Partial Identification of Treatment Response under Complementarity and Substitutability

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Motivation

Many environments in which we want to **evaluate a policy** but

- **Treatments**
  - Unobservable **heterogeneous effects**
  - **Endogenous**
  - Hard to find **IVs**

- **Interactions** are relevant (SUTVA violated) and:
  - Observably **heterogeneous** (positive/negative spillovers)
  - Complements vs Substitutes
  - **Endogenous**

- **Almost always the case in reality**
  - often no RCT
  - especially for monetary and prudential policies

- **Partial identification** approach;
- **Signs predicted** by ec. theory;

Examples

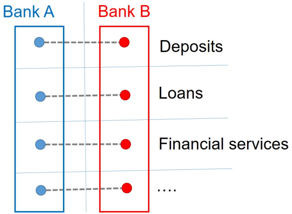
- **Social Interactions**
  - Legislative Activity
  - Conflicts and Warfare
- **Market Interactions**
  - Financial Networks
  - Production Networks

Examples

- **Social Interactions**
  - Legislative Activity
  - Conflicts and Warfare

- **Market Interactions**
  - Financial Networks

    T: (treatment) Monetary/financial policy
    Y: (outcome) Balance sheet items (q,p)
    HI: (heterogeneous interactions) By bank
    → Complementarity within /Competition between banks



Allen and Gale (2000), Allen and Babus (2009), Denbee et al. (2021), Drehmann and Tarashev (2011), Upper (2006), Anand et al. (2015), Peltonen et al. (2019), Mistrulli (2011), Fantino and Benetton (2021), Andreeva and Garcia-Posada (2021).

  - Production Networks

Contributions

- **Econometric Theory**:
  - **Partial identification** $\rightarrow$ Upper and Lower bounds for $E[Y|T]$
  - **Endogenous outcomes and treatments**
  - **Heterogeneous Interactions**
- **Economic Theory**:
  - **Microfoundation** - supermodular games
  - **Heterogeneous fixed points thrm**
  - **Monotone comparative statics**
- **Financial and Monetary Economics**:
  - **Framework to evaluate monetary and prudential policies**
  - **Bounds for bank's potential credit** under given CB funding (to it and to others)
  - 1 std increase in CB funding $\rightarrow$ up to 60% (1%) std increase in bank's credit to the NFS | other banks borrow $<$ ($>$) first (ninth) decile of the distribution.

Related Literature

- **Policy evaluation** accounting for interactions
  - Angelucci and De Giorgi (2009); Arduini et al. (2020); Auerbach and Tabord-Meehan (2021); Barrera-Osorio et al. (2011); Forastiere et al. (2021); Leung (2020, 2022).
- **Partial identification**
  - Manski (1995, 2003, 2009); Tamer (2010); Molinari (2020).
  - with interactions: Manski (2013); Lazzati (2015).
- **Game theory**
  - Noncooperative supermodular games: Topkis (1979), Vives (1985, 1990), Milgrom and Roberts (1990, 1994).
- **Finance**
  - Spillovers: Berg et al. (2021); Huber (2022); Mian et al. (2022); Pietrosanti and Rainone (2023).
  - Central bank funding: Benetton and Fantino (2021), Andreeva and Garcia-Posada (2021), Berg et al. (2021b), Carpinelli and Crosignani (2021), Garcia-Posada and Marchetti (2016), Crosignani, e Castro and Fonseca (2020),

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Outline

Structural Functions

Let S_n be a system of $n > 1$ simultaneous equations in n variables

$$\begin{aligned} y_1 &= f_1(y_2, y_3, \ldots, y_n), \\ y_2 &= f_2(y_1, y_3, \ldots, y_n), \\ &\vdots \\ y_n &= f_n(y_1, y_2 \ldots, y_{n-1}). \end{aligned} \tag{1}$$

The functions $f_1, \ldots, f_n$ are called structural functions of S_n. y_i takes value from a finite complete lattice $(\mathbb{P}_i, \leq_i, \sqcap_i, \sqcup_i)$,where $\leq_i$ is the partial order over the set $\mathbb{P}_i$, $\sqcap_i$ and $\sqcup_i$, are respectively the meet and the join of the lattice.

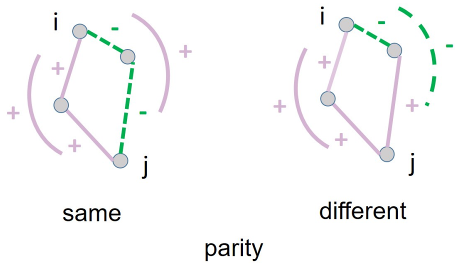
Dependence Graph and Parity

Let $D = (Y, E)$ be the dependence graph of S_n.

E represent the sign of the dependence, $+$ solid edge or - dashed edge.

The parity of a path in D is even or odd.

Dependencies in S_n are consistent if for every pair of nodes (y_i, y_j) all paths between y_i and y_j have the same parity.



Outline

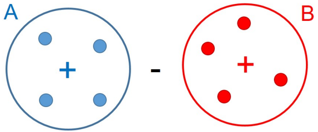
Heterogeneous Monotonicity

Consider a partition (A, B), **blocks**, of the set $\{1, \ldots, n\}$.

