Taylor projection under tail risk

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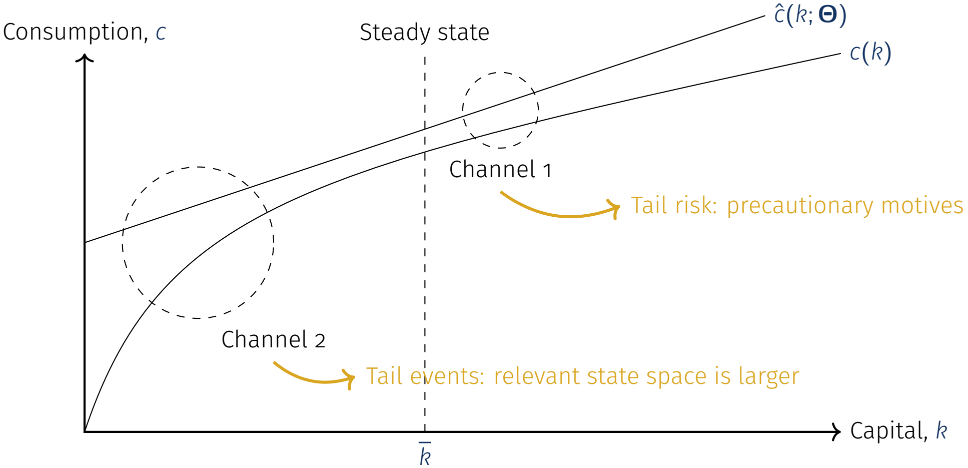
How to (accurately and efficiently) approximate the solution to stochastic dynamic models with tail events i.e., low probability but large shocks?

Examples include (but are not limited to):

- Rare disasters (e.g., Barro, 2009)
- Climate change (e.g., van den Bremer and van der Ploeg, 2021)
- Natural disasters (e.g., Douenne, 2020)
- Pandemics (e.g., Hong, Wang, and Yang, 2021)
- Wars (e.g., Federle, Meier, Müller, Mutschler, and Schularick, 2024)
- Rare booms (e.g., Bekaert and Engstrom, 2017 and Tsai and Wachter, 2015)

Challenging task! Tail events affect the degree of non-linearities through two channels:



Tail risk: precautionary motives

Tail events: relevant state space is larger

Contribution

A framework to approximate the solution of continuous-time (CT) dynamic stochastic models with tail events and tail risk (jump-diffusion)

- A CT Taylor projection (TP) algorithm (Levintal, 2018)
- Proof of convergence as the order of approximation goes to infinity
- Test TP, conditionally and unconditionally, in the Gourio (2012) economy
  - RBC with EZ preferences, time-varying disaster risk, adjustment costs of capital
  - Slow unfolding and recovery from disasters (Gourio, 2008; Nakamura et al., 2013)
- Propose globalized Taylor projection (GTP) that solves issues of
  - Inaccuracies in economies exposed unconditionally to rare disasters
  - Instabilities arising from explosive paths caused by disasters

Outline

- Framework
- Taylor projection
- An illustration: The Ramsey-Cass-Koopmans model
- An RBC model with time-varying disaster risk
- Accuracy
- Globalized Taylor projection
- Conclusion

Framework

We consider a family of continuous-time dynamic stochastic models where the rational expectations equilibrium can be summarized by the following system of equations

$$\begin{aligned} \mathbf{0} &= \mathcal{H}\left(\mathbf{x}, \mathbf{y}, \mathbf{y_x}, \mathbf{y_{xx}}\right) && (1) \\ \mathrm{d}\mathbf{x} &= \mathbf{b}\left(\mathbf{x}, \mathbf{y}\right)\mathrm{d}t + \boldsymbol{\sigma}\left(\mathbf{x}, \mathbf{y}\right)\mathrm{d}\mathbf{w} + \mathbf{f}\left(\mathbf{x}_{-}, \mathbf{y}_{-}\right)\mathrm{d}\mathbf{N}, && (2) \\ \mathbf{y} &= \mathbf{g}\left(\mathbf{x}\right), && (3) \end{aligned}$$

where:

- $\mathcal{H}$: a (deterministic) system of functional equations (second-order PDEs) that collects all the equilibrium conditions
- $\mathbf{x}$: vector of state variables (endogenous and exogenous)
- $\mathbf{y}$: vector of control variables with derivatives $\mathbf{y_x}$ and $\mathbf{y_{xx}}$ wrt the states
- $\mathbf{w}$: vector of independent Brownian motions
- $\mathbf{N}$: vector of stochastically independent Poisson processes with arrival rate vector $\boldsymbol{\lambda}$ and jump size $\mathbf{f}(\cdot)$
- $\mathbf{g}\left(\cdot\right)$: unknown vector of policy functions that solves (1) and (2)

Taylor projection

Substituting the unknown solution (3) into the operator (1) yields the new functional operator

$$\mathbf{F}(\mathbf{x}) := \mathcal{H}\left(\mathbf{x}, \mathbf{g}(\mathbf{x}), \mathbf{g}_{\mathbf{x}}(\mathbf{x}), \mathbf{g}_{\mathbf{xx}}(\mathbf{x})\right) = \mathbf{0} \tag{4}$$

