Endogenous Incumbency in Repeated Contests

Fabian Dietz & Stephan Eitel

University of Bamberg

EEA 2025, Bordeaux

Incentivizing competition

In many competitive situations, the organizer aims to incentivize participants to fight as intensely as possible: sports competitions, procurement, R&D races,...

A common wisdom is that in order to maximize intensity, competition should be fair in the sense that no participant has an advantage over another.

When competitions are repeated, it can be effort-increasing to give an advantage to past winners in order to further incentivize competition.

This generates a trade-off between fairness and rewards for the organizer.

Endogenous Incumbency

We cover competitive situations in which the size of the past winner's (incumbent's) advantage is endogenously determined by the incumbent himself.

Investing into an advantage is costly for the incumbent. The contest designer can (partially) compensate the cost of early investment.

In procurement, an *incumbent supplier* already had opportunities to implement his offer. The contest designer might reward a good implementation with an advantage in the contest for the next mandate.

We develop an infinitely repeated contest model to investigate how much an effort-maximizing contest designer should compensate the cost of early effort.

Preview of results

- Endogenous incumbency achieves substantially higher rent extraction than under independent lottery contests in optimum.
- The contest designer prefers to set compensation such that the incumbent partially discourages the contender: the contender exerts less effort than in an unbiased contest, but still positive effort.
- The contest designer can also use endogenous incumbency to eliminate the possibility of collusion between players.

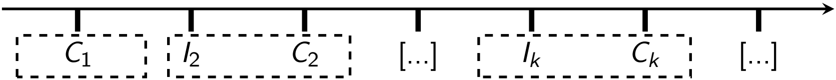
Related literature

We contribute to the literature on

- Biased contests, in particular if the asymmetry is
  - an advantage in the Contest Success Function (Fu (2006), Epstein et al. (2011), Franke et al. (2013, 2014), Fu and Wu (2020))
  - or a commitment opportunity (Dixit (1987), Linster (1993), Hinnosaar (2024), Gao et al. (2025)).
- optimal biases in contests (Lazear and Rosen (1981), Schotter and Weigelt (1992), Kirkegaard (2013)), in particular biases in ex-ante symmetric contests (Drugov and Ryvkin (2017), Barbieri and Serena (2022))
- incumbency advantages (Möller (2012), Clark and Nilssen (2018), Beviá and Corchón (2013), Deck et al. (2024))
- infinitely repeated contests and collusion (Leininger and Yang (1994), Linster (1994), Brookins et al. (2021)).

Timing of events

Each contest is partitioned into a competition stage and an investment stage.



- Competition stage 1: players play an unbiased contest
- Investment stage 2,...: the winner of contest 1 can exert early effort for Contest 2 at cost b which is observable.
- Competition stage 2,...: players play a potentially biased contest

Competition stage: biased Tullock contest

Two players i, j exert costly effort x_i in a lottery contest, where i is the incumbent and j is the contender. The contest might be additively biased in favor of the incumbent.

$$p_i = \frac{x_i + d_i}{x_i + x_j + d_i}, \qquad p_j = 1 - p_i$$

The winner gets a prize, the value of which is normalized to unity. Cost of effort is linear and takes the form $C(x_i) = x_i$.

Competition stage: biased Tullock contest

We assume that after each period, the game ends with an exogenous probability of $1 - \delta \in (0, 1)$. This reflects any shock to either the contest designer or the players causing competition to terminate.

An alternative interpretation of δ is discounting.

Players maximize utility by deciding on their effort level:

$$\max_{x_i} U_i = p_i(1 + \delta\pi_{i,t+1}) + (1 - p_i)\delta\pi_{j,t+1} - x_i$$

$$\max_{x_j} U_j = p_j(1 + \delta\pi_{i,t+1}) + (1 - p_j)\delta\pi_{j,t+1} - x_j$$

Investment stage

In investment stage k, the incumbent (player i) can exert early effort for contest k at marginal cost $b \geq 0$. The incumbent maximizes

$$\max_{d_k} \pi^{I}_{i,k} = \pi^{C}_{i,k}(d_k) - b \cdot d_k$$

where $\pi^{C}_{i,k}(d_k)$ is the continuation payoff, i.e. the payoff in the second contest given a bias of d_k.

Analysis: Contest k

Players fight for an effective prize sum of $\psi_k = 1+\delta(\pi^I_{i,k+1} - \pi^C_{j,k+1})$. Equilibrium efforts in contest k, depending on $d_{i,k}$ have the following properties:

- In the competition stage, the incumbent substitutes effort with investment.
- If investment into the bias is sufficiently cheap ($b < \frac{1}{2}$), the incumbent invests the minimum amount that is necessary to *fully discourage* the contender.
- If investment is moderately cheap ($\frac{1}{2} < b < 1$), the incumbent *partially discourages* the contender.
- If investment is expensive ($b > 1$), the incumbent invests nothing and both players play an unbiased contest.

