# Green, Greener or Neutral? Signaling Environmental Quality under Incomplete Information

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**Abstract**

How can a firm signal the environmental quality of its product to consumers when its technology is unobservable? This paper analyzes a monopoly market where the firm can be either green, using a costly non-polluting technology that generates positive externalities; or neutral, relying on a standard technology. Consumers cannot directly observe the firm's environmental performance and derive a warm glow benefit from purchasing green products. Under incomplete information, we derive a separating equilibrium where the green firm uses either one instrument (price) or two instruments (price and effort to improve the greenness of its product) to signal its environmental quality to consumers. Signaling can result in an improvement in total welfare by increasing the public good benefit of green consumption. This highlights the role of effort as a potential dual-purpose instrument: mitigating informational asymmetries and improving social surplus. Additionally, we explore how regulations such as green effort subsidies influence this equilibrium.

**JEL Codes**: D62, D42, D82, Q50, L15, L12. **Keywords**: Signaling, Green Products, Incomplete Information, Monopoly.

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# 1 Introduction

**Research Question.** In recent years, consumer preferences have increasingly shifted towards more sustainable or ethical versions of certain products. Numerous surveys show that a significant portion of consumers is willing to pay more for environmentally friendly products. For example, an IPSOS MORI survey reports that 70 % of consumers are willing to pay a premium for products they consider ethically superior (IPSOS-MORI, 2003).
However, the credence nature of a product's environmental quality entails that consumers cannot directly observe the technology used by the firm that they are purchasing from. While consumers may notice observable efforts, such as recyclable packaging or other visible green practices, these signals do not necessarily reflect genuine improvements in the product's environmental impact. For example, a polluting firm might engage in forms of greenwashing by making the product appear more environmentally friendly (*e.g.*, using packaging that appears recyclable but is not), without actually reducing the product's environmental footprint. Thus, consumers are left uncertain about whether the effort truly enhances the product's greenness or is merely a costly attempt to appear more ethical.
Therefore, how can a monopolistic firm, whether green or neutral, use price and costly effort to signal its environmental type to consumers?
This paper aims to study the optimal choice of instruments in situations where consumers cannot perfectly observe the type of product they are purchasing.

**Context and Model Overview.** In the context of this paper, the firm's "type" refers to the technology used by the firm, which can be either green or neutral, and is randomly determined by nature. Specifically, a green firm uses a non-polluting technology that creates a positive externality, but is more costly than the neutral, standard technology of production. These higher costs can arise from factors such as sourcing sustainable raw materials, adopting advanced energy-efficient processes, and investing in specialized equipment to enhance environmental performance. Notably, the positive externality generated by green production makes it an impure public good, as its consumption leads to benefits such as improved air quality and better public health, which are enjoyed by everyone regardless of their individual purchase decisions. However, the firm's type is inherently difficult for consumers to observe. Indeed, consumers often have limited or no access to information regarding the specific technologies employed by firms — such as the types of machines, raw materials,

or sources of energy used in production. One main interest of this paper therefore lies in the assumption that the technology used is private information of the firm.
However, while the underlying technology is unobservable to consumers, the firm can invest in observable but costly effort. If the firm is green, exerting a positive effort improves further the greenness of the product (hence the size of the positive externality), for instance through the provision of a recyclable packaging.
On the other hand, a neutral firm can choose to invest in effort as well to appear greener to consumers, but its effort does not lead to any real change in the environmental impact of the products, as its technology does not generate any positive externality.
Because the neutral firm would use effort solely to perform a form of greenwashing[1] (*i.e.* to be perceived as environmentally friendly by consumers), the marginal cost of effort it incurs is higher than for the green type. Indeed, it lacks the necessary technology and knowledge to make substantial, genuine improvements, and then must invest in more costly, less efficient alternative or external adjustments such as offset programs or superficial changes.
For example, Volkswagen's "Clean Diesel" campaign marketed its diesel cars as low emission vehicles. However, it was revealed in 2015 by the EPA that the company had invested in the installation of a software to cheat emissions tests, allowing the vehicles to appear greener than they were, without any reduction in emissions.
One could think of a labels or certification as a solution to incomplete information about the firm's environmental quality. In this paper, we rule out the use of such instruments, and only rely on two instruments to provide information: price and effort. Even when labels do exist to indicate the environmental quality of a product, studies have shown that the large number and vast diversity of labels causes considerable confusion among consumers.[2] In other situations, labels can be manipulated by powerful firms and therefore lose their value.[3]
A crucial aspect of our model is the assumption that consumers derive a warm glow benefit from purchasing the green product. The concept of warm glow refers to the intrinsic satisfaction that consumers experience from contributing to the public

[1]For papers investigating strategic greenwashing decisions by firms, see notably Lyon and Maxwell (2011); Schmittmann and Gao (2022). For empirical work on greenwashing in the electric utility sector, see Kim and Lyon (2011).

[2]See Giovannucci and Koekoek (2003) for a market study of coffee.

[3]There are many examples of ecolabel controversies to refer to. Among others, the Marine Stewardship Council, the Forest Stewardship Council, and the Roundtable on Sustainable Palm Oil certification violation scandals.

good, such as purchasing a product that is perceived as green. This idea was first formally introduced by Andreoni (1990). We assume that all consumers experience the same level of warm glow when consuming the green product, which adds to their utility and is independent of the other product's attributes. This uniform warm glow benefit captures the idea that green consumption is partly motivated by the consumers' desire to "do their part".[4]

**Main Results.** The results provide three main insights.
First, as a benchmark, we derive the equilibrium monopoly strategies under complete information, as well as the welfare-maximizing solution. Under complete information, firms follow a standard monopoly pricing rule (where marginal revenue equals marginal cost) and the green firm does not take into account the public good aspect of its product, resulting in zero effort — contrary to the positive effort level that a social planner would require as optimal.
Then, we describe how these strategies change when consumers cannot directly observe the environmental quality of the product (*i.e.* the type of firm they are facing). We examine the conditions for the emergence of a separating perfect Bayesian equilibrium. We show that the green firm may use two instruments to signal itself to consumers: a higher price and a positive effort level to improve the greenness of the product.
We shed light on the implications for overall welfare. We demonstrate that welfare under incomplete information can surprisingly increase towards the first-best outcome, resulting in an improvement over the full information case, when the net public good benefit from green consumption is sufficiently high. As such, this setting highlights a potential benefit of incomplete information in the presence of public goods: the effort exerted by the green firm to mitigate the informational asymmetry is costly but contributes to correcting the under-provision of the public good, leading to an overall improvement of the situation when the gains generated by the public good attribute outweigh the costs of signaling.
Finally, we investigate the effect of regulation by a benevolent social planner. Specifically, we analyze policies such as subsidies for green effort or taxes on neutral firms

[4] An extensive strand of experimental and psychology literature has investigated the precise factors generating warm glow benefits. This effect can be linked to feelings of moral satisfaction and alignment with personal values, as well as social recognition (or avoidance of guilt) for making environmentally responsible choices. See Cason and Gangadharan (2002), Elfenbein and McManus (2010), Griskevicius et al. (2010).

and assess their impact on welfare and on the structure of a separating equilibrium.

**Related Literature and Contribution.** This paper relates to two main strands of literature.
First, this paper contributes to the theoretical literature on signaling games. A classic contribution is Wolinsky (1983), which focuses on the use of prices to signal quality in markets for experience private goods. Like Wolinsky's model, this paper derives a separating perfect Bayesian equilibrium. They share important features such as higher production costs for high quality products (more specifically in our case, green products), and the idea that equilibrium prices include premiums for quality. However, our model differs in several dimensions. Indeed, it allows for the possibility of signaling with two instruments (price and effort), rather than restricting to price only. Also, the type of good we consider is different, as it can be defined as an impure public good: the "quality" of the product has a public good characteristic. Finally, the credence nature of the environmental quality implies that consumers cannot infer the product's true type through experience, which differs from Wolinsky. There are numerous papers that examine this very issue of quality provision in markets where consumers have incomplete information and are very close to Wolinsky (1983). Among them, we can cite Farrell (1981), Cooper and Ross (1984), Klein and Leffler (1981) and Shapiro (1982). Bagwell and Riordan (1991) extend this line of work by considering a durable good, allowing for the optimal price to vary over time.
A related seminal paper is Milgrom and Roberts (1986), which examines signaling with two instruments: price and advertising. Similarly, our model explores the use of multiple instruments (price and effort) to achieve separation. In both papers, there are situations in which the firm only uses the price to achieve separation, and situations in which both instruments are used. However, as mentioned above, we focus on the environmental quality as a public good attribute, which introduces an additional form of inefficiency, linked to the provision of the public good. This key feature introduces new welfare insights in the presence of incomplete information. Also, this paper differs in the nature of the signaling instruments: in Milgrom and Roberts' model, advertising is a purely dissipative signal, while effort by the green firm here directly enhances the product's environmental quality and contributes to the public good benefit.
More closely related is the working paper by Mahenc and Volle (2021), which studies the interplay between price signaling and noisy consumer monitoring to gain infor-

mation about the credence attribute of the good. While both papers explore the use of multiple instruments to signal the credence attribute of the product, our model focuses on firm-driven mechanisms (price and effort) rather than consumer-driven monitoring. Additionally, we explicitly incorporate the public good nature of environmental quality, analyzing how it affects welfare under incomplete information.
Second, this paper contributes to the literature on the private provision of public goods. Seminal works in this field include Warr (1983) and Bergstrom et al. (1986), which establish conditions under which private action can lead to the provision of public goods. More recently, Besley and Ghatak (2007) investigate the feasibility of such provision and its desirability (*e.g.* compared to government provision), and conclude that private provision generally falls short of the first-best outcome. Similar to our paper, they assume that (at least some) consumers derive a personal benefit from contributing to the public good.[5] This assumption is also used by Bagnoli and Watts (2003), who study how the market structure affects the private provision of an impure public good, when consumers enjoy a homogenous warm glow from consuming the "ethical" product. Finally, Kotchen (2006) studies the choice of consumers between consuming an impure public good or consuming the private good separately and making contributions to a pure public good. The main contribution of our paper with respect to this strand of the literature lies in the integration of incomplete information, analyzing how signaling through price and effort affects the provision of a public good.

**Structure of the Paper.** This paper is organized as follows: Section 2 describes the model; Section 3 discusses the monopoly strategies in equilibrium under complete information and the first-best solution; Section 4 characterizes and analyzes the separating perfect Bayesian equilibrium; and Section 5 discusses extensions of the model in which a benevolent social planner intervenes.