Taylor projection of order h approximates $\mathbf{g}(\mathbf{x})$ in (4) by polynomial power expansions at an arbitrary point $\bar{\mathbf{x}}$

$$\widehat{\mathbf{g}}(\mathbf{x}; \mathbf{\Theta}) = \sum_{i=0}^{h} \widehat{\mathbf{G}}_{\mathbf{x}^i} (\mathbf{x} - \bar{\mathbf{x}})^{\otimes [i]} \tag{5}$$

where $\mathbf{\Theta}$ is a collection of non-repeated coefficients of $\widehat{\mathbf{G}}_{\mathbf{x}^i}$. Replacing $\mathbf{g}(\mathbf{x})$ with (5) in the equilibrium conditions (4) yields

$$\widehat{\mathbf{F}}(\mathbf{x}; \mathbf{\Theta}) = \mathcal{H}\left(\mathbf{x}, \widehat{\mathbf{g}}(\mathbf{x}; \mathbf{\Theta}), \widehat{\mathbf{g}}_{\mathbf{x}}(\mathbf{x}; \mathbf{\Theta}), \widehat{\mathbf{g}}_{\mathbf{xx}}(\mathbf{x}; \mathbf{\Theta})\right) = \mathbf{0} \tag{6}$$

Goal: find the values of $\mathbf{\Theta}$ that satisfy (6) around the neighborhood of $\bar{\mathbf{x}}$

This can be achieved by successively computing the first h derivatives of the functional equation (6) with respect to $\mathbf{x}$ and evaluating them at $\bar{\mathbf{x}}$. Collect derivatives in residual function $\mathcal{R}(\boldsymbol{\Theta}, \bar{\mathbf{x}})$:

$$\mathcal{R}(\boldsymbol{\Theta}, \bar{\mathbf{x}}) = \begin{bmatrix} \widehat{\mathbf{F}}(\bar{\mathbf{x}}; \boldsymbol{\Theta}) \\ \operatorname{vec}\left(\widehat{\mathbf{F}}_{\mathbf{x}}(\bar{\mathbf{x}}; \boldsymbol{\Theta})\right) \\ \operatorname{vech}\left(\widehat{\mathbf{F}}_{\mathbf{xx}}(\bar{\mathbf{x}}; \boldsymbol{\Theta})\right) \\ \vdots \\ \operatorname{vech}\left(\widehat{\mathbf{F}}_{\mathbf{x}^h}(\bar{\mathbf{x}}; \boldsymbol{\Theta})\right) \end{bmatrix} = \mathbf{0} \tag{7}$$

This procedure is consistent with a Taylor series expansion of (6) around $\bar{\mathbf{x}}$

$$\underbrace{\mathbf{0} = \mathbf{F}(\mathbf{x})}_{\text{Eq. (4)}} \approx \underbrace{\mathbf{F}(\bar{\mathbf{x}})}_{=\mathbf{0}} + \underbrace{\mathbf{F}_{\mathbf{x}}(\bar{\mathbf{x}})}_{=\mathbf{0}}(\mathbf{x} - \bar{\mathbf{x}}) + \frac{1}{2}\underbrace{\mathbf{F}_{\mathbf{xx}}(\bar{\mathbf{x}})}_{=\mathbf{0}}(\mathbf{x} - \bar{\mathbf{x}})^{\otimes 2} + \cdots + \frac{1}{h!}\underbrace{\mathbf{F}_{\mathbf{x}^h}(\bar{\mathbf{x}})}_{=\mathbf{0}}(\mathbf{x} - \bar{\mathbf{x}})^{\otimes h}$$

which help us identify $\boldsymbol{\Theta}$ exactly

Remarks

- Theorem (convergence): $\widehat{\mathbf{g}}(\mathbf{x}; \mathbf{\Theta}) \to \mathbf{g}(\mathbf{x})$ as $h \to \infty$ Details
- Continuous-time versus discrete-time Taylor projection Details
  - Continuous-time Taylor projection has fewer theoretical assumptions
  - Continuous-time Taylor projection has fewer computational costs
- (CT) Taylor projection vs perturbation (workhorse solution method) Details
  - Taylor projection can be built at any point (known or unknown) on the state space
  - Taylor projection requires fewer derivatives
  - Taylor projection adjusts intercepts and slopes to risk already at first-order ($h = 1$)

An illustration:
The Ramsey-Cass-Koopmans economy

Deterministic Ramsey-Cass-Koopmans economy

Households maximize lifetime utility by choosing $\{c_t\}_{t=0}^{\infty}$

$$U_0 = \int_0^{\infty} e^{-\rho t} \frac{c_t^{1-\gamma}}{1-\gamma} \mathrm{d}t$$

subject to the evolution of capital

$$\mathrm{d}k_t = [f(k_t) - \delta k_t - c_t] \, \mathrm{d}t$$

Optimal c is characterized by the following partial differential equation (PDE)

$$\mathcal{H}(k, c, c_k) = c\,[f_k(k) - \delta - \rho] - \gamma c_k\,[f(k) - \delta k - c] = 0$$

Since $c = c(k)$, we may write

$$\mathbf{F}(k) = c(k)\,[f_k(k) - \delta - \rho] - \gamma c_k(k)\,[f(k) - \delta k - c(k)] = 0$$

Derivation

The Ramsey-Cass-Koopmans economy

As c is an unknown function, consider a first-order polynomial approximant

$$\hat{c}\left(k;\boldsymbol{\Theta}\right) = \theta_0 + \theta_1\left(k - \bar{k}\right)$$

around an arbitrary point $\bar{k}$. The coefficients $\boldsymbol{\Theta} = [\theta_0,\ \theta_1]^{\top}$ are unknown

