Are Earnings Inequality and Firm Concentration Connected? Evidence from an Assignment Model

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This version: August 6, 2025.

Abstract

This paper develops a general equilibrium assignment model with hierarchical firms to explain rising worker earnings inequality and firm concentration. The model shows that concentration can increase even as productivity dispersion declines, driven by flatter organizational structures and rising skill inequality. Calibrated to U.S. data from the 1980s and 2010s, the model replicates key distributional trends and untargeted moments. Counterfactuals highlight the role of organizational hierarchy and skill dispersion. The findings suggest that changes in labor and organizational structure—not just productivity gaps—are central to explaining recent trends in inequality and concentration.
Keywords: *Firm Concentration; Earnings Inequality; Assignment Model; Superstar firms; Wage dispersion; Monopolistic competition; Labor market sorting*
JEL Classification: E24, J31, L11

1 Introduction

Over the past four decades, the U.S. economy has undergone two widely discussed transformations: a notable rise in worker earnings inequality and a parallel surge in firm concentration. While both trends have been extensively studied separately, a fundamental question remains open: are they causally connected, and if so, through which mechanisms? This paper proposes a novel answer by developing an assignment-based general equilibrium model that jointly explains both phenomena. The model yields new insights: it shows how employment and sales concentration can rise even when productivity dispersion falls, a result that runs counter to canonical models of firm heterogeneity.

I develop an assignment-based general equilibrium model in which heterogeneous workers are matched to hierarchical positions within heterogeneous firms operating under monopolistic competition. The framework generalizes the canonical model of CEO pay by Gabaix & Landier (2008) in two key ways: firms can employ multiple workers across ranked positions with heterogeneous marginal contributions, and they compete in differentiated product markets, à la Melitz (2003). In this setting, firm size and worker earnings are determined endogenously through the interaction between firm-level productivity, the distribution of worker skills, and the structure of internal firm hierarchies. The hierarchy's structure governs how rapidly the marginal product of additional workers declines within

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the firm—the flatter the hierarchy, the slower the decline in the marginal product of an additional worker.

Unlike canonical firm heterogeneity models—such as Melitz (2003)—where larger firms are inherently more productive, my model allows for the expansion of more productive firms not because they've widened their productivity lead, but because organizational structures have evolved to make additional hiring more efficient. This shift enables concentration to rise even with stable or narrowing productivity gaps. The model attributes this divergence to two key forces: the flattening of internal firm hierarchies, which enhances scalable employment, and rising skill dispersion, which disproportionately benefits the most productive firms. By embedding these mechanisms within a unified framework, I offer a new explanation for how structural changes in labor and product markets jointly generate inequality and concentration.

The model yields closed-form expressions for the distribution of firm size—measured by both employment and sales—and for worker earnings. It delivers three core theoretical results. First, earnings inequality rises in response to greater dispersion in worker skill, greater dispersion in firm productivity, and a higher price elasticity of demand. Second, firm size concentration in employment increases with firm productivity dispersion and flatter hierarchies (i.e., lower dispersion in the marginal productivity of additional positions). Third, sales concentration increases with both skill and productivity dispersion, flatter hierarchies, as well as greater demand elasticity. In the upper tail, both firm sales and employment distributions follow a Pareto distribution with a shape parameter governed by the ratio of the steepness of firm hierarchical structure to firm productivity dispersion.

To assess the empirical relevance of these forces, I calibrate the model targeting a set of moments that reflect earnings dispersion and firm size concentration in the U.S. in the 1980s and 2010s. The model fits these targets well and provides internally consistent estimates across both time periods. Importantly, the model also replicates several untargeted features of the data, including quantitatively reasonable Pareto-shaped tails in the earnings and firm size distributions and a positive correlation between firm size, or productivity, and wages. These results support the model's usefulness for interpreting changes in the joint distribution of earnings and firm size. I then use the calibrated model to perform counterfactual decompositions that isolate the contribution of each structural force—changes in skills, productivity, hierarchical structure, and price sensitivity—to the observed trends.

Four main quantitative results emerge from the calibrated model. First, the dispersion of hierarchical structure within firms has declined over time. This flattening of firm hierarchies increases the marginal benefit of adding new positions, allowing more productive firms to expand in employment. In the model, this mechanism fully accounts for the observed increase in employment concentration and also contributes significantly to rising sales concentration. This channel offers a new explanation for employment concentration that does not rely on widening productivity gaps between firms.

Second, the skill distribution has become more skewed, leading to a greater divergence between low- and high-skill workers. This shift is the driving force of the rise in earnings inequality, explaining more that the total increase and 41% of the rise in sales concentration. These results are consistent with the literature on skill-biased technological change, but they also show that labor-side developments have meaningful effects on firm concentration.

Third, the modest increase in the price elasticity of demand has contributed slightly to the increases in both earnings inequality and sales concentration. As consumers have become more price-sensitive—possibly due to technological changes in retail and online search, or global competition—competitive pressure amplifies the market share of more productive firms, increasing the rewards to both firm efficiency and high-skilled labor.

Finally, the dispersion in firm productivity has actually declined slightly between the 1980s and 2010s, moderating the rise in inequality and firm concentration. This finding aligns with recent empirical evidence that rising concentration is not being driven by increasing heterogeneity in firm-level fundamentals, but rather by differential scalability and returns to skill (e.g., Autor et al. (2020); Hsieh & Rossi-Hansberg (2023)).

The model yields several insights that challenge common narratives. First, the rise in employment concentration among large firms is not driven by increasing productivity gaps but by a flattening of organizational hierarchies, which makes it more efficient for top firms to scale their workforce. Second, the widening of the skill distribution emerges as the dominant driver of earnings inequality and also explains much of the growth in sales concentration. Third, while changes in market competition (price sensitivity) modestly amplify both concentration and inequality, the model proposes that the joint evolution of labor and firm structures is what underlies these long-run shifts. These results suggest that rising inequality and concentration are—at least in part—two sides of the same coin, and that policies targeting only one may overlook the deeper economic forces driving both. Rather than focusing narrowly on antitrust or redistribution, effective policy may need to address structural sources of inequality—such as unequal access to skills or barriers to upward mobility within firms—that shape how talent and scale are matched in the economy.

Related literature This paper contributes to several strands of the literature that link labor market inequality, firm heterogeneity, and internal organizational structure.

First, it extends theoretical models of assignment and wage inequality (e.g., Rosen (1982); Gabaix & Landier (2008); Terviö (2008); Jung & Subramanian (2017)) by incorporating multi-worker firms with internal hierarchies and imperfect product market competition. While most assignment models focus on one-to-one matching, I follow Gabaix & Landier (2008) in modeling hierarchically structured firms and extend the framework by embedding it in a product market with endogenous firm size and pricing decisions. My model differs from canonical Melitz-type approaches by applying CES demand at the position level—rather than the firm level—which allows firms to be modeled as collections of task-specific units. This formulation preserves analytical tractability while capturing heterogeneity within firms and enabling separate pricing and demand for each position.

This approach is closely related to Jung & Subramanian (2017), who combine the Gabaix & Landier (2008) assignment model with a Melitz-style product market to study CEO compensation. While their focus is narrowly on the CEO pay, I broaden the scope to analyze inequality across the full earnings distribution and its joint evolution with firm concentration, both analytically and quantitatively.

Second, the paper connects to the literature on sorting and wage dispersion in models with heterogeneous firms and workers. In contrast to search-theoretic models that emphasize match-specific productivity shocks or frictions (e.g., Helpman et al. (2008, 2010); Bagger & Lentz (2019); Cortes & Tschopp (2024); Borovičková & Shimer (2024)), I model competitive assignment based on ex ante skill differences. Like Helpman et al. (2010), the model generates Pareto-distributed outcomes for wages, sales, and employment. However, I focus on ex ante skill heterogeneity and the equilibrium sorting of talent into firms. More productive firms derive greater returns from high-skilled workers due to production complementarities and offer higher wages to attract them. I also allow for within-firm wage dispersion and use the model to study how labor and product market forces jointly shape both earnings inequality and firm concentration.

Third, my model complements recent work on monopsonistic competition and rent-sharing (e.g., Card et al. (2018); Berger et al. (2022); Lamadon et al. (2022); Haanwinckel (2023); Lorenzini (2024)). While those models attribute wage dispersion to firm-specific preferences or frictions, my model emphasizes production-side complementarities between skill and firm efficiency. It shows that even in competitive labor markets, substantial between-firm inequality can arise from the endogenous matching of talent to scalable firms.

Finally, this paper contributes to the empirical literature on rent-sharing and the role of firms in shaping wage inequality (e.g., Abowd et al. (1999); Webber (2015); Barth et al. (2016); Card et al. (2018); Rinz (2018); Song et al. (2019); Bonhomme et al. (2019, 2023); Cortes et al. (2023)), as well as to research estimating the tail parameters of income and firm size distributions (e.g., Axtell (2001); Gabaix (2009); Jones & Kim (2018); Kondo et al. (2023)). It offers a structural framework that rationalizes several observed empirical patterns within a unified and analytically transparent model.

2 Stylized facts

Recent decades have witnessed growing interest in the relationship between firm dynamics and inequality. This section documents key empirical trends in firm concentration and labor market inequality in the United States, providing context and motivation for the structural model developed in Section 3.

I measure firm size concentration—both in employment and sales—using the commonly employed top 1 percent shares. For example, the top 1 percent employment share refers to the share of total employment accounted for by firms at the 99th percentile of the firm employment size distribution. Similarly, I use top 1 percent earnings and income shares to capture labor and total income concentration, respectively. The top 1 percent earnings share represents the share of total earnings received by earners above the 99th percentile of the earnings distribution.

Data on firm-level employment and sales concentration come from Kwon et al. (2024), who digitize historical publications from the Internal Revenue Service's Statistics of Income to construct long-run measures of firm size distributions in sales, assets, and net income. Data based on receipts (sales) are available starting in 1959, while data on top firm employment shares—derived from the Census Bureau's Business Dynamics Statistics—are available from 1978 onward. One caveat is that Kwon et al. (2024) provide comprehensive coverage only for corporations (both C- and S-corporations), but their results are robust to the inclusion of non-corporate firms when such data are available.

For earnings concentration, I use data on the top 1 percent earnings share from Kopczuk et al. (2007) for the years 1979–1990, and from the Economic Policy Institute (Gould & Kandra (2022)) for the years 1991–2018. I focus on earnings rather than total income inequality to better align with the model presented in Section 3, which focuses on the labor income distribution rather than capital income. While top income measures include both labor and capital income, the earnings inequality measure isolates labor income, providing a more consistent counterpart for model calibration.[1] For completeness, I also report the top 1 percent income share from the World Inequality Database.

Figure 1 displays the trends in top 1 percent concentration measures from 1979 to 2018. Panel A shows employment concentration, Panel B shows sales concentration, Panel C shows earnings inequality, and Panel D shows income inequality. As documented by Kwon et al. (2024), Autor et al. (2020), and Covarrubias et al. (2020), concentration in both firm sales and employment has risen over the past three decades. The top 1 percent employment share increased from 54 percent in 1987—the lowest value in the sample—to 60 percent in 2018. In other words, in 2018, the top 1 percent of firms employed three out of every five workers. Sales concentration rose even more steeply: in 1986, the top 1 percent of firms accounted for 69 percent of total firm sales; by 2018, this share had increased to 81 percent.

The top 1 percent earnings share exhibits a similar trend, rising from 7 percent in 1979 to 13 percent in 2018. Income inequality rose even more sharply, reflecting the increasing importance of capital income at the top.

Next, I examine the correlations between these concentration measures. Figure 2 presents scatter plots and binned scatter plots for all pairs of concentration measures, along with Pearson correlation coefficients (r). All measures are strongly positively correlated, with raw Pearson coefficients ranging from 0.68 to 0.96. Unsurprisingly, firm employment and sales concentration are tightly linked (Panel A), with a correlation of 0.96. More interest-

[1] It is worth noting that workers, especially at the top of the distribution, increasingly receive compensation in the form of equity (see Eisfeldt et al. (2023)). One may thus interpret the labor compensation in the model to include both cash and equity-based pay. In that case, income-based measures may also provide informative comparisons.

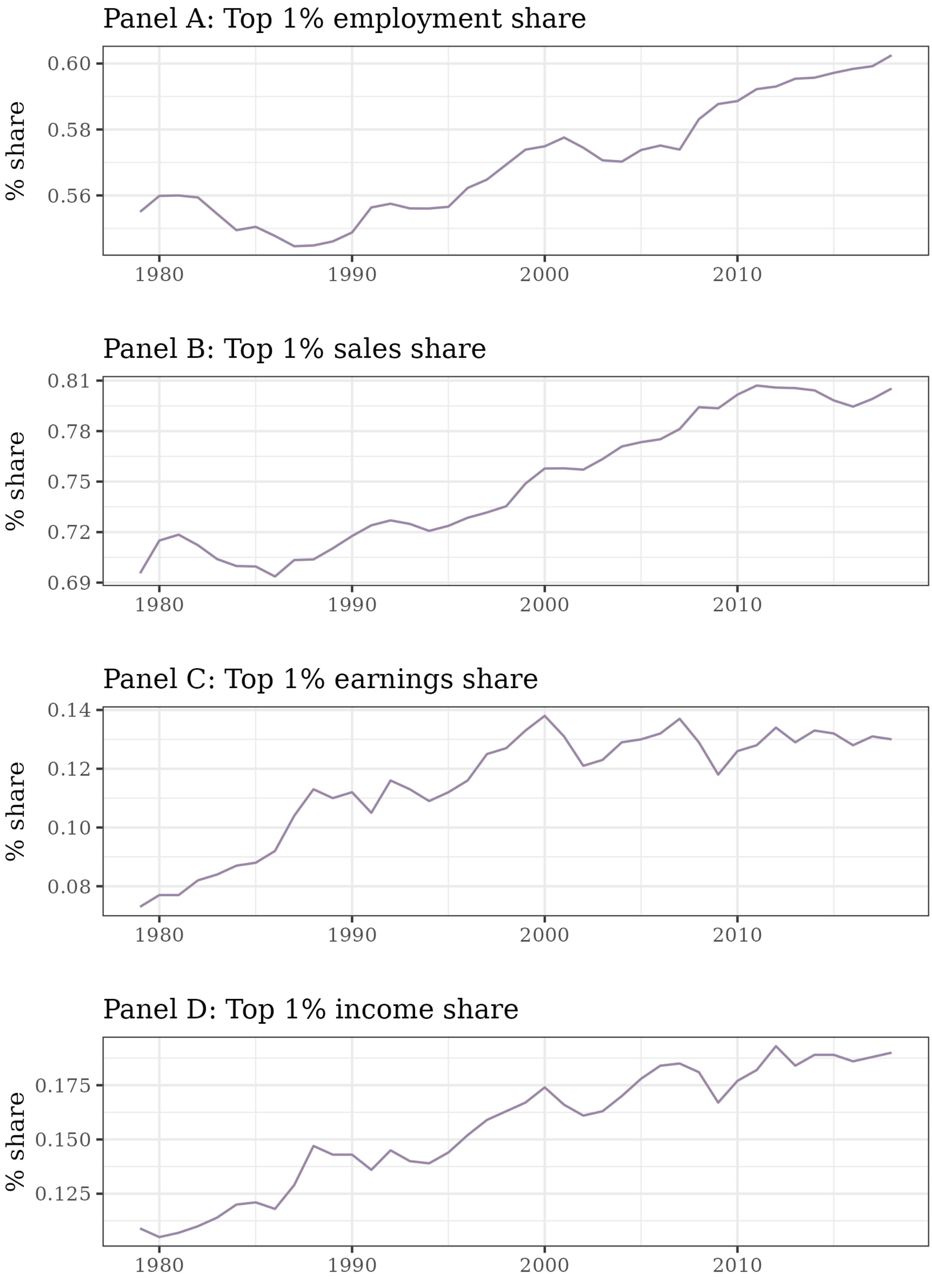


Figure 1: Employment, sales, earnings, and income concentration in the United States, 1979–2018.

Note: Panel A shows the top 1% employment share; Panel B the top 1% sales share; Panel C the top 1% earnings share; and Panel D the top 1% income share. Sources: Kopczuk et al. (2007); Gould & Kandra (2022); Kwon et al. (2024); World Inequality Database (2025).

ingly, the top 1 percent earnings share is also strongly correlated with both employment and sales concentration, suggesting a connection between firm size concentration and labor market inequality.

