Sources of consumer information *

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Abstract

A buyer can learn about a product, either through search or through the information disclosed by the seller. We analyze how this buyer-seller relationship is affected by lower search costs or an improvement in the seller's ability to fine-tune her disclosure of product information. Whereas a drop in search costs improves consumer surplus and decreases profit when the seller can resort to an optimal disclosure strategy, its impact is ambiguous if the seller is unable to provide information. When it is unlikely that the buyer's valuation is below marginal cost, the buyer does not benefit from optimal information disclosure by the seller if search costs are high. With such high search costs and no disclosure, both parties can be better off than with lower search costs and optimal information disclosure. The seller then adopts a mass market strategy where she posts a low enough price so that the buyer always purchases the product without search. By contrast, if it is sufficiently likely that the buyer's valuation is below marginal cost, the buyer can benefit from sophisticated information disclosure for relatively low search costs. The corresponding outcome is better for both parties than an environment with higher search costs and no information disclosure. The optimal seller strategy targets a niche of high-valuation buyers and prevents wasteful search by buyers with low valuations.

Keywords: information disclosure, information acquisition, advertising, search.

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1 Introduction

Search environments can differ greatly with regard to the consumers' access to product information and how this access is shaped by the sellers or intermediaries such as online platforms. Think of a basic search engine that relies solely on the content of the consumer's requests as opposed to one triggered by a targeted ad or that benefits from product steering on a marketplace or a vertical platform specialized in hotels, flights, or real estate. The search engine is an effective way to gather a wealth of information about the product from different independent sources, whereas the search directed by targeting or product steering is ideal for quickly finding a product which is reasonably suitable for the consumer's need. Whether the consumer benefits from undertaking his or her own search as opposed to relying on the information from the seller or a platform depends on a number of factors, including the level of search costs, the nature of the buyer's uncertainty about how much she or he likes the product, but also the incentives of the seller in providing information and the concomitant choice of the seller's price. As another illustration, the progressive migration of trade from offline to online with the development of the digital economy has had significant implications, both for search and for the design of information transmitted from sellers to consumers. While information acquisition has become less costly, the ability of sellers to optimally tailor information transmission has been greatly enhanced by the use of big data and artificial intelligence as well as the availability of buyer personal data.

In this paper, we study the impact on the market outcome of changes in the consumers' cost of information acquisition and in the sellers' ability to optimize their communication. We analyze the determination of price, quantity sold, and how much consumers learn about the product before they buy it. We characterize which configurations of consumer search costs and seller information transmission capacity are more favorable to either side and to total welfare.

We consider a simple buyer-seller setting where the buyer (he) has unit demand and his valuation is initially unknown to either party. The buyer can learn about the valuation through some product information provided by the seller (she) and can decide to learn more by incurring a fixed search cost, in which case he becomes perfectly informed.

We first characterize the seller's profit-maximizing strategy assuming that she can perfectly control the informative signals that reach the buyer. We show that the seller optimally designs her product information so as to completely deter the buyer from acquiring additional information. The optimal disclosure is characterized by a threshold on the match between the product and the buyer, such that the latter learns whether or not the match is above the threshold but learns no additional information. As the cost of information acquisition increases, the information disclosed to a consumer who purchases the product deteriorates (i.e., the threshold decreases). Consequently, the probability that the buyer purchases the product increases. This optimal information disclosure allows the seller to increase her price up to a point where she can extract the entire expected consumer surplus. If the information acquisition cost is high enough that the buyer has zero expected surplus, a further increase in that cost induces a decrease in price until information acquisition becomes so costly that the seller achieves a first-best outcome: the product is purchased if and only if the buyer's valuation exceeds marginal cost, and the entire surplus is extracted by the seller.

An increased cost of information acquisition for the buyer obviously increases the seller's profit until it reaches its first-best level. It also results in lower consumer welfare. This deterioration occurs because price increases whenever the buyer's surplus is strictly positive and the buyer, receiving poorer information from the seller, faces a larger expected loss from buying with a negative surplus. Total surplus improves because the deterioration of the buyer's information (resulting from a lower threshold) also induces a higher purchase probability and, because the threshold always screens out valuations below marginal cost, a higher probability that the buyer purchases the product improves social welfare.

In order to assess the impact of such sophisticated information transmission, we contrast it with an extreme benchmark where the seller cannot communicate product information and can only post a price. Without information disclosure, the seller chooses between a high price at which the buyer searches or a lower price inducing an immediate purchase without search. The latter strategy cannot be optimal if search costs are low because it would require posting a very low price whereas the buyer is willing to search even if price is at monopoly price or close to it. For higher search costs however, profit maximization can involve setting a price that deters search and the corresponding profit can even be above monopoly profit provided the expected buyer valuation is sufficiently larger than marginal cost. Contrary to what happens with optimal disclosure, profit need not be monotonically increasing in search costs. If the expected buyer valuation is not large enough, it can be negatively impacted by a search cost increase for low search costs or even for all search costs. Similarly, buyer surplus is not necessarily decreasing in search costs. Whenever the seller finds it optimal to drop the price to induce an immediate purchase, the buyer's surplus discontinuously jumps up and can be larger than his full information monopoly surplus.

It is ambiguous whether the availability of seller-provided product information benefits the buyer or not. It partially prevents wasteful search as well as purchases with a very negative surplus by screening out buyers with very low valuations (below the optimal disclosure threshold). However, it enables the seller to sell at a price typically above the monopoly price a quantity which exceeds the monopoly quantity, thus capturing a large share or the entirety of total surplus. We show that, for large search costs with an expected buyer valuation above marginal cost, no product information disclosure is always preferable for the buyer. It is then possible that both sides are better off with large search costs and no information disclosure than with very low search costs and an elaborate communication of product information by the seller. With high search costs and no product information, the seller adopts a mass market strategy where she posts a low price at which all buyers buy immediately. We also show that, when it is sufficiently likely that buyer valuation is below marginal cost, there is a range of relatively low search cost values at which the buyer benefits from the seller's communication. This suggests that for such niche products, it might be harmful to limit the use of sophisticated communication based on user data for search engines where the cost of information acquisition is relatively small. Then the combination of low search costs and elaborate communication by the seller can be an improvement for both parties over the outcome that would prevail with larger search costs and no communication capacity by the seller, which could be viewed as a crude description of offline trade.

The contrasted results for mass market products and niche products have interesting implications for platform design. A platform is most attractive to sellers if it maintains high search costs and facilitates optimal seller communication. However, such an outcome is extremely detrimental to buyers who would then turn away from the platform. To avoid such a fatal outcome the platform can resort to two alternative or combined strategies: reducing search costs and shutting down the sellers' ability to communicate optimally. For mass markets, the best option for the platform is to disable communication by sellers while keeping search costs relatively high: this results in an increase in consumer surplus while sellers can still enjoy substantial profits thanks to the large search costs. In contrast, if the items sold on the platform are niche products, it is preferable for the platform to maintain sophistication in the sellers' communication while improving the buyers' welfare through a decrease in search costs: search costs can be set at a level where the buyer's surplus is larger than what it would be without communication and profits remain relatively large.

Related literature. This paper fits most directly in the literature on the interaction between consumer search and product information disclosure by firms. Early examples are Anderson and Renault (2006) and Bar-Isaac, Caruana, and Cuñat (2010). More recent contributions include Choi, Dai, and Kim (2018), Lyu (2023), and Dogan and Hu (2022), who characterize optimal buyer information in the search setting of Wolinsky (1986) and Anderson and Renault (1999).[1] Some of the recent contributions also abstract from pricing as in Board and Lu (2018), Au and Whitmeyer (2023), or Boleslavsky and Krasteva (2025). We are closest to Wang (2017), who studies a similar problem, although he restricts the seller's information disclosure to use the information transmission setup introduced by Lewis and Sappington (1994), which is quite different from optimal disclosure.[2] This restriction on communication results in a profit-maximizing strategy for the seller which shares more features with the no disclosure setting than with the optimal disclosure solution we characterize.[3]

Our work also contributes to the broader literature on information design applied to product information disclosure[4] (e.g. Saak, 2006, Anderson and Renault, 2006, Smolin, 2023, or Hwang, Kim, and Boleslavsky, 2023 who consider oligopolistic competition). Information design applied to consumer information has also been investigated while considering providers of information other than the sellers themselves with an objective that can differ from profit maximization as in Armstrong and Zhou (2022), Roesler and Szentes (2017), and Terstiege and Wasser (2020, 2024).

Our finding that it is optimal to completely deter search is reminiscent of a result in Matyskova and Montes (2023), who introduce rational inattention to model costly learning by the receiver in a standard information design framework, under a separability assumption on the cost of information

[1] See also Biglaiser, Gu, and Li (2024) where consumers decide whether to become informed before observing prices.

[2] He also does not allow for valuations below marginal cost.

[3] In a recent working paper, Feng (2024) also provides a characterization of optimal disclosure in the setting of Wang (2017). Her characterization is similar to ours, although she does not provide an argument for uniqueness, so it is difficult to interpret the comparative statics. She does not allow valuations below marginal cost and has no results on the comparison with no information.

[4] There is also a literature on product information disclosure that uses a signaling setting (see Renault, 2015 for a survey, and Celik and Drugov, 2024 for a recent contribution).

acquisition. Without this assumption, the optimal price-disclosure strategy may instead induce information acquisition—for instance, when the consumer can acquire only partial information at some fixed cost (see, e.g., Terstiege, 2016, Yang, 2024). Relatedly, the seller's optimal information provision may not fully deter search when consumers have private information, such as privately observed search costs, as recently analyzed by Pham (2025). Finally, our analysis also uses the concepts of mass markets and niche markets along the lines of Johnson and Myatt (2006) which was introduced in a search environment by Bar-Isaac, Caruana, and Cuñat (2012).

The rest of the paper unfolds as follows. The model is presented in the next section. Section 3 shows that an optimal disclosure strategy is a threshold strategy that completely deters information acquisition. Profit-maximizing disclosure and pricing strategies are studied in Section 4, which also provides some welfare analysis. In Section 5, the results with the seller using an optimal disclosure strategy are compared to those when she provides no information.

