Coordinating Bank Dividend and Capital Regulation

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Abstract

In this paper, we examine how dividend taxes (and bans) and capital requirements that vary with the state of the economy influence a bank's optimal capital buffers and shareholder value. In the model, the bank distributes dividends and issues costly equity to maximise shareholder value, while its assets generate stochastic income under time-varying macroeconomic conditions. We solve the bank's stochastic control problem and derive the distribution of its capital buffers in closed form. Imposing dividend taxes (or bans) in bad macroeconomic states generates an intertemporal trade-off, as it encourages capital buffers accumulation in those states but promotes dividend payouts in the good ones. Furthermore, the policy undermines financial stability by reducing the bank's value and weakening its incentives to recapitalise in both good and bad states. Coordinating dividend taxes with counter-cyclical capital requirements can mitigate value losses and ease the trade-off, but it also exacerbates disincentives for recapitalisation. (JEL: C61; G21; G32; G35; G38)

Keywords: Capital requirements; dividend bans; dividend taxes; policy coordination; stochastic optimal control.

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1 Introduction

At the height of the COVID-19 crisis, motivated by the observation that banks did not adjust their dividends during the Great Financial Crisis (Acharya et al., 2011; Cziraki et al., 2024; Belloni et al., 2024), banking regulators worldwide recommended, and in some cases enforced, dividend restrictions.[1] These unprecedented measures aimed to preserve banks' credit capacity by ensuring adequate capital buffers and, more critically, preventing systemic defaults.

Empirical evidence suggests that the short-term outcome of these policies has been two-sided. On the one hand, Andreeva et al. (2023) and Sanders et al. (2024) find that dividend restrictions and their announcements have negatively impacted bank equity valuations, consistent with the notion that shareholders demand higher returns in response to lower and delayed future payouts. On the other hand, Li et al. (2020) and Hardy (2021) argue that they were effective in improving the balance sheets of banks and, ultimately, in avoiding a credit crunch. Couaillier et al. (2025) find that dividend policies played a pivotal role in sustaining bank lending, highlighting that capital buffer releases alone would not have been sufficient to achieve the same outcome.

From a theoretical standpoint, the problem of evaluating dividend regulation – such as bans and, more broadly, taxation – during adverse macroeconomic conditions has only recently received attention (see Vadasz, 2022 and Kroen, 2022) and remains poorly understood. This is because such policies often involve complex interactions with banks' decision-making and other forms of regulation. This paper develops a theory to evaluate how dividend taxes (or bans) contingent on the aggregate state of the economy affect a bank's optimal recapitalisation and dividend payout decisions in both the short and long term, as well as their interaction with traditional capital requirements.

We model the optimal control problem of a bank that holds a fixed amount of loans

[1] The ECB advised suspensions to start in March 2020; regular payments resumed in the fourth quarter of 2021. The FED imposed restrictions in June 2020, partially easing them in December 2020. The restrictions ended between June and July 2021, allowing banks to revert to pre-pandemic dividend policies.

and insured deposits.[2] Loans generate stochastic cash flows, whose expected returns and volatility depend on the aggregate state of the economy, as in Hackbarth et al. (2006).[3] The regulator imposes dividend taxes (or bans) and capital requirements, depending on the aggregate state of the economy. Similarly to Décamps et al. (2011) and Moreno-Bromberg and Rochet (2014), the bank decides whether to default or, at a cost, to issue equity (and in what amount) when capital requirements become binding; it retains cash flows as capital buffers or pays them out as dividends to avoid costly recapitalisation, aiming to maximise shareholder value.

In the first part of the paper, we solve the bank's stochastic control problem in closed form, demonstrating that optimal payouts follow a threshold strategy. In particular, we show that dividends are paid only when their marginal value, proportional to the state-contingent dividend tax rate, exceeds that of accumulating additional capital buffers. Otherwise, the bank takes no action. Notably, adopting state-contingent dividend taxes is equivalent to imposing a dividend ban when the tax rate is high enough. Next, we show that recapitalising the bank is optimal if its costs are sufficiently small ("incentive compatible"). If that is the case, when the capital constraint binds and there are no dividend taxes, the bank injects equity until the reserves reach the dividend payout threshold. In the presence of dividend taxes, the optimal recapitalisation target falls below the payout threshold.

Equipped with the bank's optimal controls, we analytically derive the stationary distribution of its capital buffers, conditional on the aggregate state of the economy. We interpret this distribution either as a measure of the bank's "ex-ante" credit capacity or as the cross-sectional distribution of buffers that a regulator should consider in an economy populated by an infinite number of identical banks. We use this distribution to evaluate the long-term outcomes of different policies.

The second part of the paper employs numerical analysis to investigate how dividend

[2] In principle, the firm we model can represent either a financial or a non-financial company. However, our model is particularly well suited to financial firms, which are typically subject to capital requirements.

[3] Chay and Suh (2009) provide empirical evidence that cash flow uncertainty is a key determinant of firms' dividend payout decisions.

taxes and capital regulation affect the bank's optimal decisions and the distribution of capital buffers, and to derive policy implications. Motivated by the COVID-19 policy case, we examine a scenario in which dividend taxes are higher during a bad economic state, characterised by high cash flow volatility and low expected returns. To isolate the effects of dividend taxes, we first consider a-cyclical capital requirements.

Consistent with its scope, the policy encourages the accumulation of reserves in the targeted state by increasing the corresponding dividend payout threshold. This happens because higher taxes lower the marginal value of dividends. At the same time, however, the policy reduces the bank's value not only in the bad state ("ex-post") but also in the good state ("ex-ante") as shareholders internalise the prospect of lower future returns. Consequently, the bank finds it optimal to increase dividend payouts (i.e., reduce capital buffers) in the good state to compensate for these losses partially. These predictions warn that regulatory uncertainty regarding dividend restrictions can backfire by generating lower capital buffers in the long run (on this point, see also Attig et al., 2021) and encourage counter-cyclical equity issuance strategies, as suggested by Baron (2020).

The second finding of the numerical analysis is that imposing higher dividend taxes (or, equivalently, restrictions) during adverse macroeconomic conditions reduces the bank's optimal recapitalisation targets across all states. Additionally, the resulting shareholder value losses lower the incentive-compatible cost threshold beyond which shareholders are unwilling to inject equity when capital constraints are binding. This result suggests that state-contingent tax policies may increase default risk, potentially amplifying concerns about financial stability.

In the final part of the paper, we examine whether coordinating dividend taxes with cyclical capital requirements can help mitigate the adverse effects of dividend policy. This analysis is particularly relevant in light of the findings of Dursun-de Neef et al. (2023), which demonstrate that the combination of recommendations to suspend dividends and relax capital requirements was crucial in sustaining lending during the COVID-19 pandemic.

According to our simulations, coordinating countercyclical capital requirements with dividend taxes can mitigate the adverse effects of the latter by redistributing value losses across states and reducing the dispersion of the capital buffer distribution in the long run. However, these benefits come at the cost of creating additional disincentives for recapitalisation.

The rest of the paper is organised as follows. Section 2 situates the paper within the existing literature. Section 3 introduces the model, and Section 4 provides its analytical solution. Section 5 analyses the model numerically and discusses its policy implications. Section 6 concludes.

2 Related literature

Our work is related to recent empirical papers that examine the (primarily short-term) effects of dividend restrictions during the COVID-19 pandemic. Andreeva et al. (2023) and Sanders et al. (2024) find that banks subject to dividend suspension policies experienced a temporary drop in equity valuation but were able to increase their lending to the economy. Hardy (2021) show that banks' CDS spreads declined after this measure, suggesting an improvement in their safety. Dautovic et al. (2023) find that dividend restrictions were effective in limiting banks' pro-cyclical behaviour, thereby improving their capital buffers. Mücke (2023) show that mutual funds permanently reduced their ownership stakes in banks under payout restrictions. We complement this literature by providing a theoretical framework that analyses and characterizes the joint effects of dividends and capital regulation on a bank's endogenous decisions in both the short and long run.

To the best of our knowledge, only a few papers have theoretically examined the effects of banks' dividend regulations. Goodhart et al. (2010) study the impact of dividend restrictions on an interbank market, showing that it may reduce defaults and improve welfare. Lindensjö and Lindskog (2020) solve the optimal control problem of a financial company facing dividend restrictions, finding that they may increase default risk. Unlike our work, these papers abstract from macroeconomic uncertainty.

Other related contributions include Vadasz (2022), which explores the ex-post intervention problem between a regulator and a bank in a two-period game, and Ampudia et al. (2023), which investigates dividend bans using a quantitative DSGE model with banks. Similar to our work, these papers highlight the trade-off between the benefits of increased lending and the losses in bank valuation resulting from policy intervention. We differentiate substantially by studying the joint effect of dividends and capital regulation on the bank's recapitalisation decisions in a traditional corporate finance setting. In this respect, we draw on the extensive literature that examines the optimal cash management of the firm, such as Décamps et al. (2011) and Gryglewicz (2011). We depart from these studies by considering macroeconomic uncertainty, as in Hackbarth et al. (2006) (which, in turn, does not consider dividend or capital requirements).

Methodologically, we build on the continuous-time stochastic control literature on dividends pioneered by Jeanblanc-Picqué and Shiryaev (1995). In particular, we tackle a problem that features Markovian regime switching, as in Jiang and Pistorius (2012) and Ferrari et al. (2022), and consider endogenous equity issuance, as in Løkka and Zervos (2008). Unlike in Radner and Shepp (1996), cash flows are exogenous. Our solution employs a guess-and-verify approach, similar to that described in Sotomayor and Cadenillas (2011).

We distinguish ourselves from these studies in three dimensions. First, we consider macroeconomic uncertainty not only in expected cash flows, as in Reppen et al., 2020, but also in their volatility. Second, we are the first to incorporate both dividend taxes (or bans) and capital regulation into an optimal dividend framework. Third, we analytically derive the stationary distribution of banks' capital buffers under the optimal control.

3 Model set up

Time is continuous and indexed by $t \in [0, \infty)$. As in Guo et al. (2005) and Hackbarth et al. (2006), we consider a bank subject to aggregate uncertainty on the state of the economy, modelled via a bi-variate Markov chain $I_t \in \{1, 2\}$, with transition intensity λ_{I_t}. A risk-neutral manager runs the bank in the best interest of its shareholders.

The bank holds a fixed amount of insured liabilities, D (deposits) and assets, A_t. Assets include a constant stock of illiquid loans, L, and time-varying liquid reserves ("capital buffers"), X_t. The book value of the bank equity at time t satisfies the balance sheet identity

$$E_t + D = X_t + L. \tag{1}$$

Deposits yield the risk-free interest rate $\rho \geq 0$, while reserves are not remunerated, for simplicity. Loans generate operating cash flows according to the following stochastic differential equation:

$$\bar{\mu}_{I_t} dt + \sigma_{I_t} dW_t, \tag{2}$$

where W_t is a standard one-dimensional Brownian motion defined in some filtered probability space $(\Omega, \mathcal{F}, \mathbb{F} := (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})$. The drift and diffusion terms of (2) are contingent on the state of the economy I_t. In particular, we assume that $\bar{\mu}_1 \geq \bar{\mu}_2 > 0$ and $\sigma_2 \geq \sigma_1 > 0$ so that states $I_t = 1$ and $I_t = 2$ represent expansions (higher expected returns and lower volatility) and recessions (lower expected returns and higher volatility), respectively. We will thus refer to $I_t = 1$ as the "good" and to $I_t = 2$ as the "bad" state throughout the paper.

The cash flows can be retained as reserves or paid out as dividends. dZ_t denotes the time-t dividend payment and is the manager's first choice variable. The government taxes dividends depending on I_t. We denote as $\beta_{I_t} \in [0, 1]$ the after-tax value of a 1\$ dividend in State I_t. Consistent with the paper's motivation, we focus on counter-cyclical dividend tax schedules and set $\beta_1 = 1$ and $\beta_1 \geq \beta_2$.

The regulator requires the bank to hold sufficient equity to repay debtors fully in the case loans are liquidated at the fire-sale price $\alpha \in [0,1]$. as in Décamps et al. (2011). Provided that $D > \alpha L$, equity must be such that:

$$E_{R,I_t} \geq L(1-\alpha) + \Gamma_{I_t}. \tag{3}$$

The parameter Γ_{I_t} captures additional capital requirements (e.g., those prescribed by regulations such as Basel III) when $\Gamma_{I_t} > 0$, or subsidies (e.g., state guarantees to cover default losses partially) when $\Gamma_{I_t} < 0$. By substituting (3) into (1), the capital requirement can be expressed as the following minimum level of the reserve:

$$X_t \geq D - \alpha L + \Gamma_{I_t} := x_{R,I_t}. \tag{4}$$

Since X_t is stochastic, (4) becomes occasionally binding at random times $(\tau_n)_{n\geq 1}$. When that happens, the manager can either liquidate or recapitalise the bank by issuing equity. The outcome is captured by the auxiliary variable b_n, which takes value 0 (liquidation) or 1 (recapitalisation). This is the bank's second choice variable. In the event of a liquidation, shareholders incur no costs but forgo all future dividends. The additional buffer $\Gamma^+_{I_t}$ is rebated to the shareholders. In the case of recapitalisation, shareholders provide the firm with new liquidity, G_n, and pay a fixed cost, $\kappa \geq 0$.

For a given equity issuance schedule, liquid reserves up to a (possibly infinite) liquidation time $\tau_\ell \coloneqq \inf\{\tau_n \geq 0 : \; b_n = 0\}$ evolve as follows:

$$\begin{cases} X_{\tau_n} = X_{\tau_n^-} + G_n, \qquad n \geq 0, \\ \\ dX_t = \underbrace{(\bar{\mu}_{I_t} - \rho D)}_{:=\mu_{I_t}} dt + \sigma_{I_t} dW_t - dZ_t, \qquad t \in [\tau_n, \tau_{n+1}), \end{cases} \tag{5}$$

with initial values $I_0 = i \in \{1, 2\}$ and $X_0 = x \geq \min\{x_{R,1}, x_{R,2}\}$.

Since reserves are not remunerated, the bank retains dividends only to avoid costly recapitalisation or liquidation. Formally, the admissible control is a triple of $\mathbb{F}_t$-measurable stochastic processes $A := (Z, b, G) = \big((Z_t)_{t\geq 0}, (b_n, G_n)_{n\geq 1}\big)$ such that:

(i) The cumulative dividend $(Z_t)_{t\geq 0}$ is right-continuous, non-decreasing and such that, setting $Z_{0^-} = 0$, each increment $\Delta Z_t := Z_t - Z_{t^-} < X_t - x_{R,i}$, $\forall t \geq 0$ and $i = 1, 2$. This condition ensures that the bank cannot issue dividends and equity simultaneously.