Taken together, these empirical regularities—rising firm concentration and its strong correlation with top earnings inequality—motivate the structural framework in the next section, which investigates the mechanisms that may underlie these trends.

3 A static assignment model with heterogeneous firms and workers

3.1 Preliminaries

I consider a static assignment problem with two-sided heterogeneity, in which heterogeneous workers are matched to hierarchical positions within heterogeneous firms. Each worker g is characterized by a skill level $T(g)$, and each firm i by its productivity $A(i)$. Production in a given position depends on both the productivity of the position within the firm and the skill level of the worker. Firms produce differentiated varieties $\omega \in \Omega$, where Ω denotes a continuum of varieties. Each position within a firm corresponds to one such variety and competes under CES product demand. This implies that CES preferences apply at the position level, with each position facing its own downward-sloping demand curve and setting its price independently.

Unlike canonical assignment models with one-to-one matching (e.g., Becker (1973); Sattinger (1993); Teulings (1995); Gabaix & Landier (2008); Terviö (2008)), I follow Gabaix & Landier (2008) in allowing firms to employ multiple workers across a hierarchy of roles. Positions differ in productivity, declining with rank within the firm. This reflects variation in autonomy, complexity, and responsibility across job levels (e.g., Bayer & Kuhn (2023)), and helps explain why top firms do not solely employ top-skilled workers—serving also as a structural limit on firm size.

Formally, the effective productivity of position h in firm i is $c(h)A(i)$, where $c'(h) < 0$ captures firm hierarchical structure. Each filled position incurs a fixed cost f_e, which, together with $c(h)$, determines the number of active positions per firm. Once matched to a worker, the position's total productivity becomes $c(h)A(i)T(g)$. Firms thus operate as collections of semi-autonomous units, each contributing separately to aggregate output and revenue.

Wages are endogenously determined in a competitive market and reflect positive assortative matching: higher-skilled workers are paired with higher-productivity positions, following the equilibrium logic of Gabaix & Landier (2008) and Terviö (2008).

Lastly, I assume that there are no complementarities between workers. This assumption enhances tractability, as it implies that the assignment problem between a given firm and each of its workers can be solved independently, in the same manner as in a one-to-one matching framework. I return to this assumption in Section 3.3.1 and Appendix B.

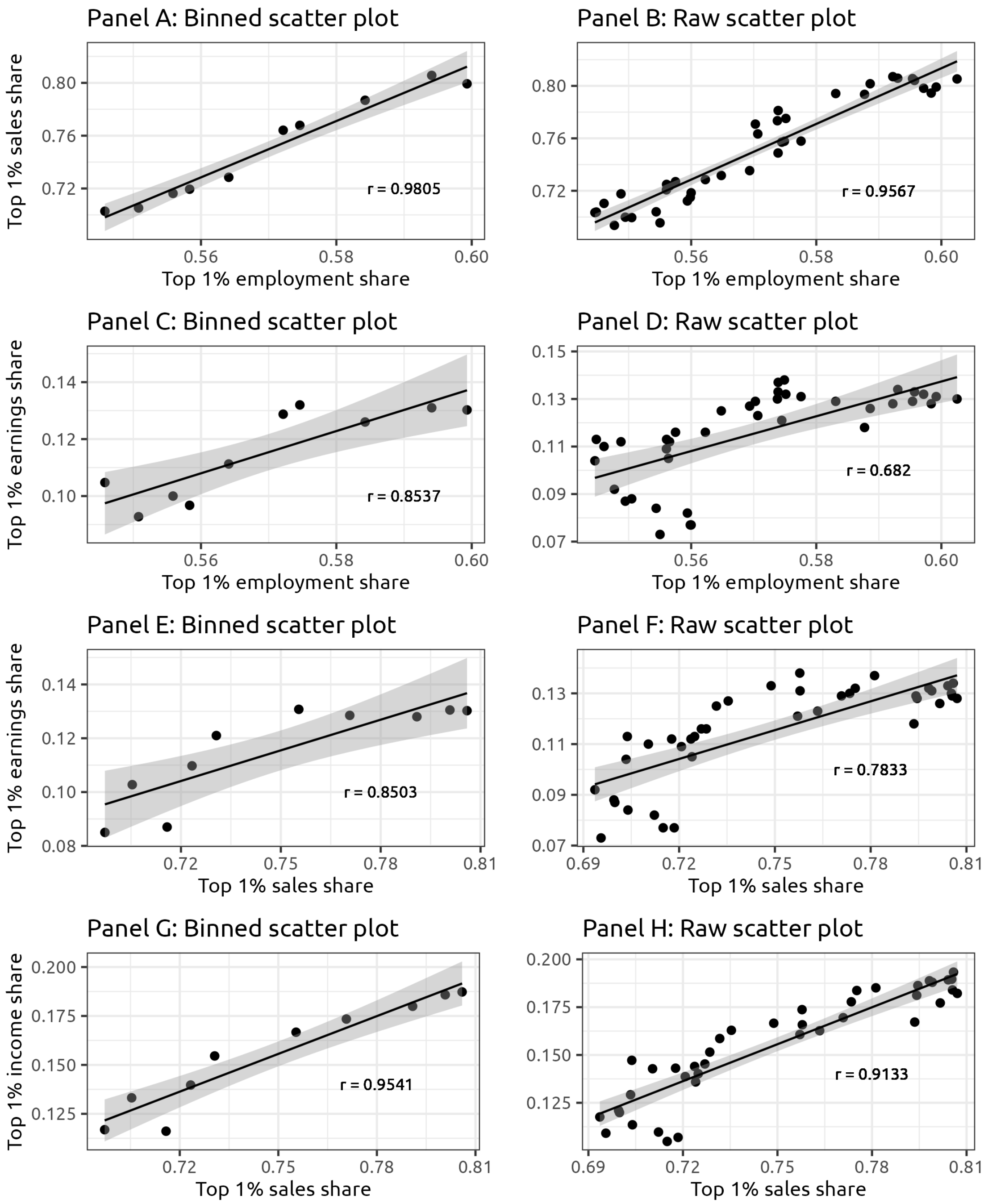


Figure 2: Correlations between top 1% employment, sales, and earnings concentration in the United States, 1979–2018.

Note: Pearson correlation coefficients (r) are based on raw annual data. Sources: Kopczuk et al. (2007); Gould & Kandra (2022); Kwon et al. (2024); Author's calculations.

Figure 3 summarizes the theoretical channels through which heterogeneity in skills and firm productivity, in combination with firm hierarchical structure and product market demand, affect inequality and concentration in the model.

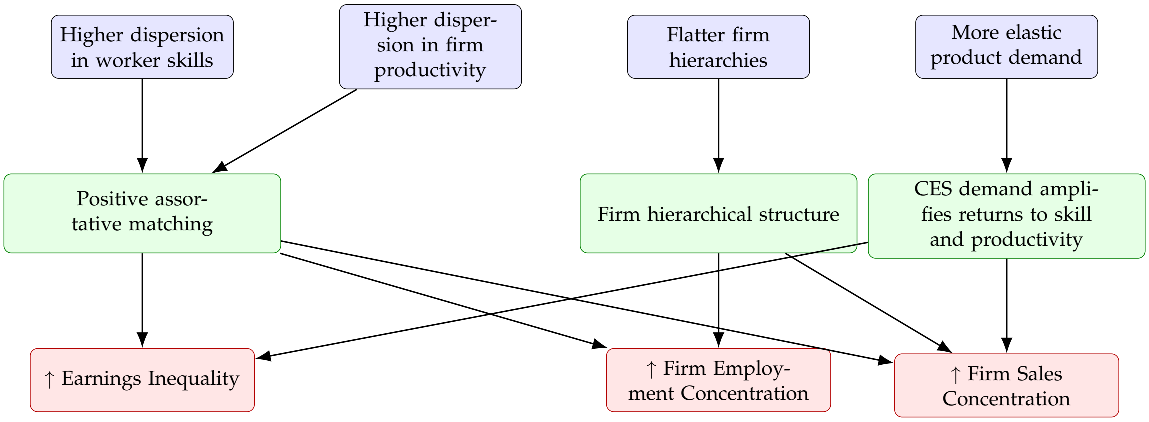


Figure 3: Main mechanisms linking heterogeneity in skills and productivity, hierarchical structure, and product demand to earnings and firm concentration.

3.2 Workers

3.2.1 Skill distribution of workers

Workers have heterogeneous skills drawn from a continuous distribution. Each worker $g \in (0, 1]$ is endowed with a skill level $T(g)$, interpreted as efficiency units of labor. The skill function is strictly decreasing, $T'(g) < 0$, implying that lower-ranked workers (i.e., lower g) are more skilled. Labor supply is inelastic.

Assumption 1: The skill distribution follows $T(g) = Bg^{-\beta}$, where B is a scale parameter and β is the shape parameter of the distribution. This distribution corresponds to a Pareto distribution with a scale parameter equal to $B^{1/\beta}$ and a shape parameter equal to $1/\beta$.[2]

The advantage of assuming that the skill distribution of workers follows a Pareto distribution lies in its analytical tractability. However, this assumption may initially appear strong. For example, many traditional measures of skill, such as intelligence quotient (IQ), are typically modeled as normally distributed. In this context, however, "skills" encompass the broad range of experiences and capabilities that workers accumulate over their life cycle. In a cross-section of workers, differences in age, education, job history, and other career attributes naturally lead to a highly skewed skill distribution. Even if innate ability is normally distributed, the accumulation process over time may follow proportional random growth. Under Gibrat's law, where each worker's skill grows proportionally and is

[2]Throughout the paper, I express Pareto-distributed variables in terms of their inverse rank-size form, i.e., $Size \sim Rank^{-1/\alpha}$. This corresponds to the standard Pareto distribution $P(Size > x) \sim x^{-\alpha}$, but presented with rank on the right-hand side to simplify closed-form expressions for skills, productivity, earnings, and firm sizes. For instance, I write the skill distribution as $T(g) = Bg^{-\beta}$, where β is the inverse of the conventional Pareto shape parameter.

bounded below, the steady-state distribution converges to a Pareto form (Champernowne (1953); Simon (1955)).

There exists a threshold skill level, G, defined in terms of the worker ranking, such that workers with $g \leq G$ will be matched with a firm and engage in production, while those with $g > G$ will remain unmatched and instead produce at home, receiving a value of home production denoted by w. There are no information asymmetries: the productivity of firms and positions, as well as the skills of workers, are known to all agents in the economy. A worker's compensation depends on her skill level, $T(g)$; in equilibrium, she will earn a wage rate $w_g(T)$. Wages are bounded below by $w_g(T) \geq w$, where w represents the minimum wage a worker is willing to accept, which is assumed to be equal to the value of home production.

3.2.2 Preferences and utility maximization problem

I assume that workers' preferences follow Pollak's additive utility functions (Pollak (1971)), and I focus on the special case of constant elasticity of substitution (CES) preferences. This form allows for substitution across varieties while maintaining tractability in aggregation. All workers share the same preferences but differ in their total incomes, denoted by W_g. I abstract from the consumption–savings decision by assuming that workers consume all of their income. The general form of Pollak's preferences is given by the additive utility function:

$$U_g = \int_{\omega \in \Omega_g} \alpha_\omega (q_g(\omega) - \gamma_g)^{1-\frac{1}{\sigma}} d\omega,$$

where $q_g(\omega)$ denotes worker g's consumption of variety ω, γ_g is a constant, and $q(\omega) > \gamma_g$. The parameter α_ω is a variety-specific demand shifter, which I henceforth assume to be equal to 1. I assume that $\Omega \subseteq \bar{\Omega}$, where $\bar{\Omega}$ is a compact set containing all potential varieties in the economy, and Ω denotes a subset of varieties actually produced by active firms. The CES form of Pollak's preferences is obtained by setting $\gamma_g = 0$, yielding:

$$U_g = \int_{\omega \in \Omega_g} q_g(\omega)^{1-\frac{1}{\sigma}} d\omega,$$

and the maximization problem of a worker g, who consumes varieties $\omega \in \Omega$, becomes:

$$\underset{q_g(\omega) \geq 0}{max}\, U_g = \int_{\omega \in \Omega} q_g(\omega)^{1-\frac{1}{\sigma}} \quad s.t. \;\; W_g \geq \int_{\omega \in \Omega} p(\omega) q_g(\omega) d\omega,$$

where $p(\omega)$ denotes the price of a variety ω. The first-order conditions can be written as:

$$q_g(\omega): \;\; (1 - \frac{1}{\sigma}) q_g(\omega)^{-\frac{1}{\sigma}} = \lambda_g p(\omega) \;\; \forall \;\; q_g(\omega) > 0,$$

$$\lambda_g: \;\; W_g = \int_{\omega \in \Omega} p(\omega) q_g(\omega) d\omega,$$

where λ_g is the Lagrange multiplier. The demand function for each variety ω by worker g is then

$$q_g(\omega) = W_g p(\omega)^{-\sigma} \int_{\Omega} p(\omega^{'})^{-1} p(\omega^{'})^{\sigma} d\omega^{'} = p(\omega)^{-\sigma} \frac{W_g}{P},$$

where $\omega^{'}$ ranges over all positively consumed varieties in the economy, and $P = \int_{\Omega} p(\omega^{'})^{1-\sigma} d\omega^{'}$ is the aggregate price index in the economy.[3] The demand for each variety is a decreasing function of its own price and an increasing function of a worker's income, W_g. The aggregate demand for each variety depends on the total income in the economy, $W = \int_g W_g \, dg$, and is given by:

$$q(\omega) = \frac{1}{P} p(\omega)^{-\sigma} \int_g W_g dg = p(\omega)^{-\sigma} \frac{W}{P}. \tag{1}$$

Given the CES preferences, the price elasticity of demand is constant and equal to $\varepsilon(\omega) = -\sigma$.

3.3 Firms

3.3.1 Production technology and productivity distribution of firms

Firms draw their productivity from a given distribution. Each firm indexed by $i \in (0, 1]$ has a productivity $A(i)$. In the same way as workers, firms can be ranked based on their productivity, where i is the rank of a firm. The lower rank i implies higher productivity, and $A(i)$ is a decreasing function of i.

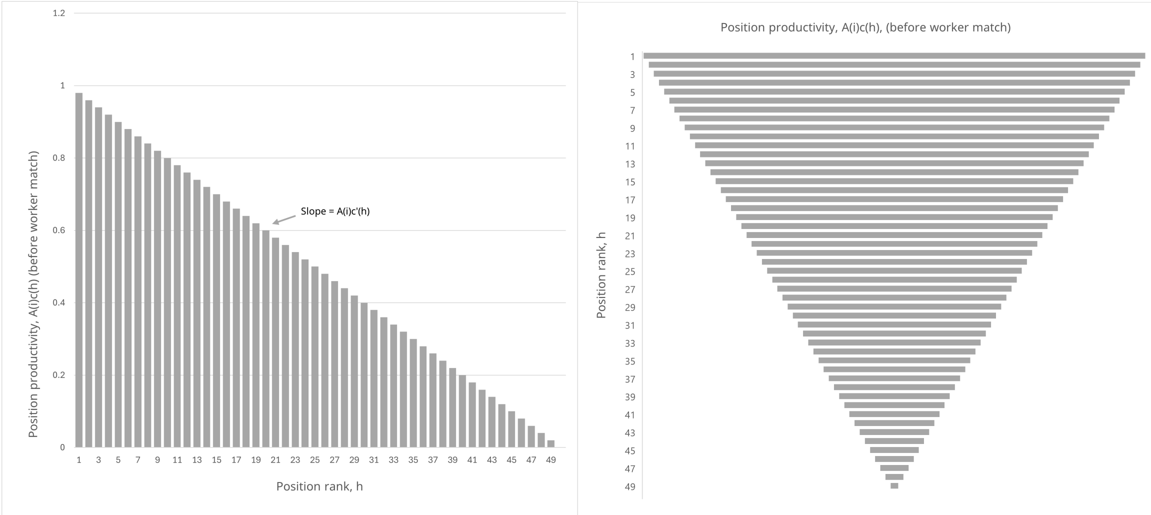
Assumption 2: The productivity distribution is given by $A(i) = Di^{-\delta}$, where D and δ are scale and shape parameters, respectively. This distribution corresponds to a Pareto distribution with a scale parameter of $D^{1/\delta}$ and a shape parameter of $1/\delta$.