First-order Taylor projection ($h = 1$): Plug $\hat{c}(k;\boldsymbol{\Theta})$ into $\mathbf{F}(k)$ to form $\widehat{\mathbf{F}}(k;\boldsymbol{\Theta})$ and then

1. Differentiate $\widehat{\mathbf{F}}(k;\boldsymbol{\Theta})$ wrt $k \longrightarrow \widehat{\mathbf{F}}_k(k;\boldsymbol{\Theta})$
2. Evaluate $\widehat{\mathbf{F}}(k;\boldsymbol{\Theta})$ and $\widehat{\mathbf{F}}_k(k;\boldsymbol{\Theta})$ at $\bar{k}$

$$\mathcal{R}(\boldsymbol{\Theta}) = \begin{bmatrix} \widehat{\mathbf{F}}(\bar{k};\boldsymbol{\Theta}) \\ \widehat{\mathbf{F}}_k(\bar{k};\boldsymbol{\Theta}) \end{bmatrix} = \begin{bmatrix} \theta_0\left(\overline{f_k} - \delta - \rho\right) - \gamma\theta_1\left(\bar{f} - \delta\bar{k} - \theta_0\right) \\ \theta_1\left(\overline{f_k} - \delta - \rho\right) + \theta_0\overline{f_{kk}} - \gamma\theta_1\left(\overline{f_k} - \delta - \theta_1\right) \end{bmatrix} = \mathbf{0}$$

That is, two nonlinear equations in two unknowns

An RBC model with time-varying disaster risk

Given initial values K_0, $z_{r,0}$, $z_{p,0}$, q_0, and λ_0, the social planner chooses sequences $\{C,\ell\}_{t=0}^{\infty}$ that maximize the household’s present discounted value of lifetime utility

$$V\left(K, z_p, z_r, q, \lambda\right) = \max_{\{C,\ell\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\int_0^{\infty} \beta\theta J \left(\left(\frac{C\,(1-\ell)^{\nu}}{((1-\gamma)\,\beta J)^{\frac{1}{1-\gamma}}} \right)^{1-\frac{1}{\psi}} - 1 \right) \mathrm{d}t \right], \quad \text{s.t}$$

$$\begin{aligned}
\mathrm{d}K &= \left(\Phi\left(\frac{K^{\alpha}\left(z_r z_p \ell\right)^{1-\alpha} - C}{K} \right) - \delta \right) K \mathrm{d}t + \xi_K K \boxed{\mathrm{d}N}, \quad \xi_K < 0,\\
\mathrm{d}z_p &= (\mu + \omega q)\, z_p \mathrm{d}t + \sigma_{z_p} z_p \mathrm{d}B_{z_p},\\
\mathrm{d}z_r &= \rho_z \left(q - \log z_r\right) z_r \mathrm{d}t,\\
\mathrm{d}q &= -\phi_q q \mathrm{d}t + \xi_z \boxed{\mathrm{d}N}, \quad \xi_z < 0,\\
\mathrm{d}\lambda &= \kappa\left(\bar{\lambda} - \lambda\right) \mathrm{d}t + \sigma_{\lambda} \sqrt{\lambda} \mathrm{d}B_{\lambda},
\end{aligned}$$

Solving the model with Taylor projection

- We induce stationarity by de-trending the model.
- $\widehat{\mathbf{F}}(\mathbf{x};\boldsymbol{\Theta})$ is a 3×1 vector of optimality conditions with $\mathbf{x} = [k,\ z_r,\ q,\ \lambda]^{\top}$ (minimal representation) Details
- We approximate three unknown decision rules

$$\mathbf{g}(\mathbf{x}) \simeq \hat{\mathbf{g}}(\mathbf{x};\boldsymbol{\Theta}) = \begin{bmatrix} \hat{v}_k(\mathbf{x};\boldsymbol{\Theta}) & \hat{c}(\mathbf{x};\boldsymbol{\Theta}) & \hat{\ell}(\mathbf{x};\boldsymbol{\Theta}) \end{bmatrix}^{\top}$$

- How to approximate for the value function, $v(\mathbf{x})$, in models with SDU? Details
- Standard RBC calibration (reference unit of time is annual) Details
- Impulse response functions to a disaster (tail event) and disaster risk (tail risk) Details

Accuracy and stability

We construct Unit free Euler errors, $\mathcal{M}(\mathbf{x})$, using stochastic simulations across the ergodic distribution to assess the accuracy of the approximation

Channel 2

Channel 1

	$\log_{10}\mathcal{M}(\mathbf{x})$		Average $\log_{10}\mathcal{M}(\mathbf{x})$ sorted by percentile of k					
	Mean	Max	≤ 10	$(10, 25]$	$(25, 50]$	$(50, 75]$	$(75, 90]$	$90 <$
TP1	-1.12	4.23	0.04	-0.33	-1.06	-2.42	-1.05	-0.52
TP2	-2.03	0.51	-0.22	-0.72	-1.86	-3.99	-2.01	-1.31
TP3	-2.87	0.29	-0.45	-1.09	-2.63	-5.51	-2.88	-1.93
GTP1	-1.78	-0.02	-1.22	-1.74	-1.99	-1.86	-1.77	-1.65
GTP2	-2.45	0.38	-0.32	-1.07	-2.69	-3.39	-3.33	-2.38
GTP3	-4.54	0.10	-2.10	-4.49	-5.14	-5.04	-4.97	-3.63

Table: Euler Errors across the population distribution.

