EEA 2025 Bordeaux

More Reasoning, Less Outcomes: A Monotonicity Result for Reasoning in Dynamic Games

Andrés Perea

EPICENTER
Research Center for
Epistemic Game Theory

Maastricht University

August 26, 2025

- To make good decisions in a dynamic game you must reason, at each of your information sets, about the opponents' rationality at some, or all, of their information sets.

- To make good decisions in a dynamic game you must reason, at each of your information sets, about the opponents' rationality at some, or all, of their information sets.

- Strong rationalizability (extensive-form rationalizability) (Pearce (1984), Battigalli (1997)):

  You believe, whenever possible, that your opponents choose rationally at all of their information sets.

Introduction

Reasoning about Opponents' Rationality

- To make good decisions in a dynamic game you must reason, at each of your information sets, about the opponents' rationality at some, or all, of their information sets.

- Strong rationalizability (extensive-form rationalizability) (Pearce (1984), Battigalli (1997)):

  You believe, whenever possible, that your opponents choose rationally at all of their information sets.

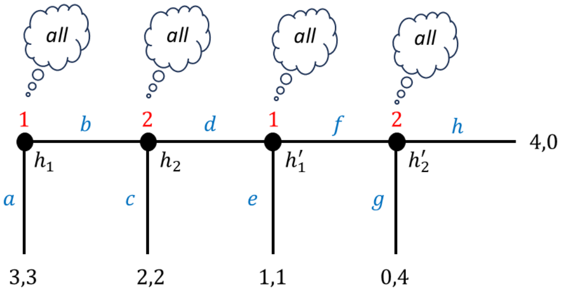
- Backward dominance (Perea (2014)), backwards rationalizability (Penta (2015), Perea (2014), Catonini and Penta (2022):

  You always believe that your opponents will choose rationally at all present and future information sets.

- Under strong rationalizability you reason about more information sets than under backward dominance.

- Under strong rationalizability you reason about more information sets than under backward dominance.
- Is strong rationalizability then more restrictive?

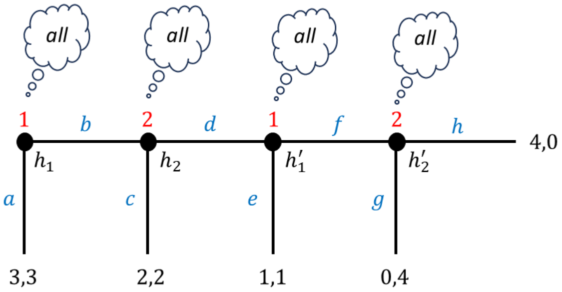
- Under strong rationalizability you reason about more information sets than under backward dominance.
- Is strong rationalizability then more restrictive?
- Not in terms of strategies.



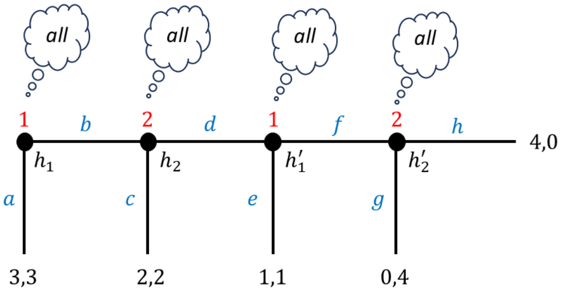
- Strong rationalizability:

Introduction

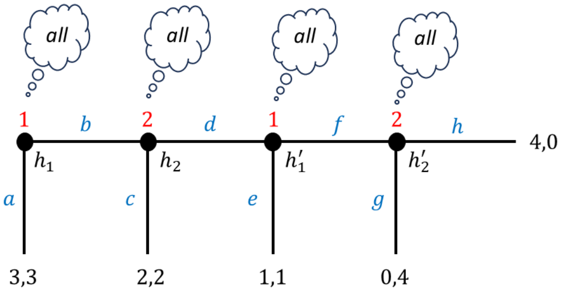
Example



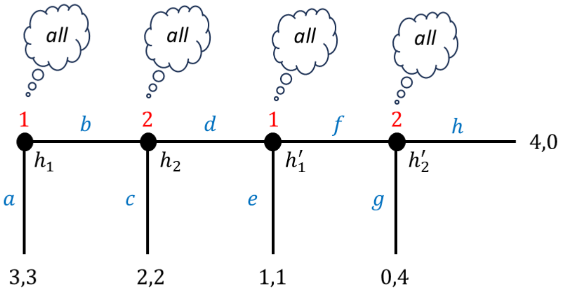
- Strong rationalizability:
- At h_2, player 2 believes, whenever possible, that player 1 chooses rationally at h_1.



