Growth with New and Old Technologies

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Motivation

- Emerging technologies attract the attention of society and policymakers.
- The National Science Foundation, for example, often supports research in new areas.

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- Should governments provide selective support to new technologies?
  ... Why not to older and mature ones which are widely used?
- Are market incentives not sufficient to optimally allocate our innovative resources?

This paper

Theory: a novel innovation-led growth model with vintage technologies.

- Technologies with high potential arrive in the frontier - but have yet to be perfected.
- Two forces of growth: improving existing techs and the development of new ones.
  - → Policy implication: R&D misallocation (distribution of R&D biased towards old techs)

Empirical evidence on the direction of innovation between old and new technologies.

- Finding: this cross-sectional distribution is hump-shaped and stationary.

Quantitative analysis: the welfare effects of R&D misallocation.

Related Literature

- Vintage capital/technology growth models; GPT models; Diffusion: Griliches (1957), Arrow (1962), Mansfield (1961, 1963), Jovanovic and Lach (1989), Chari and Hopenhayn (1991), Young (1993), Helpman and Trajtenberg (1994, 1996), Comin and Hobijn (2004, 2010), Jovanovic and Yatsenko (2012), Atkeson and Kehoe (2007), Comin and Mestieri (2018).

  Contribution: innovation on both declining and rising technologies.

- Directed technical change: Acemoglu and Zilibotti (2001), Acemoglu (2002, 2007), Acemoglu, Aghion, Bursztyn and Hemous (2012), Acemoglu, Akcigit, Hanley and Kerr (2016), Aghion, Dechezleprêtre, Hemous, Martin, Van Reenen (2016).

  Contribution: endogenous cycle of rise and obsolescence.

- Misallocation of R&D: Acemoglu (2023), Liu and Ma (2023), Zilibotti et al. (2022).

  Contribution: old vs new technologies.

Old and New Technologies in the US

- Technology vintages $\leftrightarrow$ technological classification of the US Patent Office.
- Emergence date of a technology (based on Griliches, 1957): more later.

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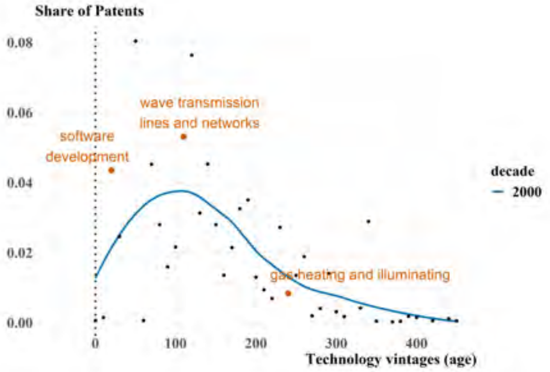

Quality weighted patents Breakthrough patents

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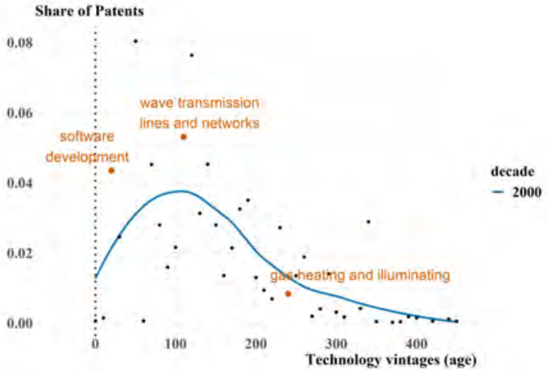

The hump-shape of US innovation

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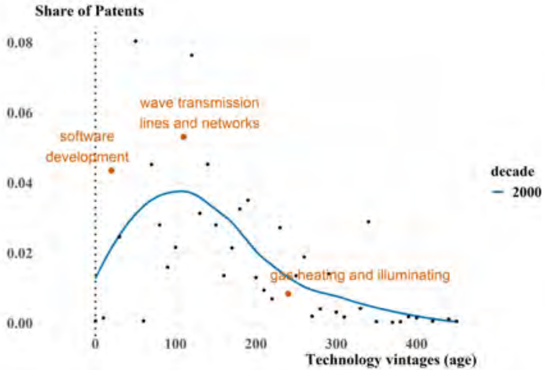

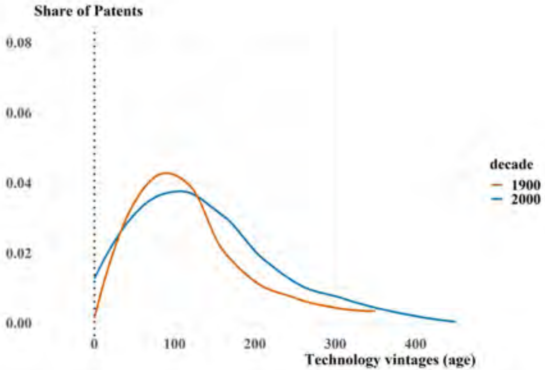

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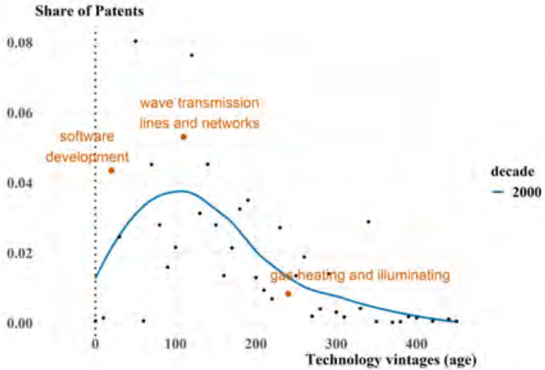

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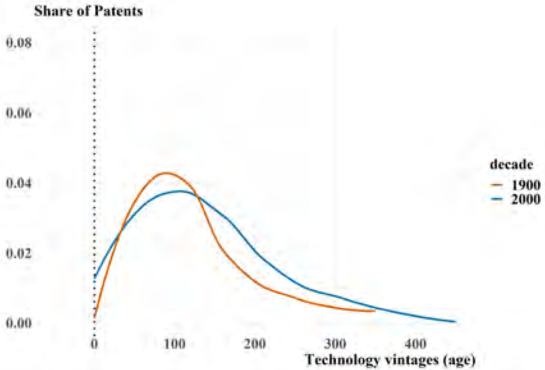

Stationary in the 20th century

Quality weighted patents
Breakthrough patents

Outline

- Theory
- Empirical Analysis and Calibration
- Quantitative Analysis

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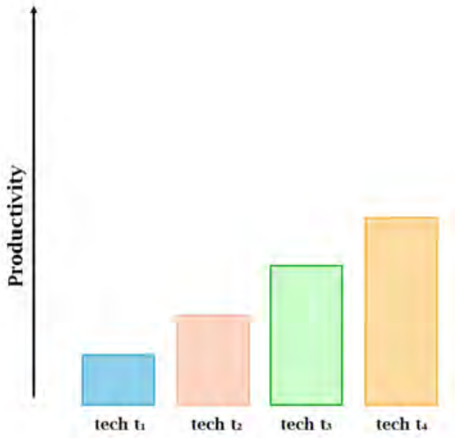
The economy: an illustration
Technology vintages across time



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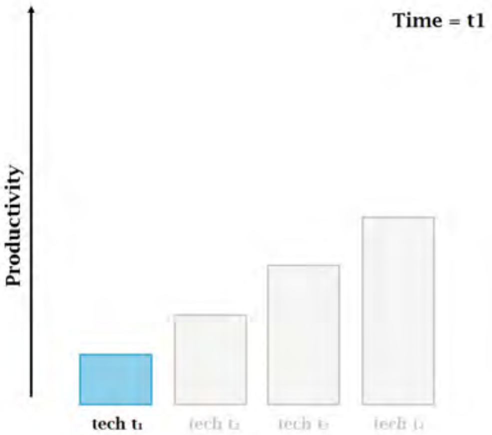

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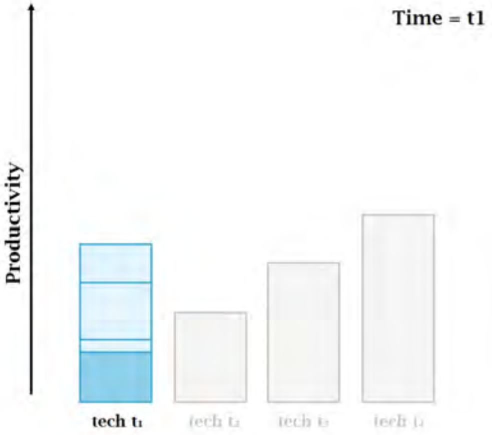

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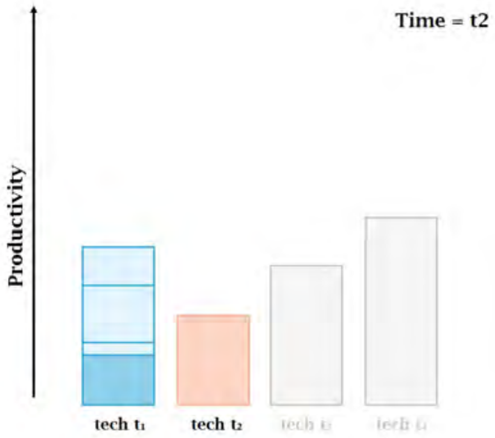

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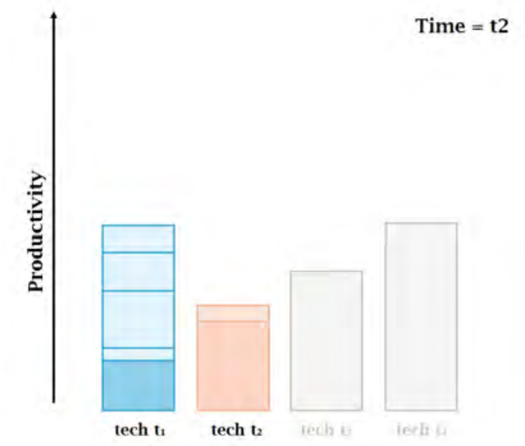

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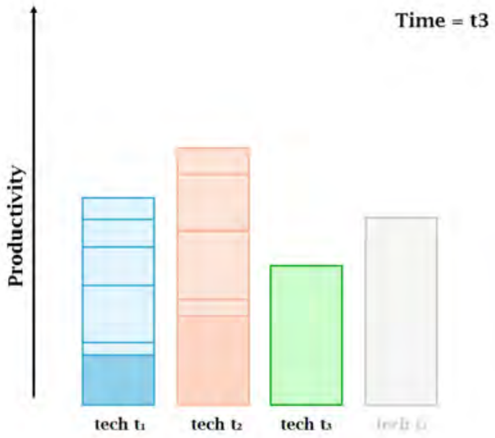