Each firm consists of a hierarchy of positions, ranked by their marginal productivity. The effective productivity of a position declines with its rank h, captured by a function $c(h) > 0$, with $c'(h) < 0$. This decreasing function proxies organizational frictions such as limited span of control or communication costs (e.g., Garicano & Rossi-Hansberg (2006, 2015)). The number of active positions in firm i, denoted h_i^*, is determined endogenously.

Larger firms experience diminishing returns to expanding their workforce: although all positions share the same firm-level productivity $A(i)$, the marginal output of additional positions declines with h. This structure helps constrain firm size and supports positive sorting in equilibrium.

Figure 4 illustrates this hierarchy. Panel (a) plots effective productivity of each position h for a given firm with productivity $A(i)$ and a given functional form of $c(h)$. In this example, effective productivity of a position decreases linearly in its rank h. Panel (b) illustrates the same organizational hierarchy, but in a common pyramid shape, emphasizing the role of ranked job levels.

[3] Refer to the appendix for algebra.



(a) Productivity of position by rank (b) Organizational hierarchy, ranked by productivity

Figure 4: Firm structure and productivity of positions before worker match

Each position is matched with a worker with skill $T(g)$, so the total productivity of a position depends jointly on the firm's productivity, the position's rank, and the worker's skill. Denoting the matched triple as (i, g, h), the production function is $y(i, g, h) = c(h)A(i)T(g)$. The output of a position is increasing in a firm's and its worker's rank (i.e., decreasing in i and g) as well as a position's rank in the firm hierarchy. The marginal cost of each position (i, g, h) is given by $c(i, g, h) = \frac{1}{c(h)A(i)T(g)}$.

A key modeling assumption is that CES product demand applies at the level of each position's output, rather than at the firm level. That is, consumers allocate expenditure across varieties produced by individual positions, not across aggregate firm output. This separability implies that each position faces a downward-sloping demand curve and chooses its price independently. While stylized, this structure preserves analytical tractability and aligns with the assumption that workers and positions are matched individually in a many-to-one framework. Crucially, it allows the assignment problem between workers and positions to remain separable, which would no longer hold if CES demand were applied to the firm-level sum of productivities.

Appendix B explores an alternative specification in which CES demand is applied to the firm-level aggregate productivity rather than to individual positions. This change introduces implicit complementarities: the marginal product of a worker now depends on their coworkers, and the firm's total revenue includes additional positive cross-terms. While these complementarities are economically interesting and potentially important, I abstract from them in the baseline model to retain closed-form solutions and preserve analytical transparency.

3.3.2 Effective productivity of each position

As in Gabaix & Landier (2008), I will need to determine the effective productivity distribution of positions to solve the assignment problem between workers and positions.[4] The firm productivity distribution is given by $A(i) = Di^{-\delta}$, which provides a one-to-one mapping between a firm's rank i and its productivity $A(i)$. The effective productivity of a position is given by $\tilde{A}(i,h) = c(h)A(i)$, where $c(h)$ is a function that measures productivity of a position ranked h^{th} in a firm.

All positions in the economy can be ranked based on their effective productivity. I need to define a mapping between effective productivity and an effective rank. Assume that i is the upper quantile (or the rank) of a firm with productivity a. Then, i satisfies $i = P(\hat{A} > a)$. Using $A(i) = Di^{-\delta}$, I have $i = P(A > a) = D^{1/\delta}A^{-1/\delta}$. Following the same logic, it follows that

$$i = P(\tilde{A} > a) = P(c(h)A_i > a) = P(A_i > a/c(h))$$

$$= E(P(A_i > a/c(h)|c(h)) = E(D^{1/\delta}(a/c(h))^{-1/\delta})$$

$$= D^{1/\delta}E(c(h)^{1/\delta})a^{-1/\delta}.$$

This result implies that effective productivity at a quantile i is $\tilde{A}(i,h) = \tilde{D}i^{-\delta}$ with $\tilde{D} = DE\left[c(h)^{1/\delta}\right]^{\delta}$. The average sensitivity, the term $\bar{c} = E[c(h)^{1/\delta}]^{\delta}$ will be an average sensitivity over all positions in the economy, $\bar{c} = \left(\frac{1}{\int_0^I h_i^* di}\int_0^I \int_1^{h_i^*} c(h)^{1/\delta} dh di\right)^{\delta}$, where I is the total number of firms in the economy.

Despite heterogeneity in position productivity within firms, the aggregate distribution of effective productivity across all positions remains Pareto-distributed, with scale adjusted by the average sensitivity across positions.

3.3.3 A position's profit-maximization problem

Given the ranked effective productivity and the assumption that there are no complementarities between workers, the profit-maximization problem of positions can now be solved using backward induction, as in Jung & Subramanian (2017). I can first solve an active position's optimal pricing rule for all (i, g, h), taking each position's productivity as given. I can then solve the optimal assignment problem of workers into positions, and last, a position's entry problem.

Pricing rule. An active position with an effective size $\tilde{A}(i,h)$ will be choosing its price to maximize its profits. Its total productivity is $\tilde{A}(i,h)T(g)$, where $T(g)$ is the skill of a worker matched with a position ranked i. Conditional on g, the optimal pricing rule is then

[4] As a position's productivity will determine the size, measured in sales, of each position, I will use the "size" and "productivity" of a position interchangeably.

$$p_{i,g,h} = \frac{\varepsilon(\omega)}{1+\varepsilon(\omega)} \frac{1}{\tilde{A}(i,h)T(g)} = \frac{\sigma}{\sigma - 1} \frac{1}{\tilde{A}(i,h)T(g)}, \tag{2}$$

which follows the typical markup pricing rule in monopolistic competition models.

The output and the maximized profits of each position with an effective size $\tilde{A}(i,h)$ are then

$$q_{i,g,h} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \left[\tilde{A}(i,h)T(g)\right]^{\sigma} \frac{W}{P}, \tag{3}$$

$$\pi^*_{i,g,h} \equiv p_{i,g,h}q_{i,g,h} - w_g - f_e = \frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left[\tilde{A}(i,h)T(g)\right]^{\sigma-1} - w_g - f_e, \tag{4}$$

where w_g is the wage of a worker with ranking g, determined in the equilibrium, and f_e is fixed cost of production.

3.3.4 Assignment problem

There will now be one-to-one matching between a worker with a rank g and a position with an effective rank of i. Formally, a firm chooses a worker to maximize

$$\max_g \ \pi^*_{i,g,h} = \frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left[\tilde{A}(i,h)T(g)\right]^{\sigma-1} - w_g - f_e.$$

The revenue from a match between a worker g and a position (i,h) is

$$R_{i,g,h} = \frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left[\tilde{A}(i,h)T(g)\right]^{\sigma-1},$$

and it is easy to show that the marginal revenue of i is increasing in g,

$$\frac{\partial^2 R}{\partial i \partial g} = \frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} (\sigma-1)\left[\tilde{A}(i,h)T(g)\right]^{\sigma-2} \tilde{A}'(i,h)T'(g) > 0, \tag{5}$$

as $\tilde{A}'(i,h)$, $T'(g) < 0$. This implies that the supermodularity condition holds, and as shown in the previous literature (Sattinger (1993); Legros & Newman (2007)), the matching equilibrium is then unique and positive assortative matching (PAM) holds. This implies that higher-skilled workers will work in higher productivity positions.

The first-order condition for optimal worker choice leads to the following assignment equation:

$$\frac{\partial \pi^*_{i,g,h}}{\partial g} = \frac{W}{P}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma-1}} \tilde{A}(i,h)\left[\tilde{A}(i,h)T(g)\right]^{\sigma-2} T'(g) - w'_g = 0. \tag{6}$$

Assuming PAM implies $i = g$, and I can write the wage gradient as

$$w'_i = -\beta \frac{W}{P}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma-1}} (B\tilde{D})^{\sigma-1} i^{-(\sigma-1)(\beta+\delta)-1}.$$

The assignment equation states that the marginal cost of hiring a slightly better worker, w_g', is equal to the marginal benefit of hiring a slightly better worker, $-\beta \frac{W}{P} \frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma-1}} (B\tilde{D})^{\sigma-1} i^{-(\sigma-1)(\beta+\delta)-1}$. Integrating with respect to i yields the wage distribution:

$$= \frac{\beta (B\tilde{D})^{\sigma-1}}{(\beta+\delta)} \frac{W}{P} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \left[i^{-(\sigma-1)(\beta+\delta)} - i^{*-(\sigma-1)(\beta+\delta)} \right] + w, \tag{7}$$

where i^* is the rank of the last active position as well as the worker in the economy. Wages are increasing in a common multiplier $\frac{\beta (B\tilde{D})^{\sigma-1}}{(\beta+\delta)} \frac{W}{P} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}$ and in the total number of positions i^* in the economy, and decreasing in i, implying that higher-skilled workers are paid more.

If I assume that there exists a sufficiently large number of positions in the economy, $i^{*-(\sigma-1)(\beta+\delta)}$ approaches to zero, and the wage distribution simplifies to

$$w_i = \frac{\beta}{\beta+\delta} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{W}{P} \left(B\tilde{D}\right)^{\sigma-1} i^{-(\sigma-1)(\beta+\delta)} + w. \tag{8}$$

Equation (8) shows that the wage level of each worker depends on four factors: first, wages are proportional to a common factor $\frac{\beta}{\beta+\delta} \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{W}{P}$, which captures the overall size of the economy through $\frac{W}{P}$. Second, the wage level depends on the effective size of the position a worker is matched with, $\tilde{D} i^{-\delta}$, to the power of $\sigma - 1$. Third, the wage level depends on the skill of the worker, $B i^{-\beta}$, to the power of $\sigma - 1$. Finally, it depends on a minimum wage level, w.

If I assume that $w = 0$, or focus on the top wages implying that w has a diminishing effect on the total wage level, the overall wage distribution follows a Pareto, consistent with empirical literature. The shape parameter of the distribution is $-\frac{1}{(\sigma-1)(\beta+\delta)}$, implying that the degree of wage inequality depends on the price elasticity of demand, σ, and the shape parameters of the skill and productivity distributions, β and δ, respectively.

Similarly, the total sales per position are given by:

$$R_i = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{W}{P} \left(B\tilde{D}\right)^{\sigma-1} i^{-(\sigma-1)(\beta+\delta)}, \tag{9}$$

mirroring the wage structure and reinforcing that both wages and revenues follow power-law distributions under PAM and Pareto assumptions.

3.3.5 Number of positions in each firm, number of firms, and a firm size

Number of positions and firm size measured in employment. I now derive the equilibrium number of positions in each firm—i.e., firm size measured by employment—as well as firm-level sales and profits. Following Gabaix & Landier (2008), I begin by expressing position-level sales as a function of firm productivity.

Proposition 1. *In equilibrium, a worker of rank i is matched with a position whose effective productivity $c(h)A_n$ also has rank i. The total sales of a position h within a firm n with size A_n can be* *written as*

$$R_{n,h} = \Xi \left[B\bar{i}^{-\beta}\right]^{(\sigma-1)} \left[\bar{c}A\left(\bar{i}\right)\right]^{-(\sigma-1)\frac{\beta}{\delta}} \left[c(h)A_n\right]^{(\sigma-1)(1+\frac{\beta}{\delta})},$$

where $\Xi = \left(\frac{\sigma-1}{\sigma}\right)^{(\sigma-1)} \frac{W}{P}$ *and* $\bar{i}$ *is the reference ranking of skill.* [5]

Proof. As $\tilde{A}\left(i\right) \equiv \tilde{D}i^{-\delta} \equiv c(h)A\left(i\right) \equiv c(h)Di^{-\delta}$; $\tilde{A}(\bar{i}) \equiv \tilde{D}\bar{i}^{-\delta} \equiv \bar{c}A(\bar{i}) \equiv \bar{c}D\bar{i}^{-\delta}$, $T(\bar{i}) \equiv B\bar{i}^{-\beta}$, and $\tilde{A}_n \equiv c(h)A_n$, I can write (8) as

$$\left[\left(\frac{\sigma-1}{\sigma}\right)^{-(\sigma-1)} \frac{P}{W}w_i\right]^{\frac{1}{\sigma-1}} = (B\tilde{D})i^{-(\beta+\delta)} = B\tilde{D}\left(i^{-\delta}\right)^{1+\beta/\delta} = B\tilde{D}^{-\beta/\delta}\left(\tilde{D}i^{-\delta}\right)^{1+\beta/\delta}$$

$$= B\bar{i}^{-\beta}\left[\tilde{D}\bar{i}^{-\delta}\right]^{-\frac{\beta}{\delta}}\left[\tilde{D}i^{-\delta}\right]^{1+\frac{\beta}{\delta}}$$

$$\Leftrightarrow R_i = \Xi\left[B\bar{i}^{-\beta}\right]^{(\sigma-1)} \left[\bar{c}A(\bar{i})\right]^{-(\sigma-1)\frac{\beta}{\delta}} \left[c(h)A(i)\right]^{(\sigma-1)(1+\frac{\beta}{\delta})}. \tag{10}$$

As equation (10) holds for any size of a firm, A_n, it can be written as

$$\Leftrightarrow R_i \equiv R_{n,h} = \Xi\left[B\bar{i}^{-\beta}\right]^{(\sigma-1)} \left[\bar{c}A(\bar{i})\right]^{-(\sigma-1)\frac{\beta}{\delta}} \left[c(h)A_n\right]^{(\sigma-1)(1+\frac{\beta}{\delta})}. \tag{11}$$

□

Sales for each position depend on a common factor $\Xi\left[B\bar{i}^{-\beta}\right]^{(\sigma-1)}\left[\bar{c}A\left(\bar{i}\right)\right]^{-(\sigma-1)\frac{\beta}{\delta}}$ and a position-specific factor $\left[c(h)A_n\right]^{(\sigma-1)(1+\frac{\beta}{\delta})}$. The sales of each position in the economy is proportional to Ξ, which captures the effect of aggregate income and the price index on wages, the skill level of the worker in a reference firm, $\left[B\bar{i}^{-\beta}\right]$, to the power of $\sigma-1$, and productivity of the reference firm, $\left[\bar{c}A\left(\bar{i}\right)\right]$, to the power of $-(\sigma-1)\frac{\beta}{\delta}$. Moreover, the sales of a position h in a firm n are proportional to the effective size of the position, to the power of $(\sigma-1)(1+\frac{\beta}{\delta})$.

Following Proposition 1, the wage for position h in firm n is:

$$w_{n,h} = \left(\frac{\beta}{\beta+\delta}\right)\Xi\left[B\bar{i}^{-\beta}\right]^{\sigma-1}\left[\bar{c}A(\bar{i})\right]^{-(\sigma-1)\frac{\beta}{\delta}}\left[c(h)A_n\right]^{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)}. \tag{12}$$

Subtracting wages and fixed cost f_e gives per-position profits:

$$\pi_{n,h} = \left(\frac{\delta}{\beta+\delta}\right)\Xi\left[B\bar{i}^{-\beta}\right]^{\sigma-1}\left[\bar{c}D\bar{i}^{-\delta}\right]^{-(\sigma-1)\frac{\beta}{\delta}}\left[c(h)Dn^{-\delta}\right]^{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)} - f_e.$$

Define the constant:

$$\Gamma \equiv \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{W}{P}\left[B\bar{i}^{-\beta}\right]^{\sigma-1}\left[\tilde{D}\bar{i}^{-\delta}\right]^{-(\sigma-1)\frac{\beta}{\delta}}D^{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)}.$$

[5]For instance, the reference skill could correspond to the median worker.

Imposing a zero-profit condition on the marginal position h_n^*, I obtain:

$$c(h_n^*) = \left(\frac{\beta + \delta}{\delta} \frac{f_e}{\Gamma} \right)^{\frac{1}{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)}} n^{\delta}.$$

Since $c(h)$ is strictly decreasing in h, the number of positions h_n^* is decreasing in firm rank n, meaning that more productive firms (lower n) employ more workers.[6]

Firm size distribution. If the organizational hierarchy follows a Pareto form $c(h) = Ch^{-\rho}$, I can solve for firm size in closed form:

$$C(h_n^*)^{-\rho} = \left(\frac{\beta + \delta}{\delta} \frac{f_e}{\Gamma} \right)^{\frac{1}{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)}} n^{\delta} \quad \Rightarrow \quad h_n^* = C^{-1/\rho} \left(\frac{\beta + \delta}{\delta} \frac{f_e}{\Gamma} \right)^{-\frac{1}{\rho(\sigma-1)\left(1+\frac{\beta}{\delta}\right)}} n^{-\delta/\rho}. \tag{13}$$

Equation (13) implies that the firm size distribution follows a Pareto distribution with shape parameter $\alpha = \rho/\delta$. This result aligns with empirical findings that firm sizes are well approximated by Pareto distributions. The shape parameter α governs the degree of concentration: a higher α corresponds to more skewness and higher concentration in firm size.

