Monetary Policy Announcements and Sacrifice Ratios*

Gene Ambrocio Markus Haavio Nigel McClung

August 20, 2025

Abstract

Sacrifice ratios (the cumulative effect on output divided by the cumulative effect on inflation in response to monetary policy) are key summary statistics for central banks aiming to reduce inflation with minimal impact on the real economy. We show that sacrifice ratios associated with announcements of the most likely course of monetary policy are lower when the implementation date is further out into the future in a class of New Keynesian models. This is not due to forward guidance puzzle effects and holds even when agents' expectations feature cognitive discounting or when policy announcements are imperfectly credible, so that the forward guidance puzzle is resolved. We also show that the benefits are concentrated at short horizons: more than one third (half) of the maximum reduction in sacrifice ratios is achieved if policy is announced one (two) period(s) in advance. Finally, we provide some empirical evidence supporting the predictions of the theory.

Keywords: monetary policy announcements, sacrifice ratio, cognitive discounting, near-term signaling
JEL Codes: E31, E52, E58

---

[*] $\ll$ Email:gene.ambrocio@bof.fi, markus.haavio@bof.fi, nigel.mcclung@bof.fi $\gg$. Bank of Finland, Snellmaninaukio, P.O. Box 160, Helsinki, 00101, Finland. The views and opinions expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Finland.

1 Introduction

> "While higher interest rates. . . will bring down inflation, they will also bring some pain to households and businesses. These are the unfortunate costs of reducing inflation." - Jerome Powell (Jackson Hole, Aug. 26, 2022)

> "Setting the right 'level' and 'length' will be critical for our monetary policy as we continue our tightening cycle." - Christine Lagarde (Sintra, Jun. 27, 2023)

The resurgence of inflation in many parts of the world in the post Covid-19 pandemic period has brought about a renewed interest in how central banks can efficiently bring inflation back down to target. As highlighted in the above quote from Federal Reserve Chairman Jerome Powell, stabilizing inflation can sometimes be *painful* to households and firms and minimizing the pain brought about by monetary policy may be considered an important metric regarding efficiency. In this regard, sacrifice ratios (the ratio of cumulative changes in output relative to inflation) tend to be quite useful as a summary statistic. In these situations, particularly when inflation pressures are expected to arise from the supply side, the central bank typically wants to disinflate the economy without generating additional downward pressure on economic activity and output. Hence, a policy path which generates a low sacrifice ratio would be preferable.

As indicated in the above quote from European Central Bank President Christine Lagarde, two elements to crafting policy paths may be considered corresponding to two approaches to disinflation - a *level* approach favoring immediate and forceful action or a *length* approach which entails a more gradual but prolonged tightening. Both approaches could feasibly generate the required disinflation over a given horizon but could have different effects on output. The key difference is the timing of when monetary tightening is relatively strongest. As we show both theoretically in a simple New Keynesian framework and through simulations of a model with additional nominal and real rigidities, it may be efficient to communicate planned policy actions in advance. Specifically, our results suggest that the *length* approach to disinflation delivers lower sacrifice ratios.

The intuition behind our finding is quite simple. In a class of New Keynesian models, inflation is more *"forward-looking"* than output. Therefore, announcing a policy change that will take place further out into the future will have stronger effects on inflation relative to output. This class of models includes the canonical (or textbook) New Keynesian model, versions of the New Keynesian model that feature additional "discounting" in the Euler equation or Phillips curve (Gabaix 2020, Angeletos and Lian 2018, Woodford 2019), the analytical heterogeneous agent New Keynesian (THANK) model (Bilbiie 2025), and the New Keynesian model with endogenous growth through R&D (Elfsbacka-Schmöller and McClung 2025).

Further, we show that our results are not due to the well-known forward guidance puzzle, whereby policy changes far out into the future deliver implausibly large effects today (Del Negro et al. 2023). Instead, our results hold even when the forward guidance puzzle is resolved by introducing i) cognitive discounting (Gabaix 2020) or finite planning horizons (Woodford 2019), ii) lack of common knowledge about future economic conditions (Angeletos and Lian 2018), iii) a heterogeneous agent structure together with procyclical inequality (Bilbiie 2025), or iv) by assuming that the monetary policy announcements are imperfectly credible. (The results also hold when the model features forward guidance puzzled effects.)

In the class of log-linearized models we consider, the effects of a policy rate path announcement can be decomposed into the sum of the effects of a sequence of news shocks about the policy rate at each date in the future. In turn, the effect of each news shock component can be obtained in a setting where the policy rate is treated as exogenous over the evaluation period while following the Taylor rule thereafter (Laséen and Svensson 2011, Barnichon and Mesters 2023, McKay and Wolf 2023). This allows us to focus our attention to simple examples of policy paths - an announcement today that the interest rate will be raised in H periods where a low H (potentially zero or one) is representative of the level approach, and a large H represents the length approach.[1] Using these simple policy examples, we show that the sacrifice ratio associated with a policy announcement that the interest rate will be raised H periods ahead is decreasing in H. This holds even with additional "discounting" in the Euler equation or Phillips curve, or when policy announcements are imperfectly credible, although the rate at which sacrifice ratios fall is attenuated by the intensity of (cognitive) discounting and by imperfect credibility of announcements. On the other hand, when households discount the future much more than firms (this can be the case for example in analytical heterogeneous agent models with procyclical inequality), the result is amplified as it makes output even less *"forward-looking"* relative to inflation.

Another important feature of our results is that sacrifice ratios are convex over the horizon H which means that most of the reduction in sacrifice ratios is achieved when increasing the horizon at low initial levels (e.g., from 1 to 2-periods ahead). We derive limits on how low the sacrifice ratio can get as the implementation horizon tends to infinity. We then establish lower bounds on the proportion that the sacrifice ratio is reduced when moving from an unannounced policy rate change to a one and two period-ahead implementation horizon. Specifically, more than one third of the reduction in

---

[1] A full-fledged scenario with a full path for policy rates can then be reconstructed from weighted sums of these simple policy paths.

sacrifice ratios is achieved by shifting the implementation horizon from zero to one and more than one half of the reduction is achieved when moving to an implementation horizon of two instead. Again, these results hold with and without forward guidance puzzle effects. These findings suggest that, even in situations where longer-term forward guidance is not a desirable option (e.g. since the central bank wants to react to new information), near-term signaling (i.e. making announcements about the most likely course of monetary policy in the near future) could be an attractive strategy, substantially improving the inflation-output trade-off.

We also show that our results mainly go through in models with additional real and nominal rigidities.[2] We add habit persistence in consumption as well as backward indexation in prices to the New Keynesian framework with cognitive discounting, following Afsar et al. (2024). Consistent with the intuitive interpretation of our results, we find that adding habit persistence tends to raise sacrifice ratios for low values of the implementation horizon H. On the other hand, adding backward-indexation in prices tend to lower sacrifice ratios at all horizons as well as the long-run limit as H tends to infinity.

Finally, we provide some empirical evidence from the US in support of our theory. We estimate sacrifice ratios using i) monetary policy surprises in short rates as proxies for short horizons H and ii) monetary policy surprises in long rates as proxies for longer horizons H. Consistent with the theory predictions, we find that the sacrifice ratio estimates with long rates are lower than the estimates with short rates.

Our results can be interpreted in the context of advanced communication of the most likely course of action that a central bank is going to take. Central bank communication

---

[2] We restrict our attention to parametrizations of the model where the effect on output and inflation do not change sign over the simulation horizon.

plays a central role in the conduct of monetary policy and in this respect two extreme views are to play coy and perhaps deliberately obfuscate planned changes to policy or to be completely transparent, telegraphing the most likely course of action in the near to medium-term.[3] Recent research tend to favor the latter. For instance, Acosta (2023) find that greater Federal Reserve transparency leads to stronger effects of monetary policy shocks on nominal and real rates. In this paper we show that the latter option is also more efficient according to at least one important metric - minimizing the effect of monetary policy on output, relative to inflation. The result provides a rationale for the desirability of early central bank communication about the most likely path of policy rates.

Even if a central bank were to strictly follow a data-driven and rules-based approach in the conduct of monetary policy, communicating ahead of time (changes in) the most likely course of action for monetary policy, or equivalently, correcting private sector misconceptions of the path of policy rates are important concerns in practice. This is because data regarding private sector expectations do not correspond well to what is typically assumed in full information rational expectation models.[4] The empirical evidence on monetary policy surprises also indicate that communication or news regarding the future path of policy rates is growing in importance and is present also outside of effective lower bound episodes (see e.g., Swanson and Jayawickrema 2023 for the United States and Altavila et al. 2019 for the euro area).

Our use of sacrifice ratios to evaluate alternative central bank communication strategies is related to the literature on large disinflations, e.g., Ascari and Ropele (2012b) and Gibbs and Kulish (2017). While this strand of the literature focuses on the output cost of permanently lowering inflation, we study the output implications of advanced central

---

[3] See, e.g., the reviews in Blinder et al. (2008), Binder (2017), and de Haan and Hoogduin (2024).

[4] See e.g., Coibion et al. (2022), Sheen and Wang (2023) for evidence from US households as well as Kryvtsov and Petersen (2021) for evidence from lab experiments.

bank communication. As Ascari and Ropele (2012a) note, there is no direct mapping between sacrifice ratios and welfare gains or losses. Nevertheless, our results potentially relate to welfare under several conditions, (i) the planned policy change is *ex-ante* optimal, and (ii) minimizing the impact on output from the period of announcement to the period of implementation is at least not welfare-reducing. For example, this could be the case when the source of inflation is an (expected) external cost-push shock in which case the central bank may want to raise the policy rate while minimizing the contractionary effects on output.

Our comparison of level and length strategies for policy paths mirrors an earlier debate in the literature with regard to *cold turkey* and *gradualist* approaches to disinflation (see, e.g., Sargent 1986, Gordon 1983, Taylor 1983). In the pioneering work of Ball (1994), sacrifice ratios were found to be smaller when (permanent) disinflation episodes were relatively quick. Others have suggested that the speed of disinflation should depend on the desired size of disinflation (Ireland 1997, Nicolae and Nolan 2006). Another relevant finding in the literature with respect to our results is that central bank credibility, and therefore the strength of the expectations channel to monetary policy transmission, lowers sacrifice ratios (Nicolae and Nolan 2006, Gibbs and Kulish 2017).

Our analysis also abstracts from some issues that are potentially relevant for welfare. In particular, we assume that there are no *delphic* effects from the policy announcement. That is, the policy announcement only updates the private sector's information set regarding the path of policy rates and that the central bank acts accordingly as well. As such, we abstract from any efficiency gains or losses that could potentially arise if this were not the case.

The remainder of the paper is structured as follows. The next section provides analytical results regarding monetary policy implementation horizons and sacrifice ratios using the basic New Keynesian model. It also covers how sacrifice ratios from these policy announcements change when cognitive discounting and imperfect credibility are introduced. Section 3 extends our results to a model with additional real and nominal rigidities. Section 4 presents some empirical evidence. Finally, Section 5 concludes with some remarks.

2 The Basic New Keynesian Framework

2.1 Inflation is more forward-looking than output

To study the effects of announcements of paths for policy rates, we take a simple linearized New Keynesian framework and introduce news shocks regarding monetary policy. Linearity of the model allows us to abstract from a baseline representing private sector *ex ante* expectations of the path of policy rates prior to the announcement and we assume that the economy would be in the steady state in the absence of policy actions (and announcements). Following Laséen and Svensson (2011), Barnichon and Mesters (2023), and McKay and Wolf (2023), these news shocks (or combinations thereof) can be used to capture the effects of a policy path generated endogenously from some rule.[5] We consider the case where the policy rate is temporarily exogenous in the interim period under evaluation and follows a Taylor-type rule thereafter.

We consider a simple New Keynesian framework, which nests the canonical (rational expectations representative agent) New Keynesian model, versions of the New Keynesian model that feature additional "discounting" in the Euler equation or Phillips curve

---

[5] Again, linearity of the model allows us to abstract from the potential sources of the endogenous response of monetary policy. Essentially, only the expected future path of the policy instrument matters (Barnichon and Mesters 2023, McKay and Wolf 2023).

(Gabaix 2020, Angeletos and Lian 2018, Woodford 2019), the analytical THANK model (Bilbiie 2025), and the New Keynesian model with endogenous growth through R&D (Elfsbacka-Schmöller and McClung 2025) among other models. The (log-linearized) dynamic IS and New Keynesian Phillips Curve (NKPC) are given by:

$$y_t = M_h \mathbb{E}_t[y_{t+1}] - \sigma(i_t - \hat{M}_h \mathbb{E}_t[\pi_{t+1}]) \tag{1}$$

$$\pi_t = M_f \beta \mathbb{E}_t[\pi_{t+1}] + \kappa y_t \tag{2}$$

where y_t is output, π_t is inflation, σ is the intertemporal elasticity of substitution, β is the discount factor, and κ is the slope of the NKPC. Setting the parameters $M_h, \hat{M}_h, M_f$ to one recovers the benchmark rational expectations representative agent New Keynesian model, while assuming that $M_h, \hat{M}_h, M_f \in (0, 1]$ captures cognitive discounting (Gabaix 2020) or finite planning horizons (Woodford 2019) by households and/or firms.[6] Alternatively, a constellation with $M_h, \hat{M}_h, M_f \in (0, 1]$ can arise in a New Keynesian model without common knowledge about future economic conditions (Angeletos and Lian 2018), or in a heterogeneous agent (THANK) model with procyclical inequality (Bilbiie 2025). We allow for (cognitive) discounting since we want to show that our results do not hinge on forward guidance puzzle effects. However, we also allow for compounding, in the sense that (some of) the parameters $M_h, \hat{M}_h, M_f$ can be greater than one. This is the case in the analytical THANK model with countercyclical inequality (Bilbiie 2025) and in the New Keynesian model with endogenous growth through R&D (Elfsbacka-Schmöller

---

[6] In Gabaix (2020), $\hat{M}_h = 1$ and $M_f = M_h \left[\theta + (1-\theta)\frac{1-\beta\theta}{1-\beta\theta M_h}\right] < M_h$ where θ is the Calvo parameter. We abstract from these conditions and assume that the cognitive discounting parameters can be set independently of each other, which also affords us the flexibility to consider other potential sources of "discounting" (e.g., Angeletos and Lian 2018, Bilbiie 2025, McKay et al. 2016b, and Woodford and Xie 2022). See also (Dupraz et al. 2024) for a model where the effective discounting on various terms in the dynamic IS curve can differ.

and McClung 2025).

Let us now consider why, and in what sense, inflation is more forward-looking than output in the New Keynesian framework. The IS curve (1) shows that output is forward-looking since households care about their expected future consumption and the real interest rate. According to the NKPC (2), current inflation π_t depends on current demand, or current output y_t, which is forward-looking, due to the IS curve. However, firms also care about their future profits in their pricing decisions since prices are sticky. In turn, these depend on their future production costs and the future prices set by their competitors. This brings in an additional source of forward-lookingness, which is captured by the term $M_f \beta \mathbb{E}_t[\pi_{t+1}]$.

Slightly rearranging equation (1) and plugging it into equation (2) allows us to see how current output and inflation depend on current monetary policy and on future output and inflation which in turn depend on future monetary policies $\{i_{t+h}\}_{h=1}^{\infty}$:

$$y_t = \sigma \left\{ \frac{M_h}{\sigma} \mathbb{E}_t[y_{t+1}] + \hat{M}_h \mathbb{E}_t[\pi_{t+1}] - i_t \right\} \tag{3}$$

$$\pi_t = \kappa\sigma \left\{ \frac{M_h}{\sigma} \mathbb{E}_t[y_{t+1}] + \left(\hat{M}_h + \frac{M_f \beta}{\kappa\sigma} \right) \mathbb{E}_t[\pi_{t+1}] - i_t \right\}. \tag{4}$$

The relative weight on current monetary policy is (normalized to) unity in both equations, and also the relative weight of future output is the same (M_h/σ). However, the relative weight on future inflation is larger in equation (4) than in equation (3), $\hat{M}_h + M_f\beta/\kappa\sigma$ as opposed to $\hat{M}_h$. Hence, as long as $M_f\beta > 0$ (i.e. unless firms are completely myopic) inflation is more forward-looking than output. In subsequent sections, we formally derive the implications of this property of the New Keynesian framework for the sacrifice ratios that result from policy announcements.