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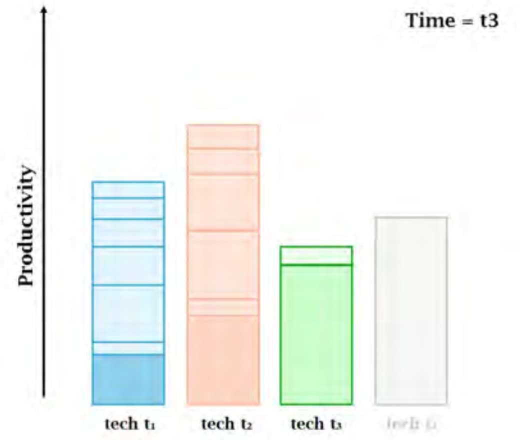

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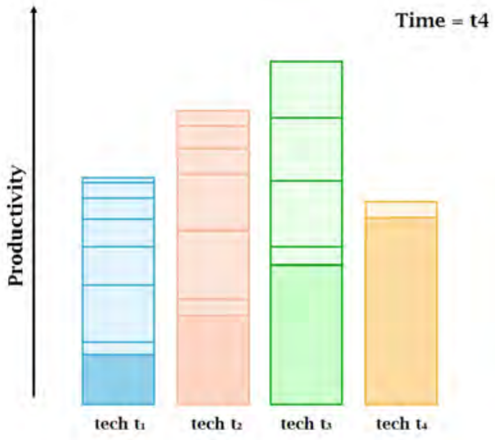

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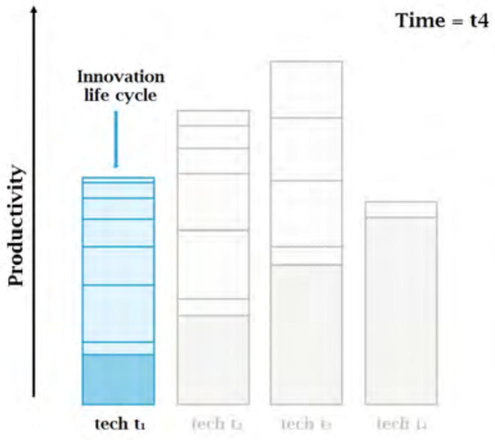

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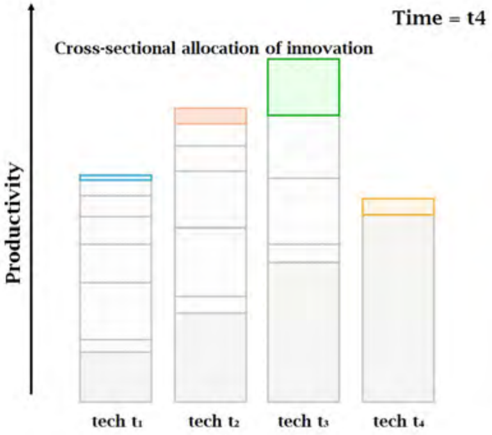

The economy

- Log preferences: $U = \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \ln C_t dt$
- Evolving set of intermediate varieties $\omega \in \Omega_t$.
- Single final-good sector: $Y_t = \frac{L^\beta}{1-\beta} \int_{\Omega_t} z(\omega) x_t(\omega)^{1-\beta} d\omega$.
- Each variety ω is produced under CRS using the final good.
  - Profits from holding the blueprint for ω: $\pi(\omega) \propto z(\omega)$.

Technology vintages

- Every period, one new technology vintage appears exogenously.
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- Newer vintages are intrinsically better: $A_\tau = e^{\gamma\tau}$.

γ: frontier growth rate

The perfection process of technologies

- $Q(t|\tau) :=$ average level of q, at time t, among varieties linked to technology τ.
- If a variety is created at time t linked to technology τ, its quality equals:

$$q = Q(t|\tau)\lambda, \text{ where } \lambda \sim H(\cdot) \text{ and } \bar{\lambda} \equiv \int \lambda dH(\lambda) \geq 1.$$

$\bar{\lambda}$: “building on the past"

Static Allocation

The Direction of Innovation

- Expected value of a successful innovation within technology τ:

$$\bar{v}(t|\tau) \equiv \mathbb{E}_\lambda \int_t^\infty \pi_t(\omega) e^{-\int_t^s r(v)dv} ds$$

- Research relative supply equation:

$$\frac{R(t|\tau)}{R(t|\tau')} = \left(\frac{\bar{v}(t|\tau)}{\bar{v}(t|\tau')}\right)^{\frac{1}{\epsilon}}$$

$R(t|\tau)$: researchers or funds targeting τ

- Consistent with several classes of models ... e.g.:
  - Congestion forces in research measured by ϵ.
  - Heterogeneous skills (dispersion measured by ϵ).
  - Lab equipment model with convex costs (degree of convexity: ϵ)

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Research allocation

- In equilibrium, *both on and off* the BGP:

$$R_t(a) = \left(\frac{e^{-\gamma a} Q_t(a)}{\bar{Q}_t}\right)^{\frac{1}{\epsilon}} R$$

where $\bar{Q}_t \equiv \left[\int_0^\infty \left(e^{-\gamma \tilde{a}} Q_t(\tilde{a})\right)^{\frac{1}{\epsilon}} d\tilde{a}\right]^\epsilon$ is an age-discounted aggregate quality index.

Notation:

age: $a = t - \tau$

$x_t(a) \equiv x(t|t-a)$

Laws of Motion

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Two competing forces:

obsolescence vs quality stock

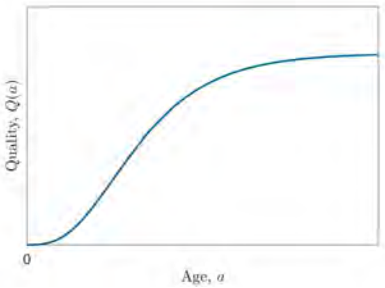
Laws of Motion

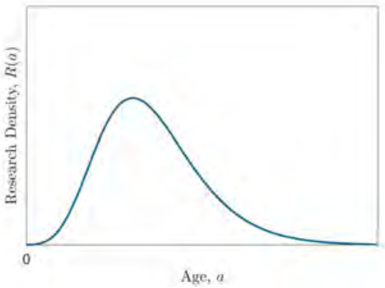
Long-run equilibrium

Proposition : Stationary Distributions

$$Q(a) = \left\{ c_1 - c_2 e^{-\frac{1-\epsilon}{\epsilon}\gamma a} \right\}^{\frac{1}{\frac{1}{\bar{\lambda}-1} - \frac{1-\epsilon}{\epsilon}}} \quad \text{and} \quad \mu(a) \propto Q(a)^{\frac{1}{\bar{\lambda}-1}}$$

c_1, c_2 are uniquely determined constants.





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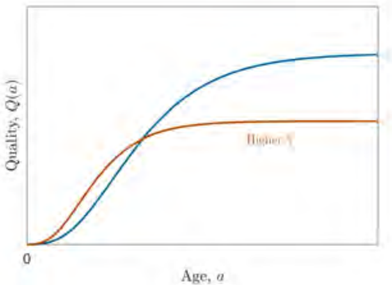

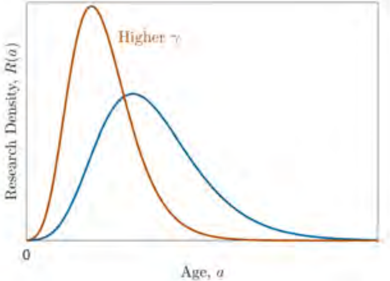

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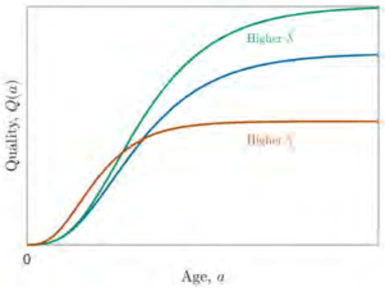

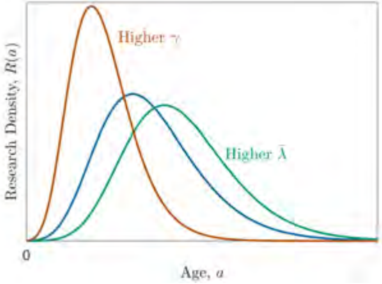

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Allocation of research solves the problem:

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$$\text{s.t} \int R(a) da \leq R$$

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Proposition : Optimal BGP allocation

Same objective function as the market ...

But internalizes:

- Law of motion for $Q(a)$:
  $\uparrow R(a) \rightarrow \uparrow Q(a')$ for $a' > a$.
  Building on giant shoulders ($\mathcal{E}_1$)
- Law of motion for $\mu(a)$:
  $\uparrow R(a) \rightarrow$ Ideas harder to find ($\mathcal{E}_2$).

Details

Too much or too little innovation in old technologies?

The optimal stationary allocation of innovation must satisfy for every $a \geq 0$:

$$R(a)^{\epsilon} \propto \underbrace{e^{-\gamma a} Q(a)}_{\text{decentralized equilibrium}} \kappa + \underbrace{\zeta_Q(a)\eta(\lambda - 1)\frac{Q(a)}{\mu(a)}}_{\mathcal{E}_1(a)} + \underbrace{\zeta_\mu(a)\eta}_{\mathcal{E}_2(a)}$$

Proposition

The optimal allocation of research features $R'(a) < 0$ and $\lim_{a \to \infty} R(a) = 0$

- Optimal allocation is not hump shaped.
- Instead: innovation biased towards new technologies.

Outline

- Theory
- Empirical Analysis and Calibration
- Quantitative Analysis

Strategy

1. Mapping model to data: Defining technologies and their date of origin:
   - Technologies = technological classification of the US patent office (USPC classes).
   - Date of origin: logistic estimation, as in Griliches (1957).
2. Calibration: new moments (universe of US patents 1836-2010)
   - Cross-section age distribution of innovation over time;
   - Age-vintage evolution of patent valuation.