Definition 1

A vector function $\mathbf{F} : \mathbb{P} \to \mathbb{P}$ is h-monotonic w.r.t. a partition (A, B), if and only if

- for any $i \in A$, f_i monotonically increases in y_j, if $j \in A$, and monotonically decreases in y_j, if $j \in B$.
- for any $i \in B$, f_i monotonically increases in y_j, if $j \in B$, and monotonically decreases in y_j, if $j \in A$.

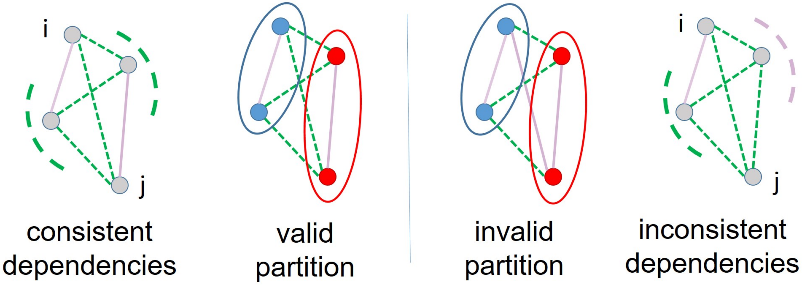


If a function vector $\mathbf{F}$ of a system S_n is *h-monotonic* w.r.t. a partition (A, B), then (A, B) is said to be a valid partition of S_n.

Consistency and Valid Partition

Lemma 2

There exists a valid partition *for a system* S_n *iff the dependences in* S_n *are* consistent.



Outline

Heterogeneous Fixed Points

Theorem 3 (Kanade et al. (2005))

If the function vector $\mathbf{F} : \mathbb{P} \to \mathbb{P}$ *of a system* S_n *is h-monotonic w.r.t. a valid partition* (A, B), *then* $HFP^{(A,B)}(\mathbf{F}), HFP^{(B,A)}(\mathbf{F}) \in FP(\mathbf{F})$.

- $HFP^{(A,B)}(\mathbf{F}) \to$ *least possible values for* A *and the greatest possible values for* B*;*
- $HFP^{(B,A)}(\mathbf{F}) \to$ *least possible values for* B *and the greatest possible values for* A*;*

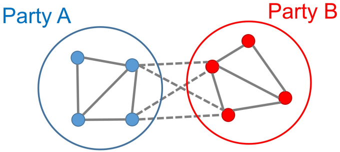
Assume that $\mathbf{F}$ is *h-monotonic* in t, the **treatment vector**.

Theorem 4

Let $\mathbb{P}$ *be a complete lattice and* $\mathbf{F} : \mathbb{P} \times T \to \mathbb{P}$. *Suppose* $\mathbf{F}$ *is h-monotonic with respect to a valid partition* (A, B), *then also* $HFP^{(A,B)}(\mathbf{F})$ *and* $HFP^{(A,B)}(\mathbf{F})$ *are h-monotonic in* $\mathbf{t}$.

- **Social Interactions**
  - Legislative Activity

    T: (treatment) Incumbency/PAC
    Y: (outcome) Re-election
    HI: (heterogenous interactions) By party
    → + within same party | - between different parties



HFP$_{(B,A)}$	HFP$_{(A,B)}$
H	L
L	H

Peoples (2008), Masket (2008), Rogowski and Sinclair (2012), Cohen and Malloy (2014), Harmon et al. (2019), Battaglini and Patacchini (2018), Fowler (2006), Kirkland (2011), Battaglini et al. (2020), Battaglini et al. (2022).

  - Conflicts and Warfare

H-Monotonic Comparative Statics

- Strategic complementarities → Supermodular games
- Tarsky's FPT → highest / lowest solution for all
- Milgrom & Roberts (90,94) → **Monotonic comparative statics**

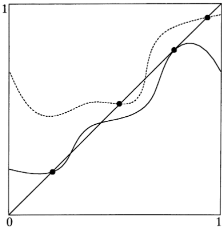


FIGURE 2. THE HIGHEST AND LOWEST FIXED POINTS OF THE HIGHER FUNCTION ARE GREATER THAN THOSE OF THE LOWER FUNCTION

- Strategic complementarities / substitutabilities
- Heterogeneous FPT → highest (lowest) / lowest (highest) solution for A (B) → **H-Monotonic comparative statics**

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Outline

Model Specification

Large groups (but finite) indexed by r where **interactions are anonymous**.

Let the units in population $\mathbb{P}$ be partitioned in A and B. Let y^A and y^B be the outcome for the two partitions.

$$y^A_{ir} = f^A_{ir}(h^0(\mathbf{t_r}),\, S^0_{ir}(\mathbf{y_r^{A/i}}),\, S^0_{ir}(\mathbf{y_r^{B}})), \tag{2}$$

$$y^B_{ir} = f^B_{ir}(h^0(\mathbf{t_r}),\; S^0_{ir}(\mathbf{y_r^{B/i}}),\, S^0_{ir}(\mathbf{y_r^{A}})).$$

where S^0_{ir} can be any function that respects stochastic dominance, e.g. $S^0_{ir} = E(\mathbf{y_r^{k/i}}(\mathbf{t_r})), E(\mathbf{y_r^{k}}(\mathbf{t_r}))$ with $k \in \mathcal{K} \equiv \{A, B\}$, and h^0 is the exposure mapping **EF**.

