# The Deadweight Loss of Short-Time Work *

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updated version

**Abstract**

Since the Great Recession, OECD countries have expanded short-time work schemes—publicly subsidized reductions in working hours—to stabilize employment over the business cycle. These expansions have involved lowering firms' costs to participate. Using new French data and local projection methods, I show that past cost reductions for firms decreased total hours worked by registered employees, while having no effect on unemployment. I develop a dynamic labor market model that captures the trade-off inherent in short-time work policies: lower participation costs incentivize firms to retain jobs, but also encourage reductions in working hours. The model identifies a strictly positive efficiency threshold for firm's participation costs, below which the fiscal cost of the program exceeds the benefits of job preservation. Since most OECD programs set participation costs at zero, they generate *per se* a deadweight loss. Calibrating the model with French data, I show that participation costs in 2023 and during the Covid-19 crisis, while above zero, remained below the efficient level. In steady state, a 50% increase in firm participation costs would raise social welfare by 25% and nearly eliminate the program's fiscal deficit—without significantly affecting employment.

**JEL Classification**: E24; H21; J22; J24; J65;

**Keywords**: Short-Time Work; Deadweight loss; Optimal Policy;

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# 1 Introduction

Following the success of Germany's Short-Time Work (STW) program during the Great Recession (Rinne and Zimmermann, 2012), STW has become a cornerstone of labor market policy in OECD countries. Governments adjust the program's incentives in response to economic conditions to help mitigate labor market fluctuations. Despite its widespread adoption, the role of monetary incentives in shaping the program's use and effectiveness remains underexplored in the literature. This paper aims to fill that gap by analyzing how financial incentives influence firms' participation in STW and their impact on employment, working hours, and public expenditure.

STW is a job retention scheme designed for firms facing temporary business difficulties. It allows firms to reduce working hours and wages while compensating employees for lost income through a combination of employer and government contributions. The portion of compensation paid by firms serves as the primary monetary incentive influencing their decision to use the program.

This paper demonstrates that changes in firms' contributions (the short-time cost) affect both extensive use (the number of workers enrolled in STW) and intensive use (the number of hours worked by those enrolled). While extensive use primarily determines the number of jobs preserved, intensive use impacts public spending and the overall output of short-time workers. I identify a threshold below which reducing the firm's contribution primarily affects intensive rather than extensive use. I estimate that most OECD countries set their ST cost below this threshold, leading to deadweight loss. This paper contributes to the literature by quantifying the trade-off between job preservation and hours worked, demonstrating the inefficiencies of current STW designs, and proposing a policy to minimize deadweight loss.

To quantify this inefficiency, the paper follows a three-step approach. First, I estimate the labor market effects of STW policy changes using a local projection method. Second, I develop a dynamic labor market model incorporating hours worked to derive the optimal STW design and predict the consequences of suboptimal policies. Third, I calibrate the model using empirical data to measure the deviation from optimal policy.

I construct a novel dataset that tracks all changes in monetary incentives for STW in France from 2008 to 2024 using legal records. France's STW program, one of the oldest and least generous, has undergone multiple reforms, making it an ideal case study for assessing the impact of varying levels of generosity. Merging this dataset with quarterly data on employment, hours worked, public spending, and STW participation, I apply a local projection method to estimate policy effects. The results indicate that reductions in firms' ST costs increase program participation by 0.04 percentage points, reduce hours worked per enrolled worker by 30 hours per quarter, and raise public expenditure by 2

I model a dynamic labor market *à la* Diamond-Mortensen-Pissarides with hours worked and endogenous layoffs triggered by idiosyncratic and random productivity shocks. In this model, wage rigidities lead to excessive layoffs and low hours for ST workers. A welfare-maximizing policymaker can intervene using three instruments: labor taxes, ST costs paid by firms, and ST compensation received by workers. The optimal policy solves the trade-off between increasing hours worked (intensive incentive) and preserving jobs at risk (extensive incentive) thanks to a combination of labor tax and ST cost program that are two functions of the productivity of the worker-firm match.

However, OECD countries set a uniform ST cost across all worker-firm pairs, creating inefficiencies. A single ST cost fails to differentiate incentives based on productivity levels, forcing policymakers to choose between increasing hours worked and saving jobs. I derive a second-best policy in which the ST cost is set at a level where the marginal social benefit of increasing participation equals the marginal social cost of reducing hours worked. In this scenario, the optimal ST cost is always strictly positive. In contrast, most OECD countries set the ST cost at zero (Figure B.1.2), leading to excess deadweight loss as fewer hours are worked without a corresponding gain in employment. These theoretical predictions align with the empirical findings from local projections.

Finally, I calibrate the model using French data from 2023 and the Covid-19 recession. I estimate the optimal ST cost for both expansionary and recessionary periods and compare it to actual policy. My results suggest that the short-time cost in France has consistently been set too low. In expansionary periods, the optimal cost should be around 60% of wages, whereas after the Covid-19 crisis, it was set at 32%. During a 15% productivity decline, similar to Covid-19, the optimal cost should be 30%, yet France reduced it to zero. In both cases, underpricing the program led to suboptimal employment gains, reduced hours worked, and increased fiscal costs. Given that France historically set a ST cost above the OECD median, these inefficiencies are likely even more pronounced in other countries.

**Related literature**. This paper is the first to examine how firms' monetary contributions (ST costs) affect the use of STW and the labor market. The STW literature has primarily evaluated the effect of access to STW on employment by comparing countries with and without STW programs (Boeri and Bruecker, 2011; Brey and Hertweck, 2020; Cahuc and Carcillo, 2011; Hijzen and Venn, 2011; Hijzen and Martin, 2013), or by exploiting firm-level variation in access within a given country (Calavrezo et al., 2009; Kruppe and Scholz, 2014; Tracey and Polachek, 2020; Kopp and Siegenthaler, 2021). While these studies provide valuable insights into the employment effects of STW, they do not explore how variations in program design influence firms' participation decisions. By analyzing changes in ST costs, this paper extends the literature by proposing new macroeconomic evidence on the impact of program generosity on STW use and on the labor market. In this sense, it complements the recent paper of Brinkmann et al. (2024), which studies the micro-level effects of STW duration extensions on wage and employment trajectories. They also find that past STW extensions had no effect on employment.

By focusing on the role of ST costs, this paper tackle the issue of the optimal design of STW programs. Prior research has largely concentrated on worker compensation—examining how different levels of benefits affect workers' utility (Tilly and Niedermayer, 2016; Giupponi et al., 2022). However, worker compensation does not directly influence firms' decisions to participate in STW. This oversight may be due to the fact that in many european programs, for instance in Germany, the ST cost was effectively zero. The optimal design issue has been approached by Teichgräber et al. (2022). They model a setting where a social planner optimally allocates working hours under asymmetric information. However, this framework does not explicitly account for firms' monetary incentives, nor does it allow for empirical evaluation of current policies against the optimal benchmark. In contrast, this paper develops a dynamic labor market model that explicitly incorporates hours worked, productivity heterogeneity, ST cost and compensation, allowing for both theoretical and empirical assessment of existing STW programs.

This paper also introduces a new measure of cost-effectiveness for STW programs by comparing observed policy choices with theoretical second-best optimal policies. Previous studies have assessed the cost-effectiveness of job retention programs by comparing the fiscal cost of STW with that of unemployment insurance (Borowczyk-Martins and Lalé, 2016; Giupponi and Landais, 2023). In the paper, I show that as long as the compensation received by the worker is lower than or equal to the unemployment benefit, STW is *per se* less costly for the government. I propose to compare the observed policy with the second-best benchmark policy to measure the inefficiency of high public spending on STW.

Finally, this paper relates to the broader literature on the adverse selection in STW programs. Several studies have shown that firms may overuse STW even when jobs are not at risk, leading to inefficient public spending. For instance, Cahuc et al. (2021) provides evidence that a large proportion of French firms entered the STW program during the Great Recession despite facing little employment risk. Similarly, Albertini et al. (2022) finds that during COVID-19, firms overconsumed subsidized hours. Balleer et al. (2016) further shows that loosening STW eligibility criteria in Germany increased the number of enrolled workers but had no effect on employment, suggesting that much of the additional take-up was unnecessary. This paper builds on these insights by demonstrating that raising ST costs can mitigate adverse selection by discouraging firms with minimal productivity losses from enrolling, reducing deadweight loss without harming employment. For high-productivity firms, an increase in ST cost acts as a *de facto* tightening of program eligibility, mirroring the effects modeled by Balleer et al. (2016).

The paper is organised as follows. Section 2 introduces motivating evidence. Section 3 present a simple model. Section 4 and 5 present the theoretical predictions. Section 6 shows the simulation results.

# 2 Short-Time Work Facts

In this section, I document STW programs and policies in Europe and then focus on the French case to estimate the effect of change in the programs on macroeconomic aggregates. I compile a new narrative dataset and run a local projection to present three main stylized facts: 1- Policymakers lower the ST cost during recessions; 2- There is an increase in public spending on STW; 3- Lowering the ST cost reduces the number of hours worked per worker, increases the number of workers on STW, increases public spending of the program and has no significant effect on employment.

## 2.1 Legal Background

STW is a labor market policy that allows employers to temporarily reduce the working hours of their employees during periods of economic downturn or temporary business disruptions. The aim is to avoid layoffs by temporarily reducing working hours rather than cutting jobs. STW programs generally include the following key elements. First, eligibility: employers must meet certain criteria to be eligible for the programs, such as experiencing a significant decline in business activity through no fault of their own. In order to access the STW program, employers must prove to a public authority that they meet the eligibility criteria. Second, reduced working hours: if access to the STW program is granted, employers can reduce the regular working hours of their employees and pay them only on the basis of this reduced number of working hours. Third, wage compensation, workers receive partial wage compensation for the hours lost due to reduced working hours (compensation co-financed by the government and employers). Fourth, duration: STW is limited to a specific period, such as several weeks or months. Fifth, employer obligations: employers participating in the program may have certain obligations, such as retaining employees for a certain period of time.

STW policies work through four channels. First, they change eligibility criteria. In 2008, for example, Spanish public policymakers abolished administrative approval for reductions in the working week for economic reasons of between 10% and 70%. Balleer et al. (2016) find that the change in eligibility criteria in Germany during the Great Recession has mostly had a windfull effect on unemployment. Second, they affect the maximum use and duration. In Germany, during the Covid-19 crisis, the maximum duration was extended from 12 to 21 months. Third, ST compensation is being increased. In Portugal, during the Covid-19 crisis, the level of ST compensation was raised from 60 to 92% of the gross hourly wage for some workers. Finally, policies usually reduce the ST cost paid by firms in order to increase the benefits of STW use for firms. In 2009 in France, the cost of STW was reduced to 0 for the majority of workers, firms were only forced to pay the ST cost for high wages. Those changes in ST compensation and ST cost are the only monetary incentives to use the program.

These four channels of STW policy aim to make the STW program more attractive, all examples given in the previous paragraph are anecdote evidence but curious readers can consult ETUC (2020) reports for an exhaustive description of policies during the Covid-19. It worked: in most European countries, the use of STW reached unprecedented levels during the Great Recession and the Covid-19 (Figure B.1.2). In Germany, for example, ST workers saw their working hours halved on average during the Great Recession. The huge increase in the use of ST compensation, as well as the difference between the cost of ST compensation and the level of ST compensation, comes at a significant public cost. Between 2008 and 2009 in France, public expenditure on ST compensation rose from less than 20 million euros to almost 300 million euros. However, as in most European countries, access to STW programs became more restricted after the great recession. The same pattern was observed during the Covid-19 with a much stronger magnitude.

STW policies then restrict access and incentives to use the program after the recession. In Figure B.1.1 we see that after Covid-19 the ST costs for companies increased in most countries. Nevertheless, there is a general trend for the program to become more generous and more used over time. Over 10 years, between 2007 and 2017, the number of ST workers in Europe increased by 61% and the amount spent by policymakers on each ST worker increased by 46% (table A.1)[1].

## 2.2 Database Construction

In this section, I estimate the effect of change in monetary incentives to use STW on the French labor market. Focusing on one country allows me to clear the heterogeneity issues due to differences between programs, institutionnal background and economic trends in cross country analysis. The French program is one of the oldest and has been modified several times so it provides sufficient variations to estimate the effect of change in ST cost and compensation on the labor market. In addition, France has followed the common European trends in STW program and policies so the observation made on France is likely to be valid externally.

This paper presents a new quaterly narrative database on major shocks on the french STW program over

[1]Computations were made for four countries: Germany, France, Italy and Spain.

2008-2024. The dataset is built in two steps. First, I record all legislative actions mentioned in the official website of the French government for the publication of legislation, regulations, and legal information[2]. The legislation website record all the change in the program including change in ST cost, ST compensation, and procedure to apply to the program. In a second step, I create two binary variables registering for an increase (+1) or a decrease (-1) in ST cost paid by firms and in ST compensation received by workers.

I count 15 reforms between 2008 and 2024 (Table A.2). In most cases, reforms of ST compensation and ST costs happened in the same quarter. The direction of the reforms is not straightforward due to the complexity of the French program. In Figure B.1.3, I plot the ST cost paid by the firm according to the wage of the ST worker in 2009. The cost paid by firms is non-linear and depends on the size of the firm, the type of STW program in which it is enrolled and the wage of the worker. In addition, before 2021 the cost of STW was a fixed cost and then changed to a proportion of the worker's remuneration. In Figure 2, I plot the cost paid by the firm and the compensation received by the worker for an hour of STW consumed when the firm has less than 250 employees and the worker has the average wage (in 2023, 17 euros per hour). Between 2009 and 2022, all but one of the reforms have increased the generosity of the program; after the Covid-19 recession, the share of compensation paid by the firm has increased. Reforms do not necessarily affect all worker-firm pairs. In my narrative database, the binary variable takes a value of +1 (-1) if the generosity of the scheme has increased (decreased) for at least one worker-firm match.

I merge the narrative database with French quarterly data on STW consumption. This dataset is produced by the Statistical Department of the French Labor Ministry (*DARES*) and is on public access. It includes the number of workers on short-time work, the number of hours consumed, the number of firms in STW, the number of STW demand, and the amount of public expenditure between 2008 and 2024. Finally, I include quarterly GDP growth from OECD.

French STW consumption follows the trend observed for European countries, it is strongly countercyclical and has increased over time (Figure 1). French data allows me to observe the extensive and intensive use of the program. The extensive use is measured as the share of ST workers in the labor force, the intensive use is measured as the number of hours not worked per workers. Both measures increased during recessions and over-time. The public expenditure per ST workers follows a similar pattern. In Figure 2, I plot the evolution of ST compensation and ST cost through years. The gap between the two captures the average public expenditure for each hour of STW consumed. This gap increased during recessions and tends to increase overtime.

[2] https://www.legifrance.gouv.fr

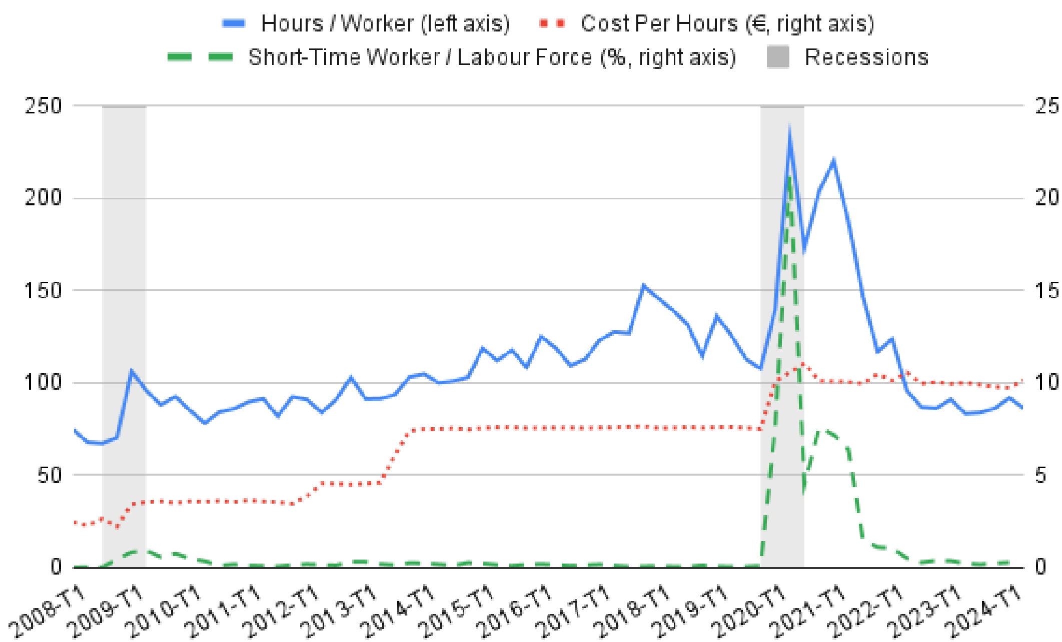


Figure 1: STW Consumption, Public Expenditure and GDP growth in France, 2008-2024

Sources: DARES, OECD. The graph plots the public STW expenditure per hour, the number of hours of STW consumed per worker, and the share of short-time worker in the labor force. Grey areas correspond to recessions.

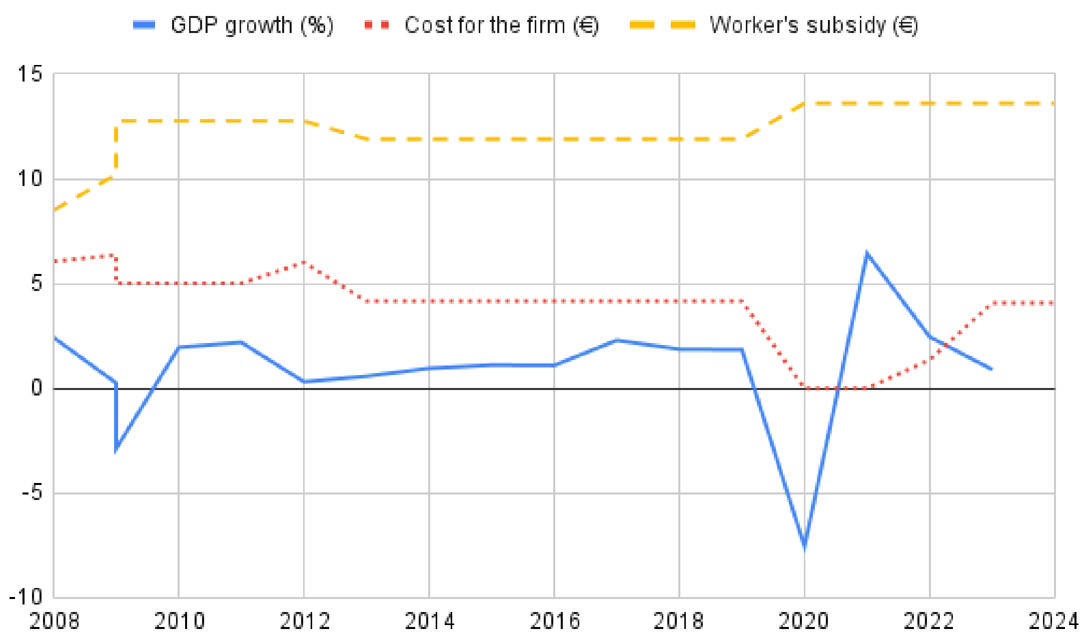


Figure 2: STW cost, consumption and GDP growth in France, 2008-2023

Sources: LegiFrance, INSEE. Own computations. The graph plots the STW cost of adding one hour of STW for a firm (red line) and the ST compensation received by the worker (yellow line) for a firm with less than 250 employees and a worker earning the average wage in 2023 (17€ per hour).

## 2.3 Local Projection Evidence

In this section, I estimate the impact of a change in monetary incentives to use STW on the number of hours consumed per workers (intensive use), the share of workers enrolled (extensive use), and the public expenditure per worker. I use the narrative database created to assess the response of my three outcomes to a change in ST costs and ST compensation.