- Strong rationalizability:
- At h_2, player 2 believes, whenever possible, that player 1 chooses rationally at h_1.
- Hence, player 2 believes at h_2 that player 1 chooses (b, f).

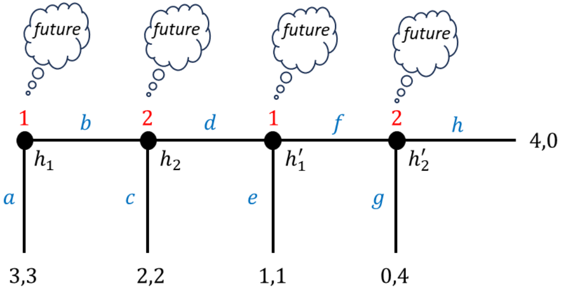


- Strong rationalizability:
- At h_2, player 2 believes, whenever possible, that player 1 chooses rationally at h_1.
- Hence, player 2 believes at h_2 that player 1 chooses (b, f).
- Player 2 chooses (d, g).



- Strong rationalizability:
- At h_2, player 2 believes, whenever possible, that player 1 chooses rationally at h_1.
- Hence, player 2 believes at h_2 that player 1 chooses (b, f).
- Player 2 chooses (d, g).
- Player 1 chooses a.

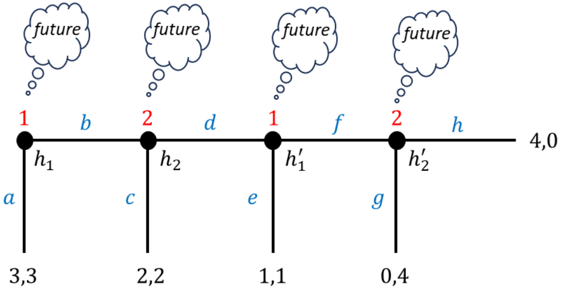
Introduction
Example



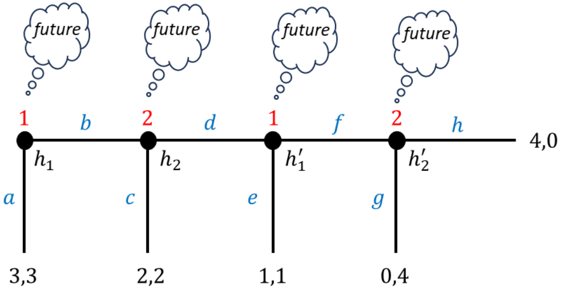
- Backward dominance:

Introduction

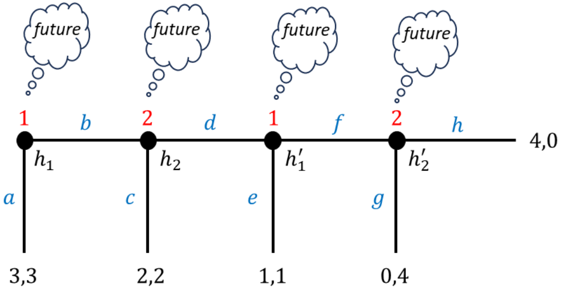
Example



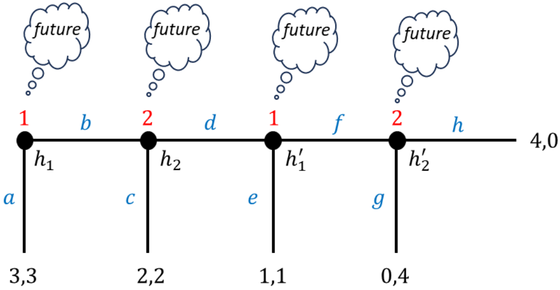
- Backward dominance:
- At h_2, player 2 must believe