The economic mechanisms behind this relationship are intuitive. A higher δ, which captures the degree of heterogeneity in firm productivity, increases concentration, as more productive firms can sustain a larger number of positions. Conversely, a higher ρ implies a steeper hierarchy in organizational productivity—meaning productivity drops more rapidly with position rank within the firm—thus limiting the size advantage of highly productive firms.

A special case of interest is when $\delta = \rho$, in which case the shape parameter becomes $\alpha = 1$, yielding a Zipf distribution. While the model does not impose Zipf's law through calibration, I evaluate this benchmark in the model validation exercises. The model produces a firm size distribution with an implied shape parameter of approximately $\alpha = 0.9$, reasonably close to Zipf's law and consistent with the empirical regularity reported by Axtell (2001). This provides evidence that the model captures key features of firm size concentration, even without directly targeting them in estimation.

In summary, the shape of the firm employment distribution in this model depends entirely on firm-level technology: specifically, on the distribution of productivity across firms (δ) and the structure of internal hierarchies (ρ). It is independent of labor market parameters β and σ, highlighting the role of organizational and technological heterogeneity in shaping macroeconomic firm employment size patterns.

Firm size measured in sales. I now derive a closed-form approximation for firm size measured in sales, integrating over positions starting from 1—i.e., the highest-ranked po-

[6] Here, n denotes the firm type or rank in the productivity distribution. Higher n corresponds to lower productivity firms.

sition in the firm (such as the CEO)—to the lowest-ranked, h_n^*. Firm n's total sales are:

$$R_n = \int_1^{h_n^*} R_n(k)\, dk. \tag{14}$$

Using equation (11) and the expression for revenue per position,

$$R_n(k) = \frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[B\bar{i}^{-\beta}\right]^{\sigma-1}\left[\tilde{D}\bar{i}^{-\delta}\right]^{-(\sigma-1)\frac{\beta}{\delta}}\left[c(k)Dn^{-\delta}\right]^{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)},$$

and assuming a Pareto form for the hierarchy function, $c(k) = Ck^{-\rho}$, I obtain:

$$R_n = \Phi n^{-(\sigma-1)(\beta+\delta)} \int_1^{h_n^*} k^{-\rho(\sigma-1)(1+\frac{\beta}{\delta})} dk,$$

where Φ determines constants:

$$\Phi \equiv \frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[B\bar{i}^{-\beta}\right]^{\sigma-1}\left[\tilde{D}\bar{i}^{-\delta}\right]^{-(\sigma-1)\frac{\beta}{\delta}} D^{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)}C^{(\sigma-1)\left(1+\frac{\beta}{\delta}\right)}.$$

Let $\lambda \equiv \rho(\sigma-1)(1+\frac{\beta}{\delta})$ and assume $\lambda \neq 1$, so the integral becomes:

$$\int_1^{h_n^*} k^{-\lambda} dk = \frac{(h_n^*)^{1-\lambda} - 1}{1-\lambda}.$$

Then

$$R_n = \Phi n^{-(\sigma-1)(\beta+\delta)} \frac{(h_n^*)^{1-\lambda} - 1}{1-\lambda} = \frac{\Phi}{1-\lambda} \cdot n^{-(\sigma-1)(\beta+\delta)}[(h_n^*)^{1-\lambda} - 1]. \tag{15}$$

The above expression shows that firm sales are higher when a constant $\frac{\Gamma}{1-\lambda}$ increases, when a firm is more highly ranked (i.e., lower n), and when a firm have a larger number of workers, given by $[(h_n^*)^{1-\lambda} - 1]$.[7]

When focusing on the top tail of the firm size distribution with a large h^* and using the expression for optimal employment size $h_n^* \propto n^{-\delta/\rho}$, I get

$$R_n \propto n^{-(\sigma-1)(\beta+\delta)}\left(n^{-\delta/\rho}\right)^{1-\lambda} = n^{-[(\sigma-1)(\beta+\delta)+\frac{\delta}{\rho}(1-\lambda)]}.$$

Then, substituting back $\lambda = \rho(\sigma-1)(1+\frac{\beta}{\delta})$, and simplifying the exponent yields

$$(\sigma-1)(\beta+\delta) + \frac{\delta}{\rho}\left[1 - \rho(\sigma-1)\left(1+\frac{\beta}{\delta}\right)\right] = \frac{\delta}{\rho}.$$

Hence,

$$R_n \propto n^{-\delta/\rho}. \tag{16}$$

This confirms that integrating from position 1 to h_n^* still yields the same theoretical prediction: the upper tail of the firm sales distribution follows a Pareto distribution with

[7]This relies on $\lambda < 1$, which I find to be the case quantitatively.

shape parameter ρ/δ, identical to that of employment in the top tail.

Comparison of employment and sales distributions. Under the model's assumptions, firm size measured in sales inherits the same Pareto exponent as employment—at least in the upper tail. Intuitively, this result arises from the interaction between the firm size (measured in sales) and the hierarchy of workers. On the one hand, for highly-ranked workers, firm sales are increasing in $n^{-(\sigma-1)(\beta+\delta)}$, as high-ranked workers generate a large share of the firm's sales. On the other hand, lower-ranked workers contribute less on the overall firm sales, reflected in the term $n^{-(\frac{\delta}{\rho}-(\sigma-1)(\beta+\delta))}$. While more productive firms have more workers, their contribution to sales is small. Thus, the shape parameter δ/ρ arises because the diminishing marginal returns to position (in the sales contribution from lower-ranked workers) counterbalance the increasing returns from higher-ranked workers. In this limit, revenue scales approximately proportionally with employment, leading to identical slopes in log-rank regressions.

This is confirmed by the quantitative model exercises. When fitting *log-rank* $\sim$ *log-size* regressions to model-simulated data, I find that both employment and sales distributions are well-approximated by power laws with nearly identical slopes in the top percentiles. For example, the estimated elasticity between $\log(R_n)$ and $\log(h_n)$ is around 1.08 among top firms, with $R^2 \approx 0.9997$, closely matching the theoretical prediction of proportionality.

However, this alignment is specific to the top tail. Away from the tail, compositional differences in the value of worker productivity or in the returns to skill can drive a wedge between sales and employment. Notably, the counterfactual experiments show that increases in β—the shape parameter of the skill distribution— can significantly increase sales concentration even when the employment distribution remains unchanged. This highlights that, although the tail behavior of firm size is governed by ρ/δ, concentration in sales is also sensitive to heterogeneity in the value of worker skills.

In sum, while the model predicts approximate tail equivalence between the employment and sales distributions, it also clarifies the mechanisms through which these distributions can diverge elsewhere. It offers a potential explanation for the empirical observation that firm concentration is typically higher when measured by sales rather than by employment (Kwon et al. (2024)). These deviations in sales and employment concentration may therefore reflect shifts in the underlying skill distribution or changes in how talent is allocated within firms.

Total number of firms. The total number of firms, I, is determined as the lowest-productivity firm that can employ at least one worker, produce, and generate non-negative profits. Formally, using equation (13), I write

$$c_1 = \left[\frac{\beta+\delta}{\delta}\frac{f_e}{\Gamma}\right]^{\frac{1}{(\sigma-1)(1+\frac{\beta}{\delta})}} I^{\delta}. \tag{17}$$

If I further assume that $c(h) = Ch^{-\rho}$, the number of firms equals

$$I = C^{\frac{1}{\rho}} \left[\frac{\beta + \delta}{\delta} \frac{f_e}{\Gamma} \right]^{-\frac{1}{(\sigma-1)(\delta+\beta)}}.$$

The number of firms is increasing in C and the aggregate size of the market Γ, while decreasing in the fixed cost of production, f_e.

3.4 Analytical predictions

Given the model results presented in the previous section, I can draw information on the model predictions related to earnings inequality and firm concentration.

Proposition 2. *Overall earnings inequality is increasing in σ, β, and δ.*

Proof. Using (8), the earnings ratio between workers with different rankings equals

$$\frac{w_i}{w_{i'}} = \left(\frac{i'}{i} \right)^{(\sigma-1)(\beta+\delta)}.$$

Without a loss of generality, assume that $i' > i$. Earnings differentials are thus increasing in σ, δ and β, indicating that more skewed skill and productivity distributions along with a higher price elasticity of substitution lead to higher earnings inequality. □

The prediction that a higher σ increases earnings inequality arises from a notion that a higher σ implies a more price-sensitive demand. In such environments, productivity gains lead to relatively larger increases in firm sales and revenues. As a result, firms are willing to pay higher wages to more productive workers, increasing inequality. Furthermore, a higher σ corresponds to lower product markups. This implies that more competitive product markets generate greater earnings inequality, all else equal.

Earnings inequality also depends on the skewness of both the skill and productivity distributions: The more unequal skill or productivity leads to higher earnings inequality.

Proposition 3. *Concentration in the firm size distribution, measured in employment, is increasing in δ and decreasing in ρ.*

Proof. Using (13), the worker ratio between firms with different rankings equals

$$\frac{h_i^*}{h_{i'}^*} = \left(\frac{i'}{i} \right)^{\frac{\delta}{\rho}}.$$

Again, assume that $i' > i$, and the worker ratio, and thus concentration in the number of workers, is increasing in δ and decreasing in ρ. □

The concentration in the firm size, measured in employment, depends on the skewness of the firm productivity distribution and the organizational hierarchy. The number of workers is determined by the zero-profit condition, given the fixed cost of hiring a new worker, f_e. More productive firms can hire more workers, as they can still cover the fixed cost of employment even when employing lower-productivity workers. Also, if the organization hierarchy, $c(h)$, is flatter (lower ρ), the benefits of hiring an additional worker are decaying at a lower rate, increasing the number of workers hired. As higher-productivity firms are benefiting more from lower ρ, the overall concentration in employment is decreasing in ρ.

Proposition 4. *The concentration of the firm size distribution, measured in sales, depends on σ, β, δ, and ρ. In the top tail, sales follow a Pareto distribution with shape parameter ρ/δ, identical to employment.*

Proof. Using equation (15), total revenue of firm i is given by

$$R_i = \frac{\Gamma}{1-\lambda} \cdot i^{-(\sigma-1)(\beta+\delta)}[(h_i^*)^{1-\lambda} - 1].$$

The sales ratio between firm with different rankings equals

$$\frac{R_i}{R_{i'}} = \left(\frac{i'}{i}\right)^{(\sigma-1)(\beta+\delta)} \frac{(h_i^*)^{1-\lambda} - 1}{(h_{i'}^*)^{1-\lambda} - 1},$$

where $\lambda = \rho(\sigma-1)(\beta+\delta)$. The first part of the sales ratio, $\left(\frac{i'}{i}\right)^{(\sigma-1)(\beta+\delta)}$ shows that, similar to earnings inequality, sales concentration depends on the shapes of the skill and productivity distributions and the price elasticity of demand. In addition, sales concentration also depends on the relative number of workers in each firm, shown by the second term. As $h_i^* > h_{i'}^*$, the ratio $\frac{(h_i^*)^{1-\lambda}-1}{(h_{i'}^*)^{1-\lambda}-1}$, and the ratio is increasing in the skewness of the employment size distribution. The skewness of the employment size distribution is increasing in δ and decreasing in ρ, as shown in equation (13).

Focusing on the top tail of the distribution and substituting in the optimal employment size $h_i^* \propto i^{-\delta/\rho}$, I can write

$$R_i \propto i^{-(\sigma-1)(\beta+\delta)} \cdot i^{\delta(\sigma-1)(1+\beta/\delta)} = i^{-\delta/\rho},$$

which implies that

$$\frac{R_i}{R_{i'}} = \left(\frac{i'}{i}\right)^{\delta/\rho}$$

Hence, in the top tail, firm sales follow a Pareto distribution with a shape parameter ρ/δ, identical to employment, and firm sales concentration increases in δ and decreases in ρ. Importantly, σ and β cancel out in the limit, implying that the concentration in the upper tail of sales is governed only by δ and ρ.

However, outside the tail—particularly in the body of the distribution—sales concentration can depend on all four parameters. This is because the shape of the employment distribution and the curvature of revenue contributions across ranks (governed by β and σ) influence the relative scale of firm sales in non-asymptotic ranges.

□

Propositions 2 and 4 imply that concentration in both earnings and sales distribution depends partly on the same parameters. This result makes intuitive sense as firms are competing for heterogeneous talent by paying the workers based on their value for the firm. Because of the complementarities between firm productivity and worker skill, more productive firms can pay more (as they benefit more from a higher skill), making the pay dependent of the firm productivity distribution and the sales dependent of the worker skills and its distribution. More competitive product markets with a high σ further amplify these effects as any increase in total productivity of firms (which depend on both worker skill and firm productivity) will increase the market share of the firm relatively more.

Finally, the model has implications on wage markdowns. A wage markdown is the ratio between the wage level and the marginal revenue product of labor. This marginal revenue product of labor measures the change in revenue that results from employing an additional unit of labor. In this model, the marginal revenue product for hiring each additional worker is exactly equal to the position revenue, defined in equation (11). Thus, a proposition follows:

Proposition 5. *The wage markdown for each worker equals* $\frac{\beta}{\beta+\delta}$.

Proof. Take the ratio of the wage and the marginal revenue product of a worker by using equations (12) and (11). Taking the ratio, it follows directly that the wage markdown equals $\frac{\beta}{\beta+\delta}$. □

This result implies that the markdown is uniform across workers and that firms pay a larger fraction of marginal revenue to workers when the skill distribution is more skewed relative to firm productivity.

3.5 Equilibrium

In equilibrium, total income is the sum of total wage income and total profits.[8] First, simplify the price index, as

$$P = \int_{i=i^*}^{0} p(i)^{1-\sigma}\, di = \int_{i=i^*}^{0} \left[\frac{\sigma}{\sigma-1} \cdot \frac{1}{(B\tilde{D})i^{-(\beta+\delta)}} \right]^{1-\sigma} di$$

$$= \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \int_{i=i^*}^{0} (B\tilde{D})^{\sigma-1} i^{-(\sigma-1)(\beta+\delta)}\, di.^{9}$$

Total income is thus defined as

$$W = \int_{i=i^*}^{0} (w_i + \pi_i)\, di + \int_{i=i^{\max}}^{i^*} w\, di$$

[8]The way profits are distributed across workers will not change the main findings as preferences are homothetic.