2 Model

A profit-maximizing seller, with costs normalized to zero, wishes to sell an object to a buyer. The buyer's valuation is v, drawn from a distribution with support $[\underline{v}, \bar{v}]$, where $\underline{v} \leq 0 < \bar{v}$ and with a distribution function G which admits a continuously differentiable density g. The buyer is risk-neutral, and his utility from buying the product at price p is $v - p$ while his utility is 0 if he does not buy. The hazard rate, $\frac{g(v)}{1-G(v)}$, is strictly increasing on $[\underline{v}, \bar{v}]$, so profit $p[1 - G(p)]$ is strictly quasi-concave in p and is maximized at a unique monopoly price $p_M \in (0, \bar{v})$, which is the solution to

$$p_M = \frac{1 - G(p_M)}{g(p_M)}.$$

Let $\pi_M = \max_{p \geq 0} p[1 - G(p)] = p_M[1 - G(p_M)]$ be the monopoly profit. The prior expected valuation is $E(v) = \int_{\underline{v}}^{\bar{v}} v \, dG(v)$. The first-best *ex ante* total surplus is the prior expected value of v conditional on the valuation exceeding marginal cost 0, denoted by

$$\mu \equiv E(v \mid v \geq 0) = \frac{\int_0^{\bar{v}} v \, dG(v)}{1 - G(0)} \geq E(v).$$

The match realization v is initially not known by either party, and they share a common prior described by G. The seller can provide certified information about its "product type" to the buyer through advertising. Such information disclosure entails no cost for either side. Though the product type conveys no information about the match to the seller (reflected in a prior on v for the seller independent of the product type), product information can generate some informative signal about v for the buyer, who knows his tastes (his "buyer type") perfectly: in particular, complete revelation of the product type informs the buyer perfectly about v.

Anderson and Renault (2006) provide examples of such settings.[5] As illustrated by the analysis in Koessler and Renault (2012), the nature of the information that can be imparted to the buyer through the disclosure of product attributes critically depends on how the sets of product types and buyer types are mapped into buyer valuations. However, additional flexibility in fine-tuning information disclosure by the seller can be achieved if a platform can collect data pertaining to the buyer's taste and implement an appropriate product steering policy. The setting studied by De Corniere (2016) provides an example where the product space is a circle and a search engine, which learns consumer tastes perfectly, steers consumers towards products located within a certain distance of their bliss point, so they learn that their valuations for the products they are matched with are above some threshold. As long as the platform is committed to its product steering policy, which is known to both the buyer and the seller, it can convey information to the buyer regarding his valuation for a product he is targeted with on the platform.

We abstract from the details of how the disclosure of product information combined with the platform's product steering policy can generate relevant information for the buyer and merely assume that the seller can select any signal structure to achieve maximal profit, in line with standard information design (Myerson, 1982, Kamenica and Gentzkow, 2011, Taneva, 2019). Formally, the seller chooses a disclosure policy, defined as a measurable function $X : [\underline{v}, \bar{v}] \to \Delta(M)$ for some signal space M.

The seller also posts a price p, which is independent of v because she has no prior information about the match value. After observing the price and the product information disclosed by the seller, the buyer can choose to learn his match realization perfectly by incurring a search cost $s \in (0, \bar{v})$ before deciding whether to purchase the product. The timing can be summarized as follows:

[5] See also Koessler and Renault (2012), and Koessler and Skreta (2016, 2019) for further elaborations on product information disclosure in related frameworks.

1. The seller chooses a price p and a disclosure policy X;

2. Nature determines the match v according to the prior G, and a signal realization $m \in M$ according to the distribution $X(v)$;

3. The buyer observes the price p, the disclosure policy X, and the realized signal m;

4. The buyer chooses $a \in \{drop, buy, search\}$;

5. If the buyer drops, he gets 0; if he buys, he gets $v - p$; if he searches, he pays s, observes v, then buys if $v \geq p$ and drops if $v < p$. The seller gets p if the buyer buys and gets 0 if the buyer drops.

We next provide a simple characterization of the firm's optimal disclosure strategies.

3 Information disclosure

We start by establishing an important result regarding the characterization of the optimal information disclosure strategy for the seller. We show that there is no loss of generality in restricting attention to a set of simple disclosure policies such that the buyer never chooses to search after observing the product information provided by the seller and he is merely informed whether his valuation exceeds some threshold value. The point of the next proposition is that, for any arbitrary disclosure policy and any price p, there exists such a simple threshold policy inducing no search, such that the probability that the buyer purchases the product at price p is the same as in the original policy.

To illustrate, consider the "truth-or-noise" disclosure policy *à la* Lewis and Sappington (1994), as popularized by Johnson and Myatt (2006): the set of signals is $M = [\underline{v}, \bar{v}]$ and for each v, the signal is $m = v$ with probability η, and is drawn from the distribution function G with probability $1 - \eta$. A higher η results in more precise product information for the buyer, with full information achieved when $\eta = 1$ and no information transmitted when $\eta = 0$. Assuming the buyer's valuation is uniformly distributed on $[\underline{v}, \bar{v}] = [0, 1]$, we have $E(v) = \mu = \frac{1}{2}$ and the conditional expected match for the buyer, given that he observes the signal m, is $E(v \mid m) = \eta m + \frac{1}{2}(1 - \eta)$. Assuming small enough search costs ($s \leq \frac{3-2\sqrt{2}}{4} < \frac{1}{8}$), the optimal truth-or-noise disclosure policy is $\eta = 1 - 8s$, and the optimal price is $p = p_M = \frac{1}{2}$ (see Wang, 2017). The equilibrium behavior of the buyer consistent with profit maximization is to buy immediately after observing $m \geq \frac{1}{2}$ and to search otherwise.[6] Total purchase probability is then $\frac{1}{2} + \frac{1}{4}(1-\eta) = \frac{1}{2} + 2s$.

A first step in simplifying the above disclosure policy is to apply the revelation principle (see, e.g., Myerson, 1982). The idea is that the same outcome (i.e., the same purchase and search probabilities for a buyer with valuation v, for all $v \in [\underline{v}, \bar{v}]$) can be achieved using a simpler "buy-or-search" information disclosure policy with only two signals, $M = \{buy, search\}$. For each v, the probability that the buyer receives the signal *buy* equals the probability that he buys immediately if he has valuation v under the original truth-or-noise policy: the signal $m = buy$ is sent with probability $\eta + \frac{1}{2}(1-\eta)$ if $v \geq \frac{1}{2}$, and with probability $\frac{1}{2}(1-\eta)$ if $v < \frac{1}{2}$. The key point is that the equilibrium conditions ensuring the buyer behaves as desired under the original disclosure policy imply that the new disclosure policy satisfies four *incentive compatibility constraints*: (BnS) and (BnD) ensure that a buyer who receives the signal *buy* has a higher expected surplus if he buys than if he searches or drops, respectively, and (SnB) and (SnD) ensure the buyer does not wish to buy immediately or drop out when observing $m = search$.

Because the seller ultimately cares about the total probability of a sale, she could bypass the search process by immediately disclosing to the buyer whether he would end up purchasing if he chose to search.[7] She can achieve this by resorting to the following "buy-or-drop" disclosure policy. In this alternative policy, $M = \{buy, drop\}$, and the buyer receives the signal $m = buy$ with probability 1 if $v \geq \frac{1}{2}$ (recall that in the original policy, a buyer with $v \geq \frac{1}{2}$ always buys, either because he receives a signal above $\frac{1}{2}$ or because, after searching, he finds out that his valuation exceeds the price $\frac{1}{2}$), and with probability $\frac{1}{2}(1-\eta)$ if $v < \frac{1}{2}$ (with $v < \frac{1}{2}$ in the truth-or-noise policy, a buyer buys only if he receives a noisy signal above $\frac{1}{2}$ and hence does not search). By construction, the purchase probability is the same as in the two disclosure policies described above: $\frac{1}{2} + \frac{1}{4}(1-\eta) = \frac{1}{2} + 2s$. However, we need to check that this policy with no search is incentive compatible, so a buyer observing a signal $m \in \{buy, drop\}$ complies with the recommendation. This is trivially the case for $m = drop$ because the buyer knows he can observe such a signal only if $v < \frac{1}{2}$ and hence will have a strictly negative expected surplus if he buys immediately or searches. For a buyer who is advised to buy, it is intuitive that this constitutes

[6] The buyer is actually indifferent between buying and searching for $m \geq \frac{1}{2}$ and between searching and dropping out for $m < \frac{1}{2}$ but the seller could break those ties by lowering the price slightly.

[7] A similar idea appears in Matyskova and Montes (2023), who show that an optimal information structure can be solved as a standard information design problem, subject to the constraint that the receiver does not have an incentive to acquire additional information.

an even more favorable signal than what it was in the buy-or-search policy. Indeed, the only situations where he observes $m = buy$ whereas he would not observe it in the buy-or-search policy is when he would have searched and decided to buy in the end: in other words, in those cases he knows that $v > p$ so that buying immediately is optimal. Hence, if the incentive compatibility constraints (BnD) and (BnS) are satisfied with the buy-or-search policy, this should be all the more the case with the buy-or-drop policy.

In the above buy-or-drop disclosure policy, for $s > 0$, there is a positive probability that the buyer is recommended to buy no matter how low the actual valuation is. As in Anderson and Renault (2006) and Saak (2006), the posterior for a buyer receiving the signal *buy* can be made more favorable by concentrating all the *buy* signals on high realizations of v. Specifically, consider the threshold $\tilde{v} = \frac{1}{2} - 2s$ and assume the buyer receives $m = buy$ if and only if $v \geq \tilde{v}$. Then the purchase probability is $1 - \tilde{v} = \frac{1}{2} + 2s$, which is the same as with the previous policies we have considered. Again, a buyer who is advised to drop out will never want to buy or search at price $\frac{1}{2}$ because he is certain that $v < \frac{1}{2}$. For a buyer who should buy, concentrating the $m = buy$ signal on higher realizations of valuation v while keeping the overall probability of such a signal constant induces a more favorable posterior for the buyer who, as a result, will be more inclined to buy. So the corresponding incentive compatibility constraints (BnS) and (BnD) are not violated.

Applying analogous arguments to a generic disclosure policy, we prove the following proposition in the appendix.