Results for JMM bounds

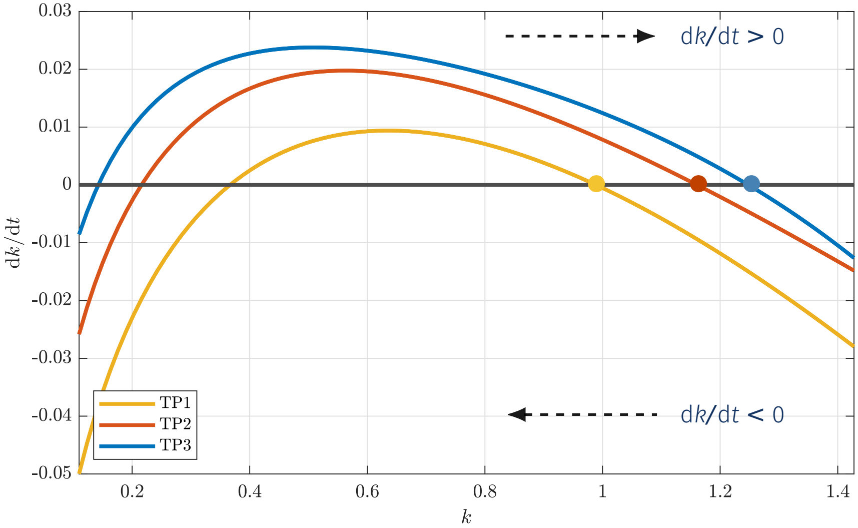
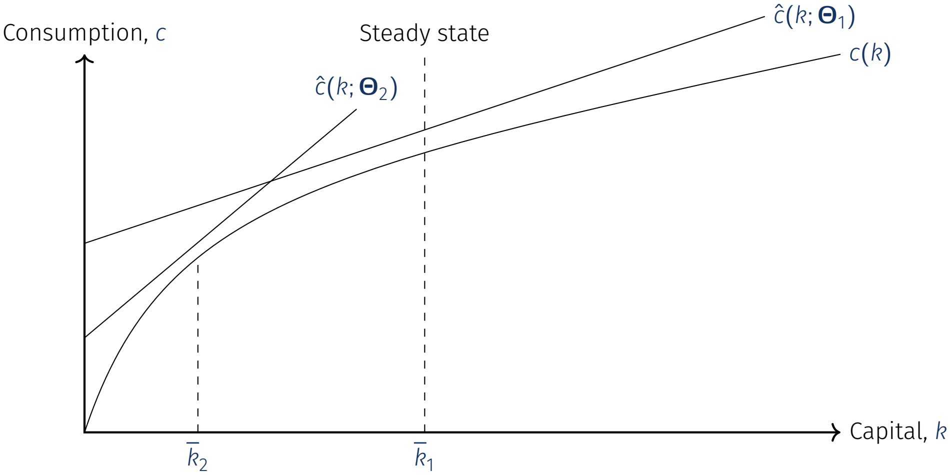


Figure: Transitional dynamics of (detrended) capital stock, $dk = \left(\Phi\left(i/k\right) - \delta - \left(\mu + \omega q\right)\right) k dt$.

Globalized Taylor projection

Inaccuracies and instabilities in tail region: Propose globalized Taylor projection (GTP)



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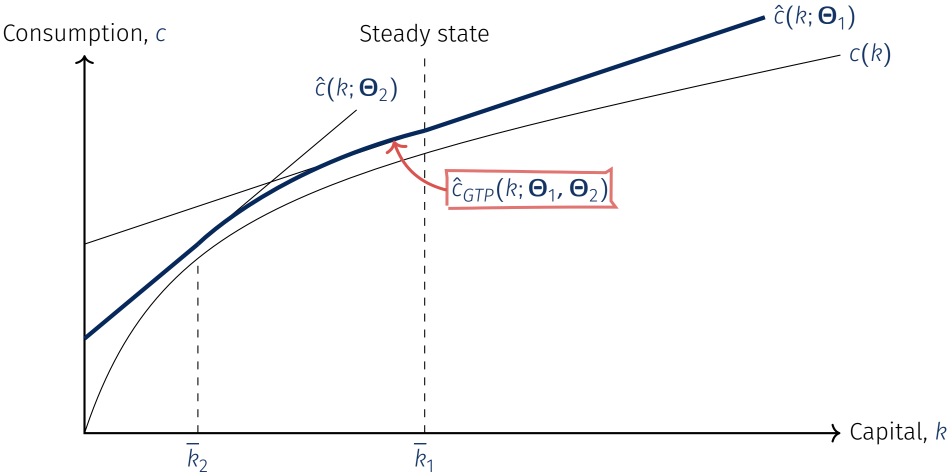

The “globalized” approximation is given by

$$\widehat{\mathbf{g}}_{GTP}\left(\mathbf{x};\mathbf{\Theta}\right)=\sum_{s=1}^{n_s}\phi\left(\mathbf{x},\bar{\mathbf{x}}_s\right)\times\widehat{\mathbf{g}}\left(\mathbf{x};\mathbf{\Theta}_s,\bar{\mathbf{x}}_s\right)$$