Contest designer: Rent extraction

The contest designer chooses the cost of early effort b to maximize rent extraction.

We define rent extraction as

$$\rho(b,\infty) = \frac{1}{\sum_{k=1}^{\infty} 1 \cdot \delta^{k-1}} \sum_{k=1}^{\infty} \delta^{k-1}(x_{i,k} + x_{j,k} + b \cdot d_k)$$

The contest designer's problem is

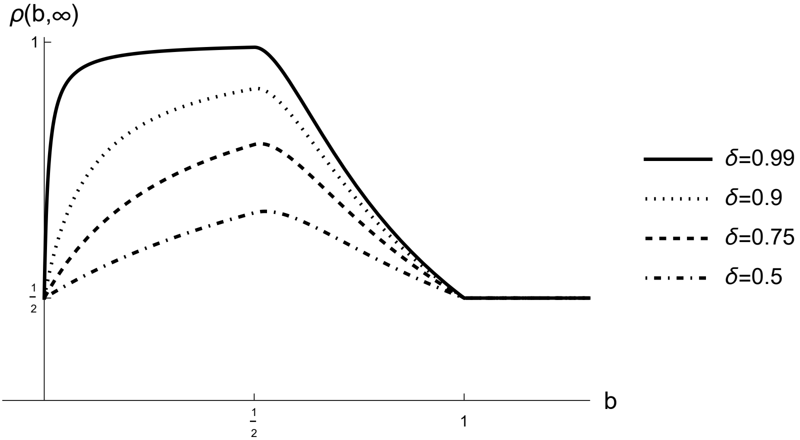
$$\max_{b} \ \rho(b,\infty)$$

Contest designer: Rent extraction

The cost of early effort b affects rent extraction in multiple ways. An increase in b has the following effects:

- Winning a contest becomes less valuable (ψ_k decreases), which decreases effort.
- Contests become less biased, which increases effort.
- The effect of b on revenue from investment ($b \cdot d_k$) is ambiguous.

Contest designer: Rent Extraction



Rent extraction in the infinite game. The designer optimally chooses, for any δ, a b^* that lies in the area of partial discouragement.

Collusion

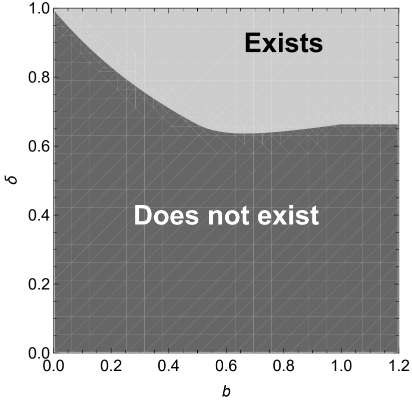
The limit equilibrium is not the unique subgame-perfect Nash equilibrium in the infinite game.

Instead, there is a continuum of equilibria in trigger strategies in which players collude as long as their counterpart does so as well.

Intuitively, these collusive equilibria only exist if the continuation probability δ is sufficiently high.

Collusion

The contest designer can use endogenous incumbency to eliminate the possibility of collusion:



Existence of the trigger Nash equilibrium

Robustness checks: unobservable early effort

Endogenous incumbency can increase total effort above the level of two unrelated lottery contests.

Endogenous incumbency combines two features: the incumbent

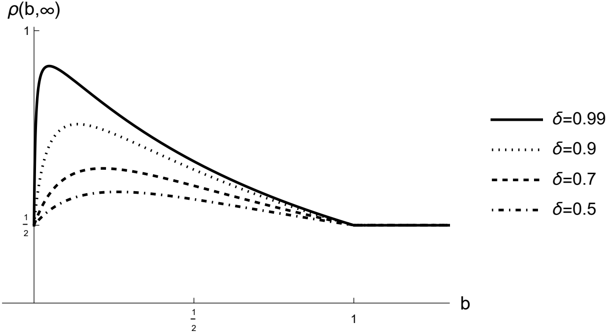
(i) can move first

(ii) has a cost advantage.

We have seen that (i) without (ii) has no effect on total effort.

Robustness checks: unobservable early effort

If the incumbent's early effort is unobservable, the incumbent loses the commitment opportunity and is left only with a cost advantage. In this case, total effort is in optimum higher than under the baseline, but lower than under observable early effort:

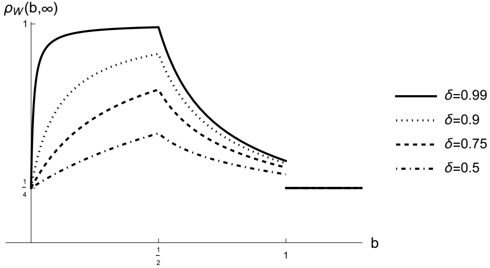


Unobservable early effort: Rent extraction depending on δ

Robustness checks: winner's effort

In some applications, the contest designer might be interested in maximizing winner's effort.