The relative forward-lookingness of inflation is distinct from the well-known forward guidance puzzle. In a setting where the policy rate is treated as exogenous over a given period, (while following the Taylor rule in the long run), the two-equation system (3) and (4) can be iterated to yield

$$x_t = -\sum_{h=0}^{\infty} A^h B i_{t+h}, \tag{5}$$

where $x_t = [y_t\,,\ \pi_t]'$, while

$$A = \begin{bmatrix} M_h & \hat{M}_h \sigma \\ M_h \kappa & \hat{M}_h \kappa \sigma + M_f \beta \end{bmatrix} \text{ and } B = \begin{bmatrix} \sigma \\ \kappa \sigma \end{bmatrix}. \tag{6}$$

The effect today of a policy announcement of a unit policy shock H periods ahead is given by: $x_t(H) = -A^H B$. As shown in Gibbs and McClung (2023) who provide conditions for a very broad class of models, the presence or absence of the forward guidance puzzle in this setting can be easily checked. To rule out the puzzle, it is sufficient that both of the eigenvalues associated with the matrix A, given by the characteristic equation $0 = \lambda^2 - tr(A)\lambda + det(A)$, are less than one.[7] Specifically, there is no forward guidance puzzle if the following conditions are met:

$$M_h M_f \beta < 1 \quad \text{and} \quad \frac{(1 - M_h)(1 - M_f \beta)}{\hat{M}_h} > \kappa \sigma \tag{7}$$

which clearly does not hold in the standard rational expectations representative agent New Keynesian model ($M_h = \hat{M}_h = M_f = 1$).

Finally, note that inflation is more forward-looking than output both when the model

---

[7] We have $tr(A) = M_h + \hat{M}_h \kappa \sigma + M_f \beta$ and $det(A) = M_h M_f \beta$. We interpret the forward guidance puzzle as $\|x_t(H)\| > \|x_t(H-1)\|$ and $\lim_{H\to\infty} \|x_t(H)\| = \infty$. Equations (7) are explicitly derived in Appendix A.1.

features the forward guidance puzzle and when the puzzle is resolved. This is the key reason why our results regarding the sacrifice ratio will hold with and without forward-guidance puzzle effects.

2.2 Simplified policy announcements

We simplify the analysis by considering announcements of a one-time policy news shock, such that the lag between the announcement and the implementation of the policy is H periods. As a metric for efficiency, we calculate and compare sacrifice ratios over a given window resulting from announcing the policy shock ahead of time. In a purely forward-looking model, it is sufficient to evaluate the paths of output and inflation from the announcement today to the period of monetary policy implementation, i.e. from period t to period $t + H$. This is because before t and after $t + H$ the impulse responses are zero. Denote by $\tilde{y}_H$ and $\tilde{\pi}_H$ the cumulative impulse responses of output and inflation to a monetary policy news shock of size unity. The sacrifice ratio associated to the news shock (SR_H) is given by

$$SR_H = \frac{\tilde{y}_H}{\tilde{\pi}_H}. \tag{8}$$

Notably, an announcement of a full path for policy rates (possibly consistent with some rule and a sequence of other shocks) can be constructed by some weighted combination of these one-time policy news shocks. To establish the link between a full-fledged policy path scenario and our simple policy path examples, we next define the sacrifice ratio SR associated with an announced policy path $\{i_{t+H}\}_{H=0}^{\tau}$ as the cumulative sum of output deviations from the steady state over the horizon t to $t + \tau$ divided by the cumulative sum of deviations in inflation over the same horizon. For a linear model, the change in output and inflation in each period can also be written down as the sum of the effects due to the deviations of the policy rate in each horizon $H \in [0, \tau]$:

$$SR(\{i_{t+H}\}_{H=0}^{\tau}) = \frac{\sum_{H=0}^{\tau} \sum_{s=0}^{\tau} y_{t+s}(H)}{\sum_{H=0}^{\tau} \sum_{s=0}^{\tau} \pi_{t+s}(H)}, \tag{9}$$

where $y_{t+s}(H)$ and $\pi_{t+s}(H)$ denote the deviations of output and inflation in period $t + s$ attributed to the policy rate deviation in period $t+H$. The expression (9) can be rewritten in the form

$$SR(\{i_{t+H}\}_{H=0}^{\tau}) = \frac{\sum_{H=0}^{\tau} i_{t+H}\tilde{y}_H}{\sum_{H=0}^{\tau} i_{t+H}\tilde{\pi}_H} = \sum_{H=0}^{\tau} \left[\frac{i_{t+H}\tilde{\pi}_H}{\sum_{J=0}^{\tau} i_{t+J}\tilde{\pi}_J} \right] SR_H, \tag{10}$$

which clearly indicates that the sacrifice ratio associated to the policy path $\{i_{t+H}\}_{H=0}^{\tau}$ is a weighted average of the sacrifice ratios SR_H associated to the news shocks. The key difference between level and length strategies is that weights are either relatively front-loaded (larger weights on low H) for level strategies or relatively back-loaded (larger weights on high H) for length strategies.

2.3 Sacrifice ratios from policy announcements

In the following analysis, we will focus our attention on simple policy rate paths such that a one-time unit policy rate deviation is announced H periods before the policy is implemented. We then study the corresponding sacrifice ratios SR_H (defined in (8)), and characterize how these sacrifice ratios change as the horizon H increases.

From now on, we find it useful to consider a setting in which the policy rate will be implemented in period T and is announced in period $T - H$, where $H = 0, 1, 2, ...$ Note that we could equally well assume that the policy is announced in period t and implemented in period $t + H$. Only the time lag between the announcement and the implementation — the horizon H — matters for the results. However, the timing convention we adopt here leads to somewhat simpler analysis (as we can work backward from a constant implementation date T, while changing H).

Next, we turn to the sacrifice ratios associated with policy announcements. First, we use the NKPC to develop some intuition. For any period $T-h$ between the announcement and implementation dates where $h = 0, .., H$, the NKPC (equation (2)) implies that

$$y_{T-h} = \frac{1}{\kappa}\left(\pi_{T-h} - M_f\beta\pi_{T-h+1}\right). \tag{11}$$

First, if the policy is implemented with no delay, i.e. $H = 0$, then (since $\pi_{T+1} = 0$) the sacrifice ratio is simply the reciprocal of the slope of the NKPC

$$SR_0 = \frac{\tilde{y}_0}{\tilde{\pi}_0} = \frac{y_T}{\pi_T} = \frac{1}{\kappa}. \tag{12}$$

If the policy is announced one period in advance, the sacrifice ratio is

$$SR_1 = \frac{\tilde{y}_1}{\tilde{\pi}_1} = \frac{y_T + y_{T-1}}{\pi_T + \pi_{T-1}} = \frac{1}{\kappa}\left[1 - M_f\beta\left(1 - \frac{\pi_{T-1}}{\pi_T + \pi_{T-1}}\right)\right]. \tag{13}$$

More generally, the sacrifice ratio associated with a unit policy shock announced H periods in advance can be expressed as follows,

$$SR_H = \frac{\tilde{y}_H}{\tilde{\pi}_H} = \frac{\sum_{h=0}^{H} y_{T-h}}{\sum_{h=0}^{H} \pi_{T-h}} = \frac{1}{\kappa}\left[1 - M_f\beta\left(1 - w_H\right)\right], \tag{14}$$

where

$$w_H \equiv \frac{\pi_{T-H}}{\sum_{h=0}^{H} \pi_{T-h}}. \tag{15}$$

Equation (14), derived in Appendix A.2, immediately reveals that, as long as $M_f\beta > 0$ (i.e. unless firms are completely myopic), $SR_H < SR_0$ for any $H > 0$. That is, an anticipated policy change delivers lower sacrifice ratios than an unanticipated policy change.

To get a more complete characterization of how the sacrifice ratio SR_H evolves as the horizon H increases, we need to study the system (3)-(4). Using the definition of the sacrifice ratio, we can re-express it in terms of sums of the ratios of output to inflation in every period (for derivation see Appendix A.4)

$$SR_H = \sum_{h=0}^{H} w_{h,H} s_h, \tag{16}$$

where $s_h = y_{T-h}/\pi_{T-h}$ is the ratio of the effect of the policy announcement on output relative to inflation in period $T-h$, and $w_{h,H} \equiv \pi_{T-h}/\sum_{j=0}^{H} \pi_{T-j}$ are weights which sum to one. Dividing equation (3) by equation (4), the ratios s_h can be recursively defined: $s_0 = 1/\kappa$, while for $h = 1, 2, ...H$

$$s_h = \left[\frac{1}{\kappa}\right] \left[\frac{s_{h-1} + \frac{\hat{M}_h}{M_h}\sigma}{s_{h-1} + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}}\right] \equiv f(s_{h-1}), \tag{17}$$

which is decreasing and convex in h whenever $M_f\beta > 0$, i.e. unless firms are completely myopic. Hence, the sacrifice ratio SR_H is a convex combination of the decreasing and convex (in h) series of ratios s_h.

Next, it is useful to again re-express the sacrifice ratio recursively as follows,

$$SR_H = w_H s_H + (1 - w_H)\, SR_{H-1}, \tag{18}$$

where SR_{H-1} is the sacrifice ratio associated with a monetary policy announcement $H-1$ periods ahead, w_H is the weight defined in equation (15), and $s_H = y_{T-H}/\pi_{T-H}$ satisfies the recursion $s_H = f(s_{H-1})$ defined in equation (17). A step-by-step derivation is provided in Appendices A.3 and A.4. Now we are ready to state our main result.

Proposition 1. *The sacrifice ratio associated with a monetary policy announcement of a change in the policy rate H periods ahead is a) decreasing and b) convex in H.*

Proof. *a*) Proof by induction. Equation (17) gives $s_H = f(s_{H-1}) \in (0, \kappa^{-1})$ for all $s_{H-1} > 0$ since $M_f \beta > 0$. Thus, we have $s_1 = f(s_0) < \kappa^{-1} = s_0$. If $s_H < s_{H-1}$ then, since $f'(s_{H-1}) > 0$, we also have that $s_{H+1} = f(s_H) < f(s_{H-1}) = s_H$. Therefore, s_H is a decreasing sequence in H. Finally, since by equation (16) SR_{H-1} is a convex combination of the sequence $\{s_h\}_{h=0}^{H-1}$ then $SR_{H-1} > s_H$ which means by equation (18) that SR_H is also a decreasing sequence in H. *b*) Proof in Appendix A.5. □

The intuition behind this result is that inflation is more forward-looking than output as we describe in Section 2.1. Therefore, announcements of future policies have a stronger effect on inflation than on output. The second part of the intuition behind Proposition 1 is the fact that the relative forward-lookingness of inflation is compounded when the announcement horizon increases. This intuition is captured by the recursion in equation (17) which illustrates that the ratio of the effect of the announced period-T policy shock on output relative to inflation is smaller in period $T-h$ than in the next period $T-(h-1)$ (i.e., $s_h < s_{h-1}$). This explains why the sacrifice ratio SR_H is decreasing in H. The convexity of the sequence SR_H means that a significant fraction of the decrease in sacrifice ratios is achieved in short horizons, i.e. at small values of H.

This property will be explored further in Section 2.4.

Importantly, Proposition 1 does not hinge on the forward guidance puzzle (or the absence of it). The result holds in the canonical New Keynesian model ($M_h = \hat{M}_h = M_f = 1$) and New Keynesian models with extra compounding (with some of the parameters $M_h, \hat{M}_h, M_f$ greater than one), featuring the forward guidance puzzle, as well as in models with extra discounting in the Euler equation and the Phillips curve ($M_h, \hat{M}_h, M_f \in (0,1)$), so that the puzzle is resolved. However, the rate at which the sacrifice ratio falls depends on the rate of extra (cognitive) discounting (or compounding) by households and firms, as our next result shows.[8]

Proposition 2. *When an announced policy change takes place at least one period ahead, stronger cognitive discounting for households lowers the sacrifice ratio, while stronger cognitive discounting for firms raises the sacrifice ratio. Furthermore, the sacrifice ratio rises with stronger uniform cognitive discounting. That is, for all H=1,2,...*

$$\textit{(a)}\ \frac{dSR_H}{dM_h} > 0\,,\ \textit{(b)}\ \frac{dSR_H}{d\hat{M}_h} > 0\,,\ \textit{(c)}\ \frac{dSR_H}{dM_f} < 0\,,\ \textit{and (d)}\ \frac{dSR_H}{d\bar{M}} < 0,$$

where $M_f = M_h = \hat{M}_h = \bar{M}$ *for (d).*

Proof in Appendix A.6.

Intuitively, cognitive discounting by households makes output even less forward-looking compared to inflation while cognitive discounting by firms makes inflation less forward-looking (items a, b and c).[9] Take the case when $\hat{M}_h = M_h$ and recall the recursion in equation (17). In this simplified case, one can easily see how the relative cognitive discounting of firms vis-a-vis households (M_f/M_h) affects the rate at which sacrifice ratios decrease as the associated policy announcement horizon increases.

---

[8] To simplify exposition, for now on we will refer to the parameters $M_h, \hat{M}_h, M_f$ as ”cognitive discounting”, with smaller values marking stronger discounting by households or firms. However, one should keep in mind the other possible interpretations and theoretical motivations of these parameters.

[9] However, inflation is always more forward-looking than output, as long as firms are not completely myopic, i.e., as long as $M_f\beta > 0$.

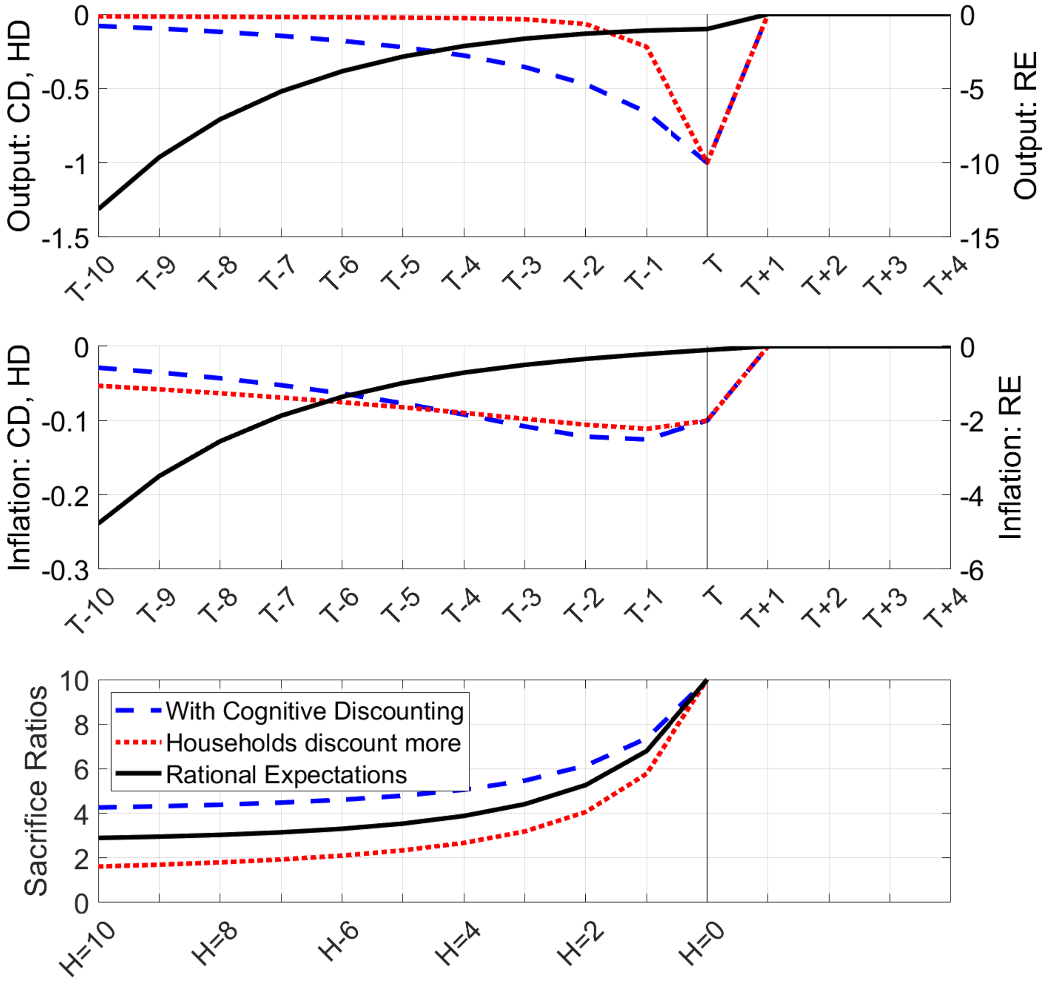


Figure 1: Sacrifice ratios, output, and inflation to an announced future policy rate change

The figure plots the dynamics of output (top panel) and inflation (middle panel) to a unit increase in the policy rate in period T under cognitive discounting (left axes) and rational expectations (right axes). The bottom panel plots the sacrifice ratios if the policy change in period T were to be announced in the period indicated on the horizontal axis (from 10 periods ahead to an unannounced policy shock). The black lines denote the paths of the variables under rational expectations (RE). The dashed blue lines are for when there is cognitive discounting (CD) such that the forward guidance puzzle is resolved ($M_h = \hat{M}_h = M_f = 0.6$)*. The dotted red lines denote paths when households have stronger cognitive discounting relative to firms (HD where* $M_h = \hat{M}_h = 0.2$, $M_f = 0.9$)*. In all cases, the discount factor is set to 0.99, the coefficient of relative risk aversion is set to one and the slope of the New Keynesian Phillips Curve is set to 0.1.*

The overall degree of cognitive discounting also matters (item d). Sacrifice ratios fall with the horizon H because announced future policies have a larger relative impact on inflation than on output. This mechanism is weaker if announced future policies have a more limited effect on the current dynamics of the economy, which is the case when all agents feature (strong) cognitive discounting. This can be seen by studying the special case where all cognitive discounting parameters take the same value ($M_h = \hat{M}_h = M_f = \bar{M}$) corresponding to item (d) of Proposition 2. In this special case, equation (17) implies that the per-period ratio of output and inflation (s_h) is independent of cognitive discounting. However, the weights $w_{h,H}$ appearing in equation (16) depend on $\bar{M}$. Intuitively, forward guidance puzzle effects mean that the response of inflation in a given period is increasing in the implementation horizon therefore putting a relatively larger weight on the effects closer to the announcement date (e.g., s_H) and relatively smaller weights on the effects closer to the implementation date (e.g., s_0 and s_1). Consequently, the level of the sacrifice ratio, fixing the horizon H, will tend to increase as cognitive discounting is introduced.