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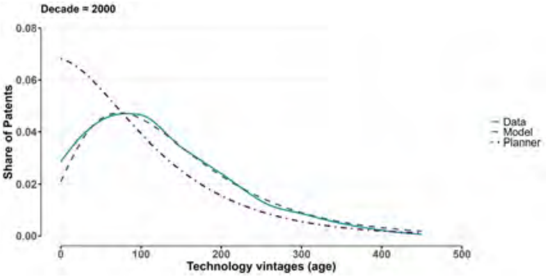
Optimal BGP research allocation



With the optimal research allocation

... consumption ↑ 30%

▸ Alternative Calibrations

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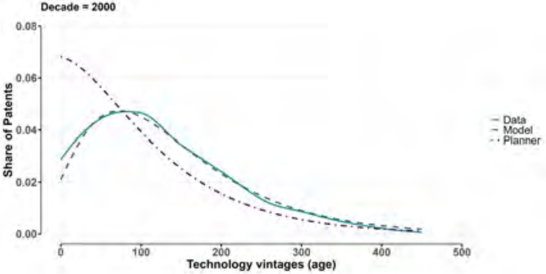

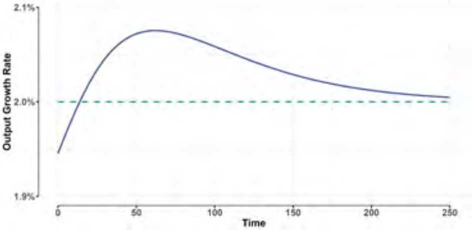

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Transition dynamics:

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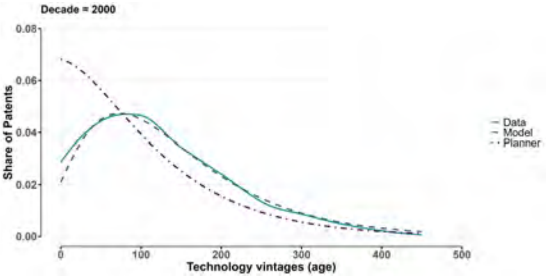

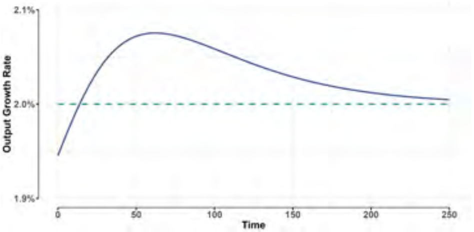

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Discount rate	$\rho = 0.01$	$\rho = 0.025$	$\rho = 0.05$
Welfare Gain	10%	3%	1%

Alternative Calibrations

Conclusion

- Novel growth model with endogenous cycles of rise and fall of technologies.
- Despite its unbounded state space, the model is tractable.
- Misallocation: distribution of R&D suboptimally biased towards old technologies.
- Drawing on two centuries of patent data...
  - The balance of innovation between new and old technologies has been stable over time.
- Quantitative model: sizable welfare gains from correcting the R&D misallocation.

Static allocation

- Let $\mu(t|\tau)$ denote the measure $\mu(\Omega(t|\tau))$ of varieties associated with technology τ.
- Static equilibrium:

$$Y_t = \frac{L}{1-\beta} \int_{-\infty}^{t} e^{\gamma\tau} Q(t|\tau) \mu(t|\tau) d\tau$$

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Transition dynamics

- State of the economy at time t: $\Gamma_t \equiv [Q_t(a), \mu_t(a)]_{a=0}^{\infty}$.
- Given the innovation possibilities frontier, the state variables evolve as:

$$\partial_t Q_t(a) = -\partial_a Q_t(a) + Q_t(a)(\bar{\lambda} - 1)\eta \frac{R_t(a)^{1-\epsilon}}{\mu_t(a)}$$

$$\partial_t \mu_t(a) = -\partial_a \mu_t(a) + \eta R_t(a)^{1-\epsilon}$$

- The transition dynamics can be easily computed numerically.
- A stationary solution is one in which $Q_t(a)$ and $\mu_t(a)$ do not depend on t.

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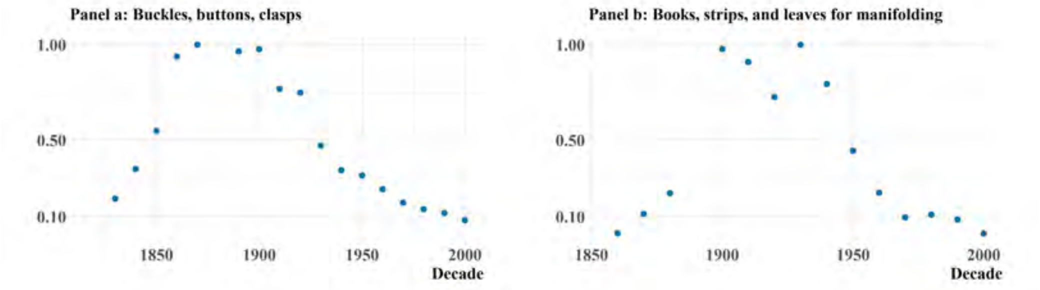
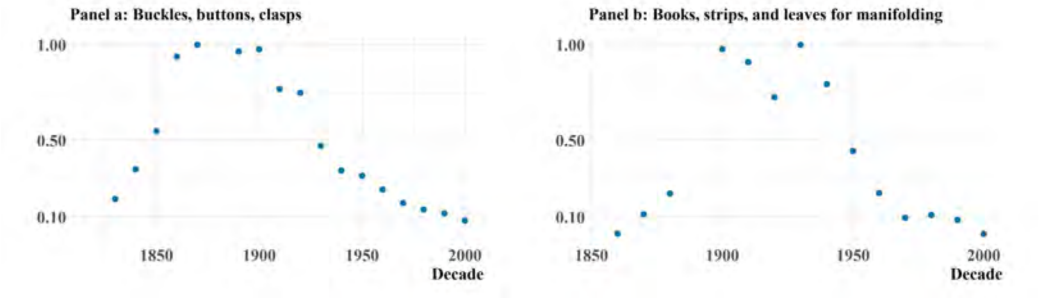
The diffusion of technologies in the *innovation* space

Figure: Normalized patenting share (1 = maximum share achieved)



The diffusion of technologies in the *innovation* space

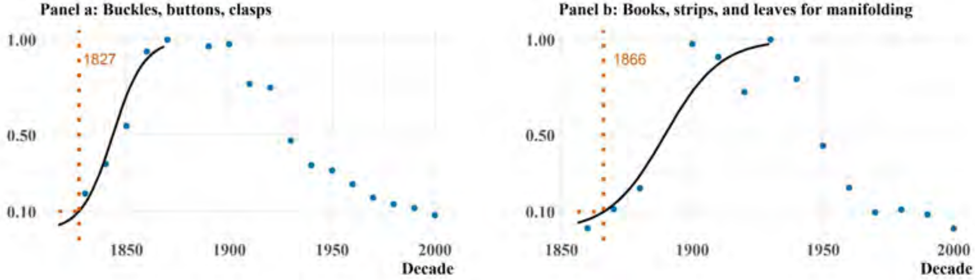
Figure: Normalized patenting share (1 = maximum share achieved)



Emergence date: logistic trend reaches 10% of its ceiling (based on Griliches, 1957).

The diffusion of technologies in the *innovation* space

Figure: Normalized patenting share (1 = maximum share achieved)



Emergence date: logistic trend reaches 10% of its ceiling (based on Griliches, 1957).

1. Additional results and robustness
2. Alternative measures for the emergence date of technologies:
   - Creation of new technology classes by the US Patent Office
   - First patents issued
3. Alternative technology definitions

Calibration: ϵ

- The observed number of patents is a (noisy) indicator of the number of new ideas:

$$Patents(t|\tau) = \kappa_\tau \times \dot{\mu}(t|\tau) \times u(t|\tau),$$

where κ_τ: patenting propensity in technology τ; and $u(t|\tau)$: scalar disturbance term.

- Use the model structure to substitute $\bar{v}(t|\tau)$ for $\dot{\mu}(t|\tau)$:

Expected (Average) value of a new patent

Back

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$$\log Patents(t|\tau) = \delta_\tau + \delta_t + \frac{1-\epsilon}{\epsilon} \log \bar{v}(t|\tau) + \log u(t|\tau).$$

where δ_τ and δ_t represent time and technology fixed effects.

Back

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$$\log Patents(t|\tau) = \delta_\tau + \delta_t + \frac{1-\epsilon}{\epsilon} \log \bar{v}(t|\tau) + \log u(t|\tau).$$

$\epsilon \approx 0.65$

Back

Calibration: $\bar{\lambda}$ and γ

- γ pins down the aggregate growth rate $\rightarrow \gamma = 2\%$.
- $\bar{\lambda}$ governs the accumulation of knowledge, $Q(a)$.

▸ Alternative approach ▸ Other parameters

Calibration: $\bar{\lambda}$ and γ

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In the model: $$\frac{Q_t(a)}{Q_t(0)} = e^{\gamma a} \frac{\bar{v}_t(a)}{\bar{v}_t(0)}$$

Alternative approach Other parameters

Calibration: $\bar{\lambda}$ and γ

- γ pins down the aggregate growth rate $\rightarrow \gamma = 2\%$.
- $\bar{\lambda}$ governs the accumulation of knowledge, $Q(a)$.

In the model: $$\frac{Q_t(a)}{Q_t(0)} = e^{\gamma a}\frac{\bar{v}_t(a)}{\bar{v}_t(0)}$$ $\longrightarrow$ Use Kogan et al (2017) data to back up the RHS.

- Choose $\bar{\lambda}$ to match the constructed moments $\{Q(a)/Q(0)\}$.

	$\log(Q(300)/Q(0))$	
$\bar{\lambda} = 2.34$	Model	Data
2.34	4.5	4.5

▸ Alternative approach ▸ Other parameters

Technology Paradigms

A technological paradigm is defined by three central characteristics:

- It is a body of knowledge related to “a ‘model’ and a ‘pattern’ of solution of selected technological problems, based on selected principles” (Dosi, 1982).
- Knowledge is cumulative within a technology: the body of knowledge is firmly based upon one or more past breakthroughs, and is extend through new research. (Thomas Khun, 1962; Mokyr, 1990)
- Exclusion effect: “the efforts and the technological imagination of engineers are focused in rather precise directions while they are, so to speak, "blind" with respect to other technological possibilities” (Dosi, 1962).

