Restoring Existence and Uniqueness at the Effective Lower Bound with Simple Fiscal Policy

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Introduction

- Ascari and Mavroeidis (2022) (AM) : (non-)uniqueness/existence are challenge for NK models subject to ZLB.
  - Existence of unique solution is non-trivial.
- Several ways to ensure solution uniqueness
  - learning (Ascari, Mavroeidis, and McClung, 2023)
  - deviation from full information rational expectations (FIRE) (Angeletos and Lian, 2023)
  - **This paper:** passive fiscal policy

This Paper

- Baseline three-equation NK model
  - Active Taylor rule
  - Rational expectations

This Paper

- Baseline three-equation NK model
  - Active Taylor rule
  - Rational expectations
- Simple Ricardian fiscal policy restores solution uniqueness
- FP needs to satisfy two properties
  1. Sufficient counter-cyclicality.
  2. High persistence.

Today

- NK Model
  - Non-uniqueness/existence of solution
- Adding fiscal policy
  - Perpetual change in spending
  - Inertial spending rule

Equilibria in Baseline NK model

- Baseline (log-linear) NK model as in Galí (2015):

$$\text{IS: } \hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \sigma^{-1}(\hat{i}_t - E_t \hat{\pi}_{t+1}) + \varepsilon_t, \tag{1}$$

$$\text{NKPC: } \hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{y}_t, \tag{2}$$

$$\text{TR: } \hat{i}_t = \max\left\{-\mu, \phi_\pi \hat{\pi}_t\right\}, \tag{3}$$

The Baseline NK model

- ε_t materialises and persists w.p. p and vanishes w.p $1-p$:

$$\varepsilon_t = \begin{cases} \frac{p}{\sigma}\hat{r}^T & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases} \tag{4}$$

- Model in transitory state is given by:

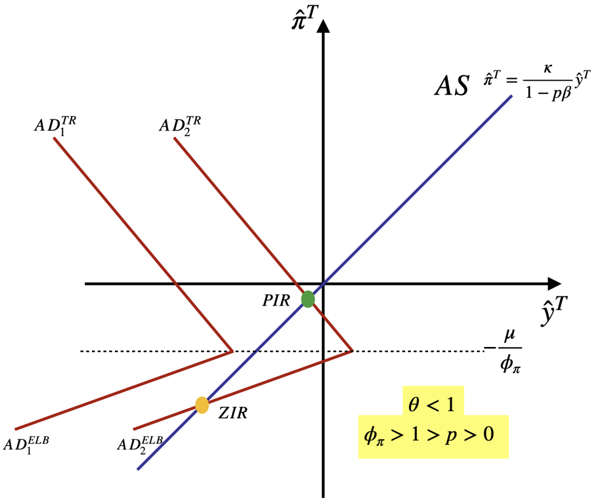
$$\hat{\pi}^T = \frac{\kappa}{1-p\beta}\hat{y}^T \quad AS, \tag{5a}$$

$$\hat{\pi}^T = \begin{cases} \frac{\sigma(1-p)}{p-\phi_\pi}\hat{y}^T - \frac{p}{p-\phi_\pi}\hat{r}^T & AD^{TR} \text{ for } \hat{\pi}^T \geq -\frac{\mu}{\phi_\pi}, \\ \frac{\sigma(1-p)}{p}\hat{y}^T - \frac{\mu}{p} - \hat{r}^T & AD^{ELB} \text{ for } \hat{\pi}^T \leq -\frac{\mu}{\phi_\pi}. \end{cases} \tag{5b}$$

- Define slope ratio of AD^{ELB} and AS:

$$\theta = \frac{\sigma(1-p)(1-p\beta)}{p\kappa} \tag{6}$$

Non-uniqueness/existence in NK Model ($\theta < 1$)



Adding Fiscal Policy

- Now include fiscal policy; government spending is funded by output and lump-sum taxes

$$\tau_t = G_t$$

- Model now is:

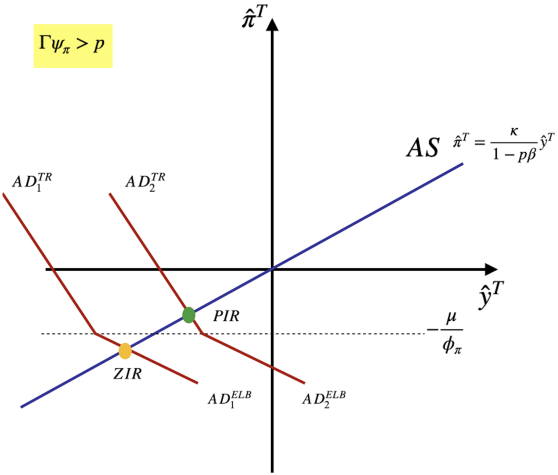
$$\hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \frac{c}{\sigma}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) - \Gamma \Delta \widehat{g}_{t+1} + \varepsilon_t, \quad (7)$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{x}_t \quad (8)$$

$$\hat{i}_t = \max\{-\mu; \phi_\pi \hat{\pi}_t\}, \quad (9)$$

$$\Delta \widehat{g}_{t+1} = \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t \quad (10)$$

Illustrating the NK-FP Model



Key Intuition

- Fiscal policy changes the slope of AD^{ZLB}
- Recall: AD^{ELB}/AS slope ratio:

$$\theta = \frac{\sigma(1-p)(1-p\beta)}{\kappa_y c(p-\Gamma\psi_\pi)} \tag{11}$$

- θ is determined by policy, ψ_π, and uncertainty, p.

Endogenous State Solution

- Analytical solution with endogenous states is infeasible.
- Model dynamics depend on all past realisations of the shock and endogenous state.
- With no endogenous states, the support of $\boldsymbol{Y}$ is time-invariant.

$$F(\boldsymbol{Y}) = \lambda(\boldsymbol{X}). \tag{12}$$

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- With endogenous states:

$$F(\boldsymbol{Y}_t) = \lambda(\boldsymbol{X}, \boldsymbol{Y}_{t-1}). \tag{13}$$

- To ensure uniqueness and existence, we use backward iterative algorithm.
- Goal: find a path from $t = 0$ to some terminal T along which the mapping is invertible.

Sketch of methodology and algorithm

Inertial gov. spending

- So far, we have considered "unit-root fiscal rules".
- We extend analysis to inertial rules $\implies$ endogenous state variables:

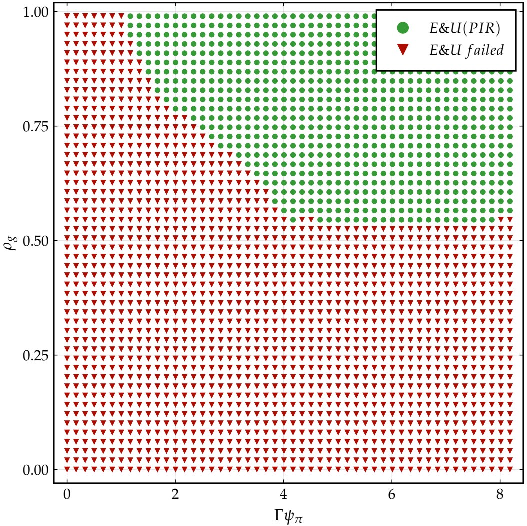
$$\hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \frac{c}{\sigma}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) - \Gamma \Delta \hat{g}_{t+1} + \varepsilon_t, \tag{14}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{x}_t, \tag{15}$$

$$\hat{i}_t = \max\{-\mu; \phi_\pi \hat{\pi}_t\}, \tag{16}$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t, \tag{17}$$

Uniqueness with Inertial Rule



Conclusion

- Ricardian fiscal policy can restore uniqueness in presence of ELB.
- FP needs to satisfy two properties: (i) sufficiently countercyclical, (ii) persistent.
- We develop operational algorithm to check uniqueness conditions with an endogenous state.