[9]I now use the effective ranking i_h of each position instead of a variety ω to define each good. These can be used interchangeably since each position produces its own variety (or the same variety as the mother firm but with a different quality).

$$= \int_{i=i^*}^{0} \left(\frac{W}{P} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} (B\tilde{D})^{\sigma-1} i^{-(\sigma-1)(\beta+\delta)} - f_e \right) di + w \cdot (i^{\max} - i^*).$$

Equilibrium. Given σ, B, β, D, δ, f_e, C, ρ, and w, a stationary equilibrium is characterized by:

- the share i^* of positions that produce,
- the number of firms I^*,
- the number of positions in each firm h_i^*,
- the aggregate price index P^*,
- total income W^*,
- weighted average sensitivity $\bar{c}^*$,
- profits $\pi^*(i)$,
- wage rates $w^*(i)$,
- output $q^*(i)$,
- and the pricing rule $p^*(i)$,

such that:

- *Position profit maximization:* Each active position i maximizes profits by producing $q^*(i)$ at price $p^*(i)$:

$$p^*(i) = \frac{\sigma}{\sigma-1} \cdot \frac{1}{(B\tilde{D}^*) i^{-(\beta+\delta)}}, \quad q^*(i) = \left(\frac{\sigma-1}{\sigma} \right)^{\sigma} \left(B\tilde{D}^* \cdot i^{-(\beta+\delta)} \right)^{\sigma} \cdot \frac{W^*}{P^*}.$$

- *Worker-position matching and compensation:* Each worker of rank i is matched with the equally ranked position. Profits and wages satisfy

$$\pi^*(i) = \frac{W^*}{P^*} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \left(B\tilde{D}^* \cdot i^{-(\beta+\delta)} \right)^{\sigma-1} - w_i^* - f_e,$$

$$w_i^* = \frac{\beta}{\beta+\delta} (B\tilde{D}^*)^{\sigma-1} \cdot \frac{W^*}{P^*} \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \left[i^{-(\sigma-1)(\beta+\delta)} - (i^*)^{-(\sigma-1)(\beta+\delta)} \right] + w.$$

- *Number of firms:*

$$I^* = C^{1/\rho} \left(\frac{\beta+\delta}{\delta} \cdot \frac{f_e}{\Gamma} \right)^{-1/[(\sigma-1)(\beta+\delta)]}.$$

- *Number of positions in each firm:*

$$h_i^* = C^{1/\rho} \left(\frac{\beta+\delta}{\delta} \cdot \frac{f_e}{\Gamma} \right)^{-1/(\rho(\sigma-1)(1+\beta/\delta))} \cdot (i^*)^{-\delta/\rho}.$$

- *Entry of positions:* Free entry pins down i^*:

$$\frac{\delta}{\beta+\delta}(B\tilde{D}^*)^{\sigma-1}\cdot\frac{W^*}{P^*}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(i^*)^{-(\sigma-1)(\beta+\delta)}=f_e+w.$$

- *Aggregate price index:*

$$P^*=\int_{i=i^*}^{0}p(i)^{1-\sigma}\,di.$$

- *Weighted average sensitivity:*

$$\bar{c}^*=\left(\frac{1}{\int_0^{I^*}h_i^*di}\int_0^{I^*}\int_1^{h_i^*}c(h)^{1/\delta}dhdi\right)^{\delta}.$$

4 Quantitative analysis

In this section, I calibrate the model and assess its ability to replicate key measures of earnings inequality and firm concentration, along with their evolution over time. The model also performs well in matching additional untargeted moments, lending support to its external validity. Building on the robustness of the calibration, I then conduct a quantitative analysis of the joint forces driving changes in both firm concentration and earnings inequality.

4.1 Data and calibration of the model parameters

This subsection outlines how I calibrate the model parameters for two periods in the U.S. economy: 1980–1986 and 2007–2013. These years represent the beginning and end of a period during which earnings inequality and firm concentration increased rapidly, as shown in Figure 1.[10] For brevity, I refer to these periods as 1980 and 2007, respectively. The goal is to identify which structural changes in the model's parameters are consistent with the empirical evolution of concentration and inequality.

I calibrate the model by targeting a set of moments that reflect earnings dispersion, worker-firm sorting, and firm size concentration. These moments are drawn from publicly available data and from published results in Mishel et al. (2015), Song et al. (2019), Gould & Kandra (2022), and Kwon et al. (2024). Table 1 reports the final parameter values.

Fixed parameters. Three scale parameters—B, C, and D—enter the model multiplicatively and affect the levels of the endogenous variables but not their dispersion. Since the focus of this study is on inequality and concentration rather than levels, I normalize

[10]The choice of these two periods also reflects the availability of data on the moments used to calibrate the model parameters. Specifically, I use results from Song et al. (2019), who estimate the contributions of worker fixed effect variance, firm fixed effect variance, and their covariance to earnings inequality. Their estimates cover five seven-year periods, with the first being 1980–86 and the last 2007–13.

$B = C = D = 1$.[11] I set the number of workers $N = 30{,}000$, reflecting the size of the simulated economy.

Joint calibration strategy. I calibrate six key parameters separately for each period $t \in \{1980, 2007\}$: the price elasticity of demand σ_t, the dispersion of worker skill β_t, the dispersion of firm productivity δ_t, the steepness of firm hierarchy ρ_t, the worker outside option w_t, and the fixed cost per position $f_{e,t}$. Each parameter is pinned down by an empirical moment that is closely tied to its theoretical role in the model. In practice, I calibrate all parameters jointly, allowing each moment to be influenced by multiple parameters simultaneously. While individual moments are closely associated with particular parameters for identification purposes, the final calibration accounts for the general equilibrium interactions among all model components.

The productivity dispersion parameter δ_t is chosen so that the model replicates the share of total sales accruing to the top 10% of firms, based on data from Kwon et al. (2024). As shown in Proposition 3, this measure is strongly increasing in δ, making it an informative target for calibration. I then calibrate ρ_t, which governs how quickly the marginal product of additional workers declines along firm hierarchy. Intuitively, a lower ρ implies that firms can scale more easily by hiring more workers, leading to greater employment concentration. Therefore, I choose ρ_t to match the observed employment share of the top 1% of firms. This moment, derived from the same data source, allows the model to capture the distributional skewness in firm employment size.

The price elasticity of demand parameter σ_t is calibrated to match the share of total earnings received by the top 10% of workers, based on data from Gould & Kandra (2022) and Kopczuk et al. (2007). In the model, a lower elasticity of demand implies steeper returns to skill, especially at the top of the earnings distribution. As shown in Proposition 2, σ directly influences the slope of the wage schedule and amplifies earnings differences across workers with varying skill levels. By targeting the top 10% earnings share, I ensure that the model captures the extent of upper-tail inequality observed in the data across the two calibration periods.

The skill dispersion parameter β_t is calibrated to match the share of earnings variance attributable to worker fixed effects (WFE), using estimates from Song et al. (2019). In the model, a higher β implies a more unequal distribution of worker skills, which in turn contributes to overall earnings dispersion. To ensure a clean comparison with the model, which abstracts from observable characteristics such as age or tenure, I adjust the empirical WFE share by excluding variance attributable to observed covariates in the AKM (Abowd et al. (1999)) decomposition. The resulting adjusted targets are 0.549 for 1980 and 0.594 for 2007. These values allow me to discipline how much of the observed rise in earnings inequality can be accounted for by greater dispersion in unobservable worker ability.

The worker outside option parameter w_t at $t = 1980$ is calibrated to match the mean-to-median earnings ratio, a standard summary measure of earnings inequality. In the model,

[11] I test the robustness of the results to this assumption by varying the parameter values. As shown in Appendix C.4, the key model moments remain invariant across alternative values of these fixed parameters.

a higher w_t raises the earnings floor, disproportionately benefiting lower-skilled workers and compressing the wage distribution. I compute the empirical target using U.S. Census Bureau data on personal income from U.S. Census Bureau (2024a) and U.S. Census Bureau (2024b), averaging across each calibration period. This moment helps discipline the model's lower tail and ensures that the degree of wage compression aligns with the data. I then calibrate w_t for $t = 2007$ such that I exactly match the documented decline in the real federal minimum wage in the U.S. between the model time periods (Mishel et al. (2015)). The federal minimum wage has declined about 2.5% between 1980 and 2007, so I set $w_{2007} = 0.975 * w_{1980}$. The federal minimum wage is a natural target for w, which measures the lowest level of wages a worker accepts.[12]

The fixed cost per position $f_{e,t}$ is calibrated to match the sales share of the top 50% of firms, using data from Kwon et al. (2024). In the model, a higher fixed cost per position reduces the scale at which firms operate, thereby potentially decreasing the relative dominance of top firms in total output. This moment serves to discipline the extensive margin of firm growth and ensures that the model reproduces the level of size concentration observed across the firm distribution.

Data adjustments and mapping. For the WFE share, I calculate adjusted targets using decomposition results from Song et al. (2019) (Table III). For 1980, I compute:

$$\begin{aligned}
\widehat{\text{var}}(y_{1980}) &= \text{var(WFE)} + \text{var(FFE)} + \text{var}(\epsilon) + 2 \times \text{cov(WFE,FFE)} \\
&= 0.330 + 0.084 + 0.154 + 0.033 = 0.601, \\
\text{Adjusted WFE Share} &= 0.330/0.601 \approx 0.549.
\end{aligned}$$

For 2007:

$$\begin{aligned}
\widehat{\text{var}}(y_{2007}) &= 0.476 + 0.081 + 0.136 + 0.108 = 0.801, \\
\text{Adjusted WFE Share} &= 0.476/0.801 \approx 0.594.
\end{aligned}$$

These targets ensure consistent comparison between the model and the empirical decomposition, isolating the structural role of skill heterogeneity and sorting.

This calibration approach disciplines each parameter with empirical evidence, grounding the quantitative analysis in key observable trends while enabling counterfactual exercises on the drivers of inequality and concentration. Additional details on moment definitions, calculations, and solution algorithm are provided in Appendix C.

[12] Quantitative results of the model are robust to alternative calibrations of w, including calibrating w_{2007} to match the mean-median wage ratio in 2007, normalizing w to 1 in both periods, or setting w_{2007} to calibrated 1980 level of w.

Table 1: Model parameters and their descriptions.

Parameter, set outside the model	**Description**	**Parameter value** $t = 1980$	$t = 2007$
B	scale parameter	1.0000	1.0000
D	scale parameter	1.0000	1.0000
C	scale parameter	1.0000	1.0000

Parameter, calibrated	**Description**	**Parameter value** $t = 1980$	$t = 2007$	$\%\Delta$
β_t	shape parameter, skill distribution	0.2288	0.2937	28.4%
δ_t	shape parameter, productivity distribution	0.1984	0.1975	-0.5%
ρ_t	shape parameter, sensitivity distribution	0.1825	0.1672	-8.4%
σ_t	price elasticity of demand	2.4955	2.5155	0.8%
w_t	threshold wage of workers	2.4600	2.3985	-2.5%
$f_{e,t}$	fixed cost of a position	0.3944	0.5035	27.7%

Note: This table presents the model parameters and their calibrated values for two periods, $t \in \{1980-86, 2007-13\}$. For conciseness, the table uses $t = 1980$ and $t = 2007$ to represent the periods $t = 1980 - 86$ and $t = 2007 - 13$, respectively. The last column of the lower panel shows the percentage change in the calibrated parameter values between the two periods.

Source: Author's estimations.

4.2 Calibration results

4.2.1 Calibrated parameter values

Table 1 presents the structural parameters of the model, their calibrated values for 1980 and 2007, and their percentage change over time. All calibrated parameters vary over time, though to differing degrees, reflecting changes in underlying distributions of worker skills, firm productivity, and product markets.

The parameter governing the dispersion of worker skill, β, increases by 28.4% over time, indicating a shift toward a more unequal distribution of perceived skills. This change may reflect two broader trends: increasing returns to higher cognitive and social skills, and growing inequality in access to high-quality education. As emphasized in the literature on skill-biased technological change (Acemoglu & Autor, 2011), modern technologies have increased the value of complex skills that are both harder to learn and rarer in the labor force. These shifts likely underlie the observed rise in β.

The dispersion of firm productivity, captured by δ, remains largely unchanged (a decline of just 0.5%), suggesting that the rising concentration in firm outcomes is not driven by widening productivity heterogeneity. This result is consistent with Bloom et al. (2018), who find declining firm-level wage premiums and suggest that inter-firm productivity gaps may have narrowed.

The parameter ρ, which governs how sensitive firm scale is to the quality of its workforce, declines by 8.4%. A lower ρ implies a flatter organizational hierarchy, enabling firms to scale more easily. This is consistent with the emergence of larger firms in recent decades.

The calibrated value of σ, the price elasticity of demand, increases slightly by 0.8%. This suggests a modest increase in price sensitivity among consumers, potentially reflecting greater global competition or technological change benefiting top firms. This interpretation aligns with evidence from Autor et al. (2020) showing increased firm dominance in concentrated markets and is also in line with the literature documenting increasing returns to skill (Acemoglu & Autor (2011)).

The fixed cost of creating a job, $f_{e,t}$, rises by 27.7%, suggesting that it has become more costly for firms to enter and create positions. This finding is broadly consistent with, for example, Hsieh & Rossi-Hansberg (2023), who document positive link between fixed costs and increased national concentrated in the United States.

Finally, the threshold wage w_t, which determines the minimum compensation required to attract workers, decreases by 2.5%, as targeted in the calibration.

4.2.2 Model fit: Targeted moments

Figure 5 and Table 2 display how well the model reproduces the calibration targets in 1980 and 2007. Overall, the model fits the data closely in both years.

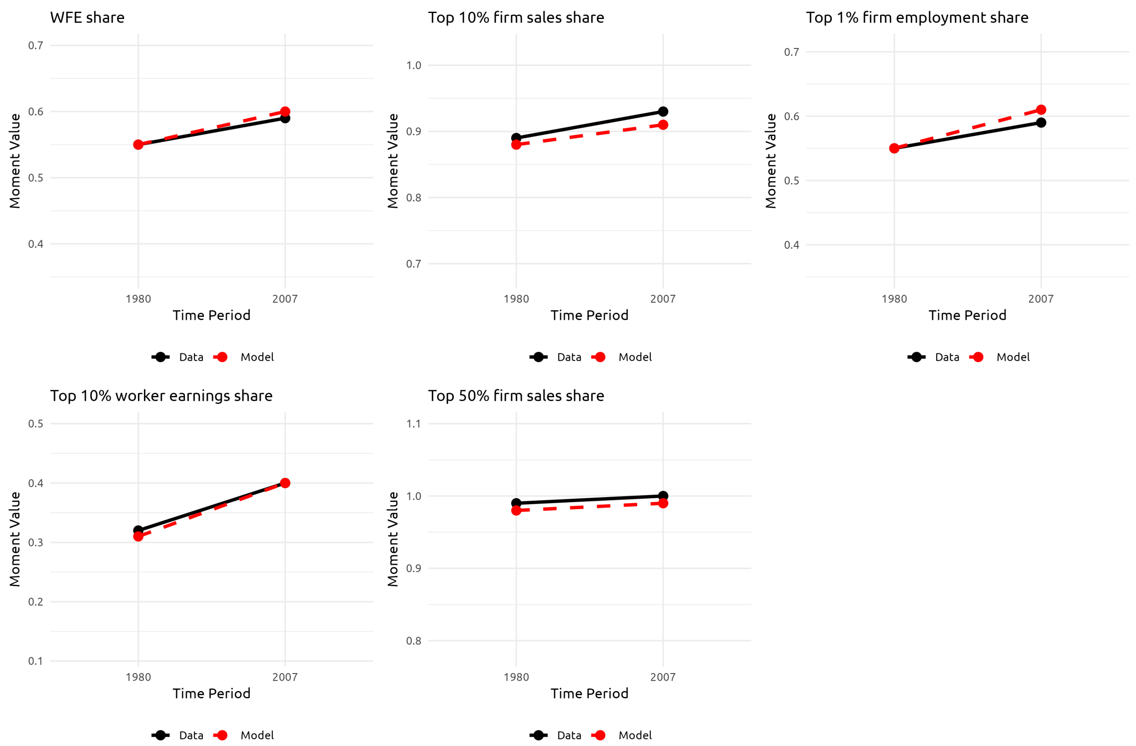


Figure 5: Model fit in 1980 and 2007.

Source: Kopczuk et al. (2007); Mishel et al. (2015); Song et al. (2019); Gould & Kandra (2022); U.S. Census Bureau (2024a,b); Kwon et al. (2024) and author's estimations.

In 1980, the model exactly matches the WFE share of total earnings variance, the top

Table 2: Model fit for targeted moments

		$t = 1980$		$t = 2007$		%Δ	
Parameter	**Target description**	**Target**	**Moment**	**Target**	**Moment**	**Data**	**Model**
β_t	WFE share of total variance	0.55	0.55	0.59	0.60	7.3%	9.1%
δ_t	Top 10% firm sales share	0.89	0.88	0.93	0.91	4.5%	3.4%
ρ_t	Top 1% firm employment share	0.55	0.55	0.59	0.61	7.3%	10.9%
σ_t	Top 10% worker earnings share	0.32	0.31	0.40	0.40	25.0%	29.0%
$f_{e,t}$	Top 50% firm sales share	0.99	0.98	1.00	0.99	1.0%	1.0%
w_t	Mean-to-median earnings ratio	1.39	1.39	-	-	-	-

Note: This table presents calibration data target and model moment values for two periods, $t \in \{1980-86, 2007-13\}$. For conciseness, the table uses $t = 1980$ and $t = 2007$ to represent the periods $t = 1980 - 86$ and $t = 2007 - 13$, respectively. The last two columns show the percentage change in the data targets and model moments between the two periods.

Source: Kopczuk et al. (2007); Mishel et al. (2015); Song et al. (2019); Gould & Kandra (2022); U.S. Census Bureau (2024a,b); Kwon et al. (2024) and author's estimations.

1% firm employment share, and the mean-to-median earnings ratio. The fit is slightly less precise for the top 10% earnings share and top 10% and top 50% firm sales shares, but discrepancies are small.