To empirically evaluate the dynamic effects of STW reforms, I rely on the local projection method of Jordà (2005) to estimate impulse response functions. The baseline specification is:

$$y_{t+k} - y_{t-1} = \tau_k + \beta_k R_t + \theta_k X_t + \varepsilon_{t+k} \tag{1}$$

in which $y$ is the dependent macroeconomic variable of interest; $\beta_k$ denotes the (cumulative) response of the variable of interest $k$ years after the shock; $\tau$ is a time fixed effects included to take account for global shocks; $R$ denotes the reform shock; and $X$ is a vector of control variables including two lags of reform shocks, two lags of real GDP growth and two lags of the relevant dependent variable. The equation is estimated using OLS. Impulse response functions (IRFs) are then obtained by plotting the estimated $\beta_k$ for k = 0, 1, ...6 with 90 percent confidence bands computed using $\beta_k$—based on robust standard errors.

Local-projection have been advocated by Auerbach and Gorodnichenko (2012) and Romer and Romer (2019) as a flexible alternative to VAR, better suited to estimating a dynamic response such as interactions between shocks and macroeconomic conditions. I also explore whether initial economic conditions at the time of the shock influence its effect on macroeconomic outcomes (equation B.1.1). As discussed in Auerbach and Gorodnichenko (2012), in this setting the local projection approach to estimating non-linear effects is equivalent to the smooth transition autoregressive (STAR) model developed by Granger (1993).

I find that a decrease in the cost paid by firms was followed by a higher enrollment in the program, a decrease in the number of hours worked by ST worker, and an increase in public expenditure. However, it is not followed by a change in unemployment. Figure 3 shows the quarterly responses of unemployment, the share of short-time workers in the labor force, the number of short-time hours consumed per short-time worker and the log of public expenditure. As expected, a reduction in the cost paid by firms to use the program is followed by an increase in take-up, which is observed at the extensive margins - more workers entered the program - and at the intensive margins - more hours were used. The higher take-up automatically leads to higher public expenditure. This shows that STW reforms aimed at making the programs more attractive to firms are having the expected effect. The reduction in hours worked by STW workers is stable, while the increase in enrolment disappears after two periods.

Past STW policies didn't have an effect on the unemployment rate. The literature generally finds that STW preserves employment, although the effect is less pronounced during expansions (Boeri and Bruecker, 2011). Here, the insignificant coefficient can either mean that the workers who entered the program were not in threatened jobs (windfall effect) or that the increase in jobs saved was too small to have a significant impact on the unemployment rate.

STW reforms have the opposite effect of output shock (Figure B.1.4). A positive shock on GDP growth decreases the number of short-time worker, the hours per short-time worker consummed and the ammount of public expenditure of the program. As intended, STW is a counter-cyclical programs and is more used during recessions periods even without STW reforms. I find that the effect of STW reforms on the labor market is only observed during recession periods (Figure B.1.7). Finally, I test to find different results if I limit the variable to change in ST cost or ST compensation only. The coefficients captured by the local projection does not change. As STW cost and ST compensation changes arise simulteneaously in most cases, I do not have sufficient variations to distinguish the effect of the two variables.

To run magnitude analysis, I test to include the cost per hour for the State as a proxy for STW reforms. Reforms in the narative database capture the change in cost of using the programms for firms, benefits of entering the program for the worker, and cost per hour consummed for the State. This ultimate variable can be observed. I divide the total amount of public expenditure by the number of hours consumed to measure the average cost per hour consummed for the state. I then re-run the local-projection estimation with this variable instead of the reform variable. I find similar results but obviously with a difference in magnitude (Figure B.1.5). The pic of effect of a decrease in ST cost happens after one quarter. An in increase in the generosity of the program by 1 euro leads to a 1 p.p. increase in the share of ST worker in the labor force, an average reduction of 10 hours worked in the quarter, and a 40% increase in public expenditure with no significant effect on unemployment. The coefficient obtained for the share of ST worker and unemployment are of similar size than the one found by Balleer et al. (2016) for Germany. In the first quarter of 2024, they were 89 thousands of workers in STW which represents 0.3% of the labor force, an increase in public spending would then raise the share of STW to 1.3% with new 300 thousands worker enrolled in the program. Workers in the program would work 30 hours less which will increase public expenses by 1.5%. In 2024, the French state spent 80 millions on STW (which equals to 0.2% of the unemployment benefit cost for the state). An increase in 1 euro of the generosity of the program would raise the public expenses by 1.2 millions.

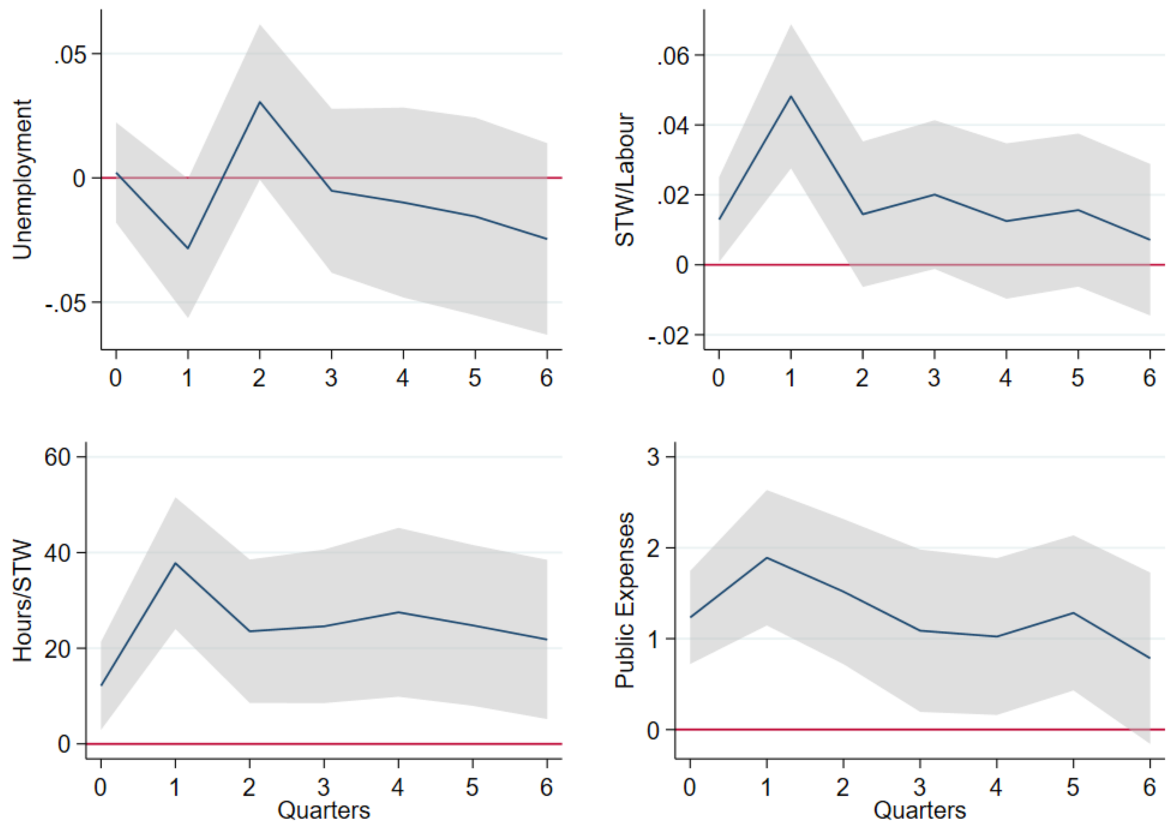


Figure 3: Local-projection: Impulse responses to a reduction in the Short-Time cost paid by firms

Impulse responses to a reduction in short-time cost paid by firms. Local projection estimated with unemployment rate (upper-left graph), log of number of short-time worker (upper-right graph), log hours consummed per workers (bottom right), and log of total public expenditure (bottom left) for 2008Q1 to 2023Q4. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

# 3 Environment

This section presents the main features on which I rely to model the effect of a change in monetary incentives to take-up the STW program. I begin to model a simple economy where a reduction in ST cost simultaneously affect the firing decision and the working hours.

The economy is populated by a continuum of infinitely lived households and a continuum of firms that can match to produce a consumption good. Each firm-worker pair differs according to their productivity $\theta$ drawn from a random distribution $F(\theta)$, the productivity is i.i.d. across workers and times.

## 3.1 Firms

A firm can produce the final consumption goods only if it successfully matchs with a worker. If a firm finds a match, it obtains a flow profit in the current period after paying the worker. The flow profit consists of three elements. First, the production function $\frac{1}{\alpha}(\theta h)^{\alpha}$, a concave increasing function with respect to the worker's productivity $\theta$ and the number of hours worked $h \in [0;1]$. Second, the linear wage $w$ and the labor tax $\tau$ for the hours worked. Third, the cost of STW, i.e. for every hour not worked $1-h$, the firm has to pay a cost $b$. In STW programs, $b$ is always lower than the wage, so that if the firm reduces the number of hours worked, its total labor costs $(\tau+w)h + b(1-h)$ are always lower than if the worker were employed full-time $h = 1$.

The firm stay in the match if its value is bigger than the vacancy value $J^v$ plus the layoff cost $c_f$. In the next period, if the match survives the firm continues ; if the match breaks down, the firm posts a new vacancy with vacancy value $J^v_{+1}$. The match survives if the post does not become obsolete with probability $(1-\rho)$ and if the workers is kept with probability $(1-\phi)$. The future value is discounted by the factor $\sigma$. The firm's match value thus satisfies the Bellman equation:

$$J^f = \frac{1}{\alpha}(\theta h)^{\alpha} - (\tau + w)h - b(1-h) + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})J^f_{+1} + \phi_{+1}J^v_{+1}\right]. \tag{1}$$

Creating new vacancies or posting existing vacancies incurs a per-period fixed cost $\kappa$. If the vacancy is filled (with probability $q^v$), the firm obtains the value of a match $J^f_{+1}$. If the vacancy remains unfilled, then the firm goes into the next period and obtains the continuation value of the vacancy, provided that the vacancy does not be obsolete. Thus, the value of an open vacancy is given by:

$$J^v = -\kappa + \sigma(1-\rho)\mathbb{E}\left[q^v J^f_{+1} + (1-q^v)J^v_{+1}\right]. \tag{2}$$

### 3.2 Workers

Employed workers receive a flow revenue minus a disutility from working in the current period. The revenue from working is equal to the wage for the hours worked $wh$ and a compensation for the reduction in hours worked $a(1-h)$. The worker faces a quadratic disutility for the hours worked $\beta(h)^2$. In the next period, if the match survives (with probability $(1-\rho)(1-\phi_{+1})$), the worker continues; if the match fails, the worker becomes unemployed. The worker's match value thus satisfies the Bellman equation:

$$W = wh + a(1-h) - \beta(h)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right]. \tag{3}$$

When unemployed, the worker receives an unemployment benefit $u_b$. If the worker finds a job (with probability $q^u$), the worker obtains the value of a match $W_{+1}$. Otherwise, the worker goes into the next period and obtains the continuation value of the unemployment. Thus, the value of unemployment is given by

$$U = u_b + \sigma\mathbb{E}\left[q^u W_{+1} + (1-q^u)U_{+1}\right]. \tag{4}$$

A worker leaves the match if his surplus from the match is negative. The productivity threshold $A$ is defined as the employment value at which the worker is indifferent between staying or leaving the match, and implicitly defined by

$$W - U = 0. \tag{5}$$

### 3.3 The Social Planner's Problem

The social planner allocates labor to maximize the surplus generated by each match across the productivity distribution, defined as $S(\theta) = W(\theta) - U + J^f(\theta) - J^v$, where $W(\theta)$ is the value of employment with productivity $\theta$, $U$ is the value of unemployment, $J^f(\theta)$ is the firm's matching value with productivity $\theta$, and $J^v$ is the value of a vacancy.

In each period, the planner decides which job matches to preserve and how many hours $h$ should be worked in each preserved match, the planner's objective is:

$$\max_{R,A,h} \int S(\theta)\, dF(\theta), \tag{6}$$

where $R$ is the productivity threshold below which workers are laid-off, $A$ is the productivity threshold below which workers leave the match, and $h$ denotes hours worked.

The objective can be expressed as maximizing match-level output, $\frac{1}{\alpha}(\theta h)^{\alpha}$, minus the disutility from effort, $\beta h^2$ (see subsection C.1). The continuation value is unaffected by the planner's policy since the ending probability $\phi_{+1}$ is determined in the next period and the the job-finding probability is taken as exogenous. The ending probability $\phi_{+1}$ is determined in the next period vecause the threshold parameters $R$ and $A$ are reset at the beginning of each period.

The social planner's allocation maximizes the current surplus at the match level, ensuring outcomes that are weakly preferred by both workers and firms, regardless of how the surplus is split. However, the planner does not internalize the effect of this allocation on market tightness. As such, while the solution is optimal at the micro level, it may be suboptimal at the macroeconomic level.

I will show that the decentralized *laissez-faire* equilibrium is inefficient even if the Hosios condition is satisfied (Hosios, 1990). That is, inefficiency arises not only from externalities but also from the fact that firms and workers are unable to maximize their own surplus.

The planner's problem can be solved using standard optimal control techniques. The optimal number of hours worked, $h^*$, is defined by a first-order condition. The productivity threshold $R^*$ is determined by the condition that the match surplus $S(\theta)$ equals zero.

# 4 First-Best Policy

The optimal policy is derived in two steps. First, I characterize the optimal allocation of resources chosen by a benevolent social planner. Then, I turn to its implementation in a decentralized economy.

## 4.1 Optimal Allocation

**Lemma 1.** *The optimal allocation chosen by a benevolent social planner is characterized by the following system*

$$h^*(\theta) = \left(\frac{\theta^{\alpha}}{2\beta}\right)^{\frac{1}{2-\alpha}} \tag{7}$$

$$R^* = A^* = \left[\frac{U + J^v - \sigma\mathbb{E}[S_{+1}]}{G(\beta)}\right]^{\frac{2-\alpha}{2\alpha}} \tag{8}$$

with $G(\beta) = \frac{1}{\alpha}(2\beta)^{\frac{-\alpha}{2-\alpha}} - \beta(2\beta)^{\frac{-2}{2-\alpha}}$

PROOF

See appendix C.1 □

The optimal hours worked $h^*(\theta)$ is a function of the ratio between the productivity of the match $\theta^\alpha$ and the marginal disutility of working $2\beta$. The optimal hours worked maximize the output of each match under the disutility constraint.

The optimal ending threshold $R^*$ is a function of the value of staying in the match relative to quitting and the disutility of staying in the match. The value of staying in the match is captured by the difference between the outside option for both the worker and the firm and the continuation value of the match. The disutility of staying in the match is a function of $G(\beta) = \frac{1}{\alpha}(2\beta)^{\frac{-\alpha}{2-\alpha}} - \beta(2\beta)^{\frac{-2}{2-\alpha}}$. The optimal ending threshold ensures that any match that generates a positive surplus is preserved.

Combining equations (7) and (8), I can express the optimal allocation of labor by a function that sets the number of hours worked for each productivity value $h^*(\theta)$. The function is 0 below the ending threshold $\theta \leq R^*$, continuously increasing in the interval $]R^*; (2\beta)^{\frac{1}{\alpha}}[$ and 1 above $(2\beta)^{\frac{1}{\alpha}}$. Then worker-firm pairs should be on STW if their productivity is in the interval $]R^*; (2\beta)^{\frac{1}{\alpha}}[$.

The solution to the planner's problem maximizes the concave production function under a quadratic disutility of labor. The surplus generated can then be fully taxed and distributed as a lump sum to firms, employed and unemployed workers. The dynamic effect of such an allocation through the labor market is not the subject of this paper. In other words, the effect of changing the final threshold on job creation is ignored. Nevertheless, by adding unemployment and labor market dynamics to the social problem, the FOC with unemployment and market tightness as state variables leads to optimality conditions that are exactly identical to those derived in Pissarides (2000, chapter 8) for net output maximization with the hours equation.

## 4.2 Inefficiency & Implementation

Having characterized the optimal allocation, I now turn to its implementation in a decentralized economy. Throughout the paper, I restrict the government to relying exclusively on the following four policy instruments: short-time costs $b$, short-time compensation $a$, labor taxes that can differ for short-time workers $\tau^{st}$ and full-time workers $\tau^f$. I choose to focus on these four because they are the most natural instruments through which the government implements the short-time work program in practice. Moreover, as we will see in this section, they are sufficient to implement the first-best allocation of resources in a benchmark case.

The government has a balanced budget and finances the expenditure on short-time work through labor taxes. The expenditure on short-time work is equal to the difference between the short-time compensation $a$

minus the short-time costs paid by the firm $b$ times the consumption of short-time work $(1-h)$ aggregated for all workers. I allow the policy maker to levy a different labor tax on short-time workers $\tau^{stw}$ and on full-time workers $\tau^{f}$. The budget should be balanced in each period, so that public spending on short-time workers is financed by full-time workers. The inclusion of an intertemporal budget constraint does not affect the derivation of the optimal allocation. With an intertemporal budget, public expenditure on short-time work is financed by current and future full-time workers.

$$\int_{R^{st}}^{1} \tau^{f} dF(\theta) = \int_{R}^{R^{st}} (a-b)(1-h) dF(\theta) - \int_{R}^{R^{st}} \tau^{st} h dF(\theta) \tag{9}$$

where $R^{st} \geq R$ is the exogenous eligibility criterion for STW, which defines the productivity threshold below which a firm-employee pair has access to STW. Above this threshold, the working time is set to the maximum $h = 1$.

In the decentralised economy, four stages of interest can be distinguished.

- Stage 1: New matchs are made, old matchs continue and the employer and employee agree on a wage rate.
- Stage 2: The idiosyncratic productivity is revealed.
- Stage 3: The government chooses the level of the four policy instruments.
- Stage 4: Firms set the hours worked, firms and workers decide to stay or leave the match.

The problem is solved backwardly. Starting at stage 4, The hours worked set by the firm is derived from the FOC of firms' matching value (1) with respect to hours,

$$h(\theta, \ell) = \left(\frac{\theta^{\alpha}}{\ell}\right)^{\frac{1}{1-\alpha}}. \tag{10}$$

The number of hours worked is a ratio between the productivity of the worker $\theta^{\alpha}$ and the cost to the firm of adding one hour of work $\ell = \tau + w - b$. I can now describe the firing decision of the firm, which depends on the working time $h$. Workers are fired if the losses they generate are higher than the firing cost $J^{f} - J^{v} \leq -c_{f}$. Inserting the FOC (equation 10) in the firm value function (1), and setting $J^{f} - J^{v} = -c_{f}$, I derive the productivity reservation:

$$R = \left[\frac{\alpha}{1-\alpha}\big(-c_{f} + b + J^{v} - \sigma(1-\rho)\mathbb{E}[(1-\phi_{+1})J^{f}_{+1} + \phi_{+1}J^{v}_{+1}]\big)\right]^{\frac{1-\alpha}{\alpha}} \ell. \tag{11}$$

The productivity reservation is the productivity level $\theta$ at which the firm is indifferent between firing and retaining a worker on STW. I can now derive the effect of the STW cost on the number of hours worked and the productivity reservation.

**Lemma 2.** *The firing threshold and the hours worked are increasing with respect to the ST cost*

$$\frac{\partial R}{\partial b} \geq 0 \ ; \ \frac{\partial h}{\partial b} \geq 0$$

PROOF

See appendix C.2 □

Raising the ST cost has a twofold direct effect, it increases the number of hours worked by the worker by making adjustment at the hours margin less flexible and so the worker will produce more, but it reduces the likelihood that a worker will be retained in a firm by increasing the total labor cost paid by the firm.

**Lemma 3.** *The leaving threshold $A$ is not binding if*

$$w \geq \beta \text{ and } a \geq u_b$$

PROOF

See appendix C.3 □

Lemma 3 outlines two conditions under which a worker will choose to remain in a job match. First, the wage must exceed the disutility from working. This ensures that full-time workers derive a positive immediate surplus from employment. Second, the ST compensation must be greater than the unemployment benefit. This guarantees that even when working zero hours, workers receive a higher immediate surplus than they would if unemployed.