- $\widehat{\mathbf{g}}\left(\mathbf{x};\mathbf{\Theta}_s,\bar{\mathbf{x}}_s\right)$ is a Taylor projection approximation of order h at $\bar{\mathbf{x}}_s\in\{\bar{\mathbf{x}}_1,\ldots,\bar{\mathbf{x}}_s,\ldots,\bar{\mathbf{x}}_{n_s}\}$
- $\phi\left(\mathbf{x},\bar{\mathbf{x}}_s\right)\in[0,1]$ are scalar weight functions
  - Centered weights at $\bar{\mathbf{x}}_1,\ldots,\bar{\mathbf{x}}_s,\ldots,\bar{\mathbf{x}}_{n_s}$
  - $\sum_{s=1}^{n_s}\phi\left(\mathbf{x},\bar{\mathbf{x}}_s\right)=1\quad\forall\mathbf{x}$

We prove convergence of GTP for any grid, $\bar{\mathbf{x}}_1,\ldots,\bar{\mathbf{x}}_s,\ldots,\bar{\mathbf{x}}_{n_s}$,

$$\widehat{\mathbf{g}}_{GTP}\left(\mathbf{x};\mathbf{\Theta}\right)\rightarrow\mathbf{g}\left(\mathbf{x}\right)\text{ as }h\rightarrow\infty$$

Illustration GTP order 1

	$\log_{10} \mathcal{M}$		Average $\log_{10} \mathcal{M}$ sorted by percentile of k					
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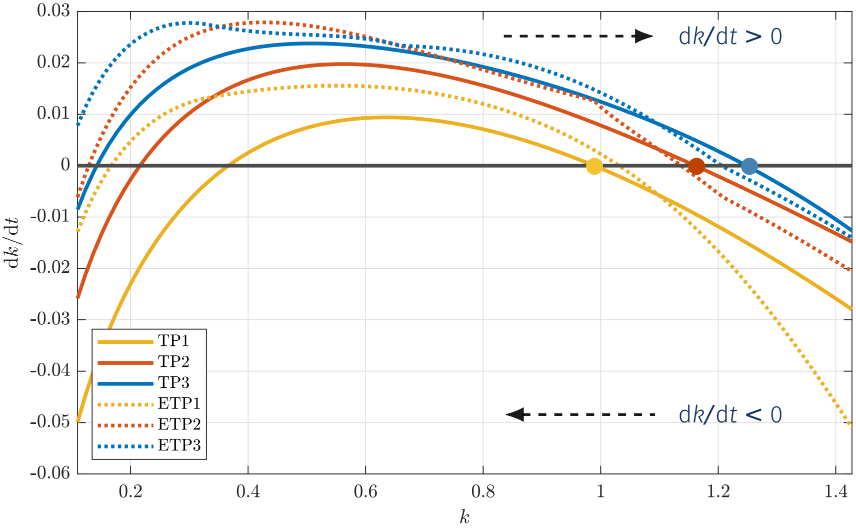

Figure: Transitional dynamics of (detrended) capital stock, $\mathrm{d}k = (\Phi(i/k) - \delta - (\mu + \omega q))\, k\mathrm{d}t$

Conclusion

Conclusion

We extend Taylor projection to approximate the solution of continuous-time dynamic stochastic models under tail risk and tail events

- We provide proof of convergence
- Test TP in an economy with infrequent disasters by gauging errors and stability
  - Captures well tail risk (channel 1): accuracy increases with the order of approximation
  - Issues with tail events (channel 2): sizable approximation errors and instability
- Propose globalized Taylor projection to ensure stable and precise approximations

Our appendix provides additional applications: a New Keynesian economy, an endogenous growth (AK) economy with disaster risk, and a resource extraction problem

Thank you!

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Appendix

Convergence of Taylor projection

Assumptions

1. $\mathcal{H}$ and $\mathbf{g}$ are analytic functions on an open set $\mathbb{X} \ni \bar{\mathbf{x}}$
2. As the order of approximation approaches infinity, $h \to \infty$, the derivatives of the policy function at the approximation point, $\mathbf{g}_{\mathbf{x}^h}(\bar{\mathbf{x}})$, go sufficiently fast to zero

Theorem

Let Assumptions 1 and 2 hold. Then, $\widehat{\mathbf{g}}(\mathbf{x}; \mathbf{\Theta})$ converges to $\mathbf{g}(\mathbf{x})$ as $h \to \infty$

Return

Continuous vs. discrete time

- Discrete-time framework (Levintal, 2018)

$$\mathbb{E}_t\left[\mathcal{H}\left(\mathbf{x}, \mathbf{x}', \mathbf{y}, \mathbf{y}'\right)\right] = \mathbf{0}.$$

  The framework requires approximations of $\mathbb{E}_t[\cdot]$

- Continuous-time framework implies (using Itô calculus) the absence of any conditional expectation operator in the equilibrium conditions

$$\mathcal{H}\left(\mathbf{x}, \mathbf{y}, \mathbf{y}_{\mathbf{x}}, \mathbf{y}_{\mathbf{xx}}\right) = \mathbf{0}.$$

  But on the flip side, the derivatives of policy functions are needed

- Implications
  - Theoretical: proof of convergence requires fewer assumptions
  - Numerical: cost of $\mathbf{y}_{\mathbf{x}^h}$ is small (exploit $\mathbf{y}$'s polynomial structure)

Return

Taylor projection vs. perturbation

Computational differences

- A perturbation approximation of order h needs information from derivatives of equilibrium equations of order $h+2$ (Gaspar and Judd, 1997)
- An h-th order Taylor projection only needs derivatives of equilibrium equations of order h
- Example: Five equilibrium equations and three state variables. A 2nd order perturbation approximation needs 125 derivatives, while Taylor projection needs 45