In 2007, the model continues to match most targets closely, although it modestly overestimates the WFE share and the top 1% employment share and slightly underestimates the firm sales concentration shares. The final two columns in Table 2 compare the model-implied growth rates in these statistics with those observed in the data. The model closely replicates the direction and magnitude of change for most moments, validating its dynamic performance.

4.2.3 Validation against untargeted moments

To assess the model's external validity, I evaluate its performance in matching a set of untargeted moments—that is, empirical statistics not used during calibration. Tables 3 and 4 report the data and model values for nine untargeted moments in 1980 and 2007.

The model matches the additional inequality and concentration measures reasonably well. In both periods, the model modestly underestimates firm concentration, measured using the top 1% firm sales share and the top 0.1% firm employment share, and slightly overestimates the top 0.1%, top 1% and 5% worker earnings shares. All measures increase over time in both the model and the data between 1980 and 2007.

Next, I assess how well the model reproduces other earnings variance decomposition results reported by Song et al. (2019). I first consider the share of total earnings variance explained by the variance of firm fixed effects (FFE) and the covariance between worker fixed effects (WFE) and FFE. I define FFE in the model as:

$$FFE_{i,t} = \log(Di^{-(\sigma_t - 1)\delta_t}) = \log(D) - (\sigma_t - 1)\delta_t \log(i),$$

Table 3: Model Fit for Untargeted Moments (1980)

Moment	Data	Model	Data source
Top 0.1% worker earnings share	0.02	0.03	Kwon et al. (2024)
Top 1% worker earnings share	0.08	0.10	Kwon et al. (2024)
Top 5% worker earnings share	0.21	0.22	Gould & Kandra (2022)
Top 0.1% firm employment share	0.36	0.30	Gould & Kandra (2022)
Top 1% firm sales share	0.71	0.63	Gould & Kandra (2022)
FFE share of total variance	0.14	0.18	Song et al. (2019)
2*cov(WFE,FFE) share of total variance	0.06	0.15	Song et al. (2019)
Within-firm WFE share of total variance	0.46	0.53	Song et al. (2019)
Mean WFE share of total variance	0.09	0.02	Song et al. (2019)

Note: This table presents untargeted data moments and their corresponding model-generated moments for the period $t \in \{1980 - 86\}$. The final column reports the data sources for the untargeted moments.

Source: Model moments: Author's estimations.

and calculate its variance as var$(FFE_{i,t})$. The FFE share is given by $\frac{\text{var}(FFE_{i,t})}{\text{var}(w_{i,t})}$. I then calculate two times the covariance between the average WFE within a firm, $\overline{WFE}_i$, and $FFE_{i,t}$, and express its share of total earnings variance as $\frac{2\times\text{cov}(\overline{WFE}_i, FFE_{i,t})}{\text{var}(w_{i,t})}$. The adjusted FFE and $2 \times$ cov$(\overline{WFE}_i, FFE_{i,t})$ shares in the data are calculated using the same methodology as described in Section 4.1.

The model reproduces the FFE share reasonably well. It slightly overestimates the role of FFE in generating earnings variance in both periods. This overestimation is particularly true if one considers recent literature studying biases in firm fixed effect estimations (Bonhomme et al. (2019); Kline et al. (2020); Bonhomme et al. (2023)). Using U.S. data from 2010 to 2015, Bonhomme et al. (2023) find that, after correcting for biases arising from limited worker mobility across firms, firm fixed effects account for only about 5 to 6% of total earnings variation. Nevertheless, the model reproduces the decline in the FFE share of total earnings variance over time, as reported by Song et al. (2019).

The model overestimates the role of covariance between worker and firm effects in 1980, but closely matches the empirical value in 2007. Covariance between worker and firm effects is typically interpreted as a measure of worker sorting: the extent to which high-skill workers are matched with high-productivity firms. Thus, the model produces too much sorting in the first period but successfully matches the sorting pattern in the later period. However, the model fails to replicate the increase in worker sorting between 1980 and 2007 reported in the data (Song et al. (2019)).

Finally, I decompose the variance in worker-firm effects (WFE) into two components: the variance arising from differences in average WFE across firms ($\overline{WFE}_i$) and the within-firm variance in WFE. The model overestimates within-firm variation and underestimates between-firm variation in WFE. In other words, it fails to generate sufficiently large differences in average worker skill across firms. This discrepancy may stem from the absence

Table 4: Model Fit for Untargeted Moments (2007)

Moment	Data	Model	Data source
Top 0.1% worker earnings share	0.05	0.06	Kwon et al. (2024)
Top 1% worker earnings share	0.13	0.16	Kwon et al. (2024)
Top 5% worker earnings share	0.28	0.31	Gould & Kandra (2022)
Top 0.1% firm employment share	0.39	0.32	Gould & Kandra (2022)
Top 1% firm sales share	0.80	0.69	Gould & Kandra (2022)
FFE share of total variance	0.10	0.12	Song et al. (2019)
2*cov(WFE,FFE) share of total variance	0.14	0.13	Song et al. (2019)
Within-firm WFE share of total variance	0.44	0.59	Song et al. (2019)
Mean WFE share of total variance	0.15	0.01	Song et al. (2019)

Note: This table presents untargeted data moments and their corresponding model-generated moments for the period $t \in \{2007 - 13\}$. The final column reports the data sources for the untargeted moments.

Source: Model moments: Author's estimations.

of complementarities between workers in the model. In Helpman et al. (2010), segregation in average worker skill across firms arises because a firm's final productivity depends on the average productivity of its entire workforce. This creates an incentive for high-productivity firms to avoid hiring low-skilled workers, as doing so would lower overall productivity. The resulting equilibrium features stronger sorting and segregation. By contrast, my model abstracts from such complementarities, leading to weaker sorting and a greater willingness by high-productivity firms to hire lower-skilled workers.

Figure 6 provides a visual comparison of untargeted model and data moments. Each subplot plots model-implied moments against their empirical counterparts for 1980 and 2007, respectively. Most moments lie close to the 45-degree line, confirming that the model captures key patterns in earnings inequality, firm concentration, and worker sorting.

Figure 7 presents log-log rank plots for three key variables—–worker earnings, firm sales, and firm employment—–in the model simulated data. For each variable, I plot the full simulated distribution (blue dots) and overlay a fitted regression line (red dashed) based on the top 1% of earners or firms. These plots show that the model generates Pareto-like upper tails across all distributions, consistent with empirical evidence. The approximate linearity in the upper range of the plots indicates that the simulated distributions closely resemble power laws.

Notably, the estimated slopes of the fitted regression lines are in the range of empirical estimates. For example, Axtell (2001), Gabaix (2009), Jones & Kim (2018), and Kondo et al. (2023) document that firm size and income distributions often follow Zipf's law or closely related Pareto distributions with exponents between 0.6 and 1.5 for firm size, and between 1.2 and 3 for earnings. The model replicates these magnitudes well, reinforcing its ability to capture key empirical regularities in the distributions of economic outcomes.

As a final cross-validation exercise, I perform regressions using simulated data, relating

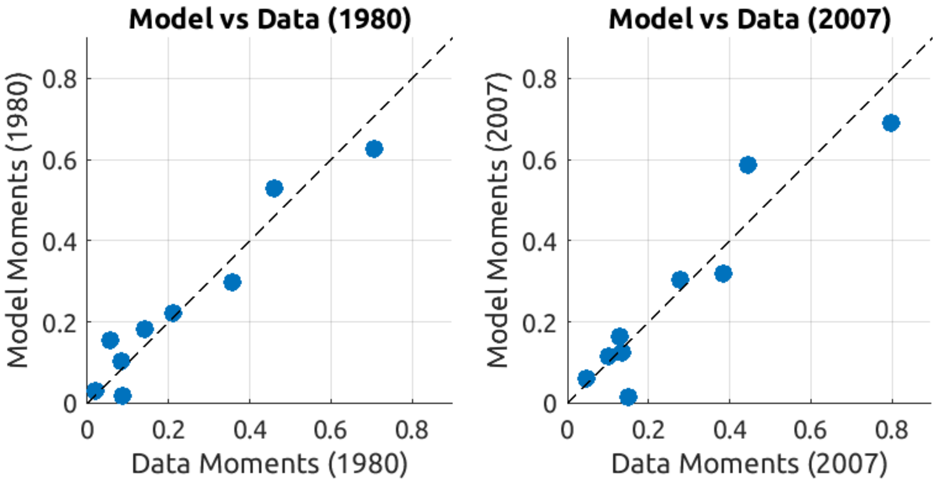


Figure 6: Model vs. data for untargeted moments (1980 and 2007).

Note: Each point represents an untargeted moment. The dashed 45-degree line indicates perfect agreement between model and data. Most moments lie close to the line, particularly after accounting for outliers.

worker earnings to firm characteristics within the economies for 1980 and 2007. First, regressing mean firm-level earnings on firm size yields elasticities of approximately 0.075 in 1980 and 0.085 in 2007. These results suggest that the model reproduces the positive relationship between firm size and earnings observed in the data, and that these relationships strengthened slightly over time, consistent with increasing concentration patterns.

Second, I look at the relationship between log firm-level earnings and log firm revenue per worker, a commonly used measure of firm productivity in empirical literature. Results show that a 1% increase in firm revenue per worker is associated with a 0.3% and 0.5% increase in mean firm-level earnings in 1980 and 2007, respectively. To examine how sensitivity to firm productivity varies across the within-firm earnings distribution, I estimate the elasticity of log earnings with respect to log firm productivity separately for each decile in the within-firm earnings distribution, while dropping firms with less than ten workers. The estimated elasticities ranging from approximately 0.014 to 1.675 in 1980 (median $\approx$ 0.08), and from 0.011 to 1.324 in 2007 (median $\approx$ 0.07). The highest values are concentrated among the top deciles. The high elasticities for the top percentiles are a feature of the model, which is designed to capture superstar dynamics: in equilibrium, the most talented workers are disproportionately concentrated in the most productive firms, and their compensation responds steeply to firm productivity. This is consistent with the span-of-control mechanism introduced by Rosen (1981) and emphasized in Gabaix & Landier (2008), where the combination of extreme firm heterogeneity and sorting generates convex returns to skill and firm performance at the top of the distribution. Such strong nonlinearities are difficult to generate in many standard models.

The empirical relevance of the model-implied elasticities can be further evaluated by

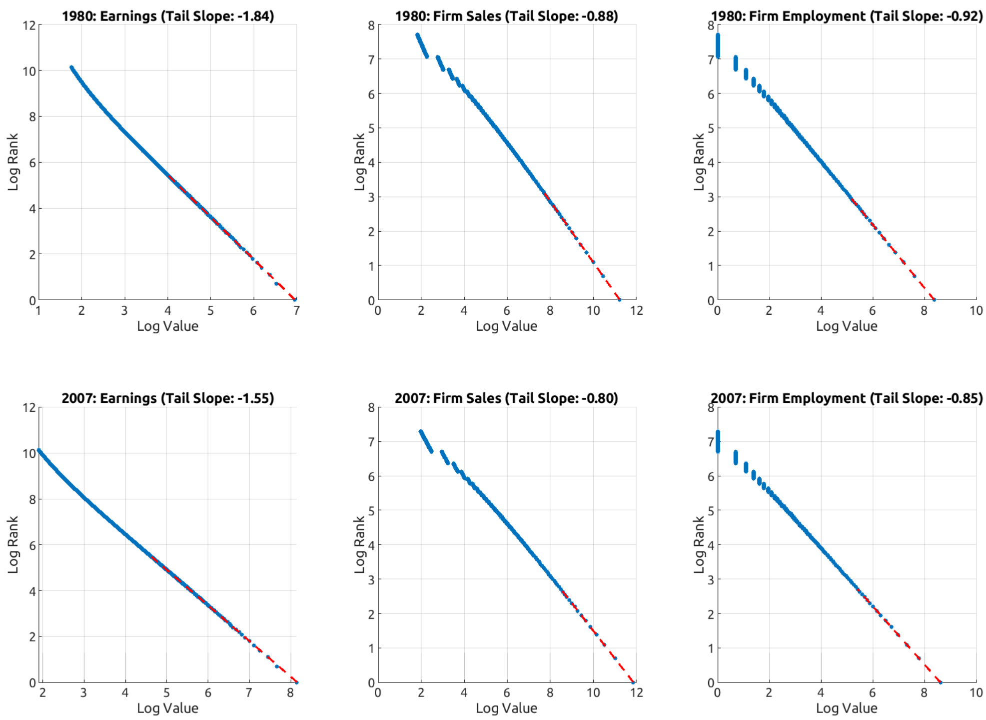


Figure 7: Log-rank plots for simulated firm sales, employment, and worker earnings.

Note: The blue dots represent the full simulated distribution of outcomes. The dashed red line shows the fitted regression line based on the top 1% of the distribution, estimated from a log rank–log outcome regression. The approximate linearity in the upper tail indicates that the model replicates Pareto-like behavior, consistent with empirical upper-tail distributions of earnings and firm size.

Source: Author's estimations.

comparing them to estimates from the existing empirical literature. Using U.S. data between 2003 and 2015, Wallskog et al. (2024) find earnings-productivity elasticities that range between 0.04 and 0.13 across the within-firm earnings percentiles, with highest earners having a stronger connection between productivity and earnings. While the model underestimates elasticities for lower-ranked workers and overestimates them for top earners, the model generates similar median elasticity as well as similar convex returns to skill and firm productivity at the top of the distribution.

In conclusion, the model successfully replicates increased concentration and earnings inequality, despite being calibrated only to a subset of the data, highlighting the model's ability to generalize to broader patterns in inequality and concentration.

5 Counterfactual analysis: Drivers of earnings inequality and firm concentration

To better understand the structural forces behind rising earnings inequality and firm concentration, I conduct a set of counterfactual exercises using the calibrated model. The goal is to isolate the contribution of each parameter to the observed changes in concentration measures between 1980 and 2007.

Specifically, I quantify how changes in six model parameters—β, δ, ρ, σ, f_e, and w—individually contribute to the growth in three key moments: the top 10% earnings share, the top 10% sales share, and the top 1% employment share. Each counterfactual sets all parameters to their 1980 values and then updates one parameter at a time to its 2007 value. The resulting change in each moment reflects the isolated impact of that parameter.

Table 5 and Figure 8 summarize the decomposition, showing both raw and normalized percentage contributions. The normalization divides each parameter's impact by the total change explained across all parameters, ensuring comparability even when individual effects sum to more or less than 100%.

Skill dispersion (β). The increase in the dispersion of worker skills, captured by β, emerges as the dominant force behind the rise in earnings inequality. It explains more than the total increase in the top 10% earnings share, with a normalized contribution of 102.1%. This modest over-explanation suggests that other changes in the model partially offset the effect of increasing skill heterogeneity. β is also the second-largest contributor to the rise in sales concentration, explaining 40.7% of the increase in the top 10% firm sales share. However, it has no effect on employment concentration.

These results underscore that a more unequal skill distribution contributes not only to higher earnings dispersion but also to greater sales concentration at the firm level. In the model, positive assortative matching between high-skill workers and high-productivity firms increases the marginal product of positions. Because earnings reflect both worker and position quality, the model implies a direct link between skill heterogeneity and firm sales concentration.

Table 5: Decomposing the changes in concentration measures between 1980 and 2007.

Parameters	Top 10% earn. share %	Top 10% earn. share Norm., %	Top 10% sales share %	Top 10% sales share Norm., %	Top 1% empl. share %	Top 1% empl. share Norm., %
β	102.3	102.1	45.4	40.7	0.0	0.0
δ	-0.1	-0.1	-3.8	-3.4	-0.4	-0.4
ρ	-10.3	-10.3	68.0	61.0	107.6	108.0
σ	6.3	6.3	3.4	3.0	0.0	0.0
w	0.2	0.2	0.5	0.4	0.1	0.1
f_e	1.8	1.8	-2.0	-1.8	-7.7	-7.7
Total explained	100.2	100.0	111.5	100.0	99.6	100.0

Note: This table reports the contribution of each structural parameter to the change in three concentration measures—top 10% earnings share, top 10% sales share, and top 1% employment share—between 1980 and 2007. The column labeled "%" shows the raw percentage contribution of each parameter to the total observed change in that moment. The "Norm., %" column normalizes these contributions so that they sum to 100%, allowing for easier comparison of relative importance across parameters. Negative values indicate that the parameter dampened the observed change in the concentration measure. For example, the parameter ρ, which governs the steepness of hierarchical decay, explains 108% of the rise in top 1% employment share, offset in part by counteracting effects from other parameters.