⏩ Exposure Mapping

We assume **H-monotonicity** (**HMI**) w.r.t. these aggregating functions.

Outline

H-Monotone Treatment Response and Selection

Following Manski and Pepper (2000):

- **Monotone Treatment Response (HMTR)**: *each person's* **wage function** *is weakly increasing in years of schooling.*

$$Y(1) - Y(0) \geq 0.$$

- **Monotone Treatment Selection (HMTS)**: *persons who select higher levels of schooling have weakly higher* **mean wage** *functions than those who don't.*

$$E[Y(1)|Z=0] \leq E[Y(1)|Z=1], \quad E[Y(0)|Z=0] \leq E[Y(0)|Z=1].$$

- Extend **(+) own treatment** assumption to **(+) treatments in the same block** members and **(-) treatments in the other block** members. Manski (2013) opposing + reinforcing interactions.

Following Lazzati (2015) we provide sufficient conditions to use realized treatments as monotone instrumental variables.

Equilibrium Selection (ES)

HFPT $\rightarrow$ multiple equilibria $\rightarrow$ force the extreme HFP

Let $\phi^k(\mathbf{d_r}, r)$ be the solution set of the system of structural equations for a given k, r, $\mathbf{d_r}$.

Assumption 1 (ES)

Within each group units in block A select either the largest (the smallest) element of $\phi^A(\mathbf{d_r}, r)$ and units in block B select the smallest (the largest) element of $\phi^B(\mathbf{d_r}, r)$ for each $\mathbf{d_r} \in \mathcal{D}$.

Lemma 5

If Assumptions **HMI**, **HMTR**, **EF**, *and* **ES** *hold, then $y_r^A(d)$ and $y_r^B(d)$ are h-monotonic in d.*

Lemma 6

If Assumptions **HMI**, **HMTS**, **EF**, *and* **ES** *hold, then for each $d \in \mathcal{D}$, $E(y^A(d)|z = s)$ and $E(y^B(d)|z = s)$ are h-monotonic in s.*

Outline

H-monotonic Treatment Response Bounds (HMTR)

Using just the **law of total probability**.

Under H-monotonic Treatment Response (HMTR) $d_1^A \geq d_2^A \implies y^A(d_1^A) \geq y^A(d_2^A)$

Observations with realized treatments above and below a value can be used to construct more informative bounds.

$$
\begin{array}{cccccc}
\overline{y}^A & P(z^A \leq d^A) & + & E(y^A|z^A > d^A) & P(z^A > d^A) \\
\vee| & & & \vee| & \\
E(y^A(d^A)|z^A \leq d^A) & P(z^A \leq d^A) & + & E(y^A(d^A)|z^A > d^A) & P(z^A > d^A) \\
\vee| & & & \vee| & \\
E(y^A|z^A \leq d^A) & P(z^A \leq d^A) & + & \underline{y}^A & P(z^A > d^A)
\end{array}
$$

Average across realized smaller (larger) treatments, $z^A \leq d^A$ ($z^A \geq d^A$) can be used to construct the lower (upper) bound of the potential outcome at d^A.

visual examples

H-monotonic Treatment Selection Bounds

Under HMTS

$$\begin{array}{lcccc} z^A < d^A & \Longrightarrow & E(y^A(t^A)|u^A = z^A) & \leq & E(y^A|u^A = d^A), \\ z^A = d^A & \Longrightarrow & E(y^A(t^A)|u^A = z^A) & = & E(y^A|u^A = d^A), \\ z^A > d^A & \Longrightarrow & E(y^A(t^A)|u^A = z^A) & \geq & E(y^A|u^A = d^A). \end{array}$$

Realized outcomes with realized treatments at d^A provide information for upper (lower) bounds of potential outcomes at d^A for individual with lower (higher) realized treatments.

$$\begin{array}{ccccc} E(y^A|z^A = d^A) & P(z^A \leq d^A) & + & \overline{y}^A & P(z^A > d^A) \\ \vee\!\mid & & & \vee\!\mid & \\ E(y^A(d^A)|z^A \leq d^A) & P(z^A \leq d^A) & + & E(y^A(d^A)|z^A > d^A) & P(z^A > d^A) \\ \vee\!\mid & & & \vee\!\mid & \\ \underline{y}^A & P(z^A \leq d^A) & + & E(y^A|z^A = d^A) & P(z^A > d^A) \end{array}$$

visual examples

H-monotonic Treatment Response and Selection Bounds

$$
\begin{array}{cccccc}
E(y^A|z^A = d^A) & P(z^A \leq d^A) & + & E(y^A|z^A > d^A) & P(z^A = d^A) \\
\vee| & & & \vee| & \\
E(y^A(d^A)|z^A \leq d^A) & P(z^A \leq d^A) & + & E(y^A(d^A)|z^A > d^A) & P(z^A > d^A) \\
\vee| & & & \vee| & \\
E(y^A|z^A \leq d^A) & P(z^A = d^A) & + & E(y^A|z^A = d^A) & P(z^A > d^A)
\end{array}
$$

Combination of HMTR and HMTS can further narrow the bounds.