# 2 The Model

Consider a market with a monopoly and consumers.

[5]For a related analysis of corporate public good provision without relying on consumer warm glow, and instead focusing on the alignment of managerial incentives with shareholder interests, see Morgan and Tumlinson (2019).

## Monopoly

The firm can be either of two types, randomly chosen by nature. It is either a green firm (type $\theta = g$), and produces a green good, or it is a neutral firm ($\theta = n$), and produces a regular neutral good. The type of the firm can be easily interpreted as the technology it uses, which determines the environmental impact of its production process. In other words, we could equivalently assume that with probability $\lambda$, the firm is green and uses an environmentally friendly technology to produce. Conversely, with probability $(1 - \lambda)$, the firm is neutral and relies on a conventional technology. Green production creates a positive externality[6] which can be further enhanced if the firm invests in some positive effort to increase the greenness of the product. The firm chooses a price $p_\theta$ and an effort level $e_\theta \geq 0$ to maximize its profits, which can be written as: $\pi_\theta(p_\theta, e_\theta) = (p_\theta - c_\theta)q_\theta(p_\theta) - \gamma_\theta e_\theta$ where $c_\theta$ is the constant marginal cost of production for type $\theta$, $q_\theta(p_\theta)$ is the demand faced by the firm, and $\gamma_\theta$ is the marginal cost of effort. We assume that the marginal cost of producing the green good is higher than the marginal cost of producing the neutral good, *i.e.* $c_g > c_n$. This assumption can be justified in various ways. Indeed, sourcing sustainable raw materials can be more costly than using conventional alternatives. Moreover, green production may rely on advanced, energy-efficient technologies, which require significant capital investment. We can simplify further by assuming that $c_n = 0$ and thus denote the marginal cost of producing the green good by $c_g = c > 0$. On the contrary, it is less costly for the green firm to exert a certain effort level: $\gamma_n > \gamma_g > 0$. This could arise from the fact that, for the green firm, effort directly enhances the greenness of its product, improving its quality and aligning with the firm's expertise in environmentally friendly practices. On the other hand, for the neutral firm, effort incurs a cost without resulting in any real improvement of the product's environmental attribute. The only incentive for a neutral firm to exert effort could arise when consumers cannot directly observe the firm's type and thus the firm might gain some profit by mimicking the behavior of a green type. This makes the marginal cost of effort heavier for the neutral firm compared to the green firm.

[6] In this paper, we assume that green production creates a positive externality (public good aspect) while neutral production does not. It would be equivalent to assume that neutral production creates a negative externality (for instance, through pollution), while green production does not.

## Consumers

Let consumers have unit demand. In this case, they decide whether they want to purchase one unit of the good or not consume. Let $v_i$ be consumer $i$'s valuation of the intrinsic features of the product, and $v_i \sim \mathcal{U}[0,1]$.
If consumer $i$ faces a neutral firm, she gets utility $u_i^n = v_i - p_n$ if she consumes and $u_0^n = 0$ if she does not.
However, if the consumer faces a green firm, she gets utility $u_i^g = v_i + b - p_g + B(q_g, e_g)$ if she consumes and $u_0^g = B(q_g, e_g)$ if she does not; where $b > 0$ represents the warm glow enjoyed from the consumer's own purchase of a green product, and $B(q_g, e_g)$ is the positive externality created by the existence of the green product, where:

$$B(q_g, e_g) = \begin{cases} 0 & \text{if } q_g = 0, \\ \beta + h(e_g) & \text{if } q_g > 0, \text{ where } \quad h(0) = 0, \quad h'(e_g) > 0, \quad h''(e_g) \leq 0. \end{cases}$$

This public good benefit is enjoyed by consumer $i$ regardless of her own purchasing decision. It exists even when the firm exerts zero effort, as long as the quantity produced is strictly positive. In this case, it is equal to a constant $\beta > 0$, which represents the benefit from having green production on this market compared to neutral production (*e.g.* less waste, less pollution, etc.). In the case where the firm invests in a positive level of effort to increase the greenness of its product, this benefit increases with $e_g$ at a decreasing rate, following function $h(\cdot)$. Assuming that the public good benefit does not increase with the quantity produced captures settings where the existence of green production (rather than its scale) creates value (*e.g.*, reduced baseline pollution). This simplification keeps the analysis simple and focuses attention on the firm's effort choice rather than output level.[7]
Notice that neutral production and consumption do not create any type of positive externality, and although a neutral monopoly may choose a positive level of costly effort, it does not have any impact on the quality of the product and utility for consumers.
Let us further assume that the warm glow, identical across consumers, exceeds the marginal cost of producing the green product: $b > c > 0$. This assumption ensures that consumers derive greater utility from the personal benefit of purchasing a green product than the cost of producing this product. Therefore, it guarantees the

[7]Appendix G explores an alternative specification where $B(q_g, e_g) = q_g(1 + h(e_g))$, allowing the public good benefit to increase with quantity. The qualitative results and main economic insights remain consistent.

optimality of this production despite the higher cost.

### Timing of the Game

Figure 1 summarizes the timing of the game, which unfolds as follows. First, the firm's type $\theta \in \{g, n\}$ is randomly drawn by Nature. The firm observes its type, and offers a price and an effort level. Consumers observe the price and effort and formulate their demand. Finally, consumption, profits and utilities are realized.

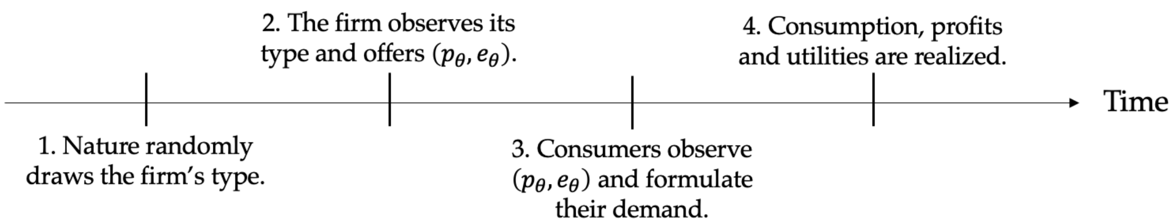


Figure 1: Timing of the game

# 3 Complete Information Benchmark and First-Best

## 3.1 Monopoly Strategy

First of all, we establish the benchmark under complete information. This will allow us to characterize the monopoly strategy chosen by each type of firm. In this context, assume that consumers possess complete information about the type of firm they are facing. They know whether the monopoly is a green or neutral producer, and they can base their consumption on this observation. Facing their demand, the firm will choose a price in order to maximize its profits.

If the firm is neutral, a consumer $i$ chooses to purchase the good if and only if $u_i^n \geq u_0^n$, that is when $v_i \geq p_n$. Given the distribution of the individual valuations, the number of active consumers (*i.e.* total demand for the neutral product) is given by $q_n(p_n) = 1 - p_n$, decreasing in the price charged by the firm. Facing this demand, the neutral monopoly chooses price $p_n^m$ and effort level $e_n^m$ to maximize its profits as follows:

$$\max_{p_n, e_n \geq 0} \pi_n(p_n, e_n) = p_n(1 - p_n) - \gamma_n e_n.$$

The solution implies zero effort exerted $e_n^m = 0$ as effort level is costly and does not change anything for consumers; and the price offered is the standard monopoly price: $p_n^m = 1/2$. In this case, the final demand is equal to $q_n(p_n^m) = 1/2$, and profits amount to $\pi_n(p_n^m, 0) = 1/4$.

If the firm is green, a consumer $i$ chooses to purchase the good if and only if $u_i^g \geq u_0^g$, that is when $v_i \geq p_g - b$. Given the distribution of the individual valuations, the number of active consumers (*i.e.* total demand for the green product) is given by: $q_g(p_g) = 1 + b - p_g$, decreasing in the price charged by the firm and increasing in the warm-glow benefit. As a consequence, the green monopoly chooses price $p_g^m$ and effort level $e_g^m$ in order to maximize its profits as follows:

$$\max_{p_g, e_g \geq 0} \quad \pi_g(p_g, e_g) = (p_g - c)(1 + b - p_g) - \gamma_g e_g.$$

The optimal solution still implies zero effort $e_g^m = 0$ and the price offered by the green monopoly is higher than the neutral price, and is given by $p_g^m = \dfrac{1 + b + c}{2} > p_n^m$. As a consequence, the quantity of green good on the market is given by $q_g(p_g^m) = \dfrac{1 + b - c}{2}$, and monopoly profits are equal to $\pi_g^m(p_g^m, 0) = \dfrac{(1 + b - c)^2}{4}$.

**Remark 1.** Consumer demand is not influenced by the effort level $e_g$ of the green firm either. Indeed, the costly effort exerted to increase the greenness of the product benefits the consumers regardless of their decision to purchase or not, as it only enters in the public good benefit. Also, the size of the positive externality received by a consumer is not (or to a negligible extent) affected by his decision to purchase or not. As a consequence, the purchase decision of the consumers does not depend on the effort undertaken by the firm, which explains why, under complete information, the firm chooses zero effort. The only influence of effort will be through pre-purchase perceptions of the product's type by consumers, which may only be useful under incomplete information (see Milgrom and Roberts, 1986).

**Remark 2.** In the same spirit, the public good attribute of the green product does not influence the price charged by the firm. Only the individual warm glow enjoyed from the consumers' own purchase has a role in the pricing strategy of the monopoly. Indeed, because the public good benefit is enjoyed by the consumers whether they decide to buy the good or not, it does not have any impact on this decision itself.

This implies that the price premium charged by green firms is induced by two dimensions: first, a higher cost of production compared to neutral production, and second, the personal preference of consumers for green consumption through the warm-glow parameter.

## 3.2 First-Best Solution

To complement the benchmark monopoly strategies, we derive the welfare-maximizing quantities[8] and efforts. A benevolent social planner would maximize the total social welfare by considering the sum of firm profits, consumer surplus, and the potential positive externality. The main insights are summarized in the following proposition.

