The Environmental Impact of Working From Home

December, 2024

Abstract

Is working from home (WFH) good for the environment? The rise of remote work has raised questions about its environmental and urban impacts. While reduced commuting can lower emissions, increased residential energy use and shifts in spatial patterns complicate its overall effects. In this paper, we develop a quantitative spatial model of Swedish cities incorporating sector-specific remote work adoption, commuting modes, and residential choices. We find that remote work leads to reductions in aggregate emissions, mainly driven by decreased commuting, despite longer trip distances. Residential emissions increase, but emissions reductions from commercial sectors dominate. Remote work also delivers positive welfare effects and modest shifts in urban population and employment, with the densest municipalities generally gaining residents and workers.

1 Introduction

Nowadays, a significant portion of employees engage in partial remote work. The Covid-19 pandemic has notably reshaped perceptions of remote work, leading both individuals and employers to choose a higher proportion of WFH compared to pre-pandemic times. A recent survey conducted in 27 countries around the world found that WFH averages 1.5 days per week, with workers expressing a desire for more days (averaging 1.7), while employers plan for a lower frequency (0.7 on average) (see Aksoy et al. (2022)). Although these figures exhibit considerable variation among countries, it is widely acknowledged that post-pandemic WFH levels will surpass those observed prior to the pandemic. The transition towards remote work patterns raises numerous questions, including the potential effects of hybrid working arrangements on the size and structure of cities, as well as whether this shift will yield environmental benefits. In this paper, we aim to address those questions using a quantitative urban model for Swedish cities.

Remote work might sound more sustainable from an environmental perspective, since workers do not have to commute to work daily. Indeed, if commuting is significantly reduced, there will be important benefits from the reduction of local and global pollution.[1] At the same time, WFH implies smaller office space and larger apartments that will include home office space. Thus, less energy will be used at the office, but more energy will be used at home. Office buildings, though, are designed to be more energy-efficient than homes. Moreover, home office space is likely to be larger than the office space per employee, which means that more energy is needed to heat or cool the home office than the office at work. Last, if people move to the outskirts of the city, commuting per trip will be longer, leading to higher emissions per trip. It is, thus, unclear if WFH will be good for the environment.

The impacts of these shifts may also exhibit significant variation across cities. Cities have different sizes and specialize in different industries, which will influence the extent of on-site work requirements (see Monte et al. (2023)). Furthermore, commuting distances and modes of transport used for these trips vary widely across cities. Additionally, workers will need to allocate space within their residences for use as a home office, while there might be an excess of unutilized office space if a substantial portion of the workforce operates from home.

To study the environmental impacts of WFH, we develop a quantitative spatial model, which can be used to study the reallocation of employment and residences across space. We then quantify our model to match the observed distributions of employment and residences in Sweden across 2,290 locations. WFH fractions depend on the industry, while workers choose where to live, where to work, as well as their commuting mode. Indeed, commuting choices vary among workers and depend on the distance and the availability of commuting modes. Three options are explored: private vehicles, public transportation, and bikes.

After calibrating the model, we conduct a counterfactual exercise where we simulate a sector-specific and uniform increase of WFH. To assess the teleworkability of different sectors, we use survey data from Sweden, reporting the fraction of WFH during the strictest Covid-19 measures and when those measures were relaxed.

Our results suggest that aggregate emissions decrease, the effect being mainly driven by reductions from commuting emissions. While commuting distances increase, the environmental benefits of lower commuting frequency dominate. Emissions from floorspace consumption increase in the residential sector due to lower rents but the effect is dominated in the aggregate by larger reductions in emissions in the commercial sector. We also find that the increase in remote working has significant and positive welfare effects. Finally, our results suggest that there is a small impact on the location decisions of workers and firms.

[1] The transport sector is the most important contributor to urban air pollution as it accounts for more than 50 percent of local pollutants (NOx and PM10) in many European cities (Font et al., 2019).

Overall, the densest municipalities gain both residents and workers. However, these results vary across cities, where the densest municipalities in Stockholm gain residents, while the opposite is true for the two second largest cities in Sweden; Gothenburg and Malmo.

Our paper contributes to the literature that studies the impacts of remote work on the structure of cities. A significant and consistent finding across multiple studies is that the rise of remote work has reduced the necessity for workers to live close to central business districts, driving a decentralization of urban populations. Recent contributions of spatial models that examine how hybrid working schemes affect the demand for housing and office space, land rent prices, urban productivity, the internal structure and the size of cities include Behrens et al. (2021); Monte et al. (2023); Brueckner (2024) and Kyriakopoulou and Picard (2023). Reduced commuting costs and the growing preference for larger homes with dedicated home-office space are the main drivers of these shifts, which lead to the spatial expansion of cities as central rents fall and suburban rents rise.

Additionally, we draw insights from empirical studies, such as Liu and Su (2021) who explore the impact of COVID-19 on housing demand, and Brueckner et al. (2023) and Gupta et al. (2022) who respectively uncover effects on house prices and rents in high-productivity areas and urban centers. These papers find that remote work weakens traditional agglomeration economies tied to urban CBDs, redistributing residential and economic activities toward suburban and secondary locations. This induces flatter housing/rent gradients and declining CBD real estate values. Mondragón and Wieland (2022) and Guglielminetti et al. (2023) document a surge in demand for suburban housing during the pandemic. Mondragón and Wieland (2022) show that over half of the national U.S. housing price growth can be attributed to remote work during the period 2019-2021, with suburban and exurban areas experiencing the sharpest increases. Guglielminetti et al. (2023) use data from the Italian housing market and show that WFH adoption increases demand for larger, single-family houses with outdoor space.

We also contribute to the literature that uses quantitative spatial equilibrium models to study the organization of economic activity across space and the choices of workplace and residence, such as Ahlfeldt et al. (2015); Heblich et al. (2020) and Monte et al. (2018). There is also a number of more recent contribution—such as Delventhal et al. (2022) and Delventhal and Parkhomenko (2023)—that build quantitative spatial models for Los Angeles and the US, respectively, to study the spatial impacts of WFH. In particular, Delventhal et al. (2022) show that remote work leads to jobs concentrating closer to the city center while residents move further out. This shift reduces traffic congestion and decreases average housing prices, while telecommuters benefit the most from these changes. For the whole of US, Delventhal and Parkhomenko (2023) show that the rise of remote work reshapes where people live and work, benefiting those who can work remotely while hurting those who can't, and reducing wage inequalities between different areas. Also, Monte et al. (2023) investigate how the ability to work remotely is changing cities of different sizes. Their research suggests that cities can settle into different stationary equilibria, where most people either commute to the city center or work remotely, while events, like Covid-19, can push cities from one equilibrium to the other. By analyzing data from US cities, they find that larger cities have experienced a more permanent shift towards remote work, while their model predicts welfare losses in those cities.

Our model uses a similar approach, but differs in allowing employees to choose, apart from their workplace and residence location, their preferred commuting mode. Commuting choices are important for many reasons including the environmental externalities that are generated by the different modes. The relocation of households to more distant locations, as a response to WFH, might imply higher car dependence which is harmful for the environment. This is the first paper that uses a quantitative spatial equilibrium model with multiple commuting choices to study the environmental impact of remote work on cities of different sizes and mixes of teleworkable occupations.

The literature exploring the environmental impacts of remote work remains scarce. There are a few papers that focus on transportation emissions, showing that WFH can reduce commuting-related emissions and vehicle miles traveled by 30-80% (Tao et al. (2023); Navaratnam et al. (2022)). However, rebound effects such as increased non-work travel and suburban relocations limit overall transportation benefits (Marz and Şen (2022); Cerqueira et al. (2020)). Pre-pandemic studies that have explored the impact of telework on energy consumption, greenhouse gases and air pollution have highlighted the fact that WFH-induced urban sprawl and pressure on transit systems require action to prevent environmental trade-offs from decentralization (Larson and Zhao (2016); Giovanis (2018)). However, the long-term environmental impacts associated with urban form shifts, commuting mode changes, and agglomeration externalities related to the different productivity levels of remote and office workers, remain largely unstudied. The paper closest to ours is Borck et al. (2024). They study the environmental effects of WFH in a similar model in Germany. The main difference to their study is that we use a finer geography and include addtional margins of adjustment such as sector and transport mode.

The rest of the paper is organized as follows. Section 2 describes the spatial framework used to model WFH. Section 3 presents data and calibration strategies to quantify the model. Section 4 describes the results from the counterfactual exercise, while Section 5 concludes the paper.

2 Model

We consider a country that includes a unit mass of individuals who choose their consumption of goods and housing floor space, their residence and job location, the sector in which they work, and their transport mode. The country spans over a set of distinct geographic locations.

2.1 Individuals

Individuals consume housing floor space and a composite good, that embeds all sectoral goods. They spend a fraction θ of work time in the office and $1 - \theta$ at home as work from home (WFH). They are endowed with the utility function

$$U(c, h) = z \frac{X}{d(\theta)} \left(\frac{c}{1 - \beta} \right)^{1-\beta} \left(\frac{h}{\beta} \right)^{\beta}, \tag{1}$$

where h and c are the use of housing floor space and consumption of a composite good, β being the expenditure share of housing consumption. X represents spatial amenities at residence, and z is an idiosyncratic preference parameter. Since we consider GHG emissions only, we do not include emissions in the utility. Since climate change is a global externality, the damage caused by GHG emissions is independent of individuals' locations or mode choices and therefore can also not influence these choices.[2] Utility decreases with commuting time t between home and work locations according to the disutility function $d(t, \theta) = \theta e^{\kappa t} + (1 - \theta)$ where θ is the fraction of work time in the office and $1 - \theta$ that spent on work from home (WFH). κ measures the disutility of commuting time per unit of distance. WFH requires an additional fraction of floor space $g(\theta)$. Assuming an open economy, the prices of sectoral goods are given by world markets. We normalize quantity units so that their prices and therefore the price of the composite good are equal to one. The budget constraint is therefore given by $c + qh[1 + g(\theta)] = w$ where q and w are the prices of land and labor.