Proposition 1 *For profit maximization, it is without loss of generality to consider a threshold disclosure policy. This policy recommends buying when $v \geq \tilde{v}$ and dropping when $v < \tilde{v}$, for some $\tilde{v} \in [\underline{v}, \bar{v}]$. Specifically, $M = \{drop, buy\}$, $X(buy \mid v) = 1$ if $v \geq \tilde{v}$, $X(drop \mid v) = 1$ if $v < \tilde{v}$, and the consumer follows the recommended action. Additionally, at the optimum for the firm, we have $p > \tilde{v}$.*

Proof. See Appendix A.1. ■

The above result identifies a relevant class of disclosure policies that the seller can resort to in order to achieve maximum profit. Each of these policies is fully characterized by the threshold value $\tilde{v}$, which should be below the selected price p. The policy should also be incentive compatible so that those who learn that $v \geq \tilde{v}$ buy immediately and those who learn that $v < \tilde{v}$ drop out. The requirement that the threshold should be below the price ensures incentive compatibility for those who should drop out. For those who should buy immediately, the two incentive compatibility constraints are

$$E(v \mid buy) - p \geq 0, \tag{BnD}$$

so they do not drop out and

$$E(v \mid buy) - p \geq \mathbb{P}\{v \geq p \mid buy\} E(v - p \mid v \geq p, buy) - s,$$

or, equivalently,

$$E(p - v \mid v < p, buy)\mathbb{P}\{v < p \mid buy\} \leq s, \tag{BnS}$$

in order for them not to want to search.[8] For a threshold disclosure policy with threshold value $\tilde{v}$ these constraints are

$$\int_{\tilde{v}}^{\bar{v}} (v - p) g(v)\, dv \geq 0, \tag{Δ}$$

for (BnD) and

$$\frac{\int_{\tilde{v}}^{p} (p - v) g(v)\, dv}{1 - G(\tilde{v})} \leq s \tag{Σ}$$

for (BnS). For any $\tilde{v} \in [\underline{v}, \bar{v}]$, a price $p > \bar{v}$ would violate (Δ). Hence the seller's problem consists in selecting a price $p \in [0, \bar{v}]$ and a threshold value $\tilde{v} \in [\underline{v}, p]$ to maximize profit $p[1 - G(\tilde{v})]$ subject to constraints (Δ) and (Σ).

Proposition 1 tells us that, when using a threshold match policy with no search, the seller can achieve at least as much profit as with any alternative policy. There is no claim that profit could not be maximized by using some disclosure policy outside this specific class. However, eliminating search and resorting to threshold match disclosure typically enables the seller to achieve a higher profit than with an alternative disclosure strategy because she can sell the same quantity at a strictly higher price. We now return to the truth-or-noise example to briefly discuss why this is the case.

In the buy-or-search policy obtained by applying the revelation principle to the truth-or-noise policy, there are two sources of price elasticity in the buyer's demand at price $\frac{1}{2}$. One stems from the

[8]The second formulation of (BnS) follows from rewriting $E(v \mid buy) - p$ as $\mathbb{P}\{v \geq p \mid buy\}E(v - p \mid v \geq p, buy) + \mathbb{P}\{v < p \mid buy\}E(v - p \mid v < p, buy)$.

purchase decisions made by buyers who search and become fully informed. The second arises because at this price, incentive compatibility constraint (BnS) is actually binding so that, with a higher price, some buyers would switch from buying immediately to searching and then would possibly find out they don't want to buy. If the seller raises her price above $\frac{1}{2}$, she would lose some demand on both these fronts. Switching to the buy-or-drop policy clearly eliminates the first effect because there is no longer any search. But it also relaxes the two incentive constraints because, as explained earlier, the additional cases where the buyer receives $m = buy$ whereas he would not have received it in the buy-or-search policy involve valuations larger than the price. This makes buying more attractive as compared to searching or dropping out. Hence demand is perfectly inelastic at price $\frac{1}{2}$ and the seller can increase her price without losing any demand.

Now consider switching from the buy-or-drop policy where the signal $m = buy$ can be observed with a strictly positive probability for any $v \in [\underline{v}, \bar{v}]$ to a threshold match disclosure policy, while keeping unchanged the probability of observing the signal $m = buy$. In the threshold match policy, the signal $m = buy$ is associated with stochastically higher valuations in the sense that the posterior distribution of v conditional on $m = buy$ stochastically dominates the posterior distribution of v in the non threshold disclosure policy. It follows that $E(v \mid buy)$ is strictly higher and $\mathbb{P}\{v < p \mid buy\}E(p - v \mid, v < p, buy)$ is strictly lower so both constraints are further relaxed. This allows the seller to raise her price even higher without altering the buyer's purchase probability.

An alternative method for increasing profit when the incentive constraints are relaxed would be to decrease the threshold value in order to sell more rather than charging a higher price while keeping quantity constant or, indeed, moving both p and $\tilde{v}$ simultaneously. This price quantity tradeoff is at the heart of the general profit maximization problem we study in the next section.

4 Profit maximization

In this section, we characterize the seller's profit-maximizing strategy, both in terms of disclosure and pricing. From our analysis in Section 3 the seller faces a price-quantity tradeoff resulting from two incentive compatibility constraints for those who buy: a no search constraint and a no drop constraint. Indeed, these constraints indicate that the product can be sold at a certain price provided that the information transmitted to the buyer is favorable enough, which in turn requires that only buyers with high enough valuations are induced to buy. Here we analyze how a change in the buyers' search cost impacts price and quantity as well as the resulting changes in profit, consumer welfare and total welfare.

Formally, from our discussion following Proposition 1, the seller chooses a price $p \in [0, \bar{v}]$ and a threshold $\tilde{v} \in [\underline{v}, p]$ that maximize $p(1 - G(\tilde{v}))$ subject to two incentive compatibility constraints: (Δ) and (Σ). The two constraints respectively ensure that, when the buyer is advised to buy, that is, when he learns that his valuation is above the threshold, he does not prefer to drop out and he does not prefer to search.

The *ex ante* first-best outcome is achieved by maximizing profit subject to (Δ) alone. It yields $p = \mu = E(v \mid v \geq 0)$ and $\tilde{v} = 0$ so the buyer buys only if his valuation exceeds the marginal cost. The resulting profit is $[1 - G(0)]\mu$ and the seller fully extracts total surplus, i.e., (Δ) binds. The seller achieves this first-best outcome if it satisfies constraint (Σ), i.e.,

$$s \geq \bar{s} := \frac{\int_0^\mu (\mu - v) g(v)\, dv}{1 - G(0)}. \tag{1}$$

If $s < \bar{s}$, then either both constraints (Δ) and (Σ) bind, or only constraint (Σ) binds.

When they bind, the no-drop and no-search constraints yield the two following equations, respectively labeled (Δ_b) and (Σ_b):

$$\int_{\tilde{v}}^{\bar{v}} (v - p) g(v)\, dv = 0, \tag{Δ_b}$$

$$\phi(\tilde{v}, p) := \frac{\int_{\tilde{v}}^{p} (p - v) g(v)\, dv}{1 - G(\tilde{v})} = s. \tag{Σ_b}$$

Note that $\phi(0, \mu) = \bar{s}$. The curves delineated by (Δ_b) and (Σ_b), denoted Δ_b and Σ_b, along with the associated incentive-compatible pricing and disclosure policies, are shown in Figure 1 for a match distribution uniform on $[0, 1]$, in which case (Σ_b) and (Δ_b) respectively yield $p = \tilde{v} + \sqrt{2s(\bar{v} - \tilde{v})}$ for $\tilde{v} \in [0, \bar{v} - 2s]$, and $p = \frac{\tilde{v}+1}{2}$, for $\tilde{v} \in [0, 1]$.

The next lemma provides some general properties of Δ_b and Σ_b, none of which require an increasing hazard rate for G.

Lemma 4.1

(i) Along the Δ_b curve, p is strictly increasing in $\tilde{v}$, with $p = \mu$ when $\tilde{v} = 0$ and $p = \bar{v}$ when $\tilde{v} = \bar{v}$.

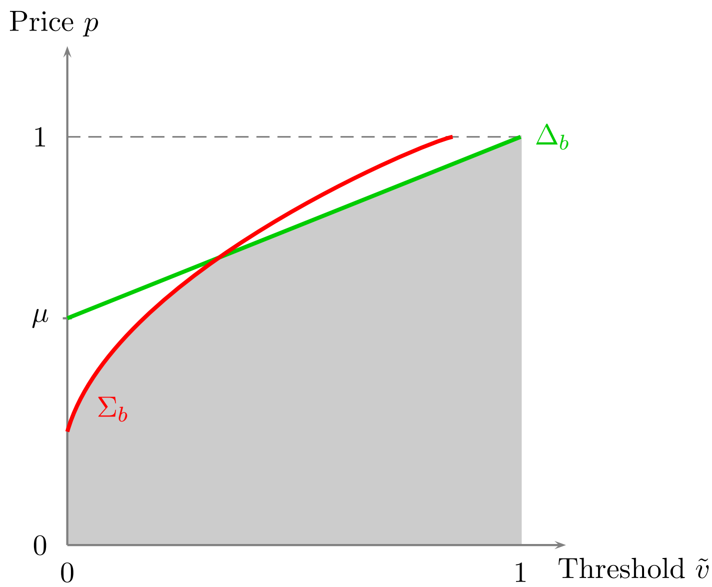

Figure 1: Pairs of disclosure thresholds and prices $(\tilde{v}, p)$ satisfying the no drop constraint (below the Δ_b curve) and the no search constraint (below the Σ_b curve) when the match distribution is uniform on $[0, 1]$.

(ii) Along the Σ_b curve, p is strictly increasing in $\tilde{v}$ and an increase in search cost s shifts the curve upwards (i.e, for any $\tilde{v}$, price is strictly increasing in s). Furthermore, if $s = \bar{s}$, $p = \mu$ when $\tilde{v} = 0$ and, if $s \leq \bar{s}$, then there exists $\tilde{v} \in (0, \bar{v})$ such that $p = \bar{v}$.

Proof. See Appendix A.2. ■

Part (ii) of the lemma implies that if $s < \bar{s}$, then the Σ_b curve crosses the price axis at $p < \mu$, and then, the curves Δ_b and Σ_b cross at least once at some $\tilde{v} \in (0, \bar{v})$, with Σ_b crossing Δ_b from below at the first such crossing point.[9] Notice that any combination $(\tilde{v}, p)$ that satisfies both (Σ) and (Δ), which is to the right of this first intersection point necessarily yields a lower profit than that first intersection point. Such points are dominated by a combination of the same $\tilde{v}$ with the price that binds (Δ). However the corresponding point, which is on Δ_b but specifies a higher threshold than the intersection point yields a lower profit than the intersection point: this is because, when (Δ) binds, the seller captures the full social surplus from the sale, which is decreasing in the threshold when it is above marginal cost, zero. It follows that when only (Σ) binds, the seller's solution is on Σ_b to the left of the first intersection point and, when both constraints bind, it is at the intersection point. Because

[9]In Step 2 of the Proof of Lemma 4.2 we show that if G has an increasing hazard rate, then this is the unique crossing point.

Σ_b crosses Δ_b from below and an increase in s shifts Σ_b upwards it can easily be seen graphically that the intersection moves to the left on the increasing curve Δ_b. It follows that, at the first crossing point an increase in search cost results in a lower price and a lower information threshold (see Figure 2).