**Proposition 1** (Complete information benchmark and first-best solution)**.** *Under complete information, the first-best solution maximizing social welfare entails the following choices by the benevolent social planner:*

*(i) If the firm is neutral, the quantity of the good provided ensures that all consumers are served. As a consequence, the socially optimal price is zero. The effort level remains zero, identically to the monopoly strategy, as neutral production provides no public good benefit.*

*(ii) If the firm is green, the socially optimal quantity of the product on the market increases compared to the monopoly strategy, resulting in a lower price equal to the marginal cost. The first-best effort level is strictly positive – exceeding the monopoly level – as it accounts for the public good benefit.*

*Proof.* See Appendix A. □

In the case where the firm is neutral, a benevolent social planner would set the first-best quantity and effort in order to maximize the sum of profits ($\pi_n(p_n, e_n)$) and consumer surplus, given by $S(q_n, p_n) = q_n - \frac{1}{2}q_n^2 - p_n q_n$. Quite obviously, the first-best effort level for the neutral firm is zero, as it implies a cost for the firm without any positive counterpart (*i.e.* no positive externality is generated by neutral consumption).[9] As the marginal cost of production is zero, the first-best price for the neutral product is zero, and in this case, all consumers are served with one unit of the good

[8] This is equivalent to deriving the prices allowing to implement these optimal quantities.

[9] With $e_n^{FB} = 0$, concavity of the program in $q_n$ is satisfied, ensuring the existence of a maximum.

($q_n = 1$).[10] Total welfare is equal to $W_n^{FB} = 1/2 > W_n^m = 3/8$.

In the case where the firm is green, the welfare-maximizing solution implies optimizing the sum of profits ($\pi_g(p_g, e_g)$), consumer surplus ($S(q_g, p_g) = q_g(1+b) - \frac{1}{2}q_g^2 - p_g q_g$), and the positive externality created by the public good aspect of the green product ($B(q_g, e_g) = \beta + h(e_g)$). As the effort to improve the greenness of the product does not only imply a cost but also a benefit, a benevolent social planner therefore chooses a positive effort level indirectly defined by $h'(e_g^{FB}) = \gamma_g$. This optimal effort level is directly influenced by the shape of function $h(\cdot)$, as it is determined by the marginal benefit associated with improved greenness. For a given cost of effort, the greater this marginal benefit, the higher the first-best effort level. Moreover, the quantity chosen by the social planner, given by $q_g^{FB} = 1 + b - c$, is greater than the monopoly quantity as the social planner accounts for consumer surplus. This results in pricing at marginal cost.

As an example, we can take $h(e_g) = \sqrt{e_g}$, which satisfies the assumptions on function $h(\cdot)$. As a result, the first-best effort level is given by $e_g^{FB} = \left(\frac{1}{2\gamma_g}\right)^2$. As a result, total welfare amounts to $W_g^{FB} = \frac{1}{2}\left(1 + b - c\right)^2 + \frac{1}{4\gamma_g} + \beta > W_g^m = \frac{3}{8}\left(1 + b - c\right)^2 + \beta$.

# 4 Signaling under Incomplete Information

Assume now that consumers do not directly observe the monopoly's type. In other words, the costs $(c_\theta, \gamma_\theta)$ are private information of the firm, and there is no credible direct way for the firm to provide information about the product. There is only one period. Consumers observe the price and effort level offered by the monopoly, and must try to infer the firm's type from this information. This illustrates the credence nature of the green attribute: consumers base their purchase decision on their belief; and even if they learn the true type of the product ex-post, they are not able to change their decision. In other words, the price and effort level observed by consumers will determine their perception of the firm's type, based on which they formulate their demand: if they believe the firm is green, their demand will be given by $q_g(p_g)$;

[10] Note that this is a standard marginal cost pricing rule, under the extreme case where marginal cost is zero. If the neutral firm has a positive marginal cost of production $c_n < c_g$, the first-best price for the neutral product would be $p_n = c_n$. In this case, the quantity served is $q_n^{FB} = 1 - c_n$, ensuring that all consumers with valuations above $c_n$ are served.

if they believe it is neutral instead, their demand will be $q_n(p_n)$. In this context, we are interested in the firm's optimal strategy, and we focus more particularly on separating strategies.[11]

## 4.1 Conditions for a Separating Equilibrium

First, note that the pair of monopoly strategies under complete information $\{(p_n^m, e_n^m); (p_g^m, e_g^m)\}$ does not yield a separating equilibrium, because it does not provide the necessary conditions for effective signaling.[12] Indeed, the neutral firm can choose $p_g^m$ and zero effort to lead consumers to believe that its product is green, and therefore enjoys higher demand and makes higher profits without incurring the cost of producing the green quality. For a separating perfect Bayesian equilibrium to arise, the following conditions need to be satisfied (Wolinsky, 1983):

(i) Type-$\theta$ firm maximizes its profits such that profits are non-negative;

(ii) Expectations are fulfilled *i.e.* in equilibrium, the perceived type by consumers given the observed price and effort corresponds to the real type of the firm;

(iii) Consumers make optimal choices by maximizing their utility.

Formally, the monopoly of type $\theta \in \{g, n\}$ maximizes its profits with respect to the price and effort level ensuring that the following necessary equilibrium constraints are simultaneously satisfied:

$$
\begin{aligned}
&(1) \quad (p_g - c)q_g(p_g) - \gamma_g e_g \geq (p_n - c)q_n(p_n) - \gamma_g e_n, \\
&(2) \quad p_n q_n(p_n) - \gamma_n e_n \geq p_g q_g(p_g) - \gamma_n e_g, \\
&(3) \quad \pi_g(p_g, e_g) \geq 0, \\
&(4) \quad \pi_n(p_n, e_n) \geq 0.
\end{aligned}
$$

Condition (1) ensures that a green firm prefers choosing $(p_g, e_g)$ and be perceived by consumers as green than be perceived as neutral with $(p_n, e_n)$. Similarly, condition (2) ensures that a neutral firm prefers offering $(p_n, e_n)$ and be perceived as neutral

[11] Here, we disregard the analysis of pooling equilibria.

[12] Note that if the warm glow ($b$) is absent, the complete information monopoly strategies are separating because the neutral firm would have no incentive to mimic the green firm. Indeed, demand would only depend on price, and the neutral firm would not achieve higher demand or profit by pretending to be green.

than choosing $(p_g, e_g)$ and be perceived as green. Conditions (3) and (4) ensure that profits for each type are non-negative, for the firm to be willing to participate in production.[13] Finally, beliefs off the equilibrium path are required to satisfy the Cho-Kreps Intuitive Criterion (Cho & Kreps, 1987), allowing to focus on the least-cost separating equilibrium.

## 4.2 Solving for a Separating Equilibrium

In a separating equilibrium, a neutral monopoly always offers the same price and zero-effort package as in full information: $(p_n^m = 1/2, e_n^m = 0)$. Taking this into account, a green monopoly will choose to offer $(p_g, e_g)$ in order to maximize its profits while satisfying the constraints for a separating equilibrium, that is to signal its type to consumers. Formally, the program writes:

$$\max_{p_g, e_g \geq 0} \quad \pi_g(\cdot) = (p_g - c)(1 + b - p_g) - \gamma_g e_g \tag{5}$$
$$\text{s.t. } (1) - (4).$$

As a result, two cases can arise. The following proposition summarizes these results under incomplete information about the firm's type.

**Proposition 2** (Separating equilibrium with price and effort)**.** *Suppose that consumers cannot observe the true type of the monopoly. In a separating perfect Bayesian equilibrium satisfying (1)-(4):*

*(i) A neutral firm offers its full information strategy, consisting of the monopoly price and zero effort:* $p_n^m = 1/2$ *and* $e_n^m = 0$.

*(ii) A green firm offers a higher-than-monopoly price* $p_g > p_g^m$ *and may offer a positive effort level* $e_g > 0$ *if the warm-glow benefit is large enough compared to the firm's marginal cost of production; and/or the costs of effort among types are not too close.*

*Proof.* See Appendix B. □

**Case 1: Positive effort.** First, if $\sqrt{2b + b^2} > c\dfrac{\gamma_n}{\Delta\gamma}$, where $\Delta\gamma = \gamma_n - \gamma_g$, the green firm uses two instruments (price and effort) to signal its type to consumers. More

[13] For sufficiency, a single-crossing type of condition needs to be added and verified. See Appendix B for more details on this matter.

precisely it offers the following price:

$$p_g = \frac{1}{2}\left(1 + b + c\frac{\gamma_n}{\Delta\gamma}\right)$$

and a positive effort to improve the greenness of its product, equal to:

$$e_g = \frac{1}{\gamma_n}\left[\frac{1}{4}b^2 + \frac{1}{2}b - \frac{1}{4}c^2\left(\frac{\gamma_n}{\Delta\gamma}\right)^2\right].$$

This price is higher than the full information monopoly price $p_g^m$. The cost of separation for the green firm in this case is equal to the loss in profits associated with charging a price above $p_g^m$ plus the total cost of providing effort $e_g$. The occurrence of such situation is facilitated by two channels. Firstly, when the warm glow $b$ enjoyed by consumers from purchasing a green product is large enough compared to the marginal cost of producing such product, $c$. And secondly, when the marginal costs of effort for the green and the neutral types, $\gamma_g$ and $\gamma_n$, are different enough.[14] Intuitively, when the warm-glow benefit $b$ is large, the green product is desirable as it is more strongly valued by consumers. As a consequence, the gain in revenues associated with being perceived as green are higher. For a true green monopoly to distinguish itself from the neutral type using solely the price, that price has to be very high. In this case, the green firm finds it preferable to use a combination of price and positive effort to reduce the distortion on the price. Secondly, using positive effort as a signaling instruments is clearly advantageous when it is relatively cheap for the green firm to increase effort compared to the neutral firm. Indeed, the neutral monopoly will be deterred from mimicking as exerting the green effort is too costly.

Technically, the green firm prefers using both effort and price to distinguish itself to consumers because, for a fixed price, increasing effort from zero to some positive level relaxes the constraint of interest for a separating equilibrium to arise, constraint (2). As a consequence, the firm is able to charge a price lower than the price required to achieve separation with zero effort (until constraint (2) becomes binding), therefore a price closer to the monopoly price. Moreover, this price $p_g$ is decreasing with the firm's own cost of effort $\gamma_g$. Indeed, as the cost of effort shrinks, the firm can achieve separation by exerting greater effort and charging lower price.

[14] If $\gamma_g = \gamma_n = \gamma$, or more generally, if the cost of effort functions are identical across types, the green firm never exerts a positive effort to improve the greenness of the product. Refer to Appendix C for more details on this matter.