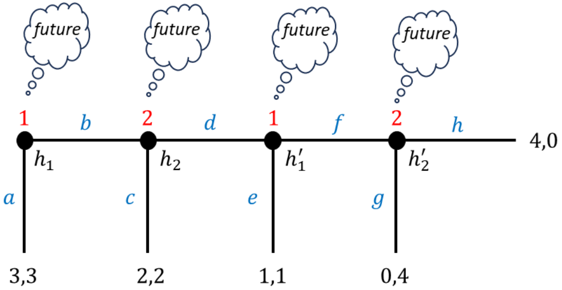
- Backward dominance:
- At h_2, player 2 must believe
- that player 1 chooses rationally at h_1', and



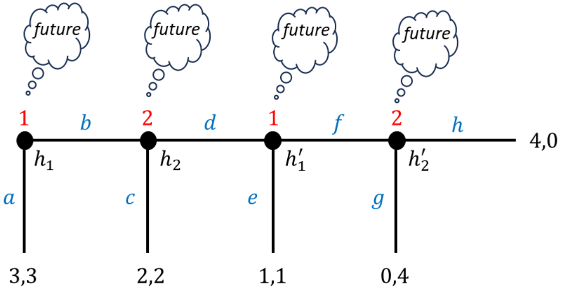
- Backward dominance:
- At h_2, player 2 must believe
- that player 1 chooses rationally at h_1', and
- that player 1 believes at h_1' that player 2 chooses rationally at h_2'.



- Backward dominance:
- At h_2, player 2 must believe
- that player 1 chooses rationally at h'_1, and
- that player 1 believes at h'_1 that player 2 chooses rationally at h'_2.
- Player 2 believes at h_2 that player 1 chooses (b, e).



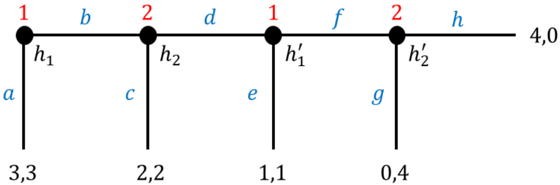
- Backward dominance:
- At h_2, player 2 must believe
- that player 1 chooses rationally at h'_1, and
- that player 1 believes at h'_1 that player 2 chooses rationally at h'_2.
- Player 2 believes at h_2 that player 1 chooses (b, e).
- Player 2 chooses c.



- Backward dominance:
- At h_2, player 2 must believe
- that player 1 chooses rationally at h_1', and
- that player 1 believes at h_1' that player 2 chooses rationally at h_2'.
- Player 2 believes at h_2 that player 1 chooses (b, e).
- Player 2 chooses c.
- Player 1 chooses a.

Introduction

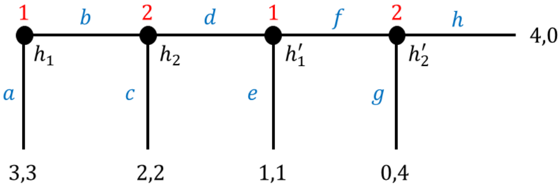
Forward Induction versus Backward Induction



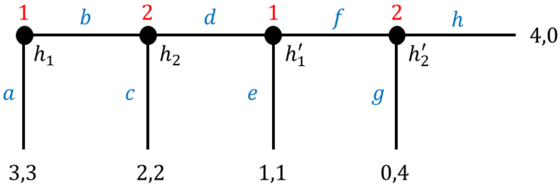
- But strong rationalizability and backward dominance lead to the same outcome: the backward induction outcome *a*.

Introduction

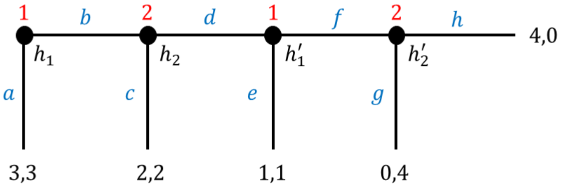
Forward Induction versus Backward Induction



- But strong rationalizability and backward dominance lead to the same outcome: the backward induction outcome *a*.
- Battigalli (1997) has shown that this is true for every game with perfect information that has no relevant ties.