Our main results, summarized in Propositions 1 and 2, are illustrated in Figure 1. The top two panels in the figure plot the dynamics of output and inflation in response to an announced unit increase in the policy rate in period T. The bottom panel reports the resulting sacrifice ratios over the period $T - 10$ to T, $T - 9$ to T, and so on which would be equivalent to the resulting sacrifice ratios if the announcement where to be made 10 periods ahead, 9 periods ahead, and so on (i.e., $H = 0, 1, ..., 10$). The solid black lines indicate the paths of output, inflation, and resulting sacrifice ratios under rational expectations. The dashed blue lines report the corresponding paths under uniform cognitive discounting such that the forward guidance puzzle is resolved ($M_h = \hat{M}_h = M_f$). The dotted red lines are for the case when households have stronger cognitive discounting relative to firms ($M_h = \hat{M}_h < M_f$).

The top two panels show forward guidance puzzle effects in the benchmark representative agent New Keynesian model under rational expectations (solid black line, right vertical axes) which can be resolved by introducing cognitive discounting (dashed blue and dotted red lines, left vertical axes). The bottom panel shows that sacrifice ratios fall with the horizon H under both rational expectations and cognitive discounting (Proposition 1). In all cases, the ratio peaks on impact and is equal to the inverse of the slope of the New Keynesian Phillips Curve. Further, cognitive discounting attenuates the rate at which the sacrifice ratio falls such that for any given horizon other than on impact, sacrifice ratios are higher under cognitive discounting than under rational expectations (Proposition 2, item d) . However, the opposite is true when households' expectations feature stronger cognitive discounting relative to firms (Proposition 2, items a,b,c). See Appendix A.11 for further analysis and some illustrations.

2.4 Gains from increasing the implementation horizon

Next we assess the gains from increasing the implementation horizon. To do so, we first derive the minimum value of the sacrifice ratio, which is attained as a limit when the implementation horizon goes to infinity. It is useful to start with the representation in equation (14). Without forward guidance puzzle effects, the announcement effect on inflation goes to zero as the implementation date is set further out into the future and so $\lim_{H\to\infty} w_H = \lim_{H\to\infty} \pi_{T-H} / \sum_{h=0}^{H} \pi_{T-h} = 0$. Then in this case where there are no forward guidance puzzle effects, we have

$$\lim_{H\to\infty} SR_H = \frac{1}{\kappa}\left(1 - \beta M_f\right). \tag{19}$$

We know that the maximum value of the sacrifice ratio (at SR_0) is pinned down by the slope of the NKPC. Interestingly, without forward guidance puzzle effects, the minimum value of the sacrifice ratio also only depends on the parameters of the NKPC.

On the other hand, with forward guidance puzzle effects, the weight w_H tends to a constant, and the limit of the sacrifice ratio tends to the limit value of the per-period ratio s_H (see equation (18)). In this case, the limit is characterized by the recursion $s_H = f(s_{H-1})$ in equation (17) which depends on the parameters from both the dynamic IS curve and the NKPC. Further, recall that under uniform cognitive discounting ($M_h = \hat{M}_h = M_f = \bar{M}$) the recursion (17) does not depend on $\bar{M}$.

We evaluate the limit value of the sacrifice ratio and its properties with and without forward guidance puzzle effects which we summarize in the following proposition.

Proposition 3. *The sacrifice ratio associated with an announcement of a unit change in the policy rate H periods ahead tends to a constant as H tends to infinity,*

$$\lim_{H\to\infty} SR_H = \left(\frac{1}{\kappa}\right)\left[1 - \frac{M_f\beta}{\max\{1, \lambda_2\}}\right], \tag{20}$$

where λ_2 is the larger eigenvalue of the matrix A defined in (6). Moreover,

(A) Suppose there is uniform cognitive discounting, so that $M_h = \hat{M}_h = M_f = \bar{M}$. Then,

$$\lim_{H\to\infty} SR_H \geq \lim_{H\to\infty} SR_H^{RE}, \tag{21}$$

where $\lim_{H\to\infty} SR_H^{RE}$ denotes the limit of the sacrifice ratio under rational expectations, and the inequality above is strict when $\lambda_2 = \bar{M}\lambda_2^{RE} < 1$, i.e., when there are no forward guidance puzzle effects.

(B) The sacrifice ratio at the limit is decreasing in household cognitive discounting and increasing in firm cognitive discounting as well as under uniform cognitive discounting,

$$(a)\ \frac{\partial \lim_{H\to\infty} SR_H}{\partial M_h} \geq 0, \qquad (b)\ \frac{\partial \lim_{H\to\infty} SR_H}{\partial \hat{M}_h} \geq 0,$$

$$(c)\ \frac{\partial \lim_{H\to\infty} SR_H}{\partial M_f} < 0, \quad and \quad (d)\ \frac{\partial \lim_{H\to\infty} SR_H}{\partial \bar{M}} \leq 0, \tag{22}$$

where the inequalities in (a) and (b) above are strict when $\lambda_2 > 1$, i.e., when there are forward guidance puzzle effects while the inequality in (d) above is strict when $\lambda_2 < 1$, i.e., when there are no forward guidance puzzle effects.

Proof in Appendix A.8.

These results echo Proposition 2, which applies to finite announcement horizons, but with a twist. The infinite horizon limit of the sacrifice ratio tends to be lower when households' expectations feature stronger cognitive discounting, and higher when firms' expectations feature stronger cognitive discounting or under uniform cognitive discounting. However, if there are no forward guidance puzzle effects, the limit is independent of household cognitive discounting (since M_h and $\hat{M}_h$ enter the IS curve, but not the NKPC), while if there are forward guidance puzzle effects, the limit is independent of uniform cognitive discounting (since the per-period ratio s_H does not depend on uniform cognitive discounting).

Proposition 3 describes the minimum value of the sacrifice ratio, attainable with infinitely long implementation horizons H. However, not only are sacrifice ratios decreasing in the implementation horizon, they are also convex (Proposition 1, item b). That is, a significant fraction of the fall in sacrifice ratios is achieved at small values for H. Define the fraction of the reduction in the sacrifice ratio achieved by announcing a policy change in period H relative to the limit as H tends to infinity with the ratio of operators $\Delta_0 SR_H / \Delta_0 SR_\infty$,

$$\frac{\Delta_0 SR_H}{\Delta_0 SR_\infty} \equiv \frac{SR_0 - SR_H}{SR_0 - SR_\infty}. \tag{23}$$

We use this fraction as a metric for the gains from increasing the horizon H. This gives us the following proposition.

Proposition 4. *More than one third and one half of the reduction in sacrifice ratios is achieved when increasing the implementation horizon from zero to one and two respectively:*

$$\frac{\Delta_0 SR_1}{\Delta_0 SR_\infty} = \frac{\max\{1, \lambda_2\}}{1 + \lambda_1 + \lambda_2} > \frac{1}{3} \tag{24}$$

$$\frac{\Delta_0 SR_2}{\Delta_0 SR_\infty} = \frac{\max\{1, \lambda_2\}}{\frac{1-\lambda_1\lambda_2}{1+\lambda_1+\lambda 2} + \lambda_1 + \lambda_2} > \frac{1}{2} \tag{25}$$

where $\{\lambda_1, \lambda_2\}$ *are the eigenvalues of the matrix* A *defined in (6) and* $\lambda_2 > \lambda_1$ *without loss of generality.*

Proof in Appendix A.9.

Note that the proposition holds both with and without forward guidance puzzle effects. The proposition states that, with any parametrization of the New Keynesian framework, the gains from advanced communication of the most likely course for monetary policy are concentrated in the shorter horizons. Simply moving from an unannounced to a one-period-ahead anticipated monetary policy rate change already reaps more than a third of the benefits in terms of reducing the sacrifice ratio and making the announcement another period earlier will already bring with it more than half of the benefits.

2.5 Imperfectly credible policy announcements

Our results extend to a situation where monetary policy announcements are imperfectly credible. Assume that the private sector believes that a change in monetary policy, announced h periods in advance, is implemented with probability q_h. We postulate that the credibility of an announcement is decreasing in the horizon, such that $q_0 = 1$, and $q_{h+1} < q_h$ for all $h = 0, 1, \ldots$. We further assume that the change in monetary policy, to be implemented in period T, is (re)announced in each period $T-h$, where $h = 0, 1, \ldots, H$.

Then for all $h = 0, 1, ..., H$ we get

$$x^*_{T-h} = -q_h A^h B = q_h x_{T-h} \tag{26}$$

where $x^*_{T-h} = [y^*_{t-h}, \pi^*_{T-h}]'$ are the reactions of output and inflation to an imperfectly credible monetary policy announcement, $x_{T-h} = [y_{T-h}, \pi_{T-h}]'$ are the corresponding reactions to a perfectly credible announcement, and the matrices A and B are defined in (6).

A particularly simple and intuitive special case is such that $q_h = \rho^h$, with $\rho \in (0, 1)$. Then the equation (26) takes the form[10]

$$x^*_{T-h} = -(\rho A)^h B = -\tilde{A}^h B \tag{27}$$

where

$$\tilde{A} = \begin{bmatrix} \rho M_h & \rho \hat{M}_h \sigma \\ \rho M_h \kappa & \rho \hat{M}_h \kappa \sigma + \rho M_f \beta \end{bmatrix}. \tag{28}$$

Hence, this form of imperfectly credible policy announcements is equivalent to perfectly credible announcements, but stronger cognitive discounting (with each of the parameters M_f, M_h and $\hat{M}_h$ multiplied by ρ). Specifically, there is no forward guidance puzzle if

---

[10] Following an approach similar to Gibbs and McClung (2023) and Haberis et al. (2019), one could alternatively assume that the policymaker promises an exogenous nominal rate path at time $T - h$. In each period between $T - h$ and T, the policymaker can stick to its commitment, or renege and go back to its normal policy. Let the 'renege' state be an absorbing state (i.e., the policymaker is expected to abandon the commitment in all future periods if it deviates in any single period). Let ρ denote the probability that a policymaker sticks to the plan in the following period conditional on sticking to the plan today. It follows that the probability of sticking to the plan h periods in the future is ρ^h. Also these assumptions give rise to equation (27).

the following conditions (corresponding to conditions (7)) are satisfied:

$$\rho^2 M_h M_f \beta < 1 \quad \text{and} \quad \frac{(1-\rho M_h)(1-\rho M_f \beta)}{\rho \hat{M}_h} > \kappa\sigma. \tag{29}$$

Clearly, if ρ is low enough, the conditions (29) may hold not only under cognitive discounting, but also in the rational expectations representative agent model (i.e. when $M_h = \hat{M}_h = M_f = 1$), and even with compounding (i.e. when some of the parameters $M_h, \hat{M}_h, M_f$ are greater than one). Indeed, there is no forward guidance puzzle, if $\rho < 1/\lambda_2$, where λ_2 is the larger eigenvalue of the matrix A given in (6).[11] Also in the more general case where $q_h \neq \rho^h$, it easy to show that there is no forward guidance puzzle if (sufficient condition) $q_{h+1}/q_h < 1/\lambda_2$ for all $h = 0, 1, \ldots$

Given the equivalence between imperfectly credible announcements and cognitive discounting in the special case with $q_h = \rho^h$, it is immediately clear that the results stated in Propositions 1 – 4 continue to hold with this form of imperfectly credible policy announcements. In addition, one can show that $dSR_H/d\rho < 0$ for all $H = 1, 2, ..$ – this is essentially a corollary of Proposition 2 d), see Appendix A.6.

The main insights of the paper also extend to the more general form of imperfect credibility, where $q_h \neq \rho^h$. Sacrifice ratios associated with announcements of the most likely course of monetary policy are lower when the implementation date is further out into the future. Nevertheless, the rate at which sacrifice ratios fall with the implementation horizon is attenuated by imperfect credibility of announcements, as our next, and final, proposition shows.

Proposition 5. *Assume that the private sector believes that a change in monetary policy, announced h periods in advance, is implemented with probability* q_h*, where* q_h

---

[11] Evidently, if $\lambda_2 < 1$, forward guidance puzzle effects are ruled out for all values of $\rho \in (0, 1)$.

is decreasing in h. Denote the sacrifice ratios associated with imperfectly credible monetary policy announcements by SR^*_H. *a) The sacrifice ratio is decreasing in the horizon H even when monetary policy announcements are imperfectly credible. That is,* $SR^*_{H+1} < SR^*_H$ *for all* $H = 0, 1, \ldots$. *b) When an announced policy change takes place at least one period ahead, the imperfect credibility of policy announcements raises the sacrifice ratio, compared to the case of fully credible announcements. That is, for all H=1,2,...* $SR^*_H > SR_H$*, where* SR_H *is the corresponding sacrifice ratio, when the policy announcements are fully credible.*

Proof in Appendix A.10.

The key intuition behind Proposition 5 is that inflation is more forward-looking than output and therefore reacts more strongly to policy announcements (compared to output), even when the announcements are imperfectly credible.

3 A Model with Additional Rigidities

The key insights from the simple New Keynesian framework carry over to a model with additional real and nominal rigidities. We extend the basic New Keynesian model with habit persistence as an additional real rigidity and backward-indexation in prices as an additional nominal rigidity.[12] We use the model to simulate the effects of one-time monetary tightening news shocks, which is announced $H \in \{0, 1, ..., 10\}$ periods ahead of implementation, and calculate sacrifice ratios over 40 periods to account for persistent deviations from the steady state induced by the additional rigidities. In these simulations, we assume that monetary policy is temporarily exogenous for 100 periods before switching to a regime with a Taylor rule thereafter.

Briefly, the two additional rigidities are introduced as follows. First, household preferences can now feature external habits. Specifically, households choose consumption

---

[12] The extended model is based on Afsar et al. (2024).

and labor to maximize the following expected lifetime utility specification,

$$\tilde{\mathbb{E}}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_{j,t} - bC_{t-1}])^{1-1/\sigma}}{1 - 1/\sigma} - \frac{N_{j,t}^{1+\psi}}{1+\psi} \right] \tag{30}$$

where $\tilde{\mathbb{E}}$ is the behavioral expectations operator following Gabaix (2020), β is the conventional discount factor, σ is the intertemporal elasticity of substitution, ψ is the inverse Frisch elasticity, b is the external habits parameter, $C_{j,t}$ is the consumption of household j, C_{t-1} is the average past consumption in the economy, and $N_{j,t}$ is hours worked for household j.

Second, we assume that firms who are unable to reoptimize prices have their prices partially indexed to past inflation, $P_{i,t} = P_{i,t-1}\Pi_{t-1}^{\omega}$, where $P_{i,t}$ is the price set by (non-optimizing) firm i, Π_{t-1} is the gross inflation rate between periods $t-1$ and t, and ω is the price indexation parameter which could also be interpreted as the fraction of backward looking firms.

Together, these two additional rigidities imply the following log-linearized dynamic IS and New Keynesian Phillips curves,

$$y_t = \frac{b}{1+b} y_{t-1} + \frac{M_h}{1+b} \mathbb{E}_t y_{t+1} - \frac{\sigma(1-b)}{1+b} \left(i_t - \hat{M}_h \mathbb{E}_t \pi_{t+1} \right) \tag{31}$$

$$\begin{aligned} \pi_t = {} & \frac{\omega}{1 + M_f \beta \omega} \pi_{t-1} + \frac{M_f \beta}{1 + M_f \beta \omega} \mathbb{E}_t \pi_{t+1} \\ & + \frac{\kappa}{1 + M_f \beta \omega} y_t - \left[\frac{\kappa}{1 + M_f \beta \omega} \right] \left[\frac{b}{1 + \sigma\psi(1-b)} \right] y_{t-1} \end{aligned} \tag{32}$$

where $\kappa = [(1-\theta)(1-\theta\beta)/\theta][\psi + 1/\sigma(1-b)]$ is the slope of the conventional NKPC with external habits under rational expectations, θ is the Calvo price rigidity parameter, and $M_h, \hat{M}_h$ and M_f are cognitive discounting parameters. Following Gabaix (2020) and Afsar et al. (2024), we set $M_f = M_h[\theta + (1-\theta)(1-\theta\beta)/(1-\theta\beta M_h)]$.