Moreover, since the disutility of labor is convex and consumption utility is linear with respect to hours worked, if the immediate surplus is positive at both the upper and lower bounds of the feasible hours interval, it must also be positive throughout the entire interval. In this case, the worker has no incentive to exit the match regardless of hours worked.

Finally, because productivity is randomly drawn in each period, the expected value of a newly formed job match equals that of continuing in the current one. Therefore, if the immediate surplus from the current match is positive, no worker will voluntarily leave their job, and the exit threshold $A$ is not binding.

The conditions specified in Lemma 3 are likely to be satisfied in practice. In many countries—France being a prime example—STW compensation typically exceeds unemployment benefits. Furthermore, wages can generally be assumed to surpass the disutility of working; otherwise, even highly productive jobs would not be viable. For these reasons, I will assume by default that both conditions outlined in Lemma 3 hold throughout the remainder of the paper.

First, I study the *laissez-faire* allocation, which means there is no STW policy at stage 3. In this situation, all policy instruments are set at 0. The condition for the *laissez-faire* allocation to result in the optimal allocation without the policy maker's intervention is stated in Lemma 4. This means the wage rate decided at stage 1, where equations (8) and (8) are satisfied.

**Lemma 4** *The first-best allocation can be obtain through the Nash-Bargaining process:*

$$\max_{(w,a=b)} \left(W - U\right)^p \left(J^f - J^v\right)^{1-p}.$$

PROOF

See appendix C.4 □

The two necessary conditions for reaching the first-best allocations are: first, that wages and ST cost are jointly bargained; second, that productivity is observed. In reality, both conditions are unlikely to be met.

First, as short-time work programs target low productivity matches, the expected productivity is likely to be higher than the observed productivity, leading to a higher wage than the optimal wage. This inadequacy is a precondition to access STW in most countries (see section 2), as the programs target firms facing an unexpected shock. Moreover, the literature observes low fluctuations in real wages during recessions (Bewley and Bewley, 2009; Fallick et al., 2016). Wage rigidities are often used in the literature to justify the use of STW (see Giupponi and Landais, 2023).

A consequence of Lemma 3 is that even with wage flexibility, the decentralized equilibrium cannot be reached without bargaining over the ST cost. In Lemma 3, there is no intervention by the policymaker, so the ST cost is equal to the ST compensation ($a = b$). Negotiating the ST cost along with the wage is equivalent to negotiating the hours along with the wage (Appendix C.4). Without this simultaneous bargaining, the Nash bargaining process cannot jointly satisfy the two equations of Lemma 2. The bargaining over ST costs and compensation is unlikely to be satisfied, since they are generally set unilaterally by the policymaker.

Although Lemma 3 is unlikely to be satisfied, it shows that the optimal allocation can be implemented without government intervention. Allowing worker-firm matchs that experience a productivity shock such that their productivity is below the STW productivity threshold $R^{st}$ to bargain over the ST cost and re-bargain over the wage leads to optimal decentralized allocation. In this section, I derive the policy maker's program when Lemma 3 is not satisfied. I keep the notation $w$ for the wage to remain as general as possible about how it is set.

Another consequence of Lemma 3 is that hours flexibility alone is not sufficient to achieve the optimal allocation. If the wage is rigid, the labor cost is suboptimal and the hours defined in equation (10) are likely to differ from the optimum defined in equation (7). The condition for the decentralized equilibrium to satisfy the hours condition is

$$w + \tau^{st}(\theta) - b(\theta) = \left[(2\beta)\theta^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{2-\alpha}}. \tag{12}$$

Equation (12) is obtained by deriving the labor cost $\ell = w + \tau^{st} - b$, where the hours worked in equations (10) and (7) are equal. Equation (7) implies that the labor tax is actually a labor subsidy $\tau^{st} < 0$. Indeed, if the wage is bargained before observing productivity, with the reasonable assumption that the productivity of short-time workers is lower than their expected productivity, then the wage is higher than the optimal labor cost $\left[(2\beta)\theta^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{2-\alpha}}$. Thus, the labor tax is a wage subsidy that corrects for excessive wages. When the wage and ST cost are set as in Lemma 3, the labor cost is optimal and it immediately follows that the labor tax for ST workers is 0 $\tau^{st} = 0$.

The value of the ST cost and the compensation $a, b$ are then derived by setting the decentralized firing and leaving thresholds (11), (5) equal to the optimal ending threshold (8). Combining the three equations leads to the following condition: for a productivity $\theta$, the surplus from a match for a worker $W(\theta) - U$ and for a firm $J^f(\theta) - J^v$ should be positive if and only if their combined surplus is positive in the optimal allocation $S^*(\theta) > 0$. Under this condition, the worker and firm surplus can be expressed as a linear function of the cumulative surplus: $W(\theta) - U = k^2 S^*(\theta)$ and $J(\theta)^f + J^v = k^1 S^*(\theta)$ with $(k^1, k^2) > (0, 0)$. The two function ensures that the decentralized firing and exit threshold are optimal and is used to derive the value for the ST cost and ST compensation:

$$b(\theta) = (1-k^1)(\theta h^*(\theta))^\alpha + k^1\beta(h^*(\theta))^2 - J^v + (1-k^1)\sigma(S_{+1}) \tag{13}$$

$$a(\theta)(1-h^*) = k^2(\theta h^*(\theta))^\alpha + (1-k^2)\beta(h^*(\theta))^2 + U - (1-k^2)\sigma(S_{+1}). \tag{14}$$

**Proposition 1.** *The first-best allocation can be implemented in a decentralized economy by choosing the values of the policy instruments $b$, $a$, $\tau^{st}$, and $\tau^f$, that jointly satisfy equations (9), (12), (13), and (14).*

PROOF

See appendix C.5 □

In Proposition 1, the wage subsidy $\tau^{st}$ ensures that the decentralized hours are optimally set for each productivity match $\theta$. Without the wage subsidy, the STW program cannot simultaneously satisfy the optimal hours and the termination threshold in Lemma 1. Then, the ST cost $b$ and the compensation $a$ ensure that all matchs that generate a positive surplus are preserved, and the labor tax on full-time workers $\tau^f$ satisfies the budget constraint. The implementation proposed in Proposition 1 is not the only possible implementation. The coefficients $k^1$ and $k^2$ can be replaced by a nonlinear function, the program can be coupled with a lump-sum transfer... Rather, it proposes a simple rule to achieve the optimal allocation described in Lemma 1. It is possible to attain the optimal allocation with fixed ST cost and compensation. However, the wage subsidy have to differ for each productivity-type $\theta$.

**Corollary 1.** *The optimal labor tax for full-time workers is*

$$[1 - G(R^{st})]\tau^f = \int_{R^*}^{R^{st}} (k^1 + k^2 - 1)S^*(\theta)dF(\theta) \begin{cases} < 0 \text{ if } k^1 + k^2 < 1 \\ = 0 \text{ if } k^1 + k^2 = 1 \\ > 0 \text{ if } k^1 + k^2 > 1 \end{cases}$$

PROOF

See appendix C.6 □

The tax on full-time worker depends on the deficit generated by the STW program. Its value then depends on the government's preference for redistribution between the firm and the worker and between pairs along the productivity distribution, captured by the constants $k^1$ and $k^2$. The redistributive property of the STW program is beyond the scope of this paper. Here I show that STW can be self-financing. If the sum of the weights for the worker and the firm in a match is less than or equal to one $k^1 + k^2 \leq 1$, then the optimal surplus generated is redistributed by the government without generating a deficit. Thus, in this case, the first-best allocation is reached without generating a tax for full-time workers.

## 5 Second-Best & Deadweight Loss

In the first-best policy, ST costs and compensation are a function of idiosyncratic productivity and are coupled with wage subsidies. No country has ever implemented such a program. The values of ST costs and compensations are always rigid and common to all worker-firm pairs. Moreover, STW programs are never coupled with a wage subsidy program. Because of these features, STW programs cannot jointly satisfy the optimum characterized by equations (7) and (8) and generate *per se* a deadweight loss.

### 5.1 Second-Best

In this section I propose a policy where the components of the STW program $(b, a)$ are constants along the productivity distribution and where no wage subsidy program is implemented $\tau^{st} = \tilde{\tau}$. This results in a second-best program that is closer to the one implemented by policymakers. This setting is similar to assuming incomplete information, i.e. the policy maker does not observe the productivity draw $\theta$ for each pair, but knows the productivity distribution $F(\theta)$. The second best policy is obtained by deriving the social welfare function (6) under the public budget constraint (9).

I start with the ST compensation $a$. Since it does not affect the working hours, the compensation should just be set so that no workers leaves a match without being laid off. A sufficient condition is that the ST compensation is greater than or equal to the unemployment benefit

$$a' \geq u_b. \tag{15}$$

Since the shock is i.i.d. across periods and workers, the worker's decision to leave the match is mainly driven by his immediate earnings. A ST cost greater than or equal to the unemployment benefit ensures that the surplus from a match for a worker is positive. Intuitively, with the i.i.d. property of a shock, the expected value of a match in the next period is the same for employed and unemployed workers. Since the probability of remaining employed is higher than that of finding a job, and productivity is randomly distributed across periods, a worker would leave a match only if his immediate earnings are higher when he is unemployed. Since the wage is higher than the unemployment benefit (otherwise no match is formed), an ST compensation higher than the unemployment benefit ensures that the earnings of employed workers are always higher than those of unemployed workers.

The derivation of the ST cost value is more complex since it simultaneously affect the firing threshold and hours worked. Without wage subsidies, the second-best value of ST cost s the result of the trade-off between satisfying the optimal condition on hours worked (7) and on the ending threshold (8). At second best, the short-time cost satisfies the equation

$$\frac{\epsilon^R}{\epsilon^S} = \frac{\int_R^{R^{st}} S(\theta) dF(\theta)}{R \cdot S(R) \cdot f(R)}. \tag{16}$$

In the second best scenario, the ST cost should be set so that the surplus gain from adding a worker to the labor force is equal to the surplus loss from reducing the hours worked by all workers on ST work. $\epsilon^R$ and $\epsilon^S$ are respectively the elasticity of the firing threshold and the surplus of workers in short-time work with respect to the ST costs. The ratio between the two measures the capacity of the government to lower the firing threshold without affecting the surplus of employed workers. $\int_R^{R^{st}} S(\theta)dF(\theta)$ is the surplus of worker-firm pairs in the program for which the match is already preserved and which will experience a loss following a reduction in ST costs. $S(R) \cdot f(R)$ is the surplus gain due to the additional jobs preserved.

**Proposition 2.** *The second-best allocation can be implemented in a decentralized economy by choosing the values of the policy instruments $a$, $b$, and $\tau^f$, that jointly satisfy equations (9), (15), and (16).*

PROOF

See appendix C.7 □

The second-best policy results in a firing threshold that is higher than in the first-best allocation and a number of hours worked for each worker that is lower than in the first-best allocation. Even assuming that one of the two elasticities is null, the second-best allocation cannot set the firing threshold to the optimal value or the hours worked to the optimal value. This inefficiency is due to the rigidity of ST costs along the productivity distribution.

Figure 4 illustrates the trade-off between hours worked and employment loss in the second best allocation. Area (2) measures the distance between optimal and second best hours for each productivity type. Area (1) captures the loss in surplus due to the higher firing threshold in second best. By lowering the ST cost, the policy maker reduces area (1) but increases area (2). The policy maker's problem in second best can then be interpreted as minimizing areas (1) and (2). Canceling both areas is only possible by coupling STW with wage subsidies.

The attentive reader will note that the second and first best solutions satisfy Mirrlees's policy of incentive compatibility Mirrlees (1971). There are no incentives for firms to misreport their productivity draw $\theta$, and the labor costs faced in the first and second best settings are such that firms would receive a lower payoff by choosing a different number of hours $\theta'$. At second-best, the program the same screening properties as the one derived by Teichgräber et al. (2022).

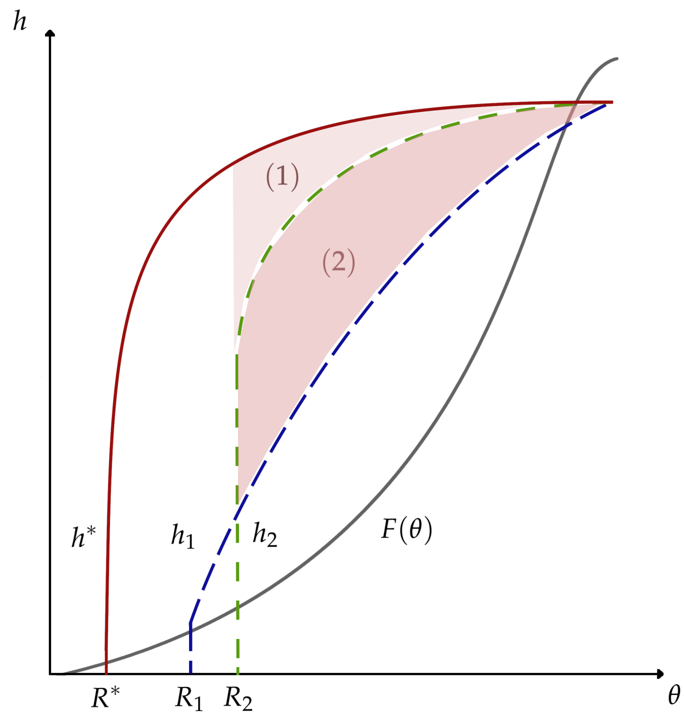


Figure 4: Hours worked with first and second-best policy

The graph displays the number of hours worked, $h$, as a function of productivity, $\theta$. The curve $h^*$ represents the optimal allocation of hours worked across productivity levels, as defined in equation (7). The threshold $R^*$ denotes the reservation productivity in the optimal allocation, described in equation (8). Curves $h_1$ and $h_2$, and points $R_1$ and $R_2$, correspond to the number of hours worked and reservation productivity levels in the decentralized equilibrium, derived from equations (10) and (11), under two short-time work cost levels with $b_1 < b_2$. Area (1) illustrates the loss in hours worked for individuals with productivity $\theta \geq R_2$ in the decentralized equilibrium with $b_1$, relative to the optimal allocation. Area (2) represents the additional loss in hours worked for the same productivity range $\theta \geq R_2$, when comparing the decentralized allocation under cost levels $b_1$ and $b_2$. The function $F(\theta)$ denotes the cumulative distribution function of worker productivity.

**Corollary 2.** *At second best, the short-time cost is greater than 0*

$$b' > 0.$$

PROOF

See appendix C.8 □

Equation (16) implies that the ST cost should be greater than 0. If the ST cost is set to 0 and the ST compensation is greater than the unemployment benefit, the firing threshold is below the first allocation, i.e. more jobs are preserved than with the first-best allocation and workers in STW works less than in the second-best. In the majority of OECD countries the ST cost is set to 0 and the ST compensation is higher than the unemployment benefit (Figure B.1.2). Thus, most STW programs are suboptimal.

## 5.2 Deadweight Loss

To measure the social welfare effect of a suboptimal STW policy, I use definition 1.

**Definition 1.** *The Deadweight Loss (DWL) is the difference between the first-best surplus and the observed surplus.*

$$DWL = \int S^*(\theta)dF(\theta) - \int S(\theta)dF(\theta)$$

Deadweight loss measures the loss of social welfare due to misallocation of labor. It takes into account the misallocation of labor between the unemployed and employed status and the misallocation of labor effort through working hours. In the literature, cost-benefit analyses of the STW program are carried out either by comparing it with the unemployment benefit (Borowczyk-Martins and Lalé, 2016; Giupponi and Landais, 2023) or by measuring the windfall effect of the program (Balleer et al., 2016; Cahuc et al., 2021; Albertini et al., 2022). The first method is uninformative because the STW program is *per se* less costly for the policymaker as long as the ST compensation is less than or equal to the unemployment benefit. Since firms pay a share of worker's revenue with STW, giving the same revenue for a worker is always less costly with STW than with unemployment benefits. The second method focuses on the leniency of the program and ignores the effect on hours of lowering the firing threshold.

Figure 5, illustrates the measurement of deadweight loss for a productivity type $\theta$. In this figure, the optimal point is characterized by the price and quantity of labor $(\ell^*, h^*)$. This point is reached by the first-best policy presented in Proposition 1. The second allocation leads to the point $(\ell', h')$. At this point, since there is no wage subsidy and the ST cost is less than or equal to the first-best ST cost, the cost of adding an hour of labor is higher $\ell' = w - b' > w + \tau^* - b^* = \ell^*$. With a higher labor cost, the number of hours worked by the firm is lower $h' < h^*$, resulting in a deadweight loss measured by area (1). The figure illustrates the deadweight loss generated by the second best policy for a productivity type $\theta \in ]R, R^{st}[$. Definition one then measures the sum of all areas (1) over all matchs belonging to the productivity interval $[R, R^{st}]$. If the policymaker lowers the ST cost below the second-best $b'$, this results in even higher labor costs $\ell''$ associated with fewer hours worked and higher deadweight loss measured by area (2).

The deadweight loss of generous STW programs does not affect all agents equally. On the one hand, it increases the match value of firms whose idiosyncratic productivity is between the firing threshold $R$ and the eligibility criteria $R^{st}$. On the other hand, it reduces the income of workers on STW if the ST

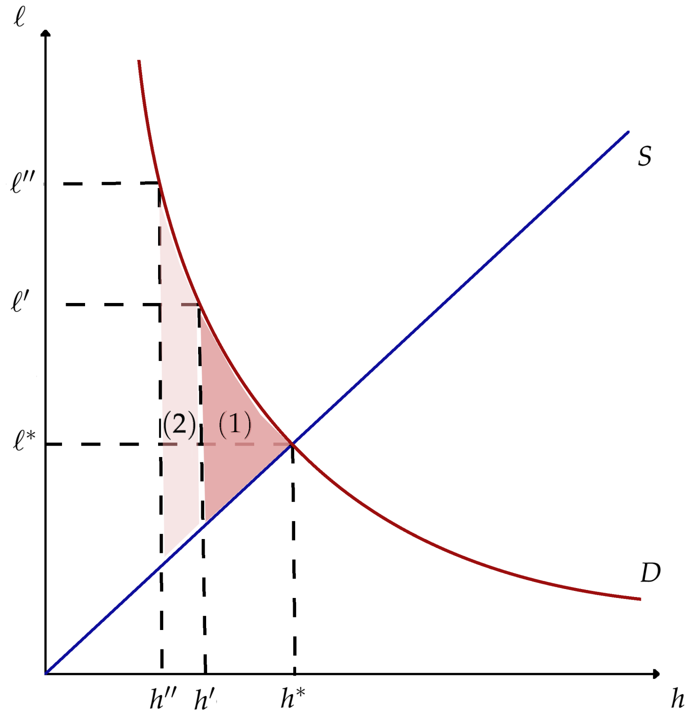


Figure 5: The Individual Deadweight Loss of Low Short-Time Cost

The graph illustrates the labor demand (D) and supply (S) for hours worked, $h$, as a function of the marginal cost of adding one hour of work, denoted $\ell$. The demand curve (D) corresponds to the FOC with respect to hours worked in the firm's value function, given in equation (1). The supply curve (S) corresponds to the FOC with respect to hours worked in the worker's value function, given in equation (3), under the assumption that short-time compensation equals the short-time cost, i.e., $a = b$. The points $h'$ and $h''$ represent the number of hours worked when the short-time work cost is below the optimal level, with $b'' < b' < b^*$, corresponding to marginal costs of $\ell'' < \ell' < \ell^*$. Area (1) measures the individual deadweight loss resulting from a decrease in the short-time cost from $b^*$ to $b'$. Similarly, area (2) represents the additional deadweight loss when the short-time cost decreases further from $b'$ to $b''$.

compensation is less than the wage[3].