Similarly, the effort level chosen by the firm is decreasing in the neutral type's cost of effort $\gamma_n$: for a given price in equilibrium, separation is achieved with lower effort for the green firm when the cost of effort for the neutral firm grows. The level of effort exerted remains below the first-best effort. By using both instruments in this case, the firm gains some profit with respect to using the price only, as long as effort and green production are not too costly.[15]

**Case 2: Zero effort.** On the other hand, when $\sqrt{2b+b^2} \leq c\frac{\gamma_n}{\Delta\gamma}$, the green firm provides zero effort ($e_g = 0$) and only uses the price to signal its type to consumers. As a result, the firm offers the following price:

$$p_g = \frac{1 + b + \sqrt{2b + b^2}}{2}.$$

This price is higher than the full information green monopoly price $p_g^m$, and the loss in profits for the green firm associated with offering a higher price is the cost of separation. As the firm tries to minimize this cost, the price chosen is the lowest price possible given the constraints for a separating equilibrium. This optimum implies that constraint (2) binds. As a consequence, the marginal cost of producing the green good $c$ does not play a role in this case. Indeed, the separating price here is determined by the constraint on the neutral firm, which does not involve such cost. It is however increasing in the warm glow $b$, as the firm enjoys higher demand when this benefit increases. Here, even if the firm tried to provide a positive costly effort level, given the constraints for separation, it would not be able to offer a more profitable price (*i.e.* lower) than the one described above.

## 4.3 Welfare Analysis

This section focuses on the welfare implications under incomplete information, in the described equilibrium. Quite obviously, welfare does not change when the firm is neutral, as the offered price and effort remain identical under complete and incomplete information. However, when the firm is green, there are conditions under which welfare is higher under incomplete information compared to the full information monopoly strategies. We also investigate the factors that influence this outcome. The main insight is summarized in the following proposition.

[15]See Appendix D.

**Proposition 3** (Welfare implications in separating equilibrium)**.** *When the green firm offers a positive effort level, total welfare is higher under incomplete information than under complete information, provided the net benefit of improved greenness is sufficiently large.*

*Proof.* See Appendix E. □

First of all, when the green firm provides zero effort in a separating equilibrium to improve the greenness of its product (*i.e*. Case 2), overall welfare under incomplete information is strictly lower than under complete information. This is because, in a separating equilibrium, the green firm's signaling price is higher than the complete information monopoly price, making both the firm and consumers worse off. Put differently, we have $q_g < q_g^m < q_g^{FB}$. In this case, we are facing two types of inefficiencies: a public good inefficiency (zero effort to improve greenness) and an informational inefficiency (higher price for differentiation).

However, when the green firm exerts a positive level of effort to signal itself (*i.e*. Case 1), two opposing effects come into play and may result in higher overall welfare compared to the full information situation.
On the one hand, and similarly to Case 1, the price offered by the green firm in a separating equilibrium is higher than the full information price, which decreases welfare on both the firm and consumers' side, moving further away from the first-best. Note, however, that the welfare loss in this case is smaller compared to the previous scenario, as price does increase but to a lesser extent than when only one signaling instrument is used.
On the other hand, the green firm's effort is costly but also improves the public good benefit associated with green production, and thus increases this dimension of total welfare.
The net effect on welfare depends on the balance between these two effects. It is possible for the positive effort (and its associated public good benefit) to outweigh the price increase negative effect, leading to an overall welfare improvement: this occurs when the net benefit from improved greenness ($h(e_g) - \gamma_g e_g$) is sufficiently high.
Therefore, the shape of function $h(\cdot)$, which governs the relationship between effort and the incremental positive externality, plays a crucial role in determining whether or not welfare under incomplete information is higher than under complete information. More specifically, if $h(\cdot)$ is sufficiently steep, meaning that the public good benefit increases significantly with effort, then the green firm's effort can generate

enough public good benefit to outweigh the costs of effort and increase in price.
To summarize, when the green firm can use price and effort to signal itself to consumers, the presence of the informational inefficiency helps correcting another form of inefficiency: the under-provision of the public good. Moreover, if welfare increases, we conclude that the gains from addressing the public good inefficiency outweigh the losses caused by informational asymmetry. This surprising result leads to an overall improvement in the situation.

## 4.4 Numerical Application

We provide examples of specific parameter values to illustrate the different scenarios that arise as a result of this model. As used previously, assume that $h(e_g) = \sqrt{e_g}$. Suppose $\beta = 2$.

First, consider the following values for the model parameters: $b = 5$, $c = 0.4$, $\gamma_n = 8$, and $\gamma_g = 0.4$. In a separating equilibrium, the green firm exerts a positive effort $e_g \approx 1.1$ to increase the greenness of its product. In this case, the benefit from improved greenness is sufficiently large compared to the total cost of effort, and welfare in incomplete information increases compared to the monopoly case in complete information. More specifically, we have $W_g \approx 14.34 > W_g^m = 13.76$.

As discussed in section 4.3, it could be the case that the green firm signals itself to consumers using a positive effort in addition to the price, but welfare decreases compared to the monopoly case in complete information. For instance, increase the cost of effort of the green firm closer to the one of the neutral firm, and shrink the gap between the warm glow benefit and the marginal cost of production: $b = 5$, $c = 0.5$, $\gamma_n = 1$, and $\gamma_g = 0.4$. In this case, effort to increase the greenness of the product is still positive ($e_g \approx 8.58$), but the total cost of providing such effort becomes too large compared to its positive public good effect, so total welfare drops to $W_g \approx 12.37 < W_g^m \approx 13.34$. With these changes in the parameter values, both the signaling price and the effort level have increased compared to the previous example. The reason is the following: as types are more similar and green production is not as profitable, the green firm has to undertake more distortions to be perceived as its true type.

There exist parameter values that induce a separating equilibrium where the green firm exerts zero effort and relies solely on the price to signal its type to consumers. As mentioned by Proposition 2, this outcome is more likely to arise when the warm-glow benefit is low, the marginal cost of production is high, and the costs of effort are relatively similar across firm types. For instance, we can take $b = 3$, $c = 0.5$, $\gamma_n = 1$, and $\gamma_g = 0.9$. In this case, the signaling price is approximately $p_g \approx 4.5$. It is higher than the signaling prices offered in the two previous examples where effort is positive, as the firm does not use effort here to reduce this distortion away from the monopoly price.

# 5 Regulation: Green Effort Subsidies

We now introduce the role of regulation in this framework. The objective of this section is to analyze how a benevolent social planner can intervene to influence the firm's behavior and improve overall welfare. More specifically, we explore the implementation of a subsidy for green effort, when the regulator does not directly observe the type of the monopoly, just like the consumers. Throughout this section, we keep the assumption that $h(e_g) = \sqrt{e_g}$.

## 5.1 Subsidizing the First-Best Green Effort

First, suppose the regulator wants to use the subsidy to implement the first-best effort levels, therefore make the green firm exert $e_g^{FB} = \left(\frac{1}{2\gamma_g}\right)^2$. Assume for now that the regulator does not have any other regulatory instruments (for instance, he cannot set the price or the quantity to be offered by each type of firm). For the green firm to be willing to set $e_g^{FB}$, the subsidy has to cover the cost of doing so. Therefore, the "full" per-unit subsidy is equal to $s = \gamma_g$.

Under complete information, the regulator is able to condition the allocation of this subsidy on the firm's type: if the firm is green, it receives the subsidy in exchange of setting the first-best green effort level; and if the firm is neutral, it does not receive anything. As a consequence, a green monopoly will optimally choose its monopoly price $p_g^m$ and make profits $\pi_g^m(p_g^m, e_g^{FB}, s) = \pi_g^m(p_g^m, 0)$ as the cost of exerting the first-best effort is totally covered by the subsidy. On the other hand, the neutral firm's strategy is identical as in section 3.1, with zero-effort level and monopoly pricing ($p_n^m = 1/2$) and profits ($\pi_n^m(p_n^m, 0) = 1/4$).

The main issue of interest here is to analyze how the addition of the subsidy in exchange of exerting $e_g^{FB}$ alters the separating equilibrium under incomplete information. When the type is private information of the firm, the regulator will give out the subsidy to a firm offering the green price and first-best green effort level; and will not grant any subsidy to a firm offering any other price and effort pair. In particular, we will demonstrate that under some conditions, the complete information strategies $\{(p_n^m, e_n^m); (p_g^m, e_g^{FB}, s)\}$ can be directly separating under incomplete information, without the neutral firm facing incentive to mimic the green type. The key insights are summarized in the following proposition.

**Proposition 4** (Separation with first-best green effort subsidy)**.** *Suppose there exists incomplete information about the monopoly's type and the regulator offers a per-unit subsidy $s = \gamma_g$ to a firm offering the green monopoly price $p_g^m$ and exerting the green first-best effort $e_g^{FB}$. The strategies $\{(p_n^m, e_n^m); (p_g^m, e_g^{FB}, s)\}$ can directly induce separation under some conditions:*

*(i) When the difference in costs of effort $\Delta\gamma = \gamma_n - \gamma_g$ is sufficiently large, making the residual cost to provide the first-best green effort level too high for the neutral firm, despite the subsidy.*

*(ii) When the warm-glow benefit $b$ is sufficiently low, inducing the gain in revenues made by the neutral type perceived as green too weak to attenuate the residual cost of effort.*

*Proof.* See Appendix F. □

The neutral firm chooses its monopoly strategy, allowing for a separating equilibrium to arise directly, if and only if $\pi_n^m(p_n^m, 0) > \pi_n(p_g^m, e_g^{FB}, s)$, where the profits from offering the green type's price and effort and receiving the subsidy are given by $\pi_n(p_g^m, e_g^{FB}, s) = p_g^m(1+b-p_g^m) - (\gamma_n - s)e_g^{FB}$. As a consequence, we can rewrite this condition as follows:

$$2b + b^2 - c^2 < \frac{\Delta\gamma}{\gamma_g^2}. \tag{6}$$

Intuitively, this condition is more easily satisfied when the costs of effort are sufficiently different, and/or, for a given marginal cost of green production, the warm glow is not too large.

First of all, suppose $b$ and $c$ are fixed. Thus, the minimum difference between the costs of efforts necessary for these strategies to be separating is $\Delta\hat{\gamma} = \gamma_g^2(2b + b^2 - c^2)$.