Properties (first-order)

- Perturbation adjusts for risk in levels but slopes are certainty equivalent
- Taylor projection adjusts for risk in both level and slope coefficients

Return

The Ramsey-Cass-Koopmans economy

HJB reads

$$\rho v = \frac{c^{1-\gamma}}{1-\gamma} + v_k \left(f\left(k\right) - \delta k - c\right)$$

FOC

$$c^{-\gamma} - v_k = 0 \qquad \Rightarrow \qquad c(k)$$

Costate

$$v_k \left(f_k\left(k\right) - \delta - \rho\right) + v_{kk} \left(f\left(k\right) - \delta k - c\right) = 0$$

Using the FOC, substitute v_k and v_{kk} with c and c_k into costate to get $\mathbf{F}(k)$

Return

Social planner problem (Gourio, 2012)

The social planner chooses sequences $\{C_t, \ell_t\}_{t=0}^{\infty}$ such that that

$$V\left(K_0, z_{r,0}, z_{p,0}, q_0, \lambda_0\right) = \max_{\{C_t,\ell_t\}_{t=0}^{\infty}} J_0,$$

subject to

$$\begin{aligned}
\mathrm{d}K_t =& \left(\Phi\left(\frac{K_t^{\alpha}\left(z_{r,t}z_{p,t}\ell_t\right)^{1-\alpha} - C_t}{K_t}\right) - \delta\right)K_t\mathrm{d}t + \xi_K K_{-t}\mathrm{d}N_t,\\
\mathrm{d}z_{p,t} =& \left(\mu + \omega q_t\right) z_{p,t}\mathrm{d}t + \sigma_{z_p} z_{p,t}\mathrm{d}B_{z_p,t},\\
\mathrm{d}z_{r,t} =& \rho_z\left(q_t - \log z_{r,t}\right) z_{r,t}\mathrm{d}t,\\
\mathrm{d}q_t =& -\phi_q q_t\mathrm{d}t + \xi_z\mathrm{d}N_t,\\
\mathrm{d}\lambda_t =& \kappa\left(\overline{\lambda} - \lambda_t\right)\mathrm{d}t + \sigma_\lambda\sqrt{\lambda_t}\mathrm{d}B_{\lambda,t},
\end{aligned}$$

Social planner problem (Gourio, 2012)

We characterize the competitive equilibrium as the evolution of the costate of capital

$$\begin{aligned} 0 = & \left[\Phi_{\iota} \iota_k k + \Phi(\iota) - \delta - \mu - \omega q + \gamma\sigma_{z_p}^2 + \left(\mu + \omega q - \frac{1}{2}\gamma\sigma_{z_p}^2\right)(1-\gamma) + f_v\right] v_k \\ & + \left[\Phi(\iota) - \delta - \mu - \omega q + \gamma\sigma_{z_p}^2\right] k v_{kk} + \sigma_{z_p}^2 k v_{kk} + \frac{1}{2}\sigma_{z_p}^2 k^2 v_{kkk} \\ & + \rho_z \left(q - \log(z_r)\right) z_r v_{z_r k} - \phi_q q v_{qk} \\ & + \kappa\left(\bar{\lambda} - \lambda\right) v_{\lambda k} + \frac{1}{2}\sigma_{\lambda}^2 \lambda v_{\lambda\lambda k} + \lambda\left[\tilde{v}_k (1+\xi) - v_k\right], \end{aligned} \tag{8}$$

along with first-order conditions for c and ℓ

$$f_u u_c + \Phi_{\iota} \iota_c k v_k = 0, \tag{9}$$

$$f_u u_{\ell} + \Phi_{\iota} \iota_{\ell} k v_k = 0, \tag{10}$$

Together, they form the equilibrium vector, $\mathbf{F}(k, z_r, q, \lambda)$

Return

Social planner problem (Gourio, 2012)

A note on the approximation of the value function

- Stochastic differential utility (SDU) ⇒ policy functions depend on the (unknown) value function $v(\mathbf{x})$
- We do not approximate the value function directly. Instead

  $$\hat{v}(\mathbf{x}) = \int \hat{v}_k(\mathbf{x}; \mathbf{\Theta})\mathrm{d}k$$

  using the approximant of the costate variable for capital
- The yet unknown constant of integration is pinned down by the maximized HJB

Return

Calibration

Parameters	Value
Household's preferences	
Coefficient of relative risk aversion, γ	3.8000
Intertemporal elasticity of substitution, $\hat{\psi}$	2.0000
Leisure preference, ν	2.3300
Subjective discount rate, β	0.0410
Production	
Capital share in output, α	0.3400
Depreciation rate of capital, δ	0.0776
Elasticity, investment-to-capital ratio, wrt Tobin's q, η	10.000
Det. steady-state value, investment-to-capital ratio, $\bar{\iota}$	$\delta + \mu$
Exogenous processes	
Trend growth of TFP, μ	0.0200
Standard deviation of TFP shock, σ_{z_p}	0.0200
Mean arrival rate of Poisson process, $\bar{\lambda}$	0.0280
Persistence of disaster risk, κ	0.2000
Standard deviation of disaster risk, σ_λ	0.1000
Permanent destruction in productivity after a jump, ω	0.1900
Destruction rate of capital/productivity, $\xi_K = \xi_z := \xi$	-0.4742
Speed of recovery, ϕ_q	0.7526
Speed of disaster realization, ρ_z	0.7527

Large destruction of capital!