The second-best program is not self-financing. A marginal reduction in the cost of STW has a triple negative impact on public finances. First, it increases public expenditure due to new entrants. Second, it increases public spending due to fewer hours worked. Third, it increases public expenditure due to a lower participation of firms in ST compensation. All these effects are additive and lead to an increase in taxes. In this model, the STW program is financed by a lump-sum transfer. Taxation models, however, highlight the potential disincentive effect that taxes can have on the effort of highly productive workers. In addition, these taxes can reduce the dynamism of job creation by reducing the payoff to entering a highly productive

[3]To date, no STW program has implemented an ST compensation higher than or equal to the wage.

match. Therefore, the deadweight loss of a generous STW program is likely to be underestimated by the model.

The literature has highlighted the windfall effect that the STW program may have. To reduce this effect, some papers recommend restricting access to the program by lowering the eligibility criteria $R^{st}$. An alternative is to play with the ST cost. With a high cost of using the program, all firms with a productivity $\theta^{\alpha} \geq \ell$ stay out of the STW program because it is not profitable for them, while firms with low productivity continue to use it. Thus, by addressing the loss of hours worked issue, the STW cost derived in Proposition 2 also reduces the deadweight loss of STW.

The majority of STW programs before and during the Covid-19 recessions involved zero ST costs for firms and ST compensation higher than unemployment benefits. Following Proposition 1, these programs generates *per se* a deadweight loss. Following Corollary 2, these programs *per se* does not reach the second-best allocation. These programs led to the lowest possible dismissal threshold and the lowest possible allocation of hours worked, with the highest possible deadweight loss. In the next session, I estimate the second best ST cost for France in 2023. France had one of the highest ST costs among OECD countries (Figure B.1.1). Therefore, if the second-best ST cost is higher than the observed ST cost for France, it is likely that all countries with lower ST costs than France have also set too generous incentives.

# 6 Numerical Simulation

In this section, I first extend the model to incorporate a standard DMP labor market and a Nash-bargaining process generating wage rigidities. Second, I describe my calibration strategy. Then I present the results of numerical simulations to compare the effect of the current STW program with the first-best and second-best policies at steady state and after a productivity shock.

## 6.1 Full Model

This section incorporates the model presented in section 3 into the standard search and matching framework of Diamond (1998) and Mortensen and Pissarides (1999).

Firms and workers bargain over wages. The Nash bargaining takes place before the realisation of the idiosyncratic shock, so they bargain over the expected value of their respective surplus from the match. The Nash bargaining problem is given by

$$\max_{(w)} \Big(\mathbb{E}[W - U]\Big)^{p} \Big(\mathbb{E}[J^{f} - J^{v}]\Big)^{1-p}, \tag{17}$$

where p $\in$ [0,1] represents the bargaining power of workers. The share of short-time worker with respect to the share of full-time worker is low in the model at the steady state as in the economy during expansion periods ($\leq$ 10%). I assume that workers and firms expect the employment to be set a full-time job $\mathbb{E}[h] = 1$. The first condition implies that

$$(1-p)(J^f - J^v)\frac{\partial J^f - J^v}{\partial w} = p(W-U)\frac{\partial W - U}{\partial w},$$

where, from the worker's value I have $\frac{\partial W - U}{\partial w} = 1$ and from the firm's value function, I have $\frac{\partial J^f - J^v}{\partial w} = -1$. The bargaining solution and the expression for employment surplus together imply that the Nash bargaining wage $w$ satisfies the Bellman equation

$$w = p\mathbb{E}[S] + \beta + \mathbb{E}[U] - \sigma(1-\rho)\mathbb{E}\left[(1-\phi)W_{+1} + \phi U_{+1}\right]. \tag{18}$$

Following lemma 3, the bargained wage defined above is inefficent.

New job matchs are formed based on the matching function

$$m = \mu u^{\alpha_m} v^{1-\alpha_m},$$

where the parameter $\mu$ is the scale of matching efficiency and the parameter $\alpha_m \in [0,1]$ is the elasticity of job matching with respect to efficiency units of seeking workers, $u$ is the number of unemployed workers and $v$ is the number of vacancies. The number of unemployed is equal to the total labor force minus the number of employed:

$$u = 1 - N.$$

Newly formed matchs add to the employment pool, while job separations and obsolescence subtract from it. Thus, aggregate employment evolves according to the law of motion

$$N = (1-\rho)(1-\phi)(N_{-1} + m_{-1}). \tag{19}$$

The total job destruction $\phi$ depends on the endogenous job destruction rate $\phi^e$ and the exogenous job destruction rate $\phi^x$:

$$\phi = \phi^e + \phi^x. \tag{20}$$

The endogenous rate of job destruction is equal to the share of workers with productivity below the firing threshold:

$$\phi^e = \int_0^R dF(\theta) \tag{21}$$

The stock of vacancies $v$ evolves according to the law of motion:

$$v = (1 - q^v)(1 - \rho)v_{-1} + n \tag{22}$$

Following Coles and Kelishomi (2011), I assume that vacancy creation entails a non-negative entry cost of x, drawn from an i.i.d. distribution $\psi(\cdot)$. A new vacancy is created if and only if $x \leq J^v \equiv x^*$, or equivalently, if and only if its net value is non-negative. Thus the number of new vacancies $n$ is equal to $\psi(J^v)$ - the cumulative density of entry costs evaluated at the vacancy value. As in Leduc and Liu (2020), I assume that the functional form of the distribution function $\psi(\cdot)$ is

$$n = \eta(J^v)^{\xi}, \tag{23}$$

where $\eta$ is a scale parameter and $\xi$ measures the elasticity of new vacancies with respect to the value of the vacancy. The special case with $\xi \to +\infty$ corresponds to the standard DMP model with free entry (i.e. $J^v$ = 0). The probability of filling the vacancy $q^v$ and the probability of finding a job $q^u$ are given by

$$q^u = \frac{m}{u} \tag{24}$$

$$q^v = \frac{m}{v}. \tag{25}$$

The labor market equilibrium is defined by equations (1), (2), (3), (4), (9), and (18)-(25).

To refine the model's calibration, I introduce two additional variables in the firm's value function: (i) a fixed cost of production, $f_c$, which captures non-labor costs such as machinery, input supplies, and infrastructure, and (ii) a cost of adjusting working hours, $h_c$, which reflects logistical and administrative costs related to altering hours worked. The fixed cost enters the firm's profit function linearly, while the adjustment cost enters as the ST cost (see Appendix D).

## 6.2 Calibration

I assume that the idiosyncratic productivity shock $f(\theta)$ follows a cubic distribution, and set the returns to scale at $\alpha = \frac{2}{3}$ to obtain a closed-form solution. A key challenge in the calibration process lies in identifying functions that are analytically integrable. To address this, the distribution function and the returns to scale parameter must be jointly specified. A return to scale below one ($\alpha < 1$) implies that the productivity distribution must take a polynomial form with positive integer powers. This assumption does not significantly affect the model's predictions, as STW policies primarily influence the left tail of the productivity distribution (see Figure 6). Consequently, reducing the probability mass in the right tail—by adopting a distribution close to normal—has negligible implications for the behavior of short-time workers.

The fixed cost of creating a vacancy, $\kappa$, and the obsolescence rate, $\rho$ are respectively set to 0.1887 and 0.0196 following the estimates in Leduc and Liu (2020). The quarterly discount factor $\beta$ is 0.99, consistent with an annual real interest rate of approximately 4.1%. As in Krause and Lubik (2007), one-third of separations are endogenous while the remaining two-thirds are exogenously determined. This calibration reflects French labor market data, where around one-third of dismissals are classified as economic dismissals (source: DARES). Consistent with Balleer et al. (2016), the matching efficiency parameter is set to 0.43, and the bargaining power is fixed at 0.5. In my baseline experiment, I set $\xi \to \infty$ to reproduce the classic DMP model, I'll then switch to 1, as in Fujita and Ramey (2007) and Coles and Kelishomi (2011).

According to the French labor Force Survey (FLFS) for 2017-19, the quarterly transition rate from unemployment to employment was 7.76%, while the employment rate was 77.94% (Chéron and Terriau, 2024). Data from DARES (2023) shows that ST workers completed 77% of their regular hours during a quarter, with 1.68% of all workers were enrolled in the program. According to Legifrance, the ST cost, the ST compensation rate, the unemployment benefits, and the layoff cost were 34%, 60%, 73%, and 25% of the wage, respectively in 2023.

Table 1: Calibration

| **Parameter** | | **Value** |
|---|---|---|
| $\kappa$ | Vacancy creation cost | 0.1887 |
| $\sigma$ | Subjective discount factor | 0.99 |
| $\mu$ | Matching efficiency | 0.43 |
| $\alpha_m$ | Elasticity matching function | 0.6 |
| $p$ | Nash bargaining weight | 0.5 |
| $\xi$ | Elasticity of vacancy creation | $\rightarrow \infty$ |
| $\alpha$ | Return to scale | $\frac{2}{3}$ |
| $f(\theta)$ | Probability distribution function of skills | $4\theta^3$ |
| **Steady states targets** | | **Value** |
| $N$ | Employment rate | 0.77 |
| $q^u$ | Job finding rate | 0.077 |
| $\phi^{st}$ | STW share | 0.0168 |
| $\tilde{h}^{st}$ | Hours worked by ST workers | 0.77 |
| $b$ | STW cost | 0.34×w |
| $a$ | Short-time compensation | 0.6×w |
| $u_b$ | Unemployment benefit | 0.73×w |
| $l_c$ | Layoff Cost | 0.25×w |

The complete system of steady-state equations is presented in Appendix D. Calibrated parameters and steady-state outcomes are summarised in Table A.3. The ST cost affects the labor market equilibrium by altering the firing threshold (Figure 6) and the number of hours worked by ST worker (5), as well as the labor tax paid firms for full-time workers. This then impacts the expected matching value for firms, leading to a change in job creation.

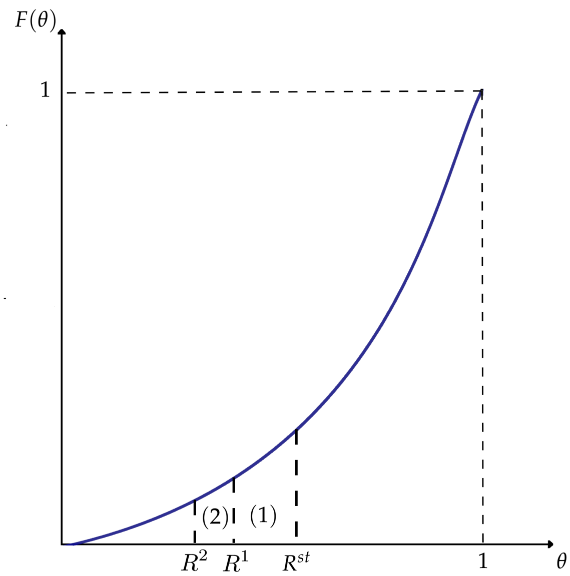


Figure 6: Cumulative Distribution Function of Productivity

The Figure displays the cumulative distribution function (CDF) of worker skills, denoted by $F(\theta)$. The threshold $R^{st}$ represents the short-time work productivity cut-off. Above this point, workers are full-time workers. $R^1$ and $R^2$ correspond to productivity reservation levels under two different short-time work costs, $b_1$ and $b_2$, with $b_1 > b_2$, as defined in equation (11). Area (1) captures the employment saved thanks to short-time work. Area (2) increase in employment resulting from a reduction in the short-time work cost from $b_1$ to $b_2$.

The targeted steady state for the job-finding and employment rates yields a job vacancy rate of 6.5%, a vacancy filling rate of 14%, and an exit rate of 22%. The job vacancy rate exceeds the observed French average of around 2.5% (source: DARES), and the quarterly vacancy filling rate is close to the measured rate of 18% (source: France Travail), and the ending probability is consistent with an annualised turnover rate of around 30% (source: INSEE).

The targeted average hours worked under STW results in a negative value for the adjustment cost, reflecting the reduced labor costs incurred by firms in the programme due to payroll tax exemptions. In France, employer contributions to social security, pensions and health insurance are reduced under STW schemes. Finally, the firing threshold derived from the one-third rule for ending contracts results in fixed production costs equalling 45% of wages, which overestimates the actual non-labor share of production costs. According to Eurostat, this is estimated to be around 25%.

### 6.3 Steady States

In this section, I estimate how a change in the short-time work program affects the steady-states. I first compute the steady-state with the target in table 1, then I substitute the value of the STW policy with the first and second-best solution. For the first-best, I substitute the labor-tax $\tau_{ss}$, the ST cost $b_{ss}$ and ST compensation $a_{ss}$ with the value that solves the first-best program when the weight of the policy-maker for workers and firms respectively equals their bargaining power in the first-best allocation.

For the second best solution, I substitute the value of the ST cost $b_{ss}$ with the one satisfying proposition 2. Using Newton's method, I find that the STW cost equals should equal 51% of the wage in the second-best case. Recalling that the ST compensation equal to 60% of the wage, it implies that nearly all the expenses on STW are covered by firms reducing drastically public deficit. Overall, an increase of the ST cost from 30% to 58% of the wage would reduce public expenses on STW by 80%. I find that this change would have a small impact on unemployment with an increase of 1.2 p.p but would increase the aggregate surplus of ST worker by 9.4%. The first-best policy would even lead to higher increase in surplus with an increase of 22% with a similar unemployment rate (a decrease of 0.15 p.p.).

Table 2: First and Second-Best STW programme vs the Current French Programme at the Steady-State

| | $2^{nd}$ Best | $1^{st}$ Best |
|---|---|---|
| Welfare of ST workers | +9.4 % | + 22 % |
| Average hours worked of ST workers | + 25 % | + 83 % |
| Budget Expenses | - 92 % | - 100 % |
| Firing rate | + 0.03 p.p | -0.06 p.p |

The table compare the economy at steady-state where the ST cost is set as in France in 2023 (36% of the wage) with the steady-state where the ST cost set as in the second-best (51% of the wage) and with the steady-state where the first best STW programme is implemented.

Figure 7 shows how the ST cost affects the worker-firm surplus $S_{ss}$, the firm surplus $J^f_{ss} - J^v_{ss}$, the public budget $G_{ss}$, the hours worked $h_{ss}$ along the productivity $\theta$. The green line is drawn for the productivity of the average worker on STW, the red line for the current ST cost and the blue line for the ST cost in the second best case. A plane is drawn at 0. The top left graph shows how the worker-firm surplus rises steadily up to the second-best ST cost. The top right graph illustrates both the firing threshold and how the ST cost affects it, it also illustrates that the firm surplus increases as the ST cost decreases. Combining the two graphs shows that a generous program benefits the firm at the expense of the aggregate surplus as predicted in Lemma 3. The bottom left-hand corner of the graph shows the public expenditure on the match. Similarly, a low ST cost has a negative impact on the government budget as predicted in lemma 4. On the bottom right of the graph is the effect of ST costs on hours worked, which shows how steeply the number of hours worked decreases with ST costs and productivity, and therefore how strongly ST costs affect the hours worked and output produced by a match as predicted in Lemma 1.

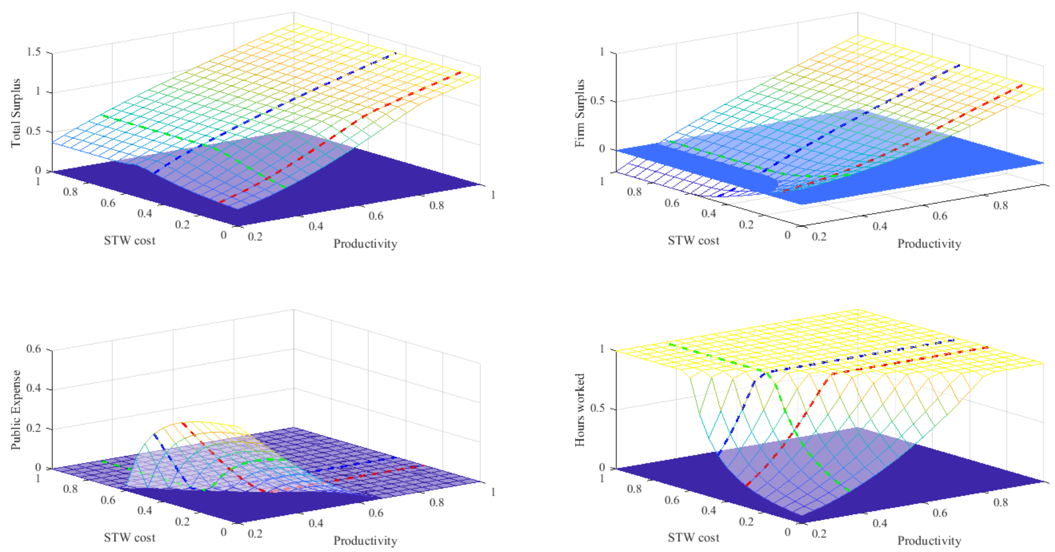


Figure 7: The effect of ST cost along the productivity distribution

The Figure shows the worker-firm surplus $S_{ss}$, the firm surplus $J^f_{ss} - J^v_{ss}$, the public budget $G_{ss}$, the hours worked $h_{ss}$ along the productivity $\theta$ and the short-time cost $b_{ss}$. The green line is drawn for the productivity of the average worker on STW, the red line for the current ST cost and the blue line for the ST cost in the second best case.

## 6.4 Dynamics

In this section I estimate how short-time work policies affect the model during recessions. STW schemes are generally made more attractive during economic downturn in order to encourage worker-firm pairs to enter and remain in the scheme. I model a half standard negative deviation of the productivity distribution during one period. The aim is to model a one-period recession and the effect of different STW policies on the deviation of the model from steady state. Figure B.2.1 shows the impulse responses to a fall in productivity during one periods for the three scenarios. The first scenario, which I call ”current program”, mimics the French policy during the Covid-19 with the ST cost reduced to 0 during the recession and then

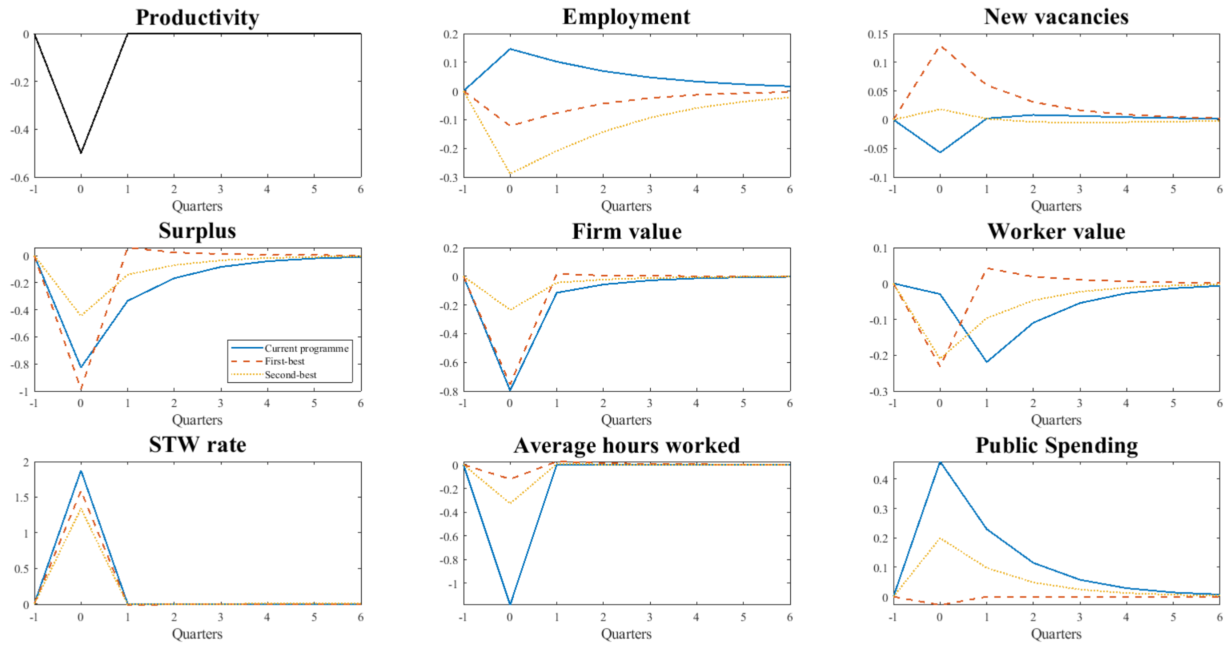


Figure 8: Impulse responses of a negative shock to aggregate productivity

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in aggregate productivity. The blue line corresponds to the scenario where the short-time cost is reduced to 0 during the recession. The yellow line correspond to a scenario where the policy-maker reduce to 30% of the wage during the recession. The red line corresponds to a scenario where the policy-maker implements a first-best policy during the recession. The top left graph shows the productivity shock (common to the 3 scenarios). The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labor force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

goes back to the steady state value. In the second scenario, which I call the ”first-best”, I apply the first-best policy described in proposition 1 during the recession with the weights for workers and firms equal to their bargaining power. In the third scenario, which I call ”second-best”, the ST cost is lowered so that the firing rate is the same as in the ”first-best” scenario during the recession. This policy is not the same as the one described in proposition 2 since it does not take into account the deadweight loss generated by a low ST cost. In this setting, all matchs that could generate a positive surplus are preserved. I choose this second-best policy because the goal of the policymaker during recessions is often to protect the level of employment rather than to maximize the surplus produced. I find that the ST cost in this setting is 32% of the wage, which was nearly the value of the French ST cost before and after the Covid-19 (34%).