- But strong rationalizability and backward dominance lead to the same outcome: the backward induction outcome *a*.
- Battigalli (1997) has shown that this is true for every game with perfect information that has no relevant ties.
- Perea (2017) and Catonini (2020) have shown that in every game with observed past choices, strong rationalizability is more, or equally, restrictive than backward dominance in terms of outcomes.



- But strong rationalizability and backward dominance lead to the same outcome: the backward induction outcome *a*.
- Battigalli (1997) has shown that this is true for every game with perfect information that has no relevant ties.
- Perea (2017) and Catonini (2020) have shown that in every game with observed past choices, strong rationalizability is more, or equally, restrictive than backward dominance in terms of outcomes.
- This paper generalizes that result: Under certain conditions, reasoning about more information sets leads to less, or the same, outcomes.

Outline

- Focus-based rationalizability procedures

Outline

- Focus-based rationalizability procedures
- Existence

Outline

- Focus-based rationalizability procedures
- Existence
- Monotonicity theorem

- A decision problem for player i at information set h_i is a pair $(D_i(h_i), D_{-i}(h_i))$ where $D_i(h_i) \subseteq S_i(h_i)$ and $D_{-i}(h_i) \subseteq S_{-i}(h_i)$.

Definition (Focus function)

A focus function f assigns to every player i, each of his information sets h_i, every opponent $j \neq i$, and every collection of decision problems D, a collection $f_{ij}(h_i, D) \subseteq H_j$ of information sets for j.

- A decision problem for player i at information set h_i is a pair $(D_i(h_i), D_{-i}(h_i))$ where $D_i(h_i) \subseteq S_i(h_i)$ and $D_{-i}(h_i) \subseteq S_{-i}(h_i)$.
- A collection of decision problems D specifies for every player i and information set h_i a decision problem for player i at h_i.

Definition (Focus function)

A focus function f assigns to every player i, each of his information sets h_i, every opponent $j \neq i$, and every collection of decision problems D, a collection $f_{ij}(h_i, D) \subseteq H_j$ of information sets for j.

- A decision problem for player i at information set h_i is a pair $(D_i(h_i), D_{-i}(h_i))$ where $D_i(h_i) \subseteq S_i(h_i)$ and $D_{-i}(h_i) \subseteq S_{-i}(h_i)$.
- A collection of decision problems D specifies for every player i and information set h_i a decision problem for player i at h_i.

Definition (Focus function)

A focus function f assigns to every player i, each of his information sets h_i, every opponent $j \neq i$, and every collection of decision problems D, a collection $f_{ij}(h_i, D) \subseteq H_j$ of information sets for j.

- Strong rationalizability: $f_{ij}(h_i, D) = H_j$.

- A decision problem for player i at information set h_i is a pair $(D_i(h_i), D_{-i}(h_i))$ where $D_i(h_i) \subseteq S_i(h_i)$ and $D_{-i}(h_i) \subseteq S_{-i}(h_i)$.
- A collection of decision problems D specifies for every player i and information set h_i a decision problem for player i at h_i.

Definition (Focus function)

A focus function f assigns to every player i, each of his information sets h_i, every opponent $j \neq i$, and every collection of decision problems D, a collection $f_{ij}(h_i, D) \subseteq H_j$ of information sets for j.

- Strong rationalizability: $f_{ij}(h_i, D) = H_j$.
- Backward dominance: $f_{ij}(h_i, D) = \{h_j \in H_j \mid h_j \text{ weakly follows } h_i\}$.

- A reduction operator r assigns to every collection of decision problems D a smaller collection of decision problems $r(D) \subseteq D$.

- A reduction operator r assigns to every collection of decision problems D a smaller collection of decision problems $r(D) \subseteq D$.
- $r^k(D)$: k-fold application of r to D

- A reduction operator r assigns to every collection of decision problems D a smaller collection of decision problems $r(D) \subseteq D$.
- $r^k(D)$: k-fold application of r to D
- D^{full}: collection of full decision problems $(S_i(h_i), S_{-i}(h_i))$.