(ii) The auxiliary function $(b_n)_{n\geq 1}$ is $\mathcal{F}_{\tau_n}$-measurable and takes values $b_n = 0$ or $b_n = 1$ if a liquidation or recapitalisation takes place at time τ_n, respectively.

(iii) The equity issuance $(G_n)_{n\geq 1}$ at time τ_n when $b_n = 1$ is strictly positive and $\mathcal{F}_{\tau_n}$-measurable.

Denoting the set of admissible strategies by $\mathcal{A}$, the bank's gain function is

$$J(x, i; A) = \mathbb{E}\left[\int_0^{\tau_\ell} e^{-\delta t}\beta_{I_t} dZ_t - \sum_{n=1}^{\ell-1} e^{-\delta\tau_n}(G_n + \kappa)\right], \forall A \in \mathcal{A}, \tag{6}$$

where $\delta > 0$ is a discount rate, with the convention $\sum_{n=1}^{0} = 0$. The bank's value function (i.e., shareholder value) is then given by

$$V(x, i) := \sup_{A\in\mathcal{A}} J(x, i; A) \tag{7}$$

$$s.t.\ \ (5) \tag{8}$$

4 Model solution

The first part of this section derives the solution to the bank's stochastic control problem analytically. For this purpose, we focus on the "baseline" case in which capital requirements are a-cyclical, i.e. not contingent on the state of the economy ($x_{R,1} = x_{R,2} = x_R$). We will consider cyclical capital requirements in Section 5.

The second part of the section derives and analyses the (stationary) distribution of the bank's reserves under the optimal control.

4.1 Shareholder value and optimal strategy

To tackle the problem (7), we follow a *guess-and-verify* approach.[4] More specifically, we formulate a set of optimality conditions for a candidate value function v based on heuristic considerations. Then, we use verification arguments to prove that $v = V$.

We expect the value function to solve (in a suitable sense) the following system of Hamilton-Jacobi-Bellman Variational Inequalities (HJBVI):

$$\max\Big\{\mathcal{L}_i v(x,i) - \lambda_i \left[v(x,i) - v(x,3-i)\right], \beta_i - v'(x,i)\Big\} = 0, \qquad i = 1,2, \quad x > x_R, \tag{9}$$

where $v : [x_R, \infty) \times \{1,2\} \to \mathbb{R}$, and $\mathcal{L}_i$ are differential operators acting on functions $\phi \in C^2(x_R, \infty)$ as follows:

$$\mathcal{L}_i\phi = \frac{1}{2}\sigma_i^2\phi'' + \mu_i\phi' - \delta\phi, \qquad i = 1,2.$$

Associated with a smooth enough solution to (9), there are the following *continuation* and *intervention* regions for $i = 1,2$:

$$\mathcal{C}_i := \left\{x > x_R : \quad v'(x,i) > \beta_i\right\}, \tag{10}$$

$$\mathcal{S}_i := \left\{x > x_R : \quad v'(x,i) = \beta_i\right\}. \tag{11}$$

Equipped with these objects, we conjecture that the optimal control has a threshold structure, as in Sotomayor and Cadenillas (2011), meaning that for each $i = 1,2$, there is a reserve level $\tilde{x}_i$ below which the bank does not pay dividends. Indeed, we expect that it

[4] This approach is the most used in the classical corporate finance literature starting from Leland (1994). For a *direct* approach to a similar problem, building on the theory of viscosity solutions and stating the optimality conditions as *necessary*, we refer to Akyildirim et al. (2014).

is optimal to accumulate reserves as long as their marginal value ($v'(\cdot, i)$) exceeds that of dividends (β_i). Formally, we guess that the above regions are $\mathcal{C}_i = (x_R, \tilde{x}_i)$ and $\mathcal{S}_i = [\tilde{x}_i, \infty)$ for $i = 1, 2$, implying that

$$\tilde{x}_i = \inf \{x \geq x_R : v'(x, i) \leq \beta_i\}. \tag{12}$$

Next, we make some conjectures to construct a sufficiently smooth solution to (9). First, we assume that $v(\cdot, i) \in C([x_R, \infty)) \cap C^2((x_R, \infty))$. Second, we postulate $x_R < \tilde{x}_1 < \tilde{x}_2$, based on the intuition that it is optimal to pay more dividends when assets yield higher returns and carry less uncertainty. Third, we impose appropriate boundary conditions at the regulatory threshold x_R. The boundary conditions determine whether recapitalisation is optimal at x_R and, if so, the amount of equity to issue.

If the manager liquidates the firm, the shareholders receive the maximum between the capital buffer and zero (Γ_i^+). Otherwise, the optimal recapitalisation policy ($\hat{G}$) must be feasible and "incentive-compatible", i.e. its benefits must be larger than or equal to its costs. Therefore, we require that $\hat{G} = \operatorname{argmax} \{v(x_R + G; i) - G - \kappa\}$, with $v(x_R + \hat{G}, i) \geq \hat{G} + \kappa$ and $\hat{G} \in \mathcal{G} := [0, \tilde{x}_i - x_R]$. As either liquidation or recapitalisation must be optimal, we impose

$$v(x_R, i) = \max \left\{ \max_{G \in \mathcal{G}} \{v(x_R + G, i) - G - \kappa\}, \Gamma_i^+ \right\} \quad \text{for } i = 1, 2. \tag{13}$$

We now construct a function that meets all these conditions.

In the intervals $[\tilde{x}_i, \infty)$, the function must satisfy $v'(\cdot, i) = \beta_i$. In the interval $(\tilde{x}_1, \tilde{x}_2)$, we define $v'(\cdot, 2)$ as the unique solution to

$$\frac{1}{2}\sigma_2^2 v'''(x, 2) + \mu_2 v''(x, 2) - (\delta + \lambda_2) v'(x, 2) + \lambda_2 = 0, \tag{14}$$

with boundary conditions $v'(\tilde{x}_2, 2) = \beta_2 > 0$ (optimality condition), and $v''(\tilde{x}_2, 2) = 0$ (super contact condition, see Dumas, 1991). In the interval $(x_R, \tilde{x}_1)$, we find the functions $v'(\cdot, i)$

as unique solutions to the following system:

$$\begin{cases} \frac{1}{2}\sigma_1^2 v'''(x,1) + \mu_1 v''(x,1) - (\delta+\lambda_1)\, v'(x,1) + \lambda_1 v'(x,2) = 0, \\ \\ \frac{1}{2}\sigma_2^2 v'''(x,2) + \mu_2 v''(x,2) - (\delta+\lambda_2)\, v'(x,2) + \lambda_2 v'(x,1) = 0. \end{cases} \tag{15}$$

with boundary conditions $v'(\tilde{x}_1, 1) = 1$ (optimality condition), $v''(\tilde{x}_1, 1) = 0$ (super-contact condition), $v'(\tilde{x}_1^-, 2) = v'(\tilde{x}_1^+, 2)$, and $v''(\tilde{x}_1^-, 2) = v''(\tilde{x}_1^+, 2)$ (continuity conditions).

Solving (14) and (15) for $v'(\cdot, i)$ and integrating over the corresponding intervals yields the following proposition.

Proposition 1 (**Solution to the HJBVI system**). *Recall that $\delta > 0$, $\lambda_i > 0$, $\mu_1 \geq \mu_2 > 0$, $\sigma_1 \leq \sigma_2$ and assume that $\beta_2 > \lambda_2/(\lambda_2 + \delta)$. Then, the following holds.*

1. *Fix $\tilde{x}_1, \tilde{x}_2$ such that $x_R < \tilde{x}_1 < \tilde{x}_2$ and define*

$$v(x,1) = K_1 + \begin{cases} (x - \tilde{x}_1), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^4 A_j \left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1), \end{cases} \tag{16}$$

$$v(x,2) = K_2 + \begin{cases} \beta_2(x - \tilde{x}_2), & x \in [\tilde{x}_2, \infty), \\ \frac{\lambda_2(x-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{j=1}^2 \tilde{A}_j \left(e^{\tilde{\alpha}_j(x-\tilde{x}_1)} - e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}\right), & x \in [\tilde{x}_1, \tilde{x}_2), \\ \frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{j=1}^2 \tilde{A}_j (1 - e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}) + \sum_{j=1}^4 B_j \left(e^{\alpha_j(x-\tilde{x}_1)} - 1\right), & x \in [x_R, \tilde{x}_1), \end{cases} \tag{17}$$

where $\alpha_j < \alpha_2 < 0 < \alpha_3 < \alpha_4$ and $\tilde{\alpha}_1 < 0 < \tilde{\alpha}_2$ are the real roots of

$$\underbrace{\left(\delta + \lambda_1 - \alpha\mu_1 - \frac{\sigma_1^2}{2}\alpha^2\right)}_{:=G_1(\alpha)} \underbrace{\left(\delta + \lambda_2 - \alpha\mu_2 - \frac{\sigma_2^2}{2}\alpha^2\right)}_{:=G_2(\alpha)} = \lambda_1\lambda_2, \tag{18}$$

$$\frac{1}{2}\sigma_2^2\tilde{\alpha}^2 + \mu_2\tilde{\alpha} = \delta + \lambda_2, \tag{19}$$

and $(A_1, A_2, A_3, A_4, \tilde{A}_1, \tilde{A}_2) \in \mathbb{R}^6$ *and* $(K_1, K_2) \in \mathbb{R}^2_+$ *solve the following linear system:*[5]

$$\begin{cases} \sum_{j=1}^4 A_j \alpha_j - 1 = 0, \\ \sum_{j=1}^4 A_j \alpha_j^2 = 0, \\ \sum_{j=1}^4 B_j \alpha_j - \frac{\lambda_2}{\delta+\lambda_2} - \sum_{h=1}^2 \tilde{A}_h \tilde{\alpha}_h = 0, \\ \sum_{j=1}^4 B_j \alpha_j^2 - \sum_{h=1}^2 \tilde{A}_h \tilde{\alpha}_h^2 = 0, \\ \sum_{h=1}^2 \tilde{\alpha}_h \tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2 - \tilde{x}_1)} - \beta_2 + \frac{\lambda_2}{\delta+\lambda_2} = 0 \\ \sum_{h=1}^2 \tilde{\alpha}_h^2 \tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2 - \tilde{x}_1)} = 0. \\ \mu_2\beta_2 - (\delta+\lambda_2)K_2 + \lambda_2(K_1 + \tilde{x}_2 - \tilde{x}_1) = 0, \\ \mu_1 - (\delta+\lambda_1)K_1 + \lambda_1 \left(K_2 + \frac{\lambda_2}{\delta+\lambda_2}(\tilde{x}_1 - \tilde{x}_2)\right) = 0, \end{cases} \tag{20}$$

with $B_j = A_j \lambda_1^{-1} G_1(\alpha_j)$*. Moreover, assume that*

$$\begin{cases} \sum_{j=1}^4 A_j \alpha_j^3 > 0, \\ \sum_{j=1}^4 B_j \alpha_j^3 > 0, \\ \sum_{j=1}^4 A_j \alpha_j^3 e^{\alpha_j(x_R - \tilde{x}_1)} > 0, \\ \sum_{j=1}^4 B_j \alpha_j^3 e^{\alpha_j(x_R - \tilde{x}_1)} > 0. \end{cases} \tag{21}$$

Then, $v''(\cdot, i) < 0$ *in* $(x_R, \tilde{x}_i)$ *and* (16) *and* (17) *solve* (9) *in a classical sense.*

2. *Consider the framework in Point 1 and assume that*

$$\sum_{j=1}^4 B_j \alpha_j e^{\alpha_j(x_R - \tilde{x}_1)} - 1 > 0. \tag{22}$$

[5] Assuming that a solution to this system exists.

Then, $x_2^* \in (x_R, \tilde{x}_2]$ *is the unique solution of*

$$\mathbf{1}_{[x_R,\tilde{x}_1]}(x_2^*)\sum_{j=1}^{4} B_j\alpha_j e^{\alpha_j(x_2^*-\tilde{x}_1)} + \mathbf{1}_{(\tilde{x}_1,\tilde{x}_2]}(x_2^*)\left(\frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2}\tilde{\alpha}_h\tilde{A}_h e^{\tilde{\alpha}_h(x_2^*-\tilde{x}_1)}\right) - 1 = 0. \tag{23}$$

3. *Consider the framework in Points 1 and 2 and set* $\Gamma_i^+ = 0$*. Moreover, assume that* $\tilde{x}_1, \tilde{x}_2$ *solve the following algebraic system:*

$$\begin{cases} \sum_{j=1}^{4} A_j\left(e^{\alpha_j(x_R-\tilde{x}_1)}-1\right) + (\tilde{x}_1 - x_R) + \kappa = 0, \\ \frac{\lambda_2\tilde{x}_1}{\delta+\lambda_2} - \sum_{j=1}^{2}\tilde{A}_j(1-e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}) - \sum_{j=1}^{4} B_j\left(e^{\alpha_j(x_R-\tilde{x}_1)}-1\right) + \\ -\left(\frac{\lambda_2\tilde{x}_1}{\delta+\lambda_2} + \sum_{j=1}^{2}\tilde{A}_j(1-e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}) + \sum_{j=1}^{4} B_j\left(e^{\alpha_j(x_2^*-\tilde{x}_1)}-1\right)\right)\mathbf{1}_{(x_R,\tilde{x}_1)} + \\ -\left(\frac{\lambda_2 x_2^*}{\delta+\lambda_2} + \sum_{j=1}^{2}\tilde{A}_j\left(e^{\tilde{\alpha}_j(x_2^*-\tilde{x}_1)} - e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}\right)\right)\mathbf{1}_{(\tilde{x}_1,\tilde{x}_2)} - (x_2^* - x_R) + \kappa = 0. \end{cases} \tag{24}$$

Then, (16) *and* (17) *satisfy the boundary conditions* (13).

Proof. See Appendix A.1. □

If we suppose that all the above assumptions hold, then v equals the value function. In that case, we can use this proposition to determine the bank's optimal strategy by solving a system of algebraic equations. The procedure is as follows.

The bank only pays dividends in the state i when its reserves reach $\tilde{x}_i$. For a given $x_2^* \in (x_R, \tilde{x}_2]$, the payout thresholds solve the system in (24). When reserves reach x_R, liquidation is never optimal ($\hat{\tau}_l = \infty$ or, equivalently, $\hat{b}_n = 1$), provided that κ is sufficiently small to ensure that (13) is positive. If that is the case, the optimal recapitalisation in state i injects equity until the reserves reach the target level x_i^*. Therefore, $\hat{G}(i) = x_i^* - x_R$.