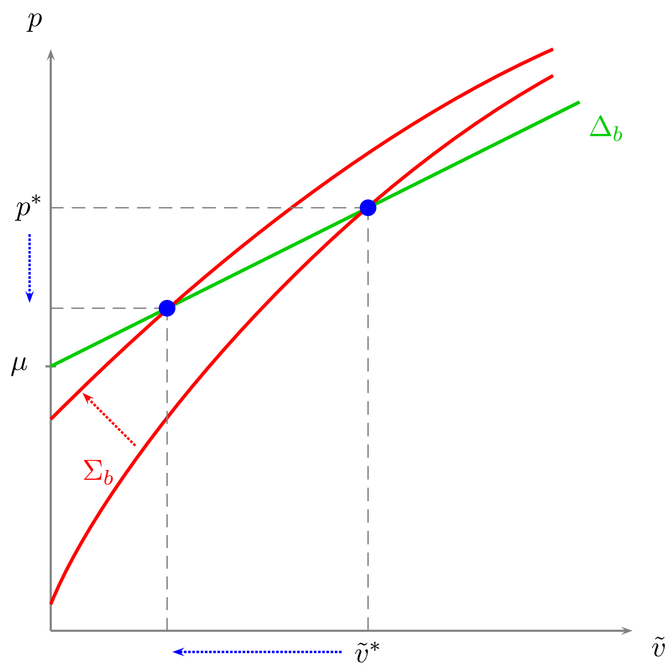

Figure 2: Impact of an increase in s when both (Δ) and (Σ) bind.

Assume now that $s \leq \bar{s}$ and consider the *relaxed program* where the seller chooses a price $p \in [0, \bar{v}]$ and a threshold $\tilde{v} \in [\underline{v}, p]$ to maximize profit $p(1 - G(\tilde{v}))$ subject to the no-search constraint (Σ) alone. Because profit is continuous and the set of valid combinations $(\tilde{v}, p)$ is compact, the relaxed program has a solution, and this solution necessarily binds (Σ).

We use a version of the first-order conditions that compares at a given point on Σ_b the slope of the iso-profit curve going through that point to the slope of Σ_b. Let us define $p'_\Sigma(\tilde{v})$ the derivative of price p with respect to threshold $\tilde{v}$ along the Σ_b curve. We also define, for any point $(\tilde{v}, p)$ on Σ_b, $p'_{ISO}(\tilde{v})$ as the derivative of price p with respect to threshold $\tilde{v}$ along the iso-profit curve going through that point: although this derivative depends on both p and $\tilde{v}$, p is a function of $\tilde{v}$ because of the requirement that Σ_b holds. The first-order conditions for a solution $(\tilde{v}^*, p^*)$ are:

- $p'_{ISO}(\tilde{v}^*) = p'_\Sigma(\tilde{v}^*)$ if $(\tilde{v}^*, p^*)$ is interior, i.e., $p^* < \bar{v}$ and $\tilde{v} > \underline{v}$;
- $p'_{ISO}(\tilde{v}^*) \geq p'_\Sigma(\tilde{v}^*)$ if $\tilde{v}^* = \underline{v}$;

- $p'_{ISO}(\tilde{v}^*) \leq p'_{\Sigma}(\tilde{v}^*)$ if $p^* = \bar{v}$.

We show below that the first-order conditions identify a unique solution to the relaxed program. Under the strictly increasing hazard rate property, $1 - G$ is log-concave, so profit $p(1 - G(\tilde{v}))$ is log-concave and therefore quasi-concave in $(\tilde{v}, p)$. As a result, the isoprofit curves are strictly convex. Hence, the relaxed program has a unique solution if the no-search constraint (Σ) is convex (i.e., if ϕ is quasi-convex), as in Figure 1. The main challenge is that even with an increasing hazard rate, (Σ) is not necessarily convex. For example when the match distribution is $G(v) = v^2$, with support $[0, 1]$, the Σ_b curve is s-shaped as illustrated in Figure 3. In such a case, the first-order conditions may have multiple solutions; for instance, we cannot exclude the possibility of multiple tangent points between the iso-profit curves and the Σ_b curve.

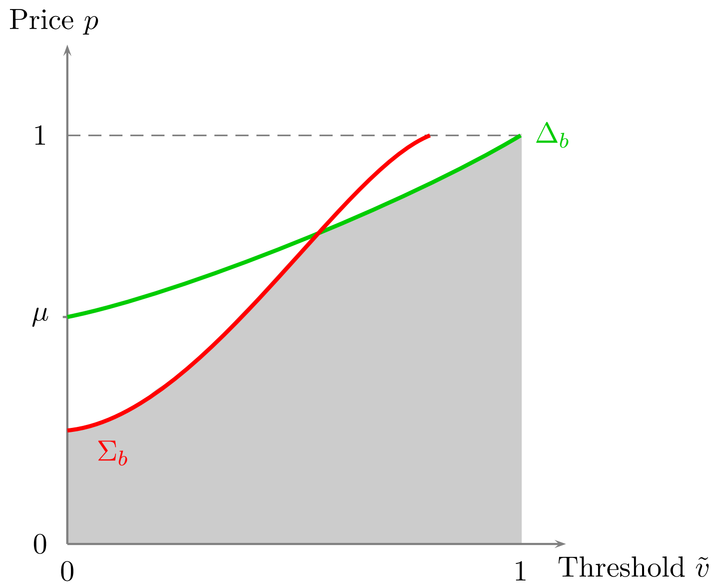

Figure 3: Pairs of disclosure thresholds and prices $(\tilde{v}, p)$ satisfying the no drop constraint (below the Δ_b curve) and the no search constraint (below the Σ_b curve) for a distribution of match values such that (Σ) is not convex.

Despite these non-convexities, we show in the next lemma that, with a strictly increasing hazard rate, the solution of the relaxed program is unique: the solution is either interior, with $p'_{ISO}(\tilde{v}) = p'_{\Sigma}(\tilde{v})$, or $p^* = \bar{v}$. This is established by showing that the function p'_{ISO} must cross the function p'_{Σ} from below. We then show that $\tilde{v}^* > 0$, so if $p^* < \bar{v}$ the solution is necessarily interior.

Lemma 4.2 *Assume $s \leq \bar{s}$. In the relaxed profit-maximization program under (Σ), the optimum*

$(\tilde{v}^*, p^*)$ *is unique and we have* $\tilde{v}^* > 0$*. If* $p^* < \bar{v}$*, then* $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$*, i.e.,*

$$p^* = \frac{\int_{\tilde{v}^*}^{p^*} 1 - G(v)\, dv}{G(p^*) - G(\tilde{v}^*)} \tag{2}$$

Proof. See Appendix A.2. ∎

Now consider an increase in the buyer's information acquisition cost, s. This relaxes constraint (Σ) because, as shown in Lemma 4.1, for any information threshold $\tilde{v}$, the constraint remains satisfied for higher price levels. Hence, sales could be maintained while raising the price but they could also be increased without having to drop the price. Lemma 4.3 below, shows that the seller does a bit of both by increasing price and decreasing the information threshold, as illustrated in Figure 4. The proof is a straightforward comparative statics argument using the increasing hazard rate property applied to the first order conditions for an interior solution, (2), so $p^* < \bar{v}$.[10]

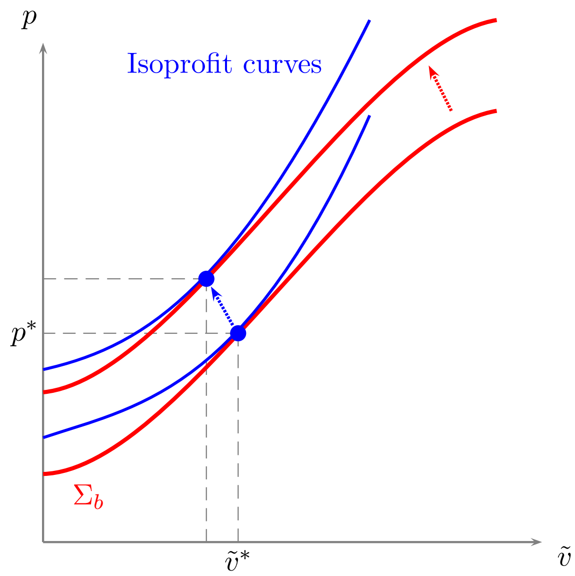

Figure 4: Impact of an increase in s when only (Σ) binds.

Lemma 4.3 *Assume* $s \leq \bar{s}$*. In the relaxed profit-maximization program under* (Σ)*, if* $p^* < \bar{v}$*, then* $\tilde{v}^*$ *is strictly decreasing in* s *and* p^* *is strictly increasing in* s*.*

[10]The argument also relies on the uniqueness of the solution to the first-order condition established in Lemma 4.2, so there cannot be any jump in the solution as s increases. It can be shown that for $p^* = \bar{v}$ an increase in s leaves price unchanged and decreases $\tilde{v}^*$ but this is irrelevant to the analysis below because the relaxed problem solution then violates constraint (Δ).

Proof. See Appendix A.2. ■

The solution to the relaxed problem solves the seller's optimization problem if and only if it satisfies constraint (Δ), in which case only constraint (Σ) binds. Else, both incentive compatibility constraints are binding. In other words, only (Σ) binds if and only if, at the solution to the relaxed program, the surplus of the buyer with valuation above $\tilde{v}^*$ is non negative. If $s = 0$, then constraint (Σ) merely requires that $p \leq \tilde{v}$, because at any higher price, the buyer would choose to search (which involves no cost in this case) to check that his valuation exceeds the price. Hence, if the constraint binds, quantity is determined by price as in the standard monopoly problem so the solution of the relaxed program is $p^* = \tilde{v}^* = p_M$, and expected consumer surplus for buyers with valuation above p_M is strictly positive. Next, for $s = \bar{s}$, from Lemma 4.1(ii), the first-best outcome $(\tilde{v}, p) = (0, \mu)$ satisfies constraint (Σ) and yet, from Lemma 4.2, the solution to the relaxed program involves $\tilde{v}^* > 0$. Since the first-best outcome yields the highest profit under constraint (Δ), this constraint is necessarily violated by the solution to the relaxed program, and the expected surplus of a buyer with valuation above $\tilde{v}^*$ is strictly negative for $s = \bar{s}$.

Using (Σ_b) the expected surplus of a buyer who learns his valuations exceeds $\tilde{v}^*$ can be written as $\frac{\int_{\tilde{v}^*}^{\bar{v}} v - p^* dG(v)}{1-G(\tilde{v}^*)} = \frac{\int_{p^*}^{\bar{v}} v - p^* dG(v)}{1-G(\tilde{v}^*)} - s$, which is strictly decreasing in s because, from Lemma 4.3, p^* is increasing in s while $\tilde{v}^*$ is decreasing in s. We have shown above that this surplus is strictly positive for $s = 0$ and strictly negative for $s = \bar{s}$, so there exists $s_1 \in [0, \bar{s}]$ such that it is strictly negative if and only if $s > s_1$. Hence constraint (Δ) binds if and only if $s > s_1$.