Individuals can freely locate over residence locations i, work at job locations j and in sectors s, and commute with transport modes f. We denote the spatial, sectoral, and modal characteristics of variables by subscripts. Individuals differ in their idiosyncratic preference parameter z_{ijsf} which combines three independent taste shocks: ξ_{ij} for the pair of residential location i and job place j, φ_s for the sector s and ζ_f for the transport mode f. Those shocks are respectively drawn from the Fréchet cumulative distribution functions $e^{-\xi^{-\gamma}}$, $e^{-\zeta^{-\nu}}$ and $e^{-\varphi^{-\rho}}$ where γ, ν and ρ measure the dispersions of the taste shocks. Individuals receive independent draws from these distributions and, having received these draws, choose residence and job location, sector, and transport mode to maximize their utility. Finally, WFH depends on sector so we label the WFH parameter as θ_s.

[2]Borck et al. (2024), by contrast, focus on local pollution, where the damage depends on individuals' location choice. In their calibration, however, the effect of pollution on relocation turns out to be small.

At given prices, individuals' indirect utility is given by $V_{ijfs} = \xi_{ij}\zeta_f\varphi_s\, v_{ijsf}$ where

$$v_{ijsf} = \frac{X_i}{d(t_{ijf}, \theta_s)} \frac{w_{js}}{[q_i(1 + g(\theta_s))]^\beta}. \tag{2}$$

Individuals choose the residences, job places, sectors and transport modes that maximize their utility. Using the properties of Fréchet distributions, the probability that an individual chooses the characteristics i, j, s, and f is equal to $\pi_{ijsf} = \pi_{ij}\, \pi_{s|ij}\, \pi_{f|ijs}$ where

$$\pi_{f|ijs} = \left[\frac{v_{ijsf}}{\mathbb{E}(v_{ijs})}\right]^\nu \quad \text{with} \quad \mathbb{E}(v_{ijs}) = \Gamma(\nu)\left[\sum_f v_{ijsf}^\nu\right]^{1/\nu}, \tag{3}$$

$$\pi_{s|ij} = \left[\frac{\mathbb{E}(v_{ijs})}{\mathbb{E}(v_{ij})}\right]^\rho \quad \text{with} \quad \mathbb{E}(v_{ij}) = \Gamma(\rho)\left[\sum_s \mathbb{E}(v_{ijs})^\rho\right]^{1/\rho}, \tag{4}$$

$$\pi_{ij} = \left[\frac{\mathbb{E}(v_{ij})}{\mathbb{E}(v)}\right]^\gamma \quad \text{with} \quad \mathbb{E}(v) = \Gamma(\gamma)\left[\sum_i \sum_j \mathbb{E}(v_{ij})^\gamma\right]^{1/\gamma}. \tag{5}$$

2.2 Production

In each location, firms produce a set of sectoral goods and sell those in the perfectly competitive world market at the prices normalized above. They produce under constant returns to scale using floor space and labor with various education levels.

More formally, in each job location j and sector s, firms produce a quantity Y_{js} of sectoral goods by combining office floor units H_{mjs} and effective labor units M_{mjs} such that

$$Y_{js} = A_{js}\left(\frac{M_{js}}{\alpha_s}\right)^{\alpha_s}\left(\frac{H_{js}}{1-\alpha_s}\right)^{1-\alpha_s}$$

where α_s and $1 - \alpha_s$ are the cost shares of effective labor and office floor and A_{js} is a location-specific productivity parameter. Because WFH allows firms to save on office floor space, we assume that WFH decreases the cost share of floor space, $1 - \alpha_s$. In particular, we assume that $1 - \alpha_s = \theta_s^{\delta_s}(1 - \overline{\alpha}_s)$ where $1 - \overline{\alpha}_s$ is the cost share of office floor in the absence of WHF ($\theta_s = 1$) and δ_s is the elasticity of floorspace expenditures with respect to the amount of remote work. In addition, labor productivity may differ in the office and at home. For instance, in the absence of effective supervision, quiet home environment and good IT connection and terminals, workers are less effective at home so that $\xi_s < 1$. In opposite situations, $\xi_s > 1$. Each worker supplies an amount ξ_s of effective labor units. Accordingly, the mass of workers N_{js} brings an amount of labor productivity units equal to $M_{js} = \xi_s N_{js}$. We assume that $\xi_s \equiv \theta_s + \overline{\xi}_s\,(1 - \theta_s)$ where $\overline{\xi}_s$ is the worker's home productivity relative to her on-site productivity. If $\overline{\xi}_s$ is greater (lower) than one implies that remote work is more (less) productive than on-site work.[3]

[3]The verdict on WFH productivity is still out. For example, Bloom et al. (2014) found positive effects of

Firms choose labor and floor space to minimize costs. Under perfect competition, output prices are equal to marginal production cost so that:

$$A_{js}^{-1}\left(\frac{w_{js}}{\xi_s}\right)^{\alpha_s} q_j^{1-\alpha_s} = 1. \tag{6}$$

2.3 Developers

In each location i, a set of competitive developers use a quantity of land l_i and composite good c_i to produce the amount of floor space H_i:

$$H_i = \left(\frac{\phi_i l_i}{\eta}\right)^{\eta}\left(\frac{c_i}{1-\eta}\right)^{1-\eta}, \tag{7}$$

where ϕ_i is a fixed land-productivity parameter and η is the share of land in developers' cost. Producers sell their floor space at the price q_i and pay a cost equal to $r_i l_i + c_i$ where r_i is the land unit price. The demand for land is therefore equal to $l_i = \eta\phi^{-\eta} r_i^{\eta-1} H_i$ and the unit cost of floor space is given by $(r_i/\phi_i)^{\eta}$. In equilibrium, the land demand equates its supply the exogenous supply of buildable land Λ_i and the unit cost equates the floor price q_i. As a result, the equilibrium supply of floor space and the land price are given by

$$H_i = \frac{\phi_i \Lambda_i}{\eta} q_i^{\frac{1-\eta}{\eta}} \tag{8}$$

$$r_i = \phi_i q_i^{\frac{1}{\eta}}. \tag{9}$$

2.4 Externalities

In every residential location i, residents benefit from an exogenous fundamental amenity x_i and an endogenous amenity based on the density of local residents

$$X_i = x_i\left[\frac{1}{\Lambda_i}\sum_j \sum_s N_{ijs}\right]^{\chi} \tag{10}$$

where $\chi < 1$ is a congestion parameter. Similarly, in every job location j, firm productivity increases with an exogenous fundamental productivity component A_{js} and an endogenous productivity component that increases with the density of local on-site and remote employment such that

$$A_{js} = a_{js}\left[\frac{1}{\Lambda_j}\sum_i \sum_s [\theta_s + \psi(1-\theta_s)] N_{ijs}\right]^{\lambda}, \tag{11}$$

WFH on productivity of call-center workers, while Gibbs et al. (2023) found negative effects in tech firms. Bloom et al. (2024) find no effect of hybrid work arrangements on performance.

where $\psi \leq 1$ measures the degree of remote workers' participation in productive externalities and $\lambda < 1$ is the agglomeration elasticity. While not much is known about the magnitude of ψ, Liu and Su (2023) find that WFH decreases agglomeration externalities. In our quantitative exercise, we follow Delventhal et al. (2022) and run the counterfactual with either $\psi = 0$ or $\psi = 1$.

2.5 Equilibrium

In the equilibrium, workers freely locate across locations, sectors, and transport modes. Given the unit mass population, the equilibrium is determined by the conditions

$$N_{ijsf} = \pi_{ijsf}. \tag{12}$$

In each location i, local residents use floorspace H_i^R for residential housing purposes and H_i^{WFH} for their home office. Local firms use H_i^{WFW} for on-site work. As a result, total floor space demand is:

$$H_i = H_i^R + H_i^{WFH} + H_i^{WFW}, \tag{13}$$

with:

$$q_i H_i^R = \beta \sum_j \sum_s \frac{1}{1 + g(\theta_s)} w_{js} N_{ijs}, \tag{14}$$

$$q_i H_i^{WFH} = \beta \sum_j \sum_s \frac{g(\theta_s)}{1 + g(\theta_s)} w_{js} N_{ijs}, \tag{15}$$

$$q_j H_j^{WFW} = \sum_s \frac{1 - \alpha_s}{\alpha_s} w_{js} \sum_i N_{ijs}. \tag{16}$$

Definition 1. Given local fundamentals a_{js}, x_i, and Λ_i; bilateral mode-specific commute times t_{ijf}; and economy-wide parameters θ_s α_s, β, γ, ν, ρ, κ, η, λ, and χ; a spatial equilibrium consists of allocations of workers to sectors, residences, workplaces, and transport modes, N_{ijfs}; productivities, A_{js}; residential amenities, X_i; wages, w_{js}; floorspace prices, q_i and floorspace quantities, H_i; such that equations 12, 11, 10, 6, 8, and 13 are satisfied.

2.6 Existence and Uniqueness

In Appendix B, we useresults from Allen et al. (2023) to check that baseline and counterfactual equilibria exist and are unique for the set of parameters we use. In particular, for the simplified case of a single sector and a single transport mode, we compute the spectral radius of the matrix $\boldsymbol{A}$ from Allen et al. (2023) and show that it is lower than 1.