Since $v(\cdot, i)$ is differentiable, we can use (13) and the boundary condition $v(\tilde{x}_1, 1) = 1$ to obtain that $x_1^* = \tilde{x}_1$ and $x_2^* \in (x_R, \tilde{x}_2]$ as the unique solution of (23). In other words, the bank finds it optimal to recapitalize exactly to its dividend payout threshold in the good state and *below* the payout threshold in the bad state ($x_2^* \leq \tilde{x}_2$) because the marginal value

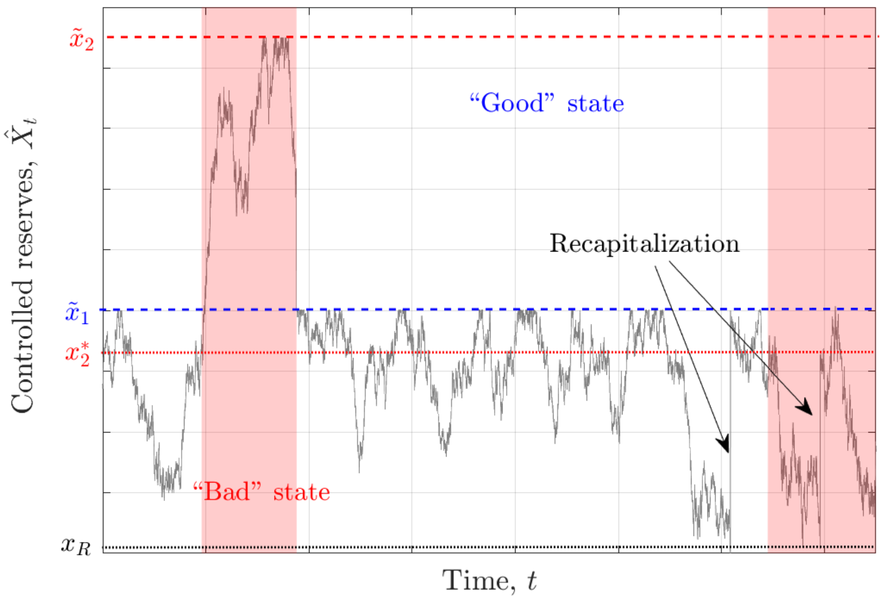

Figure 1: One possible path of the bank's controlled reserves.

of dividends (β_2) is less than one. Since v is concave, $v(\tilde{x}_2, 2) = \beta_2 \leq v(x_2^*, 2) = 1$, and each state has unique payout threshold and recapitalisation target.

For a given $\hat{G}(i)$, the following condition defines the maximum cost level that ensures that recapitalisation is incentive-compatible:

$$\bar{\kappa} = x_R + \min\left\{v(x_1^*, 1) - x_1^*, v(x_2^*, 2) - x_2^*\right\}. \tag{25}$$

To visualize the optimal strategy, Figure 1 displays one possible path of the bank's controlled reserves process $\hat{X}_t$. The process evolves as an arithmetic Brownian motion until it hits the requirement threshold x_R or a dividend payout threshold $\tilde{x}_i$. In the former case, recapitalisation generates a jump to $\tilde{x}_1$ or x_2^*, depending on whether $i = 1$ or 2. The process is "reflected" at the boundary in the latter case. When a state change occurs, dividends are immediately paid if the current reserve level is above the payout threshold of the incoming state.

The next theorem verifies that v is indeed the value function and formally expresses the optimal strategy we described above.

Theorem 1 (**Verification**). *Let all the assumptions of Proposition 1 hold and let* $v(\cdot, i)$ *be the functions constructed therein. Moreover, let* $(x, i) \in (x_R, \infty) \times \{1, 2\}$. *Then,* $v(x, i) = V(x, i)$ *and the control* $\hat{A} = (\hat{Z}, (\hat{b}, \hat{G})) \in \mathcal{A}$ *such that*

$$\begin{cases} \hat{b}_n = 1, \\ \hat{G}_n = x^*_{I_{\hat{\tau}_n^-}} - x_R, \\ \hat{Z}_t = \hat{Z}_{\tau_n} + \sup_{s \in [\hat{\tau}_n, t)} \left[x^*_{I_{\hat{\tau}_n^-}} + \int_{\hat{\tau}_n}^s (\mu_{I_r} dr + \sigma_{I_r} dW_r) - \tilde{x}_{I_{\hat{\tau}_n^-}} \right]^+, \qquad t \in [\hat{\tau}_n, \hat{\tau}_{n+1}), \end{cases} \tag{26}$$

where $\hat{\tau}_0 := 0$ *and* $\hat{\tau}_n$ *is defined recursively as* $\hat{\tau}_{n+1} = \inf\{t \geq \hat{\tau}_n : \ \hat{X}_{t^-} = x_R\}$ *being* $\hat{X}_t$ *the associated state process, is optimal.*

Proof. See Appendix A.2. □

Remark 1. ***(Dividend tax threshold)*** *The solution structure described in Proposition 1 and Theorem 1 holds under the parametric restriction that dividend taxes are not too high, i.e.* $\beta_2 > \lambda_2/(\lambda_2 + \delta)$. *In contrast, when* $\beta_2 \leq \lambda_2/(\lambda_2 + \delta)$, *an inspection of the conditions set shows that the solution structure used to obtain them breaks down. The same conditions suggest that the guess should feature an alternative structure, with* $\tilde{x}_2 = \infty$. *The correct structure has* $\mathcal{C}_2 = (x_R, \infty)$, *regardless of the value of* $\beta_2 \in [0, \lambda_2/(\lambda_2 + \delta))$. *A detailed discussion of this case appears in Appendix A.4.*

Remark 1 clarifies that, when taxes are excessively high in the bad state, the bank finds it optimal to postpone dividend payments until the good state materializes. Therefore, setting $\beta_2 \leq \lambda_2/(\delta + \lambda_2)$ is equivalent to imposing a dividend ban in the bad state. The level of β_2 that triggers the "ban-like" behaviour increases with the probability of transitioning from the bad state to the good state (λ_2) and decreases with the manager's impatience (δ).

4.2 Capital buffers distribution

This section derives the probability density function (pdf) of the bank reserves process in state $i = 1, 2$, denoted as $\pi(x, i)$.[6] We will use this object to evaluate the effects of dividend taxes and capital regulation on the bank's capital buffers in Section 5.

Since the dynamics of reserves obey the controlled process (26), standard arguments can be applied to show that $\pi(x, i)$ satisfies the following system of Kolmogorov Forward Equations (KFE) over the interval $(x_R, x_2^*) \cup (x_2^*, \tilde{x}_1)$:

$$\begin{cases} \frac{\sigma_1^2}{2}\pi''(x,1) - \mu_1\pi'(x,1) + \lambda_1\left(\pi(x,2) - \pi(x,1)\right) = 0, \\ \frac{\sigma_2^2}{2}\pi''(x,2) - \mu_2\pi'(x,2) + \lambda_2\left(\pi(x,1) - \pi(x,2)\right) = 0, \end{cases} \tag{27}$$

with boundary conditions $\pi(x_2^{*-}, i) = \pi(x_2^{*+}, i)$ (value matching), $\pi'(x_2^{*-}, 1) = \pi'(x_2^{*+}, 1)$ (smooth pasting). By the same logic, in the interval $(\tilde{x}_1, \tilde{x}_2)$ the density function in state 2 satisfies

$$\frac{\sigma_2^2}{2}\pi''(x,2) - \mu_2\pi'(x,2) - \lambda_2\pi(x,2) = 0, \tag{28}$$

with boundary conditions $\pi(\tilde{x}_1^-, 2) = \pi(\tilde{x}_1^+, 2)$ and $\pi'(\tilde{x}_1^-, 2) = \pi'(\tilde{x}_1^+, 2)$. These conditions imply that $\pi(\cdot, i)$ is C^1 in all interior regions when both $i = 1, 2$, except for $\pi(\cdot, 2)$ at x_2^*, which is the mass point where reserves accumulate after each recapitalisation when $i = 2$. To characterize the pdf at $\tilde{x}_1$ and $\tilde{x}_2$, we impose the following reflecting barriers:[7]

$$\frac{\sigma_i^2}{2}\pi'(\tilde{x}_i, i) - \mu_i\pi(\tilde{x}_i, i) = 0, \text{ for } i = 1, 2. \tag{29}$$

Consequently, we set $\pi(\cdot, i) = 0$ in the interval $(\tilde{x}_i, \infty)$ for $i = 1, 2$.

To characterize the pdf at the regulatory threshold, we impose that the reserves process is "absorbed" at x_R, in the sense that it is immediately and irreversibly transported to

[6] Throughout this section, we consider the case $x_2^* \leq \tilde{x}_1 \leq \tilde{x}_2$. Analogous formulas can be derived when $x_2^* > \tilde{x}_1$.

[7] For a formal derivation of reflecting barriers for controlled diffusion process, we refer to Cox and Miller (1965), Chapter 5.

the interior states x_i^* (see Yaegashi et al., 2019, for a discussion of similar conditions in a uni-variate setting). Thus, we set $\pi(x_R, i) = 0$ for $i = 1, 2$. Finally, we impose

$$\sum_{h=1}^{2} \frac{\lambda_{3-h}}{\lambda_1 + \lambda_2} \int_{x_R}^{\infty} \pi(x, h) dx = 1, \tag{30}$$

where $\lambda_h/(\lambda_1 + \lambda_2) = 1 - \mathbb{P}\{i = h\}$, because $\pi(x, i)$ is a pdf. Solving (27) and (28) under these conditions yields the following.

Proposition 2. ***(Capital buffers probability density function)*** *Fix $\tilde{x}_1, \tilde{x}_2$, and x_2^* such that $x_R < x_2^* < \tilde{x}_1 < \tilde{x}_2$.*[8] *Then, the pdf of the bank's reserves in state $i = 1, 2$ equals*

$$\pi(x, 1) = \begin{cases} P_1 e^{r_1 x} + P_2 e^{r_2 x} + P_3 e^{r_3 x} + P_4 e^{r_4 x}, & x \in (x_R, x^*), \\ \tilde{P}_1 e^{r_1 x} + \tilde{P}_2 e^{r_2 x} + \tilde{P}_3 e^{r_3 x} + \tilde{P}_4 e^{r_4 x}, & x \in (x^*, \tilde{x}_1), \\ 0, & x \in (\tilde{x}_1, \infty), \end{cases}$$

$$\pi(x, 2) = \begin{cases} Q_1 e^{r_1 x} + Q_2 e^{r_2 x} + Q_3 e^{r_3 x} + Q_4 e^{r_4 x}, & x \in (x_R, x^*), \\ \tilde{Q}_1 e^{r_1 x} + \tilde{Q}_2 e^{r_2 x} + \tilde{Q}_3 e^{r_3 x} + \tilde{Q}_4 e^{r_4 x}, & x \in (x^*, \tilde{x}_1), \\ H_1 e^{s_1 x} + H_2 e^{s_2 x}, & x \in (\tilde{x}_1, \tilde{x}_2), \\ 0, & x \in (\tilde{x}_2, \infty), \end{cases}$$

in which $r_1 < r_2 < 0 < r_3 < r_4$ and $s_1 < 0 < s_2$ are the real roots of

$$\underbrace{\left(\lambda_1 + r\mu_1 - \frac{\sigma_1^2}{2} r^2\right)}_{:=F_1(r)} \underbrace{\left(\lambda_2 + r\mu_2 - \frac{\sigma_2^2}{2} r^2\right)}_{:=F_2(r)} = \lambda_1 \lambda_2, \tag{31}$$

$$\frac{\sigma_2^2}{2} s^2 - \mu_2 s = \lambda_2, \tag{32}$$

[8] Similar expressions can be obtained for the case in which $x_R < \tilde{x}_1 < x_2^* < \tilde{x}_2$.

and the constants $(P_1, P_2, P_3, P_4, \tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \tilde{P}_4, H_1, H_2) \in \mathbb{R}^{10}$ *solve the linear system*

$$\begin{cases} \sum_{j=1}^{4} \tilde{P}_j \left(\frac{\sigma_1^2}{2} r_j - \mu_1\right) e^{r_j \tilde{x}_1} = 0, \\ \sum_{h=1}^{2} H_h \left(\frac{\sigma_2^2}{2} s_h - \mu_2\right) e^{s_h \tilde{x}_2} = 0, \\ \sum_{j=1}^{4} P_j e^{r_j x_R} = 0, \\ \sum_{j=1}^{4} Q_j e^{r_j x_R} = 0, \\ \sum_{j=1}^{4} e^{r_j x_2^*} \left(P_j - \tilde{P}_j\right) = 0, \\ \sum_{j=1}^{4} e^{r_j x_2^*} r_j \left(P_j - \tilde{P}_j\right) = 0, \\ \sum_{j=1}^{4} e^{r_j x_2^*} \left(Q_j - \tilde{Q}_j\right) = 0, \\ \sum_{h=1}^{2} H_h e^{s_h \tilde{x}_1} - \sum_{j=1}^{4} \tilde{Q}_j e^{r_j \tilde{x}_1} = 0, \\ \sum_{h=1}^{2} H_h s_h e^{s_h \tilde{x}_1} - \sum_{j=1}^{4} \tilde{Q}_j r_j e^{r_j \tilde{x}_1} = 0, \\ \sum_{h=1}^{2} \lambda_{3-h} \int_{x_R}^{\tilde{x}_i} \pi(x,i) dx - \lambda_1 - \lambda_2 = 0, \end{cases} \tag{33}$$

where $Q_j = \lambda_1^{-1} F_1(r_j) P_j$ *and* $\tilde{Q}_j = \lambda_1^{-1} F_1(r_j) \tilde{P}_j$.

Proof. See Appendix A.3. □

5 Policy implications

In this section, we parameterize the model and numerically analyse its policy implications by examining the effects of dividend taxes on optimal controls, shareholder value, and the resulting capital buffer distribution. We then extend the baseline model to incorporate counter-cyclical capital requirements and examine their interaction with dividend regulation.