The current program freezes the labor market during recessions. In this scenario, there is an increase in employment relative to the steady state due to a drop in layoff. The model reproduces that during the Covid-19 crisis, France reduced the ST cost to 0 and observed the lowest job destruction rate and one of the lowest job creation rates of the century. This is due to the fact that each firm can reduce the number of hours worked by its worker to 0 at no cost and the worker receives an ST compensation higher than the unemployment benefit with the insurance of having a job in the next periods. This comes at the public cost of an increase in public expenditure, which will then increase future taxes. The reduction in the mass of unemployed workers plus the future taxes reduce the number of new vacancies created. As described in lemmas 3 and 4, a ST cost set below the second-best level has an immediate and dynamic negative effect on the surplus from a match. The surplus then falls more sharply during recessions and takes longer to return to its steady-state value. The negative immediate effect is compensated by the public expenditure with low ST costs received by firms and high ST compensation received by workers. As a result, the value of the firm falls during the recession at a similar level to the first best scenario and the effect on the value of the worker is small. However, after the recession, the taxes delay the recovery of the firm value and reduce the worker value even more than during the recession. As the STW program is generous, there is a huge increase in the use of the program extensively and intensively during the recession, with an associated steep increase in public spending. The negative surplus effect after the shock captures the effect of post-recession austerity, as after the Great Recession (House et al., 2020). In the model, the severity of the austerity depends on the deficit registered during the recession.

The first-best program maximized the surplus from each match at no public cost. The content of the policy and then its effect on the labor market is very different from the other two policies modelled. In the first-best policy, the recession period has a small effect on employment thanks to a dynamic labor market. A 0.5 deviation in productivity produces a similar deviation in the firing rate (Figure B.2.1), but only a -0.1 deviation in unemployment. The recession has a small effect on unemployment thanks to an increase in the number of vacancies created. In the first-best case, the increase in the mass of unemployed workers and the maximisation of the surplus from a match stimulate the creation of new jobs, which dampens the negative effect of the fall in productivity on employment. The positive effect of the first-best policy on new vacancies is captured by a smaller fall in the value of the firm during the recession and an immediate take-up afterwards. Employee value, on the other hand, falls more sharply because employees are not paid as much as in the first-best scenario. However, it rebounds immediately afterwards with a small increase due to lower taxes in the next period. As productivity falls, the program is used more extensively and intensively during the recession. The reduction in working hours is small due to the opposite effect of the individual reduction in working hours and the recomposition of the sample of ST workers after the productivity shock. Indeed, during the recession more workers are in ST work, but the average skill of ST workers increases as the mass of workers with skills close to the STW threshold $R^{stw}$ increases. Even without this effect, each ST worker works more with the first-best policy. Finally, this policy is not costly for the policy maker, as the surplus of each match is fully redistributed without additional spending. This reduces public expenditure relative to the steady state.

The second-best policy is an intermediate point between the first-best and the current program, i.e. it preserves employment, firm and worker value at a small cost relative to the current program. The second-best policy lowers the ST cost so that the firing rate is equal to the first-best firing threshold, which is higher than the firing threshold of the current program. The second-best policy implements the lower bound of Proposition 2, I estimate the second-best ST cost during the recession to be 29% of wages, which is 5 p.p. lower than the steady state value and 29 p.p. higher than the current program value. In this setting, the level of employment is lower than in the current program due to the higher firing rate and lower than in the first best due to a less dynamic labor market captured by a small fall in vacancies created. The loss of surplus during the recession is the lowest in the second best scenario because only those matchs that generate a positive surplus at a low cost to the firm are retained, while the value of the worker is protected through ST compensation. The proportion of ST workers and the reduction in hours worked are lower than in the current program because the cost of ST is higher. The rate of ST workers is lower than in the first best because more workers are unemployed. As shown in Figure 4, the reduction in hours worked is greater than in the first-best scenario. As a result, the increase in public spending is lower than in the current program.

Overall, the ST cost was too low during and after the Covid-19 recession in France. This leads to a lower surplus generated by each worker-firm pair, which is partly compensated by public spending. In the steady state, this mainly benefits firms that reduce their labor costs and increase their profits, ceteris paribus. In the dynamic, both workers and firms are losers, as the immediate benefit of low ST costs is offset by future taxation to finance the public deficit. The reduction in ST costs, as in the last two recessions, is necessary but has been too generous. I find that by reducing the ST cost to 0, policymakers are freezing the labor market by maintaining the maximum possible number of matchs at the expense of the immediate and future surplus generated by matchs. I estimate that a -0.5 deviation in labor productivity should imply a ST cost of at most 0.29% of wages. In France, labor productivity fell by 16% in the first quarter of 2020 (Devulder et al., 2024); in this context, the gap in ST costs and deadweight loss between the second best and the current program is even larger. In Figure B.2.2, I plot the IRF for a deviation in labor productivity of -0.16, the model reproduces the French labor market features observed during Covid-19 with an 18 p.p. growth in the share of short-time workers in the model vs. a 20 p.p. in the data, and an increase in employment of 0.15 % in the model vs. 0.1% in the data and an increase in hours consumed per worker of 180% in the model vs. 120% in the data.

In Figure B.2.3 I model an autoregressive one standard negative deviation of ST costs. As in the local projection (3), it has a small effect on unemployment, but increases the extensive and intensive use of the program, resulting in a higher public expenditure effect. The small effect on employment is due to the fact that ST costs are already too low, so the mass of labor-firm pairs below the dismissal threshold is small. In the model, the increase in the share of ST workers is smaller than in the local projection, probably due to a . However, as in the local projection, the reduction in ST costs reduces the number of hours worked and public expenditure. As there is no change in ST compensation, it immediately increases the surplus through the increase in compensation received by workers and the decrease in hours worked. However, the immediate positive effect on the surplus is offset by a larger decrease in the surplus in the next periods due to the increase in taxes. In addition, when ST costs decrease, the number of new vacancies decreases due to a lower number of unemployed workers and a higher future taxation.

# 7 Conclusion

This paper proposes a theory of optimal STW in a labor market with hours. The optimal STW is defined so that the hours worked by the worker are a ratio between his productivity and his marginal disutility of work which maximizes the surplus generated by the worker-firm match. The cost paid by the firm for using the program and the compensation received by the worker should then be set to share the surplus generated by the match. This optimum implies that the program should differ between worker-firm pairs according to the productivity loss observed. One way of achieving this equilibrium is to decentralise the STW program and allow firms and workers to negotiate the costs and compensation of short-time working. If the program is rigid among workers, it can only lead to a second best with a higher unemployment rate, a lower generated surplus and a higher government deficit.

I derive a second-best policy that solves the trade-off between protecting jobs and reducing working hours. The optimal solution depends on the distribution of productivity across workers. By calibrating the model to French unemployment, short-time work consumption and the short-time work program in the steady state, I derive a numerical approximation for this second-best solution. I find that the second-best cost of short-time work is twice as high as the one currently implemented in France. As a result, short-time workers work less, produce less and generate larger public deficits for a small increase in the employment level. This finding is supported by local projections in which I estimate the effect of reducing short-time costs on employment, hours worked and public expenditure.

The structural budget deficit of STW programs increases during recessions. As productivity falls, more workers become eligible for short-time work, thus increasing public expenditure. This effect is accentuated by the fact that STW policies reduce the costs paid by firms during recessions. As a result, each short-time worker works less, more worker-firm pairs enter the program and firms contribute less to ST compensation. During the Covid-19, the cost paid by the firm in France was reduced to 0. At this point, some worker-firm pairs are maintained, even though they generate a negative surplus, only because of public support.

I focus on the French case because of the availability of data and the age of the program. However, the French program followed the European trend of a decrease in the contribution paid by the company and an increase in the compensation received by the employee. Therefore, the results obtained with French data are likely to be reproducible with other European countries. Moreover, the theoretical predictions on the deadweight loss of STW are derived from the common features of the program and would therefore apply to all countries.

Finally, the paper doesn't address the impact of STW on labor market tightness and redistribution. Instead, it focuses on maximising aggregate surplus. However, I still find that by protecting jobs, the labor market program affects the probability of finding a job and the rate of job creation. Further work should be devoted to extending the optimal policy to account for this effect. Second, the paper remains agnostic about the social weight assigned to agents by the policymaker. The STW programs affect the distribution of surplus between workers and firms, and the distribution of surplus among the labor force, through the contributions of firms and the compensation of workers. The optimal policy then depends on the social choice made by the policy maker. At present, I find that STW programs tend to favour the profit of firms, since their contribution is low compared to the optimal and second-best settings.

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# Appendix A Tables

Table A.1: Short-Time Work Evolution in Germany, France, Italy, and Spain

| Time period | | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 07-17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Unit of measure | Area | | | | | | | | | | | | |
| Public Expend. | | | | | | | | | | | | | |
| Millions | France | 19 | 15 | 319 | 282 | 67 | 103 | 186 | 212 | 203 | 171 | 141 | 638 |
| Millions | Germany | 491 | 554 | 5,166 | 3,837 | 1,327 | 829 | 1,019 | 613 | 709 | 711 | 738 | 50 |
| Millions | Italy | 1,353 | 1,485 | 4,959 | 5,796 | 4,914 | 6,148 | 6,791 | 6,112 | 4,668 | 3,721 | 2,329 | 72 |
| Millions | Spain | 34 | 63 | 570 | 453 | 442 | 718 | 718 | 407 | 222 | 175 | 159 | 363 |
| Millions | SUM | 1,897 | 2,116 | 11,014 | 10,368 | 6,750 | 7,798 | 8,714 | 7,344 | 5,802 | 4,778 | 3,367 | 77 |
| Share of GDP | France | 0.001 | 0.001 | 0.016 | 0.014 | 0.003 | 0.005 | 0.009 | 0.010 | 0.009 | 0.008 | 0.006 | 500 |
| Share of GDP | Germany | 0.020 | 0.022 | 0.211 | 0.150 | 0.049 | 0.030 | 0.036 | 0.021 | 0.023 | 0.023 | 0.023 | 15 |
| Share of GDP | Italy | 0.084 | 0.091 | 0.314 | 0.360 | 0.298 | 0.378 | 0.421 | 0.376 | 0.282 | 0.219 | 0.134 | 59 |
| Share of GDP | Spain | 0.003 | 0.006 | 0.053 | 0.042 | 0.042 | 0.070 | 0.070 | 0.039 | 0.021 | 0.016 | 0.014 | 366 |
| Share of GDP | MEAN | 0.027 | 0.030 | 0.149 | 0.142 | 0.098 | 0.121 | 0.134 | 0.112 | 0.084 | 0.067 | 0.044 | 63 |
| Share of GDP | OECD | 0.007 | 0.008 | 0.043 | 0.035 | 0.019 | 0.025 | 0.026 | 0.019 | 0.016 | 0.015 | 0.011 | 57 |
| Participants | | | | | | | | | | | | | |
| Persons | France | 12,000 | 41,700 | 227,100 | 86,100 | 36,000 | 61,800 | 70,400 | 63,800 | 61,300 | 55,500 | 35,300 | 194 |
| Persons | Germany | 59,470 | 90,684 | 1,117,533 | 474,235 | 133,786 | 99,726 | 110,711 | 79,725 | 74,925 | 115,862 | 101,281 | 70 |
| Persons | Italy | 90,540 | 107,863 | 358,888 | 319,809 | 283,929 | 329,026 | 345,166 | 267,650 | 190,855 | 119,910 | 123,892 | 36 |
| Persons | Spain | 1,794 | 3,323 | 13,270 | 11,874 | 16,230 | 25,565 | 29,842 | 17,795 | 10,476 | 7,320 | 4,662 | 159 |
| Persons | SUM | 163,804 | 243,570 | 1,716,791 | 892,018 | 469,945 | 516,117 | 556,119 | 428,970 | 337,556 | 298,592 | 265,135 | 61 |
| Share of L.F. | France | 0.042 | 0.146 | 0.788 | 0.297 | 0.124 | 0.211 | 0.240 | 0.217 | 0.208 | 0.188 | 0.119 | 183 |
| Share of L.F. | Germany | 0.143 | 0.217 | 2.670 | 1.140 | 0.325 | 0.241 | 0.266 | 0.190 | 0.178 | 0.269 | 0.234 | 63 |
| Share of L.F. | Italy | 0.371 | 0.436 | 1.460 | 1.300 | 1.150 | 1.300 | 1.370 | 1.050 | 0.749 | 0.465 | 0.478 | 28 |
| Share of L.F. | Spain | 0.008 | 0.014 | 0.057 | 0.051 | 0.069 | 0.109 | 0.129 | 0.078 | 0.046 | 0.032 | 0.020 | 150 |
| Share of L.F. | MEAN | 0.141 | 0.203 | 1.244 | 0.697 | 0.417 | 0.465 | 0.501 | 0.384 | 0.295 | 0.239 | 0.213 | 50 |
| Share of L.F. | OECD | 0.130 | 0.175 | 0.691 | 0.439 | 0.208 | 0.291 | 0.330 | 0.172 | 0.164 | 0.152 | 0.136 | 4 |
| Expend./Worker | | | | | | | | | | | | | |
| Thousands | France | 2 | 0 | 1 | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 4 | 150 |
| Thousands | Germany | 8 | 6 | 5 | 8 | 10 | 8 | 9 | 8 | 9 | 6 | 7 | -12 |
| Thousands | Italy | 15 | 14 | 14 | 18 | 17 | 19 | 20 | 23 | 24 | 31 | 19 | 25 |
| Thousands | Spain | 19 | 19 | 43 | 38 | 27 | 28 | 24 | 23 | 21 | 24 | 34 | 78 |
| Thousands | MEAN | 11 | 10 | 16 | 17 | 14 | 14 | 14 | 14 | 15 | 16 | 16 | 46 |

**Source**: LMP database, category 8.2. ('Partial unemployment benefits'). **Note**: Expend. stands for expenditure. L.F. stands for labor force.

Table A.2: Evolution of the law relative to the STW scheme in France from 2008

| Effective date | Main changes | Legal reference | R |
|---|---|---|---|
| June $14^{th}$ 1996 | Creation of the STW scheme | *Loi n° 96-502 du 11 juin 1996* | |
| | (...) | | |
| May $1^{st}$ 2008 | Redesign of the STW system that will serve as a basis for all the next reforms until today with a fixed rate of allocation and the eligibility conditions (including the economic condition that I explore in this paper) | *Création Décret n°2008-244 du 7 mars 2008* | 1 |
| January $1^{st}$ 2009 | Upgrade of STW allocation to workers from 50% to 60% of the wage<br>Raise of the maximum STW consecutive period allowed (up to 6 weeks)<br>Raise of the maximum STW period allowed<br>(up to 1 000 hours per year for some sectors)<br>Firms STW subsidy per hour is 3.84€ (vs 2.44€ before, if ≤250 employees)<br>Firms STW subsidy per hour is 3.33€ (vs 3.13€ if >250 employees) | *Avenant du 15 décembre 2008*<br>*Décret du 22 décembre 2008*<br>*Arrêté du 30 décembre 2008*<br>*Décret n°2009-110 du 29 janvier 2009* | 1 |
| May $1^{st}$ 2009 | Creation of long-time STW (*APLD*)<br>Min-Max duration: 3-12 months<br>Compensation raised at 75% of gross hourly wage<br>Firms subsidy per hours increased up to 7.74€ | *Décret n°2009-478 du 29 avril 2009 et convention Etat-Unédic du 1er mai 2009* | 1 |
| January $1^{st}$ 2010 | The maximum STW period allowed is generalized for all sectors and all workers | *Arrêté du 31 décembre 2009 et ANI du 8 juillet 2008* | 1 |
| March $1^{st}$ 2012 | Firms do not need to ask for a pre-authorization to consume STW<br>Firm subsidy per hour is 4.84€ (6.74€ for ADLP)<br>Long-period STW is now financed by *unédic*<br>The minimum time for a long-period STW take-up is lowered down to 2 months | *Décret n°2012-183 2012 et arrêté du 4 mai 2012*<br>*Décret n°2012-275* | 1 |
| November $1^{st}$ 2012 | Firms not need to wait for a pre-authorization to consume STW | *Décret n°2012-1271* | 0 |
| July $1^{st}$ 2013 | Merge of the "classic" STW program and the long-period STW program<br>Firms STW subsidy per hour is 7.74€ (if ≤250 employees)<br>Firms STW subsidy per hour is 7.23€ (if >250 employees)<br>Downgrade of STW allocation to workers down to 70% of the wage | *Loi n°2013-504 and décret n°2013-551* | 1 |

Table A.2: Evolution of the law relative to the STW scheme in France from 2008 to 2024, cont'd

| | | | |
|---|---|---|---|
| July $2^{nd}$ 2014 | The application process is now an online procedure | *Décret n° 2014-740* | 0 |
| March $25^{th}$ 2020 | The state finance the ST compensation up to 4,5 times the minimum wage | *Décret n°2020-325* | 1 |
| January $1^{st}$ 2021 | Downgrade of STW allocation to worker down to 60% of the wage | *Décret n°2020-1316* | -1 |
| February $1^{st}$ 2021 | The state finance the STW at 36% of the wage (remaining 34% is paid by the firm) | *Décret n°2020-1319* | -1 |
| May $1^{st}$ 2022 | Firms STW subsidy per hour is 7.73€ | *Décret n° 2022-654* | -1 |
| August $1^{st}$ 2022 | Firms STW subsidy per hour is 7.88€ | *Décret n° 2022-1072* | -1 |
| January $1^{st}$ 2024 | Firms STW subsidy per hour is 8.30€<br>STW allocation to worker is 36% of the wage | *Décret n° 2023-1305* | 1 |

**Notes:** This table does not display the directives sent by the government during the periods that played a major role in STW take-up. Also, this table does not present the evolution before 2008. French legislation.