- A reduction operator r assigns to every collection of decision problems D a smaller collection of decision problems $r(D) \subseteq D$.
- $r^k(D)$: k-fold application of r to D
- D^{full}: collection of full decision problems $(S_i(h_i), S_{-i}(h_i))$.
- r induces iterated reduction procedure $(r^0(D^{full}), r^1(D^{full}), r^2(D^{full}), ...)$.

- Every focus function f induces reduction operator rf.

- Every focus function f induces reduction operator rf.
- Starting from collection of decision problems D, the reduction $rf(D) \subseteq D$ is defined as follows:

Focus-based Rationalizability Procedures

Focus Function Induces Reduction Operator

- Every focus function f induces reduction operator rf.
- Starting from collection of decision problems D, the reduction $rf(D) \subseteq D$ is defined as follows:
- Consider decision problem $(D_i(h_i), D_{-i}(h_i))$.

- Every focus function f induces reduction operator rf.
- Starting from collection of decision problems D, the reduction $rf(D) \subseteq D$ is defined as follows:
- Consider decision problem $(D_i(h_i), D_{-i}(h_i))$.
- From $D_{-i}(h_i)$, only keep opponents' strategies s_j such that

  $s_j \in D_j(h_j)$ for all $h_j \in f_{ij}(h_i, D) \cap H_j(s_j)$ ...

- Every focus function f induces reduction operator rf.
- Starting from collection of decision problems D, the reduction $rf(D) \subseteq D$ is defined as follows:
- Consider decision problem $(D_i(h_i), D_{-i}(h_i))$.
- From $D_{-i}(h_i)$, only keep opponents' strategies s_j such that

  $s_j \in D_j(h_j)$ for all $h_j \in f_{ij}(h_i, D) \cap H_j(s_j)$ ...
- ... if such strategies s_j exist.

- Every focus function f induces reduction operator rf.
- Starting from collection of decision problems D, the reduction $rf(D) \subseteq D$ is defined as follows:
- Consider decision problem $(D_i(h_i), D_{-i}(h_i))$.
- From $D_{-i}(h_i)$, only keep opponents' strategies s_j such that

  $s_j \in D_j(h_j)$ for all $h_j \in f_{ij}(h_i, D) \cap H_j(s_j)$ ...
- ... if such strategies s_j exist.
- If such strategies s_j do not exist, keep all opponents' strategies in $D_{-i}(h_i)$.

- Every focus function f induces reduction operator rf.
- Starting from collection of decision problems D, the reduction $rf(D) \subseteq D$ is defined as follows:
- Consider decision problem $(D_i(h_i), D_{-i}(h_i))$.
- From $D_{-i}(h_i)$, only keep opponents' strategies s_j such that

  $s_j \in D_j(h_j)$ for all $h_j \in f_{ij}(h_i, D) \cap H_j(s_j)$ ...
- ... if such strategies s_j exist.
- If such strategies s_j do not exist, keep all opponents' strategies in $D_{-i}(h_i)$.
- From $D_i(h_i)$, eliminate all strategies s_i that have become strictly dominated in the decision problem at h_i.

- We have seen: A focus function f induces a reduction operator rf.

- We have seen: A focus function f induces a reduction operator rf.
- The sequence $(rf^0(D^{full}), rf^1(D^{full}), ...)$ is the f-rationalizability procedure.

- We have seen: A focus function f induces a reduction operator rf.
- The sequence $(rf^0(D^{full}), rf^1(D^{full}), ...)$ is the f-rationalizability procedure.
- Let $D^\infty := rf^\infty(D^{full})$.

- We have seen: A focus function f induces a reduction operator rf.
- The sequence $(rf^0(D^{full}), rf^1(D^{full}), ...)$ is the f-rationalizability procedure.
- Let $D^\infty := rf^\infty(D^{full})$.
- Strategy s_i is f-rationalizable if $s_i \in D_i^\infty(h_i)$ for all $h_i \in H_i(s_i)$.

Existence
Conditions

- Under which conditions on the focus function f do f-rationalizable strategies exist?