This is intuitive: when the costs of effort are similar, the neutral firm's incentive to behave like the green type increases, because the subsidy will cover most of the cost of exerting $e_g^{FB}$, while revenues increase as consumers perceive the firm as green. However, when the difference between the costs of effort exceeds $\Delta\hat{\gamma}$, the residual cost $(\gamma_n - s)$ that the neutral firm has to incur to provide $e_g^{FB}$ remains too high compared to the benefit it retrieves from being perceived as green.
Secondly, the warm glow generates a gain in revenues for the neutral firm perceived as green by consumers. Therefore, a low warm-glow benefit deters the neutral firm from mimicking the green type, and even facilitates satisfying the condition 6 for separation at a lower threshold value $\Delta\hat{\gamma}$. More specifically, direct separation can still be induced as marginal costs of effort converge, as long as $b$ simultaneously decreases towards $c$, with the limit of $b$ very close to but still greater than $c$. In the limiting case where $b \approx c$, direct separation remains feasible only if the difference in marginal costs of effort $\Delta\hat{\gamma}$ exceeds $2c\gamma_g^2$. This cutoff cannot be further reduced.

This subsection gives insights on the conditions under which a regulator should implement a subsidy to support first-best green effort and expect to tackle two inefficiencies: the informational and the public good provision issues. When the marginal costs of efforts differ sufficiently across types, the social planner should introduce the subsidy without concern that the neutral firm will mimic the green firm's behavior. Moreover, even when these cost differences narrow, the regulator may still have room to implement the subsidy directly, when the warm-glow benefit is low. The regulator's interest in implementing this policy when the conditions are favorable lies in the fact that total welfare is inevitably higher (or unchanged) when the subsidy leads to the emergence of a separating equilibrium rather than not intervening and letting the firm play its signaling strategy. Indeed, if the firm is green, the quantity (resp. price) offered in this context will be higher (resp. lower) compared to the signaling strategy, both for Case 1 and Case 2, representing a gain for firm profits as well as consumer surplus. In addition to that, the level of effort will be the first-best level, maximizing the public good benefit net of the cost.

## 5.2 Optimal Subsidy under Incomplete Information

In this section, we will study the optimal choice of the regulator the previous policy $\{(p_n^m, e_n^m); (p_g^m, e_g^{FB}, s)\}$ does not directly induce separation.

## 6 Conclusions

This model highlights how markets for credence impure public goods might achieve effective differentiation even in the absence of certification and labels, using firm-specific signaling instruments of price and effort. By combining the warm glow effect with the impure public good aspect of green products, this paper provides a novel perspective on signaling. Besides the characterization of a separating perfect Bayesian equilibrium, the results discuss conditions under which incomplete information improves the situation compared to the initial monopoly strategies under complete information. Higher green prices under incomplete information suggest that environmentally friendly firms might charge high prices for two key reasons: a quality premium as green is more costly to produce, and for signaling purposes. But this increase in price can be attenuated if the green firm is able to use costly effort to improve the greenness of its product. This highlights the role of effort as a dual-purpose instrument: signaling and improving social outcomes. The analysis regarding the implementation of a subsidy for the first-best green effort generates some policy recommendations: such a regulation improves the situation when the two types of firms are sufficiently different, allowing for a separating equilibrium to directly arise without the green firm making any adjustments. This project is still ongoing. Among other things, we characterize in a later version the optimal menu of contract (quantities and subsidies/efforts) chosen by a regulator to induce separation when the full subsidy is not directly separating. We also aim to extend the model to include two types of consumers ("caring" and "not caring" for the greenness of the product), and conduct an analysis of the potential pooling equilibria.

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# A Proof of Proposition 1

**Welfare function.** The total welfare function includes consumer surplus, firm profits, and possibly the public good benefit. We can either write a welfare function for each situation: the firm is green or the firm is neutral. Alternatively, we can write a single welfare function which takes into account that the firm is green with probability $\lambda \in [0,1]$, and neutral with probability $(1-\lambda)$. In this case, we have:

$$W(q_n, e_n, q_g, e_g) = \lambda\Bigg(\underbrace{q_g(1+b) - \frac{1}{2}q_g^2 - p_g q_g}_{\text{Green Consumer Surplus}} + \underbrace{p_g q_g - c q_g - \gamma_g e_g}_{\text{Green Profits}} + \underbrace{B(q_g, e_g)}_{\text{Public Good Benefit}}\Bigg)$$
$$+ (1-\lambda)\Bigg(\underbrace{q_n - \frac{1}{2}q_n^2 - p_n q_n}_{\text{Neutral Consumer Surplus}} + \underbrace{p_n q_n - \gamma_n e_n}_{\text{Neutral Profits}}\Bigg),$$

where $B(q_g, e_g) = \begin{cases} 0 \text{ if } q_g = 0, \\ \beta + h(e_g) \text{ if } q_g > 0 \text{ where } h(0) = 0,\ h'(e_g) > 0,\ h''(e_g) \leq 0. \end{cases}$

**Monopoly strategies under complete information.** The derivation of pair of monopoly strategies under complete information $\{(p_n^m, e_n^m); (p_g^m, e_g^m)\}$ is explained in Section 3.1 and is fairly straightforward. Based on the definition of welfare above, when the firm is neutral (*i.e.* $\theta = n$ is realized), total welfare in this situation is equal to $W_n^m = 3/8$. On the other hand, when the firm is green ($\theta = g$ is realized), total welfare is equal to $W_g^m = \frac{3}{8}(1+b-c)^2 + \beta$.

**Welfare-maximizing solution (first-best).** A benevolent social planner chooses maximizes quantities (equivalent to prices) and effort levels in order to maximize total welfare. This optimization problem simplifies to:

$$\max_{q_n, e_n, q_g, e_g \geq 0} \quad W = \lambda\Bigg(q_g(1+b-c) - \frac{1}{2}q_g^2 - \gamma_g e_g + \beta + h(e_g)\Bigg)$$
$$+ (1-\lambda)\Bigg(q_n - \frac{1}{2}q_n^2 - \gamma_n e_n\Bigg).$$

We can consider the situation where the firm is neutral and the situation where the firm is green as separate cases, as $(q_g, e_g)$ does not interact with $(q_n, e_n)$.
For the neutral firm, the solution implies $e_n^{FB} = e_n^m = 0$ as it only enter negatively in

the welfare function. As mentioned in Foonote 9, the program is therefore concave in $q_n$, ensuring the existence of a maximum. As explained in Section 3.2, with zero marginal cost, the first-best price is zero and we have $q_n^{FB} = 1$ (all consumers are active). If this state of nature is realized ($\theta = n$), total welfare is equal to $W_n^{FB} = 1/2$, greater than $W_n^m$.

If the firm is green, we can compute the first order conditions with respect to $q_g$ and $e_g$:

$$h'(e_g^{FB}) = \gamma_g,$$
$$q_g^{FB} = 1 + b - c \text{ which translates to } p_g^{FB} = c.$$

Using $h(e_g) = \sqrt{e_g}$ as in the discussed examples, we can derive the exact expressions for $e_g^{FB} = \left(\frac{1}{2\gamma_g}\right)^2$.

If this state of nature is realized ($\theta = g$), total welfare is equal to:

$$W_g^{FB} = \frac{1}{2}(1 + b - c)^2 + \frac{1}{4\gamma_g} + \beta$$

which is greater than $W_g^m$.

We can write the expected total maximum welfare in first best as:

$$W^{FB} = \lambda\left(\frac{1}{2}(1 + b - c)^2 + \frac{1}{4\gamma_g} + \beta\right) + \frac{1 - \lambda}{2}.$$

To conclude, we can verify the concavity of the program to ensure the existence of this maximum. The Hessian writes:

$$H_{(q_g, e_g)} = \begin{bmatrix} -1 & 0 \\ 0 & h''(e_g) \end{bmatrix}.$$

For concavity, as the Hessian is a diagonal matrix, we verify the eigenvalues which are the diagonal elements : $-1 < 0$ and $h''(e_g) \leq 0$ as assumed in the model.

# B Proof of Proposition 2

First of all, we add a single-crossing condition to the set of constraints to ensure necessity and sufficiency. With a fixed price $p$, we have $\frac{\partial \pi_g(\cdot)}{\partial p} = 1 + b + c - 2p$ and $\frac{\partial \pi_n(\cdot)}{\partial p} = 1 - 2p$. The first condition we verify is: $\frac{\partial \pi_g(\cdot)}{\partial p} > \frac{\partial \pi_n(\cdot)}{\partial p} \Rightarrow b + c > 0$ which holds by assumption. With a fixed effort level $e$, we compute $\frac{\partial \pi_g(\cdot)}{\partial e} = -\gamma_g$ and $\frac{\partial \pi_n(\cdot)}{\partial e} = -\gamma_n$. We verify the second condition: $\frac{\partial \pi_g(\cdot)}{\partial e} < \frac{\partial \pi_n(\cdot)}{\partial e} \Rightarrow \gamma_n > \gamma_g$ which is also true by assumption.

Secondly, it is straightforward to show that the green monopoly strategy is not separating when there is incomplete information. Indeed, we can compute:
$\pi_n(p_n^m, e_n^m) = 1/4 < \pi_n(p_g^m, e_g^m) = \frac{(1+b-c)(1+b+c)}{4}$.

Now, turn to the derivation of the solution. First, we can prove that, in a separating equilibrium, the neutral monopoly offers the same package as the monopoly strategy in full information, that is $\boxed{(p_n^m = 1/2, e_n^m = 0)}$. Proceed by contradiction: at a separating equilibrium, consumers who observe $p_n$ and $e_n$ believe that the good they purchase is neutral, and thus formulate demand $q_n(p_n)$. If $p_n \neq p_n^m$, the neutral firm is not maximizing its profits given $q_n(p_n)$. In this case, it deviates to offer $p_n^m = 1/2$. Similarly, providing any positive level of effort $e_n > 0$ entails a cost for the neutral firm that does not yield any benefit, so it provides zero effort $e_n^m = 0$. Notice that this ensures that constraint (4) is satisfied, with $\pi_n(p_n^m) = 1/4$.

To solve (5), we can replace $(p_n, e_n)$ by these values in the constraints. The program simplifies to:

$$\max_{p_g, e_g \geq 0} \quad \pi_g(\cdot) = (p_g - c)(1 + b - p_g) - \gamma_g e_g$$

$$\text{s.t.} \quad (1) \quad -p_g^2 + p_g(1 + b + c) - c\left(\frac{1}{2} + b\right) - \frac{1}{4} - \gamma_g e_g \geq 0,$$

$$(2) \ p_g^2 - p_g(1 + b) + \frac{1}{4} + \gamma_n e_g \geq 0,$$

$$(3), \ (4).$$

Temporarily ignore constraints (3) and (4) and check that they are satisfied ex-post. These constraints ensure that the firm's profits are non-negative. As mentioned in the first part of the proof, the neutral firm chooses its monopoly strategy, and therefore earns non-negative profits, satisfying constraint (4). For constraint (3), as the firm is a monopoly, it is likely to earn positive profits, so we can ensure that this is the case after we solve for a separating equilibrium.