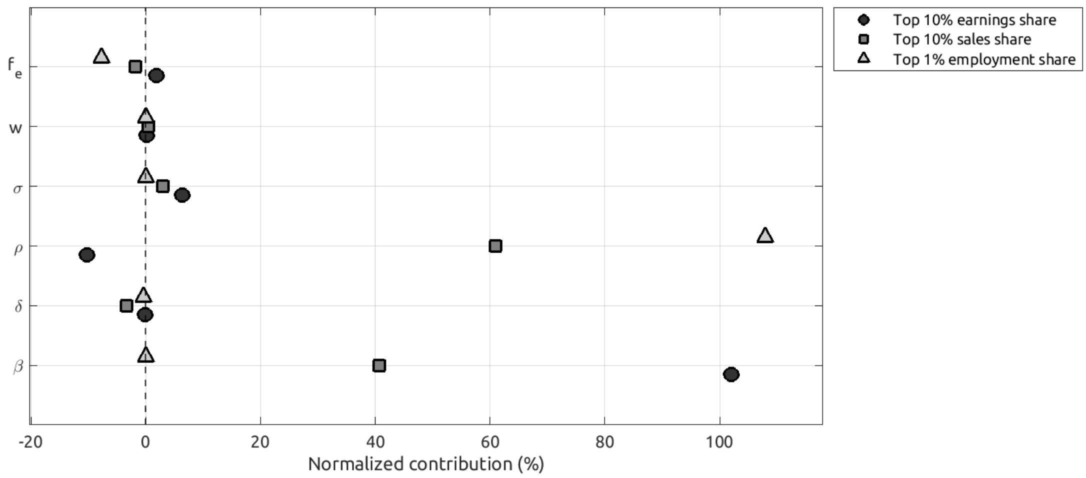


Figure 8: Parameter contributions to changes in concentration measures.

Note: This figure presents a dot plot illustrating the normalized percentage contributions of each parameter to the changes in three concentration measures between 1980 and 2007: top 10% earnings share, top 10% sales share, and top 1% employment share. Each dot represents a parameter's relative importance for a given moment. Negative values indicate parameters that mitigated the observed change.

Source: Author's calculations based on model simulations.

Firm hierarchy steepness (ρ). The decline in ρ plays a pivotal role in explaining the rise in employment concentration. A lower ρ flattens the marginal cost of adding workers to a firm, allowing more productive firms to grow larger. This generates a substantial increase—over 100%—in the top 1% employment share. ρ also accounts for about 60% of the rise in the top 10% sales share, as larger employment scales translate into higher revenues. Its contribution to earnings inequality is slightly negative, as increased firm size diminishes rent-sharing among a larger workforce.

Price elasticity of demand (σ). The modest increase in σ over time increases both inequality and sales concentration. In the model, a higher σ implies more elastic product demand, strengthening the returns to skill and productivity. This is consistent with recent literature suggesting that technological change increases σ through globalization and variety expansion (e.g., Autor et al., 2020; Cortes & Tschopp, 2024).

Productivity dispersion (δ). The parameter δ, which governs the dispersion of firm productivity, changed very little between 1980 and 2007. Consequently, its contribution to all three concentration measures is negligible or slightly negative. This result is consistent with the model calibration and suggests that the observed trends in firm concentration—perhaps surprisingly—are not driven by increased productivity dispersion. This finding also aligns with empirical evidence from Bloom et al. (2018), who document a declining role of firm fixed effects in explaining wage differences over time.

Worker outside option (w). The decline in w, which lowers earnings at the lower end of the wage distribution, modestly heighten inequality by expanding the bottom tail. Its overall effect on earnings inequality and sales concentration is small, though it slightly increases employment concentration by increasing the entry of marginal positions in high-productivity firms.

Fixed position cost (f_e). Changes in the fixed cost of maintaining a position have small, mixed effects. An increase in f_e discourages firm expansion, slightly lowering firm size at the higher end of the distribution. As a result, it mildly increases earnings concentration through the indirect effect of firm contraction, but the overall impact is limited.

Overall, two parameters—β and ρ—emerge as the dominant drivers of rising concentration and inequality. The increase in β is responsible for almost the entirety of the rise in top-earnings concentration and a substantial share of sales concentration. The decline in ρ is the primary driver of increased employment concentration and also plays a major role in sales concentration.

These findings suggest that some structural forces affect both firm-level and worker-level inequality. In particular, the increased dispersion of worker skills not only amplifies earnings inequality but also raises firm concentration through sorting and productivity effects. This channel has received limited attention in the literature on rising firm concentration, which tends to emphasize productivity dispersion or technological scale economies.

My results imply that labor market dynamics—especially those involving skill heterogeneity and sorting—may play an important role in shaping observed patterns of market concentration.

These findings also indicate that the dispersion of hierarchical productivity within firms has declined over time. This flattening of firm hierarchies raises the marginal benefit of adding new positions, enabling more productive firms to expand their workforce. This mechanism fully explains the observed increase in employment concentration and also contributes meaningfully to rising sales concentration. Importantly, it offers a novel explanation for growing employment concentration that does not depend on widening productivity differences between firms.

To assess the robustness of these results, I conduct sensitivity analyses by varying the calibrated parameters by $\pm 5\%$ around the calibrated values. Tables C.4 and C.5 in Appendix C.4 report the resulting percentage changes in the model's core moments. The qualitative patterns are robust across all specifications. For example, a 5% reduction in the decay parameter ρ leads to a 7% increase in the top 1% employment share in the 1980 calibration, underscoring the importance of organizational flattening in explaining rising concentration. Similarly, variation in β strongly influences inequality levels but also firm sales concentration, highlighting the role in skill dispersion generating both.

To complement the one-at-a-time counterfactuals, I group parameters into two blocks—worker-side and firm-side—and evaluate their joint effects on concentration outcomes. The worker block includes β, σ, and w, while the firm block includes δ, ρ, and f_e. Results, reported in Appendix Table D.1, show that worker-side factors not only explain the entire increase in earnings inequality but also account for roughly half of the rise in firm sales concentration. In contrast, firm-side factors primarily drive employment concentration and also contribute significantly to sales concentration, but have little explanatory power for rising earnings inequality.

I further report results from pairwise counterfactual exercise in which I simulate joint changes in pairs of parameters and study how combinations of parameter changes contribute to observed trends in earnings inequality and firm concentration. These results are reported in Appendix Table D.2. As expected, some joint parameter changes (e.g., in β and σ) reveal interaction effects, but the magnitude of these interaction effects is generally modest.

6 Conclusion

This paper develops a unified framework to study the joint evolution of earnings inequality and firm concentration in the U.S. economy. I extend the assignment model of Gabaix and Landier (2008) to a setting with multi-worker firms operating under monopolistic competition, featuring endogenous firm size, hierarchical production, and positive assortative matching between workers and positions. The model demonstrates how changes in underlying structural forces—skill dispersion, firm productivity heterogeneity, hierarchical structure, and product market competition—jointly shape the distributions of earnings and firm size.

Quantitatively, I show that flatter organizational hierarchies and increasingly skewed skill distributions are the primary drivers of both earnings inequality and firm sales concentration, while flatter organizational hierarchies account for increased employment concentration. The model matches several untargeted empirical patterns, including Pareto-shaped firm and earnings distributions and a positive firm-size wage premium, lending further credibility to the framework.

By showing how changes in skill distribution, organizational structure, and price elasticity can jointly drive inequality and firm concentration, this paper offers a unified explanation for the rise of "superstar firms" and the increased dispersion in earnings. The results highlight that firm concentration and labor market inequality may not only be correlated trends, but rather co-determined outcomes of structural shifts in the economy.

Future research could extend the model to incorporate dynamic firm growth, endogenous investment in skill accumulation, non-homothetic demand structures, or firm-specific wage-setting power. Empirically, linking the model to micro-level matched employer-employee data could sharpen identification and enable richer validation exercises.

Appendix A: Analytical derivations

Appendix A.1: Demand functions

The CES form of Pollak's preferences is obtained by setting $\gamma_g = 0$,

$$U_g = \int_{\omega \in \Omega_g} \alpha_\omega q_g(\omega)^{1-\frac{1}{\sigma}} d\omega,$$

and the maximization problem of a worker g buying varieties $\omega \in \Omega$ becomes

$$\underset{q_g(\omega)\geq 0}{max} \; U_g = \int_{\omega \in \Omega} \alpha_\omega q_g(\omega)^{1-\frac{1}{\sigma}} \;\; s.t. \;\; W_g \geq \int_{\omega \in \Omega} p(\omega) q_g(\omega) d\omega.$$

First-order conditions can be written as

$$q_g(\omega): \;\; (1 - \frac{1}{\sigma})\alpha_\omega q_g(\omega)^{-\frac{1}{\sigma}} = \lambda_g p(\omega) \;\; \forall \;\; q_g(\omega) > 0$$

$$\lambda_g: \;\; W_g = \int_{\omega \in \Omega} p(\omega) q_g(\omega) d\omega,$$

where λ_g is a Lagrange multiplier. Any pair of varieties, ω, $\omega^{'} \in \Omega$ gives

$$\frac{\alpha_\omega q_g(\omega)^{-\frac{1}{\sigma}}}{\alpha_{\omega^{'}} q_g(\omega^{'})^{-\frac{1}{\sigma}}} = \frac{p(\omega)}{p(\omega^{'})} \Leftrightarrow p(\omega^{'})\alpha_\omega q_g(\omega)^{-\frac{1}{\sigma}} = p(\omega)\alpha_{\omega^{'}} q_g(\omega^{'})^{-\frac{1}{\sigma}}$$

$$\Leftrightarrow \frac{p(\omega^{'})}{\alpha_{\omega^{'}}} q_g(\omega^{'})^{\frac{1}{\sigma}} = \frac{p(\omega)}{\alpha_\omega} q_g(\omega)^{\frac{1}{\sigma}}$$

$$\Leftrightarrow q_g(\omega^{'}) = \frac{\left(\frac{p(\omega^{'})}{\alpha_{\omega^{'}}}\right)^{-\sigma}}{\left(\frac{p(\omega)}{\alpha_\omega}\right)^{-\sigma}} q_g(\omega).$$

Multiply both sides by $p(\omega^{'})$ and integrate over all $\omega^{'} \in \Omega$ to get

$$\int_\Omega p(\omega^{'}) q_g(\omega^{'}) d\omega^{'} = \int_\Omega \frac{p(\omega^{'})\left(\frac{p(\omega^{'})}{\alpha_{\omega^{'}}}\right)^{-\sigma}}{\left(\frac{p(\omega)}{\alpha_\omega}\right)^{-\sigma}} q_g(\omega) d\omega^{'}$$

$$\Leftrightarrow W_g = \frac{q_g(\omega)}{\left(\frac{p(\omega)}{\alpha_\omega}\right)^{-\sigma}} \int_\Omega p(\omega^{'}) \left(\frac{p(\omega^{'})}{\alpha_{\omega^{'}}}\right)^{-\sigma} d\omega^{'}$$

$$\Leftrightarrow q_g(\omega) = \frac{\left(\frac{p(\omega)}{\alpha_\omega}\right)^{-\sigma}}{\int_\Omega p(\omega^{'}) \left(\frac{p(\omega^{'})}{\alpha_{\omega^{'}}}\right)^{-\sigma} d\omega^{'}} W_g,$$

where the budget constraint of a worker g implies that $W_g = \int_\Omega p(\omega^{'}) q_g(\omega^{'}) d\omega^{'}$.

Appendix A.2: Derivations of assignment equilibrium and wage distribution

This appendix presents the full derivations underlying the wage distribution and sales expressions from Section 3.3, under the assumption of positive assortative matching (PAM) between workers and positions.

First-order condition and assignment equation. Each firm selects a worker g for position (i, h) to maximize profits:

$$\pi^*_{i,g,h} = \frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[\tilde{A}(i,h)T(g)\right]^{\sigma-1} - w_g - f_e,$$

where $\tilde{A}(i,h)$ is the effective productivity of position h in firm i.

The first-order condition for optimal worker choice is:

$$\frac{\partial \pi^*_{i,g,h}}{\partial g} = \frac{W}{P}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma-1}}\tilde{A}(i,h)\left[\tilde{A}(i,h)T(g)\right]^{\sigma-2}T'(g) - w'_g = 0.$$

With $T(g) = Bg^{-\beta}$, we have $T'(g) = -\beta B g^{-\beta-1}$. Substituting and simplifying:

$$w'_g = -\beta\frac{W}{P}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma-1}}\tilde{A}(i,h)\left[\tilde{A}(i,h)T(g)\right]^{\sigma-2}Bg^{-\beta-1}.$$

Assuming positive assortative matching (i.e., $i = g$), I obtain:

$$w'_i = -\beta\frac{W}{P}\frac{(\sigma-1)^{\sigma}}{\sigma^{\sigma-1}}(B\tilde{D})^{\sigma-1}i^{-(\sigma-1)(\beta+\delta)-1}.$$

Wage function. Integrating w'_i with respect to i yields:

$$w_i = \int w'_i\,di = \frac{\beta(B\tilde{D})^{\sigma-1}}{\beta+\delta}\frac{W}{P}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left[i^{-(\sigma-1)(\beta+\delta)} - i^{*-(\sigma-1)(\beta+\delta)}\right] + w,$$

where i^* is the rank of the last active worker, and w is the reservation wage.

If i^* is sufficiently large, then $i^{*-(\sigma-1)(\beta+\delta)} \approx 0$, and the wage distribution simplifies to:

$$w_i = \frac{\beta}{\beta+\delta}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{W}{P}(B\tilde{D})^{\sigma-1}i^{-(\sigma-1)(\beta+\delta)} + w.$$

Sales per position. The total revenue from position i (matched with worker $g = i$) is:

$$R_i = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{W}{P}\left[\tilde{A}(i,h)T(i)\right]^{\sigma-1}.$$

Substituting $T(i) = Bi^{-\beta}$ and $\tilde{A}(i,h) = \tilde{D}i^{-\delta}$, I have:

$$R_i = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{W}{P}(B\tilde{D})^{\sigma-1}i^{-(\sigma-1)(\beta+\delta)}.$$

Interpretation. Both wages and sales follow power-law distributions due to three structural features:

1. Pareto distributions of worker skill (β) and firm productivity (δ),
2. CES product demand at the position level,
3. Positive assortative matching enabled by supermodularity.

The shape parameter of the wage distribution is $(\sigma - 1)(\beta + \delta)$. Thus, wage inequality increases with:

- a more convex demand structure (higher σ),
- greater dispersion in worker skill (β),
- greater dispersion in firm productivity (δ).

Appendix B: Firm problem with implicit complementarities from CES demand at the firm level

Assume firms have the same hierarchical structure as in the baseline profit-maximization problem. A firm's objective is to choose the total number of positions, h^*, assign a worker with a given skill $T(g)$ to each position h, and set its price, $p(\omega)$, to maximize profits. The firm's problem can be solved recursively by first optimizing over prices given an assignment of workers and then solving the assignment problem.

In the baseline model, firm revenue is computed by applying CES demand to each worker's contribution separately. Specifically, revenue takes the form:

$$R_i = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \int_{h=1}^{h^*} [c(h)T(g_h)A(i)]^{\sigma-1}\, dh \cdot \frac{W}{P},$$

where $T(g_h)$ is the talent of the worker in position h, $c(h)$ reflects the hierarchical decay in influence from position h, and $A(i)$ is the firm-specific productivity. This specification assumes that each worker's output contributes additively and independently to the firm's revenue.

In this appendix, I consider a modified formulation in which CES demand is applied to the *sum* of position-level productivities rather than to each term separately:

$$\widehat{R_i} = \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \left[\int_{h=1}^{h^*} c(h)T(g_h)A(i)\, dh\right]^{\sigma-1} \cdot \frac{W}{P}.$$

This change introduces *implicit complementarities* between workers when $\sigma > 2$. That is, total revenue becomes convex in the vector of worker skills, so the marginal return to skill in one position increases with the skill in others.

When comparing these expressions, a couple of observations arise. First, when $\sigma = 2$, the revenues are equal, $R_i = \widehat{R_i}$. It follows that the solutions to the assignment problem are also exactly the same. In contrast, $\widehat{R_i}$ exceeds R_i when $\sigma > 2$ due to the presence of positive cross-terms between position productivities. The baseline model excludes these complementarities by applying CES separately to each worker.