2.7 Emissions of GHG

In each location, consumption of floorspace (residential and commercial) and commuting emit GHG. Local emissions from floorspace consumption are given by:

$$Z_i^H = Z_i^{R+WFH} + Z_i^C = \varepsilon_i^R (H_i^R + H_i^{WFH}) + \varepsilon_i^{WFW} H_i^{WFW}, \tag{17}$$

where ε_i^k are the local emission intensities of building types residential, R, and commercial, C. We assume that these intensities are composed of an exogenous term ϵ_i^k and an endogenous component that decreases with the local density of floorspace, such that:

$$\varepsilon_i^k = \epsilon_i^k \left(\frac{H_i}{\bar{H}_i} \right)^{-\mu^k}, \quad \text{with} \quad \mu > 0. \tag{18}$$

Borck and Brueckner (2018) model residential energy use as a function of buildings' surface area, which implies that energy use per unit of floorspace decreases with building height. Below, we will estimate μ with data on building energy use.

Commuting by mode f emits pollution (Z_f^M with M like "moving") in proportion to the travel time, with an intensity depending on the mode of transport:

$$Z_f^M = \varepsilon_f^M \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} \sum_{m \in \Theta} \sum_{s \in \Omega_T} \theta_{ms} t_{ijf} N_{mijfs}. \tag{19}$$

2.8 Counterfactuals

In the quantification section, we consider counterfactual equilibria where the shares of remote working per occupation θ_s' is different from the baseline scenario value θ_s. We solve these counterfactual equilibria for changes in endogenous variables $\hat{N}_{ijfs}$; productivities, $\hat{A}_{js}$; residential amenities, $\hat{X}_i$; wages, $\hat{w}_{js}$; floorspace supply, $\hat{H}_i$; and floorspace prices, $\hat{q}_i$; where we define changes in variable v as $\hat{v} = v'/v$. We then compute implied changes in pollution emissions from floorspace consumption and commuting, $\hat{Z}_i^H$ and $\hat{Z}_f^M$. Appendix A details the equilibrium changes equations.

3 Quantification

We divide Sweden into 2,290 locations. For large cities, we use the DeSO geographic decomposition and for other areas we use the municipality level.[4] We include 12 sectors (see below) and 3 commuting modes (walking/biking, driving, and public transport).

[4]DeSO is a nation-wide breakdown that follows county and municipal boundaries. It divides Sweden into 5,984 areas that have between 700 and 2,700 inhabitants.

3.1 Baseline

We calibrate current shares of telecommuting by sector using aggregated values from the Swedish Labor Survey for the shares of workers who, in January 2021, worked remotely at least half of the working days, respectively distributed across activities according to the Swedish Standard Industrial Classification (SNI).[5] We compute commuting flows and wages for each sector and residence-workplace location pair using the individual register from the Swedish longitudinal integrated database for individual and workplaces (LISA). We use aggregated data from the Swedish National Travel survey (RVU Sweden) to allocate commuting flows across commuting modes.[6] We compute commuting time between all pairs of locations (up to a 4 hours travel time limit) using Open Route Services API for cycling and driving, and the TravelTime SDK API for public transports.

To compute local floorspace prices, we combine the apartment register with individual wage data. We assume that the ratio β times the wage of an individual divided by the size of her apartment corresponds to the rent and take the average value over all apartments in the location. We then compute total residential and commercial floorspace quantities in each location by combining total income and rents for residential floorspace and total labor and rents for commercial floorspace.

We use floorspace GHG emissions data from the National Emissions Database. The Swedish Meteorological and Hydrological Institut runs this database and collects Sweden's national territorial emissions of greenhouse gases and air pollutants broken down to local level. We use the gridded emission data for emissions from residential housing and commercial buildings.

3.2 Parametrization

We now describe how we calibrate or estimate our parameter values. Table 1 collects the values we use.

We set the consumption share of housing, β, at .291 following the aggregate value from the Household budget survey (HBS).[7] We use values from Valentinyi and Herrendorf (2008) to calibrate the share of floorspace in on-site production, $1 - \bar{\alpha}$. In particular, we assume that it corresponds to the share of land and buildings in expenditures. We use .32 for agriculture, .15 for the manufacturing and energy sector, .09 for the construction sector, and .21 for services (9 sectors). We calibrate sector-specific elasticities of floorspace expenditures

[5]The SNI is used to classify enterprises and workplaces according to the activity carried out.

[6]The data and information about the survey can be accessed **here**.

[7]https://www.scb.se/en/finding-statistics/statistics-by-subject-area/household-finances/household-expenditures/household-budget-survey-hbs/

Table 1: Externally determined and estimated parameters

Parameter	Description	Value	Comments
β	consumption share of housing	0.291	Household budget survey
γ	Fréchet elasticity of location shock	1.05	estimated (equation 22)
ν	Fréchet elasticity of transport mode shock	.55	estimated (equation 23)
ρ	Fréchet elasticity of sector shock	.14	estimated (equation 20)
κ	elasticity of commuting cost to commuting time	3.64	estimated (equation 21)
$1-\bar{\alpha}_s$	floorspace share in production w/o remote work	various	Valentinyi and Herrendorf (2008)
δ	elasticity of floorspace to remote work	$\{0, .1, .5\}$	Credible values
ψ	contribution of telecommuters to productivity externalities	$\{0, 1\}$	separate counterfactuals
$\bar{\xi}$	relative productivity of remote work	$\{.9, 1, 1.1\}$	separate counterfactuals
η	price elasticity of floorspace supply	various	estimated (equation **??**)
λ	elasticity of local productivity to employment density	0.04	Ahlfeldt and Pietrostefani (2019)
χ	elasticity of local amenity to population density	0.172	Heblich, Redding, and Sturm (2020)

Note: The table lists parameters determined externally to the calibration process and parameters calibrated or estimated using Swedish data.

with respect to the amount of remote work using arguably plausible values. We set δ to .5 for all sectors, except for manufacturing and energy where we set it to .1 and for agriculture and hotels and restaurants where we set it to 0. A value of .5 means that an additional day of remote work (θ increases by 20%) leads to a reduction of floorspace expenditure of 10%.

We then follow the approach by Monte et al. (2018) to estimate ρ, κ, γ, and ν. We assume for simplicity that there is no work-from-home in the baseline equilibrium. We use data from 2019, so before the zoom shock. From (3) to (5), we get the following estimating equations:

$$\log \pi_{js} = \rho \log w_{js} + FE_j + FE_s + \epsilon_{js}, \tag{20}$$

$$\log \pi_{ij} = -\kappa\gamma tt_{car} + FE_i + FE_j + \epsilon_{ij}, \tag{21}$$

$$\log \pi_{ij} + \widehat{\kappa\gamma} tt_{car} = \gamma \log \left[\sum_s w_{js}^{\hat{\rho}} \right]^{1/\hat{\rho}} + FE_i + \epsilon_{ij}, \tag{22}$$

$$\log \pi_{ijsf} = -\kappa\nu \log tt_{ijf} + FE_{ijs} + \epsilon_{ijsf}. \tag{23}$$

In (20), we estimate the allocation of workers among sectors based on workplace-sector specific wages, controlling for workplace and sectoral fixed effects. Equation (21) estimates the location choice among residence-workplace pairs based on bilateral distances, controlling for residence and workplace fixed effects. As usual, the coefficient on bilateral distances only allows us to recover the combined parameter $\kappa\gamma$, which combines the Fréchet elasticity γ with the aversion to commuting time, κ.

In (22), we recover the Fréchet elasticity from variations in workplace wages, setting $\kappa\gamma$ equal to its estimate from (21) and likewise ρ to its estimated value from (20) and controlling for residence fixed effects. Finally, (23) estimates the mode share elasticity ν from the difference in commute times by mode, where we fix the commuting disutility parameter at $\hat{\kappa}$, controlling for residence by workplace by sector fixed effects.

We allow price elasticities of floorspace supply, η, to vary across location types, according to the classification of Swedish municipalities. We estimate (8) by location. In particular, we regress the logarithm of price per square meter on the logarithm of the unit floorspace, controlling for different characteristics (location, construction year, number of rooms, floors). We find values for η ranging from .8 for large cities to .9 for rural municipalities.

Finally, we estimate (18) to get parameters ϵ_i^k and μ^k. In particular, we find that residential and commercial elasticities of emissions intensity to floorspace density of .5 and 1.6%.

To compute emissions from commuting, we apply:

$$Z^{\text{transport}} = \sum_{f \in \text{ modes}} \sum_{(i,j)} Factor_f Speed_f TravelTime_{ijf} 2 \times \theta \times WorkingDays \\ \times \pi_{ijf} \times Pop,$$

with $Factor_{car} = 133gCO_2/km$, $Factor_{public} = 25gCO_2/km$, and $Factor_{foot/cycle} = 0gCO2/km$. We assume 250 working days in Sweden. We assume $Speed_{car} = 80km/h$ and $Speed_{public} = 60km/h$.

4 Sector-specific Increases in WFH

In this section, we present the results of a counterfactual exercise where we simulate the change implied by a sector-specific increase in the fraction of WFH. Table 2 presents the benchmark and counterfactual on-site worktime shares across sectors. To do so, we use the WFH fractions observed in Sweden at two points in time: during December 2021 and February 2022, when the government mandated WFH except for employees whose physical presence was essential, and during March 2022 to May 2022, when the government mandate ceased to apply. In Table 2, θ is the share of employees working on-site for at least half of the working days when no restrictions are in place, and θ' is the corresponding share in January 2022 as per the Swedish Labor Force Survey (AKU), that was the period with the strictest WFH recommendations in Sweden.