Constraint (2) is binding at the optimum: separation is costly to the green firm, so at the optimum, it will choose price and effort such that separation is just attained (in other words, to minimize its loss due to the informational inefficiency). As a consequence, we have: $p_g^2 - p_g(1+b) + \frac{1}{4} + \gamma_n e_g = 0$, which gives an expression for the effort level as a function of the price: $e_g(p) = \dfrac{1}{\gamma_n}\left(-p_g^2 + p_g(1+b) - \dfrac{1}{4}\right)$. From this, we can deduce that the effort provided by the green firm to increase the greenness of its product is positive if and only if $p_g \in \left]\dfrac{1+b-\sqrt{2b+b^2}}{2}; \dfrac{1+b+\sqrt{2b+b^2}}{2}\right[$.

There are two cases to consider:

1. Interior solution for $e_g$: the green firm uses two instruments (price and positive effort level) to signal itself into a separating equilibrium.
2. Zero-effort equilibrium: the green firm uses only the price to signal itself into a separating equilibrium and provides no effort.

**Case 1**. Consider an interior solution for $e_g > 0$.
In this case, $e_g(p) = \dfrac{1}{\gamma_n}\left(-p_g^2 + p_g(1+b) - \dfrac{1}{4}\right)$. The program writes:

$$\max_{p_g} \pi_g(\cdot) = (p_g - c)(1+b-p_g) - \frac{\gamma_g}{\gamma_n}\left(-p_g^2 + p_g(1+b) - \frac{1}{4}\right)$$

$$\text{s.t.} \quad (2) - p_g^2\left(1 - \frac{\gamma_g}{\gamma_n}\right) + p_g\left[(1+b)\left(1 - \frac{\gamma_g}{\gamma_n}\right) + c\right] - c\left(\frac{1}{2} + b\right) - \frac{1}{4}\left(1 - \frac{\gamma_g}{\gamma_n}\right) \geq 0.$$

The program is concave in $p_g$. The first-order condition gives:

$$\boxed{p_g = \frac{1}{2}\left(1 + b + c\frac{\gamma_n}{\Delta\gamma}\right)}$$

where $\Delta\gamma = \gamma_n - \gamma_g > 0$, and

$$\boxed{e_g = \frac{1}{\gamma_n}\left[\frac{1}{4}b^2 + \frac{1}{2}b - \frac{1}{4}c^2\left(\frac{\gamma_n}{\Delta\gamma}\right)^2\right].}$$

Notice that $\frac{\gamma_n}{\Delta\gamma} > 1$ so $p_g > p_g^m$.

We can derive the condition under which we indeed have $p_g \in \left]\frac{1+b-\sqrt{2b+b^2}}{2}; \frac{1+b+\sqrt{2b+b^2}}{2}\right[$ as required for $e_g > 0$:

1. $p_g > \frac{1+b-\sqrt{2b+b^2}}{2} \Leftrightarrow \underbrace{c\frac{\gamma_n}{\Delta\gamma}}_{>0} > \underbrace{-\sqrt{2b+b^2}}_{<0}$, which is always satisfied;

2. $p_g < \frac{1+b+\sqrt{2b+b^2}}{2} \Leftrightarrow \frac{\sqrt{2b+b^2}}{c} > \frac{\gamma_n}{\Delta\gamma}$.
   This condition needs to be true for the green firm to be willing to choose a positive effort level given by $e_g$. This is the case when $b$ and $c$ are not too close, and/or when $\gamma_n$ and $\gamma_g$ are different enough.

Finally, verify all remaining constraints ((1) and (3)).

- First, verify that constraint (1) holds. Recall the condition:
  $-p_g^2\left(1-\frac{\gamma_g}{\gamma_n}\right) + p_g\left[(1+b)\left(1-\frac{\gamma_g}{\gamma_n}\right)+c\right] - c\left(\frac{1}{2}+b\right) - \frac{1}{4}\left(1-\frac{\gamma_g}{\gamma_n}\right) \geq 0.$
  This is a second-degree equation in $p_g$. It admits to real roots $\underline{p}$ and $\overline{p}$ given by:
  $\underline{p} = \frac{1}{2}\left(\frac{c+\alpha(1+b)-\sqrt{\delta}}{\alpha}\right)$ and
  $\overline{p} = \frac{1}{2}\left(\frac{c+\alpha(1+b)+\sqrt{\delta}}{\alpha}\right)$,
  where $\alpha = 1 - \frac{\gamma_g}{\gamma_n} \in [0,1]$ for simplification and $\delta = (\alpha b - c)^2 + 2\alpha^2 b > 0$ is the discriminant. For constraint (1) to be satisfied, it remains to show that $p_g \in [\underline{p}, \overline{p}]$.

  – We have $p_g \geq \underline{p} \Leftrightarrow -\sqrt{\delta} \leq 0$, which is true.

  – We have $p_g \leq \overline{p} \Leftrightarrow \sqrt{\delta} \geq 0$, which is true as well.

  As a result, constraint (1) is ex-post satisfied.

- Now, turn to constraint (3): under these conditions, the profits of the green firm are non-negative: $\pi_g(p_g, e_g) = \frac{1}{4} + \left(1 - \frac{\gamma_g}{\gamma_n}\right)\left(\frac{1}{4}b^2 + \frac{1}{2}b\right) - \frac{1}{2}c(1+b) + \frac{1}{4}\frac{\gamma_n}{\Delta\gamma}c^2 > 0$.

**Case 2.** Assume that the condition for an interior solution for $e_g$ is not satisfied, *i.e.* we have $\frac{\sqrt{2b+b^2}}{c} \leq \frac{\gamma_n}{\Delta\gamma}$. In this case, $\boxed{e_g = 0}$ and the green firm uses only the price to signal itself into a separating equilibrium. As mentioned above, ignore constraints (3) and (4) for the moment, and rewrite the objective and constraints (1) and (2) as:

$$\max_{p_g} \pi_g(\cdot) = (p_g - c)(1 + b - p_g)$$

$$\text{s.t. (1)} \quad -p_g^2 + p_g(1+b+c) - c\left(\frac{1}{2} + b\right) - \frac{1}{4} \geq 0,$$

$$\text{(2)} \quad p_g^2 - p_g(1+b) + \frac{1}{4} \geq 0.$$

(1) and (2) are both second-degree equations in $p_g$ with two real roots, and therefore both give a range of prices for which the constraints are satisfied. Let us characterize these ranges:

- The interval satisfying constraint (1) is given by:
  $p_g \in \left[\underline{p_1} = \frac{1+b+c-\sqrt{2b+(b-c)^2}}{2}; \overline{p_1} = \frac{1+b+c+\sqrt{2b+(b-c)^2}}{2}\right]$.
- On the other hand, values of $p_g$ which satisfy constraint (2) are:
  $p_g \leq \underline{p_2} = \frac{1+b-\sqrt{2b+b^2}}{2}$,
  $p_g \geq \overline{p_2} = \frac{1+b+\sqrt{2b+b^2}}{2}$.

It is quite straightforward to prove the ordering $\underline{p_2} > \underline{p_1} > \overline{p_2} > \overline{p_1}$ using the expressions for each and under the assumption of the model that $b > c > 0$. From the two ranges of acceptable values for $p_g$ described above and the ordering of the roots, we can demonstrate that the unique range of prices that satisfies both (1) and (2) simultaneously is $[\overline{p_2}; \overline{p_1}]$, as shown in green by the graphical representation below.

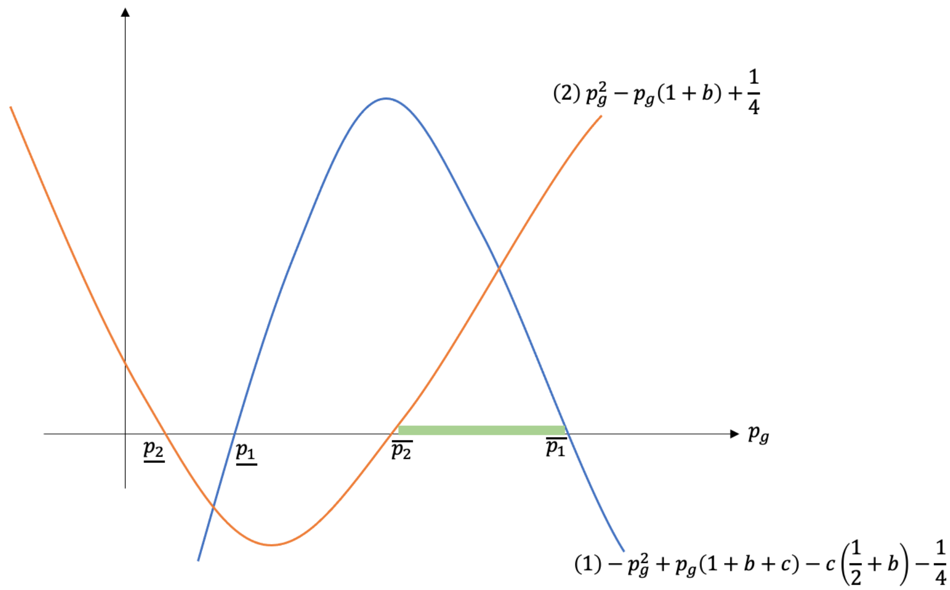


Figure 2: Graphical illustration of constraints (1) and (2) with zero effort

The optimization problem therefore boils down to: $\max_{p_g}(p_g - c)(1 + b - p_g)$ subject to $p_g \in [\overline{p_2}; \overline{p_1}]$. Notice that, for the separating equilibrium conditions to be met under zero effort, the price offered by the green firm must be higher than the full information monopoly price $p_g^m = \dfrac{1+b+c}{2} < \overline{p_2}$. Therefore, the green firm optimally chooses to offer the lowest price possible in the range of acceptable prices for a separating equilibrium, in order to get as close as possible to the full-information monopoly price (consistent with the least-cost equilibrium selection refinement, as imposed by the Intuitive Criterion). Constraint (2) will indeed be binding, and in a separating equilibrium with zero effort, we have:

$$\boxed{p_g = \frac{1 + b + \sqrt{2b + b^2}}{2}}.$$

As a consequence, notice that constraint (1) is automatically satisfied. We can verify that constraint (3) holds. When effort is zero, green profits are non-negative as long as the price belongs to the interval $[c; (1 + b)]$, which is the case with the price $p_g$ described above.