We calibrate the model as follows. We assume a sufficiently high degree of cognitive discounting ($M_h = \hat{M}_h = 0.5$) to eliminate forward guidance puzzle effects. The slope of the NKPC under rational expectations and without the additional rigidities is about 0.1, habit persistence is set to 0.8, and the backward-indexation parameter in the hybrid NKPC is set to 0.5.[13] The full set of calibration values for the model parameters are reported in Table 1.

Table 1: Extended model parameter calibration

Parameter	Value	Description
β	0.99	Discount factor
σ	1.00	Intertemporal elasticity of substitution
ψ	1.00	Inverse Frisch elasticity
b	$\{0.00, 0.80\}$	Habit persistence
ω	$\{0.00, 0.50\}$	Price backward indexation
θ	0.80	Calvo price rigidity
M_h	0.50	Household output cognitive discounting
$\hat{M}_h$	0.50	Household inflation cognitive discounting

---

[13] The reported simulations do not feature the apparently implausible inflation "reversals" discussed at length by Carlstrom et al. (2015), which can emerge in models with temporary interest rate pegs and lagged endogenous variables.

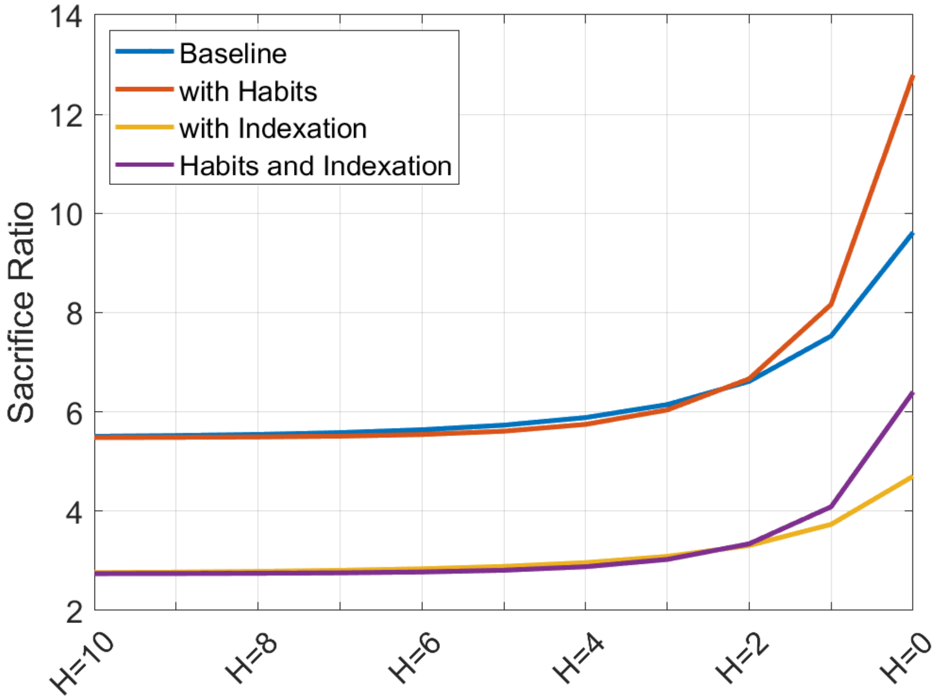


Figure 2: Sacrifice Ratios with Additional Rigidities

The figure plots the sacrifice ratio of announcements of a one-time unit policy rate increase at horizons zero to 10 (horizontal axis). The blue line is for the benchmark model (with no habit persistence or price indexation). The orange line is for the model with habits in consumption. The yellow line is for the model with price indexation. The purple line is for the model with both habits and indexation.

Figure 2 illustrates the resulting sacrifice ratios from simulations of the model where the horizontal axis reports the policy implementation horizon. Similar to what we have shown analytically in the basic New Keynesian model, we find that sacrifice ratios tend to fall the further out the horizon for the policy shock in all the cases shown in the figure. These results also do not hinge on the particular calibration we chose. We find similar patterns from simulations using alternative calibrations which are reported in Appendix A.12.

As in the basic New Keynesian model, the feature that sacrifice ratios are convex over the horizon H remains in the simulation results which means that most of the reduction in sacrifice ratios is still achieved when increasing the horizon at low initial levels. Also, it is notable that additional nominal frictions in the form of backward-indexation in prices which render inflation dynamics less forward-looking than in the basic model, tend to lower the level of the sacrifice ratio at all horizons and flatten the profile of sacrifice ratios over the policy implementation horizon.

4 Some Empirical Evidence

We provide some empirical evidence to validate the predictions of the theory. In particular, we want to investigate the main theory result (Proposition 1) which states that longer monetary policy announcement horizons H give rise to lower sacrifice ratios.

The sacrifice ratio is calculated as the cumulative impulse response of output divided by the cumulative impulse response of inflation. This suggests a two-step estimation strategy, whereby we first estimate the cumulative impulse responses to unanticipated and anticipated monetary policy shocks, and then compute the ratios. However, recent macroeconometric literature recommends a more efficient way to estimate ratios and dynamic multipliers. Following the advice of Ramey and Zubairy (2018) and Stock and Watson (2018), we estimate the sacrifice ratios in a single step. We regress cumulative output (growth) on cumulative inflation, using monetary policy surprises as (external) instruments

$$\sum_{j=0}^{J} y_{t+j} = \alpha + \delta \sum_{j=0}^{J} \pi_{t+j} + u_t. \tag{33}$$

Note that

$$\begin{aligned}
&\text{IV estimate of } \delta \\
&= \frac{\text{variation of output explained by monetary policy surprises}}{\text{variation of inflation explained by monetary policy surprises}} \\
&= \text{sacrifice ratio.}
\end{aligned}$$

We study the US, and our sample period spans from 1991Q1 to 2024Q3. As the output measure we use quarterly GDP growth, and as the inflation measure the quarterly change in CPI excluding food and energy. As instruments we apply i) surprises in short rates and ii) surprises in long rates. These high-frequency-identified monetary policy surprises are taken from Bauer and Swanson (2023), and measure the responses of interest rates over 30-minute windows around FOMC announcements. The short rate surprises are the responses of the current-quarter Eurodollar (ED) futures contract, next-quarter ED futures contract, and the two- and three-quarter-ahead ED futures contracts. The long rate surprises are 2-year, 5-year, 10-year, and 30-year Treasury yield responses, as measured from Treasury note and Treasury bond futures.

We take the short rate surprises as proxies for short horizons H and the long rate surprises as proxies for longer horizons H. We compute the sacrifice ratios over the periods of 4 quarters, 8 quarters, 12 quarters, 16 quarters and 20 quarters (J in equation (33)).

Table 2: Estimated sacrifice ratios

	Instruments	
	Surprises in short rates	Surprises in long rates
Sacrifice ratio over 4Q	6.78** (3.05)	5.06** (2.12)
Sacrifice ratio over 8Q	7.13*** (2.40)	3.23*** (1.07)
Sacrifice ratio over 12Q	7.82*** (2.51)	3.03** (1.20)
Sacrifice ratio over 16Q	7.71** (3.41)	3.36*** (1.16)
Sacrifice ratio over 20Q	8.85*** (3.00)	4.03** (1.67)

Notes: Standard deviations (HAC) in parentheses. *, ** and *** refer to statistical significance at the 10%, 5% and 1% level, respectively.

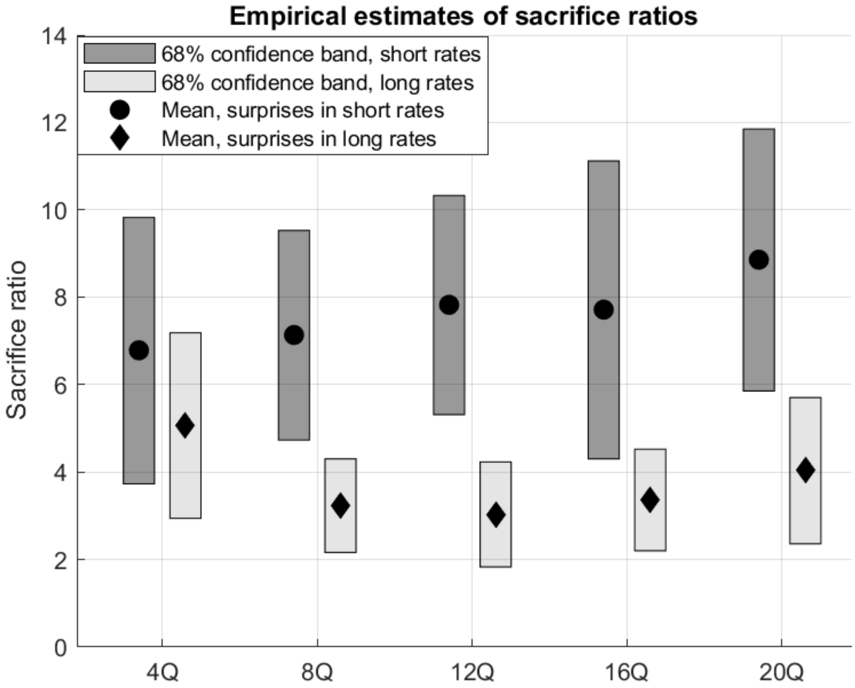


Figure 3: Empirical estimates of sacrifice ratios. The x-axis denotes the period over which the sacrifice ratio is computed (J in equation (33)).

Our theory (see Proposition 1) suggests that estimates of sacrifice ratios using surprises in long rates as instruments should be lower than estimates using surprises in short rates. The results reported in Table 2 and Figure 3 are consistent with this prediction.

Table 3: Estimated sacrifice ratios with positive monetary policy surprises

	Instruments			
	Positive surprises in short rates		Positive surprises in long rates	
Sacrifice ratio over 4Q	6.30	(3.92)	1.92	(1.94)
Sacrifice ratio over 8Q	7.43***	(2.48)	2.29	(1.93)
Sacrifice ratio over 12Q	7.10***	(2.99)	3.36**	(1.46)
Sacrifice ratio over 16Q	7.20***	(1.86)	4.63***	(1.50)
Sacrifice ratio over 20Q	8.27***	(1.44)	6.48***	(1.28)

Notes: Standard deviations (HAC) in parentheses. *, ** and *** refer to statistical significance at the 10%, 5% and 1% level, respectively.

Our paper is about monetary policy strategies that lower inflation with a minimum impact on the real economy. Hence, one could argue that sacrifice ratios associated with restrictive monetary policy shocks should be of essence. Recognizing this point, we repeat the empirical analysis, applying positive (i.e. restrictive) monetary policy surprises as instruments. The results reported in Table 3 are qualitatively similar to those shown in Table 2. The estimates of the sacrifice ratios using surprises in long rates as instruments are lower than the estimates using surprises in short rates, consistent with the predictions of the theory. (For completeness, the corresponding results with negative surprises as instruments are shown in Table A.2 in Appendix A.13.)

5 Concluding Remarks

We show that sacrifice ratios tend to fall as an announced monetary policy action is implemented further out into the future. We prove this analytically in a simple New Keynesian framework and also show that the analytical results generally carry over in a model with additional real and nominal rigidities. Finally, we present some empirical evidence that supports this prediction of the theory.

It is important to note that our findings relate to the efficient conduct of monetary policy with respect to minimizing the response of output vis-à-vis inflation. Sacrifice ratios are imperfect metrics of the desirability of monetary policy actions but are nevertheless informative regarding the key concerns that central bankers have in mind when fighting inflation. As such, our analysis abstracts from several related issues such as uncertainty regarding gaps that need to be closed in the future (and therefore the optimality of a given announced policy path) as well as the trade-off between conditionality and commitment from early announcements of future monetary policy actions.

Notably, our results suggest that the benefits in terms of lower sacrifice ratios are concentrated in short announcement horizons. Hence, for a central bank aiming to curb inflation, near-term signaling (i.e. making announcements about the most likely course of monetary policy in the near future) could be a viable strategy, improving the inflation-output trade-off, while retaining the flexibility to react to new information.

References

Acosta, M. (2023). A new measure of central bank transparency and implications for the effectiveness of monetary policy. *International Journal of Central Banking*, 19(3):49–97.

Afsar, A., Gallegos, J.-E., Jaimes, R., and Silgado-Gomez, E. (2024). A behavioral hybrid new keynesian model: Quantifying the importance of belief-formation frictions. *Economic Modelling*, 132.

Altavila, C., Brugnolini, L., Gurkaynak, R. S., Motto, R., and Ragusa, G. (2019). Measuring euro area monetary policy. *Journal of Monetary Economics*, 108:162–179.

Angeletos, G.-M. and Lian, C. (2018). Forward guidance without common knowledge. *American Economic Review*, 108(9):2477–2512.

Ascari, G. and Ropele, T. (2012a). Disinflation in a dsge prespective: Sacrifice ratio or welfare gain ratio? *Journal of Economic Dynamics and Control*, 36(2):169–182.

Ascari, G. and Ropele, T. (2012b). Sacrifice ratio in a medium-scale new keynesian model. *Journal of Money, Credit and Banking*, 44(2-3):457–467.

Ball, L. (1994). What determines the sacrifice ratio? In Mankiw, N. G., editor, *Monetary Policy*, pages 155–193. The University of Chicago Press.

Barnichon, R. and Mesters, G. (2023). A sufficient statistics approach for macro policy. *American Economic Review*, 113(11):2809–2845.

Bauer, M. and Swanson, E. (2023). A reassessment of monetary policy surprises and high-frequency identification. *NBER Macroeconomic Annual 23*.

Bilbiie, F. (2025). Monetary policy and heterogeneity: An analytical framework. *Review of Economic Studies*.

Binder, C. (2017). Fed speak on main street: Central bank communication and household expectations. *Journal of Macroeconomics*, 52:238–251.

Blinder, A. S., Ehrmann, M., Fratzscher, M., De Haan, J., and Jansen, D.-J. (2008). Central bank communication and monetary policy: A survey of theory and evidence. *Journal of Economic Literature*, 46(4):910–945.

Carlstrom, C. T., Fuerst, T. S., and Paustian, M. (2015). Inflation and output in new keynesian models with a transient interest rate peg. *Journal of Monetary Economics*, 76:230–243.

Coibion, O., Gorodnichenko, Y., and Weber, M. (2022). Monetary policy communication and their effects on household inflation expectations. *Journal of Political Economy*, 130(6):1537–1584.

de Haan, J. and Hoogduin, L. (2024). Ecb communication policies: An overview and comparison with the federal reserve. *Journal of International Money and Finance*, 142.

Del Negro, M., Giannoni, M. P., and Patterson, C. (2023). The forward guidance puzzle. *Journal of Political Economy Macroeconomics*, 1(1):43–79.

Dupraz, S., Le Bihan, H., and Matheron, J. (2024). Make-up strategies with finite planning horizons but infinitely forward-looking asset prices. *Journal of Monetary Economics*, 143.

Elfsbacka-Schmöller, M. and McClung, N. (2025). Can growth stabilize debt? a fiscal theory perspective. Technical report, Bank of Finland.

Gabaix, X. (2020). A behavioral new keynesian model. *American Economic Review*, 110(8):2271–2327.

Gibbs, C. G. and Kulish, M. (2017). Disinflations in a model of imperfectly anchored expectations. *European Economic Review*, 100:157–174.

Gibbs, C. G. and McClung, N. (2023). Does my model predict a forward guidance puzzle? *Review of Economic Dynamics*, 51:393–423.

Gordon, R. J. (1983). Why stopping inflation may be costly: Evidence from fourteen historical episodes. In Hall, R. E., editor, *Inflation: Causes and Effects*, pages 11–40. University of Chicago Press, Chicago.

Haberis, A., Harrison, R., and Waldron, M. (2019). Uncertain policy promises. *European Economic Review*, 111:459–474.

Ireland, P. N. (1997). Stopping inflations, big and small. *Journal of Money, Credit and Banking*, 29(4):759–775.

Kryvtsov, O. and Petersen, L. (2021). Central bank communication that works: Lessons from lab experiments. *Journal of Monetary Economics*, 117:760–780.

Laséen, S. and Svensson, L. E. (2011). Anticipated alternative policy rate paths in policy simulations. *International Journal of Central Banking*, 7(3):1–35.

McKay, A., Nakamura, E., and Steinsson, J. (2016a). The discounted euler equation: A note. Technical report, National Bureau of Economic Research.

McKay, A., Nakamura, E., and Steinsson, J. (2016b). The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58.

McKay, A. and Wolf, C. (2023). What can time-series regressions tell us about policy counterfactuals? *Econometrica*, 91(5):1695–1725.

Nicolae, A. and Nolan, C. (2006). The impact of imperfect credibility in a transition to price stability. *Journal of Money, Credit and Banking*, 38(1):47–66.

Ramey, V. and Zubairy, S. (2018). Government spending multipliers in good times and in bad. *Journal of Political Economy*, 126:850–901.

