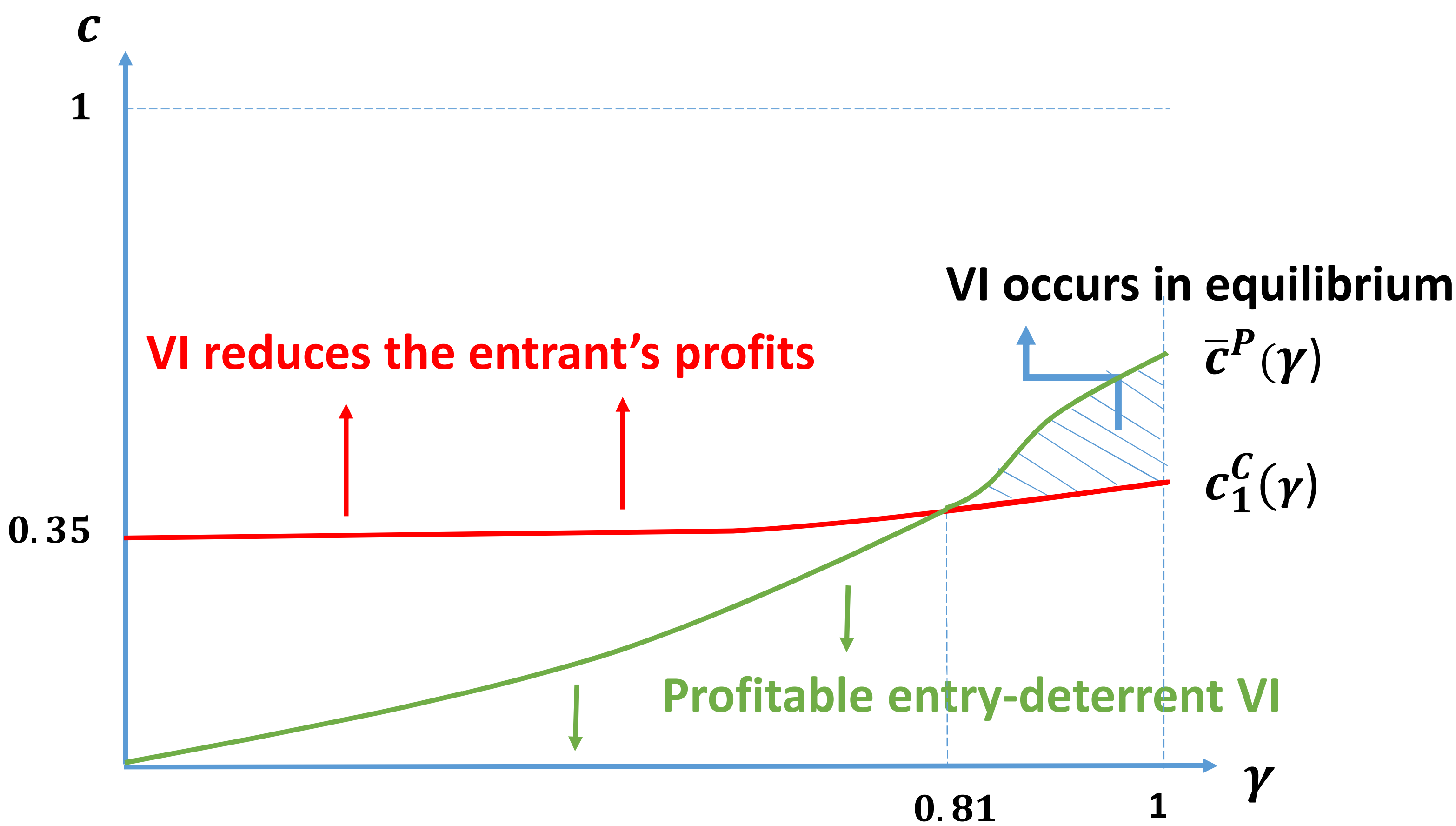

Figure 1. Equilibrium entry-deterrent VI

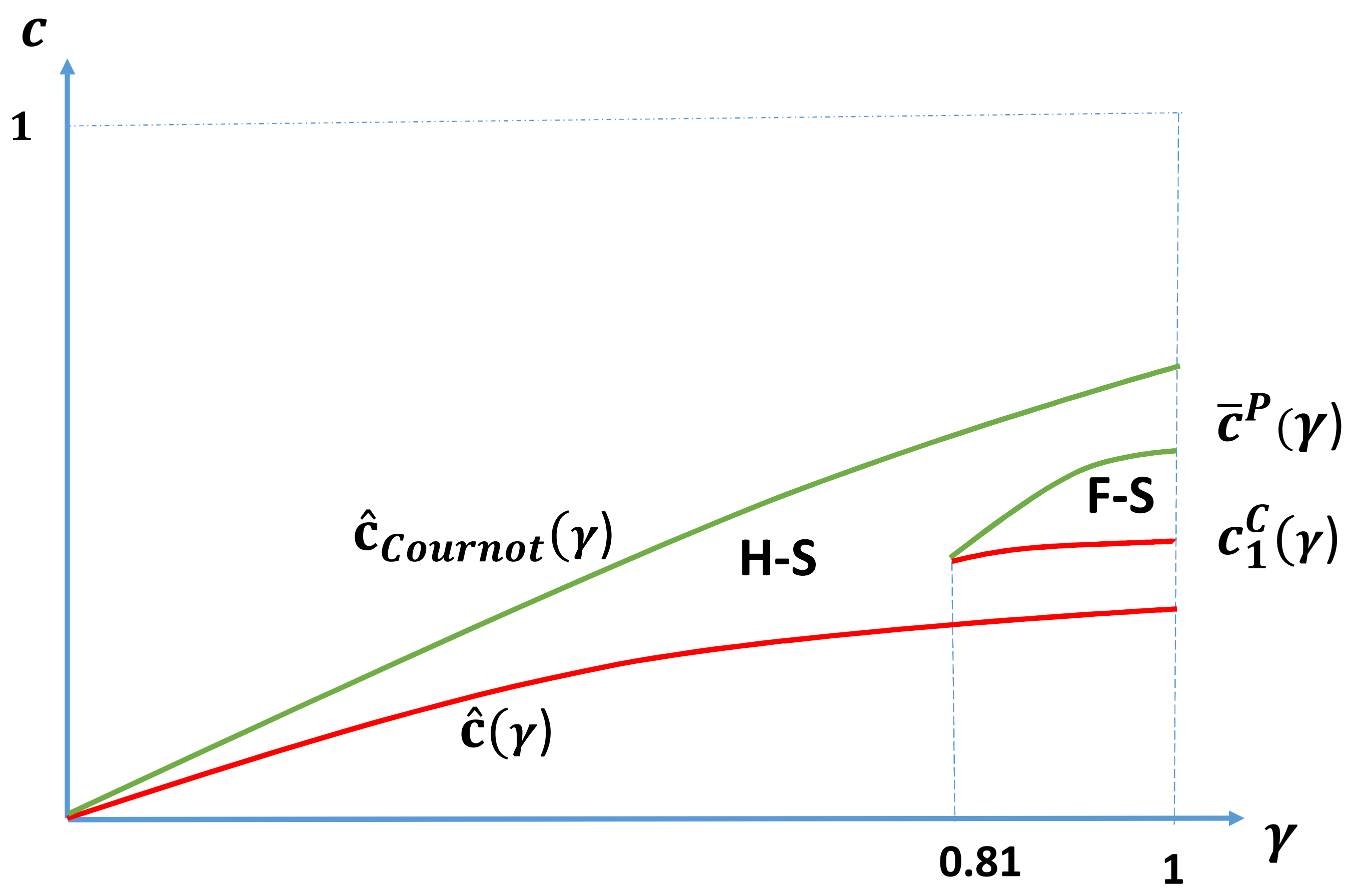

Figure 2. Comparing equilibrium entry-deterrent VI in Hunold and Schad (2023) and Fauli-Oller and Sandonis (2025)