From our discussion following Lemma 4.1, when both constraints bind (i.e. for $s \in (s_1, \bar{s})$), the seller's optimum must be at the first point of intersection of Σ_b with Δ_b, at which an increase in s causes price and information threshold to fall. Collecting the above results, and now letting $(\tilde{v}^*, p^*)$ denote the solution to the full fledged profit maximization problem, we have the following proposition.

Proposition 2 *For all $s \geq 0$, the seller's profit is maximized at some unique combination of price and information threshold, $(p^*, \tilde{v}^*)$. Furthermore, there exists $s_1 \in (0, \bar{s})$ such that:*

- *If $s \leq s_1$, then only (Σ) is binding, $(p^*, \tilde{v}^*)$ is the unique solution to (2), $\tilde{v}^*$ is strictly decreasing in s, and p^* is strictly increasing in s;*
- *If $s \in (s_1, \bar{s})$, then both (Δ) and (Σ) are binding, $\tilde{v}^*$ and p^* are strictly decreasing in s;*

- *If $s \geq \bar{s}$, then only (Δ) is binding, and the seller achieves the first-best outcome, $\tilde{v}^* = 0$ and $p^* = \mu$.*

Figure 5 illustrates the seller's optimal strategies depending on the buyer's information acquisition cost s. In the first two figures, $s < \bar{s}$ and as seen in Lemma 4.1, Σ_b crosses Δ_b from below. In Figure (a), $s \in (0, s_1)$ and only (Σ) binds. In Figure (b), $s \in (s_1, \bar{s})$ and both (Δ) and (Σ) bind. In Figure (c), we have $s \geq \bar{s}$, only (Δ) binds, so the seller extracts all the surplus, which is maximized by the first-best solution $(\tilde{v}^*, p^*) = (0, \nu)$.

Proposition 2 provides a crisp characterization of the seller's optimal disclosure and pricing strategy. When information acquisition is costly for the buyer, $s > 0$, profit exceeds its full information monopoly level. Quantity is always larger than the monopoly quantity even though the seller typically prices above the monopoly price: the only instance when she does not is, for $\mu < p_M$, when search cost s is sufficiently large (close to or above $\bar{s}$). As information acquisition becomes more costly for the buyer, quantity increases. This is because it becomes easier to deteriorate the product information provided to the buyer while still preventing him from becoming informed through search. By lowering threshold $\tilde{v}^*$, the seller sells more to buyers who know less. As long as the buyer's rent is positive, (i.e., for $s < s_1$) she can do this while raising the price. When s rises above s_1, she captures the entire surplus so the no-drop constraint binds. However, for $s < \bar{s}$, she cannot achieve the first-best outcome $(\tilde{v}^*, p^*) = (0, \mu)$ which violates the no-search constraint (Σ). She can still increase her sales by lowering $\tilde{v}^*$, but she must lower the price to keep the no-drop constraint satisfied, so the buyer keeps purchasing the product despite a less favorable information. Price is therefore non-monotonic in the search cost: it rises with the search cost for $s < s_1$ and then falls when s increases beyond s_1 until it reaches μ for $s = \bar{s}$.

Profit obviously rises as information acquisition becomes more costly thus loosening the no-search constraint until it no longer binds for $s \geq \bar{s}$. The implications for total welfare and consumer welfare are, somewhat surprisingly, just as straightforward. Threshold information transmission selects all valuations above the threshold and $\tilde{v}^*$ never falls below marginal cost. The drop in $\tilde{v}^*$ as s increases from 0 to $\bar{s}$ therefore unambiguously improves total welfare. It should be expected that the seller would never want to serve buyers with valuations below marginal cost if she captures the entire surplus (i.e. (Δ) binds) but we show in Lemma 4.2 that this is also the case when only (Σ) binds. Regarding

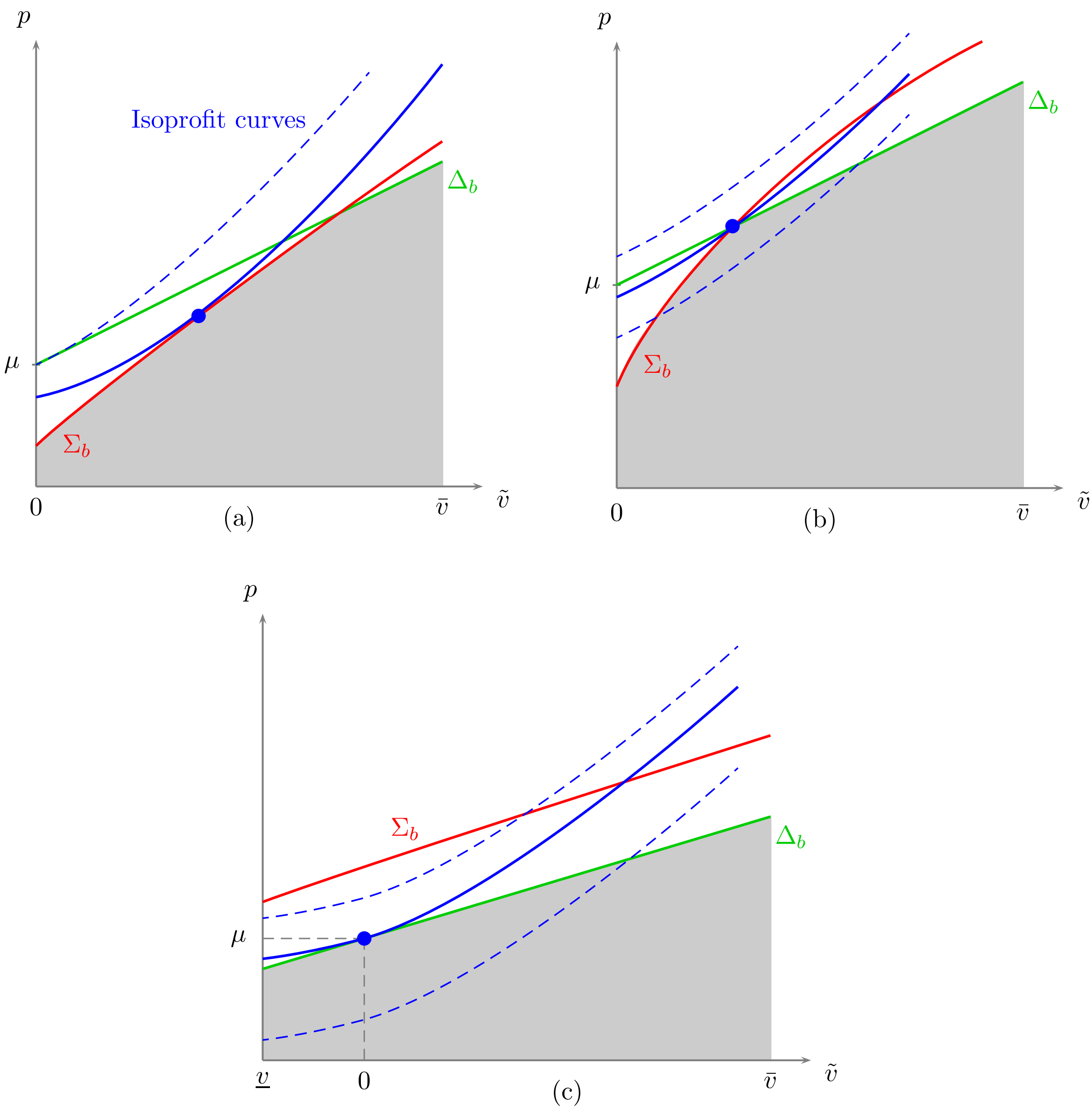

Figure 5: Optimal solution of the seller when (a) $s \in (0, s_1)$ and only (Σ) binds, (b) $s \in (s_1, \bar{s})$ and both (Δ) and (Σ) bind, (c) $s > \bar{s}$ and only (Δ) binds.

consumer welfare, for $s < s_1$ consumer surplus is non-zero. Although the buyer purchases the product with a higher probability when the search cost increases, he pays a higher price. Furthermore, because $\tilde{v}^* < p^*$, all additional values of v for which he buys are below the price so that consumer surplus necessarily decreases. Hence consumer surplus is strictly decreasing in search cost for $s \in [0, s_1]$ and is zero thereafter. The following result summarizes these welfare implications.

Proposition 3 *For $s \in [0, \bar{s}]$, an increase in the information acquisition cost s strictly increases profit and total welfare, strictly decreases consumer surplus if $s < s_1$, and leaves consumer surplus unchanged at 0 if $s \geq s_1$.*

Finally, we compare the outcome with optimal disclosure to what would happen if the seller were restricted to the "truth-or-noise" disclosure policy of Lewis and Sappington (1994), as in Wang (2017). He assumes $\underline{v} = 0$ so match exceeds marginal cost with certainty. We base our discussion on his results for a standard uniform match value distribution with $[\underline{v}, \bar{v}] = [0, 1]$, which we also use at the start of Section 3. Profit maximization with truth-or-noise disclosure involves charging the monopoly price $\frac{1}{2}$ while deteriorating the buyer's information as s increases (the signal reveals the true valuation with probability $\eta = 1 - 8s$), for $s < \frac{3-2\sqrt{2}}{4}$, and resorting to no information disclosure for larger search costs. For $s \geq \bar{s} = \frac{1}{8}$, the seller captures the first-best social surplus by charging a price equal to the expected valuation, $\frac{1}{2}$.[11] When the seller switches to no disclosure, price is set at a level at which the buyer chooses to purchase immediately: price initially falls discontinuously and then increases with s to reach $E(v) = \frac{1}{2}$ for $s = \frac{1}{8}$. This evolution of the price as search costs increase is very different from its counterpart for optimal disclosure, where price initially rises above monopoly price and then falls until it reaches μ for $s \geq \frac{1}{8}$.

The difference in the pricing profile does not impact the behavior of profit as a function of search costs: lower search costs unambiguously decrease profit strictly for $s \in [0, \frac{1}{8}]$, both with optimal and truth-or-noise disclosure. By contrast, the impact of search costs on consumer welfare differs substantially between the two disclosure policies. Because, for large enough search costs with truth-or-noise, the seller drops her price to induce an immediate purchase, the buyer's surplus discontinuously jumps up and can even be larger than what it would be with full information. If, on the contrary, the seller can optimally disclose product information, a drop in search costs benefits the buyer.

We next compare optimal disclosure with the outcome when the seller can reveal no information.

[11] Such a first-best outcome cannot be achieved with truth-or-noise if $\underline{v} < 0$.