Sargent, T. J. (1986). Stopping Moderate Inflations: The Methods of Poincare and Thatcher. In Sargent, T. J., editor, *Rational Expectations and Inflation*, pages 110–157. Harper & Row, New York.

Sheen, J. and Wang, B. Z. (2023). Do monetary condition news at the zero lower bound influence household's expectations and readiness to spend. *European Economic Review*, 152.

Stock, J. and Watson, M. (2018). Identification and estimation of dynamic causal effects in macroeconomics using external instruments. *Economic Journal*, 128:917–948.

Swanson, E. T. and Jayawickrema, V. (2023). Speeches by the fed chair are more important than fomc announcements: An improved high-frequency measure of u.s. monetary policy shocks. *Working Paper*.

Taylor, J. B. (1983). Union wage settlements during a disinflation. *American Economic Review*, 73(5):981–993.

Woodford, M. (2019). Monetary policy analysis when planning horizons are finite. *NBER Macroeconomics Annual*, 33:1–50.

Woodford, M. and Xie, Y. (2022). Fiscal and monetary stabilization policy at the zero lower bound: Consequences of limited foresight. *Journal of Monetary Economics*, 125:18–35.

Derivations and Proofs Appendix

A.1 Derivation of Equations (7)

The conditions are obtained by restricting the eigenvalues of the matrix A in equation (6) to both be less than one. Let λ_1 and λ_2 denote the smaller and larger eigenvalue, respectively. The characteristic equation of A is

$$\chi(\lambda) = \lambda^2 - tr(A)\lambda + det(A) = 0. \tag{A.1}$$

Since $tr(A) > 0$ and $det(A) > 0$, it is clear that both roots are positive, i.e. $\lambda_1, \lambda_2 > 0$. Moreover, if $det(A) = M_h M_f \beta < 1$, we know that at least one of the roots is smaller than one, i.e. $\lambda_1 < 1$ (whereas if $det(A) = M_h M_f \beta > 1$, at least one of the roots is greater than one). Now, if $\chi(1) > 0$, also $\lambda_2 < 1$. Evaluating the characteristic equation at $\lambda = 1$ yields

$$1 - tr(A) + det(A) > 0 \tag{A.2}$$

$$1 - (M_h + \hat{M}_h \kappa\sigma + M_f \beta) + M_h M_f \beta > 0 \tag{A.3}$$

$$(1 - M_h)(1 - M_f \beta) - \hat{M}_h \kappa\sigma > 0 \tag{A.4}$$

$$\frac{(1 - M_h)(1 - M_f \beta)}{\hat{M}_h} > \kappa\sigma. \tag{A.5}$$

A.2 Derivation of Equation (14)

To obtain this formulation of the sacrifice ratio, we note that for an announcement in period $T - H$ of a unit increase in the policy rate in period T, $\pi_{T+1} = 0$. Moreover, we can re-express output solely as a function of inflation by inverting the New Keynesian

Phillips Curve (NKPC),

$$y_{T-h} = \frac{1}{\kappa} \left[\pi_{T-h} - M_f \beta \pi_{T+1-h} \right]. \tag{A.6}$$

This allows us to express the sacrifice ratio as,

$$\begin{aligned}
SR_H \equiv \frac{\sum_{h=0}^{H} y_{T-h}}{\sum_{h=0}^{H} \pi_{T-h}} &= \left(\frac{1}{\kappa}\right) \left[\frac{\sum_{h=0}^{H} \left(\pi_{T-h} - M_f \beta \pi_{T+1-h}\right)}{\sum_{h=0}^{H} \pi_{T-h}} \right] && \text{(A.7)} \\
&= \left(\frac{1}{\kappa}\right) \left[1 - M_f \beta \frac{\sum_{h=0}^{H} \pi_{T+1-h}}{\sum_{h=0}^{H} \pi_{T-h}} \right] && \text{(A.8)} \\
&= \left(\frac{1}{\kappa}\right) \left[1 - M_f \beta \left(1 - \frac{\pi_{T-H}}{\sum_{h=0}^{H} \pi_{T-h}} \right) \right] && \text{(A.9)} \\
&= \left(\frac{1}{\kappa}\right) \left[1 - M_f \beta \left(1 - w_H\right) \right] && \text{(A.10)}
\end{aligned}$$

A.3 Derivation of Equation (17)

To arrive at the impact effect of an announced policy which will occur H periods ahead in period T, it is useful to first define x_{T-h} where $h \in [0, H]$ and $x \in \{y, \pi\}$ as the value that variable x takes in time $T - h$ when the interest rate is equal to 1 in period T and is zero otherwise. As nothing else occurs after period T, the terminal values are given by:

$$y_T = -\sigma \tag{A.11}$$

$$\pi_T = -\kappa\sigma \tag{A.12}$$

which results in an impact sacrifice ratio of $s_T \equiv s_0 = \kappa^{-1}$, the inverse of the slope of the NKPC. Further, the dynamics of output and inflation in between period $T - H$ and T is given by

$$y_{T-h} = M_h y_{T-h+1} + \hat{M}_h \sigma \pi_{T-h+1} \tag{A.13}$$

$$\pi_{T-h} = \kappa M_h y_{T-h+1} + (M_f \beta + \hat{M}_h \kappa \sigma) \pi_{T-h+1} \tag{A.14}$$

dividing the left-hand and right-hand sides of the first equation above with the respective left-hand and right-hand sides of the second equation yields,

$$s_{T-h} = \frac{y_{T-h}}{\pi_{T-h}} = \frac{M_h y_{T-h+1} + \hat{M}_h \sigma \pi_{T-h+1}}{\kappa M_h y_{T-h+1} + (M_f \beta + \hat{M}_h \kappa \sigma) \pi_{T-h+1}} \tag{A.15}$$

$$= \frac{M_h s_{T-h+1} + \hat{M}_h \sigma}{\kappa M_h s_{T-h+1} + M_f \beta + \hat{M}_h \kappa \sigma} \tag{A.16}$$

$$= \left[\frac{1}{\kappa}\right] \left[\frac{M_h s_{T-h+1} + \hat{M}_h \sigma}{M_h s_{T-h+1} + \hat{M}_h \sigma + \frac{M_f \beta}{\kappa}}\right] \tag{A.17}$$

To arrive at equation (17) in the main text, we define $s_h \equiv s_{T-h}$,

$$s_h = \left[\frac{1}{\kappa}\right] \left[\frac{M_h s_{h-1} + \hat{M}_h \sigma}{M_h s_{h-1} + \hat{M}_h \sigma + \frac{M_f \beta}{\kappa}}\right] \tag{A.18}$$

which yields equation (17) in the main text after dividing both the numerator and the denominator with M_h.

In the case where the central bank were to announce a unit policy rate increase in H periods but maintain a Taylor-type rule in between period t and $t+H$, $i_t = \phi_y y_t + \phi_\pi \pi_t + \epsilon_t$, then equation (5) for the same period becomes:

$$x_t = \frac{1}{1+\sigma(\phi_y+\kappa\phi_\pi)}\begin{bmatrix} M_h & \hat{M}_h\sigma - M_f\beta\phi_\pi\sigma \\ \kappa M_h & \hat{M}_h\kappa\sigma + M_f\beta(1+\sigma\phi_y) \end{bmatrix}\mathbb{E}_t[x_{t+1}] - \frac{1}{1+\sigma(\phi_y+\kappa\phi_\pi)}\begin{bmatrix}\sigma \\ \kappa\sigma\end{bmatrix}\epsilon_t \tag{A.19}$$

where ϵ_t is a monetary innovation ($\epsilon_T = 1$ and zero everywhere else). Note that, unlike the baseline scenario, the dynamics of inflation and output represented above capture the full path of interest rates from period $T-H$ to T. Specifically, these dynamics take into account both the exogenous monetary policy innovation at the end of the period and the endogenous response of monetary policy starting from the beginning of the period onwards. In turn, equation (17) becomes

$$s_h = \begin{bmatrix}\frac{1}{\kappa}\end{bmatrix}\left[\frac{M_h s_{h-1} + \sigma(\hat{M} - M_f\beta\phi_\pi)}{M_h s_{h-1} + \sigma(\hat{M} - M_f\beta\phi_\pi) + \frac{M_f\beta}{\kappa}\left(1+\sigma(\phi_y+\kappa\phi_\pi)\right)}\right] \tag{A.20}$$

which would still be decreasing in H for as long as $M_f\beta > 0$ and that the eigenvalues of the first matrix in equation (A.19) have real roots (the second condition rules out cyclical dynamics wherein s_h would oscillate between positive and negative values).

A.4 Derivation of Equations (16) and (18)

Following the definition of the sacrifice ratio SR_H we have,

$$SR_H = \sum_{h=0}^{H}\frac{y_{T-h}}{\sum_{j=0}^{H}\pi_{T-j}} \tag{A.21}$$

$$= \sum_{h=0}^{H}\frac{\pi_{T-h}}{\sum_{j=0}^{H}\pi_{T-j}}\frac{y_{T-h}}{\pi_{T-h}} \tag{A.22}$$

$$= \sum_{h=0}^{H} w_{h,H} s_h \tag{A.23}$$

where $w_{h,H} \equiv \pi_{T-h}/(\sum_{j=0}^{H} \pi_{T-j})$. This is equation (16) of the main text. We can split the sum into the first element and the sum of the remaining elements to get,

$$SR_H = w_{H,H} s_H + \sum_{h=0}^{H-1} w_{h,H} s_h \tag{A.24}$$

$$= w_H s_H + \sum_{h=0}^{H-1} \frac{\pi_{T-h}}{\sum_{j=0}^{H} \pi_{T-j}} s_h \tag{A.25}$$

$$= w_H s_H + \left[\frac{\sum_{j=0}^{H-1} \pi_{T-j}}{\sum_{j=0}^{H-1} \pi_{T-j}}\right] \sum_{h=0}^{H-1} \frac{\pi_{T-h}}{\sum_{j=0}^{H} \pi_{T-j}} s_h \tag{A.26}$$

$$= w_H s_H + \left[\frac{\sum_{j=0}^{H-1} \pi_{T-j}}{\sum_{j=0}^{H} \pi_{T-j}}\right] \sum_{h=0}^{H-1} \frac{\pi_{T-h}}{\sum_{j=0}^{H-1} \pi_{T-j}} s_h \tag{A.27}$$

$$= w_H s_H + \left[1 - \frac{\pi_{T-H}}{\sum_{j=0}^{H} \pi_{T-j}}\right] \sum_{h=0}^{H-1} \frac{\pi_{T-h}}{\sum_{j=0}^{H-1} \pi_{T-j}} s_h \tag{A.28}$$

$$= w_H s_H + [1 - w_H] \sum_{h=0}^{H-1} w_{h,H-1} s_h \tag{A.29}$$

$$= w_H s_H + [1 - w_H]\, SR_{H-1} \tag{A.30}$$

which yields equation (18) in the main text.

A.5 Proof of Proposition 1 b

We prove that the sacrifice ratio SR_H is a convex sequence. To do so, we first need to prove that the sequence s_H is convex. Let us denote $\Delta s_H \equiv s_H - s_{H-1}$ for $H = 1, 2, \ldots.$

Then $\Delta s_H = g(s_{H-1})$, where

$$g(s) = f(s) - s \tag{A.31}$$

and

$$f(s) = \left[\frac{1}{\kappa}\right]\left[\frac{s + \frac{\hat{M}_h}{M_h}\sigma}{s + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}}\right] \tag{A.32}$$

(see equation (17)).

We know (by item a of this Proposition) that $s_H < s_{H-1}$ for all $H = 1, 2, ...$. Now we need to show that $\Delta s_{H+1} = g(s_H) > g(s_{H-1}) = \Delta s_H$ for all $H = 1, 2, ...$. To do so, first note that

$$f'(s) = \left[\frac{1}{\kappa}\right]\left[\frac{\frac{M_f\beta}{M_h\kappa}}{\left(s + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}\right)^2}\right] > 0 \tag{A.33}$$

and

$$f''(s) = -\left[\frac{1}{\kappa}\right]\left[\frac{\frac{M_f\beta}{M_h\kappa}}{\left(s + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}\right)^3}\right] < 0 \tag{A.34}$$

while

$$g'(s) = f'(s) - 1 \tag{A.35}$$

and

$$g''(s) = f''(s) < 0. \tag{A.36}$$

Next, let us denote by s^* a fixed point such that $f(s^*) = s^*$, or equivalently $g(s^*) = 0$. Since $f(s)$ is an increasing and concave function, with $f(0) > 0$ and $\lim_{s\to\infty} f(s) = 1/\kappa$, it is clear that the equation $f(s^*) = s^*$ has exactly one solution over the interval $s \in (0, \infty)$. Furthermore, since $f(s)$ is an increasing function, it is clear that $f(s) > s^*$ for all $s > s^*$. Next, we use proof by induction to show that $s_H > s^*$ for all $H = 0, 1, \ldots$. First step. Since $f(s) < 1/\kappa$ for all $s > 0$, it is clear that $s^* = f(s^*) < 1/\kappa = s_0$. Induction step. Assume that $s_{H-1} > s^*$. But then since $f(s)$ is an increasing function, $s_H = f(s_{H-1}) > f(s^*) = s^*$. Hence, $s_H > s^*$ for all $H = 0, 1, \ldots$

Next, evaluating the derivative $g'(s)$ at $s = s^*$ gives

$$g'(s^*) = \left[\frac{1}{\kappa}\right]\left[\frac{\frac{M_f\beta}{M_h\kappa}}{\left(s^* + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}\right)^2}\right] - 1 \tag{A.37}$$

$$= \left[\frac{\frac{M_f\beta}{M_h\kappa}s^*}{\left(s^* + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}\right)\left(s^* + \frac{\hat{M}_h}{M_h}\sigma\right)}\right] - 1 \tag{A.38}$$

$$= -\left[\frac{\left(s^* + \frac{\hat{M}_h}{M_h}\sigma\right)^2 + \frac{M_f\beta}{M_h\kappa}\frac{\hat{M}_h}{M_h}\sigma}{\left(s^* + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}\right)\left(s^* + \frac{\hat{M}_h}{M_h}\sigma\right)}\right] < 0 \tag{A.39}$$

where the second equality follows since $s^* = f(s^*)$ implies that

$$\left[\frac{1}{\kappa}\right]\left[\frac{1}{s^* + \frac{\hat{M}_h}{M_h}\sigma + \frac{M_f\beta}{M_h\kappa}}\right] = \frac{s^*}{s^* + \frac{\hat{M}_h}{M_h}\sigma} \tag{A.40}$$

But then $g'(s^*) < 0$ together with (A.36) implies that $g'(s) < 0$ for all $s \geq s^*$.

Finally, since $s^* < s_H < s_{H-1}$ for all $H = 1, 2, ...$, we have shown that

$$\Delta s_{H+1} = g(s_H) > g(s_{H-1}) = \Delta s_H \tag{A.41}$$

for all $H = 1, 2, ...$ This completes the first part of the proof.

In the second part of the proof we need to show that the sequence SR_H is convex. Let us define $\Delta SR_H \equiv SR_H - SR_{H-1}$ for $H = 1, 2, ...$ We know that $\Delta SR_H < 0$ for all H. To prove convexity, we need to show that $\Delta SR_{H+1} > \Delta SR_H$ for all H.