5.1 Parameters

We normalize the stock of deposits $D = 1$ and choose L to match the US bank deposit-to-loan ratio in Q4 2022 (about 1.5385), according to S&P Global. According to FRED, we set

Parameter	Meaning	Value
μ_1	CF drift, good state	0.05
μ_2	CF drift, bad state	0.02
σ_1	CF vol, good state	0.25
σ_2	CF vol, bad state	0.3
κ	recapitalisation cost	0.087
δ	Discount rate	0.03
x_R	Capital requirement	0.0769
ρ	Return on deposits	0.0043
α	Haircut	0.6
β_2	Dividend regulation	0.87
$\frac{L}{D}$	Loan-to-deposit ratio	1.5385
$1/\lambda_1$	Avg duration, good state	10
$1/\lambda_2$	Avg duration, bad state	6.7
Γ_1, Γ_2	Capital buffers	0

Table 1: Baseline parameters

the rate of return on deposits at $\rho = 0.0043$. Consistently, we obtain $x_R = 1 - 0.6 \times 1.5385 = 0.0769$. We calibrate the fixed recapitalisation cost $\kappa = 0.087$ to yield a price-to-book ratio at the dividend payout threshold in the good state $(v(\tilde{x}_1, 1)/(L+\tilde{x}_1 - D))$ of about 1.04. This is the value observed across US commercial banks, according to the NYU Stern database. The drift and diffusion parameters, $\bar{\mu}_i$ and σ_i, and the discount rate δ are similar to Guo et al. (2005). The intensities of regime changes λ_i and the hair cut α come from Hackbarth et al. (2006). Lastly, we set $\beta_2 = 0.87$, corresponding to a 10 p.p. increment in the dividend tax rate from 0.3 in the good regime to 0.4 in the bad one.[9]

5.2 The effects of dividend taxes

5.2.1 Bank's optimal control and shareholder value

To evaluate the effect of state-contingent dividend taxes, we compare the bank's optimal control and shareholder value when $\beta_2 < \beta_1 = 1$ with the "no-policy" scenario where $\beta_2 = \beta_1 = 1$. The red dotted and solid blue lines in Figure 2 display $v(x, i)$ in the good (panel (a))

[9] After normalizing the value of $\beta_1 = 1$, the parameter β_2 can be obtained by solving: $1 - 0.3 = \frac{1-0.4}{\beta_2}$.

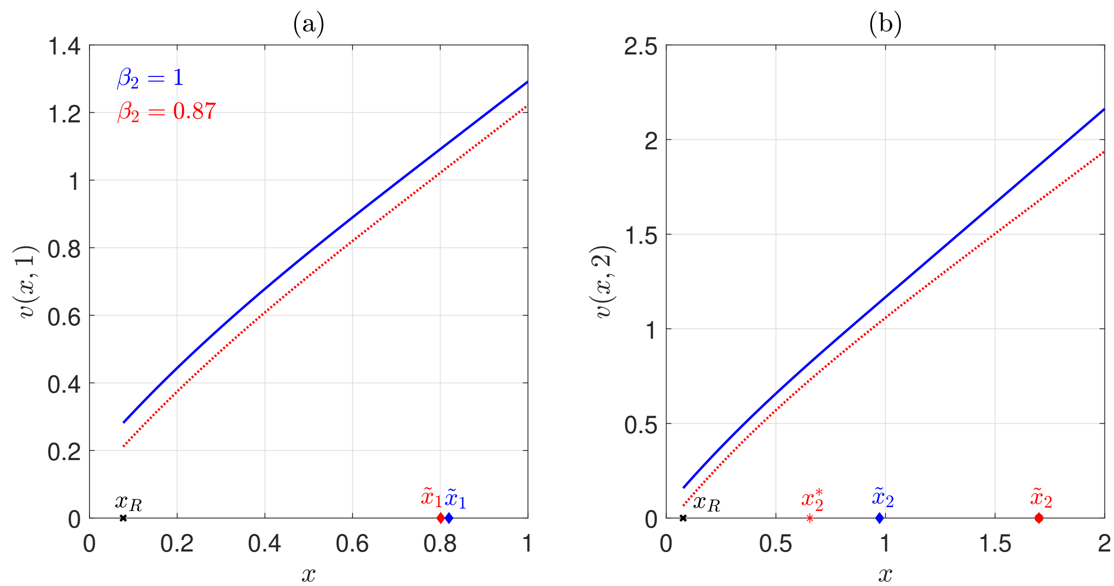

Figure 2: Optimal controls and shareholder value in the good (Panel (a)) and bad (Panel (b)) states when $\beta_2 = 1$ (solid blue) and $\beta_2 = 0.87$ (dotted red).

and bad (panel (b)) states as a function of the reserve level in these two cases. The black crosses and the blue (red) diamonds on the x axis show the regulatory threshold (x_R) and the optimal dividend payout ($\tilde{x}_i$) in the good (bad) state. The red star in panel (b) indicates the optimal recapitalisation target. Four key patterns emerge from the comparison.

First, dividend taxes reduce shareholder value blueat all levels of x and economic states. This is expected because the policy introduces an additional distortion beyond the capital requirement. The value losses are more severe in percentage terms for any reserve value in the bad state than in the good state, as the policy affects the latter only indirectly. Furthermore, value losses are always more severe when capital buffers are lower, except in the bad state, where they increase slightly after reaching a tipping point around $x = 1$.

Second, we discuss the effects of dividend taxes on the banks' optimal control. On the one hand, consistent with its intended purpose, the policy encourages the accumulation of additional capital buffers (i.e., fewer dividend payments) in the bad state, shifting $\tilde{x}_2$ from approximately 0.95 to 1.7. The higher the tax, the more willing the bank is to delay dividend payments. This outcome is apparent in Table 2, which shows the optimal $\tilde{x}_i$ and

β_2	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*
1.000	0.820	0.950	0.950
0.990	0.819	0.941	0.815
0.970	0.814	1.061	0.747
0.950	0.811	1.142	0.724
0.900	0.804	1.421	0.667
0.870	0.804	1.700	0.655
0.850	0.800	2.663	0.649
0.834	0.800	17.024	0.649

Table 2: Optimal dividend threshold and recapitalisation target as functions of β_2.

x_2^* as functions of β_2.[10] On the other hand, the policy "backfires" in the good state, where the bank reduces $\tilde{x}_1$ from 0.82 to 0.80. The reason is that the bank compensates for the tax in the bad state by anticipating dividends in the good one. However, notice that there is a significant asymmetry in the magnitude of the threshold shifts between the good and bad states. In our numerical analysis, enforcing dividend restrictions reduces the payout threshold in the good state by only 2.5%, while it increases it by more than 30% in the bad state.

Third, dividend taxes reduce the bank's optimal recapitalisation targets (x_i^*) in every state. In the good state, this result directly follows the change in the payout dividend threshold $\tilde{x}_1$ because it coincides with the recapitalisation target ($\tilde{x}_1 = x_1^*$). In contrast, $\tilde{x}_2 > x_2^*$ in the bad state because the marginal cost of paying dividends (β_2) is always lower than that of injecting liquid reserves and $v''(x, 2) < 0$ (see (23)). Notably, x_2^* decreases with β_2 (up to a limit value as $\beta_2 \to \frac{\delta+\lambda_2}{\lambda_2}$) and lies *below* the payout threshold in the good state $\tilde{x}_1$. Hence, dividend taxes reduce the bank's willingness to issue equity when reserves hit the regulatory threshold, x_R.

Fourth, the overall influence of the dividend tax policy on the bank's capital buffers ex-ante (i.e., independent of i) is ambiguous because the ways it affects its optimal control are conflicting: while the increase in $\tilde{x}_2$ yields higher capital buffers, the reduction in $\tilde{x}_1$ and

[10]Note that, consistent with Remark 1, the last rows of the table highlight that the bad-state payout threshold $\tilde{x}_2$ "explodes" when $\beta_2 \to \lambda_2/(\delta + \lambda_2)$ ($\approx 0.3\bar{3}$ with our parameters).

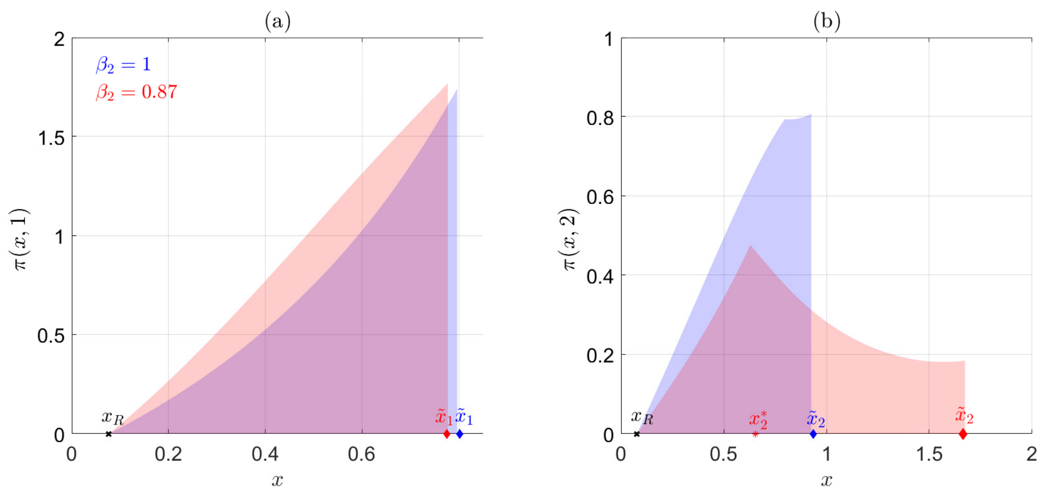

Figure 3: Capital buffers distribution and optimal controls in the good (Panel (a)) and bad (Panel (b)) states when $\beta_2 = 1$ (blue) and $\beta_2 = 0.87$ (red).

x_2^* may curb them. To determine which effect prevails, the following section analyses the impact of the policy on the bank's stationary capital buffers distribution.

5.2.2 Long-run capital buffers and recapitalisation incentives

This section uses the probability density function derived in Proposition 2 to construct a proxy for the bank's long-run "credit capacity". We use such a proxy to evaluate how credit capacity is influenced by dividend policies, both conditionally and unconditionally on the aggregate state of the economy. Although the model features a fixed loan supply, the long-run capital buffers are the amount of available resources that could potentially support new lending. From this point onward, we use the terms "long-run capital buffers" and "credit capacity" interchangeably.

As a first step in the analysis, we examine the shape of the bank's capital buffer distribution, $\pi(x, i)$, across different reserve levels x and aggregate states i. For this purpose, Figure 3 displays $\pi(x, i)$ and the optimal control when $\beta_2 = \beta_1 = 1$ (blue) and $\beta_2 < \beta_1 = 1$ (red).

As a first observation, higher dividend taxes in the bad state do not substantially alter the shape of the pdf in the good state (panel (a)). However, they transfer some probability density to lower reserve levels because the reduction of β_2 shifts the optimal dividend payout threshold $\tilde{x}_1$ to the left, even though the probability of $i = 1$ remains unchanged. In contrast, the tax policy largely affects the shape of the pdf in the bad state (panel (b)). In particular, the dispersion of $\pi(x, 2)$ increases sharply because a lower β_2 shifts $\tilde{x}_2$ to the right, widening the support of the reserves' distribution. Additionally, dividend taxes generate a steep mass point at the recapitalisation target x_2^* because $x_2^* < \tilde{x}_1 < \tilde{x}_2$.[11]

To evaluate the long-run effect of the dividend tax policy, we use $\pi(x, i)$ to define the following measure of the bank's credit capacity, conditional on state i:

$$\mathbb{E}_i^\pi [x] := \int_{x_R}^{\tilde{x}_i} x\pi(x, i)dx. \tag{34}$$

We also define the dispersion of the credit capacity as $\mathbb{V}_i^\pi [x] := \mathbb{E}_i^\pi [x^2] - \mathbb{E}_i^\pi [x]^2$. Consistently, we define the bank's unconditional (or ex-ante) credit capacity as

$$\mathbb{E}^\pi [x] = \sum_{h=1}^{2} \mathbb{E}_h^\pi [x] \underbrace{\frac{\lambda_{3-h}}{\lambda_1 + \lambda_2}}_{=\mathbb{P}\{i=h\}}, \tag{35}$$

with dispersion $\mathbb{V}^\pi [x] = \mathbb{E}^\pi [x^2] - \mathbb{E}^\pi [x]^2$. Table 3 collects the value of these objects using the parameters in Table 1 and different levels of β_2. The analysis delivers the following implications.

First, state-contingent dividend taxes reduce the bank's credit capacity in the good state but enhance it in the bad state. However, the loss in the former is always more than offset by the gains in the latter (Columns 1 and 2). As a result, imposing dividend taxes increases overall credit capacity (Column 3). Second, higher dividend taxes slightly reduce the dispersion of reserves in the good state but significantly increase it in the bad state. Hence, bank credit capacity dispersion increases overall (Columns 5-7).

The relatively small negative impact of dividend taxes on credit capacity in the good state compared to their large positive effects in the bad state, and overall, suggests that the

[11]Notice that, due to the regime-switching, the pdf of x in state 2 displays a small but interior mass point even when $\beta_2 = 1$, coinciding with the dividend payout threshold $\tilde{x}_1$ in State 1.

β_2	$\mathbb{E}_1^\pi[x]$	$\mathbb{E}_2^\pi[x]$	$\mathbb{E}^\pi[x]$	$\mathbb{V}_1^\pi[x]$	$\mathbb{V}_2^\pi[x]$	$\mathbb{V}^\pi[x]$	$\bar{\kappa}$
1.00	0.6007	0.6525	0.6214	0.0372	0.0387	0.0389	0.1351
0.95	0.5758	0.7473	0.6444	0.0284	0.0609	0.0485	0.1187
0.90	0.5698	0.8289	0.6734	0.0280	0.1015	0.0735	0.1090
0.87	0.5677	0.8913	0.6971	0.0279	0.1496	0.1017	0.1057
0.85	0.5669	0.9448	0.7181	0.0278	0.2118	0.1357	0.1043

Table 3: Bank's long-run capital buffers, capital buffers dispersion, and the maximal incentive-compatible recapitalisation cost for different dividend tax parameters β_2.

benefits of the policy outweigh its costs. Indeed, one can verify that dividend taxes make recapitalisation events less frequent in each state i and overall. To this aim, Table 4 reports a numerical approximation of the average waiting time between subsequent recapitalisation events, formally defined as $\hat{\tau}(i) := \inf\{t \geq \tau_n : \hat{X}_t = x_{R,i}, \hat{X}_{\tau_n} = x_i^*\}$ (state-contingent) and $\hat{\tau} := \sum_{i=1}^{2} \hat{\tau}(i) \cdot \lambda_{3-i}/(\lambda_1 + \lambda_2)$ (ex-ante) for different values of β_2.[12] Despite the tax, the average recapitalisation time is longer in the good state than in the bad state. However, the effect of adjusting the recapitalisation policy to a lower level on credit capacity is sizable. Specifically, if we were to ignore the bank's endogenous response in terms of the recapitalisation threshold x_2^* and instead keep it fixed at $\tilde{x}_2$, credit capacity would be nearly 20% higher.