5 No product information disclosure

Here we characterize the market outcome when the seller cannot implement optimal information transmission. We study the properties of the alternative extreme benchmark where the seller cannot provide any product information and can only post a price.

5.1 Profit maximization with no disclosure of product information

No information disclosure coincides with threshold information disclosure for $\tilde{v} = \underline{v}$. Hence, from constraints (Σ) and (Δ), for a search cost s and posted price p, the buyer prefers to buy if $\varphi(p) := \phi(\underline{v}, p) \leq s$ and $E(v) - p \geq 0$. The seller can no longer rule out that allowing the buyer to search maximizes profit, because, for low search costs, preventing it would be too costly. When he searches, the buyer anticipates that he will buy if and only if his valuation exceeds the price, so he will obtain the corresponding consumer surplus. Letting $\gamma(p)$ denote consumer surplus at price p, it is optimal for the buyer to search rather than drop if $\gamma(p) \geq s$.

Using integration by parts we have

$$\varphi(p) = \int_{\underline{v}}^{p} (p - v) g(v) dv = \int_{\underline{v}}^{p} G(v) dv, \tag{3}$$

and

$$\gamma(p) = \int_{p}^{\bar{v}} (v - p) g(v) dv = \int_{p}^{\bar{v}} 1 - G(v) dv, \tag{4}$$

so φ is strictly increasing (from $\varphi(\underline{v}) = 0$ to $\varphi(\bar{v}) = \bar{v} - E(v)$), while γ is strictly decreasing (from $\gamma(\underline{v}) = E(v) - \underline{v}$ to $\gamma(\bar{v}) = 0$). We can therefore summarize the buyer's choice for search cost, s, as a function of the seller's price, p, as follows:

$$\begin{cases} \textit{buy} & \text{if } p \leq \min\{E(v), \varphi^{-1}(s)\} \\ \textit{search} & \text{if } \varphi^{-1}(s) < p \leq \gamma^{-1}(s) \\ \textit{drop} & \text{if } p > \max\{E(v), \gamma^{-1}(s)\}. \end{cases}$$

This means that the buyer drops for high prices, buys immediately for low prices, and searches for intermediate prices if the search cost is not too high. To see this it is useful to note that

$$\gamma(E(v)) = \varphi(E(v)). \tag{5}$$

As depicted in Figure 6,[12] if $\varphi^{-1}(s) > \gamma^{-1}(s)$, i.e., $s > \gamma(E(v)) = \varphi(E(v))$, then the buyer never searches, regardless of the price: he buys for $p \leq E(v)$ and drops for $p > E(v)$. If $s < \varphi(E(v))$, he searches for prices in $(\varphi^{-1}(s), \gamma^{-1}(s)]$, drops for $p > \gamma^{-1}(s)$, and buys for $p \leq \varphi^{-1}(s)$.

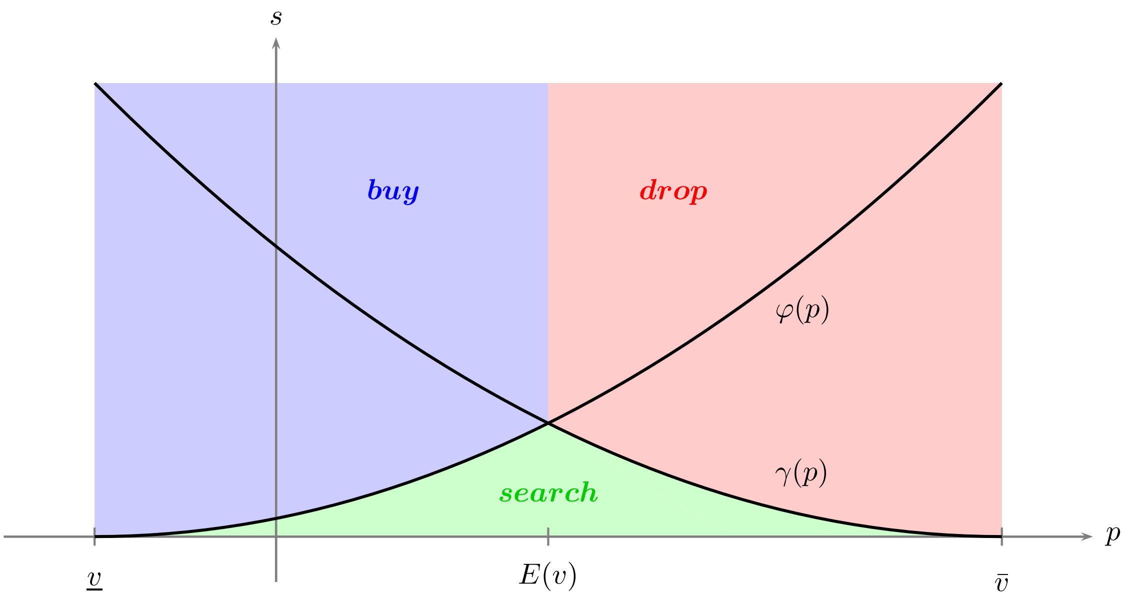

Figure 6: Optimal buyer choice as a function of p and s under no disclosure.

Using the above characterization of the buyer's behavior, we can determine the seller's optimal pricing. Suppose first that search costs are very large so that $s \geq \min\{\varphi(E(v)), \gamma(0)\}$. For $E(v) > 0$, in which case $s \geq \varphi(E(v))$, the buyer's choice at any positive price is either to buy immediately or to drop out. Hence the profit-maximizing price p^N is the largest price at which the buyer prefers to buy immediately, that is $p^N = E(v)$, and the profit is $E(v)$. If $E(v) \leq 0$ and hence $s \geq \gamma(0)$, profit is zero because the buyer drops out for any positive price.

If, on the contrary, search costs are sufficiently low, i.e. $s \leq \min\{\varphi(\pi_M), \gamma(p_M)\}$, the seller can obtain monopoly profit π_M by charging monopoly price p_M, and the buyer searches because

[12]Figure 6 assumes $E(v) \geq 0$. Sliding the φ and γ curves sufficiently to the left shows that, if $E(v) \leq 0$, there is no strictly positive price at which the buyer buys immediately.

$\varphi^{-1}(s) \le \pi_M < p_M \le \gamma^{-1}(s)$ (using φ^{-1} increasing and γ^{-1} decreasing). Her profit cannot exceed π_M because, in order to induce the buyer to purchase immediately, she must charge at most $\varphi^{-1}(s) \le \pi_M$.

For intermediate search costs, $\min\{\varphi(\pi_M), \gamma(p_M)\} < s < \min\{\varphi(E(v)), \gamma(0)\}$, the seller can always secure a positive profit because $\bar{v} > 0$ and $s < \gamma(0)$ so there is a strictly positive price at which the buyer will search and buy. She then chooses between charging the highest price such that the buyer purchases her product immediately $\varphi^{-1}(s)$ and the highest price at which the buyer searches, $\gamma^{-1}(s)$ (if that price is below p_M) or p_M. The latter strategy yields profit $\gamma^{-1}(s)[1 - G(\gamma^{-1}(s))]$ or π_M to be compared to a profit of $\varphi^{-1}(s)$ if the product is bought immediately. First notice that for $E(v) \le 0$, if $s \le \varphi(E(v))$, then $\varphi^{-1}(s) \le 0$. The seller then chooses price $\gamma^{-1}(s)$ and has the buyer search, which yields a strictly positive profit because $s < \gamma(0)$.

For $E(v) > 0$, $\min\{\varphi(E(v)), \gamma(0)\} = \varphi(E(v))$. We show in the appendix that, if $\varphi(\pi_M) \le \gamma(p_M)$, then the seller charges price $\varphi^{-1}(s)$, and the buyer chooses to buy immediately. We also show that, if $\gamma(p_M) < \varphi(\pi_M)$, then the seller prefers to charge $\gamma^{-1}(s)$ so the buyer searches, if and only if the search cost is below some critical value $\hat{s} \in (\gamma(p_M), E(v))$ at which the seller is indifferent between charging $\gamma^{-1}(s)$ and charging $\varphi^{-1}(s)$: hence

$$\varphi^{-1}(\hat{s}) = \gamma^{-1}(\hat{s})(1 - G(\gamma^{-1}(\hat{s}))). \tag{6}$$

The following result summarizes the seller's behavior with no product information.

Proposition 4 (Profit maximization under no disclosure) *Under no disclosure, the optimal price p^N and profit π^N are described as follows as a function of the search cost s.*

1. *If $s \le \min\{\varphi(\pi_M), \gamma(p_M)\}$, then $p^N = p_M$, $\pi^N = \pi_M$ and the buyer searches;*

2. *If $\min\{\varphi(\pi_M), \gamma(p_M)\} < s < \min\{\varphi(E(v)), \gamma(0)\}$:*

   2a. If $\varphi(\pi_M) < \gamma(p_M)$, then $p^N = \pi^N = \varphi^{-1}(s) > \pi_M$ and the buyer purchases immediately;

   2b. If $\varphi(\pi_M) > \gamma(p_M)$ and

   - *$E(v) \le 0$, then $p^N = \gamma^{-1}(s) < p_M$, $\pi^N = \gamma^{-1}(s)(1 - G(\gamma^{-1}(s))) < \pi_M$ and the buyer searches;*

- $E(v) > 0$ *and* $s < \hat{s}$*, then* $p^N = \gamma^{-1}(s) < p_M$*,* $\pi^N = \gamma^{-1}(s)(1 - G(\gamma^{-1}(s))) < \pi_M$ *and the buyer searches;*
- $E(v) > 0$ *and* $s > \hat{s}$*, then* $p^N = \pi = \varphi^{-1}(s)$ *and the buyer purchases immediately; in this range,* $\pi^N = \varphi^{-1}(s) < \pi_M$ *small* s*, and* $\pi^N = \varphi^{-1}(s) > \pi_M$ *for large* s*.*

3. *If* $s \geq \min\{\varphi(E(v)), \gamma(0)\}$*, then* $p^N = \pi^N = \max\{0, E(v)\}$ *and the buyer purchases immediately if* $E(v) \geq 0$*, and drops otherwise.*

Whether 2a or 2b arises depends on how monopoly price and profit relate to $E(v)$. If $p_M \leq E(v)$, then $\varphi(\pi_M) < \gamma(p_M)$ and only 2a is possible. If $\pi_M \geq E(v)$, then only 2b is possible, and both cases can arise if $\pi_M < E(v) < p_M$.

We now turn to the comparison of the outcome described in Proposition 4 with the solution chosen by the seller when she can implement an optimal product information disclosure.