Return

Impulse-response functions

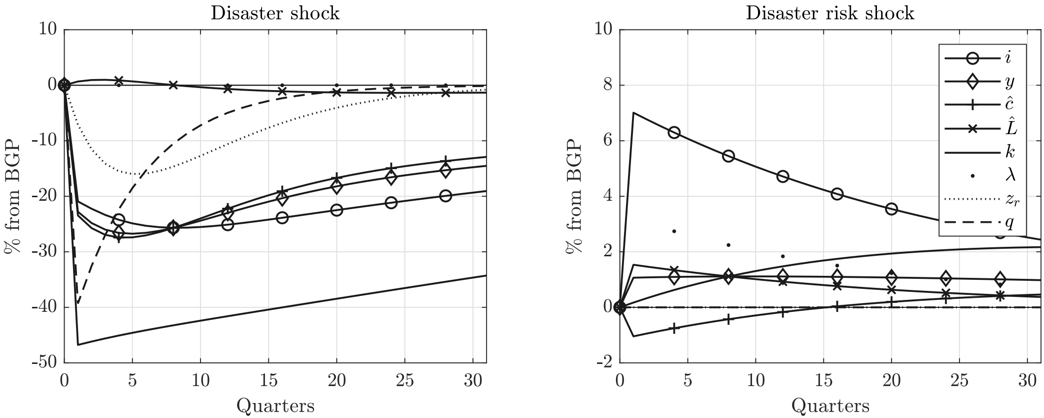


Figure: A third-order Taylor projection's impulse response (detrended variables)

Return

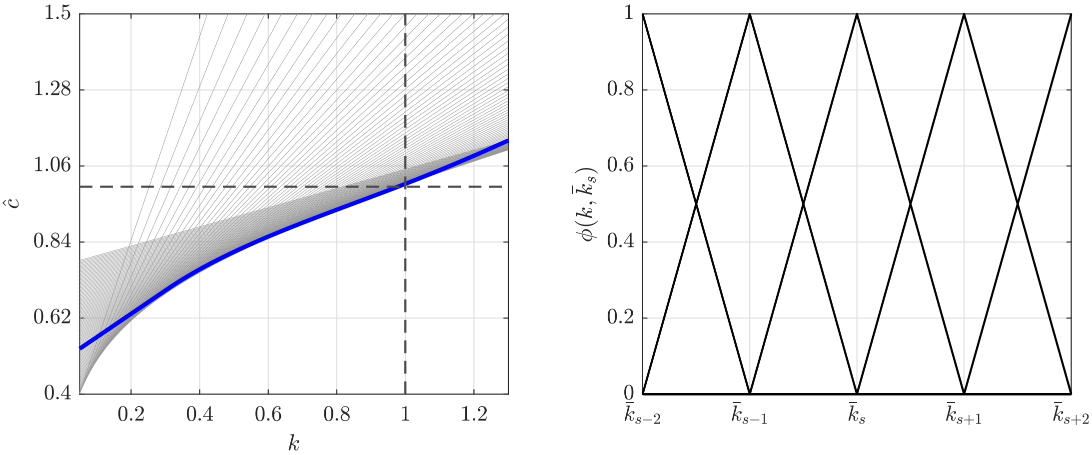


Figure: A first-order globalized Taylor projection

Return

JMM Bounds

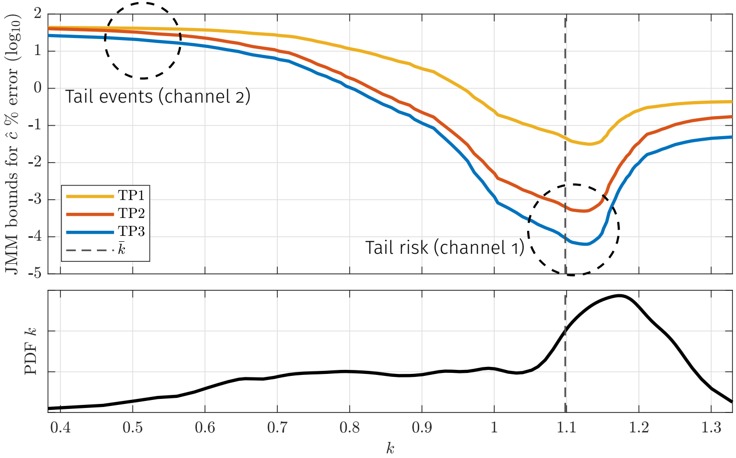


Figure: JMM bounds for $\hat{c}$ along k.

JMM Bounds

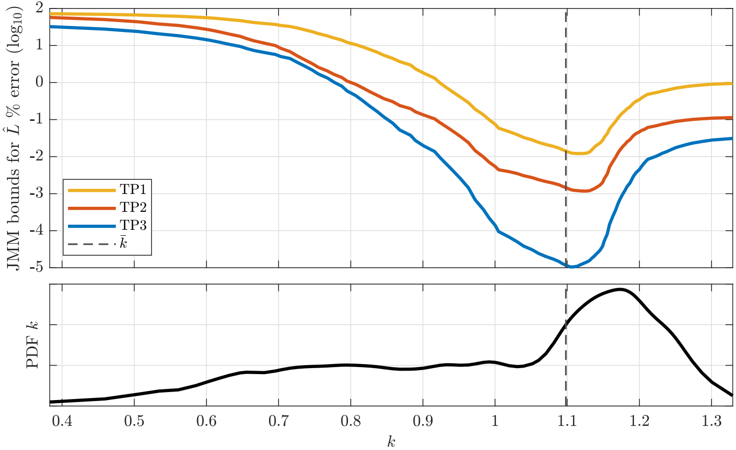


Figure: JMM bounds for $\hat{\ell}$ along k.