The global evaluation of the dividend tax policy becomes even less straightforward when noticing that, as a result of value losses, higher dividend taxes reduce the maximal equity issuance cost that is incentive-compatible ($\bar{\kappa}$, see (25)). The last column of Table 3 displays this phenomenon, showing that, for example, a 10% increase in dividend taxes is associated with a 28% reduction in the level of $\bar{\kappa}$.

It is relevant to stress that all the figures reported in the table exceed the baseline cost level adopted in our parametrisation ($\kappa = 0.087$), ensuring that the bank always finds it optimal to recapitalise when $x = x_R$. We nevertheless interpret this tightening of the incentive-compatibility constraint as evidence that dividend restrictions may endogenously

[12] The numbers are obtained by averaging 5,000 Monte Carlo simulations of the bank's reserves process under the optimal strategy (26).

β_2	$\hat{\tau}(1)$	$\hat{\tau}(2)$	$\hat{\tau}$
1.000	11.89	11.56	11.76
0.970	12.44	11.99	12.26
0.870	13.06	12.66	12.90
0.834	13.12	12.74	12.97

Table 4: Average waiting time (years) between subsequent recapitalisations for different levels of β_2.

increase default risk.

5.2.3 Comparative statics

Table 5 presents a comparative static analysis which shows that our results hold qualitatively under substantial variations in the main parameters of the model. A higher drift in the good or bad states lowers the payout thresholds because it releases the bank's precautionary motive (Rows 3-4). Higher levels of cash flow volatility generate the opposite effect (see, e.g., Row 2). Decreasing the probability of visiting the bad state (λ_1) fosters dividend payouts without the tax while mitigating the additional payout incentive in the good state induced by the tax. Moreover, it increases the maximum incentive-compatible cost level while leaving the optimal recapitalisation target unaffected. The probability of transitioning from the bad to the good state (λ_2) has similar effects on $\tilde{x}_1$, x_2^*, and $\bar{\kappa}$, while also leading to a further postponement of dividends in the bad state (Row 6).

Another aspect worth highlighting concerns the effects of a change in the regulatory parameter defined in (3), Γ_i, for $i = 1, 2$. The last two rows of the Table 5 examine the cases where $\Gamma_1 = \Gamma_2 = \Gamma = \pm 0.05$.[13] Figure 4 displays the effect of changing Γ on the value function.

As intuition suggests, tighter (looser) capital requirements are associated with higher (lower) recapitalisation thresholds and lower (higher) bank valuations. Furthermore, changes in Γ do not affect the maximum incentive-compatible recapitalisation costs. This occurs be-

[13]This analysis will also serve as a benchmark when we discuss cyclical capital regulation in Section 5.3.2.)

	$\tilde{x}_1$		$\tilde{x}_2$		x_2^*		$\bar{\kappa}$	
β_2	1.00	0.87	1.00	0.87	1.00	0.87	1.00	0.87
Baseline	0.821	0.801	0.954	1.700	0.954	0.655	0.135	0.106
$\sigma_2 = 0.35$	0.827	0.811	1.051	1.938	1.051	0.710	0.118	0.087
$\mu_1 = 0.07$	0.794	0.776	0.952	1.698	0.952	0.653	0.288	0.251
$\mu_2 = 0.03$	0.820	0.798	0.941	1.651	0.941	0.637	0.194	0.157
$\lambda_1 = 0.05$	0.816	0.806	0.954	1.700	0.954	0.655	0.186	0.160
$\lambda_2 = 0.2$	0.821	0.804	0.950	3.324	0.950	0.672	0.164	0.141
$\Gamma_1 = \Gamma_2 = 0.05$	0.871	0.851	1.007	1.750	1.007	0.705	0.133	0.106
$\Gamma_1 = \Gamma_2 = -0.05$	0.769	0.751	0.923	1.650	0.923	0.605	0.133	0.106

Table 5: Comparative statics analysis.

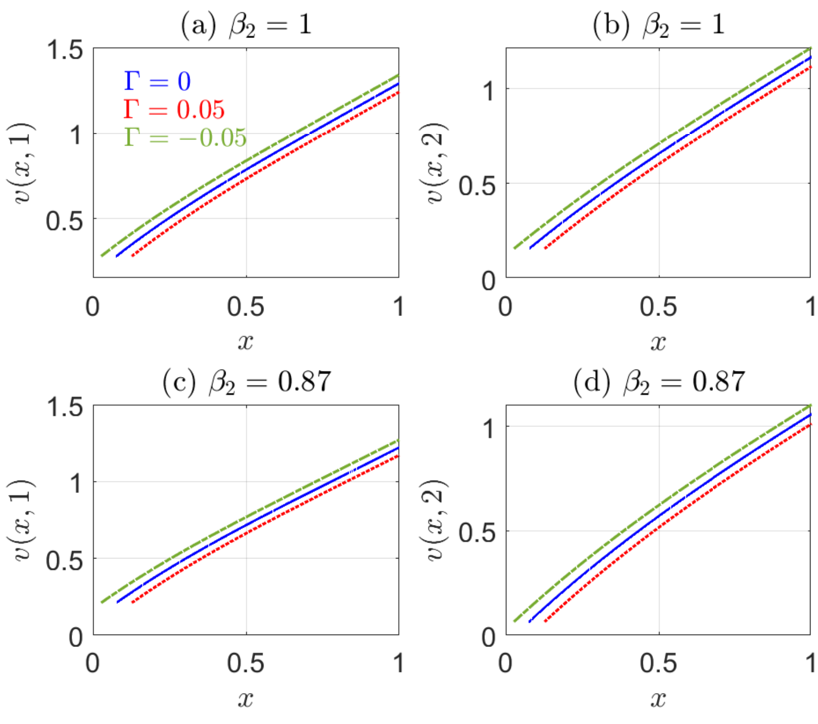

Figure 4: Shareholder value in the good (left panels) and the bad states (right panels) for different combinations of Γ when $\beta_2 = 1$ (top panels) and $\beta_2 = 0.87$ (bottom panels).

cause shareholder value shifts in the opposite direction of the change in capital requirements.

Although relatively straightforward, these effects are significant as they suggest regulators can mitigate shareholder value losses–one of the adverse impacts of the dividend tax policy–by adjusting capital requirements simultaneously. We explore this aspect further in the next section.

5.3 Coordinating dividend taxes and capital regulation

This section extends the model to incorporate counter-cyclical capital requirements and discusses how this modification affects the structure of the solution. It then explores numerically the policy challenge of coordinating dividend taxation and capital regulation. This analysis is motivated by the fact that, while recommending dividend suspensions, the ECB temporarily eased capital requirements for banks during the COVID-19 crisis (Matyunina and Ongena, 2022).

While our framework can accommodate pro-cyclical capital requirements, we focus on the special case in which they are counter-cyclical. This choice is justified by the Basel III framework and by the extensive literature that shows how pro-cyclical buffers may destabilise financial institutions and thus exacerbate crises (see, for example Repullo and Suarez, 2013; Valencia and Bolanos, 2018, and the references therein). In the supplementary material, we demonstrate how to incorporate pro-cyclical capital requirements into our model. There, we also show that, consistent with the aforementioned literature, counter-cyclical capital requirements can mitigate the unintended consequences of dividend taxes, whereas pro-cyclical requirements can only exacerbate them.

5.3.1 Solution structure with counter-cyclical capital requirements

To model counter-cyclical capital requirements, we consider $\Gamma_1 > \Gamma_2$ or, equivalently, $x_{R,1} > x_{R,2}$. This assumption requires adjusting the solution structure of the model by considering the additional region $x \in (x_{R,2}, x_{R,1})$ in the state space, as shown in Figure

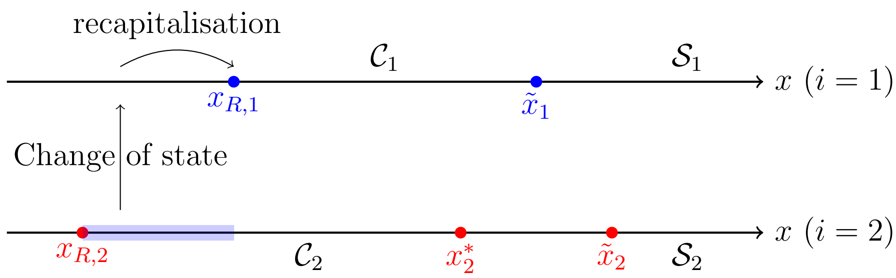

Figure 5: Solution structure with counter-cyclical capital requirements.

5. The regulator requires the bank to immediately recapitalise (or liquidate) in this region should the economy shift from state 2 to state 1. Accordingly, we set

$$v(x,1) = \max\left\{v(\tilde{x}_1,1) - (\tilde{x}_1 - x) - \kappa, 0\right\}, \tag{36}$$

and find $v(x,2)$ as the unique solution of

$$\frac{1}{2}\sigma_2^2 v''(x,2) + \mu_2 v'(x,2) - (\delta + \lambda_2)\, v(x,2) + \lambda_2 v(x,1) = 0, \tag{37}$$

with boundary conditions $v(x_{R,1}^+,2) = v(x_{R,1}^-,2)$ and $v'(x_{R,1}^+,2) = v'(x_{R,1}^-,2)$.

Since the bank always finds it optimal to recapitalize when κ is small enough (see (36)), in the remaining regions (i.e., when $x > x_{R,1}$), the value function equals the one described in Section 4.1 after setting $x_{R,1} = x_R$. Hence, we can still characterise the model's solution analytically by solving a system of algebraic equations. We report on the details in the supplementary material.

5.3.2 The effects of counter-cyclical capital requirements

We now numerically examine the effects of counter-cyclical capital requirements on the bank's optimal controls and their interaction with dividend taxes. We adopt as a benchmark the a-cyclical case discussed in Section 4 ($\Gamma_1 = \Gamma_2 = \Gamma$), assuming that the regulator sets Γ_i to maintain a constant *mean* capital requirement x_R across states. In particular, we choose

$x_{R,1}$	$x_{R,2}$	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*	$\mathbb{E}_1^\pi[x]$	$\mathbb{E}_2^\pi[x]$	$\mathbb{E}^\pi[x]$	$\mathbb{V}_1^\pi[x]$	$\mathbb{V}_2^\pi[x]$	$\mathbb{V}^\pi[x]$	$\bar{\kappa}$
0.077	0.077	0.801	1.700	0.655	0.568	0.891	0.697	0.028	0.150	0.102	0.106
0.087	0.062	0.806	1.688	0.643	0.571	0.884	0.696	0.028	0.148	0.099	0.102
0.097	0.047	0.812	1.677	0.631	0.579	0.868	0.694	0.028	0.150	0.097	0.098
0.117	0.017	0.824	1.656	0.609	0.600	0.847	0.693	0.027	0.151	0.092	0.087

Table 6: Bank optimal controls, credit capacity and dispersion, and maximal incentive-compatible recapitalisation costs for different levels of Γ ($x_{R,1}$ and $x_{R,2}$).

$\Gamma_1 = \Gamma > 0$ and $\Gamma_2 = -\Gamma_1$, such that[14]

$$x_R = \underbrace{(D - \alpha L + \Gamma)}_{=x_{R,1}} \underbrace{\frac{\lambda_2}{\lambda_1 + \lambda_2}}_{=\mathbb{P}\{i=1\}} + \underbrace{(D - \alpha L - \Gamma)}_{=x_{R,2}} \underbrace{\frac{\lambda_1}{\lambda_1 + \lambda_2}}_{=\mathbb{P}\{i=2\}}. \tag{38}$$

Table 6 reports the bank's optimal controls, credit capacity and dispersion, and the maximal incentive-compatible recapitalisation cost for different levels of Γ. Row 1 displays the benchmark case where $\Gamma = 0$. We obtain the following predictions.

First, adopting counter-cyclical capital requirements increases the bank's credit capacity in the good state (Columns 3 and 6), while reducing it in the bad state (Columns 4 and 6) and overall (Column 8), relative to the benchmark. When both dividend taxes *and* capital regulation are used, the bank's unconditional credit capacity remains higher than when dividend regulation is not applied (i.e., $\beta_2 = \beta_1 = 1$). Therefore, coordinating the two policies enables the imposition of dividend taxes without curtailing credit capacity in the good state. Table 6 supports this claim.

The second effect of the policy is to reduce the dispersion of capital buffers, even after accounting for its (lower) expected value. For example, our simulations show that the coefficient of variation of the reserves distribution ($\sqrt{\mathbb{V}^\pi[x]}/\mathbb{E}^\pi[x]$) decreases by approximately 3.2% when $x_{R,1} = 0.117$ and $x_{R,2} = 0.017$, compared to when $x_{R,1} = x_{R,2} = 0.077$.

[14] The appendix examines a simpler case where the regulator relaxes capital requirements only in the bad state (i.e., $x_{R,1} = x_R$ and $x_{R,2} = x_{R,1} - \Gamma_2$ with $\Gamma_2 > 0$). As expected, the policy lowers the dividend payout threshold, diminishes recapitalisation incentives, and reduces average credit capacity and dispersion across states. Furthermore, it enhances the bank's value in both states.