Back

Integration constants

The stationary quality-age schedule (distribution) at any given t is:

$$Q(a) = \left\{ c_1 - c_2 e^{-\frac{1-\epsilon}{\epsilon}\gamma a} \right\}^{\phi} \tag{1}$$

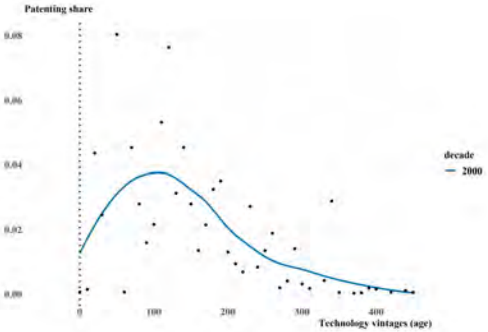
where

$$\phi \equiv \frac{1}{\frac{1}{\bar{\lambda}-1} - \frac{1-\epsilon}{\epsilon}}, \quad c_2 \equiv \frac{Q_0^{\frac{1}{\bar{\lambda}-1}}(\bar{\lambda}-1)\eta}{\mu_0 \phi \frac{1-\epsilon}{\epsilon} \gamma \bar{Q}^{\frac{1-\epsilon}{\epsilon}}}, \quad c_1 \equiv Q_0^{\frac{1}{\phi}} + c_2.$$

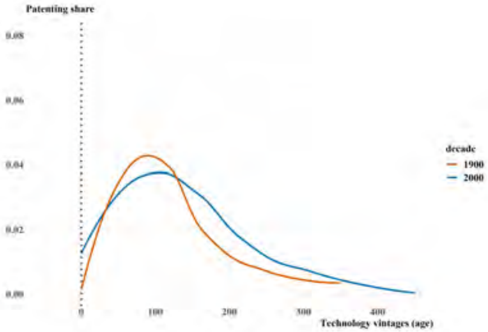
Back

Additional Figures

Patents weighted by their importance index (Kelly et al., 2021)



(a)

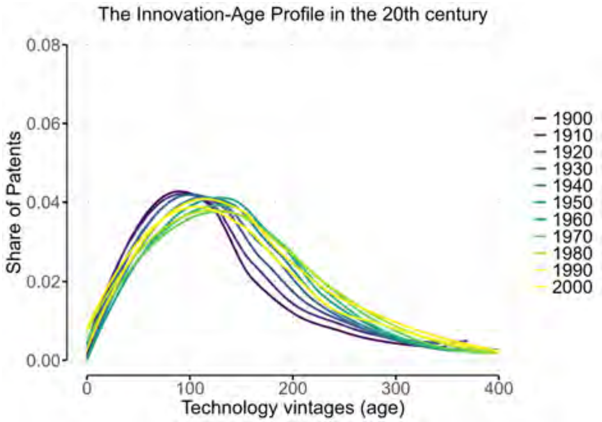


(b)

Back

Additional Figures

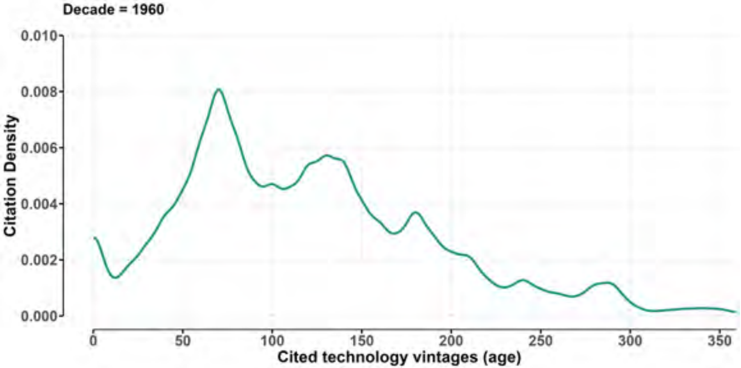
Figure: Patenting shares per technology vintages



▸ Back

Citation Distribution Across Technologies

- Technologies represent the technological classes of the USPTO.



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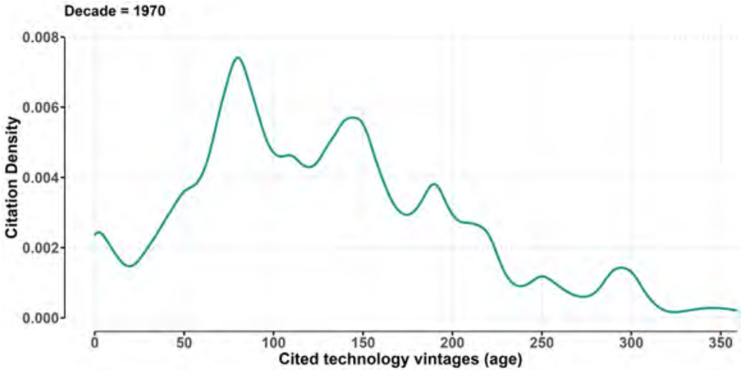

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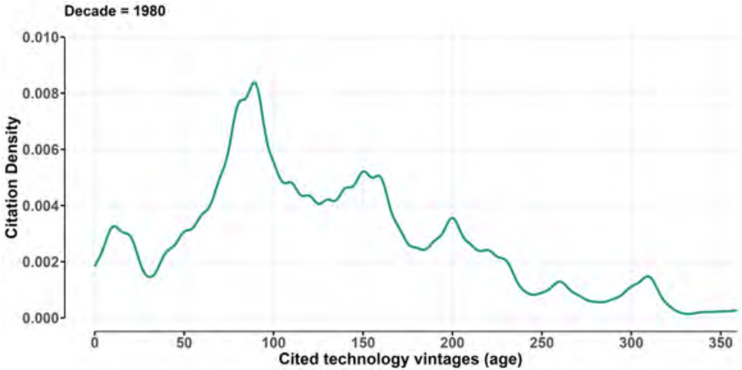

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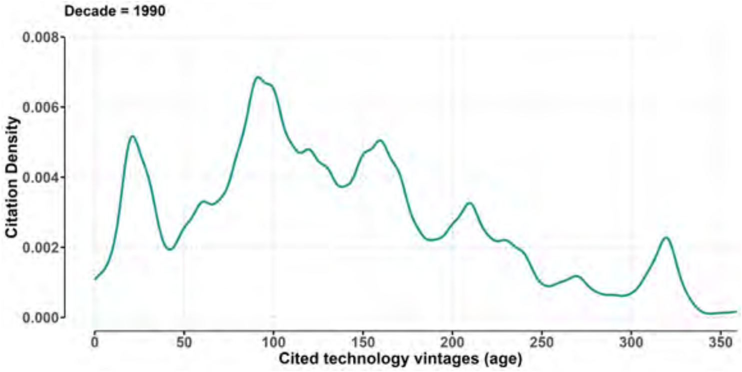

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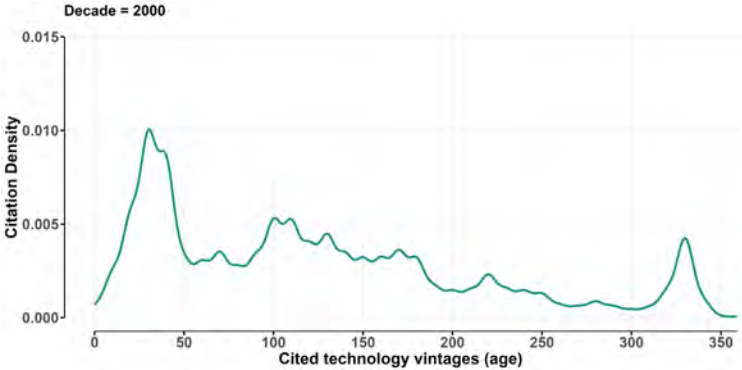

Alternative approach to define technologies

- Kelly et al. (2021) $\rightarrow$ Use text analysis to identify *breakthrough patents*.
  (e.g. Samuel Morse's telegraphy wire patent; Google's PageRank algorithm patent)

Table: The Hump-Shaped Innovation age Profile (Alternative Technology Definition)

		Share of Patents Building on Breakthroughs of Age:			
		[0,10)	[0,20)	50+	70+
Decade	1960	6%	31%	15%	6%
	1970	5%	22%	29%	8%
	1980	11%	26%	32%	11%
	1990	18%	43%	27%	14%
	2000	9%	54%	19%	10%

Notes: Breakthrough patents are identified by Kelly et al. (2021). All remaining patents are assigned to breakthroughs using the citation network (see the text of this session for details).

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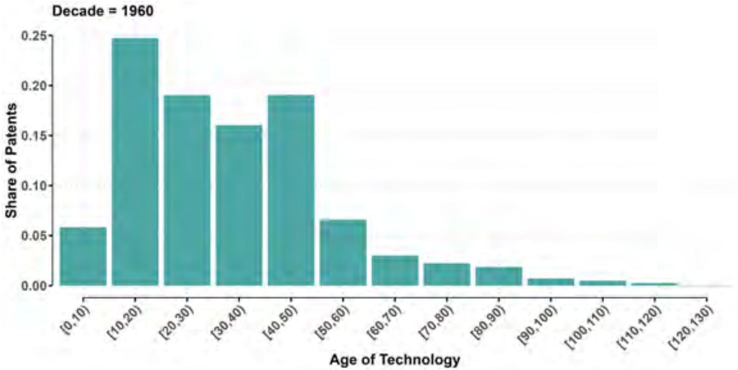
- In this exercise, I assume that a breakthrough patent defines a technology.



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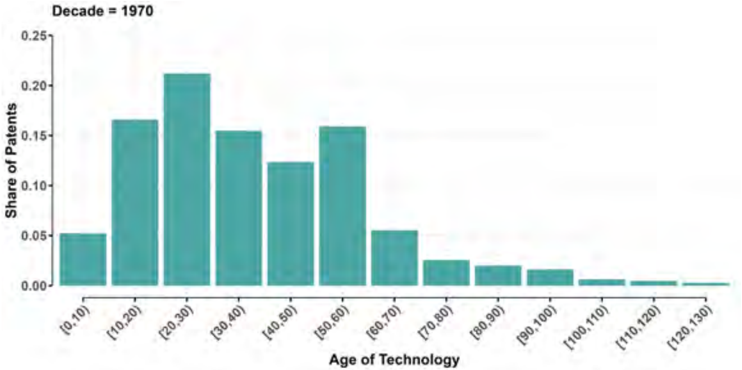

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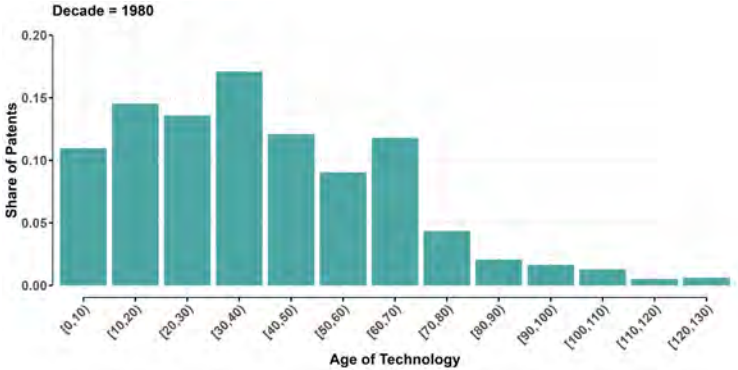

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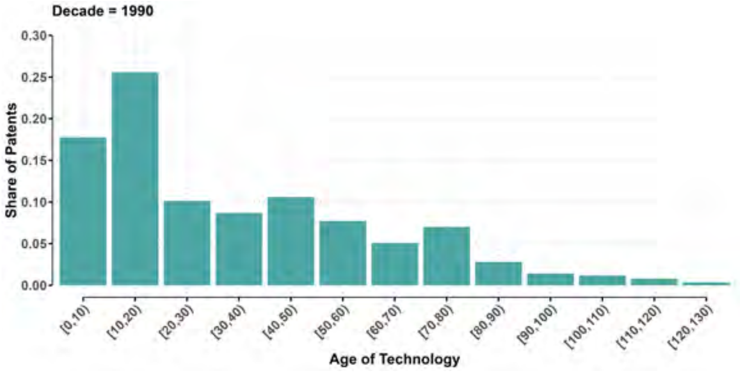