Using the recursion (18), we get

$$\begin{aligned}
\Delta SR_{H+1} &= w_{H+1} \left(s_{H+1} - SR_H\right) && \text{(A.42)}\\
&= w_{H+1} \left(\Delta s_{H+1} - \Delta SR_H + s_H - SR_{H-1}\right) && \text{(A.43)}\\
&= w_{H+1} \left(\Delta s_{H+1} - \Delta SR_H + \frac{\Delta SR_H}{w_H}\right) && \text{(A.44)}\\
&= \frac{w_{H+1}}{w_H} \left[w_H \Delta s_{H+1} + (1 - w_H) \Delta SR_H\right] && \text{(A.45)}\\
&= \frac{w_{H+1}}{w_H} z_H && \text{(A.46)}
\end{aligned}$$

where

$$z_H = w_H \Delta s_{H+1} + (1 - w_H) \Delta SR_H \tag{A.47}$$

for $H = 1, 2, ...$, while

$$z_0 = \Delta s_1. \tag{A.48}$$

Clearly $z_H < 0$ for all H. Next, combining (14) and (18) gives

$$SR_0\left[1 - \beta M_f(1 - w_{H+1})\right] = w_{H+1}s_{H+1} + (1 - w_{H+1})SR_0\left[1 - \beta M_f(1 - w_H)\right] \tag{A.49}$$

and solving for w_{H+1} yields

$$\begin{aligned}
w_{H+1} &= \frac{SR_0\beta M_f w_H}{SR_0\beta M_f + \Delta SR_H - \Delta s_{H+1} + SR_{H-1} - s_H} && \text{(A.50)}\\
&= \frac{SR_0\beta M_f w_H}{SR_0\beta M_f + \Delta SR_H - \Delta s_{H+1} - \frac{\Delta SR_H}{w_H}} && \text{(A.51)}\\
&= \frac{SR_0\beta M_f w_H^2}{SR_0\beta M_f w_H - [w_H\Delta s_{H+1} + (1 - w_H)\Delta SR_H]} && \text{(A.52)}\\
&= \frac{SR_0\beta M_f w_H^2}{SR_0\beta M_f w_H - z_H}. && \text{(A.53)}
\end{aligned}$$

Then plugging (A.53) into (A.46) yields

$$\Delta SR_{H+1} = z_H\left[\frac{SR_0\beta M_f w_H}{SR_0\beta M_f w_H - z_H}\right] \equiv q(z_H, w_H) \tag{A.54}$$

Clearly

$$\Delta SR_{H+1} > z_H, \tag{A.55}$$

since $z_H < 0$, and the term between the square brackets in (A.54) is between 0 and 1. Also note that

$$\frac{\partial q(z_H, w_H)}{\partial z_H} = \left[\frac{SR_0\beta M_f w_H}{SR_0\beta M_f w_H - z_H}\right]^2 > 0 \tag{A.56}$$

and

$$\frac{\partial q(z_H, w_H)}{\partial w_H} = \frac{SR_0 \beta M_f z_H^2}{\left(SR_0 \beta M_f w_H - z_H\right)^2} < 0. \tag{A.57}$$

Next,

$$\Delta SR_{H+1} - \Delta SR_H = \frac{(SR_0 \beta M_f)^2 w_H w_{H-1} (z_H - z_{H-1}) - SR_0 \beta M_f \Delta w_H z_H z_{H-1}}{(SR_0 \beta M_f w_H - z_H)(SR_0 \beta M_f w_{H-1} - z_{H-1})}, \tag{A.58}$$

where

$$\Delta w_H = w_H - w_{H-1} = \frac{SR_H - SR_{H-1}}{SR_0 \beta M_f} < 0, \tag{A.59}$$

where the second form follows from equation (14), and the sign follows since $SR_H < SR_{H-1}$, by Proposition 1 item (a). Then it is clear from (A.58) that $\Delta SR_{H+1} - \Delta SR_H > 0$ if (sufficient condition) $z_H > z_{H-1}$.

We use proof by induction to show that z_H is an increasing sequence. First step. Given (A.41), (A.47), (A.48) and (A.55), it is clear that

$$z_1 = w_1 \Delta s_2 + (1 - w_1) \Delta SR_1 \tag{A.60}$$

$$> w_1 \Delta s_2 + (1 - w_1) z_0 \tag{A.61}$$

$$= w_1 \Delta s_2 + (1 - w_1) \Delta s_1 \tag{A.62}$$

$$> \Delta s_1 = z_0. \tag{A.63}$$

Induction step. Assume that $z_H > z_{H-1}$. In this final step of the proof, we need to consider two cases. i) First, suppose that $\Delta SR_{H+1} < \Delta s_{H+2}$. Then (A.47) and (A.55) imply that

$$
\begin{aligned}
z_{H+1} &= w_{H+1}\Delta s_{H+2} + (1 - w_{H+1})\Delta SR_{H+1} && \text{(A.64)} \\
&> \Delta SR_{H+1} > z_H. && \text{(A.65)}
\end{aligned}
$$

ii) Second, suppose that $\Delta SR_{H+1} > \Delta s_{H+2}$. Then

$$
\begin{aligned}
z_{H+1} &= w_{H+1}\Delta s_{H+2} + (1 - w_{H+1})\Delta SR_{H+1} && \text{(A.66)} \\
&= \Delta w_{H+1}(\Delta s_{H+2} - \Delta SR_{H+1}) + w_H\Delta s_{H+2} + (1 - w_H)\Delta SR_{H+1} && \text{(A.67)} \\
&> w_H\Delta s_{H+2} + (1 - w_H)\Delta SR_{H+1} && \text{(A.68)} \\
&> w_H\Delta s_{H+1} + (1 - w_H)\Delta SR_H = z_H && \text{(A.69)}
\end{aligned}
$$

where the first inequality follows from (A.59) and the assumption that $\Delta SR_{H+1} > \Delta s_{H+2}$, while the second inequality follows from (A.41), (A.54), (A.56) and (A.57), together with $z_H > z_{H-1}$ (by the induction assumption). Hence, we have shown that z_H is an increasing sequence and, by implication (recall equation (A.58)), ΔSR_H is an increasing sequence and SR_H is a convex sequence. This completes the proof.

A.6 Proof of Proposition 2

In this Appendix we prove Proposition 2. Furthermore, we show that in the special case where imperfect credibility of policy announcements (see Section 2.5) takes the form $q_h = \rho^h$, we get $dSR_H/d\rho < 0$ for all $H = 1, 2, ..$ Here we exploit the fact that a model with this form of imperfectly credible policy announcements is (technically) equivalent to a model with perfectly credible announcements, but stronger cognitive discounting. With slight abuse of notation, we capture this relationship by setting

$M_f = \rho m_f$, $M_h = \rho m_h$, and $\hat{M}_h = \rho \hat{m}_h$, where $m_f, m_h, \hat{m}_h \in (0,1]$. This is item e) of the proof.

The proof consists of two parts. In the first part we show that for all $H = 1, 2, ...$ we have a) $ds_H/dM_f < 0$, b) $ds_H/dM_h > 0$, c) $ds_H/d\hat{M}_h > 0$, d) $ds_H/d\bar{M} = 0$ if $M_f = M_h = \hat{M}_h = \bar{M}$, and e) $ds_H/d\rho = 0$ if $M_f = \rho m_f, M_h = \rho m_h, \hat{M}_h = \rho \hat{m}_h$.

Let us denote

$$s_H = \left[\frac{1}{\kappa}\right]\left[\frac{M_h s_{H-1} + \hat{M}_h \sigma}{M_h s_{H-1} + \hat{M}_h \sigma + \frac{M_f \beta}{\kappa}}\right] \equiv f(s_{H-1}; M_f, M_h, \hat{M}_h). \tag{A.70}$$

Then for $H = 1, 2...$

$$\frac{ds_H}{dM_j} = \frac{\partial f(s_{H-1}; M_f, M_h, \hat{M}_h)}{\partial M_j} + \frac{\partial f(s_{H-1}; M_f, M_h, \hat{M}_h)}{\partial s_{H-1}} \frac{ds_{H-1}}{dM_j} \tag{A.71}$$

for $M_j \in \{M_f, M_h, \hat{M}_h, \bar{M}\}$. Next, it is straightforward to show that

$$\frac{\partial f(s_{H-1}; M_f, M_h, \hat{M}_h)}{\partial M_h} > 0, \frac{\partial f(s_{H-1}; M_f, M_h, \hat{M}_h)}{\partial \hat{M}_h} > 0, \frac{\partial f(s_{H-1}; M_f, M_h, \hat{M}_h)}{\partial M_f} < 0. \tag{A.72}$$

Proof by induction. First step. Since $s_0 = \kappa^{-1}$ it is clear that $ds_0/dM_j = 0$. Then it follows from (A.71) and (A.72) that

$$\frac{ds_1}{dM_h} > 0, \frac{ds_1}{d\hat{M}_h} > 0, \frac{ds_1}{dM_f} < 0 \tag{A.73}$$

Induction step. Assume that for $H = 2, 3, ...$

$$\frac{ds_{H-1}}{dM_h} > 0, \frac{ds_{H-1}}{d\hat{M}_h} > 0, \frac{ds_{H-1}}{dM_f} < 0 \tag{A.74}$$

Then it follows from (A.71) and (A.72) that

$$\frac{ds_H}{dM_h} > 0, \frac{ds_H}{d\hat{M}_f} > 0, \frac{ds_H}{dM_f} < 0 \tag{A.75}$$

Furthermore, it is immediately clear from (A.70) that

$$\frac{ds_H}{d\bar{M}} = 0, \frac{ds_H}{d\rho} = 0 \tag{A.76}$$

for all $H = 0, 1, \ldots$ This completes the first part of the proof.

In the second part of the proof, we characterize and sign the derivatives of the sacrifice ratio SR_H. Plugging (14) into (18) and differentiating yields

$$\frac{dw_H}{dM_j} = \frac{(1 - w_H)\frac{dSR_{H-1}}{dM_j} + w_H \frac{ds_H}{dM_j} + \beta(1 - w_H)SR_0 \frac{dM_f}{dM_j}}{\beta M_f SR_0 + SR_{H-1} - s_H} \tag{A.77}$$

where

$$\frac{dM_f}{dM_j} = \begin{cases} 1 & \text{if } M_j \in \{M_f, \bar{M}\} \\ 0 & \text{if } M_j \in \{M_h, \hat{M}_h\} \\ m_f & \text{if } M_j = \rho \end{cases} \tag{A.78}$$

$$\tag{A.79}$$

Note that

$$SR_{H-1} > s_H \tag{A.80}$$

(see the proof of Proposition 1). Then differentiating (14) and using (A.77) yields

$$\begin{aligned}\frac{dSR_H}{dM_j} &= \beta \times SR_0 \left[M_f \frac{dw_H}{dM_j} - (1-w_H)\frac{dM_f}{dM_j} \right] \\ &= \beta \times SR_0 \left[\frac{(1-w_H)\left[M_f \frac{dSR_{H-1}}{dM_j} - (SR_{H-1} - s_H)\frac{dM_f}{dM_j}\right] + w_H M_f \frac{ds_H}{dM_j}}{\beta M_f SR_0 + SR_{H-1} - s_H} \right] \end{aligned} \tag{A.81}$$

Proof by induction. First step. Since $SR_0 = \kappa^{-1}$, it is clear that $dSR_0/dM_j = 0$. Then it follows from (A.73), (A.76), (A.79), (A.80) and (A.81) that

$$\frac{dSR_1}{dM_h} > 0, \frac{dSR_1}{d\hat{M}_h} > 0, \frac{dSR_1}{dM_f} < 0, \frac{dSR_1}{d\bar{M}} < 0, \frac{dSR_1}{d\rho} < 0. \tag{A.82}$$

Induction step. Assume that for $H = 2, 3, ..$

$$\frac{dSR_{H-1}}{dM_h} > 0, \frac{dSR_{H-1}}{d\hat{M}_h} > 0, \frac{dSR_{H-1}}{dM_f} < 0, \frac{dSR_{H-1}}{d\bar{M}} < 0, \frac{dSR_{H-1}}{d\rho} < 0. \tag{A.83}$$

Then (A.75), (A.76), (A.79), (A.80), (A.81) and (A.83) imply that

$$a)\ \frac{dSR_H}{dM_h} > 0,\ b)\ \frac{dSR_H}{d\hat{M}_h} > 0,\ c)\ \frac{dSR_H}{dM_f} < 0, d)\ \frac{dSR_H}{d\bar{M}} < 0,\ e)\ \frac{dSR_H}{d\rho} < 0. \tag{A.84}$$

This completes the proof.

A.7 The sacrifice ratio as a function of eigenvalues of the matrix A

The sacrifice ratio can also be expressed in terms of the eigenvalues of the matrix A,

$$SR_H = \left(\frac{1}{\kappa}\right)\left[1 - M_f \beta \frac{\sum_{h=0}^{H} \lambda_2^h - \sum_{h=0}^{H} \lambda_1^h}{\sum_{h=0}^{H} \lambda_2^{h+1} - \sum_{h=0}^{H} \lambda_1^{h+1}}\right] \tag{A.85}$$

where λ_1 and λ_2 are the eigenvalues of the matrix A and, without loss of generality, $\lambda_1 < \lambda_2$. The sacrifice ratio can again be recursively defined as in equation (18) where the ratio of output to inflation s_H as well as the weight w_H are given by,

$$s_H = \left(\frac{1}{\kappa}\right)\left[1 - M_f \beta \frac{d_H}{d_{H+1}}\right] \tag{A.86}$$

$$w_H = \frac{d_{H+1}}{D_{H+1}} \tag{A.87}$$

where $d_H \equiv \lambda_2^H - \lambda_1^H$ while $D_H = \sum_{h=0}^{H} d_h$.

These expressions are derived as follows. Applying an eigenvalue decomposition to matrix $A = Q\Lambda Q^{-1}$ where,

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \tag{A.88}$$

$$Q = \begin{bmatrix} \hat{M}_h \sigma & \hat{M}_h \sigma \\ -(M_h - \lambda_1) & -(M_h - \lambda_2) \end{bmatrix} \tag{A.89}$$

we can re-express equation (5) where a unit policy increase which will take place in period T is announced in period $T - H$ as,

$$x_{T-h} = -Q\Lambda^h Q^{-1} B \tag{A.90}$$

$$\Rightarrow$$

$$y_{T-h} = -\frac{\sigma}{\lambda_2 - \lambda_1}\left[\lambda_2^{h+1} - \lambda_1^{h+1} - M_f \beta (\lambda_2^h - \lambda_1^h)\right] \tag{A.91}$$

$$\pi_{T-h} = -\frac{\kappa\sigma}{\lambda_2 - \lambda_1}\left[\lambda_2^{h+1} - \lambda_1^{h+1}\right] \tag{A.92}$$

The above expressions also illustrate what we mean when we say that inflation is more "*forward-looking*" than output. Taking sums over $h \in [0, H]$ and dividing the cumulative change in output over inflation yields the sacrifice ratio as a function of the eigenvalues of the matrix A:

$$SR_H = \frac{\sum_{h=0}^{H} y_{T-h}}{\sum_{h=0}^{H} \pi_{T-h}} = \left(\frac{1}{\kappa}\right) \frac{\sum_{h=0}^{H} \lambda_2^{h+1} - \lambda_1^{h+1} - M_f \beta \sum_{h=0}^{H} \lambda_2^{h} - \lambda_1^{h}}{\sum_{h=0}^{H} \lambda_2^{h+1} - \lambda_1^{h+1}} \tag{A.93}$$

$$= \left(\frac{1}{\kappa}\right) \left[1 - M_f \beta \frac{\sum_{h=0}^{H} \lambda_2^{h} - \lambda_1^{h}}{\sum_{h=0}^{H} \lambda_2^{h+1} - \lambda_1^{h+1}}\right] \tag{A.94}$$

Similarly, equation (A.86) is obtained by dividing output with inflation:

$$s_H = \frac{y_{T-H}}{\pi_{T-H}} = \left(\frac{1}{\kappa}\right) \left[1 - M_f \beta \frac{\lambda_2^{H} - \lambda_1^{H}}{\lambda_2^{H+1} - \lambda_1^{H+1}}\right] \tag{A.95}$$

$$= \left(\frac{1}{\kappa}\right) \left[1 - M_f \beta \frac{d_H}{d_{H+1}}\right] \tag{A.96}$$

where $d_H \equiv \lambda_2^H - \lambda_1^H$. Finally, the weight on the announcement impact effect is obtained by dividing the response of inflation on announcement over its cumulative effect:

$$w_H = \frac{\lambda_2^{H+1} - \lambda_1^{H+1}}{\sum_{h=0}^{H} \lambda_2^{h+1} - \lambda_1^{h+1}} \tag{A.97}$$

$$= \frac{d_{H+1}}{D_{H+1}} \tag{A.98}$$

where $D_H \equiv \sum_{h=0}^{H} d_h$.

A.8 Proof of Proposition 3

We first re-express the sacrifice ratio or the weights in terms of the eigenvalues of the matrix A as described in Appendix A.7. The limit result is obtained by evaluating the expression D_H / D_{H+1} and noting that the sum of powers of the eigenvalues is a geometric series,

$$\lim_{H\to\infty} \frac{D_H}{D_{H+1}} = \lim_{H\to\infty} \frac{d_1 - \lambda_2^{H+1}(1-\lambda_1) + \lambda_1^{H+1}(1-\lambda_2)}{d_1 - \lambda_2^{H+2}(1-\lambda_1) + \lambda_1^{H+2}(1-\lambda_2)} \tag{A.99}$$

$$= \begin{cases} 1 & \text{if } \lambda_2 \leq 1 \\ \frac{1}{\lambda_2} & \text{if } \lambda_2 > 1 \end{cases} \tag{A.100}$$

$$= min\{1, \lambda_2^{-1}\} \tag{A.101}$$

The limit of SR_H is just the appropriate transformation of the limit shown above. Note that $w_H = d_{H+1}/D_{H+1}$ such that $1 - w_H = D_H/D_{H+1}$.

$$SR_H = \kappa^{-1}\left[1 - M_f\beta\frac{D_H}{D_{H+1}}\right] \tag{A.102}$$

$$\lim_{H\to\infty} SR_H = \kappa^{-1}\left[1 - M_f\beta min\{1, \lambda_2^{-1}\}\right] \tag{A.103}$$

$$= \kappa^{-1}\left[1 - \frac{M_f\beta}{max\{1, \lambda_2\}}\right] \tag{A.104}$$

Hence, the result in equation (20).