4.1 Main Results

We summarize the main results in Table 3. We run four alternative versions of our counterfactual assuming different values for parameters ψ and ξ. Columns (1) to (3) show the absolute change in GHG emissions from commuting and residential and commercial floorspace consumption. Column (4) shows the social value of the total GHG emissions change, using a Social Cost of Carbon value of 1,200 SEK Column (5) shows the equivalent variation in

Sector	θ	θ'	**Share**
Agriculture	78%	70%	1.8%
Artothers	85%	68%	4.6%
Construction	92%	90%	7.1%
Education	91%	79%	11.4%
Finance	72%	49%	15.0%
Healthsocial	96%	90%	15.9%
Hotelrestau	100%	100%	4.1%
Infocomm	45%	18%	3.4%
Manufenergy	91%	77%	13.3%
Publicadmin	76%	44%	6.3%
Trade	87%	76%	12.2%
Transport	91%	90%	4.8%

Note: θ and θ' respectively correspond to one minus the share of workers usually working from home in the baseline and counterfactual equilibrium. The third column *Share* indicates the baseline share of the population working in each sector.

Table 2: Shares of on-site work across sectors.

income.[8] For each alternative version of the counterfactual, we present absolute values and relative changes in italics below.

Table 3 reveals that the higher fractions of WFH shown in Table 2 lead to reductions in GHG emissions from commuting and commercial floorspace and to increases in emissions from residential floorspace consumption. Emissions from commuting decrease by around 11%, or about .47 million tons of CO2 equivalent. Emissions from residential floorspace increase by 3%, or about .01 million tons of CO2 equivalent. Emissions from commercial floorspace decrease by around 6%, about .03 million tons of CO2 equivalent. Overall, reductions in emissions from commuting drive most of the aggregate environmental effect of remote working. The aggregate social value of the GHG emissions reductions is around .58 billion SEK. This represents .1% of the total income perceived by workers. Turning to the welfare impact of the increase in remote work, column (5) shows that workers' welfare increases across all different scenarios. The aggregate equivalent variation ranges from 30 to 50 billion SEK. In relative terms, the welfare gain ranges from 2.8 to 4.5% of total income. The largest gains are observed when we assume either that remote work is relatively more productive than on-site work ($\xi > 1$) or when remote workers contribute to the productive externality ($\Psi = 1$).

[8] We compute the equivalent variation as the additional per-worker income needed to make the average worker as well off in the baseline as in the counterfactual. We solve for this value in partial equilibrium.

Table 3: Environmental & Welfare Impacts of Remote Work

Scenario	**GHG Emissions Changes**			**Total**	**Equivalent**
	Commuting	**Residential**	**Commercial**	**SCC**	**Variation**
	MtonsCO2 (%)			BnSEK (%)	
	(1)	(2)	(3)	(4)	(5)
$\Psi = 0$ & $\xi = 1$	-0.47	+0.01	-0.03	+0.58	+40.32
	-11.7%	*+3.0%*	*-5.9%*	*+0.1%*	*+3.7%*
$\Psi = 0$ & $\xi = 1.1$	-0.47	+0.01	-0.03	+0.59	+49.65
	-11.7%	*+3.1%*	*-6.1%*	*+0.1%*	*+4.5%*
$\Psi = 0$ & $\xi = 0.9$	-0.47	+0.01	-0.03	+0.58	+30.49
	-11.7%	*+2.9%*	*-5.7%*	*+0.1%*	*+2.8%*
$\Psi = 1$ & $\xi = 1$	-0.47	+0.01	-0.03	+0.58	+45.80
	-11.7%	*+3.1%*	*-6.1%*	*+0.1%*	*+4.2%*

GHG emissions changes are in millions of tons of CO2 equivalent GHG.
We compute the social cost of carbon of emissions changes on the basis of 1,200 SEK per ton of CO2eq.

4.2 Spatial Shifts

Figures 1 and 3 show the reallocation of the population across space following the increase of remote working. Figure 1 aggregates data at the municipality level (a little bit less than 300 for Sweden) while Fig. 3 focuses on the three largest municipalities and disaggregates reallocation across DeSO within these particular municipalities. In particular, each panel shows the absolute change in residents or workers as a function of the baseline density of workers or residents. The left panel of Figure 1 shows that people would move from cities in the middle of the density distribution to the densest municipalities. The right panel shows a similar picture, showing that the densest municipalities would also gain in population of workers. These findings differ from Delventhal et al. (2022) who find that, on average, denser cities would lose residents and jobs.

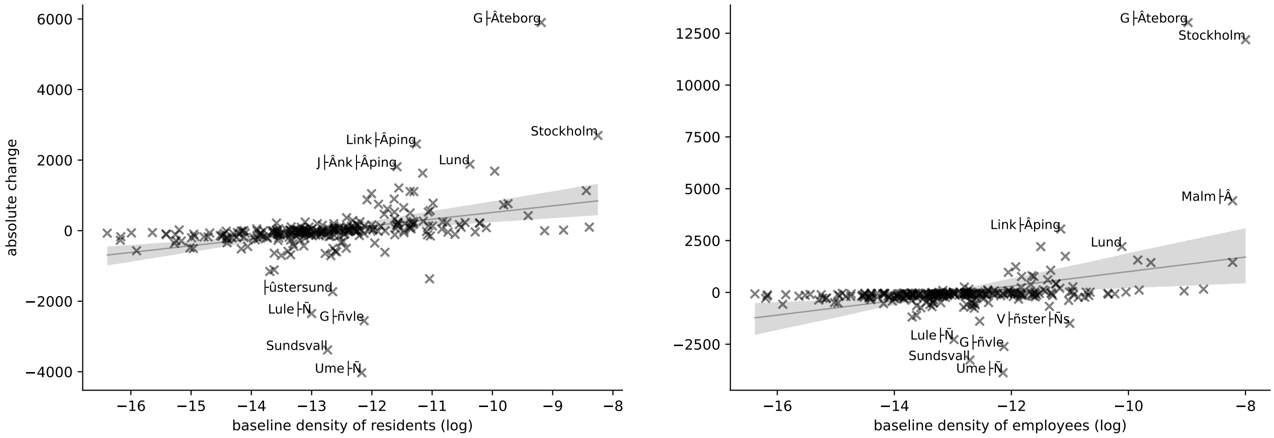

Figure 1: Population reallocation across municipalities

To explain these patteNrs, Figure 2 plots the average change in the share of time spent working on-site (θ). While this change is the same for all workers within a given sector, spatial heterogeneity in sectoral composition implies that the average fraction of remote work differs across space. Figure 2 shows that the average increase in remote work (negative variations of work-from-work time) is significantly larger in denser municipalities.

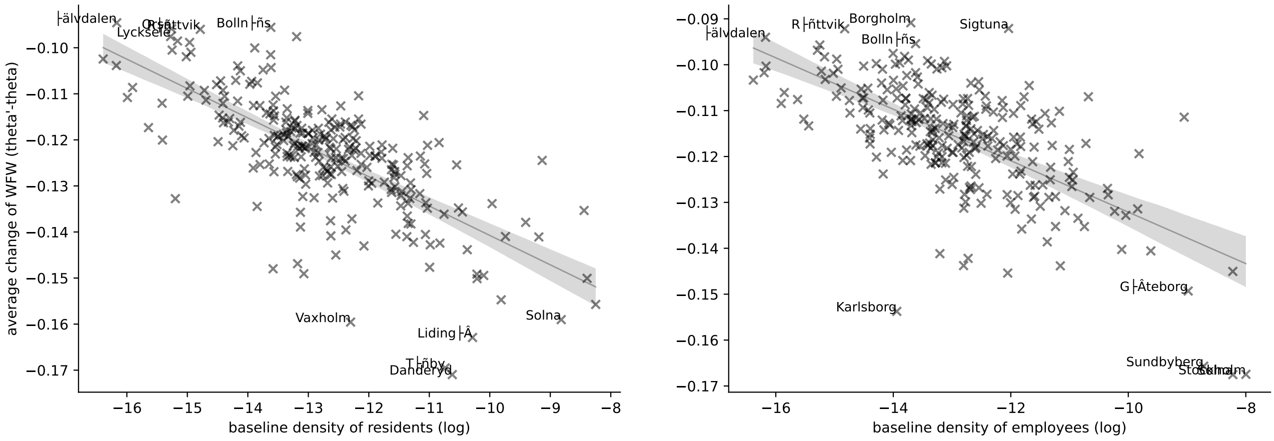

Figure 2: Spatial variation in the WFH share change

Figure 3 explains what happens within these dense municipalities, focusing on the top three, Stockholm, Gothenburg, and Malmo. For the three cities, we see on the left panels that the locations that gain the most new residents are the least dense locations. Conversely, the right panel shows us that most of the new residents of these large municipalities start working in locations where most of the workers are concentrated. This is more in line with Delventhal et al. (2022). Like them, we find that in big cities, households on average move to less dense locations. This seems intuitive, since WFH allows an even stronger separation of residences and workplace, so households can live where housing is cheap and work (partly remotely) where wages are high. This is consistent with the right panel, which shows that employment increases most in the densest locations.

4.3 Emissions from Commuting

Table 4 details how emissions from commuting change with the increase in WFH. The first column presents the full effect for all modes as well as disaggregated by mode. The next two columns decompose the effect into the reallocation of commuters (workers who change residence and/or workplace) and changes in emissions via the lower commuting frequency due to the increase of remote working. The last column contains the second-order effects.