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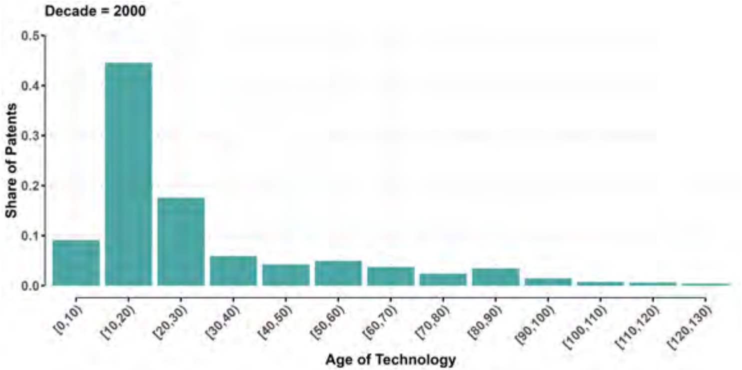
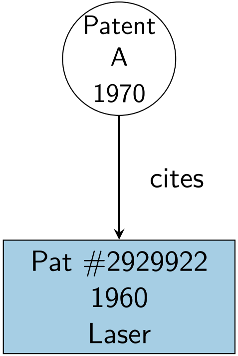
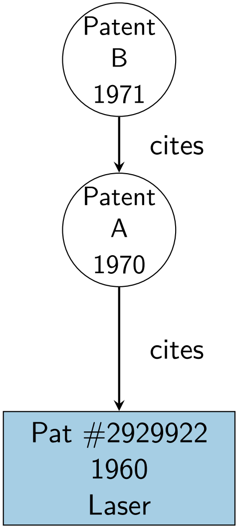


Pat #2929922
1960
Laser



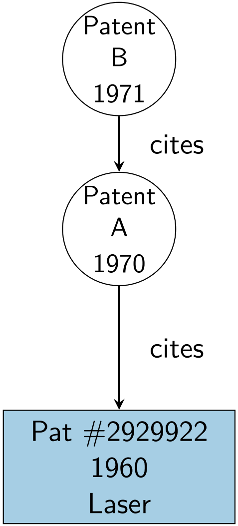
Patent A: linked to a 10 year old technology



Patent B: does **not cite any breakthrough**

... but cites A

B linked to a 11 year old technology



Patent B: does **not cite any breakthrough**

... but cites A

B linked to a 11 year old technology

For each patent, I find its *closest* breakthrough in the citation network

Additional Tables: Vintage Measure Validation

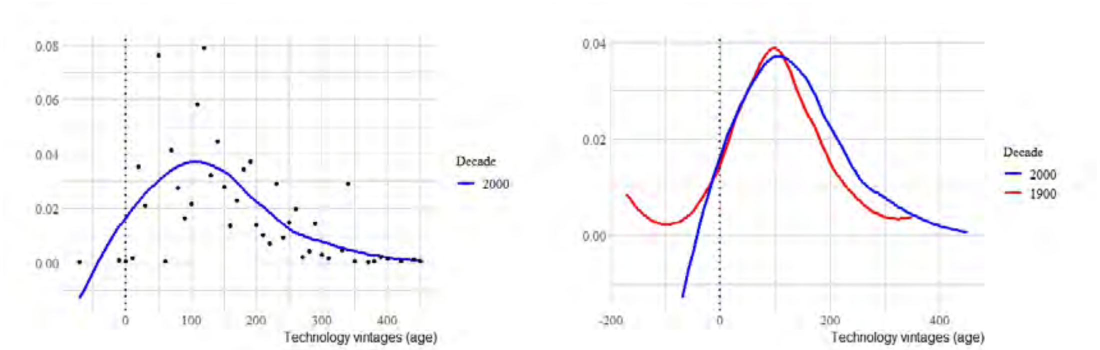
Dependent Variable:	$\mathbb{1}\{$*Patent is a Breakthrough* (*Kelly et al.*, 2021)$\}$		
Model:	(1)	(2)	(3)
Variables			
Constant	0.1739***		
	(0.0002)		
Vintage	-0.0006***	-0.0007***	-0.0007***
	(1.26×10^{-6})	(0.0001)	(0.0001)
Citations			0.0009***
			(8.12×10^{-5})
Fixed-effects			
Time		Yes	Yes
Fit statistics			
Observations	7,624,687	7,624,687	7,623,697
R^2	0.02888	0.05841	0.07281
Within R^2		0.03441	0.04917

*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Back Back

Additional Figures

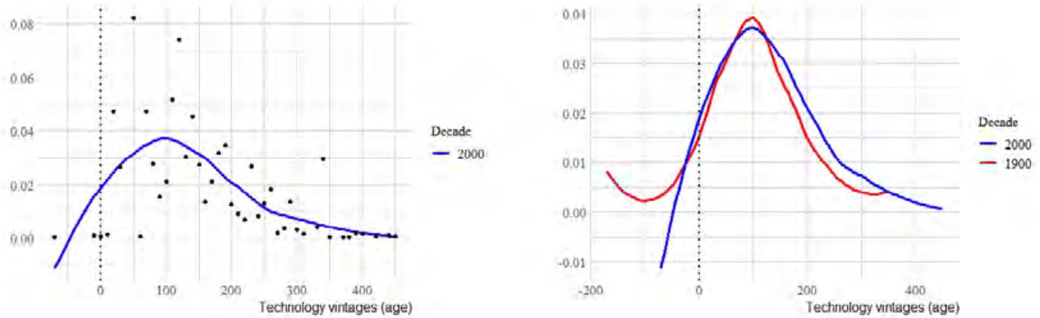
Figure: Patenting shares per technology vintages at different points in time



Notes: The age of each technology is estimated by a logistic regression as in Griliches (1957). The dots (in the left) represent the share accrued by each technology among all patents issued. The solid lines summarize the data in a smooth curve based on local regression models from Cleveland et al. (1992).

Additional Figures

Figure: Patenting shares per technology vintages at different points in time

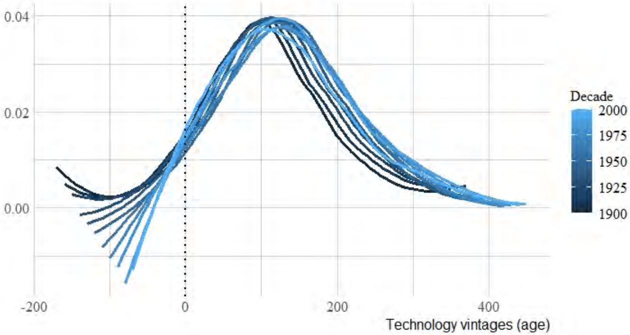


Patents weighted by their importance index (Kelly et al., 2021).

▸ Back

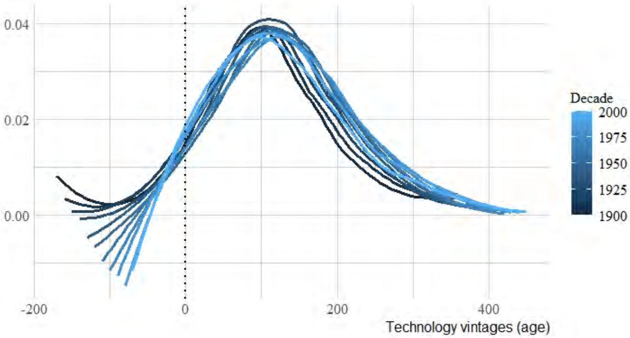
Additional Figures

Figure: Patenting shares per technology vintages



Notes: Each curve represents the allocation of innovation in a specific decade across vintages of different ages. The curves are computed as a smooth trend summarizing the data, using local regression methods as in Cleveland et al. (1992). The age of each technology is estimated by a logistic regression as in Griliches.

Figure: Patenting shares per technology vintages



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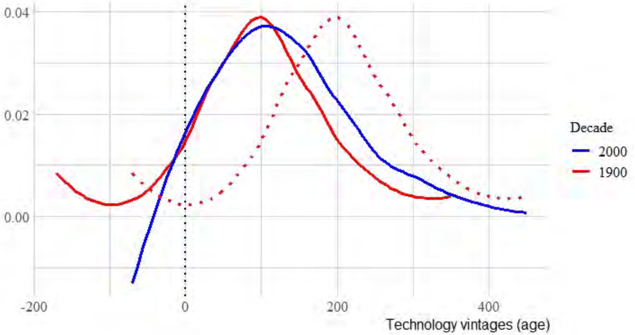
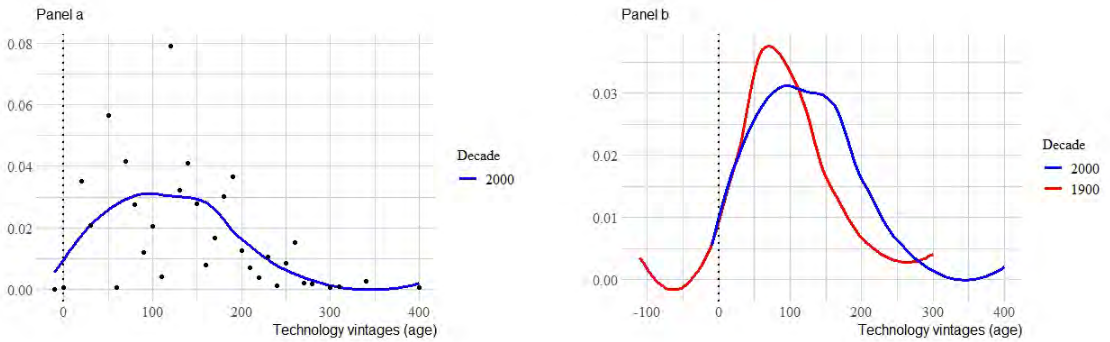


The dashed line represents the counterfactual scenario in which the innovation shares across technologies observed in 1900 would remain the same in 2000.

Back

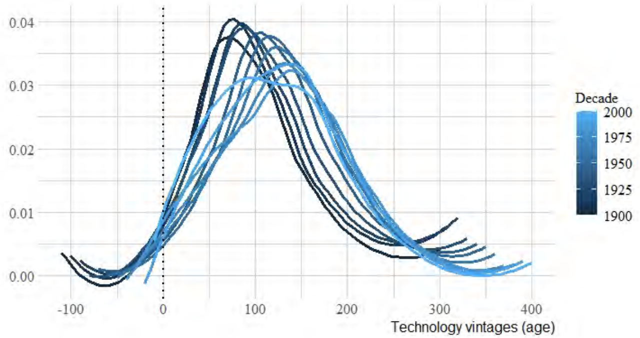
Additional Figures

Figure: Patenting shares per technology vintages



Only data for selected technologies: the ones whose logistic trend fit has $R^2 > 0.5$.