To prove the result under uniform cognitive discounting, we first note that under uniform cognitive discounting the trace and determinant of the matrix A and the eigenvalues are given by,

$$tr(A) = \bar{M}(1 + \kappa\sigma + \beta) \tag{A.105}$$

$$det(A) = \bar{M}^2\beta \tag{A.106}$$

$$\lambda = \bar{M}\left[\frac{1+\kappa\sigma+\beta}{2} \pm \left(\left[\frac{1+\kappa\sigma+\beta}{2}\right]^2 - \beta\right)^{\frac{1}{2}}\right] \tag{A.107}$$

Under rational expectations $\bar{M} = 1$ such that the last equation also means that $\lambda = \bar{M}\lambda^{RE}$ when keeping all the other model parameters constant and where λ^{RE} are the eigenvalues of the matrix A under rational expectations. Evaluating equation (20) under uniform cognitive discounting ($M_h = \hat{M}_h = M_f = \bar{M}$), noting that $\lambda_2^{-1} = \lambda_1/det(A)$, and letting $\lambda = \bar{M}\lambda^{RE}$ yields

$$\lim_{H\to\infty} SR_H = \kappa^{-1}\left[1 - \bar{M}\beta min\{1, \lambda_2^{-1}\}\right] \tag{A.108}$$

$$= \kappa^{-1}\left[1 - \frac{\lambda_1}{\bar{M}} min\{1, \lambda_2\}\right] \tag{A.109}$$

$$= \kappa^{-1}\left[1 - \lambda_1^{RE} min\{1, \bar{M}\lambda_2^{RE}\}\right] \tag{A.110}$$

Noting that $\lim_{H\to\infty} SR_H^{RE} = \kappa^{-1}\left(1 - \lambda_1^{RE}\right)$, we get equation (21). Moreover, since λ^{RE} do not depend on $\bar{M}$, the derivative of the limit with respect to uniform cognitive discounting is negative when $\lambda_2 = \bar{M}\lambda_2^{RE} < 1$ and zero otherwise. This yields item (d) in equation (22).

Finally, to obtain the rest of the derivatives in equation (22), we first note that when $\lambda_2 < 1$ then the limit of the sacrifice ratio only depends (negatively) on the parameters κ, β, and M_f (see equation (20)). Therefore, in this case the derivatives in parts (a) and (b) are equal to zero and the derivative in part (c) is negative.

For the other case we have $\lambda_2 > 1$ and that the limit value of the sacrifice ratio also depends on λ_2. Therefore we first obtain derivatives of the larger eigenvalue with respect to the cognitive discounting parameters. To do so we make use of the characteristic equation,

$$\chi \equiv \lambda^2 - \lambda tr(M_j) + det(M_j) = 0 \tag{A.111}$$

$$\Rightarrow$$

$$\frac{\partial \chi}{\partial M_j} = 2\lambda \frac{\partial \lambda}{\partial M_j} - tr(M_j)\frac{\partial \lambda}{\partial M_j} - \lambda \frac{\partial tr(M_j)}{\partial M_j} + \frac{\partial det(M_j)}{\partial M_j} = 0 \tag{A.112}$$

$$\Rightarrow$$

$$\frac{\partial \lambda}{\partial M_j} = \frac{\lambda \frac{\partial tr(M_j)}{\partial M_j} - \frac{\partial det(M_j)}{\partial M_j}}{2\lambda - tr(M_j)} \tag{A.113}$$

where $M_j \in \{M_h, \hat{M}_h, M_f\}$. Evaluating the last equation for the larger eigenvalue and each of the cognitive discounting parameters yield,

$$\frac{\partial \lambda_2}{\partial M_h} = \frac{\lambda_2 - M_f\beta}{2\lambda_2 - \lambda_1 - \lambda_2} = \frac{\lambda_2 - M_f\beta}{\lambda_2 - \lambda_1} > 0 \tag{A.114}$$

$$\frac{\partial \lambda_2}{\partial \hat{M}_h} = \frac{\lambda_2 \kappa \sigma}{2\lambda_2 - \lambda_1 - \lambda_2} > 0 \tag{A.115}$$

$$\frac{\partial \lambda_2}{\partial M_f} = \frac{\lambda_2\beta - M_h\beta}{2\lambda_2 - \lambda_1 - \lambda_2} = \frac{(\lambda_2 - M_h)\beta}{\lambda_2 - \lambda_1} > 0 \tag{A.116}$$

Therefore, we get that,

$$\frac{\partial \lim_{H\to\infty} SR_H}{\partial M_h} = \left(\frac{M_f\beta}{\kappa\lambda_2^2}\right)\left[\frac{\partial \lambda_2}{\partial M_h}\right] > 0 \tag{A.117}$$

$$\frac{\partial \lim_{H\to\infty} SR_H}{\partial \hat{M}_h} = \left(\frac{M_f\beta}{\kappa\lambda_2^2}\right)\left[\frac{\partial \lambda_2}{\partial \hat{M}_h}\right] > 0 \tag{A.118}$$

$$\begin{aligned}
\frac{\partial \lim_{H\to\infty} SR_H}{\partial M_f} &= \left(\frac{M_f\beta}{\kappa\lambda_2^2}\right)\left[\frac{\partial \lambda_2}{\partial M_f}\right] - \frac{\beta}{\kappa\lambda_2} \\
&= \left(\frac{\beta}{\kappa\lambda_2}\right)\left[\frac{M_f\beta(\lambda_2 - M_h)}{\lambda_2(\lambda_2 - \lambda_1)} - 1\right] \\
&= \left(\frac{\beta}{\kappa\lambda_2}\right)\left[\frac{M_f\beta\lambda_2 - M_hM_f\beta - \lambda_2^2 + \lambda_1\lambda_2}{\lambda_2(\lambda_2 - \lambda_1)}\right] \qquad (A.119) \\
&= \left(\frac{\beta}{\kappa\lambda_2}\right)\left[\frac{M_f\beta\lambda_2 - \lambda_1\lambda_2 - \lambda_2^2 + \lambda_1\lambda_2}{\lambda_2(\lambda_2 - \lambda_1)}\right]
\end{aligned}$$

$$= \left(\frac{\beta}{\kappa\lambda_2}\right)\left[\frac{M_f\beta - \lambda_2}{(\lambda_2 - \lambda_1)}\right] < 0$$

where the last inequality is due to $\lambda_2 > 1 > M_f\beta$. Thus the stronger the households cognitive discounting (lower values for M_h and $\hat{M}_h$), the lower the sacrifice ratio at the limit when there are forward guidance puzzle effects. The opposite is true for cognitive discounting among firms.

A.9 Proof of Proposition 4

From Proposition 3, we get the following values for sacrifice ratios,

$$\lim_{H\to\infty} SR_H = \frac{1}{\kappa}\left[1 - M_f\beta\frac{1}{\max\{1,\lambda_2\}}\right] \tag{A.120}$$

$$= SR_0\left[1 - M_f\beta\frac{1}{\max\{1,\lambda_2\}}\right]$$

$$\Rightarrow$$

$$\Delta_0 SR_\infty \equiv SR_0 - \lim_{H\to\infty} SR_H = \frac{M_f\beta}{\kappa}\left[\frac{1}{\max\{1,\lambda_2\}}\right] \tag{A.121}$$

This gives us the maximum attainable reduction in the sacrifice ratio. In turn, we use the sacrifice ratio defined in terms of eigenvalues in equation (A.85) to get,

$$SR_1 = \frac{1}{\kappa}\left[1 - M_f\beta\frac{1}{1+\lambda_1+\lambda_2}\right] \tag{A.122}$$

$$\Rightarrow$$

$$\Delta_0 SR_1 = \frac{M_f\beta}{\kappa}\left[\frac{1}{1+\lambda_1+\lambda_2}\right] \tag{A.123}$$

which yields the numerator in the left-hand side of equation (24). The expression in equation (24) is bigger than one third since $1 + \lambda_1 + \lambda_2$ is less than 3 when $\lambda_1 < \lambda_2 < 1$

and $1 + \lambda_1/\lambda_2 + 1/\lambda_2$ is also less than 3 when $\lambda_1 < 1 < \lambda_2$.

For the next equation we have that,

$$SR_2 = \frac{1}{\kappa}\left[1 - M_f\beta\frac{1+\lambda_1+\lambda_2}{(1+\lambda_1+\lambda_2)(\lambda_1+\lambda_2)+1-\lambda_1\lambda_2}\right] \tag{A.124}$$

$$\Rightarrow$$

$$\Delta_0 SR_2 = \frac{M_f\beta}{\kappa}\left[\frac{1}{\lambda_1+\lambda_2+\frac{1-\lambda_1\lambda_2}{1+\lambda_1+\lambda 2}}\right] \tag{A.125}$$

Here, it is first worth noting that $\lambda_1 < 1$ and $\lambda_1 < \lambda_2$ implies the following:

$$\lambda_1^2 < \lambda_2 \tag{A.126}$$

$$1 + \lambda_1 + \lambda_1^2 < 1 + \lambda_1 + \lambda_2 \tag{A.127}$$

$$\lambda_1(1+\lambda_1+\lambda_2) + 1 - \lambda_1\lambda_2 < 1 + \lambda_1 + \lambda_2 \tag{A.128}$$

$$\Rightarrow$$

$$\lambda_1 + \left[\frac{1-\lambda_1\lambda_2}{1+\lambda_1+\lambda_2}\right] < 1 \tag{A.129}$$

Therefore, if $\lambda_2 < 1$ then the denominator inside the square brackets for $\Delta_0 SR_2$ is less than two while the denominator inside the square brackets of $\Delta_0 SR_\infty$ is one. Hence the ratio in equation (25) is larger than one half for the case when $\lambda_2 < 1$.

On the other hand, if $\lambda_2 > 1$, then the denominator inside the square brackets of $\Delta_0 SR_\infty$ is λ_2. In turn, dividing the denominator in $\Delta_0 SR_2$ by λ_2 yields,

$$\frac{1}{\lambda_2}\left[\lambda_1+\lambda_2+\frac{1-\lambda_1\lambda_2}{1+\lambda_1+\lambda_2}\right] = 1 + \frac{1}{\lambda_2}\left[\lambda_1+\frac{1-\lambda_1\lambda_2}{1+\lambda_1+\lambda_2}\right] \tag{A.130}$$

which is again less than two since we have shown that term inside the square brackets above is less than one and $1/\lambda_2$ is also less than one. Consequently, the ratio in equation (25) is also larger than one half for the case when $\lambda_2 > 1$.

A.10 Proof of Proposition 5

a) Denote by π^*_{T-h} and y^*_{T-h} inflation and output in period $T-h$ when monetary policy announcements are imperfectly credible. It is easy to see that $\pi^*_{T-h} = q_h \pi_{T-h}$ and $y^*_{T-h} = q_h y_{T-h}$ where π_{T-h} and y_{T-h} are the corresponding values with perfectly credible policy announcements. Then

$$s^*_h = \frac{y^*_{T-h}}{\pi^*_{T-h}} = \frac{y_{T-h}}{\pi_{T-h}} = s_h \tag{A.131}$$

and the per-period ratio $s^*_h = s_h$ is decreasing in h, by the proof of Proposition 1 a).

Next, let us denote by SR^*_H the sacrifice ratio associated with an imperfectly credible policy, announced H periods in advance. Using exactly the same steps as in Appendix A.4 one can show that

$$SR^*_H = \sum_{h=0}^{H} w^*_{h,H} s_h, \tag{A.132}$$

and

$$SR^*_H = w^*_H s_H + \left(1 - w^*_H\right) SR_{H-1}, \tag{A.133}$$

where $w^*_{h,H} \equiv \pi^*_{T-h} / \sum_{j=0}^{H} \pi^*_{T-j}$ are weights which sum to one, and $w^*_H \equiv \pi^*_{T-H} / \sum_{j=0}^{H} \pi^*_{T-j}$. But then it follows from the proof of Proposition 1 a) that SR^*_H is a decreasing sequence.

b) Note that SR_H and SR^*_H are given by $SR_H = \sum_{h=0}^{H} w_{h,H} s_h$ and $SR^*_H = \sum_{h=0}^{H} w^*_{h,H} s_h$, respectively. Let $H = 1, 2, \ldots.$ Now, we can interpret $\{w_{h,H}\}_{h=0}^{H}$ and $\{w^*_{h,H}\}_{h=0}^{H}$ as two probability distributions. Next, we show that the distribution $\{w_{h,H}\}_{h=0}^{H}$ first order stochastically dominates the distribution $\{w^*_{h,H}\}_{h=0}^{H}$ in the sense that, for all $J = 0, 1, \ldots H - 1$,

$$\sum_{j=0}^{J} w^*_{j,H} > \sum_{j=0}^{J} w_{j,H}. \tag{A.134}$$

To show this, first note that

$$w^*_{j,H} \equiv \frac{\pi^*_{T-j}}{\sum_{h=0}^{H} \pi^*_{T-h}} = \frac{q_j \pi_{T-j}}{\sum_{h=0}^{H} q_h \pi_{T-h}}. \tag{A.135}$$

Then

$$\begin{aligned} \frac{\sum_{j=0}^{J} w_{j,H}}{\sum_{j=0}^{J} w^*_{j,H}} &= \left[\frac{\sum_{j=0}^{J} \pi_{T-j}}{\sum_{h=0}^{H} \pi_{T-h}}\right] \left[\frac{\sum_{h=0}^{H} q_h \pi_{T-h}}{\sum_{j=0}^{J} q_j \pi_{T-j}}\right] \\ &= \left[\frac{\sum_{j=0}^{j} \pi_{T-j}}{\sum_{j=0}^{J} q_j \pi_{T-j}}\right] \left[\frac{\sum_{h=0}^{H} q_h \pi_{T-h}}{\sum_{h=0}^{H} \pi_{T-h}}\right] \\ &= \frac{\sum_{h=0}^{H} w_{h,H} q_h}{\sum_{j=0}^{J} w_{j,J} q_j} \end{aligned} \tag{A.136}$$

where $w_{j,J} \equiv \pi_{T-j} / \sum_{h=0}^{J} \pi_{T-h}$, with $J < H$, are weights than sum to one. Next,

$$\begin{aligned} \sum_{h=0}^{H} w_{h,H} q_h &= \frac{\sum_{j=0}^{J} q_j \pi_{T-j}}{\sum_{h=0}^{H} \pi_{T-h}} + \frac{\sum_{k=J+1}^{H} q_k \pi_{T-k}}{\sum_{h=0}^{H} \pi_{T-h}} \\ &= \Omega_{J,H} \sum_{j=0}^{J} w_{j,J} q_j + \left(1 - \Omega_{J,H}\right) \sum_{k=J+1}^{H} \hat{w}_{k,J} q_k \end{aligned} \tag{A.137}$$

where $\Omega_{J,H} \equiv \sum_{j=0}^{J} \pi_{T-j} / \sum_{h=0}^{H} \pi_{T-h} \in (0,1)$, and $\hat{w}_{k,J} \equiv \pi_{T-k} / \sum_{h=J+1}^{H} \pi_{T-h}$. Then

$$\frac{\sum_{j=0}^{J} w_{j,H}}{\sum_{j=0}^{J} w_{j,H}^{*}} = \Omega_{J,H} + \left(1 - \Omega_{J,H}\right) \frac{\sum_{k=J+1}^{H} \hat{w}_{k,J} q_k}{\sum_{j=0}^{J} w_{j,J} q_j} < 1, \tag{A.138}$$

where the inequality follows since q_j is a decreasing sequence. Hence, we have proved that the distribution $\{w_{h,H}\}_{h=0}^{H}$ first order stochastically dominates the distribution $\{w_{h,H}^{*}\}_{h=0}^{H}$ in the sense given in expression (A.134). But given this stochastic dominance, and given that s_h is a decreasing sequence, we can conclude that for $H = 1, 2, \ldots$

$$SR_H^{*} = \sum_{h=0}^{H} w_{h,H}^{*} s_h > \sum_{h=0}^{H} w_{h,H} s_h = SR_H. \tag{A.139}$$

A.11 Sacrifice ratios under rational expectations and cognitive discounting

In this appendix, we examine the conditions under which the sacrifice ratios under cognitive discounting (SR_H) can be lower than under rational expectations (SR_H^{RE}). Throughout, for expositional simplicity, we assume that $\hat{M}_h = M_h$.

The main findings are illustrated in Figures A.1 and A.2. For each horizon H, $SR_H < SR_H^{RE}$ if the parameter pair $\{M_h, M_f\}$ lies above the curve C_H. In Figure A.1 we show the curves C_H for four horizons $H \in \{1, 2, 10, \infty\}$. The curves C_H are computed using the equations from appendices A.7 and A.8. In particular, there are simple closed-form characterizations for $H = 1$ and $H = \infty$: When $H = 1$, C_1 is given by

$$M_f = \frac{1 + (1 + \sigma\kappa) M_h}{2 + \sigma\kappa} \tag{A.140}$$

while when $H = \infty$, C_∞ is given by

$$M_f = max\left\{M_h, \frac{1}{\lambda_2^{RE}}\right\}, \tag{A.141}$$

where λ_2^{RE} is the larger eigenvalue of the matrix A (defined in (**??**)) under rational expectations. Figure A.1 also shows the parameter ranges with and without the forward guidance puzzle (in gray and white, respectively), characterized by equation (7).