Table 4 shows that transport emissions decrease by .47 Mtons of CO2eq or 11.7%. This large decrease is driven to a large extent by the lower frequency of commuting, since workers do not have to go to work as often. This leads to a 15% reduction in emissions (see column

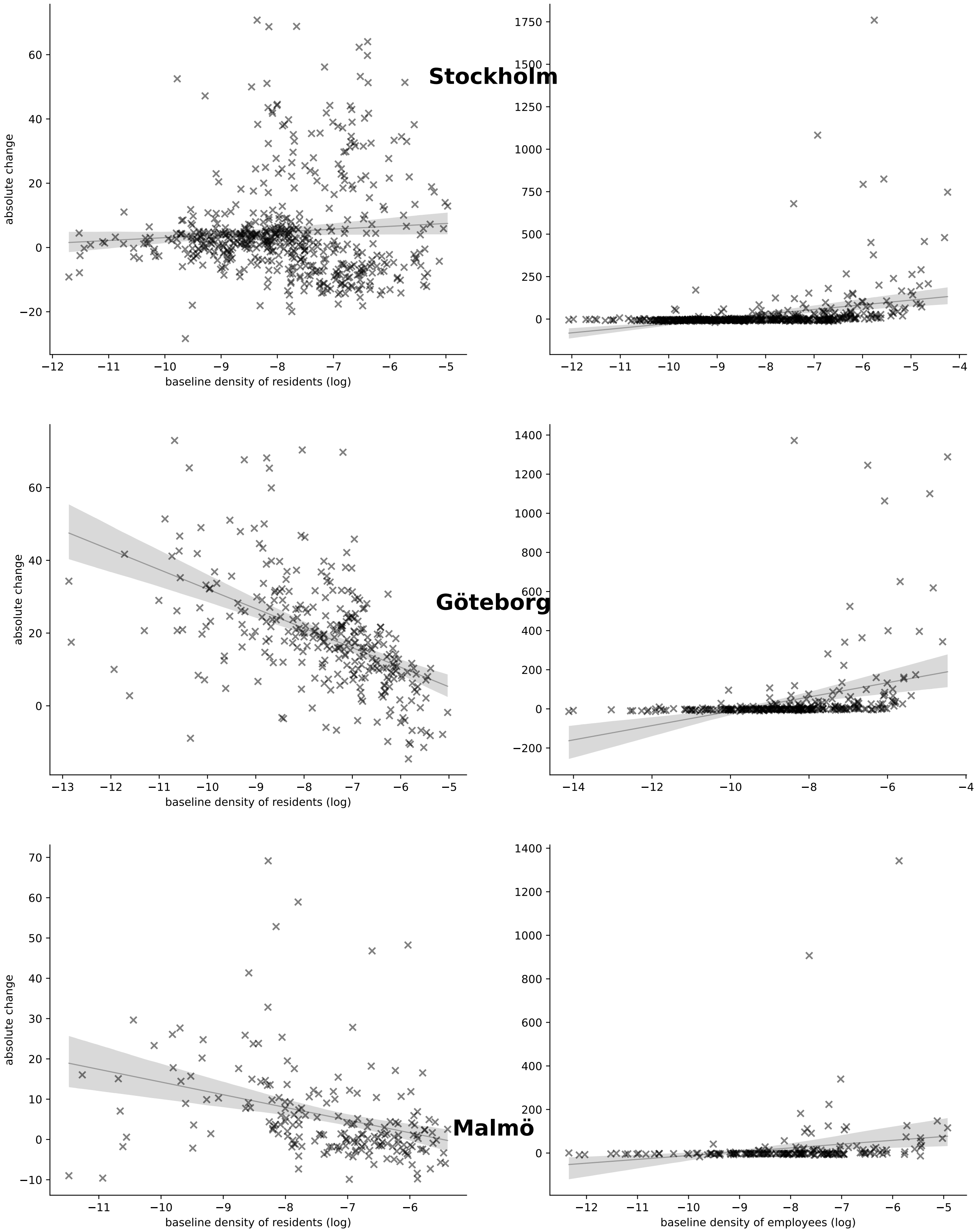

Figure 3: Population reallocation within large municipalities

(3)). However, the relocation of jobs and residences leads to a 5% increase in commuting emissions. This can be explained by workers moving further away from their workplace. Decomposing these effects across commuting modes shows that the effects are largely driven by car commuting.

Table 4: Emissions from commuting

	Full Effect	**Relocation Effect** ($\Delta\pi$)	**Frequency Effect** ($\Delta\theta$)	**Second Order**
		MtonsCO2 (%)		
	(1)	(2)	(3)	(4)
All modes	-0.47	0.22	-0.62	-0.06
	-11.7%	*5.4%*	*-15.6%*	*-1.6%*
By car	-0.43	0.19	-0.56	-0.06
	-10.7%	*4.9%*	*-14.1%*	*-1.4%*
By public	-0.04	0.02	-0.06	-0.01
	-1.0%	*0.6%*	*-1.4%*	*-0.2%*
By footcycle	0.00	0.00	0.00	0.00
	0.0%	*0.0%*	*0.0%*	*0.0%*

4.4 Residential and Commercial Floorspace Emissions

Table 5 summarizes the changes in emissions attributed to floorspace consumption. The first row presents aggregated values for the residential and commercial sectors, while the subsequent rows provide a detailed breakdown of emissions for each sector individually. The first columns present the full effect, and the other columns decompose between what we call the scale effect, the composition effect, and the technique effect. The first effect corresponds to the direct effect of increasing the total floor space consumed across the country. In contrast, the composition effect corresponds to the reallocation of floorspace across locations, switching different emission intensities. The technique effect corresponds to variations in emission intensity due to changes in local density.

The increase in WFH leads to a reduction in total floorspace emissions by 2.6%. This is the combination of a positive effect on residential emissions (+1.0%) and a negative effect on commercial emissions (-3.6%), with the latter dominating.

Table 6 further decomposes the channels behind the changes in floor space consumption. Each column isolates one channel that explains the changes in floorspace consumption (residential and commercial). Aggregate floorspace decreases by 1.3%. Lower rents explain most of the increase in residential floorspace and compensate for the loss in income. The reduction in commercial floorspace results from the lower expenditure share on floorspace (by assumption), and the reduction in wages, which makes labor relatively more productive compared to floorspace and therefore leads to substitution of floorspace by labor.

Table 5: Emissions floorspace

	Full Effect	Scale (ΔH)	Composition	Technique ($\Delta \varepsilon$)	Second Order
			MtonsCO2 (%)		
	(1)	(2)	(3)	(4)	(5)
Total	-0.02	-0.02	-0.04	0.01	0.04
	-2.7%	*-3.4%*	*-6.0%*	*1.8%*	*4.9%*
Residential	0.01	0.01	0.00	0.00	0.00
	1.1%	*0.8%*	*0.0%*	*0.2%*	*0.0%*
Commercial	-0.03	-0.03	-0.04	0.01	0.04
	-3.8%	*-4.2%*	*-6.1%*	*1.6%*	*4.9%*

Table 6: Floorspace change decomposition

Scenario	$\Delta\alpha$	$\Delta\pi$	Δw	Δq	Second Order	Total
				kmsq (%)		
	(1)	(2)	(3)	(4)	(5)	(6)
Total	-41.36	-0.78	-84.41	123.13	-9.82	-13.24
	-4.4%	*-0.1%*	*-9.0%*	*13.2%*	*-1.1%*	*-1.4%*
Residential		0.46	-49.54	68.57	-6.74	12.75
		0.0%	*-5.3%*	*7.3%*	*-0.7%*	*1.4%*
Commercial	-41.36	-1.24	-34.87	54.56	-3.08	-25.98
	-4.4%	*-0.1%*	*-3.7%*	*5.8%*	*-0.3%*	*-2.8%*

5 Conclusion

Our study contributes to the ongoing discussion related to the future of cities and the increasing demand for flexible working arrangements. Understanding the potential environmental impacts of WFH is crucial for shaping sustainable urban planning policies and addressing climate change concerns. Our findings show that increasing WFH days can significantly reduce GHG emissions from commuting and floorspace consumption. This highlights WFH potential to promote sustainable urban development and reduce cities' environmental impact.

It is important to note that the results differ across cities and the extent to which WFH impacts a city's environmental footprint depends on various factors, including urban density, industry composition, and commuting patterns. Our analysis reveals distinct patterns in population reallocation between Stockholm, Gothenburg, and Malmo, the three largest Swedish cities. In Stockholm, the densest municipalities gain residents, while in Gothenburg and Malmo, the opposite is true. These variations highlight the importance of considering city-specific characteristics when assessing the impact of WFH on a city's environmental footprint.

References

Ahlfeldt, G. M., S. J. Redding, D. M. Sturm, and N. Wolf (2015). The economics of density: Evidence from the berlin wall. *Econometrica 83*(6), 2127–2189.

Aksoy, C. G., J. M. Barrero, N. Bloom, S. J. Davis, M. Dolls, and P. Zarate (2022). Working from home around the world. Technical report, National Bureau of Economic Research.

Behrens, K., S. Kichko, and J.-F. Thisse (2021). Working from home: Too much of a good thing?

Bloom, N., R. Han, and J. Liang (2024, June). Hybrid working from home improves retention without damaging performance. *Nature 630*(8018), 920–925. Publisher: Nature Publishing Group.

Bloom, N., J. Liang, J. Roberts, and Z. J. Ying (2014, 11). Does working from home work? evidence from a chinese experiment *. *The Quarterly Journal of Economics 130*(1), 165–218.

Borck, R. and J. K. Brueckner (2018). Optimal energy taxation in cities. *Journal of the Association of Environmental and Resource Economists 5*(2), 481–516.

Borck, R., M. Kalkuhl, and K. Lessmann (2024). Is working from home good for the environment? mimeo.

Brueckner, J. (2024). Work-from-home and cities: An elementary spatial model. *SSRN Electronic Journal*.

Brueckner, J. K., M. E. Kahn, and G. C. Lin (2023). A new spatial hedonic equilibrium in the emerging work-from-home economy? *American Economic Journal: Applied Economics 15*(2), 285–319.