▸▸ visual examples

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Empirical Application

- Focus on **banks balance sheet's items** (complements and substitutes).
- Define **H-monotonicity assumptions** among them.
- Verify **interpretability** as comparative statics (valid partition).

- Focus on the effect of **central bank funding** (treatment) on **credit to NFS** (outcome).
- Estimate **bounds for potential credit** at the single bank level.
- **Data**: unique, proprietary dataset of balance sheet items at bank level (Individual Balance-Sheet Items or IBSI) for the Italian banking system quarterly data from 2011 to 2023 with 600 to 400 banks each year.

Banks Balance Sheets and Dependencies

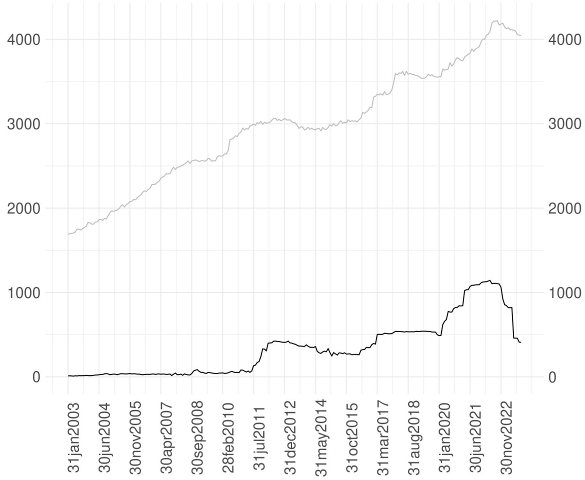
Monotonic Assumptions - MC Interactions among Banks' Assets and Liabilities

		A						L					
		c_i	c_{-i}	l_i	l_{-i}	a_i	a_{-i}	d_i	d_{-i}	b_i	b_{-i}	s_i	s_{-i}
A	c_i		$-$ [M]					$+$ [C]		$+$ [C]		$+$ [C]	
	c_{-i}								$+$ [C]		$+$ [C]		$+$ [C]
	l_i				$-$ [M]			$+$ [C]		$+$ [C]		$+$ [C]	
	l_{-i}								$+$ [C]		$+$ [C]		$+$ [C]
	a_i						$-$ [M]	$+$ [C]		$+$ [C]		$+$ [C]	
	a_{-i}								$+$ [C]		$+$ [C]		$+$ [C]
L	d_i								$-$ [M]				
	d_{-i}												
	b_i										$-$ [M]		
	b_{-i}												
	s_i												$-$ [M]
	s_{-i}												

Notes. A and L stand respectively for assets and liabilities. i is a bank and $-i$ is the rest of the system. l_i and b_i are respectively the amount of lending and borrowing in the interbank market. c_i is the amount of credit provided by bank i. d_i is the amount of deposits held at bank i. a_i is the amount of securities held by bank i. s_i is the amount of bonds issued by bank i. $+$ and $-$ indicate respectively positive and negative interactions. [M]: same side of the market. [C]: balance sheet expansion complementarity.

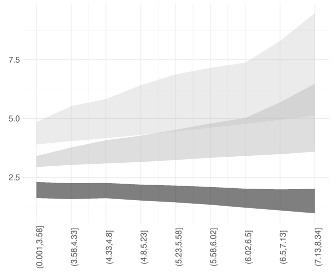
MOCS $\longrightarrow$ valid partition

Credit to NFS and central bank funding

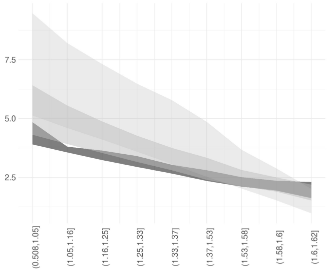


Monthly data of average stock of loans to the NFS and central bank funding for the IBSI sample of the Italian banking system in million euro. The stock of loans to the NFS is reported in grey. The stock of central bank funding is reported in black.

Partial Identification under HMIRS - Estimated Bounds



(a) Own treatment



(b) Others' treatment

1 std increase in CB funding $\rightarrow$ up to 60% (1%) std increase in bank's credit to the NFS | other banks borrow $<$ ($>$) first (ninth) decile of the distribution.

A significant portion of the positive effect stemming from one's own treatment can be mitigated by competitive interactions.

Conclusions

- Provide a new methodology to evaluate policies
- Under substitutabilities and complementarities
- Complement existing tools used
- Providing bounds of potential outcomes

- Using the heterogeneous fixed point theorem
- Interpretable as comparative statics exercises

- Suitable for fairly complex environments
- Narrow and meaningful estimates for the effect of central bank funding on credit to NFS

THANK YOU!

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Lattice and Complete Lattice

Definition 7 (Lattice)

A partially ordered set $(\mathbb{P}, \leq, \sqcap, \sqcup)$ is a lattice if for any two elements $a, b \in \mathbb{P}$ there exist:

- a greatest lower bound (the *meet*), denoted by $a \sqcap b$,
- a least upper bound (the *join*), denoted by $a \sqcup b$.