Table A.3: Calibrated parameters and French values

| Parameter | Calibration |
|---|---|
| $R^{st}$ | 0.709 |
| $f_c$ | 0.3858 |
| $h_c$ | -0.3619 |
| **Steady-State** | **Calibration** |
| $J^f$ | 1.2764 |
| $R$ | 0.5192 |
| $\ell$ | 0.8936 |
| $\phi$ | 0.2179 |
| $q^v$ | 0.1493 |
| $\tau$ | 0.1064 |
| $v$ | 0.054 |
| $w$ | 0.8057 |

# Appendix B Figures

## B.1 Short-Time Work facts

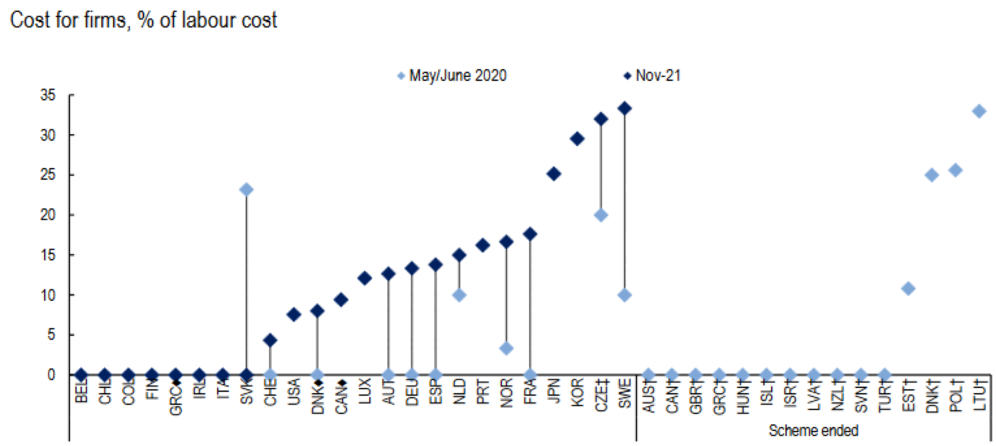



Figure B.1.1: Short-time cost paid by firms

Source: OECD. Country answers and ad hoc updates to OECD Policy Questionnaire on Working Time Regulation and Short-Time Work Schemes. Note: ‡ Czech Republic: For November 2021, refer to Antivirus, regime B. During the Crisis, Antivirus, regime 3A. Canada: There are two schemes, the Canada Emergency wage subsidy, ended in November 2021, and the Work-sharing program (indicated as ♦ ), ongoing. Denmark: There are two schemes, the system of division of labor (Arbejdsfordeling, indicated as ♦), that was temporary redesigned and the Wage compensation scheme (Lønkompensation), ended in June 2021. Greece: There are two schemes, the Special purpose compensation, restricted to some specific sectors and Syn-Ergasia, (indicated as ♦), ongoing. † Schemes no longer operational or not widely available. Mandatory employer contributions for private insurance are not taken into account (consistent with the OECD methodology of Taxing Wages). Norway: for the first 3 months (60 days).

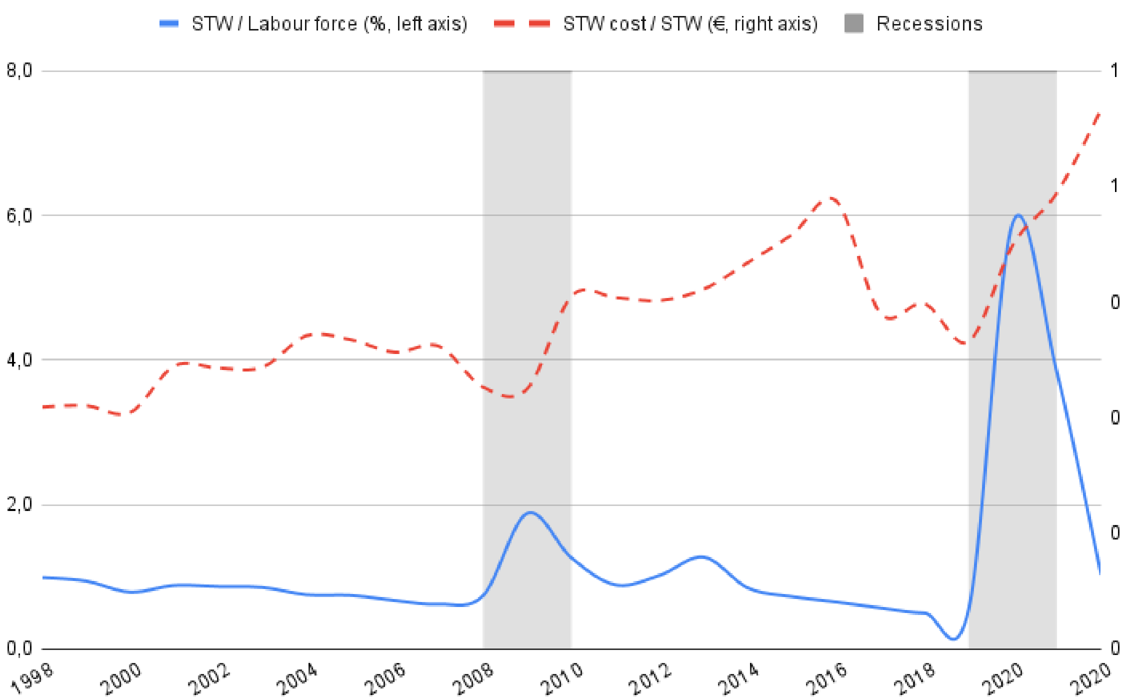
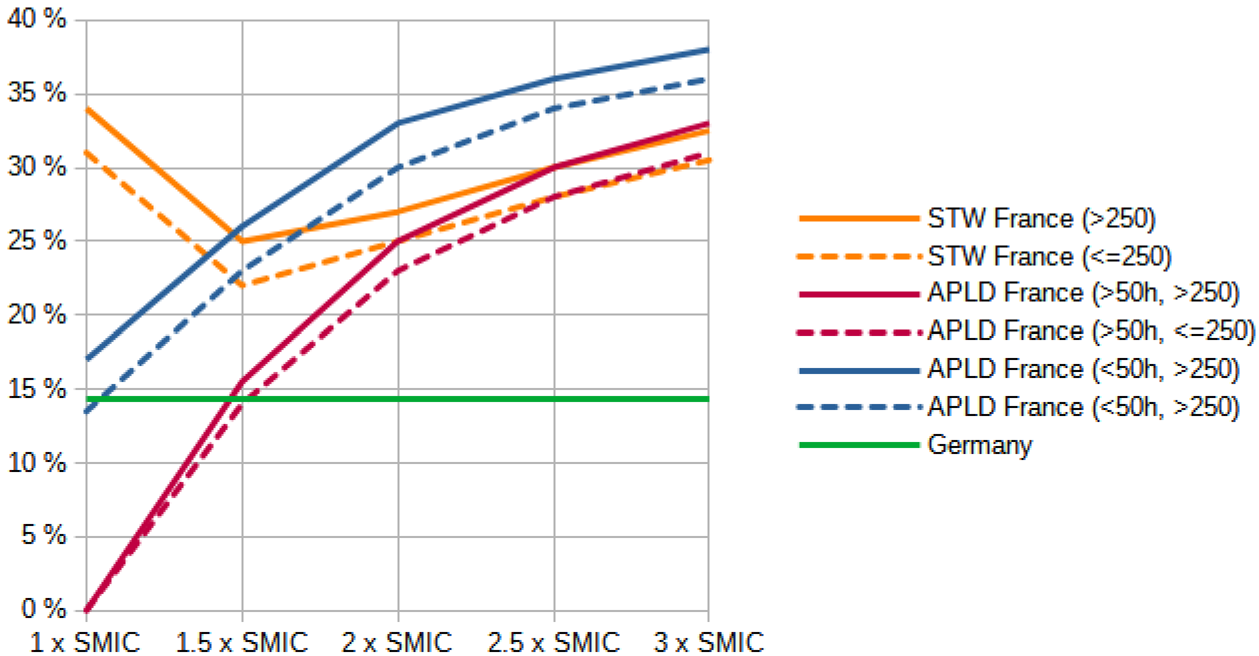



Figure B.1.2: Evolution of STW use in Europe

Source: OECD, Public expenditure and participant stocks on LMP. The graph plots the average GDP growth (blue line, right axis), public STW expenditure per worker (green line, right axis), and share of short-time worker in the labor force (yellow line, left axis) for 6 European countries: France, Germany, Italy, Portugal, Spain, and Belgium. Grey areas correspond to recessions.

Figure B.1.3: Share of ST subsidy paid by the firm in France and Germany in 2009




The y-axis is the share of STW subsidy received by the worker that is paid by the firm. The x-axis is the level of wage of the worker measured according to the French minimum wage (SMIC). The solid blue line corresponds to the *classic* French STW program for firms with more than 250 employees. The dashed blue line corresponds to the *classic* French STW program for firms with less than 250 employees. The solid orange line corresponds to the French long-term STW (ADLP) for the 50-first hours for firms with more than 250 employees. The dashed orange line corresponds to the French long-term STW (ADLP) for the 50-first hours for firms with less than 250 employees. The solid magenta line corresponds to the long-term STW (ADLP) after the 50-first hours for firms with more than 250 employees. The dashed magenta line corresponds to the French long-term STW (ADLP) after the 50-first hours for firms with more than 250 employees. The green line corresponds to the German STW program (after the 07.09)

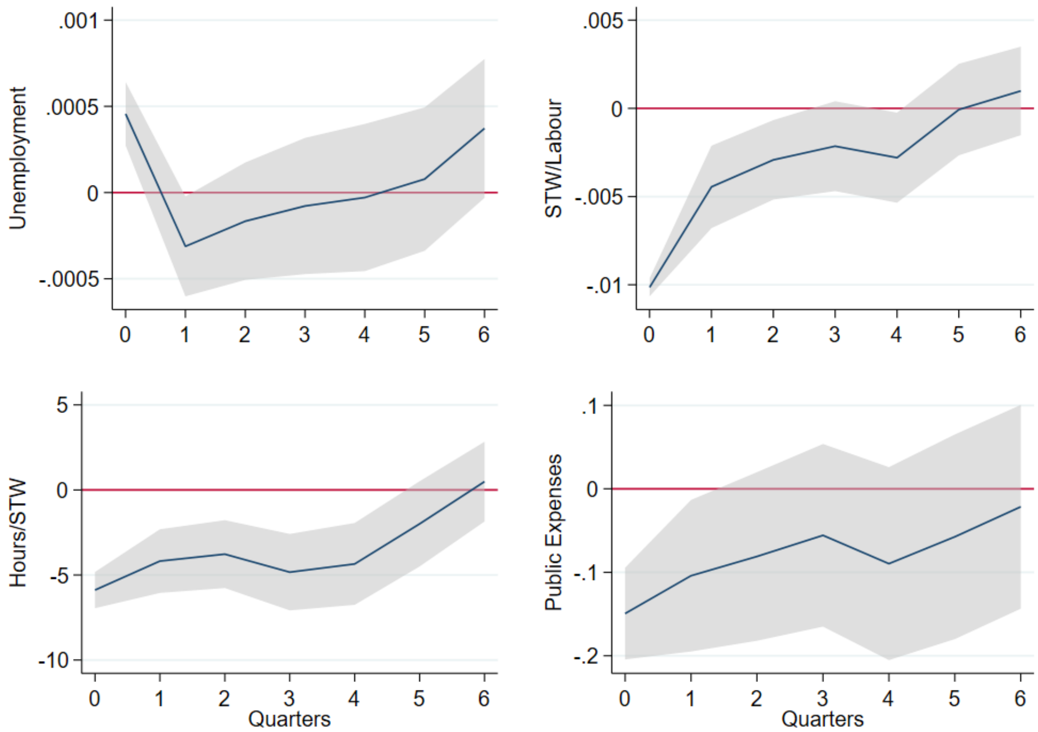



Figure B.1.4: Local-projection: Impulse responses to output shock

Impulse responses to a GDP growth shock. Local projection estimated with unemployment rate, number short-time worker divided by the labor force, the number of hours consummed per workers, and the log of total public expenditure for 2008Q1 to 2024Q1. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

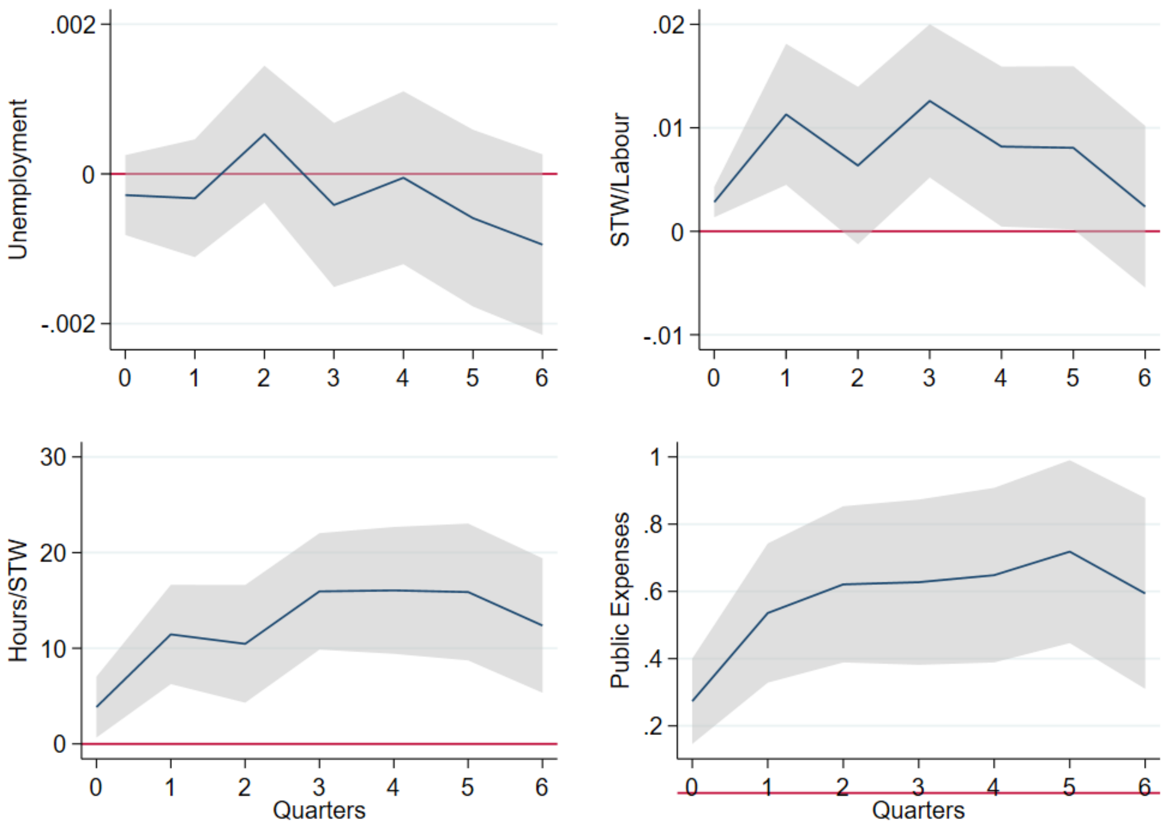



Figure B.1.5: Local-projection: Impulse responses to STW policies

Impulse responses to a public expenditure per hour of STW shock. Local projection estimated with unemployment rate, number short-time worker divided by the labor force, the number of hours consummed per workers, and the log of total public expenditure for 2008Q1 to 2024Q1. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

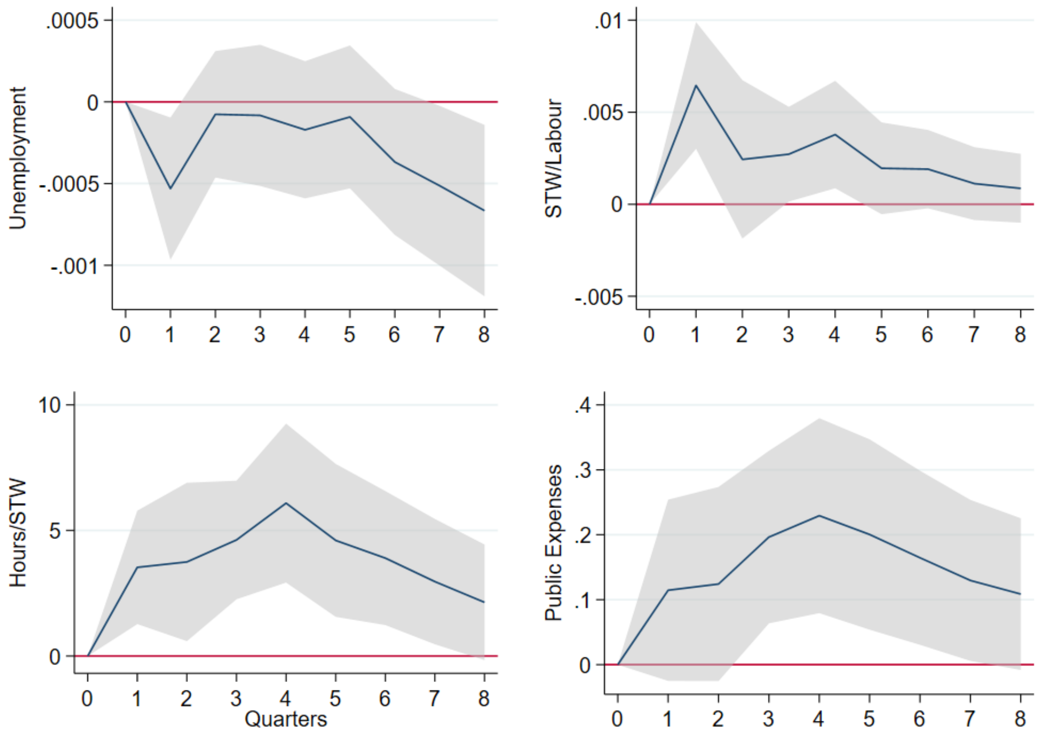



Figure B.1.6: VAR: Impulse responses to STW policies

Impulse responses to a public expenditure per hour of STW shock. VAR estimated with unemployment rate, number short-time worker divided by the labor force, the number of hours consummed per workers, and the log of total public expenditure for 2008Q1 to 2024Q1. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

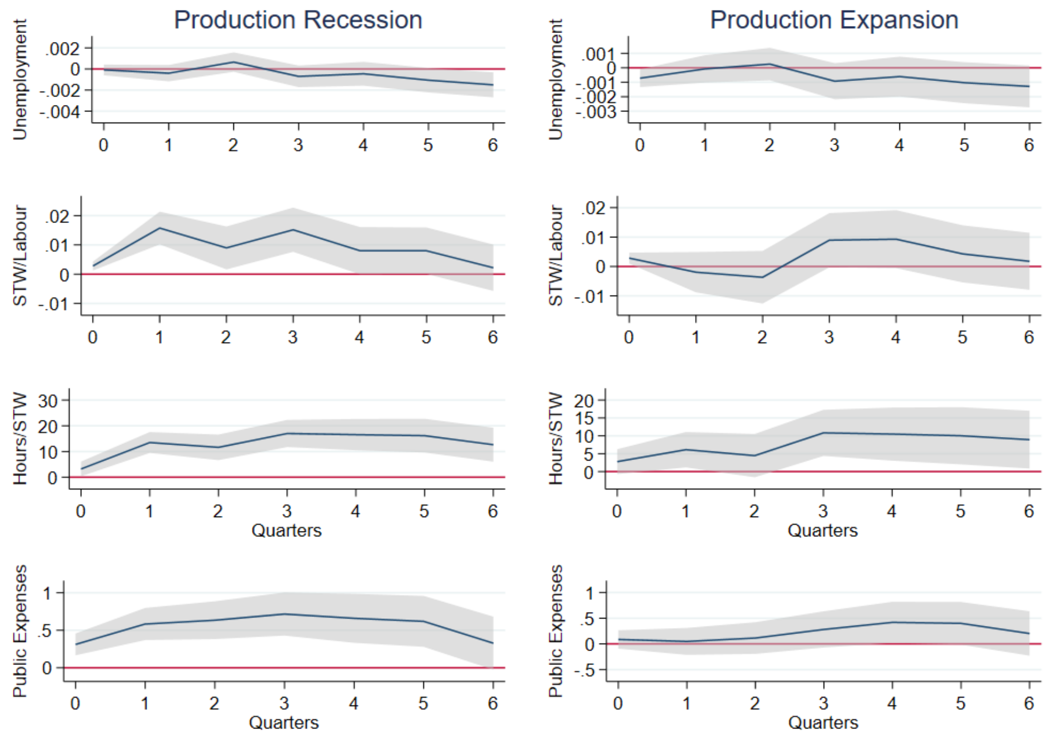



Figure B.1.7: Local-projection: Impulse responses to STW public expenditure per hour

Impulse responses to a STW policy shock. Local projection estimated with unemployment rate, log of number of short-time worker, log hours consummed per workers, and log of total public expenditure for 2008Q1 to 2023Q4. Quarterly responses to a positive one-standard deviation shock. Solid blue lines denote the response to a reform shock, grey area denotes 90 percent confidence bands based on standard errors.