In this case, full discouragement becomes optimal for the contest designer. Partial discouragement entails an efficiency loss because some effort is not valued by the contest designer.



Winner's effort: rent extraction depending on δ

Conclusion

- Endogenous incumbency achieves substantially higher rent extraction than under independent lottery contests in optimum.
- The contest designer prefers to set compensation such that the incumbent partially discourages the contender: the contender exerts less effort than in an unbiased contest, but still positive effort.
- The contest designer can also use endogenous incumbency to eliminate the possibility of collusion between players.
- If the contest designer aims to maximize winner's effort, he prefers to induce full discouragement.

Thank you for your attention!

stephan.eitel@uni-bamberg.de

Analysis: investment stage k

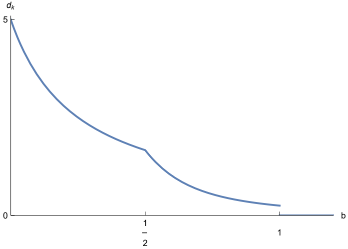
Given that players fight for a prize of ψ_k in competition stage C_k, the incumbent's investment in I_k is given by

$$d_k^* = \begin{cases} \psi_k & \text{if } b \leq \frac{1}{2}, \\ \frac{\psi_k}{4b^2} & \text{if } \frac{1}{2} < b < 1, \\ 0 & \text{if } b \geq 1 \end{cases}$$

- If investment into the bias is sufficiently cheap ($b < \frac{1}{2}$), the incumbent invests the minimum amount that is necessary to *fully discourage* the contender.
- If investment is moderately cheap ($\frac{1}{2} < b < 1$), the incumbent *partially discourages* the contender in the second contest.
- If investment is expensive ($b > 1$), the incumbent invests nothing and both players play an unbiased second contest.

Analysis: investment stage k

$$d_k^* = \begin{cases} \psi_k & \text{if } b \leq \frac{1}{2}, \\ \frac{\psi_k}{4b^2} & \text{if } \frac{1}{2} < b < 1, \\ 0 & \text{if } b \geq 1 \end{cases}$$



Investment decision in investment stage I_k for $\delta = 0.8$

Contest designer: rent extraction

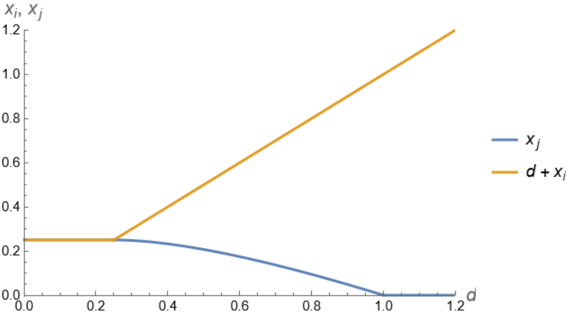
Proposition: In the limit equilibrium, rent extraction in the infinite game is given by

$$\rho(b,\infty) = \begin{cases} \frac{1}{2} & \text{if } b \geq 1 \\ \frac{1}{2} + \frac{\delta b(-8b+11)-3\delta}{8b^2+\delta(8b^2-10b+2)} & \text{if } \frac{1}{2} < b < 1 \\ \frac{1}{2} + \frac{\delta b}{2(1-\delta+\delta b)} & \text{if } 0 < b \leq \frac{1}{2}. \end{cases}$$

In particular, the contest designer optimally sets $b^* = \frac{\delta+\sqrt{9-5\delta}+3}{\delta+11}$. The optimal b^* lies in the area of partial discouragement for any δ.

$K = 2$ game: contest 2

Effective equilibrium efforts (including d_1) in the second contest:

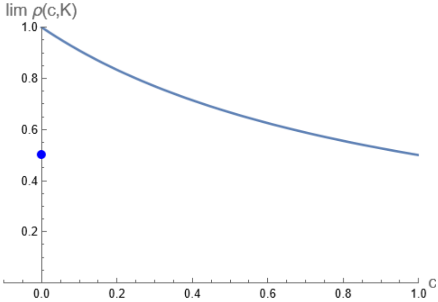


Effective equilibrium efforts of the incumbent (orange) and the contender (blue). These go into the lottery CSF.

⏩ back

Asymptotic rent extraction: exogenous incumbency

Asymptotic rent extraction for the exogenous incumbency game:



Solution concept

Since the game has no certain last period, we cannot solve the infinite game by backward induction.

Instead, we use a result from Fudenberg and Levine (1983): Under some conditions (which hold for our model), the subgame-perfect Nash equilibria in a repeated, finite game converge to a subgame-perfect Nash equilibrium in the infinite game, when the number of periods tends to infinity.

Therefore, we first find the unique subgame-perfect Nash equilibrium of the finite, K-contest game via backward induction, and then calculate a subgame-perfect Nash equilibrium (which we call the limit equilibrium) of the infinite game by letting $K \to \infty$.