5.2 Comparison

We start with the following uniform example to illustrate how profit maximization without product information differs from the fully optimal solution for the seller involving product information. Assume the match value v is uniformly distributed with $\bar{v} - \underline{v} = 1$, $\bar{v} \in (0, 1]$, so $g(v) = 1$ and $G(v) = v - \underline{v}$ for $v \in [\underline{v}, \bar{v}]$, $E(v) = \bar{v} - \frac{1}{2}$, and $\mu = \frac{\bar{v}}{2}$.

Monopoly price and profit are $p_M = \frac{\bar{v}}{2}$ and $\pi_M = \frac{\bar{v}^2}{4}$. The different regions for search costs in Proposition 4 are delineated by the following parameters:

$$\varphi(\pi_M) = \frac{(2 - \bar{v})^4}{32}, \quad \gamma(p_M) = \frac{\bar{v}^2}{8}, \quad \varphi(E(v)) = \gamma(E(v)) = \frac{1}{8}, \quad \gamma(0) = \frac{(\bar{v})^2}{2},$$

$$\hat{s} = \frac{1}{8}\left((\bar{v} - 1 + \sqrt{(\bar{v} - 1)(\bar{v} - 5)})\right)^2.$$

Lowering $\bar{v}$ from 1 to 0 allows exploration of the various possible configurations in Proposition 4. It results in a larger value for $G(0)$ so the buyer's match is more likely to be below the zero marginal cost. It is equivalent to keeping the match distribution unchanged and raising marginal cost.

First, the low search cost region corresponding to Proposition 4 (1), where the seller posts the monopoly price and the buyer searches always exists and prevails for search costs below

$$\min\{\varphi(\pi_M), \gamma(p_M)\} = \begin{cases} \gamma(p_M) & \text{if } \bar{v} \leq \hat{v} \in (\frac{1}{2}, 1) \\ \varphi(\pi_M) & \text{if } \bar{v} \geq \hat{v}, \end{cases}$$

where the crossing point at $\hat{v} \in (\frac{1}{2}, 1)$ indeed exists and is unique because $\varphi(\pi_M)$ is $\frac{1}{32}$ at $\bar{v} = 1$ and increases to $\frac{81}{512}$ for $\bar{v} = \frac{1}{2}$, while $\gamma(p_M)$ decreases from $\frac{1}{8}$ to $\frac{1}{32}$ over the same range.

For a larger search cost, two cases arise. For $\bar{v} \geq \hat{v}$, Proposition 4 (2a) applies for $s \in (\varphi(\pi_M), \varphi(E(v)))$ so the seller charges $p^N = \varphi^{-1}(s)$ and the buyer buys immediately. We have $E(v) > 0$ because $\bar{v} > \hat{v} > \frac{1}{2}$ so that, for $s \geq \varphi(E(v))$, price is $E(v)$ and the buyer purchases without search. For $\bar{v} < \hat{v}$, from Proposition 4 (2b), the seller's pricing behavior branches in two directions depending on whether $E(v) > 0$ or not, which depends on $\bar{v}$ being above or below $\frac{1}{2}$. If $E(v) \leq 0$, then for $\gamma(p_M) < s < \gamma(0)$, from the first bullet in 2b, price is $\gamma^{-1}(s)$ and the buyer searches while, from item 3, the market breaks down for higher search costs (with a negative expected valuation, the buyer does not want to buy without search). If $E(v) > 0$, so the second and third bullets in 2b are relevant, then there is search for $\gamma(p_M) < s < \hat{s}$ and the buyer buys immediately for $s \geq \hat{s}$ and, from item 3, this remains true for $s \geq \varphi(E(v))$.

In contrast to the optimal disclosure solution characterized in Section 4, costly information acquisition for the buyer does not guarantee the seller a profit above the full information monopoly level. The only instance when this can happen without product information is when, as search costs increase, the seller drops its price to switch from inducing search to inducing immediate purchase. From Proposition 4 in case (2a), after the drop profit rises above the monopoly level: this arises for $\bar{v} > \hat{v}$ in the uniform example. In case (2b) (i.e., $\bar{v} < \hat{v}$ in the uniform example) profit increases after the price drop from a level below monopoly to reach $E(v)$, which can exceed monopoly profit (e.g., for $\bar{v}$ close enough to $\hat{v}$ in the uniform example). Case (2b) also illustrates that without product information disclosure, profit can be strictly decreasing in search costs, (on the interval $[\gamma(p_M), \gamma(0)]$, if $E(v) \leq 0$ and $[\gamma(p_M), \hat{s}]$ if $E(v) > 0$), because the seller decreases her price below p_M to induce search. Recall that, with optimal disclosure, profit is always strictly increasing in search costs for $s \leq \bar{s}$, and constant thereafter.

The impact on the buyer's welfare is also quite different from the findings for optimal information disclosure, where a drop in search costs unambiguously increases consumer surplus. In order for this to be the case with no information disclosure, we need $\bar{v} \leq 1/2$, so that $E(v) \leq 0$, in which case consumer welfare is $\gamma(p_M) - s$, for $s \leq \gamma(p_M)$ and then zero for higher search costs. For $E(v) > 0$, consumer welfare is always discontinuous in s because it jumps up when the seller switches from inducing search to inducing an immediate purchase, which happens at $s = \varphi(\pi_M)$ for $\bar{v} > \hat{v}$ or at $s = \hat{s}$ for $1/2 < \bar{v} < \hat{v}$. In both cases, consumer welfare can exceed the full information level $\gamma(p_M)$ right after the jump though it eventually falls to 0 as search costs increase.

Holding the cost of information acquisition constant, profit is obviously higher with optimal disclosure than with no disclosure. Whether the buyer benefits from optimal disclosure by the seller is a more complicated matter. On the one hand, threshold information disclosure prevents him from searching or buying the product immediately when his valuation is too low. On the other hand, from our characterization of profit maximization with optimal disclosure in Proposition 2, whenever the buyer's surplus is strictly positive, price exceeds the monopoly price, and hence, the price with no information disclosure. Figure 7 illustrates three different configurations in the uniform example, with different values for $\bar{v}$, 0.5, 0.7 and 1. They suggest that the buyer is better off with no disclosure if it is sufficiently unlikely that his match falls below marginal cost and search costs are relatively high ($\bar{v} = 1$ or $\bar{v} = 0.7$ and s close to and below $\varphi(E(v))$), and he benefits from optimal information disclosure if his match is likely to be below marginal cost and search costs are relatively low ($\bar{v} = 0.7$ or $\bar{v} = 0.5$ and $s < s_1$). The next two propositions show that these insights hold more generally.

First, consider a search cost which is large, but below $\varphi(E(v))$. Further assume that $E(v) > 0$. From Proposition 4 (2), with no information, the seller charges price $p^N = \varphi^{-1}(s) < E(v)$ so the buyer purchases the product immediately and has a strictly positive surplus. With optimal disclosure, the buyer's surplus is zero if $s \geq s_1$ (see Proposition 3). The next result shows that $\bar{s} \leq \varphi(E(v))$, and since $s_1 < \bar{s}$, for s sufficiently close to $\varphi(E(v))$, the buyer's surplus is larger with no information disclosure than with optimal disclosure.

Proposition 5 *We have $\bar{s} \leq \varphi(E(v))$. Therefore, if $E(v) > 0$, the buyer is strictly better off with no information than with optimal disclosure by the seller for search costs s sufficiently close to and below $\varphi(E(v))$.*

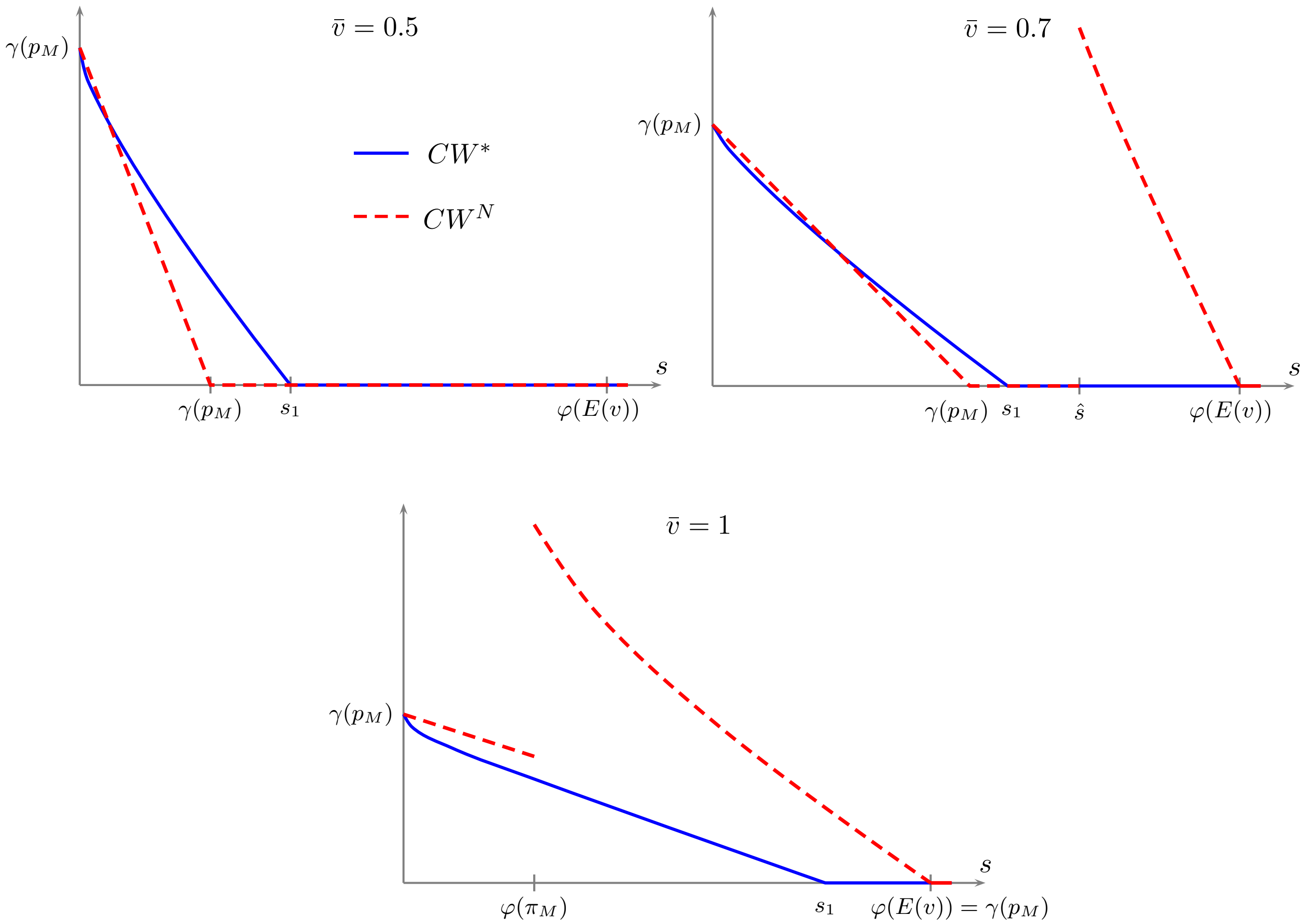

Figure 7: Comparison of consumer welfare under no disclosure (CW^N, in red dashed lines) and optimal disclosure (CW^*, in blue) when the match distribution is uniform on $[\underline{v}, \bar{v}]$, with $\underline{v} = \bar{v} - 1$, as a function of the search cost s for $\bar{v} = 0.5, 0.7, 1$.