Appendix

Sketch of Methodology (Forward Looking) I

$$\underbrace{\overbrace{\overbrace{\mathbf{0} = \boldsymbol{A}_{s_t}\boldsymbol{Y}_t + \boldsymbol{C}_{s_t}\boldsymbol{X}_t}^{\text{Gourieroux, Laffont, and Monfort (1980)}} \quad + \boldsymbol{B}_{s_t}\boldsymbol{Y}_{t+1|t} + \boldsymbol{D}_{s_t}\boldsymbol{X}_{t+1|t}}^{\text{Ascari and Mavroeidis (2022)}} + \boldsymbol{H}_{s_t}\boldsymbol{Y}_{t-1}}_{\text{This paper}}, \tag{18a}$$

$$s_t = \mathbb{1}\left\{\boldsymbol{a}^\top \boldsymbol{Y}_t + \boldsymbol{c}^\top \boldsymbol{X}_t + \boldsymbol{b}^\top \boldsymbol{Y}_{t+1|t} + \boldsymbol{d}^\top \boldsymbol{X}_{t+1|t} + \boldsymbol{h}^\top \boldsymbol{Y}_{t-1} > 0\right\}. \tag{18b}$$

- Our contribution: Extend (18) with endogenous states using GLM Theorem.

Sketch of Methodology (Forward Looking) II

Theorem (GLM Theorem 1)

A system of piecewise-linear equations satisfies coherency and completeness conditions if its matrix form defined in (18), represented as $F(\mathbf{Y}) = \lambda(\mathbf{X})$, *where* $F(\mathbf{Y}) = \sum_J \mathcal{A}_J \mathbf{1}_{\mathfrak{C}_J} vec(\mathbf{Y})$, *is invertible; which requires that all determinants* $\mathcal{A}_J, J \subset \{1, ..., k\}$ *share the same sign.*

- Assume that shocks are two-state stationary Markovian

$$\boldsymbol{K} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix} \tag{19}$$

- Along an MSV solution, we have

$$\mathbb{E}_t[\boldsymbol{Y}_{t+1} | \boldsymbol{Y}_t = \mathbf{Y}\boldsymbol{e}_i] = \mathbb{E}_t[\boldsymbol{Y}_{t+1} | \boldsymbol{X}_t = \mathbf{X}\boldsymbol{e}_i] = \mathbf{Y}\mathbf{K}^\top \boldsymbol{e}_i \tag{20}$$

Sketch of Methodology (Forward Looking) III

- This allows to rewrite the general form (with no endogenous states, i.e. $\boldsymbol{h} = \mathbf{0}$) as

$$\mathbf{0} = (\boldsymbol{A}_{s_i}\mathbf{Y} + \boldsymbol{B}_{s_i}\mathbf{Y}\boldsymbol{K}^\top + \boldsymbol{C}_{s_i}\mathbf{X} + \boldsymbol{D}_{s_i}\mathbf{X}\boldsymbol{K}^\top)\boldsymbol{e}_i,$$
$$s_i = \mathbb{1}\{(\boldsymbol{a}^\top\mathbf{Y} + \boldsymbol{b}^\top\mathbf{Y}\boldsymbol{K}^\top + \boldsymbol{c}^\top\mathbf{X} + \boldsymbol{d}^\top\mathbf{X}\boldsymbol{K}^\top)\boldsymbol{e}_i > 0\},\ i = 1, \ldots, k. \tag{21}$$

- This can be rewritten as

$$F(\mathbf{Y}) = \sum_J \mathcal{A}_J \mathbf{1}_{\mathcal{C}_J} \mathrm{vec}(\mathbf{Y}), \tag{22}$$

  where $\mathcal{C}_J = \{\mathbf{Y} : \mathbf{Y} \in \mathbb{R}^{n\times k}, s_i = \mathbf{1}_{\{i\in J\}}\}$
- (22) must be invertible for the system to have a unique solution.

Sketch of Methodology (Forward Looking) IV

- This is the case if determinants of $\mathcal{A}_J$ have the same sign:

$$\begin{aligned} \mathcal{A}_{J_1} &= \boldsymbol{A}_1 \boldsymbol{I}_2 + \boldsymbol{B}_1 \boldsymbol{K}, \quad J_1 = \{1, 2\}, \\ \mathcal{A}_{J_2} &= \boldsymbol{e}_1 \boldsymbol{e}_1^\top \mathcal{A}_{J_4} + \boldsymbol{e}_2 \boldsymbol{e}_2^\top \mathcal{A}_{J_1}, \quad J_2 = \{2\}, \\ \mathcal{A}_{J_3} &= \boldsymbol{e}_2 \boldsymbol{e}_2^\top \mathcal{A}_{J_4} + \boldsymbol{e}_1 \boldsymbol{e}_1^\top \mathcal{A}_{J_1}, \quad J_3 = \{1\}, \\ \mathcal{A}_{J_4} &= \boldsymbol{A}_0 \boldsymbol{I}_2 + \boldsymbol{B}_0 \boldsymbol{K}, \quad J_4 = \emptyset. \end{aligned} \tag{23}$$