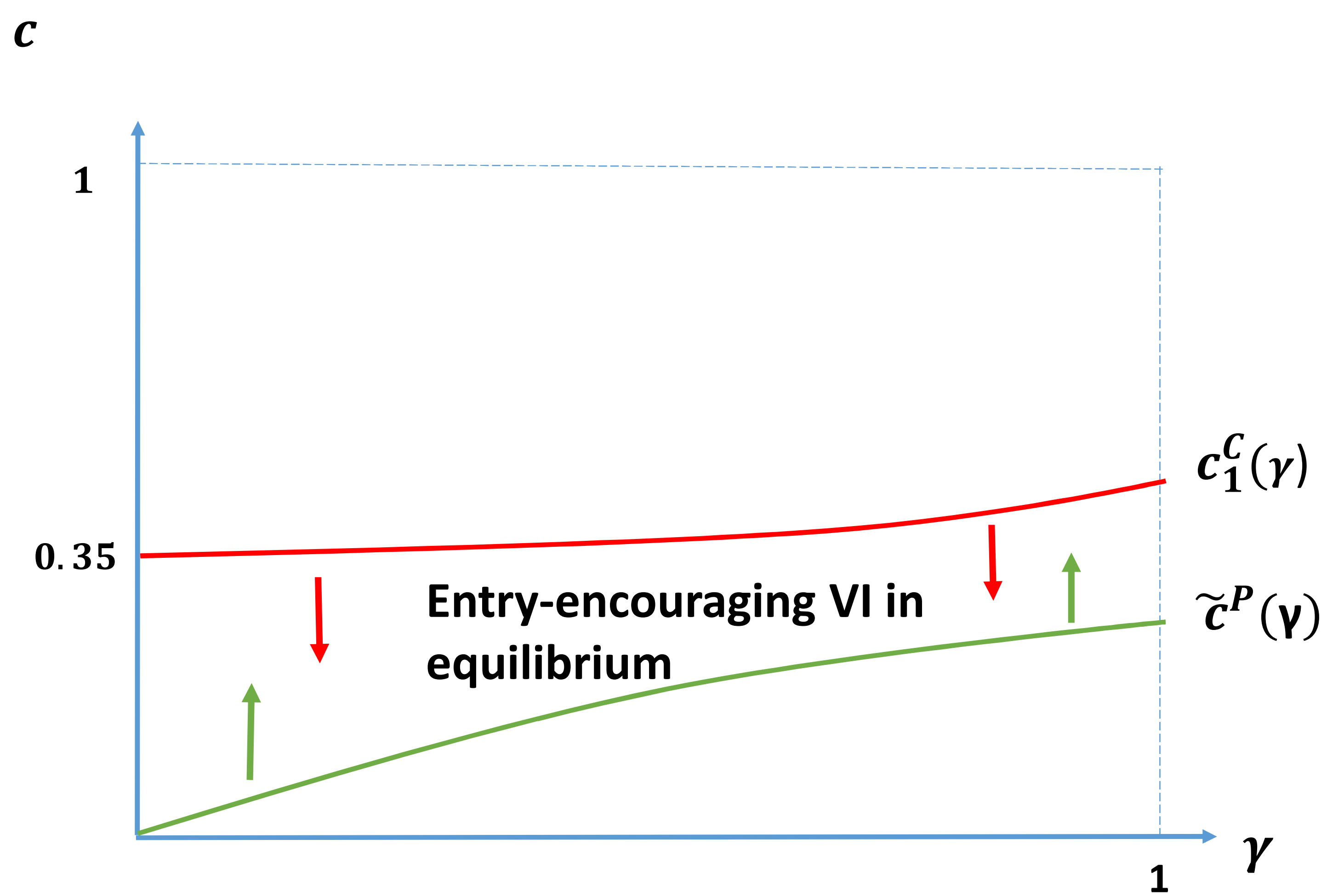

Figure 3. Equilibrium entry-encouraging vertical integration