Cerqueira, E. V., B. Motte-Baumvol, L. Chevallier, and O. Bonin (2020). Does working from home reduce co2 emissions? an analysis of travel patterns as dictated by workplaces. *Transportation Research Part D: Transport and Environment*.

Delventhal, M. J., E. Kwon, and A. Parkhomenko (2022). Jue insight: How do cities change when we work from home? *Journal of Urban Economics 127*, 103331.

Delventhal, M. J. and A. Parkhomenko (2023). Spatial implications of telecommuting. unpublished paper.

Font, A., L. Guiseppin, M. Blangiardo, V. Ghersi, and G. W. Fuller (2019). A tale of two cities: is air pollution improving in paris and london? *Environmental Pollution 249*, 1–12.

Gibbs, M., F. Mengel, and C. Siemroth (2023). Work from home and productivity: Evidence from personnel and analytics data on information technology professionals. *Journal of Political Economy Microeconomics 1*(1), 7–41.

Giovanis, E. (2018). The relationship between teleworking, traffic and air pollution. *Atmospheric Pollution Research 9*(1), 1–14.

Guglielminetti, E., M. Loberto, G. Zevi, and R. Zizza (2023). How work from home changes housing demand: Evidence from online search. *AEA Papers and Proceedings*.

Gupta, A., V. Mittal, J. Peeters, and S. Van Nieuwerburgh (2022). Flattening the curve: pandemic-induced revaluation of urban real estate. *Journal of Financial Economics 146*(2), 594–636.

Heblich, S., S. J. Redding, and D. M. Sturm (2020). The making of the modern metropolis: evidence from london. *The Quarterly Journal of Economics 135*(4), 2059–2133.

Kyriakopoulou, E. and P. M. Picard (2023). The zoom city: working from home, urban productivity and land use. *Journal of Economic Geography*, lbad025.

Larson, W. and W. Zhao (2016). Telework: Urban form, energy consumption, and greenhouse gas implications. *Economic Inquiry 55*(2), 714–735.

Liu, S. and Y. Su (2021). The impact of the covid-19 pandemic on the demand for density: Evidence from the us housing market. *Economics letters 207*, 110010.

Liu, S. and Y. Su (2023). The effect of working from home on the agglomeration economies of cities: Evidence from advertised wages. *Available at SSRN 4109630*.

Marz, W. and S. Şen (2022). Does telecommuting reduce commuting emissions? *CESifo: Economics of Digitization (Topic)*.

Mondragón, J. and J. Wieland (2022). Housing demand and remote work. *Federal Reserve Bank of San Francisco, Working Paper Series*.

Monte, F., C. Porcher, and E. Rossi-Hansberg (2023). Remote work and city structure. *NBER Working Paper No. 31494*.

Monte, F., S. J. Redding, and E. Rossi-Hansberg (2018). Commuting, migration, and local employment elasticities. *American Economic Review 108*(12), 3855–3890.

Navaratnam, S., A. Jayalath, and L. Aye (2022). Effects of working from home on greenhouse gas emissions and the associated energy costs in six australian cities. *Buildings 12*.

Tao, Y., L. Yang, S. Jaffe, F. Amini, P. Bergen, B. Hecht, and F. You (2023). Climate mitigation potentials of teleworking are sensitive to changes in lifestyle and workplace rather than ict usage. *Proceedings of the National Academy of Sciences of the United States of America 120*.

Valentinyi, A. and B. Herrendorf (2008). Measuring factor income shares at the sectoral level. *Review of Economic Dynamics 11*(4), 820–835.

A Counterfactual equations

A.1 Hat equations

We have:

$$\hat{N}_{ijsf} = \hat{\pi}_{ij}\hat{\pi}_{s|ij}\hat{\pi}_{f|ijs}, \tag{24}$$

$$\hat{\pi}_{f|ijs} = \frac{\hat{d}_{ijsf}^{-\nu}}{\sum_{f'} \pi_{f'|ijs}\hat{d}_{ijsf'}^{-\nu}}, \tag{25}$$

$$\hat{\pi}_{s|ij} = \frac{\widehat{\mathbb{E}(v_{ijs})}^{\rho}}{\sum_{s'} \pi_{s'|ij}\widehat{\mathbb{E}(v_{ijs'})}^{\rho}} \qquad \text{with} \qquad \widehat{\mathbb{E}(v_{ijs})} = \frac{\hat{X}_i\hat{w}_{js}}{\hat{q}_i^{\beta}\widehat{(1+g[\theta_s])}^{\beta}}\left[\sum_f \pi_{f|ijs}\hat{d}_{ijsf}^{-\nu}\right]^{1/\nu} \tag{26}$$

$$\hat{\pi}_{ij} = \frac{\widehat{\mathbb{E}(v_{ij})}^{\gamma}}{\sum_{i'}\sum_{j'} \pi_{i'j'}\widehat{\mathbb{E}(v_{i'j'})}^{\gamma}} \qquad \text{with} \qquad \widehat{\mathbb{E}(v_{ij})} = \left[\sum_s \pi_{s|ij}\widehat{\mathbb{E}(v_{ijs})}^{\rho}\right]^{1/\rho}. \tag{27}$$

In the baseline, we assume $\widehat{(1+g[\theta_s])} = 1$.

For wages we have:

$$\hat{w}_{js} = A_{js}^{\frac{1}{\alpha_s'}-\frac{1}{\alpha_s}} q_j^{-\frac{1-\alpha_s'}{\alpha_s'}+\frac{1-\alpha_s}{\alpha_s}} \hat{\xi}_s \hat{A}_{js}^{\frac{1}{\alpha_s'}} \hat{q}_j^{-\frac{1-\alpha_s'}{\alpha_s'}}, \tag{28}$$

where we replace unobserved A_{js} with there expression in terms of observables: $A_{js} = q_j^{1-\alpha_s}\left(\frac{w_{js}}{\xi_s}\right)^{\alpha_s}$.

For the floorspace prices:

$$\hat{q}_i = \hat{H}_i^{\frac{\eta}{1-\eta}}. \tag{29}$$

For floorspace demand:

$$\hat{H}_i = \frac{1}{q_i H_i}\frac{1}{\hat{q}_i}\left[\beta\sum_s\sum_j w_{js}N_{ijs}\hat{w}_{js}\hat{N}_{ijs} + \sum_s \frac{1-\alpha_s'}{\alpha_s'} w_{is}\hat{w}_{is}\sum_{i'} N_{i'is}\hat{N}_{i'is}\right]. \tag{30}$$

Finally:

$$\hat{A}_{js} = \frac{\left[\sum_i \sum_s [\theta_s' + \psi(1-\theta_s')]N_{ijs}\hat{N}_{ijs}\right]^{\lambda}}{\left[\sum_i \sum_s [\theta_s + \psi(1-\theta_s)]N_{ijs}\right]^{\lambda}}, \tag{31}$$

and

$$\hat{X}_i = \frac{\left[\sum_j \sum_s N_{ijs} \hat{N}_{ijs}\right]^{\chi}}{\left[\sum_j \sum_s N_{ijs}\right]^{\chi}}. \tag{32}$$

B Existence and unicity

In this section, we use Allen et al. (2023), "On the Equilibrium Properties of Spatial Models", AAL hereafter, to find a sufficient condition for our WFH model, in the case of a single sector and a single transport mode, to have a unique solution.

B.1 Our model

In this section, I write the equilibrium equations.

- Workers choose locations so that:

$$N_{ij} = \left(\frac{X_i}{d(t_{ij}, \theta)} \frac{w_j}{[q_i(1+g(\theta))]^{\beta}}\right)^{\gamma} / \sum_{m,n} \left(\frac{X_m}{d(t_{mn}, \theta)} \frac{w_n}{[q_m(1+g(\theta))]^{\beta}}\right)^{\gamma}. \tag{33}$$

- Wages are written as:

$$w_j = A_j q_j^{-\frac{1-\alpha(\theta)}{\alpha(\theta)}}. \tag{34}$$

- Developers build floorspace in response to rent levels according to:

$$H_i = \frac{\phi_i \Lambda_i}{\eta} q_i^{\frac{1-\eta}{\eta}}. \tag{35}$$

- Local floorspace markets clear and we have:

$$q_i H_i = \beta \sum_j w_j N_{ij} + \frac{1-\alpha(\theta)}{\alpha(\theta)} \sum_j w_i N_{ji}. \tag{36}$$

  Externalities are defined by:

$$A_j = a_j \left[\frac{1}{\Lambda_j} \sum_i [\theta + \psi(1-\theta)] N_{ij}\right]^{\lambda} \text{ and} \tag{37}$$

$$X_i = x_i \left[\frac{1}{\Lambda_i} \sum_j N_{ij}\right]^{\chi}. \tag{38}$$

In what follows, for simplicity and without loss of generality, I drop all θ indications and note $d(t_{ij})(1+g)^{\beta} = d_{ij}$

