Ex ante individually rational trade

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Efficiency and individual rationality

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- d'Aspremont and Gérard-Varet (1979) show that efficiency is always achievable without running a deficit if agents must decide whether to participate at the *ex ante* stage

Ex ante individual rationality interpretation

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- truthful mechanisms (under private values) simplify the participation decision
  - higher-order beliefs are irrelevant for each agent's participation decision

Our contribution

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  - show that for general distributions, we cannot achieve full efficiency in bilateral trade
  - provide a sufficient condition under which efficiency can be achieved in bilateral trade
  - in a partnership dissolution model, full efficiency can be achieved regardless of the initial property rights

Outline

- Bilateral trade : an example which shows efficiency is not always achievable
- Bilateral trade : a sufficient condition for efficient implementation
- Application to partnership dissolution

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definition

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Q: can we find a truthful, ex ante IR, efficient mechanism that has no deficit, for general distributions ?

Lemmas

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a Groves scheme has **no deficit** iff $h_B(\theta_S) + h_S(\theta_B) - \mathbb{1}(\theta_B \geq \theta_S)(\theta_B - \theta_S) \geq 0$.

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lemma

a Groves scheme is **ex-ante IR** iff $\mathbb{E}_{\theta_B}[h_S(\theta_B)] \leq GT$ and $\mathbb{E}_{\theta_S}[h_B(\theta_S)] \leq GT$ where $GT = \mathbb{E}_{\theta_B,\theta_S}[\mathbb{1}(\theta_B \geq \theta_S)(\theta_B - \theta_S)]$

Example

- $F_B \sim 0.8\delta_0 + 0.1\delta_1 + 0.1U([0,1])$
- $F_S \sim 0.8\delta_1 + 0.1\delta_0 + 0.1U([0,1])$

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- no deficit : $h_B(\theta_S) + h_S(\theta_B) - \mathbb{1}(\theta_B \geq \theta_S)(\theta_B - \theta_S) \geq 0$
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- ex ante gains from trade are $GT = \frac{13}{600}$

- ex ante individual rationality requires that $\mathbb{E}_{\theta_B}[h_S(\theta_B)] \leq GT$ and $\mathbb{E}_{\theta_S}[h_B(\theta_S)] \leq GT$

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- combining the last points, and summing the ex ante IRs, we have

$$0.1[h_B(0) + h_S(1)] + 0.8[h_B(1) + h_S(0)] + 0.1\int_0^1 [h_S(\theta) + h_B(\theta)]\, d\theta \leq \frac{7}{300}$$

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left-hand side ≥ 0.1 $\rightarrow$ contradiction !

Lessons from the example

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can we provide a sufficient condition to ensure that full efficiency is achievable ?

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Sufficient condition for efficient ex ante IR trade

theorem

if $Med(F_B) \geq Med(F_S)$, there exists an efficient, truthful, ex ante individually rational mechanism that has no deficit

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if $Med(F_B) \geq Med(F_S)$, there exists an efficient, truthful, ex ante individually rational mechanism that has no deficit

- interpretation of the result
  - Myerson and Satterthwaite (1983) imply that we cannot strengthen ex ante IR to interim IR in the theorem
  - d'Aspremont and Gérard-Varet (1979) imply that the result holds if truthfulness is relaxed to Bayesian IC

Understanding the theorem

	Interim IR	Ex ante IR
Bayesian IC	✗ Myerson and Satterthwaite, 1983	✓ d'Aspremont and Gérard-Varet, 1979
Truthful	✗ Myerson and Satterthwaite, 1983	✓ if $Med(F_B) \geq Med(F_S)$ this paper

Table: (Non)existence of efficient mechanisms for bilateral trade (that do not run deficits) under various combinations of incentive compatibility and individual rationality constraints.

Only aggregate ex ante IR matters

- since agents are risk-neutral, we can find **lump-sum** transfers between the buyer and the seller such that
  - ex-ante IR is not only satisfied in aggregate but individually
  - this doesn't change the aggregate payment, incentives or efficiency

Construction of the mechanism

- let p^* be the large-market market-clearing price, that is p^* is the only solution to $F_B(p^*) + F_S(p^*) = 1$

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- propose the following Groves scheme
  - $q(\theta) = \mathbb{1}(\theta_B \geq \theta_S)$
  - $h_B(\theta_S) = \max(p^* - \theta_S, 0)$
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Determination of the replica economy p^*

- a key object in the Groves scheme above is the replica economy market-clearing price, we illustrate its determination below

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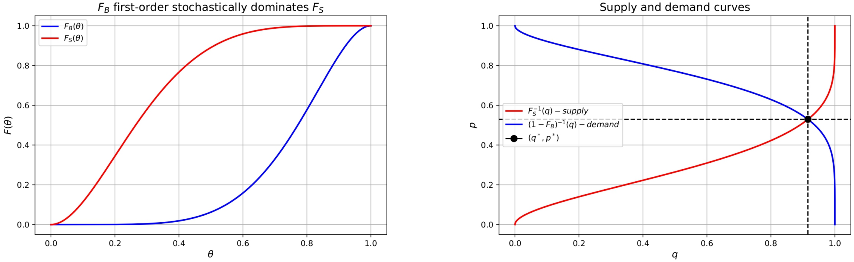

