Robust procurement design

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Procurement

Procurement and regulation are widespread: both public and private

Example: private firm procuring AI services

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Designing challenges: at the contracting stage

- uncertain demand (value)
- private and uncertain cost

Two approaches

Standard mechanism design approach: SEU

- buyer has a conjecture (belief) over cost and demand
- maximize expected welfare wrt conjecture: Baron and Myerson 1982; Lewis and Sappington 1988; Armstrong 1999

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Computer science (and more and more in econ) approach: worst-case

- buyer has no conjecture (belief-free)
- maximize the worst-case: Garrett 2014; Bergemann, Heumann, and Morris 2023; Bergemann and Schlag 2008, 2011; Guo and Shmaya 2024

Our approach

Buyer has a conjecture but does not fully trust it

Prepares for the worst: by short-listing mechanisms that max worst-case

Hopes for the best: by using conjecture to select optimal from short-list

Two-stage approach:

- Political/hierarchical organizational constraints
- attitude towards ambiguity of buyer

Model

Players: buyer (principal); seller (agent)

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Actions: quantity $q \geq 0$ and transfer t

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Actions: quantity $\mathrm{q} \geq 0$ and transfer t

Payoffs:

- Profits of the seller: $t - \theta \mathrm{q}$ for providing q units
- Buyer gets a value of $V^{\star}(\mathrm{q})$ if q units of the good are procured:

$$V^{\star}(\mathrm{q}) = \int_{0}^{\mathrm{q}} P^{\star}(s) ds$$

- Ex-post welfare from buying q units and transferring t to seller is

$$V^{\star}(\mathrm{q}) - \mathrm{t} + \alpha(\mathrm{t} - \theta \mathrm{q})$$

For this talk: $\alpha = 0$; buyer is only interested in consumer surplus

Model

Information:

- Marginal cost θ is private information of seller
- Marginal cost θ is drawn from ab. cont. $F^\star$ (technology) with positive density $f^\star$ and support $\Theta \equiv [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_{++}$

A mechanism

A (direct) mechanism is a pair:

- a quantity map: $q : \Theta \to \Re_+$
- a transfer map: $t : \Theta \to \Re$

A mechanism (q, t) is incentive compatible (IC) if

$$u(\theta) \equiv t(\theta) - \theta q(\theta) \geq t(\theta') - \theta q(\theta') \qquad \forall\ \theta, \theta' \in \Theta$$

and individually rational (IR) if

$$u(\theta) \geq 0 \qquad \forall\ \theta \in \Theta$$

(Characterization)

Baron and Myerson, 1982

Optimization problem of the buyer:

$$\max_q \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \Big] f^\star(\theta) \mathrm{d}\theta$$

$$\text{subject to} \quad q \text{ non-increasing}$$

where $z^\star(\theta) = \theta + \frac{F^\star(\theta)}{f^\star(\theta)}$ is the virtual cost

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Definition

A cdf $F^\star$ is regular if it is absolutely continuous with positive density and virtual cost $z^\star(\theta)$ is continuous and increasing over Θ.

BM optimal mechanism

BM (second-best) schedule:

$$q^{\mathrm{BM}}(\theta) = (P^\star)^{-1}(z^\star(\theta)) = (P^\star)^{-1}\left(\theta + \frac{F^\star(\theta)}{f^\star(\theta)}\right)$$

BM optimal mechanism

BM (second-best) schedule:

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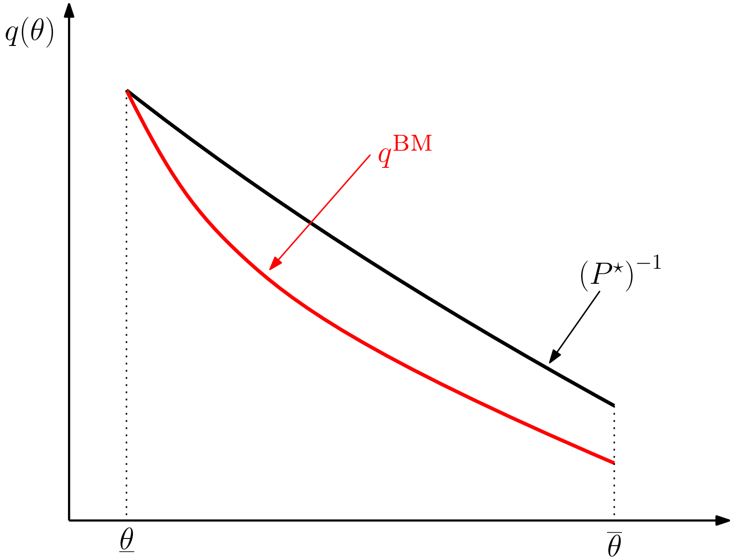
First-best schedule: $(P^\star)^{-1}(\theta)$