JMM Bounds

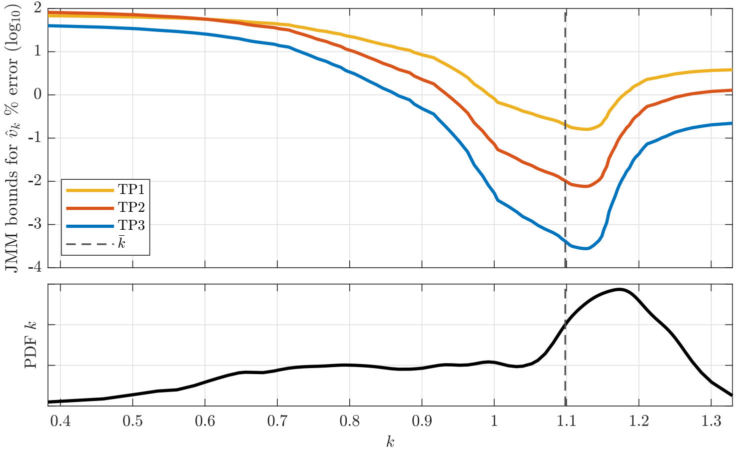
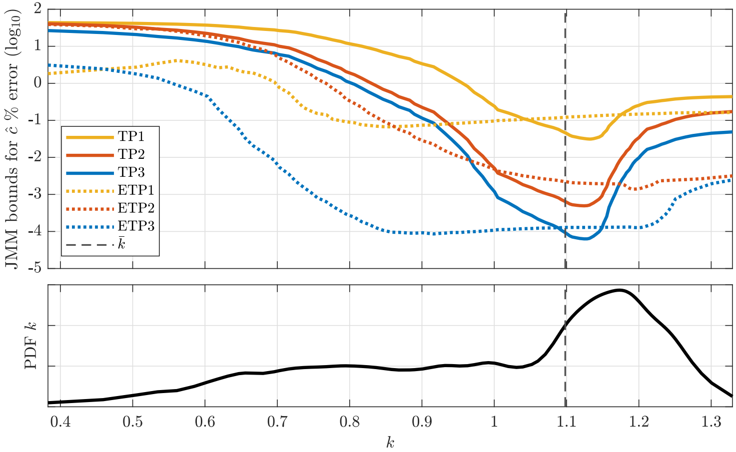


Figure: JMM bounds for $\hat{v}_k$ along k.

Return

JMM Bounds (ETP)



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JMM Bounds (ETP)

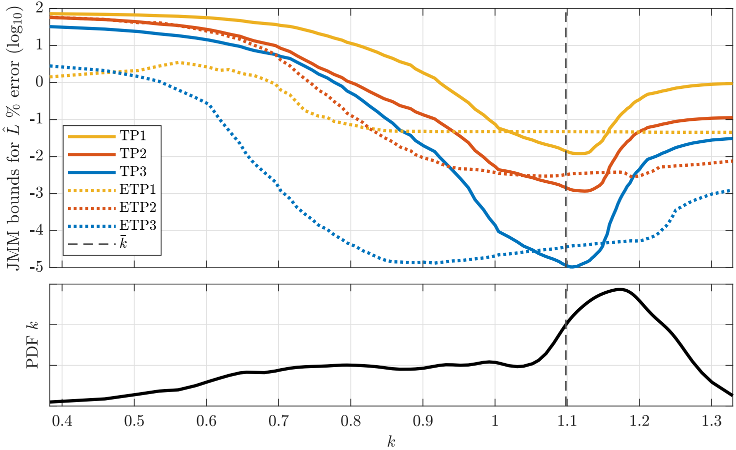

Figure: JMM bounds for $\hat{\ell}$ along k.

JMM Bounds (ETP)

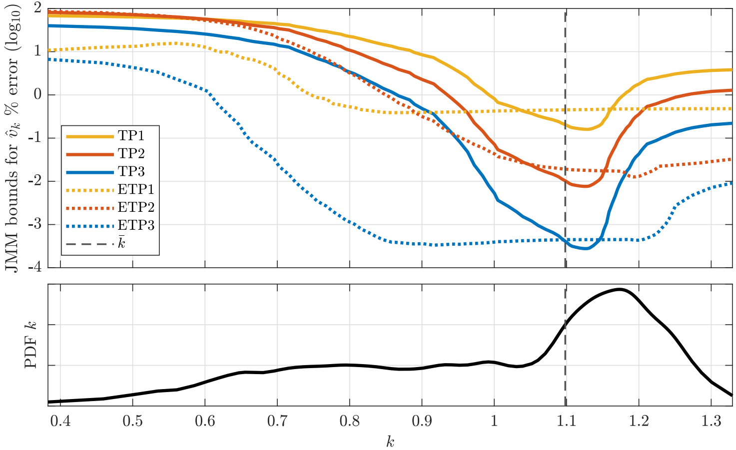

Figure: JMM bounds for $\hat{v}_k$ along k.

Return

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