Back

Methodology with an Endogenous State I

- Consider the simple case of a single endogenous state. For a date T whereby $t \geq T$, an MSV solution $f(y_{t-1}, \boldsymbol{X}_t)$ can be written as

$$\mathbf{Y}_t = \boldsymbol{G}y_{t-1} + \boldsymbol{Z},$$

- Again, assume $k = 2$
- General form:

$$\begin{aligned} \mathbf{0} = &\left(\boldsymbol{A}_{s_{t,i}}\boldsymbol{G}\boldsymbol{e}_i + \boldsymbol{h}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{G}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{G}\boldsymbol{e}_i\right) y_{t-1} \\ &+ \left(\boldsymbol{A}_{s_{t,i}}\boldsymbol{Z} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{G}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{Z} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{Z}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{t,i}}\mathbf{X} + \boldsymbol{D}_{s_{t,i}}\mathbf{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i, \end{aligned} \tag{24}$$

for all $i = 1, ..., k$.

Methodology with an Endogenous State II

- For a given regime J corresponding to the k states and their transitions, a slackness condition for the constraint $s_{t,i}$ is determined which gives a system of $2nk$ polynomial equations in the $2nk$ unknowns $\boldsymbol{G}$ and $\boldsymbol{Z}$ by equating the coefficients on y_{t-1} and the constant terms to zero, respectively.
- As these conditions are polynomial and not piecewise linear in $\boldsymbol{G}$ and $\boldsymbol{Z}$, the algorithm and theorem of Gourieroux, Laffont, and Monfort (1980) is no longer suitable to check coherency.
- Build a "brute force" algorithm which essentially goes through all possible 2^k J regime configurations to check if there are any feasible solutions that satisfy the inequality constraints.

Methodology with an Endogenous State III

- We know that at T, the solution to the model takes the following form

$$\boldsymbol{Y}_T = \boldsymbol{G}_{J_0} y_{T-1} + \boldsymbol{Z}_{J_0},$$

where $J_0 \in J$ defines the configuration of regimes in T. $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$ can be solved for from (24):

$$0 = \boldsymbol{A}_{s_{t,i}} \boldsymbol{G} \boldsymbol{e}_i + \boldsymbol{h}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}} \boldsymbol{G} \boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{G} \boldsymbol{e}_i, \tag{25}$$

$$0 = \left(\boldsymbol{A}_{s_{t,1}} \boldsymbol{Z} + \boldsymbol{B}_{s_{t,i}} \boldsymbol{G} \boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{Z} + \boldsymbol{B}_{s_{t,i}} \boldsymbol{Z} \boldsymbol{K}^\top + \boldsymbol{C}_{s_{t,i}} \boldsymbol{X} + \boldsymbol{D}_{s_{t,i}} \boldsymbol{X} \boldsymbol{K}^\top \right) \boldsymbol{e}_i, \tag{26}$$

$\forall i = 1, \ldots, k$.

Methodology with an Endogenous State IV

- $\boldsymbol{Y}_T$ is a function of $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$. Thus, $\boldsymbol{Y}_T$ is known and we can solve for $\boldsymbol{Y}_{T-1}$ from

$$\boldsymbol{0} = \left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{Y}_{T-1}\boldsymbol{e}_i\right) + \left(\boldsymbol{B}_{s_{T-1},i}\boldsymbol{Z}_{J_0}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{T-1},i}\boldsymbol{X} + \boldsymbol{D}_{s_{T-1},i}\boldsymbol{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i + \boldsymbol{h}_{s_{T-1},i}y_{T-2}.$$

- For every t the determinants relevant for CC conditions are given by

$$|\mathcal{A}_{J_0 J_1}| = \prod_i^k \det\left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right).$$