Figure A.2 shows the upper envelope of the curves $\{C_H\}_{H=1}^{\infty}$, denoted by $\bar{C}$, and the lower envelope of the curves $\{C_H\}_{H=1}^{\infty}$, denoted by $\underline{C}$. If the parameter pair $\{M_h, M_f\}$ is above the curve $\bar{C}$, $SR_H < SR_H^{RE}$ for all $H > 0$, while if $\{M_h, M_f\}$ is below the curve $\underline{C}$, $SR_H > SR_H^{RE}$ for all $H > 0$. Notably, the parameter combination corresponding to the case *"Households discount more"* shown in Figure 2 ($M_h = \hat{M}_h = 0.2, M_f = 0.9$) lies in the region above the curve $\bar{C}$.

Figure A.2 suggests that modeling approaches which impose $M_f \leq M_h$, such as Gabaix (2020) (which imposes $M_f = M_h(\theta + (1-\theta)\frac{1-\beta\theta}{1-\beta\theta M_h}) < M_h$) or Woodford (2019) and Dupraz et al. (2024) (which restrict $M_f = M_h$) will lead to relatively large sacrifice ratios compared to the benchmark rational expectations case. However, the opposite prediction may obtain if $M_h < M_f$ (e.g., McKay et al. (2016a) impose $M_h = 0.97$ and $M_f = 1$).

A.12 Extended model simulations under alternative calibrations

We also simulate the sacrifice ratios that would result from alternative calibrations of the model. Specifically, we consider three values for cognitive discount ($M_h = \hat{M}_h = \{1, 0.75, 0.5\}$) and for each of these cases we also consider four calibrations: the benchmark model ($b = 0$, $\omega = 0$), habit persistence only ($b = 0.25$, $\omega = 0$), price

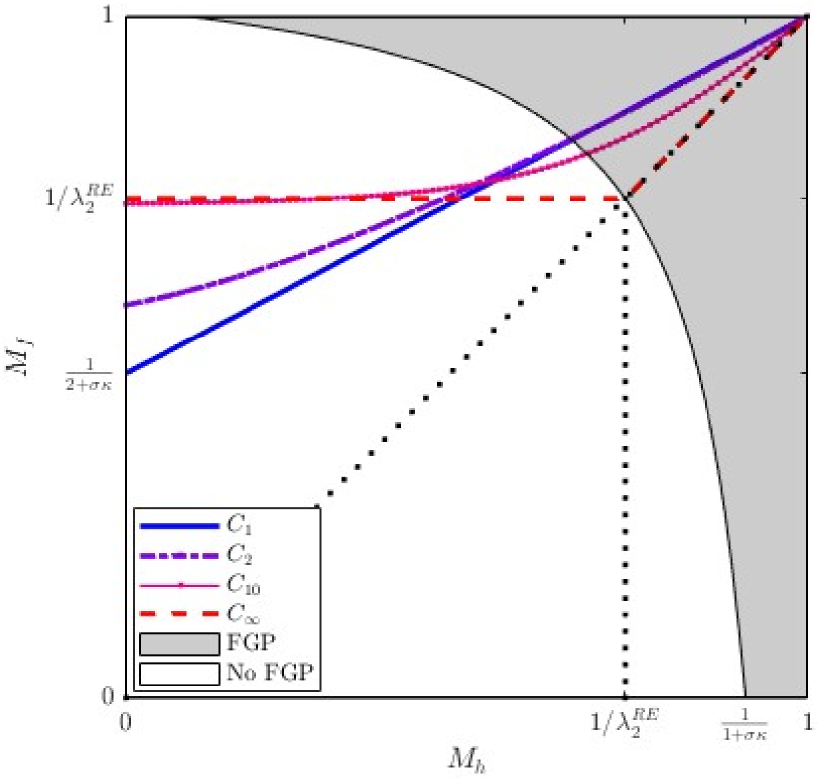


Figure A.1: Sacrifice ratios under cognitive discounting and rational expectations. If $\{M_h, M_f\}$ is above the curve $C_H, SR_H < SR_H^{RE}$. The figure shows the results for the horizons $H \in \{1, 2, 10, \infty\}$. The figure also shows the parameter ranges with and without the forward guidance puzzle (in gray and white, respectively).

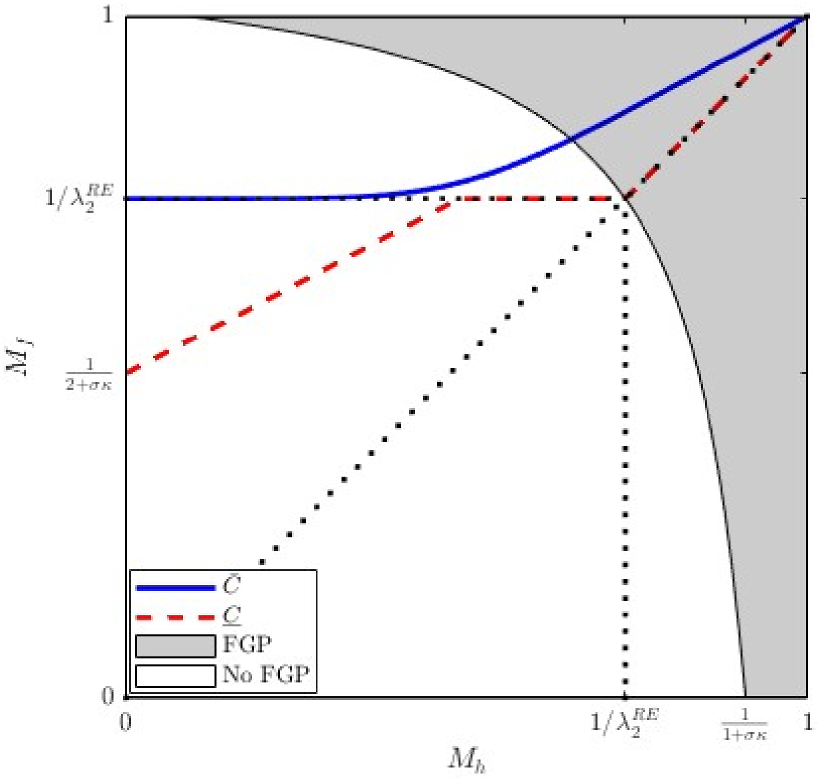


Figure A.2: Sacrifice ratios under cognitive discounting and rational expectations. If $\{M_h, M_f\}$ is above the curve $\bar{C}$, $SR_H < SR_H^{RE}$ for all $H > 0$, while if $\{M_h, M_f\}$ is below the curve $\underline{C}$, $SR_H > SR_H^{RE}$ for all $H > 0$. The figure also shows the parameter ranges with and without the forward guidance puzzle (in gray and white, respectively).

indexation only ($b = 0$, $\omega = 0.25$), and the model with both habit persistence and price indexation ($b = 0.25$, $\omega = 0.25$). To these we add a final specification taken from the estimates in Afsar et al. (2024).[14] The various parameter calibrations are reported in Table A.1

Table A.1: Extended model alternative parameter calibration

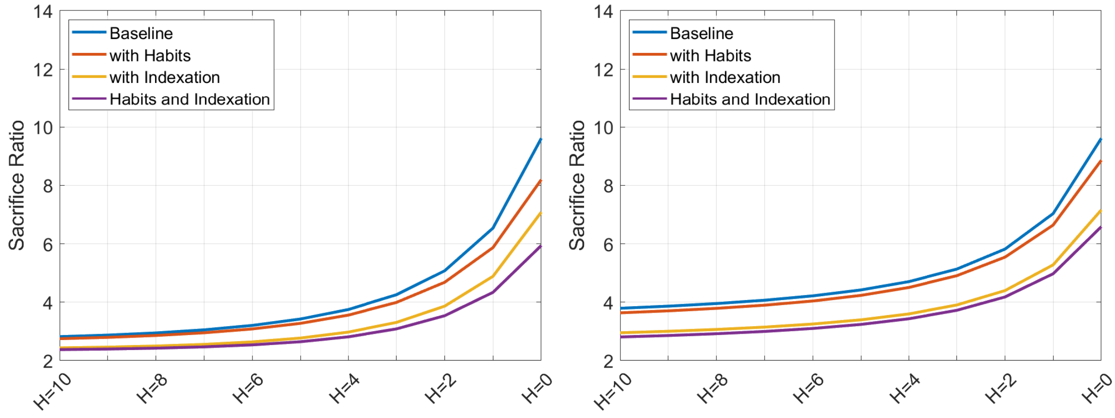
Parameter	Values	Afsar et al. (2024)	Description
β	0.99	0.99	Discount factor
σ	1.00	0.78	Intertemporal elasticity of substitution
ψ	1.00	1.44	Inverse Frisch elasticity
b	$\{0, 0.25\}$	0.65	Habit persistence
ω	$\{0, 0.25\}$	0.78	Price backward indexation
θ	0.80	0.91	Calvo price rigidity
$M_h = \hat{M}_h$	$\{1, 0.75, 0.50\}$	0.46	Household output cognitive discounting

The resulting sacrifice ratios (calculated over 50 periods) for announcements of a unit policy rate increase in H periods (horizontal axes) are reported in Figure A.3.[15]

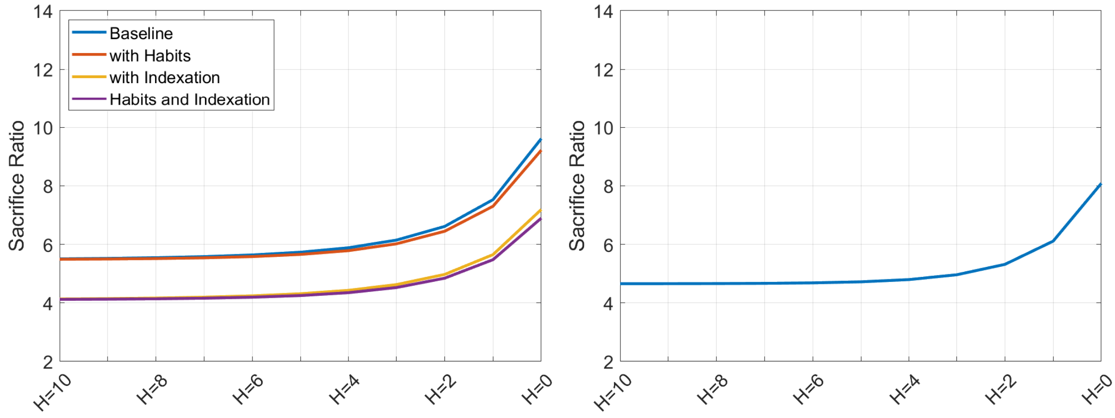
---

[14] See the last column of Table 1 in Afsar et al. (2024). We only take the relevant parameters in this table, i.e., we do not take the monetary policy rule parameters or the persistence and variance parameters that are also estimated in Afsar et al. (2024).

[15] As with our other simulations, we assume that monetary policy is temporarily exogenous for 100 periods before switching to a regime with a Taylor rule thereafter.



(a) Rational expectations ($M_h = 1$) **(b)** Moderate cognitive discounting ($M_h = 0.75$)



(c) Strong cognitive discounting ($M_h = 0.5$) **(d)** Parameters from Afsar et al. (2024)

Figure A.3: Sacrifice Ratios with Additional Rigidities

The figures plot the sacrifice ratio from announcements of a one-time unit policy rate increase at horizons one to 10 periods ahead (horizontal axes). The top left panel plots simulation results under rational expectations. The top right panel plots simulation results under moderate cognitive discounting. The bottom right panel plots simulation results under strong cognitive discounting. For the first three panels, the blue line is for the benchmark model. The orange line is for the model with habits in consumption. The yellow line is for the model with price indexation. The purple line is for the model with both habits and indexation. The bottom right panel reports simulation results using the estimated parameters in Afsar et al. (2024).

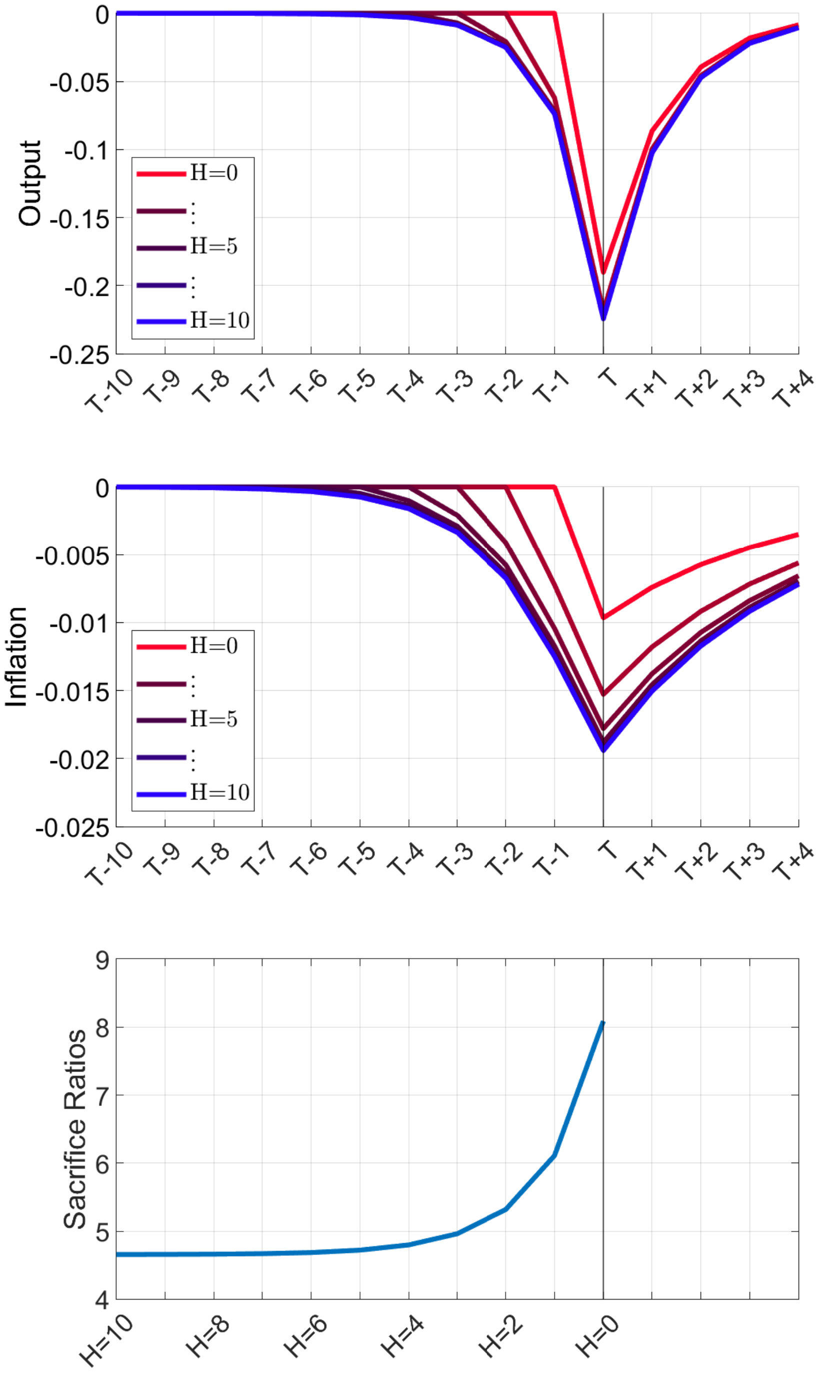


Figure A.4: Sacrifice ratios, output, and inflation using parameters from Afsar et al. (2024)

The figure plots the dynamics of output (top panel) and inflation (middle panel) to a unit increase in the policy rate in period T announced H=0 (red) to H=10 (blue) periods ahead. The bottom panel plots the sacrifice ratios if the policy change in period T were to be announced in the period indicated on the horizontal axis (from 10 periods ahead to an unannounced policy shock). The simulation results are from the extended model using the estimated parameters in Afsar et al. (2024).

A.13 Some additional estimation results

In Table 3 we report sacrifice ratio estimates with positive (i.e. restrictive) monetary policy surprises as instruments. For the sake of completeness, we report here corresponding estimates with negative (i.e. accommodative) monetary policy surprises as instruments.

Table A.2: Estimated sacrifice ratios with negative monetary policy surprises

	Instruments			
	Negative surprises in short rates		Negative surprises in long rates	
Sacrifice ratio over 4Q	3.39	(4.03)	1.22	(1.85)
Sacrifice ratio over 8Q	2.82	(3.15)	1.46	(2.23)
Sacrifice ratio over 12Q	9.77***	(4.08)	1.63	(1.44)
Sacrifice ratio over 16Q	8.06	(4.90)	2.70**	(1.37)
Sacrifice ratio over 20Q	8.55**	(4.11)	3.31**	(1.64)

Notes: Standard deviations (HAC) in parentheses. *, ** and *** refer to statistical significance at the 10%, 5% and 1% level, respectively.