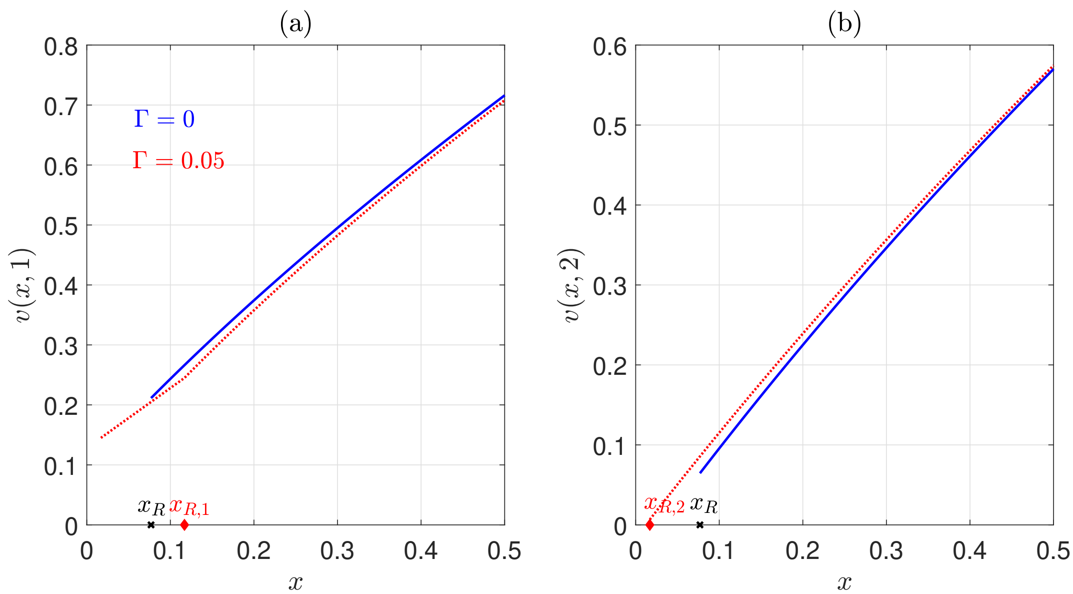

Figure 6: Effect of counter-cyclical capital requirements on shareholder value in the good (Panel (a)) and bad (Panel (b)) states.

Figure 6 displays the model's third prediction by showing the bank's value function for $\Gamma = 0$ ($x_{R,1} = x_{R,2} = 0.077$) and $\Gamma = 0.05$ ($x_{R,1} = 0.117$, $x_{R,2} = 0.017$).[15] The plot reveals that relaxing the capital requirement in the bad state helps mitigate shareholder value losses, which can be substantial when dividends are taxed (panel (b)), as discussed in the previous sections. However, the gain comes at the cost of reducing bank value in the good state (panel (a)). These outcomes arise because tighter (looser) capital requirements curb (encourage) banks' precautionary motives in the good (bad) state, leading them to anticipate (delay) their dividend payments (columns 3 and 4).

These results suggest that coordinating dividend taxes (or bans) with counter-cyclical capital regulation can mitigate some of the adverse effects of the former policy. From a policy standpoint, this provides a theoretical foundation for the regulator's decision to pair dividend restrictions with looser capital requirements during the COVID-19 crisis. However, the positive outcomes of policy coordination come with a cautionary note. Specifically, the model's fourth policy prediction shows that adopting counter-cyclical capital requirements

[15]Computing plots for different values of Γ produces qualitatively similar results.

$\tilde{x}_1$	$\tilde{x}_2$	x_2^*	$\mathbb{E}_1^\pi[x]$	$\mathbb{E}_2^\pi[x]$	$\mathbb{E}^\pi[x]$	$\mathbb{V}_1^\pi[x]$	$\mathbb{V}_2^\pi[x]$	$\mathbb{V}^\pi[x]$	$\bar{\kappa}$
0.801	1.700	0.655	0.568	0.891	0.697	0.028	0.150	0.102	0.106
0.801	1.700	0.954	0.570	0.910	0.706	0.028	0.150	0.102	0.106
0.801	1.700	1.70	0.570	1.212	0.827	0.028	0.150	0.102	0.106

Table 7: Bank optimal controls, credit capacity and dispersion, and maximal incentive-compatible recapitalisation costs for different levels of Γ ($x_{R,1}$ and $x_{R,2}$).

$x_{R,1}$	$x_{R,2}$	$\hat{\tau}(1)$	$\hat{\tau}(2)$	$\hat{\tau}$
0.077	0.077	13.06	12.66	12.90
0.087	0.062	13.11	12.97	13.05
0.097	0.047	13.22	12.94	13.10
0.117	0.017	13.38	12.84	13.16

Table 8: Average waiting time (years) between each subsequent recapitalisation, in each State i and overall, for different levels of Γ when $\beta_2 = 0.87$.

lowers the optimal recapitalisation target in the bad state (x_2^*). Moreover, the maximal incentive-compatible recapitalisation cost, $\bar{\kappa}$ (see Columns 5 and 12), decreases. These effects occur because, although the policy increases the bank's value for all states above x_R, it reduces the value *at* $x_{R,2} < x_R$, which plays a critical role in determining the optimal recapitalisation (see (13)).

The final step of the analysis assesses the effect of counter-cyclical capital regulation on the average waiting time (in years) after each recapitalisation. Table 8 presents a numerical approximation of these quantities conditional on each state i and overall for various levels of Γ when $\beta_2 = 0.87$. According to these simulations, coordinating counter-cyclical dividend taxes and capital requirements can effectively reduce the frequency of recapitalisation (see columns 3-5). However, when the policy becomes excessively cyclical, its effectiveness during downturns diminishes (see column 4). This occurs because when $x_{R,2}$ becomes too low, the positive effect on capital buffers from a higher $\tilde{x}_1$ in the good state is partially offset by the increasingly negative impact of having lower $\tilde{x}_2$ and x_2^* in the bad state (see Table 6).

6 Conclusion

We have modelled and solved the optimal control problem of a bank which takes dividends and re-capitalisation decisions under macroeconomic uncertainty, cyclical capital requirements, and dividend taxes (or bans). Our framework provides several testable policy implications, complementing recent empirical literature on the (short-term) effects of bank dividend suspension policies in the EU and the US.

First, the model predicts that state-contingent dividend taxes (or bans) negatively affect shareholder value not only during crises (ex post) but also in good times (ex ante). Second, taxing dividends in bad macroeconomic states incentivises the bank to pay out more in good times, reducing its corresponding capital buffers (credit capacity). This creates a trade-off between maintaining capital buffers (credit capacity) in good versus bad macroeconomic conditions. Third, dividend taxes may lead to dispersion in the bank's capital buffers over the long term. Furthermore, they may undermine financial stability by diminishing the bank's recapitalisation incentives. Policymakers can coordinate dividend restrictions with counter-cyclical capital requirements to reallocate value losses and credit capacity between good and bad states. However, lower capital requirements could potentially exacerbate disincentives for recapitalisation.

Similar to other studies in the literature, our tractability assumptions carry a few limitations. For example, our bank's optimisation problem does not include investments and assumes fixed loans and deposits. As a result, even though our analysis of credit capacity proxies the potential support the bank may provide to the real economy under different policies, it does not fully capture how that interacts with its risk-taking incentives. Another limitation is that the bank's optimal responses in our model perfectly anticipate the policy the regulator will enact in each possible state. A non-trivial extension of the model could consider the case where, if the economy deteriorates, the regulator may refrain from intervening with some probability. Although such extensions are beyond the scope of this paper, they open promising avenues for future research.

Finally, we believe this paper can serve as a first step towards understanding the general equilibrium implications of the joint adoption of dividend and capital regulations by focusing on the endogenous response of a single bank to a system-wide restriction. Although we interpret the stationary distribution of reserves (capital buffers) as representing heterogeneity across similar banks, analysing how a pool of heterogeneous banks responds to this policy mix is a complex and interesting research question that naturally follows from our contribution.

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A Proofs and derivations

A.1 Proof of Proposition 1

1. We divide the proof into the following steps.

   (i) Integrating $v'(\cdot, i) = \beta_i$ in $[\tilde{x}_i, x]$ yields the expression of $v(\cdot, i)$ in the interval $[\tilde{x}_i, \infty)$ with $K_i := v(\tilde{x}_i, i) > 0$.

   (ii) In the interval $(\tilde{x}_1, \tilde{x}_2)$, our guess is that $v(\cdot, 1)$ solves (14). Hence, it has the following structure:

$$v'(x, 2) = \frac{\lambda_2}{\delta + \lambda_2} + \sum_{h=1}^{2} \tilde{\alpha}_h \tilde{A}_h e^{\tilde{\alpha}_h (x - \tilde{x}_1)}. \tag{39}$$

   with $\tilde{\alpha}_i$ as in (19). By imposing $v(\cdot, 2) \in C^2$ at $x = \tilde{x}_2$, we obtain that $v'(\tilde{x}_2, 2) =$

β_2 and $v''(\tilde{x}_2, 2) = 0$, that reflect in the conditions

$$\sum_{h=1}^{2} \tilde{\alpha}_h \tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2 - \tilde{x}_1)} = \beta_2 - \frac{\lambda_2}{\delta + \lambda_2}, \qquad \sum_{h=1}^{2} \tilde{\alpha}_h^2 \tilde{A}_h e^{\tilde{\alpha}_h(\tilde{x}_2 - \tilde{x}_1)} = 0. \tag{40}$$

(iii) To show that $v''(\cdot, 2) < 0$ in $(\tilde{x}_1, \tilde{x}_2)$, we set $u(s) = v'(\tilde{x}_2 - s, 2)$ and $w(s) = v''(\tilde{x}_2 - s, 2)$. Then, from (14) and the related boundary conditions, we have that

$$\begin{cases} u'(s) = -w(s), \quad u(0) = \beta_2, \\ w'(s) = \frac{2}{\sigma_2^2}\left[\mu_2 w(s) - (\delta + \lambda_2)u(s) + \lambda_2\right], \quad w(0) = 0. \end{cases}$$

Given that $(\delta + \lambda_2)\beta_2 > \lambda_2$, an analysis of this system shows that $w'(s) < 0$ for $s > 0$. This means that $v'''(\cdot, 2) > 0$ in $(\tilde{x}_1, \tilde{x}_2)$. We verify our claims by taking into account that $v''(\tilde{x}_2, 2) = 0$.

(iv) In the interval $(x_R, \tilde{x}_1)$, the functions $v'(\cdot, i)$ satisfy the coupled ODE system (44) with boundary conditions

$$v'(\tilde{x}_1, 1) = 1, \;\; v''(\tilde{x}_1, 1) = 0, \;\; v'(\tilde{x}_1, 2) = \frac{\lambda_2}{\lambda_2 + \delta} + \sum_{h=1}^{2} \tilde{\alpha}_h \tilde{A}_h, \;\; v''(\tilde{x}_1, 2) = \sum_{h=1}^{2} \tilde{\alpha}_h^2 \tilde{A}_h. \tag{41}$$

By plugging in (41) the guesses

$$v'(x, 1) = \sum_{j=1}^{4} A_j \alpha_j e^{\alpha_j(x - \tilde{x}_1)}, \;\; v'(x, 2) = \sum_{j=1}^{4} B_j \alpha_j e^{\alpha_j(x - \tilde{x}_1)}, \tag{42}$$

and matching coefficients, we obtain the characteristic equations

$$A_j\left(\alpha_j \mu_1 + \frac{\sigma_1^2}{2}\alpha_j^2 - (\delta + \lambda_1)\right) + \lambda_1 B_j = 0$$

and

$$B_j\left(\alpha_j \mu_2 + \frac{\sigma_2^2}{2}\alpha_j^2 - (\delta + \lambda_2)\right) + \lambda_2 A_j = 0,$$

for $j=1,2,3,4$. Solving the first equation yields B_j. Substituting B_j in the latter equation and rearranging yields (18). To verify that (18) has four real roots, let us define

$$f(\theta) := \underbrace{\left(\delta+\lambda_1-\theta\mu_1-\frac{\sigma_1^2}{2}\theta^2\right)}_{:=G_1(\theta)}\underbrace{\left(\delta+\lambda_2-\theta\mu_2-\frac{\sigma_2^2}{2}\theta^2\right)}_{:=G_2(\theta)}-\lambda_1\lambda_2,$$

and let θ_j^i be the roots of $G_i(\theta_j)$. It is straightforward to verify that $f(0)>0$, $f(\infty)>0$, $f(-\infty)>0$, and $f(\theta_j^i)=-\lambda_1\lambda_2<0$ for $i=1,2$ and $j=1,2,3,4$. Then, by continuity and using that $\theta_1^i\theta_2^i=-2\left(\delta+\lambda_i\right)/\sigma_i^2<0$, (18) has four different four roots, two positive and two negative. Then, (41) reads as

$$\begin{cases}\sum_{j=1}^4 A_j\alpha_j=1,\\ \sum_{j=1}^4 A_j\alpha_j^2=0,\\ \sum_{j=1}^4 B_j\alpha_j=\frac{\lambda_2}{\delta+\lambda_2}+\tilde{A}_1\tilde{\alpha}_1+\tilde{\alpha}_2\tilde{A}_2,\\ \sum_{j=1}^4 B_j\alpha_j^2=\tilde{A}_1\tilde{\alpha}_1^2+\tilde{\alpha}_2^2\tilde{A}_2.\end{cases} \tag{43}$$

We obtain then the expressions of $v(\cdot,i)$ by integrating (41) in $[x,\tilde{x}_1]$ and using that, by value matching,

$$v(\tilde{x}_1,2)=K_2+\frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2}+\sum_{j=1}^{2}\tilde{A}_j\left(1-e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)}\right)$$

and having also set $K_1:=v(\tilde{x}_1,1)>0$.

(v) To show that $v''(\cdot,i)<0$ in $(x_R,\tilde{x}_1)$ under (21) we use that, in this interval, the functions $u(\cdot,i) := v'''(\cdot;i)$ solve the following ODE system:

$$\begin{cases} \frac{1}{2}\sigma_1^2 u''(x,1) + \mu_1 u'(x,1) - (\delta+\lambda_1)\, u(x,1) + \lambda_1 u(x,2) = 0, \\ \\ \frac{1}{2}\sigma_2^2 u''(x,2) + \mu_2 u'(x,2) - (\delta+\lambda_2)\, u(x,2) + \lambda_2 u(x,1) = 0. \end{cases} \tag{44}$$

The Feynman-Kac representation of u provides

$$u(x,i) = \mathbb{E}\left[e^{-\delta\tau} u(X_\tau^{x,i,\circ}, I_\tau^i)\right], \tag{45}$$

where $\tau = \inf\{t \geq 0 :\ X_t^{x,i,\circ} \notin (x_R, \tilde{x}_1)\}$, being $X_t^{x,i,\circ}$ the solution to

$$dX_t = \mu_{I_t} dt + \sigma_{I_t} dW_t, \qquad X_0^{x,i,\circ} = 0.$$

Condition (21) entails $u(x_R, i) > 0$ and $u(\tilde{x}_1, i) > 0$ for all $i = 1, 2$. Thus, from (45) we get $u(\cdot, i) > 0$. Hence, $v''(\cdot, i)$ is strictly increasing on $(x_R, \tilde{x}_1)$ for $i = 1, 2$. Since $v''(\tilde{x}_1^-, 1) = 0$ and $v''(\tilde{x}_1^-, 2) < 0$, we get that v'' is negative on $(x_R, \tilde{x}_1)$ and the claim follows.