## C Footnote 14

When the neutral and green types have identical costs of effort, the green firm will always offer zero effort in a separating equilibrium. In other words, the green firm only uses price as an instrument to signal its type to consumers, and does not find it worth it to provide a costly effort. The intuition behind this is the following: the benefit from choosing a positive effort level for the green monopoly arises from the possibility of offering a lower price than $p = \frac{1+b+\sqrt{2b+b^2}}{2}$, which is the price offered in a separating equilibrium with zero effort. Thus, effort is costly but enables the firm to charge a price closer to its full information monopoly price (which increases revenues). The firm will effectively choose to provide a positive effort if this gain is important enough compared to the loss in profits due to the cost of exerting this effort level. However, when the cost of effort is identical for the green and neutral types, for a positive effort to be worth it for the green firm, the price would need to decrease a lot, to an extent that is not permitted by constraint (2). As a consequence, the green firm cannot remain in a separating equilibrium by choosing a package $(p_g, e_g)$ where $e_g > 0$.

To see this formally, suppose that $\gamma_n = \gamma_g = \gamma$. First analyze constraint (2), and determine how $p_g$ decreases when $e_g$ increases to keep the Ccnstraint binding. Recall (2):

$$p_g^2 - p_g(1+b) + \frac{1}{4} + \gamma e_g = 0.$$

Thus, we have:

$$\underbrace{\frac{dp_g}{de_g} = \frac{-\gamma}{2\left(p_g - \frac{1+b}{2}\right)}}_{\star} < 0 \text{ as } p_g > \frac{1+b}{2}.$$

Now turn to the change in profits associated with a change in price and effort:

$$d\pi = \frac{\partial \pi}{\partial p_g} dp_g + \frac{\partial \pi}{\partial e_g} de_g = (1+b+c-2p_g)dp_g - \gamma de_g.$$

To get a gain from simultaneously increasing effort and decreasing the price, we would need:

$$d\pi \geq 0 \Leftrightarrow \frac{dp_g}{de_g} \leq \frac{-\gamma}{2\left(p_g - \frac{1+b+c}{2}\right)}.$$

Now we can prove that this cannot be reconciled with condition ⋆ on constraint (2). Indeed, we would need the following to be true:

$$\frac{-\gamma}{2\left(p_g - \frac{1+b}{2}\right)} \leq \frac{-\gamma}{2\left(p_g - \frac{1+b+c}{2}\right)} \Leftrightarrow c \leq 0$$

which is a contradiction. Therefore, increasing effort to a positive level and in exchange lowering the price closer to the full information monopoly price never yields a gain in profits for the green firm when the marginal costs of effort are identical, and this results in zero effort level in separating equilibrium.

# D Footnote 15

When $\gamma_n > \gamma_g > 0$, we can see how the gain in profits generated by using both the price and effort to achieve separation can be facilitated depending on the parameters of the model, by proceeding in a similar manner. Constraint (2) binding implies:

$$\frac{dp_g}{de_g} = \frac{-\gamma_n}{2\left(p_g - \frac{1+b}{2}\right)}.$$

A gain in profits by increasing $e_g$ and decreasing $p_g$ is possible if:

$$d\pi \geq 0 \Leftrightarrow \frac{dp_g}{de_g} \leq \frac{-\gamma_g}{2\left(p_g - \frac{1+b+c}{2}\right)}.$$

Therefore, combining, we need to have the following:

$$\frac{-\gamma_n}{2\left(p_g - \frac{1+b}{2}\right)} \leq \frac{-\gamma_g}{2\left(p_g - \frac{1+b+c}{2}\right)}.$$

This condition is more easily satisfied when the difference between the costs of effort, $\Delta\gamma$, is high. Moreover, it is also facilitated by a small marginal cost of production of the green product, $c$.

# E Proof of Proposition 3

We provide an intuitive reasoning to support the validity of this argument. When the firm is green, total welfare under complete information monopoly strategies can be written as a function of the monopoly quantity:

$$W_g^m = \underbrace{q_g^m(1+b-c) - \frac{1}{2}q_g^{m^2}}_{(A)} + \beta.$$

When there is incomplete information and parameters of the models allow for Case 1, total welfare in a separating equilibrium can also be written in terms of separating quantities and efforts:

$$W_g = \underbrace{q_g(1+b-c) - \frac{1}{2}q_g^2}_{(B)} \quad + \quad \underbrace{h(e_g) - \gamma_g e_g}_{\text{Net benefit of improved greenness}} \quad + \beta.$$

As $q_g^m > q_g$ (or, equivalently, the separating price is higher than the full information monopoly price), we necessarily have $(A) > (B)$. This illustrates the loss in welfare due to the decrease (resp. increase) in quantity (resp. price) when moving from complete to incomplete information. However, there is an extra component in the welfare function under incomplete information when effort is positive: the net benefit of improved greenness (positive externality minus the private cost of exerting such effort). It could be the case that this dimension gets large enough so that it outweighs the loss from $(A)$ to $(B)$. This is more likely to occur as the cost of effort decreases (low $\gamma_g$), and as the public good benefit from improved greenness increases. The latter heavily depends on the shape of function $h(\cdot)$ (a steeper slope facilitates an increase in total welfare when under incomplete information).

# F Proof of Proposition 4

Suppose that the regulator does not directly observe the type of the monopoly. The regulator gives out a subsidy of $s = \gamma_g$ per unit of effort, to a firm offering $p_g^m = \dfrac{1+b+c}{2}$ and $e_g^{FB} = \left(\dfrac{1}{2\gamma_g}\right)^2$. It gives out zero subsidy for any level of effort below this one.

The question of interest is the following: does a neutral firm facing this policy offer its usual monopoly strategy (monopoly price and zero effort), or does it face an incentive to behave like a green firm to receive the subsidy?

In other words, we examine whether the menu $\{(p_n^m, 0); (p_g^m, e_g^{FB}, s)\}$ is directly separating or not.

To do so, neutral monopoly profits must be compared to neutral profits when perceived as green. We have:

$$\pi_n(p_n^m, 0) = 1/4,$$
$$\pi_n(p_g^m, e_g^{FB}, s) = \frac{1}{4} + \frac{1}{4}\left(2b + b^2 - c^2 - \frac{\Delta\gamma}{\gamma_g^2}\right).$$

The policy offered directly induces a separating equilibrium where the neutral firm has no incentive to mimic the green firm if and only if $\pi_n(p_n^m, 0) > \pi_n(p_g^m, e_g^{FB}, s)$, *i.e.* when the following condition holds:

$$2b + b^2 - c^2 - \frac{\Delta\gamma}{\gamma_g^2} < 0. \quad (6)$$

For given values of $b$ and $c$, notice that when $\gamma_g = \gamma_n = s$, condition (6) is necessarily violated as the left-hand side becomes $\frac{1}{4}(2b + b^2 - c^2) > 0$. Therefore, when the marginal costs of effort are identical, the menu cannot be separating in incomplete information, as the neutral firm offers $e_g^{FB}$ at zero cost (fully covered by the subsidy) and enjoys higher profits.

Let us now analyze how the expression changes when $\gamma_n$ increases above $\gamma_g$. The left-hand side of condition (6) is linear and decreasing in $\gamma_n$. There exists a cutoff point $\hat{\gamma_n}$ above which (6) is satisfied, given by:

$$\hat{\gamma_n} = \gamma_g + \underbrace{\gamma_g^2(2b + b^2 - c^2)}_{\alpha}$$

where $\alpha$ is the minimum extent of the difference between $\gamma_g$ and $\gamma_n$ required for the policy to directly induce a separating equilibrium, for $b > c > 0$ fixed. In other words, we need $\Delta\gamma > \hat{\Delta\gamma} = \gamma_g^2(2b + b^2 - c^2)$ for (6) to be satisfied. The figure below provides a graphical representation of this cutoff.

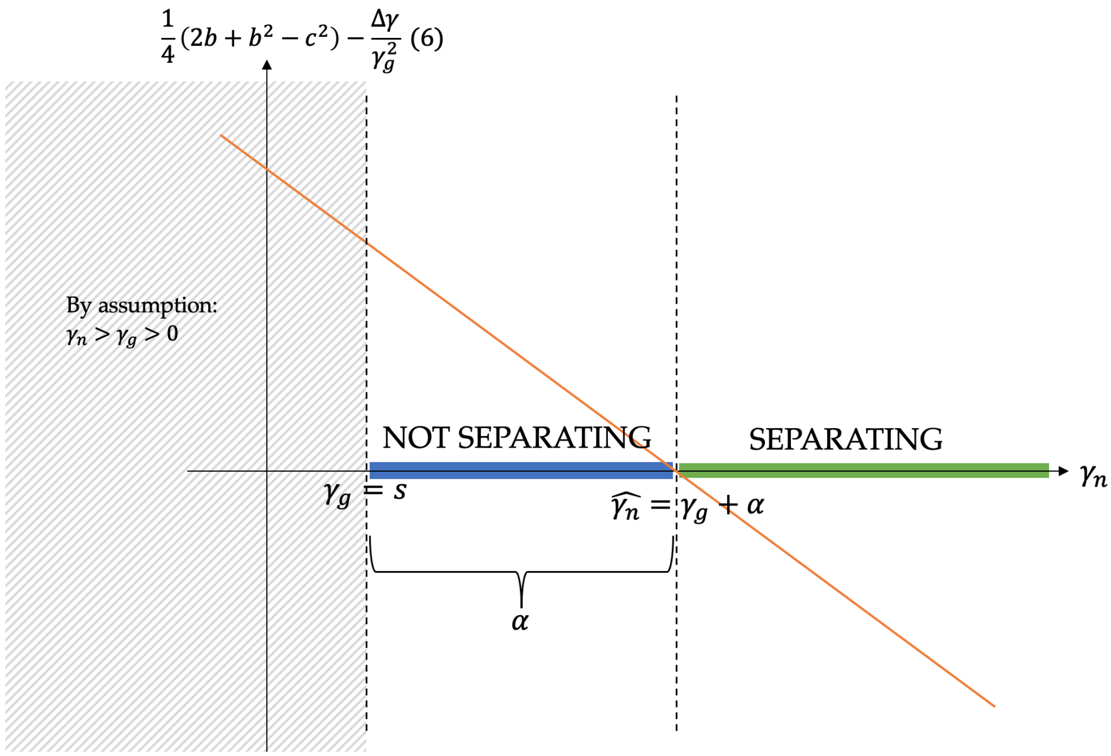


Figure 3: Graphical illustration of condition (6) as a function of the neutral firm's marginal cost of effort

We first assumed $b$ and $c$ fixed with $b > c > 0$. Now consider the effect of a variation in $b$. Recall the minimum difference between $\gamma_g$ and $\gamma_n$ required to directly achieve separation: $\hat{\Delta\gamma} = \gamma_g^2(2b + b^2 - c^2)$. Notice that this cutoff can be decreased by a lower value of $b$. For a fixed $\gamma_g$, such variation in $b$ expands the range of values of $\gamma_n$ for which a separating equilibrium is directly obtained. On figure 3, this corresponds to moving the point $\hat{\gamma_n}$ to the left, shrinking the extent of $\alpha$ and extending the green interval. However, the warm glow $b$ can take values as low as very close but greater than $c$, as assumed by the model. In the limiting case, assume that $b = c + \varepsilon$ with $\varepsilon > 0$ very small such that $b \approx c$. Then, the difference between marginal costs of efforts has to be at least $\hat{\Delta\gamma} = 2c\gamma_g^2$ in order to get separation.