Some intuition for the result

- the participation charges have a kink point at type p^* and require the buyer (resp. seller) to make payments to the mechanism if the seller has a low value (resp. buyer has a high value)

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  - expected participation charges are the replica economy per-agent consumer and producer surpluses
  - total surplus per agent in the replica economy is at most $2GT$ in the single-transaction economy

Proof

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- no-deficit can be checked easily observing that the payments sum up to

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- all that remains is to check aggregate ex ante IR

Proof

- key objects to calculate
  - expected gains from trade $GT = \mathbb{E}_{\theta_B,\theta_S}\left[\mathbb{1}(\theta_B \geq \theta_S)(\theta_B - \theta_S)\right]$
  - expected participation charges $\mathbb{E}_{\theta_S}\left[h_B(\theta_S)\right]$, $\mathbb{E}_{\theta_B}\left[h_S(\theta_B)\right]$

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\mathbb{E}_{\theta_B}\left[h_S(\theta_B)\right] = \int_{p^*}^1 (1 - F_B(\theta_B))\, d\theta_B
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Proof

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&\geq (1 - F_B(p^*))\mathbb{E}_{\theta_S}\left[h_B(\theta_S)\right] + F_S(p^*)E_{\theta_B}\left[h_S(\theta_B)\right]
\end{aligned}
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- by the condition on medians, we have $F_S(p^*) = 1 - F_B(p^*) \geq \frac{1}{2}$

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&= \int_0^{p^*} F_S(y)(1 - F_B(y))\, dy + \int_{p^*}^1 F_S(y)(1 - F_B(y))\, dy \\
&\geq (1 - F_B(p^*)) \int_0^{p^*} F_S(y)\, dy + F_S(p^*) \int_{p^*}^1 (1 - F_B(y))\, dy \\
&\geq (1 - F_B(p^*))\mathbb{E}_{\theta_S}[h_B(\theta_S)] + F_S(p^*) E_{\theta_B}[h_S(\theta_B)]
\end{aligned}
$$

- by the condition on medians, we have $F_S(p^*) = 1 - F_B(p^*) \geq \frac{1}{2}$
- hence the aggregate ex ante IR is satisfied, that is $2GT \geq \mathbb{E}_{\theta_S}[h_B(\theta_S)] + \mathbb{E}_{\theta_B}[h_S(\theta_B)]$

Outline

- Bilateral trade : an example which shows efficiency is not always achievable
- Bilateral trade : a sufficient condition for efficient implementation
- Application to partnership dissolution

Canonical partnership dissolution

- Cramton et al. (1987), N agents are involved in a partnership, each agent initially holds a share $r_i \geq 0$, $\sum_{i=1}^{N} r_i = 1$

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  - semi-positive result: if the initial partnership shares are not too asymmetric, one can dissolve the partnership efficiently using a Bayesian IC, interim IR, budget-balanced mechanism

Any partnership is dissolvable

- consider the setup of Cramton et al. (1987), initial shares are common knowledge and agents have private values for the good at partnership and are **ex ante symmetric**

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theorem

for all initial ownership shares $(r_i)_{1 \leq i \leq N}$, there exists an efficient, truthful, ex ante individually rational mechanism that has no deficit.

- crucial result here is we don't need to impose any structure on the ownership shares to dissolve the partnership efficiently

Proof sketch

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if the partnership has a single owner (i.e., there exists an agent j such that $r_j = 1$), then there exists an efficient, truthful, ex ante IR mechanism that has no deficit

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  - use the lemma to dissolve the single-owner partnership efficiently with a truthful, no-deficit, ex ante IR mechanism
  - grand mechanism is clearly efficient, truthful, no deficit
  - from the point of view of agent i, he is the seller in one mechanism and a potential buyer in $N - 1$ mechanisms, on average his ex-ante IR is satisfied

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- proof of the lemma itself is quite involved
  - seller's participation charge is a piece-wise linear function of the highest of the buyer's values
  - key complication : the buyer's participation charges must depend on the values of other buyers in addition to the value of the seller

Proof sketch

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if the partnership has a single owner (i.e., there exists an agent j such that $r_j = 1$), then there exists an efficient, truthful, ex ante IR mechanism that has no deficit

- again, we display a Groves scheme which satisfies all properties

Conclusion

- provide an example showing that efficiency is not always attainable in the bilateral trade setting with a truthful, ex ante IR mechanism that has no deficit

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- provide a simple sufficient condition for efficiency to be achievable
- apply our concepts to the partnership dissolution model and show that the structure of property rights doesn't matter

Thank you!