Figure: Patenting shares per technology vintages



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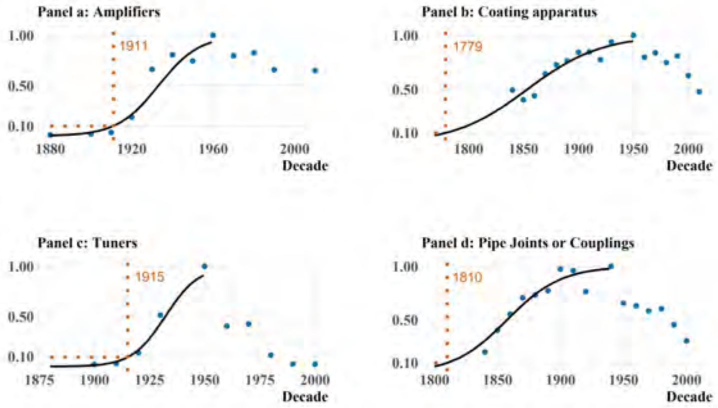
Back

Measuring the Emergence of New Technologies

Figure: Normalized patenting share (1 = maximum share achieved by the respective technology)



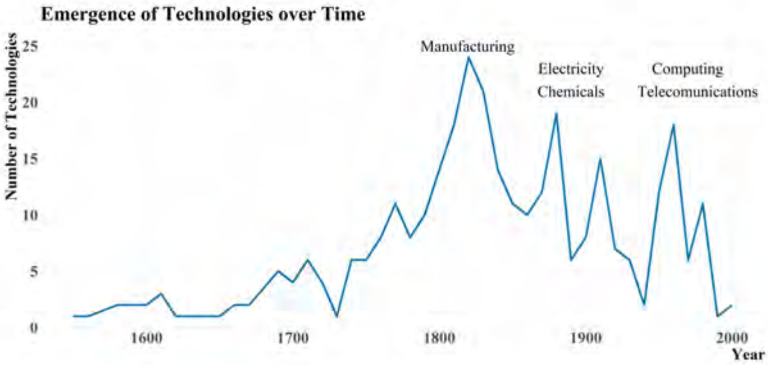
Back

Table: The emergence of key technologies

Technology class	Origin	Patenting share (%)	
		1950s	2000s
Gas: heating and illumination	1766	0.14	0.02
Aeronautics	1871	0.55	0.22
Railway draft appliances	1826	0.10	0.00
Electric lamp	1878	0.06	0.08
Television	1929	0.36	0.81
Semiconductor device manufacturing	1964	0.05	3.07
Data Processing: artificial intelligence	1968	0.00	0.15
Software development, and installation	1985	0.00	0.34
Information security	1987	0.00	0.25

Back

Measuring the Emergence of New Technologies: Results



Back

- Model the patenting share growth of a technology with the logistic function:

$$share(t) = \frac{K}{1 + e^{-(\alpha+\beta t)}}$$

where K is the ceiling value (maximum share achieved by a technology)

- Transform it:

$$\log\left(\frac{share(t)}{K - share(t)}\right) = \alpha + \beta t$$

- Recover $\hat{\alpha}$ and $\hat{\beta}$ from OLS.

Back

Additional details

$$\delta_\tau = \log \kappa_\tau + \log \eta - \frac{1-\epsilon}{\epsilon} \log(\beta \bar{\lambda})$$

$$\delta_t = -\frac{1-\epsilon}{\epsilon} \gamma t - \frac{1-\epsilon}{\epsilon} \log(D_t \bar{Q}_t).$$

Back

Calibration: ϵ

$$\log Patents(t|\tau) = \delta_\tau + \delta_t + \frac{1-\epsilon}{\epsilon} \log \bar{v}(t|\tau) + \log u(t|\tau).$$

Table: Estimates for the research supply elasticity ϵ

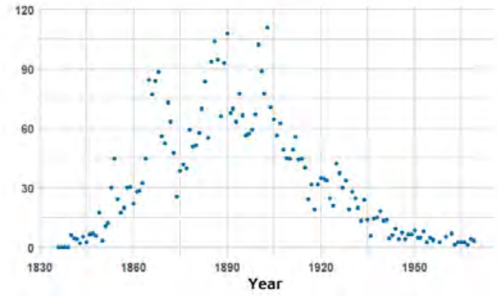
	New patents: $\log Patents(t\|\tau)$	
	(1)	(2)
Avg Patent Value $\log \bar{v}(t\|\tau)$	0.381	0.690
	(0.190)	(0.170)
	$\epsilon = 0.724$	$\epsilon = 0.591$
Year FE	✓	✓
Tech FE	✓	✓
N	205	187
F-Stat	4.028	16.44

$\bar{v}(t|\tau)$ = Kogan et al (2017) valuation measure for new patents in period t (column 1) or $t-1$ (column 2).

Back

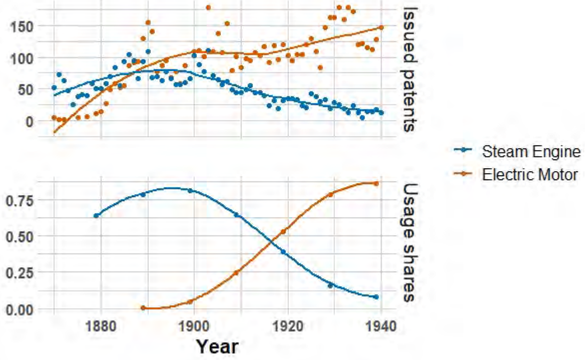
Additional Figures

Figure: Issued patents per year (Steam Engine)



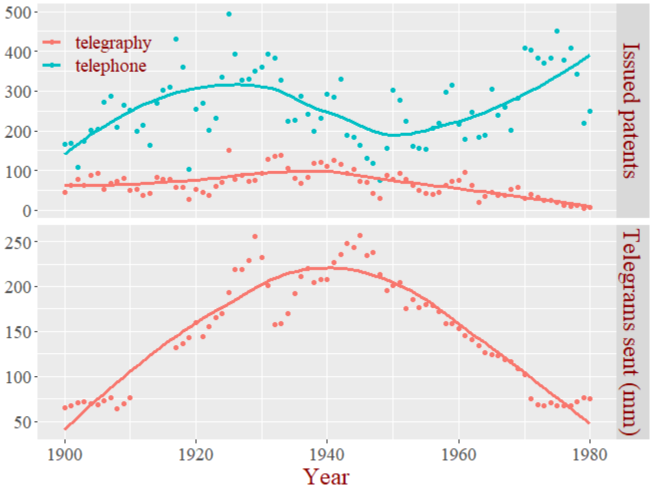
Back

Additional Figures

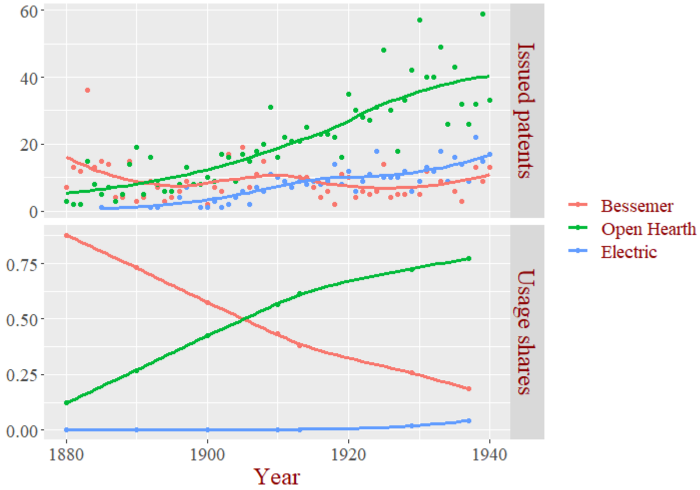


Back

Additional Figures

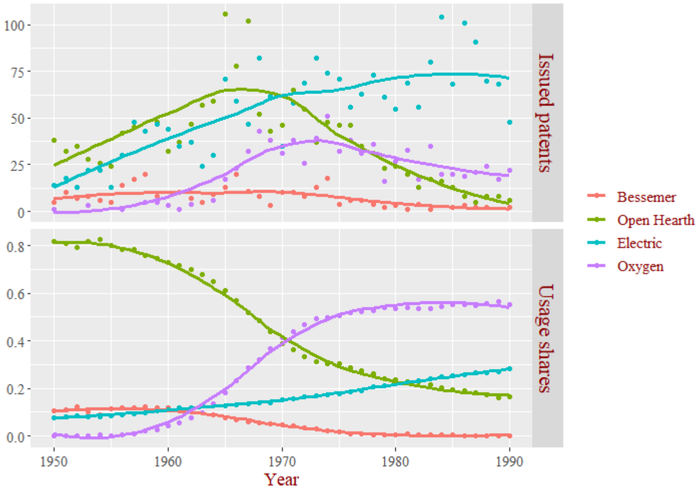


Back


60
40
20
0
0.75
0.50
0.25
0.00
1880
1900
1920
1940
Year
Issued patents
Usage shares
Bessemer
Open Hearth
Electric

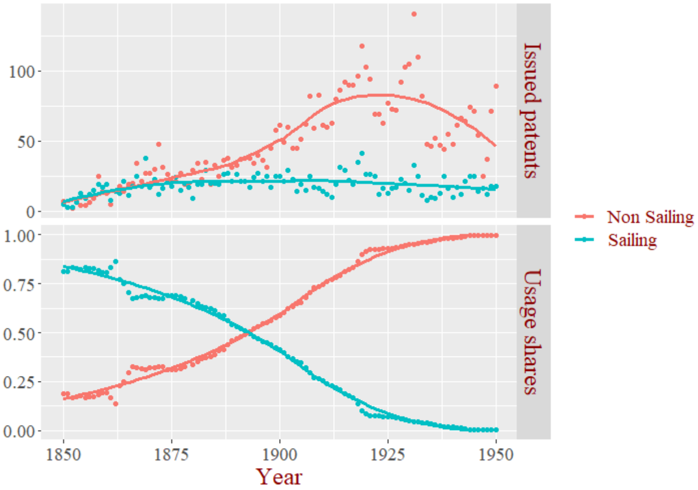
Back

Additional Figures



▸ Back

Additional Figures



Back

Additional Figures

▸ Back

Usage Shares

$s(t|\tau)$: total expenditure on varieties linked to technology τ divided by the aggregate expenditure, $P_t Y_t$:

$$s(t|\tau) \propto \frac{e^{\gamma\tau} Q(t|\tau)\mu(t|\tau)}{Y_t}$$

Relative expenditure shares, $s_t(\cdot)$, on technologies a' and a'':

$$\log s_t(a') - \log s_t(a'') = -\gamma(a' - a'') + [\log Q_t(a') - \log Q_t(a'')] + [\log \mu_t(a') - \log \mu_t(a'')]$$

▸ Back

Too much or too little innovation in old technologies?

$$\underset{[R(a)]_{a>0}}{\text{Max}} \; \hat{C} = \kappa \int_0^\infty e^{-\gamma a} Q(a) R(a)^{1-\epsilon} da$$

s.t.

$$Q'(a) = Q(a)(\lambda - 1)\frac{\eta}{1-\epsilon}\frac{R(a)^{1-\epsilon}}{\mu(a)} \qquad \text{(‘Giants Shoulders’ } \mathcal{E}_1\text{)}$$

$$\mu'(a) = \frac{\eta}{1-\epsilon} R(a)^{1-\epsilon} \qquad \text{(‘Dilution’ } \mathcal{E}_2\text{)}$$

$$R = \int_0^\infty R(a) da$$

Back

Too much or too little innovation in old technologies?