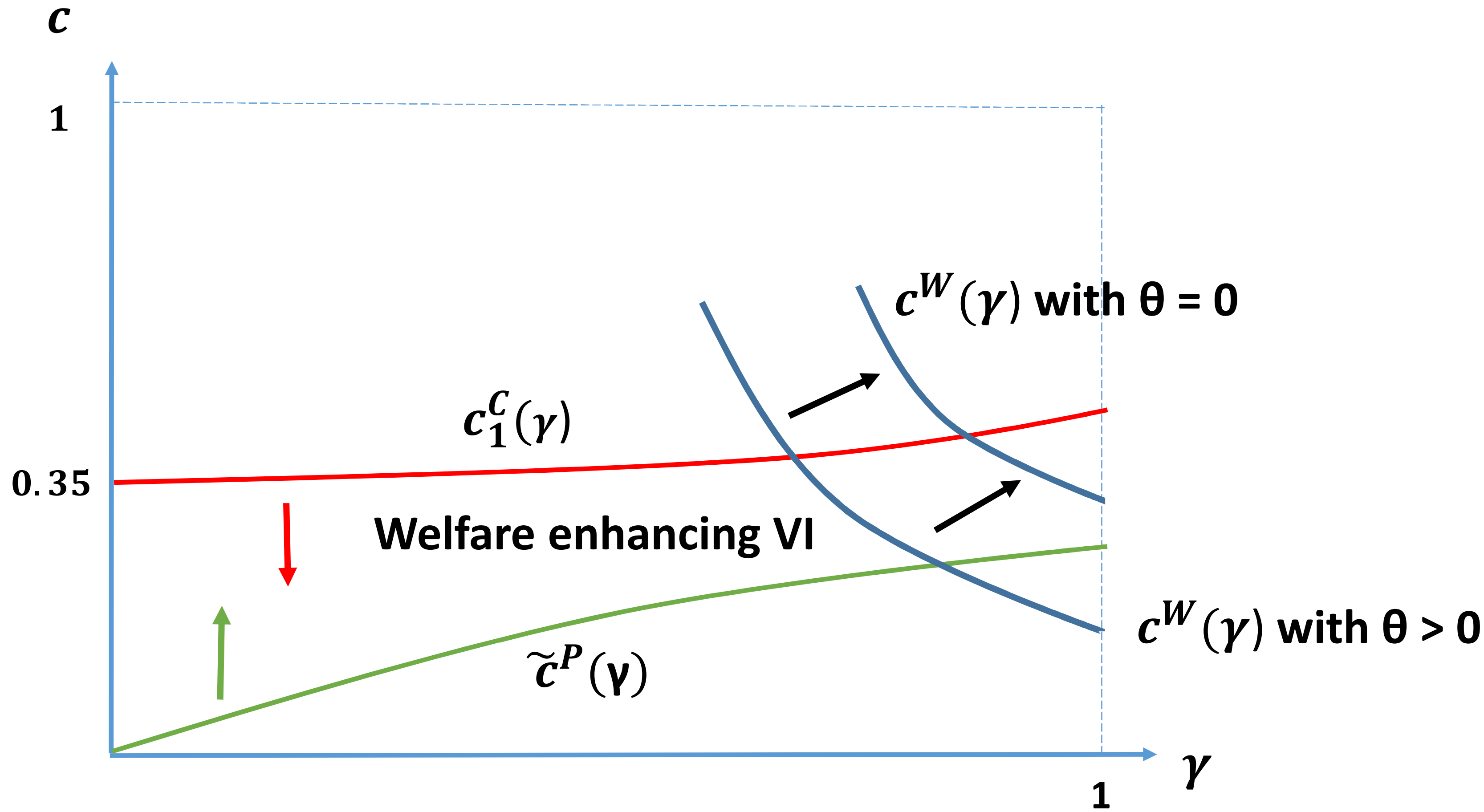

Figure 4. Welfare effects of entry-encouraging vertical integration