The arguments above provide evidence that condition (6) is more easily satisfied when $\Delta\gamma$ is high and $b$ is low.

# G Footnote 7: Model with Public Good Benefit Increasing in Quantity

This section presents a version of the model where the public good benefit generated by green production increases with the quantity produced. Specifically, assume that the public good benefit is given by:

$$B(q_g, e_g) = q_g \left(1 + h(e_g)\right),$$

where $h(0) = 0$, $h'(e_g) > 0$, and $h''(e_g) \leq 0$. If the green firm does not exert a costly effort to increase the greenness of its product ($e_g = 0$), this "public good" benefit is equal to the total quantity of green goods consumed: $B(q_g) = q_g(p_g)$. In the case where the green firm improves the greenness of its product, this benefit increases with effort at a decreasing rate. All other assumptions and elements of the model remain the same as in the main specification.

**Rationale.** This alternative captures situations in which a greater scale of green production generates larger aggregate environmental benefits. It reflects the idea that the more consumers a green monopoly serves, the fewer consumers will turn to alternatives (possibly polluting) sources to meet their demand, such as suppliers in adjacent sectors or foreign markets, which are not directly modeled here but could exist. For example, if a small green producer supplies only a few units, the overall environmental benefit may be limited compared to a large firm producing many units, which could meaningfully displace less sustainable options.

**Complete Information Monopoly Strategies and First-Best Solution.** The monopoly strategies under complete information are identical to the main model (zero effort, monopoly pricing). The welfare-maximizing solution solves the following problem:

$$\begin{aligned} \max_{q_n, e_n, q_g, e_g \geq 0} \quad W =& \lambda \left( q_g(1 + b - c) - \frac{1}{2} q_g^2 - \gamma_g e_g + q_g(1 + h(e_g)) \right) \\ &+ (1 - \lambda) \left( q_n - \frac{1}{2} q_n^2 - \gamma_n e_n \right) \end{aligned}$$

The first-best neutral effort and price are identical to the main model. If the firm is green, we can compute the first order conditions with respect to $q_g$ and $e_g$:

$$q_g^{FB} h'(e_g^{FB}) = \gamma_g,$$
$$q_g^{FB} = 1 + b - c + \left(1 + h(e_g^{FB})\right)$$

which indirectly define the first-best quantity and effort level for the green firm. Using $h(e_g) = \sqrt{e_g}$ as in the discussed examples, we can derive the exact expressions for $q_g^{FB}$ and $e_g^{FB}$:

$$q_g^{FB} = \frac{2\gamma_g}{2\gamma_g - 1}\left(2 + b - c\right)$$
$$\text{and } e_g^{FB} = \left(\frac{2 + b - c}{2\gamma_g - 1}\right)^2$$

where we impose $\gamma_g > 1/2$. If this state of nature is realized ($\theta = g$), total welfare is equal to $W_g^{FB} = \frac{\gamma_g}{2\gamma_g - 1}\left(2 + b - c\right)^2$, greater than $W_g^m = \frac{1}{8}\left(1 + b - c\right)\left(7 + 3(b - c)\right)$.

We can write the expected total maximum welfare in first best as:

$$W^{FB} = \lambda\left(\frac{\gamma_g}{2\gamma_g - 1}\left(2 + b - c\right)^2\right) + \frac{1 - \lambda}{2}.$$

To conclude, we can verify the concavity of the program to ensure the existence of this maximum. The Hessian writes:

$$H_{(q_g, e_g)} = \begin{bmatrix} -1 & h'(e_g) \\ h'(e_g) & q_g h''(e_g) \end{bmatrix}.$$

For concavity, we verify the diagonal elements : $-1 < 0$ and $q_g h''(e_g) \leq 0$, and require $\det(H_{(q_g, e_g)}) \geq 0 \Leftrightarrow -q_g h''(e_g) \geq (h'(e_g))^2$. Using the example $h(e_g) = \sqrt{e_g}$, we verify: $q_g \geq \sqrt{e_g}$. At the computed solution, this translates to $\gamma_g \geq 1/2$, which is assumed to hold.

Here, the first-best quantity is higher than the green firm's monopoly strategy, for two main reasons:

1. As in the main model, in order to take into account the consumer surplus.
2. To reflect the positive externality generated by the green product, which grows with the size of production (new mechanism compared to the main model).

**Separating Strategies.** As the public good benefit does not play a role in determining the firms' separating strategies in incomplete information, the resulting expressions (and cases) are identical as in the main model.

**Welfare Implications.** Welfare does not change in incomplete information when the firm is neutral, as the price and effort level remain the same as the complete information monopoly strategy. However, similar to the main model, when the firm is green, there are conditions under which welfare is higher under incomplete information. The intuition is identical as before: when the green firm exerts a positive level of effort to signal itself, it is possible that the gain in welfare associated with the public good benefit exceeds the loss associated with charging higher price and costly effort. For this to be the case, the net benefit from improved greenness $h(e_g)q_g - \gamma_g e_g$ needs to be sufficiently large.

**Numerical Application.** We adapt the examples of specific parameter values to illustrate the different scenarios that arise as a result. Keep the assumption that $h(e_g) = \sqrt{e_g}$.

First, consider the following values for the model parameters: $b = 6$, $c = 0.4$, $\gamma_n = 2$, and $\gamma_g = 1$. In a separating equilibrium, the green firm exerts a positive effort $e_g = 5.92$ to increase the greenness of its product. In this case, the benefit from improved greenness is sufficiently large compared to the total cost of effort, and welfare in incomplete information increases compared to the monopoly case in complete information. More specifically, we have $W_g \approx 20.4 > W_g^m = 19.635$.

As discussed previously, it could be the case that the green firm signals itself to consumers using a positive effort in addition to the price, but welfare decreases compared to the monopoly case in complete information. For instance, take the same parameters but increase the cost of effort for the green firm: $b = 6$, $c = 0.4$, $\gamma_n = 2$, and $\gamma_g = 1.8$. In this case, effort to increase the greenness of the product is still positive ($e_g = 4$), but the total cost of providing such effort becomes too large compared to its positive public good effect, so total welfare drops to $W_g = 6.075 < W_g^m = 19.635$.

There exist parameter values that induce a separating equilibrium where the green firm exerts zero effort and relies solely on the price to signal its type to consumers. As mentioned before, this outcome is more likely to arise when the warm-glow benefit is low, the marginal cost of production is high, and the costs of effort are relatively similar across firm types. For instance, we can take $b = 2$, $c = 0.5$, $\gamma_n = 2$, and $\gamma_g = 1.7$.

**Subsidizing the First-Best Green Effort.** In a similar fashion as the main model, assume that the objective of the regulator is to use a subsidy $s = \gamma_g$ per unit of effort to implement the first-best green effort level, $e_g^{FB} = \left(\frac{2+b-c}{2\gamma_g - 1}\right)^2$. Under incomplete information, the regulator offers this subsidy to a firm setting $e_g^{FB}$ and $p_g^m$. In line with the main model's conclusions, under some conditions, the complete information strategies $\{(p_n^m, e_n^m); (p_g^m, e_g^{FB}, s)\}$ can be directly separating under incomplete information. This happens when the following condition is satisfied:

$$\frac{2b + b^2 - c^2}{4} < \frac{\Delta\gamma}{(2\gamma_g - 1)^2}(2 + b - c)^2. \qquad (6')$$

For this condition to be true, the costs of effort need to be sufficiently different (*i.e.* $\Delta\gamma$ large enough). For given values of $b$ and $c$, the minimum difference between the costs of efforts necessary for these strategies to be separating is $\Delta\hat{\gamma} = \dfrac{(2b + b^2 - c^2)(2\gamma_g - 1)^2}{4(2 + b - c)^2}$. The intuition is identical as the main model. The role of the warm-glow benefit in condition (6') is slightly more complex than in the main version of the model. Indeed, the warm-glow creates two opposite effects in the neutral profits from mimicking the green type. First, a positive effect from the gain in revenues from being perceived as green by consumers and getting higher demand and charging a higher price, increasing in $b$. Second, a negative effect (which did not exist in the previous version), as the level of first-best green effort required by the regulator increases with $b$, increasing the cost of providing such effort for the neutral firm. Following the same intuition as above, when the costs of effort across types are sufficiently different ($\Delta\gamma > \frac{1}{4}(2\gamma_g - 1)^2$), regardless of the value of $b$, the gain in revenues will not be sufficient to offset the residual cost of exerting effort $e_g^{FB}$, meaning that the complete information strategies will always be separating. However, when $\Delta\gamma \in \left[\frac{c(2\gamma_g-1)^2}{8(1+c^2)}; \frac{1}{4}(2\gamma_g - 1)^2\right]$, separation becomes more difficult to achieve as the subsidy covers most of the neutral's cost of effort, but it can still arise if $b$ is low enough to mitigate the gain in revenues from mimicking green. More specifically, in this case, for the complete information strategies to directly induce separation, we need $b < \frac{2\lambda(2+c) - \frac{1}{2} + \sqrt{\delta}}{\frac{1}{2} - 2\lambda}$, where $\lambda = \frac{\Delta\gamma}{(2\gamma_g - 1)^2}$, and $\delta = 32c\lambda^2 - 6c\lambda + \frac{1}{4}(1 + c^2)$.