The optimal stationary allocation of innovation must satisfy for every $a \geq 0$:

$$R(a)^{\epsilon} \propto \underbrace{e^{-\gamma a} Q(a)}_{\text{decentralized equilibrium}} \kappa + \underbrace{\zeta_Q(a)\eta(\lambda - 1)\frac{Q(a)}{\mu(a)}}_{\mathcal{E}_1(a)} + \underbrace{\zeta_\mu(a)\eta}_{\mathcal{E}_2(a)}$$

where $\zeta_Q(a)$ and $\zeta_\mu(a)$ are the coestate functions.

1. If $\lambda = 1$, the decentralized equilibrium path is optimal.
2. For every a: $\mathcal{E}_1(a) > 0$ and $\mathcal{E}_1'(a) < 0$.
3. For every a: $\mathcal{E}_2(a) < 0$ and $\mathcal{E}_2'(a) > 0$.
4. The optimal allocation of research features $R'(a) < 0$ and $\lim_{a \to \infty} R(a) = 0$

Back

The main inneficiency

'Standing on the shoulders of giants':

$$\mathcal{E}_1(a) > 0 \quad \forall a \geq 0$$

Innovation *social* value $>$ *private* value

This gap is *relatively* higher for new technologies:

$$\mathcal{E}_1'(a) < 0 \quad \forall a \geq 0$$

$\rightarrow$ *R&D* misallocation Back

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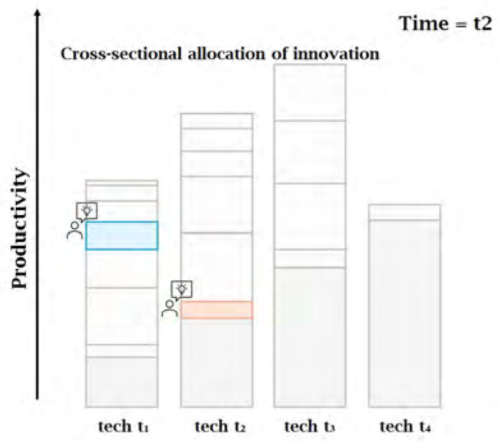
→ *R&D* misallocation ▸ Back



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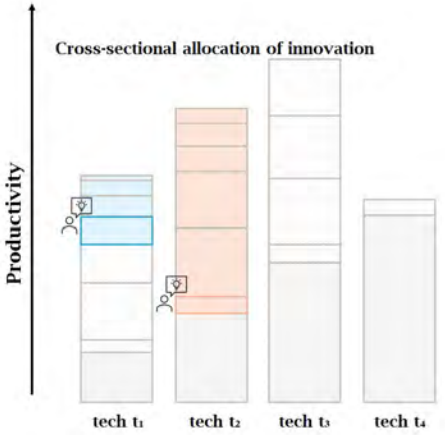
$\rightarrow$ *R&D* misallocation Back



The main ineficiency

'Standing on the shoulders of giants':

$$\mathcal{E}_1(a) > 0 \quad \forall a \geq 0$$

Innovation *social* value > *private* value

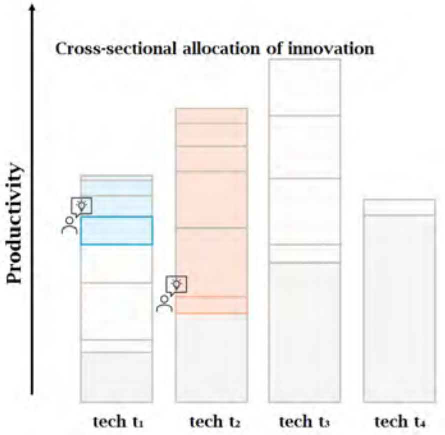
This gap is *relatively* higher for new technologies:

$$\mathcal{E}_1'(a) < 0 \quad \forall a \geq 0$$

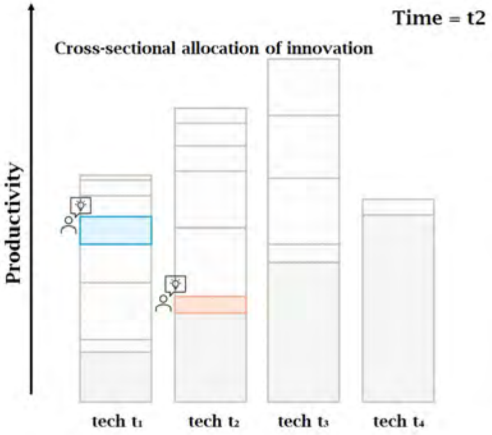
→ *R&D* misallocation ▸ Back

The multiplier on $Q(a)$ can be written as:

$$\zeta_Q(a) \approx \int_a^\infty e^{-\gamma s} \left(Q(s)/Q(a)\right) R(s)^{1-\epsilon} ds$$



The economy: an illustration
Technology vintages across time



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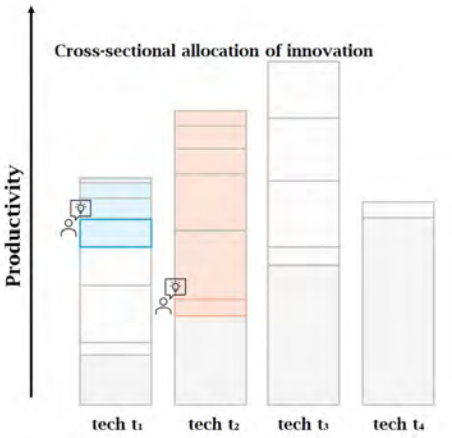
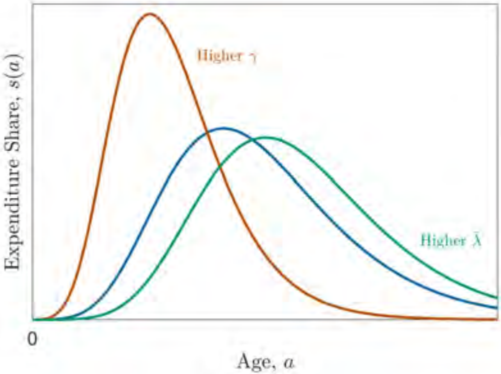
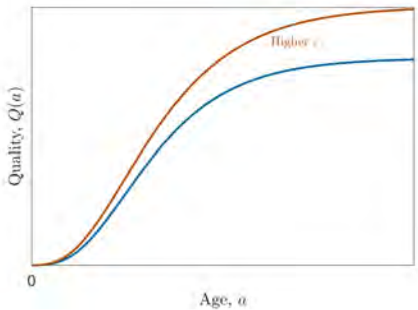


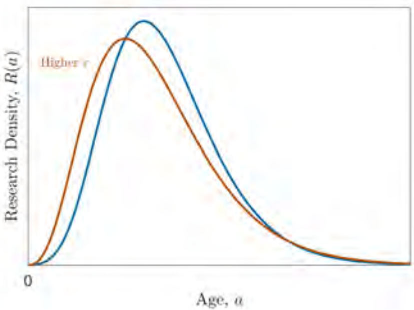
Additional Model Figures



Back

Additional Model Figures





Back

Corollary (Innovation in the frontier)

For any vector of parameters, there exists $\underline{b} > 0$ *(where* $\underline{b}$ *is a function of the parameter vector) such that, if* $\left(\bar{\lambda} - 1\right)/\gamma < \bar{b}$*, then* $R'(0) < 0$*, whereas, if* $\left(\bar{\lambda} - 1\right)/\gamma \geq \bar{b}$*, then* $R'(0) \geq 0$*.*

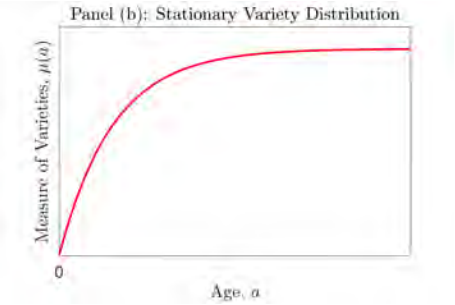
Back

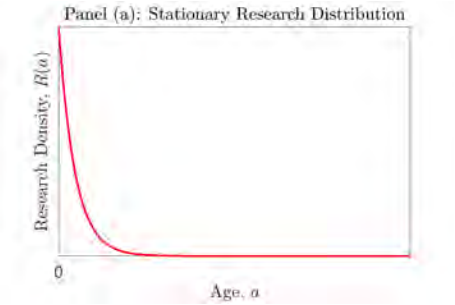
Case with $\bar{\lambda} = 1$

Proposition : Stationary Distributions if $\bar{\lambda} = 1$

$$\mu(a) = c_3 - c_4 e^{-\frac{1-\epsilon}{\epsilon}\gamma a} \quad \text{and} \quad Q(a) = Q_0$$

where c_3, c_4 are uniquely determined constants.





Back

Calibration - all parameters

Table: Baseline Calibration

Parameter	Description	Value
γ	Frontier growth rate	2%
$\bar{\lambda}$	Knowledge spillover	2.34
ϵ	Research elasticity	0.65
β	Labor share	0.66
ρ	Discount rate	5%
η	Research productivity	1
Q_0	Initial average quality	1
μ_0	Initial measure of varieties	1

Back

Implementing the Optimal BGP research allocation

Long-Run Consumption Differences

$\bar{\lambda}$	$\frac{\hat{C}_{\text{planner}}}{\hat{C}_{\text{model}}}$	ϵ	$\frac{\hat{C}_{\text{planner}}}{\hat{C}_{\text{model}}}$
2.7	1.60	0.60	1.34
2.0	1.13	0.70	1.24
1.5	1.03	0.80	1.13

Baseline calibration: $\bar{\lambda} = 2.34$, $\epsilon = 0.65$

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Welfare Gains along the Transition Dynamics

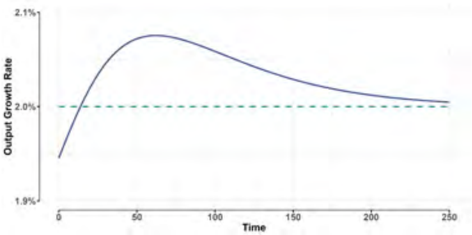
	$\epsilon = 0.8$	$\epsilon = 0.7$	$\epsilon = 0.6$
Welfare Gain	1%	3%	4%
	$\bar{\lambda} = 1.5$	$\bar{\lambda} = 2.0$	$\bar{\lambda} = 2.7$
Welfare Gain	1%	2%	5%

Back

A simple policy experiment

$$R_t(a) = (1-\alpha)R_t^{\text{private}}(a) + \alpha R(a)^{\text{planner}}$$

$\Rightarrow$ Gradually increase (decrease) research where the market underinvest (overinvest).