Definition 8 (Complete Lattice)

A lattice is complete if *every* subset is also a lattice.

Lattice Examples

In the **real line** $\mathbb{R}$, the real numbers form a complete lattice. This is because every subset of $\mathbb{R}$ has both a supremum (least upper bound) and an infimum (greatest lower bound).

- **Join** (least upper bound):
  - For any two real numbers a and b, their join is the greater of the two values:
  - $a \sqcup b = max(a, b)$.
- **Meet** (greatest lower bound):
  - For any two real numbers a and b, their meet is the lesser of the two values:
  - $a \sqcap b = min(a, b)$.

Since every subset of the real line $\mathbb{R}$ has both a supremum (least upper bound) and an infimum (greatest lower bound), $\mathbb{R}$ forms a complete lattice under the usual order relation.

back

Lattice Examples (con't)

The **set of all divisors of 12** is: $D = 1, 2, 3, 4, 6, 12$. We can order the set of divisors according to the relation $a \geq b$ if and only if a divides b. This order forms a lattice.

- **Meet** (greatest lower bound):
  - For any two elements a and b in the set D, their meet is the greatest common divisor (gcd) of a and b.
  - $gcd(2, 3) = 1$, so $2 \sqcup 3 = 1$.
  - $gcd(4, 6) = 2$, so $4 \sqcup 6 = 2$.
- **Join** (least upper bound):
  - For any two elements a and b in the set D, their join is the least common multiple (lcm) of a and b;
  - $lcm(2, 3) = 6$, so $2 \sqcup 3 = 6$;
  - $lcm(4, 6) = 12$, so $4 \sqcup 6 = 12$.

These operations of meet (gcd) and join (lcm) on the set of all divisors of 12 satisfy the properties of a lattice: each pair of elements has a meet and a join.

back

Dependence Graph and Parity

Let $D = (V, E)$ be the dependence graph of S_n, if y_i depends positively on y_j then we have that in the set E this is represented by a solid edge. On the contrary if y_i depends negatively on y_j, then there is a dashed edge from y_i to y_j.

The parity of a path in D is even if the path has an even number of dashed edges (or whatever number of solid ones), otherwise its parity is odd.

back

Within Group Opposing Interactions

Theorem 9

Given a partition (A, B) of the units, let us suppose that interactions are opposing within the same group and reinforcing between groups. Dependencies are consistent if $[G]_{ij}^{k} = 0$ for all values of $k = 2, 4, \ldots$.

Opposing between

Exposure Mapping

- **Assumption**: structural functions depends on $\mathbf{t_r}$ through a (true) discrete exposure mapping $h^0 : \mathcal{T} \to \mathcal{D}^0 \subset \mathbb{R}^3$.
- Own treatment status and the **fraction (or average) of treated units** in group r: $h^0(\mathbf{t_r}) = (t_{ir}, \frac{1}{n_r^A} \sum_{j \in r, A/i} t_{jr}, \frac{1}{n_r^B} \sum_{j \in r, B/i} t_{jr}) = \{d_{ir}^{0I}, d_{ir}^{0A}, d_{ir}^{0B}\} = d_{ir}^0 \in \mathcal{D}^0$ (Leung, 2020).
- A solution to the system (2) is a vector of potential outcomes $y_r^k(\mathbf{d_r^0}) : \mathcal{D}^0 \to Y_r^k$, which is indexed by the **effective treatment** ($\mathbf{d_r^0}$), (Manski, 2013).

Assumption 2 (EF)

$h^0(\cdot)$ *is coarser than or equal to* $\mathbf{h}(\cdot) = d_{ir} \in \mathcal{D}$ *(the exposure function selected by the researcher) and it is a function which respects stochastic dominance w.r.t.* d.

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Examples

- **SUTVA**: $h_0(\cdot)$ is constant, *i.e.*, $y_{ir}(\mathbf{t_r}) = y_{ir}(t_i)$. Individual potential outcomes depend solely on individual treatment.
- **Exchangeability**: $h_0(\mathbf{t_r}) = (t_{ir}, \sum_{j \in r} t_{jr})$, individual potential outcomes depend on the sum of treated peers, regardless of their identity.
- **Reference groups**: $h_0(\cdot) : \{0,1\}^{n_r} \rightarrow \{0,1\}^{k_r}, \; k_r < n_r$, with n_r denotes group sizes, individual outcome depend on a subset of peers (e.g., closest friends in a class).
- **Non-excheangeable peers**: $h_0(\mathbf{t}_r) = \mathbf{t}_r$ (identity function), individual outcomes depend on the whole vector of assignment, with no dimensionality reduction.

(decreasing coarseness)

Back

H-monotonic Interactions (HMI)

Assumption 3 (H-Monotonic Interactions)

For each $i \in \mathcal{P}$, $k \in \mathcal{K}$, $r \in \mathcal{R}$, *and* $d_{ir} \in \mathcal{D}$, $f_{ir}^{k}(d_{ir}, E(\mathbf{y_r^{A/i}}), E(\mathbf{y_r^{B}}))$ *are increasing in* $E(\mathbf{y_r^{A/i}})$ *and decreasing in* $E(\mathbf{y_r^{B}})$.