(vi) Here we show that

$$v'(\tilde{x}_1, 2) < \frac{\lambda_1 + \delta}{\lambda_1}. \tag{46}$$

In the interval $(x_R, \tilde{x}_1)$, the function $v'(\cdot;1)$ solve

$$\frac{1}{2}\sigma_1^2 v'''(x,1) + \mu_1 v''(x,1) - (\delta+\lambda_1)\, v'(x,1) + \lambda_1 v'(x,2) = 0. \tag{47}$$

By (21), we have $v'''(\tilde{x}_1^-, 1) > 0$. Recalling also that $v''(\tilde{x}_1, 1) = 0$ and $v'(\tilde{x}_1, 1) = 1$, plugging all these information into (47), and passing to the limit as $x \to \tilde{x}_1^-$, we get $\lambda_1 v'(\tilde{x}_1, 2) > (\lambda_1 + \delta)$, which verifies the claim.

(vii) To identify the free parameters K_1 and K_2, we impose the following conditions:

$$\mathcal{L}_i v(\tilde{x}_i, i) - \lambda_i \left[v(\tilde{x}_i, i) - v(\tilde{x}_i, 3-i) \right] = 0, \quad i = 1, 2,$$

which are obtained by passing the equality $\mathcal{L}_i v(x,i) - \lambda_i \left[v(x,i) - v(x, 3-i) \right] = 0$ to the limit as $x \to \tilde{x}_i^-$. Since $v'(\tilde{x}_i, i) = \beta_i$ and $v''(\tilde{x}_i, i) = 0$, they rewrite as

$$\mu_2 \beta_2 - (\delta + \lambda_2) K_2 + \lambda_2 (K_1 + \tilde{x}_2 - \tilde{x}_1) = 0, \tag{48}$$

and

$$\mu_1 - (\delta + \lambda_1) K_1 + \lambda_1 \left(K_2 + \frac{\lambda_2}{\delta + \lambda_2} (\tilde{x}_1 - \tilde{x}_2) \right) = 0, \tag{49}$$

which completes the system (20).

(viii) Next, we show that the solution constructed in Points (i)-(vii) solves (9). Most of the work has already been done. Indeed, looking at the previous steps, we see that we have constructed v such that all the following are met: (a) $v(\cdot, i) \in C^2((x_R, \infty); \mathbb{R})$; (b) $v'(x,i) = \beta_i$ in $[\tilde{x}_i, \infty)$; (c) $\mathcal{L}_i v(\cdot, i) - \lambda_i (v(\cdot, i) - v(\cdot, 3-i)) = 0$ in $(x_R, \tilde{x}_i)$; (d) $v(\cdot, i)$ are concave, which entails $v'(\cdot, i) \geq \beta_i$ for $i = 1, 2$. So, it remains to show that

$$H(x,i) := \mathcal{L}_i v(x,i) - \lambda_i (v(x,i) - v(x, 3-i)) \leq 0, \qquad \forall x \in [\tilde{x}_i, \infty), \quad i = 1, 2.$$

First, we prove that $H(x, 2) \leq 0$ in $[\tilde{x}_2, \infty)$. Indeed, by (48), we have $H(\tilde{x}_2, 2) = 0$. Recalling that $v'(x,i) = \beta_i$ for $x \geq \tilde{x}_2$ and using that $\beta_2 > \lambda_2 / (\lambda_2 + \delta)$, we have

$$H'(x, 2) = -(\lambda_2 + \delta)\beta_2 + \lambda_2 < 0, \quad \forall x \geq \tilde{x}_2.$$

Next, we prove that $H(x,1) \leq 0$ in $[\tilde{x}_1, \infty)$. Indeed, by (49), we have $H(\tilde{x}_1, 1) = 0$. Recalling that $v'(x, 1) = 1$ for $x \in [\tilde{x}_1, \infty)$ and using concavity of $v(\cdot, 2)$ and (46), we get

$$H'(x,1) = -(\lambda_1 + \delta) + \lambda_1 v'(x,2) < 0, \quad \forall x \geq \tilde{x}_1.$$

2. The fact that (23) admits a unique solution in $(x_R, \tilde{x}_1]$ is due to the structure of $v(\cdot, 1)$ defined in Point 1 (notably the strict concavity of $v(\cdot, 1)$ in that interval) and the fact that $v'(\tilde{x}_2) = \beta_2 \leq 1$ together with (22) entailing $v'(\tilde{x}_R^+, 2) > 1$.

3. This is immediate as (24) is nothing but the rewriting of (13) given the structure determined in the previous points.

A.2 Proof of Theorem 1

We only sketch the proof, as a rigorous argument would be highly technical. We refer to two papers that deal with similar problems and provide complete proofs: Løkka and Zervos (2008), in the case of no regime switching and no recapitalisation; and Ferrari et al. (2022), in the case of regime switching but with no recapitalisation or dividend taxes.

As a first step, we prove that $v(x,i) \geq V(x,i)$. Let $A = (Z, (b, G)) \in \mathcal{A}$ be an arbitrary control and define $\tau_0 := 0$ and, recursively on $n \geq 0$, $\tau_{n+1} = \inf\{t \geq \tau_n : \; X_{t^-} = x_R\}$, being X_t the associated state process. Then, we have, by verification arguments in the interval $[0, \tau_1)$ (see Ferrari et al., 2022)

$$v(x,i) \geq \mathbb{E}\left[\int_0^{\tau_1^-} e^{-\delta t}\beta_{I_t} dZ_t + e^{-\delta \tau_1} v(x_R, I_{\tau_1})\right]. \tag{50}$$

By using (50) and (13) we get

$$v(x,i) \geq \mathbb{E}\left[\int_0^{\tau_1^-} e^{-\delta t}\beta_{I_t} dZ_t + e^{-\delta \tau_1}\left(v(x_R + G_1; I_{\tau_1}) - G_1 - \kappa\right)\right].$$

Iterating the argument yields

$$v(x,i) \geq \mathbb{E}\left[\int_0^{\tau_n^-} e^{-\delta t}\beta_{I_t} dZ_t - \sum_{k=1}^{n} e^{-\delta\tau_k}\left(G_k+\kappa\right) + e^{-\delta\tau_n} v(x_R, I_{\tau_n})\right].$$

Letting $n \to \infty$ and observing that $\tau_n \to \infty$, we get $v(x,i) \geq J(x,i;A)$. By arbitrariness of A, we conclude this part of the proof.

As a second step, we take $A = \hat{A}$ into Step 1. Then, by construction, all previous inequalities become equalities, which allows us to conclude that $v(x,i) = J(x,i;\hat{A})$. Together with Step 1, this last condition entails $J(x,i;\hat{A}) = v(x,i) = V(x,i)$, which verifies our first claim.

A.3 Proof of Proposition 2

To obtain $\pi(\cdot,i)$ over the intervals $(x_R, \tilde{x}_i)$ for $i = 1,2$, we plug in (27) the following guess. In the region $(x_R, \tilde{x}_1)$ we guess and verify that the density takes the form:

$$\pi(x,1) = \sum_{j=1}^{4} P_j e^{r_j x},\ \pi(x,2) = \sum_{j=1}^{4} Q_j e^{r_j x}.$$

Matching coefficients and solving for P_j and Q_j yields the characteristic equation (31) and the relationship $Q_j = \lambda_1^{-1} F_1(r_j) P_j$. Similarly, we obtain $\pi(\cdot,2)$ over the interval $(\tilde{x}_1, \tilde{x}_2)$ and (32) by plugging in (28) the guess

$$\pi(x,2) = \sum_{j=1}^{2} H_j e^{s_j x}$$

and matching coefficients. Using these equations to impose the boundary and mass preservation conditions as they appear in the main text yields (33).

A.4 Solution structure with dividend bans

First, we show that when $\beta_2 \leq \lambda_2/(\lambda_2+\delta)$, the solution structure in Proposition 1 does not hold. For this purpose, we solve the linear system (40) to obtain

$$\begin{bmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{bmatrix} = \left(\beta_2 - \frac{\lambda_2}{\delta+\lambda_2}\right) \begin{bmatrix} \frac{e^{\tilde{\alpha}_1(\tilde{x}_1-\tilde{x}_2)}}{\tilde{\alpha}_1(\tilde{\alpha}_2-\tilde{\alpha}_1)} \\ -\frac{e^{\tilde{\alpha}_2(\tilde{x}_1-\tilde{x}_2)}}{\tilde{\alpha}_2(\tilde{\alpha}_2-\tilde{\alpha}_1)} \end{bmatrix}.$$

Plugging these coefficients in (39) and rearranging, we get

$$v'(x,2) = \frac{\lambda_2}{\delta+\lambda_2} + \underbrace{\frac{e^{-\tilde{\alpha}_1(\tilde{x}_2-x)} - e^{-\tilde{\alpha}_2(\tilde{x}_2-x)}}{\tilde{\alpha}_2-\tilde{\alpha}_1}}_{>0} \underbrace{\left(\beta_2 - \frac{\lambda_2}{\delta+\lambda_2}\right)}_{\leq 0}.$$

which violates the optimality condition that $v'(x,2) > \beta_2$ in $(\tilde{x}_1, \tilde{x_2})$.

Second, we build an alternative solution of (9) in which $\tilde{x}_2 = \infty$, and show that it is consistent with the parametric restriction $\beta_2 \leq \lambda_2/(\lambda_2+\delta)$. Following the same steps of Section 4.1, we look for a function v such that, for $i = 1, 2$, the following hold:

a) $v(\cdot, i) \in C([x_R, \infty)) \cap C^1((x_R, \infty))$;

b) The associated continuation and intervention regions have the following structure:

$$\mathcal{C}_2 = (x_R, \infty), \quad \mathcal{C}_1 = (x_R, \tilde{x}_1); \quad \mathcal{S}_1 = [\tilde{x}_1, \infty);$$

c) $v(\cdot, 1) \in C^2(\mathcal{C}_1)$ and $v(\cdot, 2) \in C^2(\mathcal{C}_2 \setminus \{\tilde{x}_1\})$.

Accordingly, we have $\tilde{x}_1 = \inf\{x > x_R : v'(x,1) \leq 1\}$ and $\tilde{x}_2 = \infty$. The boundary conditions at x_R are specified as in the main text.

Following the same approach of Appendix A.1, we now construct a function that fulfils all these guesses and translates them into a list of algebraic requirements. In the interval

$[\tilde{x}_1, \infty)$, we set $v'(\cdot, i) = 1$ and

$$v'(x,2) = \frac{\lambda_2}{\delta + \lambda_2} + \tilde{\alpha}_1 \tilde{A}_1 e^{\tilde{\alpha}_1 (x - \tilde{x}_1)},$$

where $\tilde{\alpha}_1 < 0$ is the negative root of (19) and $\tilde{A}_1 < 0$. In the interval $(x_R, \tilde{x}_1)$, we set the functions $v'(\cdot, i)$ as unique solutions to the system (15). This entails the same structure as in (42), whose coefficients solve the following linear system:

$$\begin{cases} \sum_{j=1}^{4} A_j \alpha_j = 1, \\ \sum_{j=1}^{4} A_j \alpha_j^2 = 0, \\ \sum_{j=1}^{4} B_j \alpha_j = \frac{\lambda_2}{\delta+\lambda_2} + \tilde{A}_1 \tilde{\alpha}_1, \\ \sum_{j=1}^{4} B_j \alpha_j^2 = \tilde{\alpha}_1^2 \tilde{A}_1. \end{cases}$$

By integrating $v'(\cdot, i)$ we get

$$v(x,1) = K_1 + \begin{cases} (x - \tilde{x}_1), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^{4} A_j \left(e^{\alpha_j (x - \tilde{x}_1)} - 1 \right), & x \in [x_R, \tilde{x}_1), \end{cases}$$

$$v(x,2) = K_2 + \begin{cases} \frac{\lambda_2}{\delta+\lambda_2}(x - \tilde{x}_1) + \tilde{A}_1 \left(e^{\tilde{\alpha}_1 (x - \tilde{x}_1)} - 1 \right), & x \in [\tilde{x}_1, \infty), \\ \sum_{j=1}^{4} B_j \left(e^{\alpha_j (x - \tilde{x}_1)} - 1 \right), & x \in [x_R, \tilde{x}_1). \end{cases}$$

Under similar assumptions as Proposition 1 and using that $v(\cdot, i)$ is differentiable, one gets that $G(1)^* = \tilde{x}_1 - x_R$ and $G(2)^* = x_2^* - x_R$, where $x_2^* \in (x_R, \infty)$ is the unique solution of

$$\mathbf{1}_{[x_R, \tilde{x}_1]} \sum_{j=1}^{4} B_j \alpha_j e^{\alpha_j (x_2^* - \tilde{x}_1)} + \mathbf{1}_{(\tilde{x}_1, \infty)} \left(\frac{\lambda_2}{\delta + \lambda_2} + \tilde{\alpha}_1 \tilde{A}_1 e^{\tilde{\alpha}_1 \left(x_2^* - \tilde{x}_1 \right)} \right) - 1 = 0.$$

To pin down the remaining coefficients ($K_1 > 0$, $K_2 > 0$, and $\tilde{A}_1 < 0$) and the dividend threshold ($\tilde{x}_1 > x_R$) we enforce that $\mathcal{L}_1 v(\tilde{x}_1, 1) - \lambda_1 \left[v(\tilde{x}_1, 1) - v(\tilde{x}_1, 2) \right] = 0$ and $\mathcal{L}_2 v(\tilde{x}_2, 2) -$

$\lambda_2 \left[v(\tilde{x}_1, 2) - v(\tilde{x}_1, 1) \right] = 0$ as $x \to \tilde{x}_1^-$, which rewrites as

$$\begin{cases} \mu_2 \sum_{j=1}^4 B_j \alpha_j + \frac{\sigma_2^2}{2} \sum_{j=1}^4 B_j \alpha_j^2 - (\delta + \lambda_2) K_2 + \lambda_2 K_1 = 0, \\ \mu_1 - (\delta + \lambda_1) K_1 + \lambda_1 K_2 = 0, \end{cases}$$

and impose the two boundary conditions in (13).

B Supplementary material (for online publication)

B.1 Solution structure with counter-cyclical capital requirements

Let us fix $\tilde{x}_1, \tilde{x}_2$ such that $x_{R,2} < x_{R,1} < \tilde{x}_1 < \tilde{x}_2$, Then, the model's solution structure encompasses the following four regions: (i) $x \in (x_{R,2}, x_{R,1})$, (ii) $x \in (x_{R,1}, \tilde{x}_1)$, (iii) $x \in (\tilde{x}_1, \tilde{x}_2)$, and (iv) $x \in (\tilde{x}_2, \infty)$.