Methodology with an Endogenous State V

- If $k = 2$, the determinants can be rewritten as

$$\begin{aligned}
|\mathcal{A}_{J_0\{1,2\}}| &= \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right), \\
|\mathcal{A}_{J_0\{2\}}| &= \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right), \\
|\mathcal{A}_{J_0\{1\}}| &= \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_1 + \boldsymbol{B}_1 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right), \\
|\mathcal{A}_{J_0\{\emptyset\}}| &= \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_1 \boldsymbol{g}^\top\right) \det\left(\boldsymbol{A}_0 + \boldsymbol{B}_0 \boldsymbol{G}_{J_0} \boldsymbol{K}^\top \boldsymbol{e}_2 \boldsymbol{g}^\top\right).
\end{aligned}$$

Methodology with an Endogenous State VI

- If the model satisfies CC, the solution is given by

$$\boldsymbol{Y}_{T-1}\boldsymbol{e}_i = -\left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right)^{-1}$$
$$\left[\left(\boldsymbol{B}_{s_{T-1},i}\boldsymbol{Z}_{J_0}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{T-1},i}\boldsymbol{X} + \boldsymbol{D}_{s_{T-1},i}\boldsymbol{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i + \boldsymbol{h}_{s_{T-1},i}y_{T-2}\right],$$

$\forall i = 1, \ldots, k.$

- Iterating the solution backwards implies that all the determinants $\mathcal{A}_{J_0,\ldots,J_{T-t}}$ must have the same sign. The recursive solution will be given by

$$\boldsymbol{Y}_t = \boldsymbol{G}_{J_0,\ldots,J_{T-t}}y_{t-1} + \boldsymbol{Z}_{J_0,\ldots,J_{T-t}},$$

Methodology with an Endogenous State VII

where $\boldsymbol{G}_{J_0,\ldots,J_{T-t}}$ and $\boldsymbol{Z}_{J_0\ldots,J_{T-t}}$ can be computed recursively using

$$\boldsymbol{Z}_{J_0\ldots,J_{T-t},i} = -\left(\boldsymbol{A}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{G}_{J_0,\ldots,J_{T-t-1}}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right)^{-1} \left(\boldsymbol{B}_{s_{t,i}}\boldsymbol{Z}_{J_0,\ldots,J_{T-t-1}}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{t,i}}\mathbf{X} + \boldsymbol{D}_{s_{t,i}}\mathbf{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i, \tag{27}$$

$$\boldsymbol{G}_{J_0,\ldots,J_{T-t},i} = -\left(\boldsymbol{A}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}}\boldsymbol{G}_{J_0,\ldots,J_{T-t-1}}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top\right)^{-1} \boldsymbol{h}_{s_{t,i}}. \tag{28}$$

- The recursive solution from terminal T solves the model backwards to $t = 1$ and implies up to $2^{(T-1)k}$ solution paths.
- Given some initial condition, y_0, and conditional on satisfaction of CC conditions, the recursive solution is unique. If the CC conditions are not satisfied, there can be either no or multiple solutions. Back

Inertial NK-FP Model I

- The MSV solution is given by

$$\boldsymbol{Y}_T = \boldsymbol{G}_{J_0} y_{T-1} + \boldsymbol{Z}_{J_0},$$

- where $J_0 \in J$ defines the configuration of regimes in T
- $\boldsymbol{G}_{J_0}$ and $\boldsymbol{Z}_{J_0}$ can be solved for from

$$\mathbf{0} = \boldsymbol{A}_{s_{t,i}} \boldsymbol{G} \boldsymbol{e}_i + \boldsymbol{h}_{s_{t,i}} + \boldsymbol{B}_{s_{t,i}} \boldsymbol{G} \boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{G} \boldsymbol{e}_i, \tag{29}$$

$$\mathbf{0} = \left(\boldsymbol{A}_{s_{t,1}} \boldsymbol{Z} + \boldsymbol{B}_{s_{t,i}} \boldsymbol{G} \boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{Z} + \boldsymbol{B}_{s_{t,i}} \boldsymbol{Z} \boldsymbol{K}^\top + \boldsymbol{C}_{s_{t,i}} \boldsymbol{X} + \boldsymbol{D}_{s_{t,i}} \boldsymbol{X} \boldsymbol{K}^\top \right) \boldsymbol{e}_i, \tag{30}$$