Under HMI we can apply HFP theorem

H-monotonic Treatment Response (HMTR)

Assumption 4 (HMTR)

For each $i \in \mathcal{P}$, $r \in \mathcal{R}$, $E(\mathbf{y_r^{A/i}}) \in \Delta_y^{A/i}$, *and* $E(\mathbf{y^B}_r) \in \Delta_y^B$,
$f_{ir}^A(d_{ir}^I, \cdot) \geq f_{ir}^A(d_{ir}^{I'}, \cdot)$ *and*
$f_{ir}^A(d_{ir}^A, \cdot) \geq f_{ir}^A(d_{ir}^{A'}, \cdot)$, *while*
$f_{ir}^A(d_{ir}^B, \cdot) \leq f_{ir}^A(d_{ir}^{B'}, \cdot)$
for all $d_{ir}^P \geq d_{ir}^{P'}$, *where* $P = \{I, A, B\}$.

Specular for B.

H-monotonic Treatment Selection (HMTS)

Let $\mathbf{z}$ be the realized effective treatment vector $(\mathbf{d})$.

Assumption 5 (H-Monotone Treatment Selection)

$P\left(\mathbf{F}_{ir}^{k} = f_{ir}^{k} \mid \mathbf{z} = \mathbf{s}\right)$ *is non-decreasing in* s_{ir}^{l} *and* s_{ir}^{k}*, and non-increasing in* s_{ir}^{-k}*, with* $k \in \mathcal{K} \equiv \{A, B\}$ *and* $k \neq l$*.*

Manki and Pepper (2018) Example

- Use **Manski and Pepper (2018)'s Example** on how right-to-carry laws affect crime rates.

$$ATE_{dx} = E[Y_d(t=1)|X] - E[Y_d(t=0)|X] \tag{3}$$

t = treatment, d = year. State *VA* (Virginia) is treated in 1990 and state *MD* (Maryland) no. Suppose we are interested in

$$\begin{aligned} ATE_{90,VA} &= E[Y_{90}(1)|X=VA] - E[Y_{90}(0)|X=VA] \quad (4) \\ &= Y_{VA,90}(1) - Y_{VA,90}(0) \end{aligned}$$

- Having **more data does not reduce the dependence of empirical findings on the assumptions**
- A linear model with a homogeneous treatment effect and fixed effects:

$$Y_{jd} = \theta t + X_{jd}\beta + \alpha_j + \gamma_d + \epsilon_{jd} \tag{5}$$

partial ID back-wrong

Commonly Used (Strong) Invariance Assumptions

- The data may be used to compute three simple but different findings on the counterfactual 1990 murder rate in Virginia, $Y_{VA,90}(0)$, under alternative invariance assumptions.
- **Time invariance:**

$$Y_{VA,90}(0) = Y_{VA,88}(0) = Y_{VA,88} = 7.75 \to ATE_{90,VA} = 1.06$$

- **Interstate invariance:**

$$Y_{VA,90}(0) = Y_{MD,90}(0) = Y_{MD,90} = 11.55 \to ATE_{90,VA} = -2.74$$

- **Difference-in-difference invariance:**

$$\begin{aligned} Y_{VA,90}(0) &= [Y_{MD,90}(0) - Y_{MD,88}(0)] + Y_{VA,88}(0) \quad (6) \\ &= (Y_{MD,90} - Y_{MD,88}) + Y_{VA,88} = 9.67 \\ &\to ATE_{90,VA} = -0.86 \end{aligned}$$

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Bounded Variation

- **Invariance assumptions are strong**
  - **Why should they hold exactly?**
  - Why should the **treatment effect be constant** across units and time (as assumed in linear models)?
- **Bounded variation is much weaker than invariance assumption**.

  Bounded time variation:

  $$|Y_{jd}(t) - Y_{je}(t)| < \delta_{j(de)}$$

  Bounded interstate variation:

  $$|Y_{jd}(t) - Y_{kd}(t)| < \delta_{(jk)d}$$

  Bounded DID variation:

  $$|[Y_{jd}(t) - Y_{je}(t)] - [Y_{kd}(t) - Y_{ke}(t)]| < \delta_{(jk)(de)}$$

These bounded-variation assumptions have identifying power.

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Upper and Lower Bounds for the Outcome

- Under a joint interstate and time bounded variation assumptions, the **counterfactual** murder rate $Y_{VA,90}(0)$ **could be bounded** as follows:

$$max(Y_{MD,90} - \delta_{(MD,VA)90}, Y_{VA,88} - \delta_{VA(90,88)})$$
$$\leq Y_{VA,90}(0) \leq$$
$$min(Y_{MD,90} + \delta_{(MD,VA)90}, Y_{VA,88} + \delta_{VA(90,88)})$$

- A **necessary condition** for this assumption to be valid is that $\delta_{VA(90,88)} + \delta_{(MD,VA)90} \geq |Y_{MD,90} - Y_{VA,88}|$. Otherwise, the lower bound exceeds the upper bound.
- While the invariance assumptions are inconsistent with the observed pre-1989 data, the bound parameters can be chosen to **ensure internal consistency**.