- Under which conditions on the focus function f do f-rationalizable strategies exist?
- Individually forward decreasing:

  If $h_i, h_i' \in H_i$ and h_i' follows h_i, then

$$f_{ij}(h_i', D) \subseteq f_{ij}(h_i, D).$$

- Under which conditions on the focus function f do f-rationalizable strategies exist?
- Individually forward decreasing:

  If $h_i, h_i' \in H_i$ and h_i' follows h_i, then

  $$f_{ij}(h_i', D) \subseteq f_{ij}(h_i, D).$$

- Individually preserves focus on past information sets:

  If $h_i, h_i' \in H_i$ and h_i' follows h_i, then

  $$f_{ij}(h_i, D) \cap H_j^{before\ h_i} \subseteq f_{ij}(h_i', D).$$

- Under which conditions on the focus function f do f-rationalizable strategies exist?
- Individually forward decreasing:

  If $h_i, h_i' \in H_i$ and h_i' follows h_i, then

$$f_{ij}(h_i', D) \subseteq f_{ij}(h_i, D).$$

- Individually preserves focus on past information sets:

  If $h_i, h_i' \in H_i$ and h_i' follows h_i, then

$$f_{ij}(h_i, D) \cap H_j^{before\ h_i} \subseteq f_{ij}(h_i', D).$$

- Monotone:

  For every two collections of decision problems $D \subseteq E$,

$$f_{ij}(h_i, E) \subseteq f_{ij}(h_i, D).$$

Theorem (Existence)

Let f be a focus function that is

individually forward decreasing,

individually preserving focus on past information sets, and

monotone.

Then, for every player there is at least one f-rationalizable strategy.

Monotonicity Theorem
Conditions

- Say that $g \subseteq f$ if
$$g_{ij}(h_i, D) \subseteq f_{ij}(h_i, D)$$
for all i, j, h_i and D.

Monotonicity Theorem
Conditions

- Say that $g \subseteq f$ if
$$g_{ij}(h_i, D) \subseteq f_{ij}(h_i, D)$$
for all i, j, h_i and D.

- Suppose that $g \subseteq f$.

  Under which conditions does f-rationalizability lead to less, or the same, outcomes as g-rationalizability?

- Define

$$f_{ii}(h_i, D) := \{h_i\} \cup \{h_i' \in H_i \mid \text{there is some } j \neq i \text{ and } h_j \in f_{ij}(h_i, D) \text{ such that } h_i' \in f_{ji}(h_j, D)\}.$$

Own information sets that player i indirectly reasons about when being at h_i.

Monotonicity Theorem
Conditions

- Define

$$f_{ii}(h_i, D) := \{h_i\} \cup \{h_i' \in H_i \mid \text{there is some } j \neq i \text{ and } h_j \in f_{ij}(h_i, D) \text{ such that } h_i' \in f_{ji}(h_j, D)\}.$$

  Own information sets that player i indirectly reasons about when being at h_i.

- Collectively forward decreasing:

  If h_j weakly follows h_i, then

$$f_{jk}(h_j, D) \subseteq f_{ik}(h_i, D).$$

Monotonicity Theorem
Conditions

- Define

$$f_{ii}(h_i, D) := \{h_i\} \cup \{h_i' \in H_i \mid \text{there is some } j \neq i \text{ and } h_j \in f_{ij}(h_i, D) \text{ such that } h_i' \in f_{ji}(h_j, D)\}.$$

  Own information sets that player i indirectly reasons about when being at h_i.

- Collectively forward decreasing:

  If h_j weakly follows h_i, then

$$f_{jk}(h_j, D) \subseteq f_{ik}(h_i, D).$$

- Collectively preserves focus on past information sets:

  If h_j weakly follows h_i, then

$$f_{ik}(h_i, D) \cap H_k^{before\ h_i} \subseteq f_{jk}(h_j, D).$$

Monotonicity Theorem

Conditions

- Transitively closed:

  If $h_j \in f_{ij}(h_i, D)$ and $h_k \in f_{jk}(h_j, D)$ then $h_k \in f_{ik}(h_i, D)$.

- Transitively closed:

  If $h_j \in f_{ij}(h_i, D)$ and $h_k \in f_{jk}(h_j, D)$ then $h_k \in f_{ik}(h_i, D)$.

- Consider two focus functions f, g and two collections of decision problems D, E where $E = rf^k(rg^m(D^{full}))$.