B.2 Application of AAL

We can rewrite our equilibrium as a system of three sets of equations to which AAL's theorem is applicable. In this system, the unknowns are the vectors of local rents, q_i, residents N_i^R, and workers N_j^W:

$$N_i^R = \sum_j \frac{d_{ij}^{-\gamma} X_i^\gamma A_j^\gamma q_j^{-\frac{1-\alpha}{\alpha}\gamma} q_i^{-\beta\gamma}}{\sum_m \sum_n d_{mn}^{-\gamma} X_m^\gamma A_n^\gamma q_n^{-\frac{1-\alpha}{\alpha}\gamma} q_m^{-\beta\gamma}}, \tag{39}$$

$$N_i^W = \sum_j \frac{d_{ji}^{-\gamma} X_j^\gamma A_i^\gamma q_i^{-\frac{1-\alpha}{\alpha}\gamma} q_j^{-\beta\gamma}}{\sum_m \sum_n d_{mn}^{-\gamma} X_m^\gamma A_n^\gamma q_n^{-\frac{1-\alpha}{\alpha}\gamma} q_m^{-\beta\gamma}}, \tag{40}$$

$$q_i^{\frac{1}{\eta}} = \sum_j \frac{\frac{\eta}{\phi_i \Lambda_i}\left[\beta d_{ij}^{-\gamma} X_i^\gamma A_j^{1+\gamma} q_j^{-\frac{1-\alpha}{\alpha}(1+\gamma)} q_i^{-\beta\gamma} + \frac{1-\alpha}{\alpha} d_{ji}^{-\gamma} X_j^\gamma A_i^{1+\gamma} q_i^{-\frac{1-\alpha}{\alpha}(1+\gamma)} q_j^{-\beta\gamma}\right]}{\sum_m \sum_n d_{mn}^{-\gamma} X_m^\gamma A_n^\gamma q_n^{-\frac{1-\alpha}{\alpha}\gamma} q_m^{-\beta\gamma}}, \tag{41}$$

where:

$$X_i = x_i \left(N_i^R / \Lambda_i\right)^\chi, \tag{42}$$

$$A_j = a_j \left([\theta + \psi(1-\theta)] N_j^W / \Lambda_j\right)^\lambda := \tilde{a}_j \left(N_j^W / \Lambda_j\right)^\lambda. \tag{43}$$

$$\tag{44}$$

The idea of the theorem is to find a sufficient condition for the equilibrium to be a contraction. To do that we need to find the 3-by-3 square matrix (because of the 3 sets of equations) defined by $(\boldsymbol{A})_{hh'} = \sup_{i,j}\left(\left|\frac{\partial \ln f_{ijh}}{\partial \ln x_{jh'}}\right|\right)$.

Notations:

- Subscripts i and j denote locations.
- Subscripts $h \in \{1, 2, 3\}$ denote the interaction, in our case three: numbers of local residents, number of local workers, and local rents.
- We note:

$$x_{ih} = \begin{cases} N_i^R & \text{if } h = 1 \\ N_i^W & \text{if } h = 2 \\ q_i^{\frac{1}{\eta}} & \text{if } h = 3, \end{cases}$$

- and:

$$f_{ijh} = \begin{cases} c_{ij1} x_{i1}^{\chi\gamma} x_{j2}^{\lambda\gamma} x_{j3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{i3}^{-\beta\eta\gamma} & \text{if } h = 1 \\ c_{ij2} x_{j1}^{\chi\gamma} x_{i2}^{\lambda\gamma} x_{i3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{j3}^{-\beta\eta\gamma} & \text{if } h = 2 \\ \underbrace{c_{ij3}^{R} x_{i1}^{\chi\gamma} x_{j2}^{\lambda(1+\gamma)} x_{j3}^{-\frac{1-\alpha}{\alpha}\eta(1+\gamma)} x_{i3}^{-\beta\eta\gamma}}_{=f_{ij3}^{A}} + \underbrace{c_{ij3}^{W} x_{j1}^{\chi\gamma} x_{i2}^{\lambda(1+\gamma)} x_{i3}^{-\frac{1-\alpha}{\alpha}\eta(1+\gamma)} x_{j3}^{-\beta\eta\gamma}}_{f_{ij3}^{B}} & \text{if } h = 3, \end{cases}$$

with $c_{ij1} = c_{ji2} = (d_{ij}^{-1} x_i \Lambda_i^{-\chi} \tilde{a}_j \Lambda_j^{-\lambda})^{\gamma}$, $c_{ij3}^{R} = \beta \tilde{a}_j \Lambda_j^{-\lambda} c_{ij1}$, and $c_{ij3}^{W} = \frac{1-\alpha}{\alpha} \tilde{a}_i \Lambda_i^{-\lambda} c_{ij2}$,

- such that we have the AAL formulation of the equilibrium:

$$x_{ih} = \frac{1}{r} \sum_j f_{ijh}(\boldsymbol{x}), \tag{45}$$

with $r = \sum_m \sum_n c_{mn1} x_{m1}^{\chi\gamma} x_{n2}^{\lambda\gamma} x_{n3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{m3}^{-\beta\eta\gamma}$.

We need to find the three-by-three matrix defined by coefficients: $a_{hh'} = \left(\sup_{i,j} \sum_j |\frac{\partial \ln \sum_k f_{ikh(x)}}{\partial \ln x_{jh'}}| + \sum_j |\frac{\partial \ln r}{\partial \ln x_{jh'}}| \right)_{hh'}$.

First we look at the derivatives of the common scalar r:

- for $h' = 1$:

$$\begin{aligned} \frac{\partial \ln r}{\partial \ln x_{j1}} &= \sum_m \sum_n \frac{c_{mn1} x_{m1}^{\chi\gamma} x_{n2}^{\lambda\gamma} x_{n3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{m3}^{-\beta\eta\gamma}}{r} \chi\gamma 1(m = j) \\ &= \sum_n \frac{c_{jn1} x_{j1}^{\chi\gamma} x_{n2}^{\lambda\gamma} x_{n3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{j3}^{-\beta\eta\gamma}}{r} \chi\gamma \\ \sum_j |\frac{\partial \ln r}{\partial \ln x_{j1}}| &= \sum_j \sum_n \frac{c_{jn1} x_{j1}^{\chi\gamma} x_{n2}^{\lambda\gamma} x_{n3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{j3}^{-\beta\eta\gamma}}{r} \chi\gamma \\ \sum_j |\frac{\partial \ln r}{\partial \ln x_{j1}}| &= \chi\gamma, \end{aligned}$$

- for $h' = 2$:

$$\frac{\partial \ln r}{\partial \ln x_{j2}} = \sum_m \sum_n \frac{c_{mn1} x_{m1}^{\chi\gamma} x_{n2}^{\lambda\gamma} x_{n3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{m3}^{-\beta\eta\gamma}}{r} \lambda\gamma 1(n = j)$$
$$= \sum_m \frac{c_{mj1} x_{m1}^{\chi\gamma} x_{j2}^{\lambda\gamma} x_{j3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{m3}^{-\beta\eta\gamma}}{r} \lambda\gamma \leq \lambda\gamma,$$
$$\sum_j \left|\frac{\partial \ln r}{\partial \ln x_{j2}}\right| = \sum_j \sum_m \frac{c_{mj1} x_{m1}^{\chi\gamma} x_{j2}^{\lambda\gamma} x_{j3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{m3}^{-\beta\eta\gamma}}{r} \lambda\gamma$$
$$\sum_j \left|\frac{\partial \ln r}{\partial \ln x_{j2}}\right| = \lambda\gamma,$$

- and for $h' = 3$:

$$\frac{\partial \ln r}{\partial \ln x_{j3}} = \sum_m \sum_n \frac{c_{mn1} x_{m1}^{\chi\gamma} x_{n2}^{\lambda\gamma} x_{n3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{m3}^{-\beta\eta\gamma}}{r} \left(- \frac{1-\alpha}{\alpha}\eta\gamma 1(n = j) - \beta\eta\gamma 1(m = j)\right)$$
$$= \sum_m \frac{c_{mj1} x_{m1}^{\chi\gamma} x_{j2}^{\lambda\gamma} x_{j3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{m3}^{-\beta\eta\gamma}}{r} \left(- \frac{1-\alpha}{\alpha}\eta\gamma\right) + \sum_n \frac{c_{jn1} x_{j1}^{\chi\gamma} x_{n2}^{\lambda\gamma} x_{n3}^{-\frac{1-\alpha}{\alpha}\eta\gamma} x_{j3}^{-\beta\eta\gamma}}{r} \left(- \beta\eta\gamma\right)$$
$$\sum_j \left|\frac{\partial \ln r}{\partial \ln x_{j3}}\right| = \frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma,$$

then we the partial derivatives of functions f:

- $h = 1, h' = 1, \forall(i, j)$:

$$\frac{\partial \ln \sum_k f_{ik1}(x)}{\partial \ln x_{j1}} = \sum_k \frac{f_{ik1}(x)}{\sum_{k'} f_{ik'1}(x)} \frac{\partial \ln f_{ik1}(x)}{\partial \ln x_{j1}}$$
$$= \sum_k \frac{f_{ik1}(x)}{\sum_{k'} f_{ik'1}(x)} \chi\gamma 1(i = j)$$
$$= \chi\gamma 1(i = j),$$
$$\sum_j \left|\frac{\partial \ln \sum_k f_{ik1}(x)}{\partial \ln x_{j1}}\right| = \chi\gamma,$$

  so $a_{11} = 2\chi\gamma$.

- $h=1, h'=2, \forall(i,j)$:

$$\begin{aligned}\frac{\partial \ln \sum\limits_k f_{ik1}(x)}{\partial \ln x_{j2}} &= \sum_k \frac{f_{ik1}(x)}{\sum\limits_{k'} f_{ik'1}(x)} \frac{\partial \ln f_{ik1}(x)}{\partial \ln x_{j2}} \\ &= \sum_k \frac{f_{ik1}(x)}{\sum\limits_{k'} f_{ik'1}(x)} \lambda\gamma 1(k=j) \\ &= \frac{f_{ij1}(x)}{\sum\limits_{k'} f_{ik'1}(x)} \lambda\gamma,\end{aligned}$$

$$\sum_j \left| \frac{\partial \ln \sum\limits_k f_{ik1}(x)}{\partial \ln x_{j2}} \right| = \lambda\gamma,$$

so $a_{12} = 2\lambda\gamma$.