I illustrate this difference using a simple example. Suppose a firm has two workers ($h^* = 2$) and that $\sigma = 3$. Then:

$$\widehat{R_i} = \left(\frac{2}{3}\right)^2 \left[\sum_{h=1}^{2} c(h)T(g_h)A(i)\right]^2 \cdot \frac{W}{P}$$

$$= \left(\frac{2}{3}\right)^2 \cdot \frac{W}{P} \cdot [c(1)T(g_1)A(i) + c(2)T(g_2)A(i)]^2$$

$$= \left(\frac{2}{3}\right)^2 \cdot \frac{W}{P} \cdot A(i)^2 \left[(c(1)T(g_1))^2 + 2c(1)c(2)T(g_1)T(g_2) + (c(2)T(g_2))^2\right].$$

The cross-term $2c(1)c(2)T(g_1)T(g_2) > 0$ shows that worker productivity is multiplicative across positions. This means that a skilled worker is even more valuable when placed alongside other skilled workers. Applying CES demand to the aggregate productivity sum introduces convexity in team output, which implies that a worker's marginal product increases with the skill of their colleagues. This creates implicit complementarities in hiring decisions, even though the underlying productivity function remains additive.

While the model retains closed-form expressions for firm prices and revenue, the convexity of revenue in worker skill implies that equilibrium wages are no longer analytically solvable. That is, the marginal revenue product of a worker depends on the full composition of coworkers. This breaks the separable structure that allows for closed-form wage schedules in the baseline model. As a result, matching patterns and the distribution of firm sizes may also shift, but the precise impact on employment or revenue concentration depends on the full equilibrium solution and is not analytically determined.

Appendix C. Calibration targets and data sources

This appendix provides additional documentation on the empirical moments used to calibrate the model, including their definitions, sources, and, where applicable, any adjustments made to align them with the model's structure.

C.1. Calibration targets

Table C.1: Summary of calibration targets and sources

Parameter	Description	Targeted Moment	Source	1980	2007
δ_t	Productivity dispersion	Top 10% firm sales share	Kwon et al. (2024)	0.89	0.93
ρ_t	Steepness of firm hierarchy	Top 1% firm employment share	Kwon et al. (2024)	0.55	0.59
$f_{e,t}$	Fixed cost per position	Top 50% firm sales share	Kwon et al. (2024)	0.99	1.00
σ_t	Price elasticity of demand	Top 10% earnings share	Gould & Kandra (2022), Kopczuk et al. (2007)	0.32	0.40
w_t	Worker outside option	Mean-to-median earnings ratio	U.S. Census Bureau	1.39	-
w_t	Worker outside option	% decline in the real U.S. federal minimum wage	Mishel et al. (2015)	-	1.36
β_t	Skill dispersion	WFE share of earnings variance	Song et al. (2019)	0.549	0.594

C.2. Notes on empirical construction

Firm concentration measures: All moments derived from Kwon et al. (2024) are computed using digitized IRS and Business Dynamics Statistics data, based on firm-level revenue and employment distributions. Each concentration measure is averaged over 1980–1986 and 2007–2013.

Earnings distribution measures: Top decile earnings shares are obtained from Gould & Kandra (2022) and Kopczuk et al. (2007), reflecting pre-tax individual earnings. Mean and median earnings are obtained from the U.S. Census Bureau. Final measures are averaged over each period. The decline in the real U.S. federal minimum wage rate is calculate using data from Mishel et al. (2015).

WFE share adjustment: To align the empirical WFE share with the model (which excludes observable worker characteristics), I exclude the variance contribution from $X_t^i\beta^p$ in the AKM decomposition from Song et al. (2019). The adjusted total variance is:

$$\widehat{\mathrm{var}}(y_t) = \mathrm{var}(\theta^i) + \mathrm{var}(\psi^j) + \mathrm{var}(\epsilon^{ij}) + 2\mathrm{cov}(\theta^i, \psi^j),$$

yielding adjusted WFE shares of:

$$\text{1980: } 0.330/0.601 = 0.549, \quad \text{2007: } 0.476/0.801 = 0.594.$$

All targets are matched in levels.

C.3. Solution algorithm

Given the set values of model parameters B, β, D, δ, σ, C, ρ, w, and f_e, and initial guesses for the equilibrium number of firms (I^*) and positions (i^*), price index P, and total income W, I solve the model as follows:

Step 1: Generate grids of worker skills, $T(g) = Bg^{-\beta}$, firm productivity, $A(i) = Di^{-\delta}$, and sensitivity function for job positions, $c(h) = Ch^{-\rho}$. In implementation, i, g, and h are discretized from 1 to $N = 30,000$. Results are robust to starting the grid at values closer to zero (e.g., 0.01), due to the scale-invariant properties of the power law distributions.

Step 2: Calculate effective productivity of positions, $\tilde{A}(i, h) = c(h)A(i)$ and rank all positions based on their effective productivity.

Step 3: Solve the matching problem by using positive assortative matching to assign workers g to positions (i, h) and computing equilibrium wages $w(i, h, g)$ using the assignment equation.

Step 4: Solve for optimal pricing using the optimal pricing rule. Then solve for output, revenue, and profits for each position.

Step 5: Determine firm and market equilibrium by solving for the number of active positions and firms. Then, compute equilibrium firm size measured in employment (or positions) and revenue, and compute aggregate price index P and total income W by summing up the total wages, total profits, and total value of home production.

Step 6: Continue iteration until the model converges to the equilibrium number of firms and positions, as well as equilibrium price index and total wages.

Step 7: Compute model outcomes related to earnings and firm size distributions.

C.4. Robustness results - sensitivity of the model moments to alternative parameter values

Scale parameters of the distributions. To assess sensitivity of the model outcomes to changes in scale parameters of the skill, productivity, and hierarchical productivity decay distributions (B, D, and C, respectively), I vary these parameter values and track the response of key model moments. The key model moments are unaffected by the values of these parameters. The results are reported in Tables C.2 and C.3.

Table C.2: Robustness of model moments to parameter variation (1980)

Parameter	Value	WFE share	Top 10% sales	Top 1% emp.	Top 10% earn.	Top 50% sales
B	1	0.55	0.88	0.55	0.31	0.98
B	334	0.55	0.88	0.55	0.31	0.98
B	667	0.55	0.88	0.55	0.31	0.98
B	1000	0.55	0.88	0.55	0.31	0.98
D	1	0.55	0.88	0.55	0.31	0.98
D	334	0.55	0.88	0.55	0.31	0.98
D	667	0.55	0.88	0.55	0.31	0.98
D	1000	0.55	0.88	0.55	0.31	0.98
C	1	0.55	0.88	0.55	0.31	0.98
C	334	0.55	0.88	0.55	0.31	0.98
C	667	0.55	0.88	0.55	0.31	0.98
C	1000	0.55	0.88	0.55	0.31	0.98

Note: This table reports model-implied moments for 1980 under alternative values of the fixed parameters B, D, and C, which scale the worker skill distribution, firm productivity distribution, and hierarchical productivity decay, respectively. The results show that the key model moments—worker-firm effect (WFE) share of wage variance, top 10% sales concentration, top 1% employment concentration, top 10% earnings concentration, and top 50% sales concentration—are invariant to these parameter changes. This reflects the model's scale-invariance properties and confirms that moment outcomes are governed by shape parameters rather than scale parameters.

Source: Author's calculations based on model simulations.

Table C.3: Robustness of model moments to parameter variation (2007)

Parameter	Value	WFE share	Top 10% sales	Top 1% emp.	Top 10% earn.	Top 50% sales
B	1	0.60	0.91	0.61	0.40	0.99
B	334	0.60	0.91	0.61	0.40	0.99
B	667	0.60	0.91	0.61	0.40	0.99
B	1000	0.60	0.91	0.61	0.40	0.99
D	1	0.60	0.91	0.61	0.40	0.99
D	334	0.60	0.91	0.61	0.40	0.99
D	667	0.60	0.91	0.61	0.40	0.99
D	1000	0.60	0.91	0.61	0.40	0.99
C	1	0.60	0.91	0.61	0.40	0.99
C	334	0.60	0.91	0.61	0.40	0.99
C	667	0.60	0.91	0.61	0.40	0.99
C	1000	0.60	0.91	0.61	0.40	0.99

Note: This table reports model-implied moments for 2007 under alternative values of the fixed parameters B, D, and C, which scale the worker skill distribution, firm productivity distribution, and hierarchical productivity decay, respectively. The results show that the key model moments—worker-firm effect (WFE) share of wage variance, top 10% sales concentration, top 1% employment concentration, top 10% earnings concentration, and top 50% sales concentration—are invariant to these parameter changes. This reflects the model's scale-invariance properties and confirms that moment outcomes are governed by shape parameters rather than scale parameters.

Source: Author's calculations based on model simulations.

Calibrated parameter values. To assess local sensitivity, I vary each structural parameter by ±5% around its baseline value and track the response of key model moments. The results are summarized in Tables C.4 and C.5.

Table C.4: Percent change in model moments from ±5% parameter perturbations (1980)

Parameter	*WFE share*		*Top 10% sales*		*Top 1% empl.*		*Top 10% earn.*		*Top 50% sales*		*Mean/median*	
	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$
β	-1.2	1.2	-0.3	0.3	0.0	0.0	-4.7	4.9	-0.1	0.1	-2.5	2.7
δ	0.1	0.2	-2.2	1.9	-6.4	6.4	-0.4	0.2	-0.5	0.4	-0.2	0.1
ρ	3.2	-2.7	1.8	-1.9	6.9	-6.6	-1.7	1.6	0.4	-0.4	-0.9	0.9
σ	3.1	-2.8	-0.9	0.9	-0.0	0.0	-10.7	12.0	-0.2	0.2	-5.5	6.8
w	-0.3	0.3	0.0	0.0	0.0	-0.1	0.1	-0.1	-0.0	0.0	0.1	-0.1
f_e	0.3	-0.3	0.0	-0.0	-0.1	0.0	-0.1	0.1	0.0	-0.0	-0.1	0.1

Note: This table reports the sensitivity of model-generated moments to $\pm5\%$ changes in each structural parameter. For each parameter, I compute the percent deviation in each moment relative to its baseline (calibrated) value. A positive (negative) value indicates that the moment increases (decreases) when the parameter is increased by 5%. Results are shown for all six model moments: the share of variance explained by worker fixed effects (WFE share), the top 10% firm sales share, the top 1% firm employment share, the top 10% worker earnings share, the top 50% firm sales share, and the mean-to-median earnings ratio.

Table C.5: Percent change in model moments from ±5% parameter perturbations (2007)

Parameter	*WFE share*		*Top 10% sales*		*Top 1% empl.*		*Top 10% earn.*		*Top 50% sales*		*Mean/median*	
	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$	-5%	$+5\%$
β	-1.1	1.1	-0.3	0.4	0.0	0.0	-5.5	5.7	-0.1	0.1	-4.2	4.7
δ	-0.1	0.3	-1.5	1.2	-5.3	4.5	-0.3	0.2	-0.3	0.3	-0.2	0.1
ρ	2.9	-2.5	1.2	-1.2	3.0	-5.9	-1.8	1.7	0.2	-0.3	-1.4	1.3
σ	2.4	-2.1	-0.9	0.9	0.0	0.0	-12.0	13.2	-0.2	0.2	-8.6	11.3
w	-0.2	0.2	-0.0	0.0	0.0	-0.0	0.1	-0.1	-0.0	0.0	0.1	-0.1
f_e	0.2	-0.2	0.0	-0.0	-0.0	0.0	-0.1	0.1	0.0	-0.0	-0.1	0.1

Note: This table reports the sensitivity of model-generated moments to $\pm5\%$ changes in each structural parameter. For each parameter, I compute the percent deviation in each moment relative to its baseline (calibrated) value. A positive (negative) value indicates that the moment increases (decreases) when the parameter is increased by 5%. Results are shown for all six model moments: the share of variance explained by worker fixed effects (WFE share), the top 10% firm sales share, the top 1% firm employment share, the top 10% worker earnings share, the top 50% firm sales share, and the mean-to-median earnings ratio.

Appendix D. Additional quantitative results

Block-level counterfactuals. To assess the joint contribution of related mechanisms, I conduct a block-level counterfactual analysis in which I group the calibrated parameters into two functional blocks. The worker-side block consists of parameters governing the skill distribution, price elasticity of substitution, and outside option: β, σ, and w. The firm-side block includes parameters shaping the productivity distribution, firm hierarchy, and fixed costs: δ, ρ, and f_e.

For each block, I replace the 1980 values of the parameters with their 2007 counterparts, holding all other parameters fixed at their 1980 levels. I then evaluate the resulting change in model-implied moments and compare it to the actual change between 1980 and 2007. This allows me to quantify the extent to which each block of mechanisms contributes to the observed increase in earnings inequality and firm concentration.

Table D.1 reports the contributions of grouped parameter changes to the observed rise in earnings inequality and firm concentration between 1980 and 2007. The worker-side block explains the full increase in earnings inequality (111.7%) and also accounts for roughly half of the rise in firm sales concentration (50.4%). These results indicate that changes in worker-side fundamentals not only drove the widening of the earnings distribution but also contributed meaningfully to the concentration of economic activity among top firms.

By contrast, the firm-side block explains nearly all of the rise in employment concentration (99.6%) and a majority of the increase in sales concentration (62.7%), but contributes negatively to earnings inequality (8.6%). This pattern underscores that firm-side forces primarily shape the distribution of firm sizes and employment, while worker-side changes are essential for understanding both wage dispersion and the concentration of output. The results are reported in Table D.2.

Table D.1: Block counterfactual contributions (as percent of total change)

Block	Top 10% sales share	Top 1% employment share	Top 10% earnings share
Worker Block (β, σ, w)	50.4	0.1	111.7
Firm Block (δ, ρ, f_e)	62.7	99.6	-8.6

Note: Each row reports the contribution of a parameter block to the total change in the respective moment between 1980 and 2007, expressed as a percentage. Negative values indicate that the block muted the observed change. See Appendix D for methodological details.

Source: Author's calculations based on model simulations.

Pair-wise counterfactuals. To assess the interaction effects between parameters, I conduct a series of pairwise counterfactual exercises. In each case, I jointly change two parameters from their 1980 values to their 2007 values while holding all others fixed. This approach captures potential complementarities between parameters—particularly how they may reinforce or offset each other in shaping concentration and inequality. For each pair, I compute the resulting model moments and compare them to the baseline 1980 and 2007 outcomes. This analysis helps clarify whether two parameters jointly account for a disproportionately large share of the observed changes, thereby highlighting key channels of interaction across worker-side and firm-side mechanisms.

Table D.2: Pairwise counterfactual decomposition

Type	Param 1	Param 2	Top 10% sales	Top 1% employment	Top 10% earnings
Single	β	–	45.4%	0.0%	102.3%
Single	δ	–	-3.8%	-0.4%	-0.1%
Single	ρ	–	68.0%	107.6%	-10.3%
Single	σ	–	3.4%	0.0%	6.3%
Single	w	–	0.5%	0.1%	0.2%
Single	f_e	–	-2.0%	-7.7%	1.8%
Pairwise	β	δ	42.2%	-0.4%	102.1%
Pairwise	β	ρ	102.0%	107.6%	89.4%
Pairwise	β	σ	50.0%	0.0%	111.5%
Pairwise	β	w	45.9%	0.1%	102.4%
Pairwise	β	f_e	43.6%	-7.7%	103.8%
Pairwise	δ	ρ	64.2%	97.5%	-10.3%
Pairwise	δ	σ	-0.4%	-0.4%	6.2%
Pairwise	δ	w	-4.2%	-10.2%	0.0%
Pairwise	δ	f_e	-5.8%	-7.6%	1.7%
Pairwise	ρ	σ	70.5%	107.6%	-4.3%
Pairwise	ρ	w	68.3%	107.8%	-10.2%
Pairwise	ρ	f_e	66.4%	92.9%	-8.5%
Pairwise	σ	w	4.0%	0.1%	6.5%
Pairwise	σ	f_e	1.4%	-7.7%	8.1%
Pairwise	w	f_e	-2.5%	-7.6%	2.0%

Note: Each row reports the contribution of a parameter pair to the total change in the respective moment between 1980 and 2007, expressed as a percentage. Negative values indicate that the pair muted the observed change. See Appendix D for methodological details.

Source: Author's calculations based on model simulations.

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