  Then, D is a (g, f)-semi reduction of E if

  $$\text{either } D = rf^k(rg^{m+1}(D^{full})) \text{ or } D = rf^{k+1}(rg^{m-1}(D^{full})).$$

Monotonicity Theorem

Conditions

- Transitively closed:

  If $h_j \in f_{ij}(h_i, D)$ and $h_k \in f_{jk}(h_j, D)$ then $h_k \in f_{ik}(h_i, D)$.

- Consider two focus functions f, g and two collections of decision problems D, E where $E = rf^k(rg^m(D^{full}))$.

  Then, D is a (g, f)-semi reduction of E if

  $$\text{either } D = rf^k(rg^{m+1}(D^{full})) \text{ or } D = rf^{k+1}(rg^{m-1}(D^{full})).$$

- Monotone with respect to g:

  If D is a (g, f)-semi reduction of E then

  $$f_{ij}(h_i, E) \subseteq f_{ij}(h_i, D).$$

Theorem (Monotonicity theorem)

Let f, g be two focus functions with $g \subseteq f$ that are

individually forward decreasing, individually preserving focus on past information sets and *monotone.*

Assume moreover that f is

collectively forward decreasing, collectively preserving focus on past information sets, transitively closed, and monotone with respect to g.

Then,

(a) the set outcomes under f-rationalizability is the same as the set of outcomes obtained if we first apply g-rationalizability and then f-rationalizability,

(b) every outcome under f-rationalizability is also an outcome under g-rationalizability, and

(c) if in f, players only reason about present and future information sets, then every f-rationalizable strategy is also g-rationalizable.

Theorem (Monotonicity theorem)

Let f, g be two focus functions with $g \subseteq f$ that are

individually forward decreasing, individually preserving focus on past information sets and *monotone.*

Assume moreover that f is

collectively forward decreasing, collectively preserving focus on past information sets, transitively closed, and *monotone with respect to g.*

Then,

(a) the set outcomes under f-rationalizability is the same as the set of outcomes obtained if we first apply g-rationalizability and then f-rationalizability,

(b) every outcome under f-rationalizability is also an outcome under g-rationalizability, and

(c) if in f, players only reason about present and future information sets, then every f-rationalizable strategy is also g-rationalizable.

Theorem (Monotonicity theorem)

Let f, g be two focus functions with $g \subseteq f$ that are

individually forward decreasing, individually preserving focus on past information sets and *monotone.*

Assume moreover that f is

collectively forward decreasing, collectively preserving focus on past information sets, transitively closed, and *monotone with respect to g.*

Then,

(a) the set outcomes under f-rationalizability is the same as the set of outcomes obtained if we first apply g-rationalizability and then f-rationalizability,

(b) every outcome under f-rationalizability is also an outcome under g-rationalizability, and

(c) if in f, players only reason about present and future information sets, then every f-rationalizable strategy is also g-rationalizable.

Theorem (Monotonicity theorem)

Let f, g be two focus functions with $g \subseteq f$ that are

individually forward decreasing, individually preserving focus on past information sets and monotone.

Assume moreover that f is

collectively forward decreasing, collectively preserving focus on past information sets, transitively closed, and monotone with respect to g.

Then,

(a) the set outcomes under f-rationalizability is the same as the set of outcomes obtained if we first apply g-rationalizability and then f-rationalizability,

(b) every outcome under f-rationalizability is also an outcome under g-rationalizability, and

(c) if in f, players only reason about present and future information sets, then every f-rationalizable strategy is also g-rationalizable.

Theorem (Monotonicity theorem)

Let f, g be two focus functions with $g \subseteq f$ that are

individually forward decreasing, individually preserving focus on past information sets and monotone.

Assume moreover that f is

collectively forward decreasing, collectively preserving focus on past information sets, transitively closed, and monotone with respect to g.

Then,

(a) the set outcomes under f-rationalizability is the same as the set of outcomes obtained if we first apply g-rationalizability and then f-rationalizability,

(b) every outcome under f-rationalizability is also an outcome under g-rationalizability, and

(c) if in f, players only reason about present and future information sets, then every f-rationalizable strategy is also g-rationalizable.

Thanks for your attention