- $h=1, h'=3, \forall(i,j)$:

$$\begin{aligned}\frac{\partial \ln \sum\limits_k f_{ik1}(x)}{\partial \ln x_{j3}} &= \sum_k \frac{f_{ik1}(x)}{\sum\limits_{k'} f_{ik'1}(x)} \frac{\partial \ln f_{ik1}(x)}{\partial \ln x_{j3}} \\ &= \sum_k \frac{f_{ik1}(x)}{\sum\limits_{k'} f_{ik'1}(x)} \left(-\frac{1-\alpha}{\alpha}\eta\gamma 1(k=j) - \beta\eta\gamma 1(i=j) \right) \\ &= -\frac{1-\alpha}{\alpha}\eta\gamma \frac{f_{ij1}(x)}{\sum\limits_{k'} f_{ij'1}(x)} - \beta\eta\gamma 1(i=j)\end{aligned}$$

$$\sum_j \left| \frac{\partial \ln \sum\limits_k f_{ik1}(x)}{\partial \ln x_{j3}} \right| = \frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma,$$

so $a_{13} = 2\left(\frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma\right)$.

- $h=2, h'=1, \forall(i,j)$:

$$\begin{aligned}\frac{\partial \ln \sum\limits_k f_{ik2}(x)}{\partial \ln x_{j1}} &= \sum_k \frac{f_{ik2}(x)}{\sum\limits_{k'} f_{ik'2}(x)} \frac{\partial \ln f_{ik2}(x)}{\partial \ln x_{j1}} \\ &= \sum_k \frac{f_{ik2}(x)}{\sum\limits_{k'} f_{ik'2}(x)} \chi\gamma 1(k=j) \\ &= \frac{f_{ij2}(x)}{\sum\limits_{k'} f_{ik'2}(x)} \chi\gamma\end{aligned}$$

$$\sum_j \left| \frac{\partial \ln \sum\limits_k f_{ik2}(x)}{\partial \ln x_{j1}} \right| = \chi\gamma,$$

so $a_{21} = 2\chi\gamma$.

- $h = 2, h' = 2, \forall(i, j)$:

$$\begin{aligned}
\frac{\partial \ln \sum_k f_{ik2}(x)}{\partial \ln x_{j2}} &= \sum_k \frac{f_{ik2}(x)}{\sum_{k'} f_{ik'2}(x)} \frac{\partial \ln f_{ik2}(x)}{\partial \ln x_{j2}} \\
&= \sum_k \frac{f_{ik2}(x)}{\sum_{k'} f_{ik'2}(x)} \lambda\gamma 1(i = j) \\
&= \lambda\gamma 1(i = j),
\end{aligned}$$

$$\sum_j \left| \frac{\partial \ln \sum_k f_{ik2}(x)}{\partial \ln x_{j2}} \right| = \lambda\gamma,$$

so $a_{22} = 2\lambda\gamma$.

- $h = 2, h' = 3, \forall(i, j)$:

$$\begin{aligned}
\frac{\partial \ln \sum_k f_{ik2}(x)}{\partial \ln x_{j3}} &= \sum_k \frac{f_{ik2}(x)}{\sum_{k'} f_{ik'2}(x)} \frac{\partial \ln f_{ik2}(x)}{\partial \ln x_{j3}} \\
&= \sum_k \frac{f_{ik2}(x)}{\sum_{k'} f_{ik'2}(x)} \left(-\frac{1-\alpha}{\alpha}\eta\gamma 1(i = j) - \beta\eta\gamma 1(k = j) \right) \\
&= -\frac{1-\alpha}{\alpha}\eta\gamma 1(i = j) - \beta\eta\gamma \frac{f_{ij2}(x)}{\sum_{k'} f_{ik'2}(x)}
\end{aligned}$$

$$\sum_j \left| \frac{\partial \ln \sum_k f_{ik2}(x)}{\partial \ln x_{j3}} \right| = \frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma,$$

so $a_{23} = 2\left(\frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma\right)$.

- $h = 3, h' = 1, \forall(i, j)$:

$$\begin{aligned}
\frac{\partial \ln \sum_k f_{ik3}(x)}{\partial \ln x_{j1}} &= \sum_k \frac{f_{ik3}(x)}{\sum_{k'} f_{ik'3}(x)} \frac{\partial \ln f_{ik3}(x)}{\partial \ln x_{j1}} \\
&= \sum_k \frac{f_{ik3}(x)}{\sum_{k'} f_{ik'3}(x)} \left[\frac{f^A_{ik3}}{f_{ik3}} \frac{\partial \ln f^A_{ik3}(x)}{\partial \ln x_{j1}} + \frac{f^A_{ik3}}{f_{ik3}} \frac{\partial \ln f^B_{ik3}(x)}{\partial \ln x_{j1}} \right] \\
&= \sum_k \frac{f_{ik3}(x)}{\sum_{k'} f_{ik'3}(x)} \left[\frac{f^A_{ik3}}{f_{ik3}} \chi\gamma 1(i = j) + \frac{f^A_{ik3}}{f_{ik3}} \chi\gamma 1(k = j) \right] \\
&= \chi\gamma 1(i = j) \frac{\sum_k f^A_{ik3}(x)}{\sum_{k'} f_{ik'3}(x)} + \chi\gamma \frac{f^B_{ij3}(x)}{\sum_{k'} f_{ik'3}(x)},
\end{aligned}$$

$$\sum_j \left| \frac{\partial \ln \sum_k f_{ik3}(x)}{\partial \ln x_{j1}} \right| = \chi\gamma \frac{\sum_k f^A_{ik3}(x) + \sum_j f^B_{ij3}(x)}{\sum_{k'} f_{ik'3}(x)} = \chi\gamma,$$

so $a_{31} = 2\chi\gamma$.

- $h = 3, h' = 2, \forall(i,j)$:

$$\frac{\partial \ln \sum_k f_{ik3}(x)}{\partial \ln x_{j2}} = \sum_k \frac{f_{ik3}(x)}{\sum_{k'} f_{ik'3}(x)} \left[\frac{f^A_{ik3}}{f_{ik3}} \frac{\partial \ln f^A_{ik3}(x)}{\partial \ln x_{j2}} + \frac{f^A_{ik3}}{f_{ik3}} \frac{\partial \ln f^B_{ik3}(x)}{\partial \ln x_{j2}}\right]$$
$$= \dots$$
$$\sum_j \left| \frac{\partial \ln \sum_k f_{ik3}(x)}{\partial \ln x_{j2}} \right| = \lambda(\gamma+1),$$

so $a_{32} = 2\lambda\gamma + \lambda$.

- $h = 3, h' = 3, \forall(i,j)$:

$$\frac{\partial \ln \sum_k f_{ik3}(x)}{\partial \ln x_{j3}} = \sum_k \frac{f_{ik3}(x)}{\sum_{k'} f_{ik'3}(x)} \left[\frac{f^A_{ik3}}{f_{ik3}} \frac{\partial \ln f^A_{ik3}(x)}{\partial \ln x_{j3}} + \frac{f^A_{ik3}}{f_{ik3}} \frac{\partial \ln f^B_{ik3}(x)}{\partial \ln x_{j3}}\right]$$
$$= \dots$$
$$\sum_j \left| \frac{\partial \ln \sum_k f_{ik3}(x)}{\partial \ln x_{j3}} \right| = \beta\eta\gamma + \frac{1-\alpha}{\alpha}\eta(\gamma+1),$$

so $a_{32} = 2\left(\frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma\right) + \frac{1-\alpha}{\alpha}\eta$.

The condition to have a contraction is then $\rho(\boldsymbol{A}) < 1$ with:

$$\boldsymbol{A} = \begin{pmatrix} 2\chi\gamma & 2\lambda\gamma & 2\left(\frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma\right) \\ 2\chi\gamma & 2\lambda\gamma & 2\left(\frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma\right) \\ 2\chi\gamma & 2\lambda\gamma + \lambda & 2\left(\frac{1-\alpha}{\alpha}\eta\gamma + \beta\eta\gamma\right) + \frac{1-\alpha}{\alpha}\eta \end{pmatrix}.$$

To compute the spectral radius of $\boldsymbol{A}$, we compute its eigenvalues by solving the characteristic polynom $det(A - xI) = 0$:

- 0 is always solution ($\boldsymbol{A}$ has two identical rows;
- the two other eigenvalues are:

$$ev_\pm = [\chi + \lambda + (\beta + \frac{1-\alpha}{\alpha})\eta]\gamma + \frac{1-\alpha}{\alpha}\frac{\eta}{2}$$
$$\pm \sqrt{\left([\chi + \lambda + (\beta + \frac{1-\alpha}{\alpha})\eta]\gamma + \frac{1-\alpha}{\alpha}\frac{\eta}{2}\right)^2 + 2\left((\chi+\lambda)\gamma\frac{1-\alpha}{\alpha}\eta + \lambda(\beta + \frac{1-\alpha}{\alpha})\eta\gamma\right)}$$

.

So the spectral radius of $\boldsymbol{A}$ is equal to:

$$\rho(\boldsymbol{A}) = [\chi + \lambda + (\beta + \frac{1-\alpha}{\alpha})\eta]\gamma + \frac{1-\alpha}{\alpha}\frac{\eta}{2}$$
$$+ \sqrt{\left([\chi + \lambda + (\beta + \frac{1-\alpha}{\alpha})\eta]\gamma + \frac{1-\alpha}{\alpha}\frac{\eta}{2}\right)^2 + 2\left((\chi + \lambda)\gamma\frac{1-\alpha}{\alpha}\eta + \lambda(\beta + \frac{1-\alpha}{\alpha})\eta\gamma\right)}.$$

We note that when $\eta \to 0$, and the floorspace equilibrium is shut down, we are back to the condition that $2(\lambda + \chi)\gamma < 1$, which is what AAL find.