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Bounding Expected Values using LTP

- Often relying on the law of total probability (LTP).

$$\begin{aligned} E[Y(T)|X] &= E[Y(T)|X, Z = S]P(Z = S) \\ &+ E[Y(T)|X, Z \neq S]P(Z \neq S) \end{aligned} \tag{7}$$

where T is the (hypothetical) treatment, Z is the realized treatment and S is a value it can assume.

- Bounds can be proposed for the expected values:

$$\begin{aligned} E[Y(T)|X] &\leq E[Y(T)|X, Z = S]P(Z = S) \\ &+ \overline{Y}P(Z \neq S) = UB \end{aligned} \tag{8}$$

- Eventually assuming only monotonocity.

▸▸ partial ID ▸▸ back-wrong

A Simple Bank Asset-Liability Model

For each bank i:

$$\begin{aligned} \text{maximize} \quad & \pi_{i,t} = r_c(c_i^t) + r_l(l_i^t) + r_a(a_i^t) + \\ & -[h_d(d_i^t) + h_b(b_i^t) + h_s(s_i^t) + h_e(e_i^t)] \\ \text{subject to} \quad & c_i^t + l_i^t + a_i^t = d_i^t + b_i^t + s_i^t + e_i^t, \end{aligned}$$

where

- c_i^t is the credit to the real economy,
- l_i^t is the credit to other banks,
- a_i^t is the securities held on the asset side.
- d_i^t is the deposits from the real economy,
- b_i^t is the credit from other banks,
- s_i^t is the debt issued, and
- e_i^t is the equity.

For simplicity we do not include reserves.

A Simple Bank Asset-Liability Model

A typical regression with banks exposed to some policy x_i^t.

$$\begin{aligned} c_i^t &= x_i^t\beta + l_i^t\alpha_1 + a_i^t\alpha_2 + d_i^t\alpha_3 + b_i^t\alpha_4 + s_i^t\alpha_5 + e_i^t\alpha_6 \\ &+ c_j^t\iota + \gamma_i + \gamma_t + \epsilon \end{aligned}$$

- RHS controls could be LHS endogenous variables

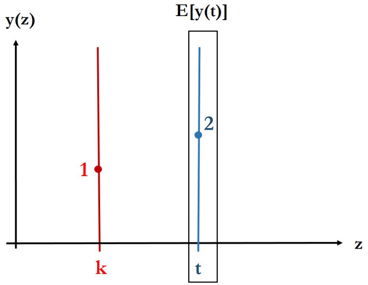
$$\begin{aligned} a_i^t &= x_i^t\beta^a + l_i^t\alpha_1^a + c_i^t\alpha_2^a + d_i^t\alpha_3^a + b_i^t\alpha_4^a + s_i^t\alpha_5^a + e_i^t\alpha_6^a \\ &+ a_j^t\iota^a + \gamma_i^a + \gamma_t^a + \epsilon \end{aligned}$$

- Using lags does not help because these processes are persistent and you still have simultaneous omitted vars

Visual Example

Suppose we observe $k < t$ and

- $k, y(k)$ for individual 1;
- $t, y(t)$ for individual 2.



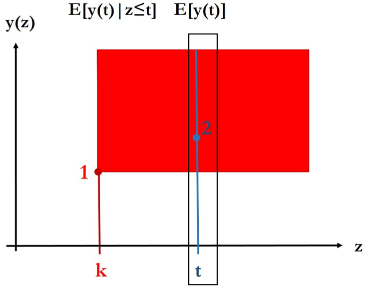
Examples:

- Social HI $\rightarrow$ y: Legislator effectiveness; z:party mates committee chairs.
- Market HI $\rightarrow$ y: Credit granted; z: competitors decreased funding.
- Simpler $\rightarrow$ y: Worker Earnings; z: years of schooling.

Let us focus on the potential outcome of y at t. Back

Visual Example - HMIR

Let us focus on $E[y(t)|z \leq t]$. The red area represents the potential outcome of 1 at $z = t$ under *HMIT*. So *HMIR* informs the LB for $E[y(t)]$ because $y_1(k)$ can be used to construct a LB for $E[y(t)|z \leq t]$, as the outcome of observations with $k \leq t$ can not be greater than their outcome at t.



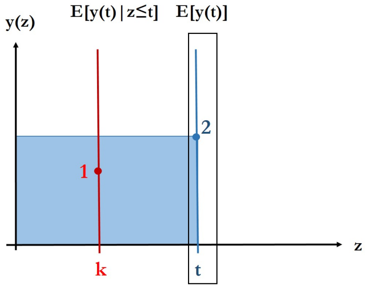
More formally *HMIT*,

$$
\begin{aligned}
k < t \quad &\Longrightarrow \quad y(k) < y(t) \\
&\Longrightarrow \quad E[y(k)] < E[y(t)] \\
&\Longrightarrow \quad E[y(k)|z \leq t] < E[y(t)|z \leq t] \\
&\Longrightarrow \quad E[y(k)|z = k]P(z = k) + E[y(t)|z = t]P(z = t) \leq E[y(t)|z \leq t]
\end{aligned}
$$

Back

Visual Example - HMIS

The blue area represents the potential outcome of 1 at $z = t$ under *HMIS*. So *HMIS* informs the UB for $E[y(t)]$, because $y_1(t)$ can be used to construct a UB for $E[y(t|z \leq t)]$, as the outcome of observations at t can not be smaller than their outcome at $z \leq t$.