- No distortion at top (at $\underline{\theta}$: the most efficient type)
- Downward distortion for all $\theta > \underline{\theta}$

BM optimal mechanism



Model

- Buyer not sure about conjecture $(V^\star, F^\star)$
- Concerned value function and cost distribution (technology) $(V, F) \neq (V^\star, F^\star)$

Admissible sets

- $\mathcal{V}$: set of possible value functions
  - each $V \in \mathcal{V}$ strictly increasing, strictly concave, differentiable
- $\mathcal{P}$: set of corresponding inverse demand functions
- $P^\star \in \mathcal{P}$ and $V^\star \in \mathcal{V}$
- lowest inverse demand function: $\underline{P}$ (alternatively, $\underline{V}$)
  - any $\mathrm{q} \geq 0$ and $P \in \mathcal{P}$

$$P(\mathrm{q}) \geq \underline{P}(\mathrm{q})$$

  - $\underline{P}$: strictly decreasing, continuous, and s.t.

$$\lim_{\mathrm{q} \to 0^+} \underline{P}(\mathrm{q}) > \overline{\theta}$$

- $\mathcal{F}$ is the set of all cdfs over Θ (later generalized)

Worst-case optimality

Let

$$W(M;V,F)=\int\Big[V(q(\theta))-t(\theta)\Big]F(\mathrm{d}\theta)$$

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$$G(M) = \inf_{V \in \mathcal{V}, F \in \mathcal{F}} W(M; V, F)$$

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The buyer prepares a short-list of worst-case optimal (guarantee max) mechanisms:

$$\mathcal{M}^{\mathrm{SL}} = \{ M \text{ is IC and IR} : G(M) \geq G(M') \text{ for all IC and IR } M' \}$$

Robust optimality

A robustly optimal mechanism maximizes ex-ante welfare under conjecture $(V^\star, F^\star)$ over the shortlist $\mathcal{M}^{\mathrm{SL}}$.

Definition

A mechanism $M^\star$ is robustly optimal if $M^\star \in \mathcal{M}^{\mathrm{SL}}$ and for every $M \in \mathcal{M}^{\mathrm{SL}}$ we have

$$W(M^\star; V^\star, F^\star) \geq W(M; V^\star, F^\star)$$

What does the two-stage approach give?

Robust procurement mechanism:

- Uncertainty only over the cost of supplier: Baron-Myerson-with-floor is robustly optimal
- Uncertainty over both cost and demand, robustly optimal mechanism adjusts Baron-Myerson optimal mechanism:
  - upward quantity adjustment for high cost
  - downward quantity adjustment for intermediate cost

In the paper

- General cost uncertainty
- Application to regulation design: quantity versus price regulation

Characterization of maximal guarantee

Lemma

For any IC and IR mechanism $M \equiv (q, t)$,

$$G(M) = \inf_{\theta} \left[\underline{V}(q(\theta)) - t(\theta) \right]$$

Further, $G(M) \leq G^* = \underline{V}(\mathrm{q}_\ell) - \overline{\theta}\mathrm{q}_\ell$, *where*

$$\mathrm{q}_\ell = \underline{P}^{-1}(\overline{\theta})$$

Short list

The buyer prepares a short-list of worst-case optimal (guarantee max) mechanisms:

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Short list

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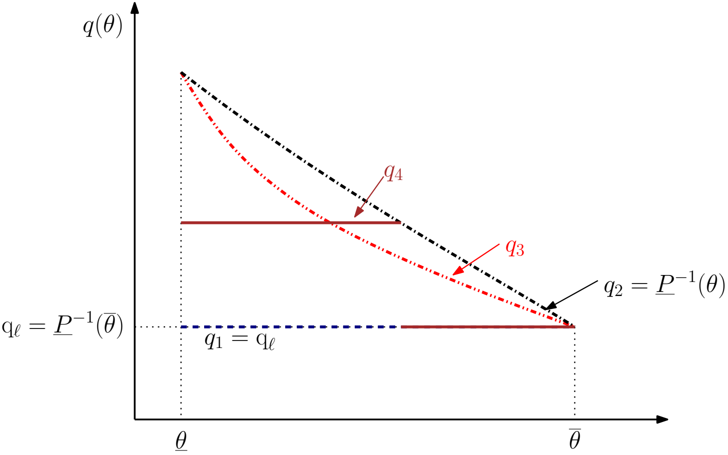
Lemma