$\forall i = 1, \ldots, k.$

Inertial NK-FP Model II

- This yields $\boldsymbol{Y}_T$, we then can solve for $\boldsymbol{Y}_{T-1}$

$$\begin{aligned}\mathbf{0} = &\left(\boldsymbol{A}_{s_{T-1},i} + \boldsymbol{B}_{s_{T-1},i}\boldsymbol{G}_{J_0}\boldsymbol{K}^\top \boldsymbol{e}_i \boldsymbol{g}^\top \boldsymbol{Y}_{T-1}\boldsymbol{e}_i\right) \\ &+ \left(\boldsymbol{B}_{s_{T-1},i}\boldsymbol{Z}_{J_0}\boldsymbol{K}^\top + \boldsymbol{C}_{s_{T-1},i}\boldsymbol{X} + \boldsymbol{D}_{s_{T-1},i}\boldsymbol{X}\boldsymbol{K}^\top\right)\boldsymbol{e}_i + \boldsymbol{h}_{s_{T-1},i} y_{T-2}.\end{aligned} \tag{31}$$

- NK-IFP model:

$$\hat{x}_t = \mathbb{E}_t \hat{x}_{t+1} - \frac{c}{\sigma}\left(\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^n\right), \tag{32a}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa_y \hat{x}_t - \kappa_g \widehat{GR}_t, \tag{32b}$$

$$\hat{i}_t = \max\{-\mu; \phi_\pi \hat{\pi}_t + \phi_y \hat{x}_t\}, \tag{32c}$$

$$\widehat{GR}_t = \rho_\tau \widehat{GR}_{t-1} + \psi_\pi \hat{\pi}_t + \psi_y \hat{x}_t, \tag{32d}$$

Inertial NK-FP Model III

with

$$\hat{r}_t^n = -\frac{\sigma}{c}(g\mathbb{E}_t\Delta\widehat{GR}_{t+1} + \varepsilon_t).$$

- Can be evaluated about two absorbing states, either PIR or ZIR.
  - PIR: $\{\hat{x}, \hat{\pi}, \hat{i}, \widehat{GR}\} = \{0, 0, 0, 0\}$
  - ZIR $\implies \hat{i} = -\mu$:

$$\{\hat{x}, \hat{\pi}, \hat{i}, \widehat{GR}\} = \left\{-\frac{(1-\beta)(1-\rho_\tau) + \kappa_g\psi_\pi}{\kappa_y(1-\rho_\tau) - \kappa_g\psi_y}\mu, -\mu, -\mu, \frac{\psi_y\hat{x} + \psi_\pi\hat{\pi}}{1-\rho_\tau}\right\}$$

- Under certain fiscal policy rules, the above ZIR equilibrium is not consistent with the constraint on the TR and, thus, the ZIR equilibrium is ruled out.

Inertial NK-FP Model IV

- Specifically, we require that $\hat{x}$ be sufficiently large such that the ZLB constraint on $\hat{i}_t$ is not binding:

$$-\mu < -\phi_\pi \mu + \phi_y \hat{x} \implies -\frac{\mu(1-\phi_\pi)}{\phi_y} < \hat{x}, \quad \phi_y \neq 0$$

- This implies

$$\frac{1-\phi_\pi}{\phi_y} > \frac{(1-\beta)(1-\rho_\tau)+\kappa_g \psi_\pi}{\kappa_y(1-\rho_\tau)-\kappa_g \psi_y},$$

with the LHS being negative under conventional restrictions on TR coefficients.

Inertial NK-FP Model V

- As ρ_τ tends to unity, we get

$$\lim_{\rho_\tau \to 1} \frac{(1-\beta)(1-\rho_\tau) + \kappa_g \psi_\pi}{\kappa_y(1-\rho_\tau) - \kappa_g \psi_y} = -\frac{\psi_\pi}{\psi_y} < \frac{1-\phi_\pi}{\phi_y},$$

  which holds under countercyclical fiscal policy, $\psi_y < 0$, $\psi_\pi < 0$ with ψ_π sufficiently large in absolute value.
- Thus, under countercyclical fiscal policy, if ρ_τ is sufficiently large there cannot exist a ZIR absorbing state.

Back