I explore whether initial economic conditions at the time of the shock influence its effect on macroeconomic outcomes. I implement this by allowing the response to vary as follows:

$$Y_{t+k} = \tau_k + \sum_{i}^{k} \beta_i^L F(z_i) R_{t-1-i} + \sum_{i}^{k} \beta_i^H (1 - F(z_i)) R_{t-1-i} + \sum_{i}^{k} \beta_i Y_{t-1-i} + \varepsilon \tag{B.1.1}$$

with $F(z_i) = \frac{exp(-\gamma z_i)}{1+exp(-\gamma z_i)}$

in which $z_i$ is an indicator of economic activity (proxied by GDP growth) normalized to have zero mean and unit variance. $R_{t-1-i}$ denotes the reform shock. The coefficients $\beta_i^L$ and $\beta_i^H$ capture the trade impact of reform shocks at each horizon k in cases of recessions ($F(z_i) \approx 1$ when z goes to minus infinity) and expansions ( $1 - F(z_i) \approx 1$ when z goes to plus infinity), respectively. I follow Duval et al. (2020) and choose $\gamma = 1.5$.

## B.2 Numerical simulation

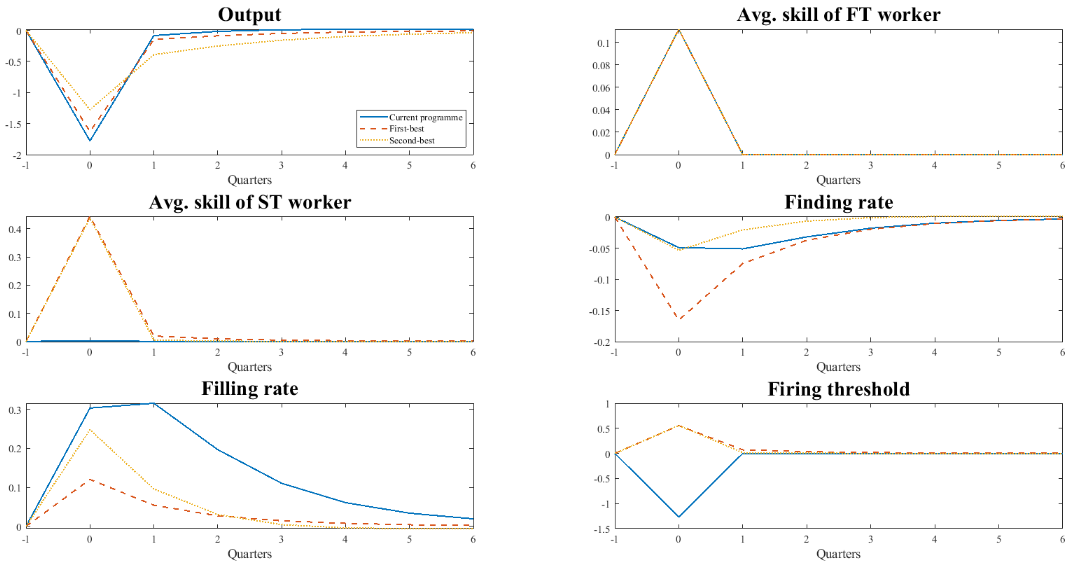



Figure B.2.1: Impulse responses of a negative productivity shock

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in aggregate productivity. The blue line corresponds to the scenario where the short-time cost is reduced to 0 during the recession. The yellow line correspond to a scenario where the policy-maker reduce to 30% of the wage during the recession. The red line corresponds to a scenario where the policy-maker implements a first-best policy during the recession. The top left graph shows the productivity shock (common to the 3 scenarios). The top left graph shows the short-time cost shock. The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labor force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

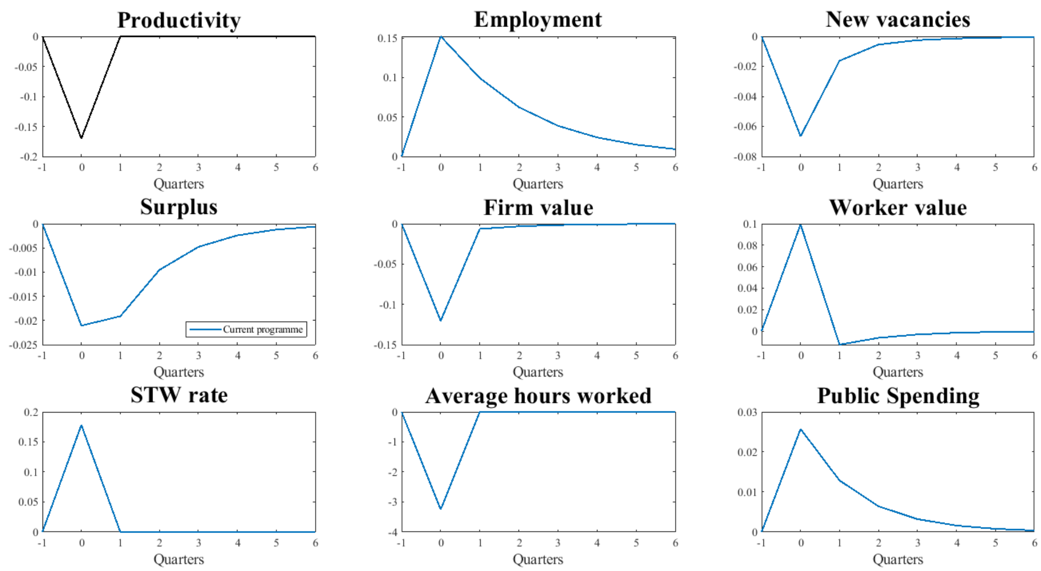



Figure B.2.2: Impulse responses of a negative shock to productivity shock

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in aggregate productivity. The top left graph shows the output response function. The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labor force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

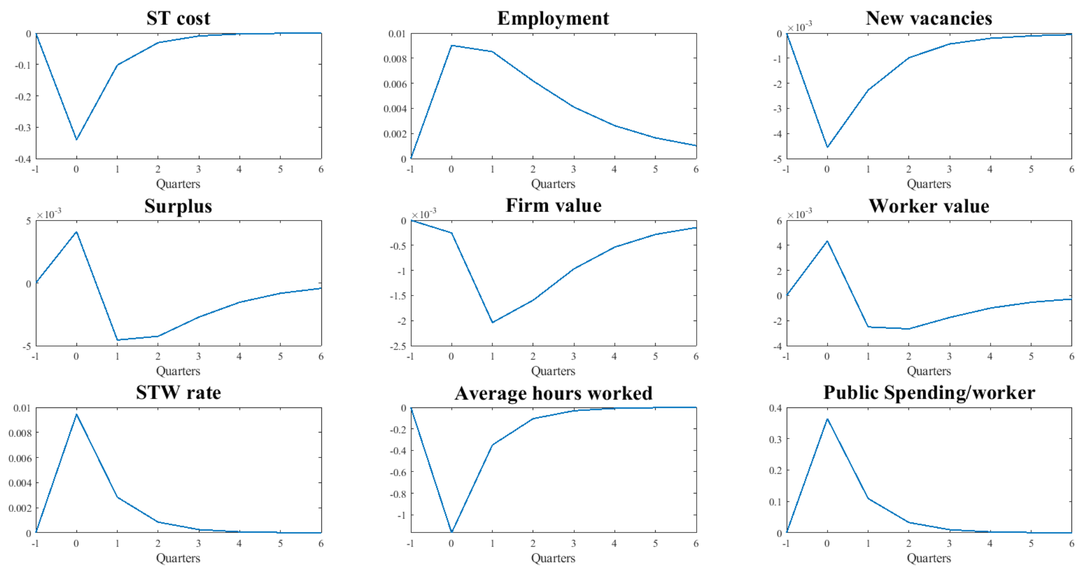



Figure B.2.3: Impulse responses of a negative shock to short-time cost

The impulse responses are given as deviations from the steady state. The shock is implemented as a temporary reduction in short-time cost. The top middle graph shows the employment response function. The top right graph shows the vacancy creation response function. The middle left graph shows the surplus response function. The middle middle graph plots the aggregate firm value response function. The middle right graph plots the aggregate worker value response function. The bottom left graph plots the rate of short-time worker in the labor force. The bottom middle graph plots the average number of working hours. The bottom right graph plots the public spending on short-time work.

# Appendix C Proof

## C.1 Lemma 1

[Back to section 4]

The Social-planner problem is the Hamiltonian

$$\mathcal{H} = \frac{1}{\alpha}(\theta h)^{\alpha} - \beta(h)^2 + \sigma(1-\rho)\mathbb{E}[J^f_{+1} + W_{+1}] - U - J^v$$

The optimal number of hours worked $h^*(\theta) = \left(\frac{\theta^{\alpha}}{2\beta}\right)^{\frac{1}{2-\alpha}}$ is derived from the FOC of the Hamiltonian with respect to $h$: $(\theta^{\alpha}h^{\alpha-2} - 2\beta) = 0$.

The SWF reaches its maximum level if and only if every and only worker-firm pairs for which the surplus is positive $S^f(\theta) \geq 0$ are preserved. With the optimal number of hours worked (7) the surplus is

$$\begin{aligned} S^*(\theta) &= \frac{1}{\alpha}(\theta h^*)^{\alpha} - \beta(h^*)^2 + \sigma(1-\rho)\mathbb{E}[J^f_{+1} + W_{+1}] - U - J^v \\ &= \frac{1}{\alpha}\left[\theta^{\frac{2\alpha}{2-\alpha}}(2\beta)^{\frac{-\alpha}{2-\alpha}}\right] - \beta\theta^{\frac{2\alpha}{2-\alpha}}(2\beta)^{\frac{-2}{2-\alpha}} + \sigma(1-\rho)\mathbb{E}[J^f_{+1} + W_{+1}] - U - J^v. \end{aligned}$$

The optimal ending cut-off $R^*$ is then defined by $S^*(R) = 0$, which gives

$$R^* = \left[\frac{U + J^v - \sigma\mathbb{E}[S_{+1}]}{(2\beta)^{\frac{-\alpha}{2-\alpha}} - \beta(2\beta)^{\frac{-2}{2-\alpha}}}\right]^{\frac{2-\alpha}{2\alpha}}.$$

A match is preserved if the leaving threshold (5) $W - U \geq 0$ and the firing threshold are satisfied (11) $J^f - J^v \geq 0$. Those three conditions are satisfied for equation (8) $\square$

## C.2 Lemma 2

[Back to section 4]

Recalling the firing threshold, $J^f - J^v = -c_f$:

$$R = \left[\frac{\alpha}{1-\alpha}\big(-c_f + b + J^v + \sigma(1-\rho)\mathbb{E}[(1-\phi_{+1})J^f_{+1} + \phi_{+1}J^v_{+1}]\big)\right]^{\frac{1-\alpha}{\alpha}} \ell$$

Where $R$ is the lowest level of productivity $\theta$ at which, all else being equal, the worker is retained. I use the implicit function theorem to find the sign of the first derivative with respect to $b$. I write $F(\theta, b) =$

$J^f - J^v = -c_f$ and examine the sign of the first derivative at the point described by the equation above.

$$\frac{\partial J^f - J^v}{\partial \theta}\Big|_{(R,b)} = -\theta^{\frac{\alpha}{1-\alpha}-1}\ell^{-\frac{\alpha}{1-\alpha}}$$
$$= -R^{\frac{\alpha}{1-\alpha}-1}\ell^{-\frac{\alpha}{1-\alpha}}$$

I assume $R > 0$ and $\ell > 0$. With $R > 0$ implying that there is endogenous firing in the model and $\ell > 0$ implying that the ST cost is lower than the wage paid by the firm $\tau + w$, the reverse would imply that there is no monetary incentive for the firm to participate in the STW program (to the best of my knowledge, no country has ever set the ST cost equal to or greater than the wage). Then $\frac{\partial J^f - J^v}{\partial \theta}|_{(R,b)} < 0$. Now the first derivative with respect to $b$ is

$$\frac{\partial J^f - J^v}{\partial b}|_{(R,b)} = \theta^{\frac{\alpha}{1-\alpha}}\ell^{-\frac{\alpha}{1-\alpha}-1} - 1$$
$$= h - 1$$

By definition the number of hours worked is lower or equal to one so $\frac{\partial J^f - J^v}{\partial b}|_{(R,b)} < 0$. Thus

$$\frac{\partial R}{\partial b} = -\frac{\frac{\partial J^f - J^v}{\partial b}|_{(R,b)}}{\frac{\partial J^f - J^v}{\partial \theta}|_{(R,b)}} \leq 0 \, \square$$

## C.3 Lemma 3

[Back to section 4]

The leaving threshold is not satisfied as long as $a \geq u_b$. The leaving threshold is binding when

$$wh + a(1-h) - \beta(h)^2 + \mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right] \leq u_b + \mathbb{E}\left[q^u W_{+1} + (1-q^u)U_{+1}\right].$$

Since the probability of losing one's job is lower than the probability of finding one's job, it follows that the leaving threshold is not binding if $wh + a(1-h) - \beta(h)^2 \geq u_b$. When hours worked is 0, the condition is $a \geq u_b$. I assume that the worker-firm match expects to be a full-time match at the beginning of the period, i.e. entry into STW is unexpected. Then the leaving threshold is not binding for full-time workers, otherwise the match is not realized. It follows that

$$w - \beta + \mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right] \geq u_b + \mathbb{E}\left[q^u W_{+1} + (1-q^u)U_{+1}\right].$$

Then, since the disutility of working is convex and the utility of consuming is linear. If the leaving threshold is not met at the upper and lower bounds of the hour interval, then the leaving threshold is not met for the entire interval between the two bounds. Then, if $a \geq u_b$, the exit threshold is never met.

## C.4 Lemma 4

[Back to section 4]

The optimal Nash bargaining program after log-linearisation:

$$\max_{(w,a)} p \ln(W-U) + (1-p)\ln(J^f - J^v)$$

The F.O.C. with respect to the wage is:

$$\begin{aligned} p\frac{W'_w}{W-U} + (1-p)\frac{{J^f}'_w}{J^f - J^v} &= 0 \\ -\frac{W'_w}{(W-U)}\frac{J^f - J^v}{{J^f}'_w} &= \frac{1-p}{p} \end{aligned}$$

With $W'_w = \frac{\partial W}{\partial w}$ and ${J^f}'_w = \frac{\partial J^f}{\partial w}$. The F.O.C. with respect to the short-time compensation is:

$$\begin{aligned} p\frac{W'_a}{U} + (1-p)\frac{{J^f}'_a}{J^f - J^v} &= 0 \\ -\frac{W'_a}{(W-U)}\frac{J^f - J^v}{{J^f}'_a} &= \frac{1-p}{p} \\ -\frac{W'_a}{(W-U)}\frac{J^f - J^v}{{J^f}'_a} &= -\frac{W'_w}{(W-U)}\frac{J^f - J^v}{{J^f}'_w} \\ -\frac{W'_a}{{J^f}'_a} &= -\frac{W'_w}{J^f_{'w}} \\ W'_a {J^f}'_w - {J^f}'_a W'_w &= 0 \end{aligned}$$

Recalling that:

$$W = wh + a(1-h) - \beta(h)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right]$$

and

$$J^f = \frac{1}{\alpha}(\theta h)^\alpha - (\tau + w)h - b(1-h) + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})J^f_{+1} + \phi J^v_{+1}\right]$$

and

$$h = \left(\frac{\theta^\alpha}{\tau + w - b}\right)^{\frac{1}{1-\alpha}}$$

Then with $\ell = w - b$ and $a = b$ I have

$$W = \left(\frac{\theta}{\ell}\right)^{\frac{\alpha}{1-\alpha}} + a - \beta(h)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right]$$

and

$$J^f = \frac{1-\alpha}{\alpha}\left(\frac{\theta}{\ell}\right)^{\frac{\alpha}{1-\alpha}} - a + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})J^f_{+1} + \phi J^v_{+1}\right].$$

I write $\frac{\partial h^2}{\partial x} = h'_{2x}$. Thus, the F.O.C. w.r.t. $a$ becomes

$$\begin{aligned}
W'_a {J^f}'_w - {J^f}'_a W'_w &= 0 \\
\left(\frac{\alpha}{1-\alpha}h + 1 + \beta h'_{2w}\right)(h) - \left(\frac{\alpha}{1-\alpha}h + \beta h'_{2w}\right)(h-1) &= 0 \\
\frac{\alpha}{1-\alpha}h + 1 + \beta h'_{2w} - \frac{\alpha}{1-\alpha}h - \beta h'_{2w} - \frac{\alpha}{1-\alpha} - \frac{\beta h'_{2w}}{h} &= 0 \\
1 - \frac{\alpha}{1-\alpha} + \beta\frac{h'_{2w}}{h} &= 0 \\
\frac{1}{1-\alpha} + \beta\frac{2}{1-\alpha}(\theta)^{\frac{2\alpha}{1-\alpha} - \frac{\alpha}{1-\alpha}}\ell^{-\frac{2}{1-\alpha} - 1 + \frac{1}{1-\alpha}} &= 0 \\
\ell^{\frac{2-\alpha}{1-\alpha}} &= 2\beta\theta^{\frac{\alpha}{1-\alpha}} \\
\ell^* &= \left(2\beta\theta^{\frac{\alpha}{1-\alpha}}\right)^{\frac{1-\alpha}{2-\alpha}}
\end{aligned}$$

Re-inserting this into $h$, I have:

$$h^* = \left(\frac{\theta^\alpha}{2\beta}\right)^{\frac{1}{2-\alpha}}$$

With $h^*$, the F.O.C. w.r.t. $w$ is now:

$$
\begin{aligned}
p\frac{W'_w}{W-U} + (1-p)\frac{J^{f\prime}_w}{J^f - J^v} &= 0 \\
p\frac{h}{W-U} + (1-p)\frac{-h}{J^f - J^v} &= 0 \\
p(J^f - J^v) &= (1-p)(W-U) \\
W - U &= pS^f \\
J^t_f - J^v &= (1-p)S^f
\end{aligned}
$$

Lemma 3 is proved $\square$

## C.5 Proposition 1

[Back to section 4]

The first-best allocation can be implemented in a decentralized economy by choosing the values of the policy instruments $b$, $a$, $\tau^{st}$, and $\tau^f$, that jointly satisfy the system:

$$
\begin{aligned}
\ell(\theta) = w + \tau^{st}(\theta) - b(\theta) &= \left[(2\beta)\theta^{\frac{\alpha}{1-\alpha}}\right]^{\frac{1-\alpha}{2-\alpha}} \\
b(\theta) &= (1-k^1)(\theta h^*(\theta))^\alpha + k^1\beta(h^*(\theta))^2 - J^v + (1-k^1)\sigma(S_{+1}) \\
a(\theta) &= k^2(\theta h^*(\theta))^\alpha + (1-k^2)\beta(h^*(\theta))^2 + U - (1-k^2)\sigma(S_{+1}) \\
\int_{R^{st}}^1 \tau^f dF(\theta) &= \int_R^{R^{st}} (a-b)(1-h)dF(\theta) - \int_R^{R^{st}} \tau^{st} h dF(\theta).
\end{aligned}
$$

The first-line of the equation is obtained by equalizing the optimal hours worked with the hours worked set in the decentralized economy $h^*(\theta) = h(\theta, \ell)$:

$$
\left(\frac{\theta^\alpha}{2\beta}\right)^{\frac{1}{2-\alpha}} = \left(\frac{\theta^\alpha}{\ell}\right)^{\frac{1}{1-\alpha}}
$$

This result can also be obtained by maximizing the social welfare under the public constraint w.r.t. the set of policy-maker tools, i.e. solving the following Hamiltonian problem:

$$
\mathcal{H} = S(\theta)f(\theta) + \lambda(\theta)\left[\tau^{st}h(\theta) + (b-a)(1-h(\theta))\right]f(\theta)
$$