A mechanism $M \in \mathcal{M}^{\mathrm{SL}}$ if and only if

- *q is non-increasing*
- *$u(\theta) = \int_{\theta}^{\overline{\theta}} q(s)ds$ for all θ*
- *for all θ*

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(s)ds \geq G^* \qquad \forall\ \theta$$

Short list $\mathcal{M}^{\mathrm{SL}}$



Robust optimality: full program

$$\max_{q} \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^{\star}(q(\theta) - z^{\star}(\theta) q(\theta) \Big] F^{\star}(\mathrm{d}\theta)$$

subject to $\quad q$ non-increasing

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(s) ds \geq G^{*} \qquad \forall\ \theta$$

where $z^{\star}(\theta) = \theta + \frac{F^{\star}(\theta)}{f^{\star}(\theta)}$ is virtual cost under $(V^{\star}, F^{\star})$

Baron-Myerson-with-floor

Recall q^{BM} is

$$P^\star(q^{\mathrm{BM}}(\theta)) = z^\star(\theta) \qquad \forall\ \theta$$

Definition

Baron-Myerson-with-floor mechanism $M^\star = (q^\star, t^\star)$ *is*

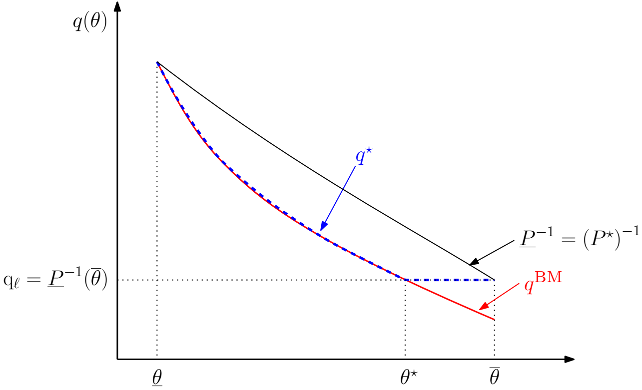
$$q^\star(\theta) = \max(q^{\mathrm{BM}}(\theta), \mathrm{q}_\ell) \qquad \forall\ \theta$$

$$u^\star(\theta) = t^\star(\theta) - \theta q^\star(\theta) = \int_{\theta}^{\overline{\theta}} q^\star(s) ds$$

Optimality of Baron-Myerson-with-floor

Theorem

Suppose $F^\star$ regular and $V^\star = \underline{V}$. Then, Baron-Myerson-with-floor is robustly optimal.



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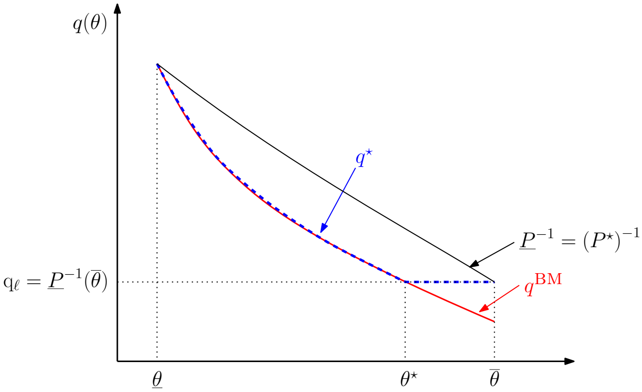
Interpretation: only uncertainty is over cost



Robust optimality of BM-with-floor

When only uncertainty is over cost:

- efficiency at both top and bottom
- buyer worried that cost is higher than conjectured
  - Advantages of lowering rent for low cost type is gone
  - $\rightarrow$ lower distortion at high θ

Flat mechanism q_ℓ at all θ makes the short-list: will be worst-case optimal (in the standard robustness sense) but it is never robustly optimal

Robust optimality: general case

Define $\theta^\star$ is the type where q^{BM} crosses q_ℓ (if it does not cross $\theta^\star = \overline{\theta}$)

Define θ^m is a largest minimizer of $\underline{W}(\theta, q^\star)$:

$$\theta^m = \max\{\theta : \theta \in \arg\min_{\theta'} \underline{W}(\theta', q^\star)\}$$