More formally *HMIS*,

$$
\begin{aligned}
&y(t|z = k) \leq y(t|z = t) \\
&E[y(t|z = k)] \leq E[y(t|z = t)] \implies E[y(t|z \leq t)] \leq E[y(t|z = t)]
\end{aligned}
$$

▸▸ Back

- *HMIT* can not be used to construct a UB for $E[y(t)|z \leq t)]$ as it does not imply that the outcome of observations at t can not be smaller than their outcome at $z \leq t$, because observations at k can have different structural functions.
- *HMIS* can not be used to construct a LB for $E[y(t)|z \leq t)]$ as it does not imply that the outcome of observations at k can not be greater than their outcome at t, which requires *HMIT*.

Back

Banks Balance Sheets MOCS Dependencies

Table: Monotonic Assumptions - MOCS

		A						L					
		c_i	c_j	l_i	l_j	a_i	a_j	d_i	d_j	b_i	b_j	s_i	s_j
	c_i		$-$ [M]	$-$ [S]		$-$ [S]		$+$ [C]	$+$ [C]	$+$ [C]		$+$ [C]	
	c_j				$-$ [S]		$-$ [S]	$+$ [C]	$+$ [C]		$+$ [C]		$+$ [C]
A	l_i				$-$ [M]	$-$ [S]		$+$ [C]		$+$ [C]	$+$ [O]	$+$ [C]	
	l_j						$-$ [S]		$+$ [C]	$+$ [O]	$+$ [C]		$+$ [C]
	a_i						$-$ [M]	$+$ [C]		$+$ [C]		$+$ [C]	$+$ [O]
	a_j								$+$ [C]		$+$ [C]	$+$ [O]	$+$ [C]
	d_i								$-$ [M]	$-$ [S]		$-$ [S]	
	d_j										$-$ [S]		$-$ [S]
L	b_i										$-$ [M]	$-$ [S]	
	b_j												$-$ [S]
	s_i												$-$ [M]
	s_j												

Notes. A and L stand respectively for assets and liabilities. i and j are two banks. l_i and b_i are respectively the amount of lending and borrowing in the interbank market. c_i is the amount of credit provided by bank i. d_i is the amount of deposits held at bank i. a_i is the amount of securities held by bank i. s_i is the amount of bonds issued by bank i. $+$ and $-$ indicate respectively positive and negative interactions. [M]: same side of the market. [O]: opposite side of the market. [S]: asset liability substitution. [C]: balance sheet expansion complementarity.

Back

- *MOCS* interactions do not guarantee per se a valid partition.
- O and S interactions may generate indirect effects with opposite sign w.r.t. the direct effects.

- In addition, we need $R^{MOCS} = D^{MOCS}$.
- Constraints the indirect effects to not offset the first order effect of one outcome variable on the other
- Provides immediately a valid partition.

Commonly Used Micro Estimates - Khwaja and Mian (2008) AER

- For each bank a:

$$c^t_{ai} = d^t_a + s^t_a \qquad (BSconst)$$
$$\bar{r}_i - \alpha_c c^t_{ai} = \alpha_s s^t_a \qquad (FOC)$$

  where c^t_{ai} is the credit to firm i, d^t_a is the deposits of bank a and s^t_a is a bond issued by bank a.
- Banks do **not compete**.
- **No simultaneity** among BS items.
- **Exogenous shocks** only for deposits.

$$d^{t+1}_a = d^t_a + \bar{\eta} + \delta_a, \quad s^{t+1}_a = s^t_a.$$

- Disentangle supply and demand contributions:

$$\Delta c_{ai} = \frac{1}{\alpha_c + \alpha_s}(\alpha_s \bar{\delta} + \bar{\eta}) + \frac{\alpha_s}{\alpha_c + \alpha_s}\Delta d_a + \frac{1}{\alpha_c + \alpha_s}\eta_i \qquad (9)$$

Advantages of a Partial Identification Approach

$$\Delta c_{ai} = \frac{1}{\alpha_c + \alpha_s}(\alpha_s \bar{\delta} + \bar{\eta}) + \frac{\alpha_s}{\alpha_c + \alpha_s}\Delta d_a + \frac{1}{\alpha_c + \alpha_s}\eta_i + \Delta s_i \alpha_5 + \iota \Delta c_{bi}(\Delta d_b \mu) \quad (10)$$

- What is the problem with **omitted endogenous variables**?
- What is the problem with **endogenous regressors and treatments**? endogenous
- What is the problem with **DiD**? DiD
- **Regressions with a parametric model** capturing all these aspects would be a nightmare!
- **Exploit prior knowledge** and evidences about monotonic relationships
  - Benetton and Fantino (2021) and Andreeva and Garcia-Posada (2021) showed the relevance of competitive interactions (Δd_b).