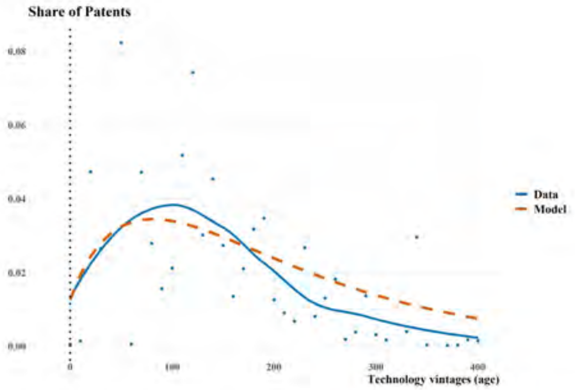


Transition path with $\alpha = 0.6$.

Discount rate	$\rho = 0.01$	$\rho = 0.025$	$\rho = 0.05$
Welfare Gain	10%	3%	1%

Back

Model Fit



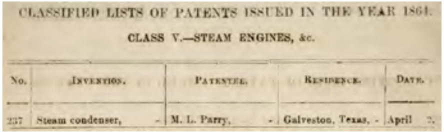
$$\epsilon = 0.75$$

▸ Back

US Patent Office classification documents: a rich picture of technological progress

- 1830s: the US Congress issued its first classification scheme.

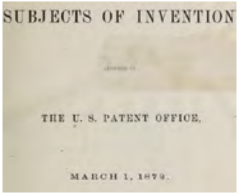
  e.g.: "Wheel carriages", "Steam and Gas Engine".



CLASSIFIED LISTS OF PATENTS ISSUED IN THE YEAR 1864.

CLASS V.—STEAM ENGINES, &c.

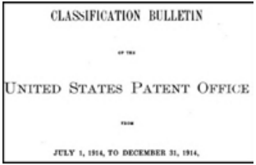
No.	Invention.	Patentee.	Residence.	Date.
237	Steam condenser,	M. L. Parry,	Galveston, Texas,	April 2,



SUBJECTS OF INVENTION

THE U. S. PATENT OFFICE.

MARCH 1, 1879.



CLASSIFICATION BULLETIN

OF THE

UNITED STATES PATENT OFFICE

FROM

JULY 1, 1914, TO DECEMBER 31, 1914.

- 1870s: the number of classes increased from 36 to 145.

  e.g.: "Railways", "Photography".

- 1898: 'Classification Division' is established; birth of the modern classification code.

- Class schedule: "periodically amended to accommodate new technologies" (USPTO,2012).

Tracking the emergence of new technology classes

New dataset: evolution of the technology classification at the USPTO. Sources:

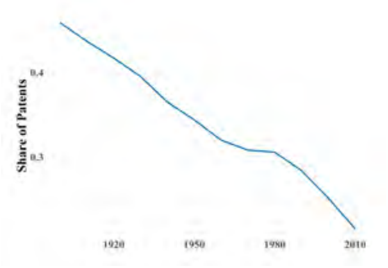
- Report of the Secretary of State to Congress (1839), Classification Index of Subjects of Invention (1872,1895), The Official Gazette (1878,1912), Manual of Classification of Subjects of Invention (1912, 1916, 1920, 1923, 1947), Classifications Bulleting (1912-1945), Index to the US Patent System (1980,1987); Official USPC class creation dates (2023).

- Emergence date of a technology = when it is first introduced in the USPTO records.
- Problem: a newly introduced technology may be simply a relabel of previous classes.
- Solution: a new technology must have low cosine similarity to previous used terms.

Age of technologies and innovation

Old technologies = technology classes traced back to the 19th century USPTO documents.

Innovation in Old Technologies

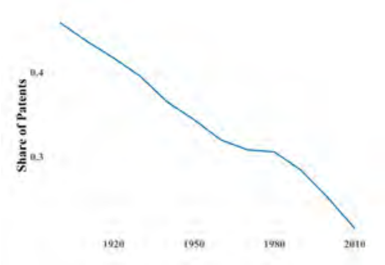


Old technologies = technology classes traced back to the 19th century USPTO documents.

For the remaining technologies, estimate:

$$PatentShare_{tech,t} = \gamma_{tech} + \gamma_t + \sum_k \beta_k \mathbf{1}\{Age_{tech,t} = k\} + \epsilon_{tech,t}$$

Innovation in Old Technologies



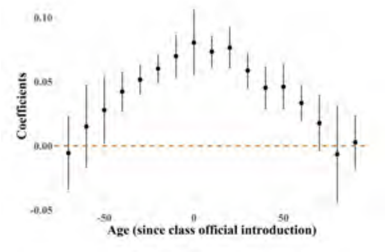
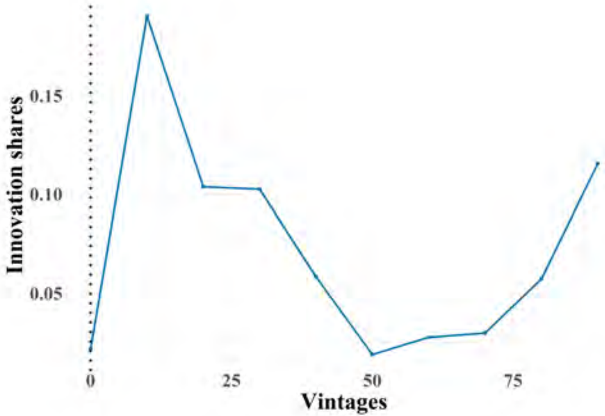
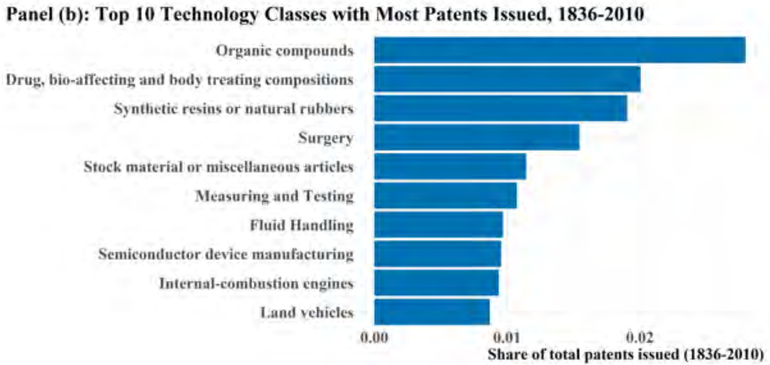


Figure: Innovation shares in the 2000 decade



USPC Technology classes



▸ Back

Model environment

- Intermediate varieties at time t: Ω_t
- Each $\omega \in \Omega_t$ is linked to one technology $\tau \in \mathcal{T}$ and has its its own quality $q \in \mathcal{Q}$.
- We have thus a total measure $\mu(\Omega_t)$ of varieties defined on $\mathcal{T} \times \mathcal{Q}$.
- Density of this measure: $f_t(\tau, q)$.

Technology-specific measure:

$$\mu(t|\tau) = \int_{\mathcal{Q}} f_t(\tau, q) dq$$

Technology average quality:

$$Q(t|\tau) = \frac{\int_{\mathcal{Q}} q f_t(\tau, q) dq}{\mu(t|\tau)}$$

Static Allocation

- Final good defintion:

$$Y_t = \frac{L^\beta}{1-\beta} \int_{\Omega_t} z(\omega) x_t(\omega)^{1-\beta} d\omega.$$

- Plugging in household demand for $x(\omega)$ + monopolist pricing of $p(\omega)$ gives

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$$Y_t = \frac{L}{1-\beta} \int_{\Omega_t} z(\omega) d\omega.$$

- Since all varieties are symmetric except for τ and q:

$$\begin{aligned} Y_t &= \frac{L}{1-\beta} \int_{-\infty}^{t} \int_{0}^{\infty} z(\tau, q) f(\tau, q) dq d\tau \\ &= \frac{L}{1-\beta} \int_{-\infty}^{t} e^{\gamma\tau} Q(t|\tau) \mu(t|\tau) d\tau \end{aligned}$$

Environment and definitions

- Every period, one new technology vintage appears exogenously.
- At period t, technologies $\tau \in (-\infty, t]$ are known.
- Therefore, the technology set is the real line $\mathcal{T} = \mathbb{R}$,
- ... and the support of f_t sastisfies $\text{supp}(f_t) \subseteq (-\infty, t] \times \mathcal{Q}$.

$\bar{\lambda}$ calibration: Growth rate for the value of newly issued patents

Calibrate $\bar{\lambda}$ so that the model BGP is consistent with ...

- the average growth rate, within vintage, of new patents' value $\rightarrow \bar{v}(t|\tau)$.
- In the theory:

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$$\frac{d}{dt}\left(\mathbb{E}_{\tau} \log \bar{v}(t|\tau) - \log D_t\right) = \frac{d}{dt}\mathbb{E}_{\tau} \log Q(t|\tau)$$

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Proxied by $\hat{\alpha}$ from the fixed effects model: $\log \bar{v}(t|\tau) = \alpha t + u_\tau + \nu(t|\tau)$

$D_t \propto$ stock market capital to GDP ratio

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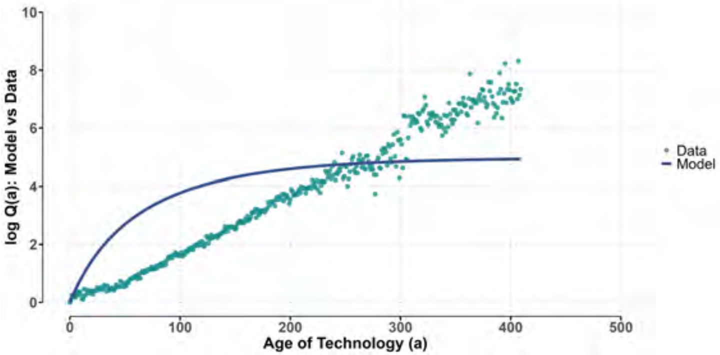
$D_t \propto$ stock market capital to GDP ratio

		Avg Growth in $Q(t\|\tau)$	
ϵ	$\bar{\lambda}$	Data	Model
0.65	2.445	0.175%	0.175%

Back

Calibrating λ



- $Q(a)$ in the data is backed up from patent valuation data from Kogan et al (2017).

Back

The Direction of Innovation

- Fixed mass R of researchers; If targeting τ, an idea arrives to a researcher at rate:

$$\mathcal{I}(t|\tau) = \eta R(t|\tau)^{-\epsilon}$$

where $R(t|\tau) =$ mass of reseachers targeting τ while ϵ parametrizes congestion.

- Aggregate innovation rate in τ: $\eta R(t|\tau)^{1-\epsilon}$ Back