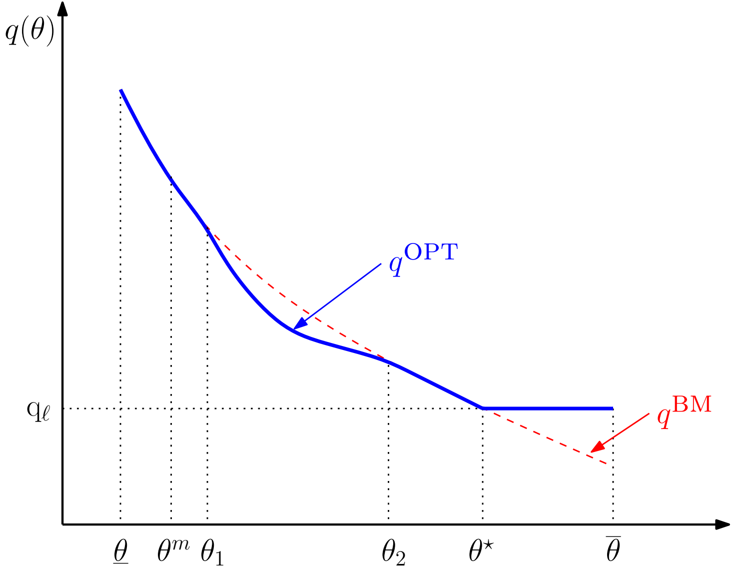
Robust optimality: general case

Theorem

Suppose $F^\star$ is regular. Then, the robustly optimal schedule q^{OPT} satisfies the following:

1. *Baron-Myerson-with-floor is robustly optimal if and only if $\theta^m = \bar{\theta}$*
2. *If $\theta^m < \bar{\theta}$, then $\theta^m \leq \theta^\star$ and*
   (a) $q^{\mathrm{OPT}}(\theta) = \mathrm{q}_\ell$ *for all* $\theta \in [\theta^\star, \bar{\theta}]$
   (b) $q^{\mathrm{OPT}}(\theta) \leq q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\theta^m, \theta^\star)$ *with strict inequality for a positive measure of types*
   (c) $q^{\mathrm{OPT}}(\theta) = q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in [\underline{\theta}, \theta^m]$

Robust optimality: general case

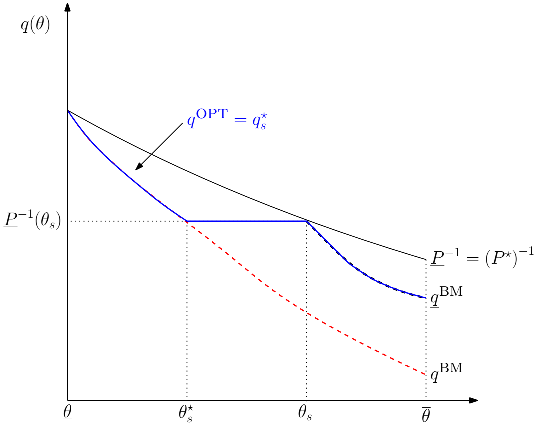


Robust Optimality: general case

- Upward quantity adjustments for high costs
  - Avoid inefficiencies motivated by rent extraction (for lower types)
- Downward adjustments for intermediate costs
  - Avoid over-procurement in case demand lower than expected
- Low costs
  - Welfare higher than G^* even when demand lower than conjectured
  - No adjustment necessary

General cost uncertainty

- $\mathcal{F}$: a cdf $\underline{F}$ and all cdfs F such that $F(\theta) \geq \underline{F}(\theta)$ for all $\theta \in \Theta$.
- So far, $\underline{F}(\theta) = \mathbb{I}(\theta \geq \overline{\theta}) \implies \mathcal{F} = \mathrm{CDF}(\Theta)$.
- Now, $\underline{F}$ has support $[\theta_s, \overline{\theta}]$.



Application to monopoly regulation

Price regulation: set price rather than quantity

- $$\mathcal{D} := \{D : \mathbb{R}_+ \to \mathbb{R}_+ \text{ s.t. } \underline{D}(q) \leq D(q) \leq \overline{D}(q) \ \forall q\}.$$

  be set of demand functions associated with $\mathcal{P}$ (bijection between $\mathcal{P}$ and $\mathcal{D}$)

- Price regulation $\widetilde{M} = (p, t)$

$$p : \Theta \to \mathbb{R}_+$$

$$t : \Theta \times \mathcal{D} \to \mathbb{R}_+$$

- Robustly optimal price regulation: Baron-Myerson with price-cap
- When is price regulation better than quantity regulation?

Thank you!

Baron and Myerson, 1982

Lemma

A mechanism (q, t) is IC and IR if and only if

- *q is non-increasing*
- *for every $\theta \in \Theta$*

$$u(\theta) = u(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(s)ds$$

- $u(\overline{\theta}) \geq 0$

(Return)