From Lemma 2, a match should end if it generates a negative surplus. A firm and a worker leaves their match if their respective surplus are negative. Thus, by setting the worker's and firm's surplus as a linear function of their cumulative surplus the policy-maker can implement the first-best allocation. Then the value of the ST compensation are derived as follows $a$:

$$
\begin{aligned}
W(\theta) - U &= k^1 S^*(\theta) \\
wh^*(\theta) + a(1-h^*(\theta)) - \beta(h^*(\theta))^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right] & \\
= k^1 \left(\frac{1}{\alpha}(\theta h^*)^\alpha - \beta(h^*)^2 + \sigma(1-\rho)\mathbb{E}[J^f_{+1} + W_{+1}] - (U - J^v)\right) & \\
a(\theta)(1-h^*(\theta)) = k^1 \left((\theta h^*(\theta))^\alpha - J^v + \sigma(1-\rho)(J^f_{+1} + W_{+1} - \phi_{+1}S_{+1})\right) + (1-k^1)\left(\beta(h^*(\theta))^2 + U\right) & \\
-\sigma(1-\rho)\left((1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right) - wh^*(\theta) &
\end{aligned}
$$

Similarly for the ST cost $b$:

$$
\begin{aligned}
J^f(\theta) - J^v &= k^2 S^*(\theta) \\
\frac{1}{\alpha}(\theta h^*(\theta))^\alpha - (\tau^{st} + w)h^*(\theta) - b(1-h^*(\theta) + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})J^f_{+1} + \phi J^v_{+1}\right] - J^v & \\
= k^2(\theta h^*(\theta))^\alpha + (1-k^2)\beta(h^*(\theta))^2 + U - (1-k^2)\sigma(S_{+1}) & \\
b(\theta) = (1-k^2)\left((\theta h^*(\theta))^\alpha - J^v\right) + k^2\left(\beta(h^*(\theta))^2 + U - \sigma(1-\rho)(J^f_{+1} + W_{+1} - \phi_{+1}S_{+1})\right) + \ell^*(\theta)h^*(\theta)\square &
\end{aligned}
$$

## C.6 Corollary 1

[Back to section 4]

From proposition 1:

$$
\begin{aligned}
a^*(\theta) &:= W - U = k^1 S^f(\theta) \\
b^*(\theta) &:= J^f - J^v = k^2 S^f(\theta) \\
\tau^{stw*} &= \ell^* + b^* - w
\end{aligned}
$$

With the firm's and worker's match value (1) (3):

$$
\begin{aligned}
a^*(\theta) &= \frac{1}{1-h}\left(k^1 S^f(\theta) - wh + \beta(h)^2 - \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right] + U\right) \\
-b^*(\theta) &= k^2 S^f(\theta) - \frac{1}{\alpha}(\theta h)^\alpha + \ell h + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})J^f_{+1} + \phi J^v_{+1}\right] + J^v \\
\tau^{stw*} &= \ell^* + b^* - w
\end{aligned}
$$

Then the deficit generated for a worker with productivity draw $\theta$:

$$
\begin{aligned}
&\tau^{stw}h - (a-b)(1-h) \\
&= (\ell^* + b^* - w)h - a - b + bh \\
&= (\ell^* - w)h - \left(k^1 S^f(\theta) + wh - \beta(h)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right] + U\right) \\
&- \left(k^2 S^f(\theta) - \frac{1}{\alpha}(\theta h)^\alpha + \ell h + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})J^f_{+1} + \phi J^v_{+1}\right] + J^v\right) \\
&= \frac{1}{\alpha}(\theta h)^\alpha - \beta(h)^2 + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}\right] - U \\
&+ \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{+1})J^f_{+1} + \phi J^v_{+1}\right] - J^v - k^1 S^f(\theta) - k^2 S^f(\theta) \\
&= S^f(\theta) - (k^1 + k^2) S^f(\theta)
\end{aligned}
$$

Then it follows that

$$
\tau^{stw}h - (a-b)(1-h) = S^f(\theta) - (k^1+k^2)S^f(\theta)
\begin{cases}
< 0 \text{ if } k^1 + k^2 < 1 \\
= 0 \text{ if } k^1 + k^2 = 1 \\
> 0 \text{ if } k^1 + k^2 > 1
\end{cases}
\tag{C.6.1}
$$

Thus

$$
\tau^f = \left[\int_R^{R^{stw}} (a^* - b^*)(1-h^*)dF(\theta) - \int_{R_i}^{R^{stw}} \tau^{stw*} h^* dF(\theta)\right]
\begin{cases}
< 0 \text{ if } k^1 + k^2 < 1 \\
= 0 \text{ if } k^1 + k^2 = 1 \\
> 0 \text{ if } k^1 + k^2 > 1
\end{cases}
\tag{C.6.2}
$$

Corollary 1 is proved □

## C.7 Proposition 2

[Back to section 5]

In second-best, the policy maker's problem is now a Lagrangian one:

$$\mathcal{L} = \int \{S(\theta)\} \, dF(\theta) + \lambda \left[ \int_R^{R^{stw}} (a-b)(1-h)dF(\theta) \right]$$

From subsection C.3, with $a \geq u_b$ the leaving threshold is not binding. Then, the ST compensation does not affect the hours worked or the end of the match. So the FOC with respect to $a$ gives

$$1 - h(\theta) + \lambda(\theta)(-(1-h(\theta))) = 0.$$

The Lagrange multiplier $\lambda$ captures the weight of the public budget constraint and is equal to 1. This is a consequence of the linear social welfare function, linear utility from consumption and the fact that the policymaker spends all the budget collected through taxes without loss, every money in the pocket of the policy maker has the same social value as a money in the pocket of a worker. It follows that the Lagrangian equals the surplus from all match with the ST compensation paid by the firm $a = b$ and without labor tax $\tau^{stw} = 0$. I rearrange the Lagrangian with the value of $\lambda$ and derive it according to the short-time cost $b$ using the Leibniz rule to find the FOC

$$-S(R)\frac{d}{db}R + \int \frac{d}{db}\{S(\theta)\}dF(\theta) = 0.$$

I write $\epsilon^R = \frac{b}{R}\frac{\partial R}{\partial b}$ and $\epsilon^S = \frac{b}{\int_R^{R^{st}} S(\theta)dF(\theta)} \frac{\partial \int_R^{R^{st}} S(\theta)dF(\theta)}{\partial b}$ it gives:

$$\frac{\epsilon^R}{\epsilon^S} = \frac{\int_R^{R^{st}} S(\theta)dF(\theta)}{R \cdot S(R) \cdot f(R)}.$$

## C.8 Corollary 2

[Back to section 5]

From proposition 2, I have:

$$\frac{\epsilon^R}{\epsilon^S} = \cdot \frac{\int_R^{R^{st}} S(\theta)dF(\theta)}{R \cdot S(R) \cdot f(R)}.$$

I can rewrite the first term as follows:

$$\frac{dR}{db}\frac{b}{R} = b((R\ell)^{\frac{\alpha-1}{\alpha}} - \ell^{-1}) = \frac{1}{\ell}(1 + \epsilon^R_{c_f}\frac{\ell}{c_f}).$$

with $\epsilon^R_{c_f} = \frac{\partial R}{\partial c_f}\frac{c_f}{R}$, I can rewrite proposition 2:

$$\frac{\epsilon^R}{\epsilon^S} = \frac{\int_R^{R^{st}} S(\theta)dF(\theta)}{R \cdot S(R) \cdot f(R)} = \frac{b}{c_f}\left(\frac{c_f}{\ell} + \epsilon^R_{c_f}\right)$$

Thus, the ST cost is equal to

$$b = \frac{e^S \cdot \int S(\theta)dF(\theta) \cdot c_f}{R \cdot S(R) \cdot f(R)(\frac{c_f}{\ell} + \epsilon^R_{c_f})} > 0 \ \square$$

$$b' \in ]J^f(\chi^{f*}) - J^v; b^*(\tilde{\theta})[\ ; \text{ with } \tilde{\theta} = \int_{\chi^f}^{\chi^{stw}} \theta dF(\theta) \text{ and } b^*(\theta) = \ell^*(\theta) - w - \tilde{\tau}.$$

# D Numerical Simulation

## D.1 Equilibrium Condition

The search equilibrium is a system of 11 equations for 11 variables summarized in the vector:

$$[J^v, R, w, u, N, \phi, \phi^e, n, q^u, q^v, \tau]$$

The unemployment level

$$u = 1 - N \tag{D.1.1}$$

The employment level

$$N = (1-\rho)(1-\phi)(N_{t-1} + m_{t-1}) \tag{D.1.2}$$

The total job destruction

$$\phi = \phi^e + \phi^x \tag{D.1.3}$$

The endogenous rate of job destruction

$$\phi^e = \int_0^R dF(\theta) = R^4 \tag{D.1.4}$$

Finding and filling rate

$$q^u = \frac{m}{u} = \mu \left(\frac{v}{u}\right)^{1-\alpha_m} \tag{D.1.5}$$
$$q^v = \frac{m}{v} = \mu \left(\frac{v}{u}\right)^{\alpha_m} \tag{D.1.6}$$

The firing threshold

$$R = \left[\frac{\alpha}{1-\alpha}\left(w(b-l_c) + h_c + f_c + \sigma(1-\rho)\mathbb{E}[(1-\phi_{+1})J^f_{+1}]\right)\right]^{\frac{1-\alpha}{\alpha}} \ell \tag{D.1.7}$$

The average filling value

$$J^f = \int_R^{R^{st}} \left\{\frac{1}{\alpha}(\theta h)^\alpha - wh - (b+h_c)(1-h) - f_c\right\} dF(\theta) + \int_{R^{st}}^1 \left\{\frac{1}{\alpha}\theta^\alpha - (\tau + w - f_c)\right\} dF(\theta)$$
$$- \phi^e l_c + \sigma(1-\rho)\mathbb{E}\left[(1-\phi_{t+1})J^f_{+1} + \phi J^v_{+1}\right]$$

with $h = \left(\frac{\theta^{\alpha}}{\ell}\right)^{\frac{1}{1-\alpha}}$, $f(\theta) = 4\theta^3$, and $J^v = 0$, it becomes:

$$\begin{aligned} J^f = & \frac{1-\alpha}{\alpha}\ell^{\frac{-\alpha}{1-\alpha}}\frac{8}{9}\left[(R^{st})^{\frac{5}{2}} - R^{\frac{5}{2}}\right] - (b \cdot w + h_c)\left[(R^{st})^4 - R^4\right] + \frac{1}{\alpha}\frac{12}{7}\left[1 - (R^{st})^{\frac{7}{3}}\right] \\ & - (\tau + w)\left[1 - (R^{st})^4\right] - (1-\phi^e)f_c - \phi^e \cdot l_c \cdot w + \sigma(1-\rho)(1-\phi_{+1})J^f_{+1} \end{aligned} \tag{D.1.8}$$

The vacancy value, with $\xi \to 0$:

$$\begin{aligned} J^v &= -\kappa + \sigma(1-\rho)\mathbb{E}\left[q^v J^f_{+1} + (1-q^v)J^v_{+1}\right] = 0 \\ \mathbb{E}[J^f_{+1}] &= \frac{\kappa}{\sigma(1-\rho)q^v} \end{aligned} \tag{D.1.9}$$

The working value of a full-time worker

$$W = w - \beta + \sigma(1-\rho)\mathbb{E}[(1-\phi_{+1})W_{+1} + \phi_{+1}U_{+1}]$$

The unemployment value

$$U = u_b + \sigma\mathbb{E}\left[q^u W_{t+1} + (1-q^u)U_{t+1}\right]$$

The sharing rule:

$$\begin{aligned} W - U &= \frac{p}{1-p}J^f \\ w &= \beta - u_b + \sigma\frac{p}{1-p}J^f(1 - \phi - q^u - \frac{1}{\sigma}) \end{aligned} \tag{D.1.10}$$

The budget constraint

$$\begin{aligned} \int_{R^{stw}}^{1}\tau^f dF(\theta) &= \int_{R}^{R^{stw}}(a-b)\cdot w \cdot (1-h)dF(\theta) - \int_{R}^{R^{stw}}\tau^{stw}h dF(\theta) \\ \tau &= \left[1 - (R^{st})^4\right]^{-1}(a-b)\cdot w \cdot \left[(R^{st})^4 - R^4 + \ell^{\frac{-1}{1-\alpha}}\frac{8}{9}\left((R^{st})^{\frac{9}{2}} - R^{\frac{9}{2}}\right)\right] \end{aligned} \tag{D.1.11}$$

The aggregate output

$$
\begin{aligned}
Y = N \int_{R}^{R^{st}} & \left\{ \frac{1}{\alpha} (\theta h)^{\alpha} - \tau h - f_c \right\} dF(\theta) \\
& + N \int_{R^{st}}^{1} \left\{ \frac{1}{\alpha} \theta^{\alpha} - \tau - f_c \right\} dF(\theta) \\
& - N \phi^{e} l_f - u u_b - v \kappa
\end{aligned}
$$

## D.2 Steady state

Unemployment rate

$$
u = 1 - N \tag{D.2.1}
$$

Vacancy post

$$
\begin{aligned}
q^{u} &= \mu \left( \frac{v}{u} \right)^{\alpha_m} \\
v &= u \left( \frac{q^{u}}{\mu} \right)^{\frac{1}{\alpha_m}}
\end{aligned} \tag{D.2.2}
$$

Job filling rate

$$
q^{v} = \mu \left( \frac{v}{u} \right)^{1-\alpha_m} \tag{D.2.3}
$$

Firm match value

$$
J^{f} = \frac{\kappa}{\sigma q^{v}} \tag{D.2.4}
$$

Ending probability

$$
\begin{aligned}
N &= N(1-\phi)N + (1-\phi) q^{u} \cdot u \\
\phi &= \frac{q^{u} \cdot u}{N + q^{u} \cdot u}
\end{aligned} \tag{D.2.5}
$$

Endogenous and exogenous ending

$$
\phi^{x} = \frac{2}{3} \phi \; ; \; \phi^{e} = \frac{1}{3} \phi
$$

Firing threshold

$$
\begin{aligned}
\phi^e &= \int_0^R dF(\theta) = \int_0^R 4\theta^3 d\theta = R^4 \\
R &= (\phi^e)^{\frac{1}{4}}
\end{aligned}
\tag{D.2.6}
$$

STW threshold

$$
\begin{aligned}
\phi^{st} &= \int_R^{R^{st}} dF(\theta) = (R^{st})^4 - R^4 \\
R^{st} &= \left(\phi^{st} - R^4\right)^{\frac{1}{4}}
\end{aligned}
\tag{D.2.7}
$$

Labor tax

$$
\tau = \left[1 - (R^{st})^4\right]^{-1} (a-b) \cdot w \cdot \left[(R^{st})^4 - R^4 + \ell^{\frac{-1}{1-\alpha}} \frac{8}{9}\left((R^{st})^{\frac{9}{2}} - R^{\frac{9}{2}}\right)\right] \tag{D.2.8}
$$

Labor cost

$$
\begin{aligned}
\mathbb{E}(h^{st}) &= \frac{\int_R^{R^{st}} h(\theta) dF(\theta)}{\int_R^{R^{st}} dF(\theta)} \\
\int_R^{R^{st}} h(\theta) dF(\theta) &= \int_R^{R^{st}} \left(\frac{\theta^\alpha}{\ell}\right)^{\frac{1}{1-\alpha}} 4\theta^3 d\theta = \ell^{\frac{-1}{1-\alpha}} \frac{8}{11}\left[(R^{st})^{\frac{11}{2}} - R^{\frac{11}{2}}\right] \\
\mathbb{E}(h^{st}) &= \ell^{\frac{-1}{1-\alpha}} \frac{8}{11}\left[(R^{st})^{\frac{11}{2}} - R^{\frac{11}{2}}\right] / \phi^{st} \\
\ell &= \left[\frac{8}{11} \frac{(R^{st})^{\frac{11}{2}} - R^{\frac{11}{2}}}{\phi^{st} \cdot \mathbb{E}(h^{st})}\right]^{1-\alpha}
\end{aligned}
$$

Adjusting hours cost

$$
\begin{aligned}
\ell &= (1-b) \cdot w - h_c \\
h_c &= (1-b) \cdot w - \ell
\end{aligned}
\tag{D.2.9}
$$

Fixed cost

$$
\begin{aligned}
J^f(R) &= \frac{1-\alpha}{\alpha}\left(\frac{R}{\ell}\right)^{\frac{\alpha}{1-\alpha}} - (b + h_c) \cdot w - f_c + \sigma(1-\phi) J^f = -l_c \cdot w \\
f_c &= \frac{1-\alpha}{\alpha}\left(\frac{R}{\ell}\right)^{\frac{\alpha}{1-\alpha}} - (b + h_c - l_c) \cdot w_c + \left(\sigma(1-\phi) - 1\right) J^f
\end{aligned}
\tag{D.2.10}
$$

labor tax

$$\tau = \left[1-(R^{st})^4\right]^{-1}(a-b)\cdot w\cdot\left[(R^{st})^4 - R^4 + \ell^{\frac{-1}{1-\alpha}}\frac{8}{9}\big((R^{st})^{\frac{9}{2}} - R^{\frac{9}{2}}\big)\right] \tag{D.2.11}$$

The wage

$$\begin{aligned}
J^f &= \frac{1-\alpha}{\alpha}\ell^{\frac{-\alpha}{1-\alpha}}\frac{8}{9}\left[(R^{st})^{\frac{5}{2}} - R^{\frac{5}{2}}\right] - (b\cdot w + h_c)\left[(R^{st})^4 - R^4\right] + \frac{1}{\alpha}\frac{12}{7}\left[1-(R^{st})^{\frac{7}{3}}\right] \\
&\quad - (\tau + w)\left[1-(R^{st})^4\right] - R^4\cdot l_c\cdot w + \sigma(1-\rho)(1-\phi)J^f \\
J^f &= \frac{1-\alpha}{\alpha}\ell^{\frac{-\alpha}{1-\alpha}}\frac{8}{9}\left[(R^{st})^{\frac{5}{2}} - R^{\frac{5}{2}}\right] - (b\cdot w + (1-b)\cdot w - \ell)\left[(R^{st})^4 - R^4\right] \\
&\quad + \frac{1}{\alpha}\frac{12}{7}\left[1-(R^{st})^{\frac{7}{3}}\right] - \left(\left[1-(R^{st})^4\right]^{-1}(a-b)\cdot w\cdot\left[(R^{st})^4 - R^4 + \ell^{\frac{-1}{1-\alpha}}\frac{8}{9}\big((R^{st})^{\frac{9}{2}} - R^{\frac{9}{2}}\big)\right] + w\right)\left[1-(R^{st})^4\right] \\
&\quad - (1-\phi^e)f_c - \phi^e\cdot l_c\cdot w + \sigma(1-\phi)J^f \\
0 &= (a) - w\cdot(b) \\
(a) &= \frac{1-\alpha}{\alpha}\ell^{\frac{-\alpha}{1-\alpha}}\frac{8}{9}\left[(R^{st})^{\frac{5}{2}} - R^{\frac{5}{2}}\right] - \ell\left[(R^{st})^4 - R^4\right] + \frac{1}{\alpha}\frac{12}{7}\left[1-(R^{st})^{\frac{7}{3}}\right] - (1-\phi^e)f_c + \big(\sigma(1-\phi)-1\big)J^f \\
(b) &= (R^{st})^4 - R^4 + \left(\frac{a-b}{1-(R^{st4}}\left[(R^{st})^4 - R^4 + \ell^{\frac{-1}{1-\alpha}}\frac{8}{9}\big((R^{st})^{\frac{9}{2}} - R^{\frac{9}{2}}\big)\right] + 1\right)[1-(R^{st})^4] + \phi^e l_c \\
w &= \frac{(a)}{(b)}
\end{aligned} \tag{D.2.12}$$