In Region (i), under the assumption that κ is small enough (i.e., it is always optimal to recapitalize), we set

$$v(x,1) = v(\tilde{x}_1, 1) - (\tilde{x}_1 - x) - \kappa > 0. \tag{51}$$

Given (51) and (37), we guess and verify that $v(\cdot, 2)$ has the following form:

$$v(x,2) = \frac{\mu_2 \lambda_2}{(\delta + \lambda_2)^2} + \frac{\lambda_2}{\delta + \lambda_2} \left(v(\tilde{x}_1, 1) - (\tilde{x}_1 - x) - \kappa \right) + C_1 e^{\tilde{\alpha}_1 (x - \tilde{x}_1)} + C_2^{\tilde{\alpha}_2 (x - \tilde{x}_1)},$$

where $(C_1, C_2) \in \mathbb{R}^2$ are two constants coefficients given below and $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ are the real roots of (19).We find $v(\cdot, i)$ in Regions (ii)-(iv) by using (16) and (17) with $x_R = x_{R,1}$.

By using these equations to impose the usual boundary (value matching, smooth pasting, and super-contact) conditions, we get the following linear system:

$$\begin{cases}
\sum_{h=1}^{4} A_h\alpha_h - 1 = 0, \\
\sum_{h=1}^{4} A_h\alpha_h^2 = 0, \\
\sum_{h=1}^{4} B_h\alpha_h - \sum_{h=1}^{2} \tilde{A}_h\tilde{\alpha}_h - \frac{\lambda_2}{\delta+\lambda_2} = 0, \\
\sum_{h=1}^{2} \tilde{A}_h\tilde{\alpha}_h^2 e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)} = 0, \\
\sum_{h=1}^{4} B_h\alpha_h^2 - \sum_{h=1}^{2} \tilde{A}_h\tilde{\alpha}_h^2 = 0, \\
\frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2} \tilde{A}_h\tilde{\alpha}_h e^{\tilde{\alpha}_j(\tilde{x}_2-\tilde{x}_1)} - \beta_2 = 0, \\
\frac{\mu_2\lambda_2}{(\delta+\lambda_2)^2} + \sum_{h=1}^{2} \left[C_h e^{\tilde{\alpha}_h\left(x_{R,1}-\tilde{x}_1\right)} - \tilde{A}_h(1 - e^{\tilde{\alpha}_h(\tilde{x}_2-\tilde{x}_1)})\right] - K_1 + \\
+\frac{\lambda_2}{\delta+\lambda_2}\left(K_1 - (\tilde{x}_1 - x_{R,1}) - \kappa\right) - \frac{\lambda_2(\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2} - \sum_{h=1}^{4} B_h\left(e^{\alpha_h(x_{R,1}-\tilde{x}_1)} - 1\right) = 0, \\
\frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2} \tilde{\alpha}_h e^{\tilde{\alpha}_h\left(x_{R,1}-\tilde{x}_1\right)} - \sum_{j=1}^{4} B_j\alpha_j e^{\alpha_j(x_{R,1}-\tilde{x}_1)} = 0, \\
\mu_2\beta_2 - (\delta+\lambda_2)K_2 + \lambda_2(K_1 + \tilde{x}_2 - \tilde{x}_1) = 0, \\
\mu_1 - (\delta+\lambda_1)K_1 + \lambda_1\left(K_2 + \frac{\lambda_2}{\delta+\lambda_2}(\tilde{x}_1 - \tilde{x}_2)\right),
\end{cases}$$

whose solution (if it exists) yields $(A_1, A_2, A_3, A_4, \tilde{A}_1, \tilde{A}_2, C_1, C_2) \in \mathbb{R}^8$ and $(K_1, K_2) \in \mathbb{R}^2_+$.

By using these coefficients to compute $v(\cdot, i)$, we can obtain the recapitalisation target x_2^* and the payout thresholds $\tilde{x}_1 = x_1^*$ and $\tilde{x}_2$ by imposing the boundary conditions (13) and the optimality condition $v'(x_2^*, 2) = 1$, which requires to solve the following non-linear system:

$$\begin{cases} \mathbf{1}_{[x_{R,2},\tilde{x}_1]} \frac{\lambda_2}{\delta+\lambda_2} + \sum_{h=1}^{2} \left[\mathbf{1}_{[x_{R,2},x_{R,1})} C_h e^{\tilde{\alpha}_h (x-\tilde{x}_1)} + \mathbf{1}_{(x_{R,1},\tilde{x}_1]} \tilde{A}_h \tilde{\alpha}_h \left(e^{\tilde{\alpha}_h (x_2^*-\tilde{x}_1)} \right) \right] + \\ + \mathbf{1}_{(\tilde{x}_1,\infty)} \sum_{h=1}^{4} B_h \alpha_h e^{\alpha_h (x_2^*-\tilde{x}_1)} - 1 = 0 \\ \sum_{h=1}^{4} A_h \left(e^{\alpha_h (x_{R,1}-\tilde{x}_1)} - 1 \right) + \tilde{x}_1 - x + \kappa = 0 \\ \frac{\mu_2 \lambda_2}{(\delta+\lambda_2)^2} + \frac{\lambda_2}{\delta+\lambda_2} (K_1 + x - \tilde{x}_1 - \kappa) + \sum_{h=1}^{2} C_h e^{\tilde{\alpha}_h \left(x_{R,1}-\tilde{x}_1 \right)} + \\ + \mathbf{1}_{[x_{R,2},x_{R,1})} \left[\frac{\mu_2 \lambda_2}{(\delta+\lambda_2)^2} + \frac{\lambda_2}{\delta+\lambda_2} (K_1 - \tilde{x}_1 + x_2^* - \kappa) + \sum_{h=1}^{2} C_h e^{\tilde{\alpha}_h \left(x_2^*-\tilde{x}_1 \right)} \right] - \\ + \mathbf{1}_{(x_{R,1},\tilde{x}_1]} \left[K_2 + \frac{\lambda_2 (x_2^*-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{h=1}^{2} \tilde{A}_h \left(e^{\tilde{\alpha}_h (x_2^*-\tilde{x}_1)} - e^{\tilde{\alpha}_h (\tilde{x}_2-\tilde{x}_1)} \right) \right] - \\ + \mathbf{1}_{(\tilde{x}_1,\infty)} \left[K_1 + \frac{\lambda_2 (\tilde{x}_1-\tilde{x}_2)}{\delta+\lambda_2} + \sum_{h=1}^{2} \tilde{A}_h (1 - e^{\tilde{\alpha}_h (\tilde{x}_2-\tilde{x}_1)}) + \sum_{h=1}^{4} B_h \left(e^{\alpha_h (x_2^*-\tilde{x}_1)} - 1 \right) \right] + \\ + (x_2^* - x) + \kappa = 0. \end{cases}$$

B.2 Solution structure with pro-cyclical capital requirements

Pro-cyclical capital requirements associate with the parametric condition $\Gamma_1 < \Gamma_2$ or, equivalently, $x_{R,1} < x_{R,2}$. Thus, modelling this case requires us to expand the support of the bank's reserves to include the region $x \in (x_{R,1}, x_{R,2})$. The rest of the state space when $x > x_{R,2}$ is that described in Section 4.1 after setting $x_{R,2} = x_R$.

When $x \in (x_{R,1}, x_{R,2})$ and there is a random transition from State 1 to State 2, the regulatory constraint $x > x_{R,2}$ is not satisfied. Thus, we assume the regulator requires the bank to immediately recapitalise or default and, following the logic of Section 5.3.1, set

$$v(x, 2) = \max \left\{ v(x_2^*, 2) - (x_2^* - x) - \kappa, \Gamma_2^+ \right\}. \tag{52}$$

Given (52), one can find $v(x, 1)$ as the unique solution of

$$\frac{1}{2} \sigma_1^1 v''(x, 1) + \mu_1 v'(x, 1) - (\delta + \lambda_1) v(x, 1) + \lambda_1 v(x, 2) = 0,$$

with boundary conditions $v(x_{R,2}^+, 1) = v(x_{R,2}^-, 1)$ and $v'(x_{R,2}^+, 1) = v'(x_{R,2}^-, 1)$.

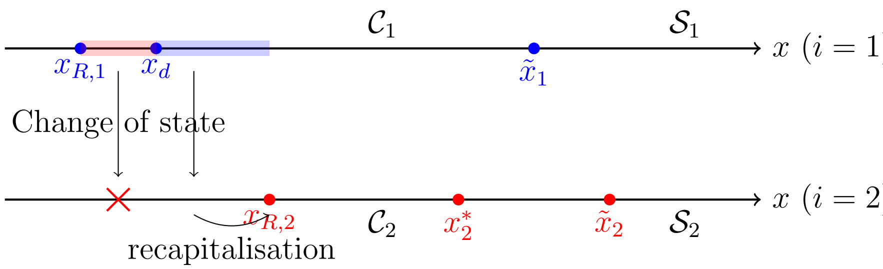

Figure 7: Solution structure with pro-cyclical capital requirements.

Unlike the case of counter-cyclical capital requirements, the parametric condition (36) does *not* ensure that the bank is always willing to recapitalise after a change of state. In other words, there may be some reserves level $x_d \in (x_{R,1}, x_{R,2})$ such that (52) equals zero. Figure 7 visually represents this issue.

To adapt the solution structure, we split the region $x \in (x_{R,1}, x_{R,2})$ into the following two sub-intervals: $x \in (x_{R,1}, x_d)$ and $x \in (x_d, x_{R,2})$. In the former, we find $v(x, 1)$ by solving

$$\frac{1}{2}\sigma_1^1 v''(x,1) + \mu_1 v'(x,1) - (\delta + \lambda_1)\, v(x,1) + \lambda_1 \left(v(x_2^*, 2) - v(x_2^*, 2) - x_2^* + x\right) = 0$$

with boundary conditions $v(x_{R,2}^+, 1) = v(x_{R,2}^-, 1)$ and $v'(x_{R,2}^+, 1) = v'(x_{R,2}^-, 1)$. In the latter, we solve

$$\frac{1}{2}\sigma_1^1 v''(x,1) + \mu_1 v'(x,1) - (\delta + \lambda_1)\, v(x,1) = 0$$

with boundary conditions $v(x_d^+, 1) = v(x_d^-, 1)$ and $v'(x_d^+, 1) = v'(x_d^-, 1)$. We find the endogenous threshold x_d by finding the reverse level so that (52) equals zero, which yields $x_d = \kappa + x_2^* - v(x_2^*, 2)$.

B.3 Counter-cyclical capital requirements: comparative statics

Table 9 reports the bank's optimal control, its credit capacity and dispersion, and the maximal incentive-compatible recapitalisation cost for different levels of Γ_2 ($x_{R,2}$).

The results of this analysis are broadly consistent with those of the mean-preserving

$x_{R,1}$	$x_{R,2}$	$\tilde{x}_1$	$\tilde{x}_2$	x_2^*	$\mathbb{E}_1^\pi[x]$	$\mathbb{E}_2^\pi[x]$	$\mathbb{E}^\pi[x]$	$\mathbb{V}_1^\pi[x]$	$\mathbb{V}_2^\pi[x]$	$\mathbb{V}^\pi[x]$	$\bar{\kappa}$
0.077	0.069	0.799	1.691	0.646	0.565	0.968	0.726	0.028	0.162	0.130	0.104
-	0.057	0.797	1.687	0.637	0.564	0.958	0.722	0.027	0.163	0.119	0.102
-	0.049	0.795	1.674	0.629	0.562	0.950	0.717	0.027	0.163	0.118	0.101
-	0.037	0.794	1.666	0.622	0.561	0.941	0.713	0.027	0.163	0.116	0.099
-	0.027	0.792	1.657	0.612	0.559	0.932	0.709	0.027	0.163	0.115	0.098

Table 9: Bank's optimal control, credit capacity and dispersion, and the maximal incentive-compatible recapitalisation cost for different levels of Γ_2 ($x_{R,2}$).

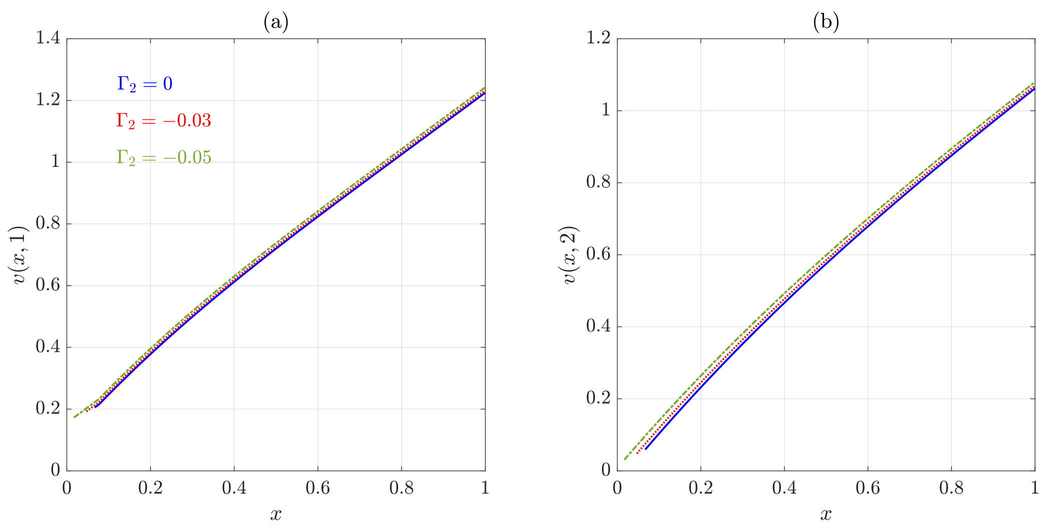

Figure 8: Effect of counter-cyclical capital requirements on shareholder value in the good (Panel (a)) and bad (Panel (b)) states.

counter-cyclical capital requirements discussed in the main text. However, the policy's effects are more straightforward, as it does not impose tighter capital requirements in the good state. Specifically, relaxing capital requirements in the bad state lowers the dividend payout threshold in both states (Columns 2 and 3). At the same time, it diminishes the bank's recapitalisation incentives (see Columns 5 and 12). Consequently, the policy reduces the average credit capacity and its dispersion across states. Figure 8 reports $v(x,i)$ for different values of Γ_2 (corresponding to Row 3 and 5 of Table 9) in State 1 (Panel (a)) and 2 (Panel (b)), showing that relaxing capital requirements increases bank value in both states.