Proof. See Appendix A.3. ■

Proposition 5 merely states that for $s < \varphi(E(v))$, if search costs are large enough, then consumer surplus with optimal disclosure must be zero whereas it is strictly positive with no information. A comparison of $\bar{v} = 1$ and $\bar{v} = 0.7$ in the uniform example suggests that the requirement that s is sufficiently large is more stringent if the probability that the buyer's valuation is below marginal cost is smaller: actually, for $\bar{v} = 1$, the buyer is strictly better off without information disclosure for all search cost levels up to $\varphi(E(v)) = \frac{1}{8}$.

Assume now that it is very likely that the buyer's match is below marginal cost so $E(v) \leq 0$. The buyer's surplus is then strictly positive if and only if $s < \gamma(p_M)$ with no disclosure or $s < s_1$ with optimal disclosure. In the uniform example with $\bar{v} = \frac{1}{2}$, $\gamma(p_M) < s_1$. Hence the buyer benefits from an optimal information disclosure by the seller in the interval $[\gamma(p_M), s_1)$. More generally, a lower $\bar{v}$ means that, with no information, the buyer is more likely to search without buying or to buy a product at a price above his valuation. Concurrently, the loss in surplus due to the higher price with optimal disclosure tends to become smaller because the potential overall surplus shrinks. Intuition suggests this makes the optimal threshold information provided by the seller more valuable for the buyer. This intuition is born out by the next proposition.

Proposition 6 *Consider a distribution function F with support $[0, \bar{w}]$ that admits a continuously differentiable density and has a strictly increasing hazard rate. Let $G(v) = F(v - \underline{v})$, with $\underline{v} \in [-\bar{w}, 0]$. If $\underline{v}$ is sufficiently low, then $\gamma(p_M) < s_1$, and there exists an interval of information acquisition costs, including $[\gamma(p_M), s_1)$, such that the buyer is strictly better off with optimal disclosure rather than no information provided by the seller.*

Proof. See Appendix A.3 ■

The idea of the proposition is to slide down the support of the match value distribution while retaining its original shape, as we have done in the uniform example by moving $\bar{v}$. This is equivalent to increasing marginal cost. The result can hold even if $E(v) > 0$ as illustrated with the uniform distribution for $\bar{v} = 0.7$, for which $s_1 > \gamma(p_M)$ as shown in Figure 7. If marginal cost is sufficiently large relative to match realizations, there is a range of search costs below s_1 at which the buyer benefits from optimal information disclosure. This range corresponds to lower values of search costs as marginal cost increases because s_1 decreases: it is $\frac{\bar{v}}{10}$ in our uniform application. This means that, in this configuration, if the buyer has access to an efficient search technology, he prefers that the seller can use sophisticated communication.

Now consider a drop in search costs combined with an improvement in the seller's ability to communicate. Our discussion around Propositions 5 and 6 suggests that the prediction for the impact on consumer welfare is contrasted depending on where marginal cost lies relative to the match distribution. If it is low enough, then the analysis in Proposition 5 is relevant so the buyer can obtain a strictly positive surplus for large search costs as long as they are below $\varphi(E(v))$. When marginal cost is sufficiently low (corresponding to $\bar{v}$ close to 1 in the uniform example), buyer surplus with a high search cost can be larger than what the buyer would obtain with full information. This is illustrated by the uniform example with $\bar{v} = 1$: at $s = \varphi(\pi_M) = \frac{1}{32}$, with no information disclosure, consumer surplus jumps up from $\frac{3}{32}$ to $\frac{1}{4}$ and remains strictly larger than its monopoly level $\frac{1}{8}$ for some interval of search costs above $\varphi(\pi_M)$. If there is an improvement in the seller's communication technology so she can resort to optimal disclosure, the buyer's surplus falls below $\frac{1}{8}$ even if there is a simultaneous drop in search costs. By contrast, in the scenario of Proposition 6 where it is very likely that marginal cost exceeds the buyer's valuation, a sufficient drop in search costs always improves the buyer's welfare and the benefit for the buyer is reinforced if there is a switch from no information to optimal disclosure.

To discuss the impact of such a joint improvement in the search and communication technologies on the seller's profit, note that, in the terminology of Johnson and Myatt (2006), the product is a mass market product when it is unlikely that marginal cost exceeds the buyer's valuation, and a niche market product in the reverse situation. The seller of a mass market product prefers selling to an uninformed buyer rather than charging the monopoly price to a buyer who knows his match perfectly. If search costs are large enough, then profit maximization with no disclosure is close to the situation where the buyer cannot acquire any information. If, on the contrary, search costs are too low, the seller, even if she can disclose information optimally, cannot raise her profit much above monopoly profit. She is therefore better off without any ability to optimize buyer information along with high search costs, than with low search costs and optimal disclosure.

In the uniform example with $\bar{v} = 1$, at $s = \varphi(\pi_M) = \frac{1}{32}$, with no information disclosure, the seller earns monopoly profit $\frac{1}{4}$ and profit rises above this level as search costs increase. By contrast, if search costs drop substantially and become close to zero while the seller can switch to optimal information disclosure profit is close to its monopoly levels. This, along with our above discussion of consumer welfare, illustrates that a drop in search costs concomitant with increased sophistication in the seller's communication, which could result, for instance, from trading on digital platforms, can be detrimental to both parties. For a niche product, the seller prefers trading with an informed buyer which is reflected in the monotonicity of her profit with respect to search costs with no information and, when search costs are low, she can do even better if she can resort to optimal disclosure. Hence, the joint improvement in technologies benefits both sides.[13] This suggests that trade on online platforms that combine low search costs with sophisticated marketing is particularly desirable for niche products.

6 Concluding remarks

The paper provides a full characterization of a seller's optimal disclosure and pricing strategy when the buyer has access to independent sources of information. The optimal information design solution highlights that the seller benefits from higher costs of information acquisition for the buyer, both by charging a higher price and selling a larger quantity. This hurts the buyer but improves overall welfare.

By comparing the optimal disclosure outcome with a situation where the seller can provide no product information, the analysis provides valuable insights into the impact of improvements in the search and communication technology on profits and consumer welfare. In particular, improved sophistication in seller communication can benefit consumers only if their search costs are low enough. Such a beneficial impact on consumers arises when it is very likely that consumer valuations are below marginal costs, in which case both buyers and sellers can benefit if the search and communication technologies jointly improve.

[13]With the uniform match distribution, such a Pareto improvement in trade arises for a low enough search cost and a high enough marginal cost, except if s is too close to zero: in our numerical examples, this neighborhood shrinks as it becomes more likely that match falls below marginal cost.

A Appendix

A.1 Proof of Proposition 1

First, according to the revelation principle (Myerson, 1982, Proposition 2), we can without loss of generality set $M = A$, making the information disclosure policy a direct recommendation system $X : [\underline{v}, \bar{v}] \to \Delta(A)$. Let $X(a \mid s)$ represent the probability that the consumer receives recommendation (signal) a given the match value v. Additionally, we can, again without loss of generality, require obedience from the consumer, meaning the consumer chooses action a upon receiving recommendation a. Thus, $X(a \mid s)$ denotes the probability that the consumer plays action a when the match value is v. Let $X(a)$ denote the unconditional probability of signal a under policy X. Let $\mathbb{P}_X$ be the probability distribution over match values and signals induced by X, and let $E_X(v \mid a)$ represent the expected match value according to the posterior induced by X when the recommendation is a.

The incentive compatibility (obedience) constraints for consumers receiving the recommendation to *buy* are:

$$E_X(v - p \mid buy) \geq 0, \tag{BnD}$$

so they don't drop, and

$$s \geq \mathbb{P}_X(v < p \mid buy)E_X(p - v \mid buy, v < p), \tag{BnS}$$

so they don't search. Similarly, incentive compatibility constraints for consumers receiving the recommendation to *drop* are:

$$E_X(v - p \mid drop) \leq 0, \tag{DnB}$$

so they do not buy, and

$$s \geq \mathbb{P}_X(v \geq p \mid drop)E_X(v - p \mid drop, v \geq p), \tag{DnS}$$

so they do not search. Finally, incentive compatibility constraints for consumers receiving the recommendation to *search* are:

$$\mathbb{P}_X(v \geq p \mid search)E_X(v - p \mid search, v \geq p) - s \geq 0, \tag{SnD}$$

so they do not drop, and

$$\mathbb{P}_X(v \geq p \mid search)E_X(v - p \mid search, v \geq p) - s \geq E_X(v - p \mid search), \tag{SnB}$$

so they do not buy.

Now, consider an arbitrary direct disclosure policy X_0 that satisfies all the incentive compatible conditions above. We show that there exists an incentive compatible disclosure policy Y such that $Y(search) = 0$ yielding the same ultimate probabilities of buying and dropping as X_0. Take Y defined as follows:

$$Y(buy \mid v) = X_0(buy \mid v) + X_0(search \mid v) \text{ and } Y(drop \mid v) = X_0(drop \mid v), \text{ if } v \geq p,$$

$$Y(buy \mid v) = X_0(buy \mid v) \text{ and } Y(drop \mid v) = X_0(drop \mid v) + X_0(search \mid v), \text{ if } v < p.$$

Then we have

$$E_Y(v \mid buy) = \frac{X_0(buy)}{Y(buy)} E_{X_0}(v \mid buy) + \frac{X_0(search \mid v \geq p)[1 - G(p)]}{Y(buy)} E_{X_0}(v \mid search, v \geq p).$$

The first expectation is at least p from the incentive compatibility condition (BnD) of X_0, and the second is at least p by construction, so a buyer receiving signal *buy* from the disclosure policy X_0 does not deviate to *drop*, i.e., the incentive compatibility condition (BnD) is also satisfied for Y. He will not search either because his benefit from searching (which only arises when $v < p$) is smaller than with disclosure policy X_0. Formally, we have

$$\mathbb{P}_{X_0}(v < p \mid buy) = \frac{\mathbb{P}_{X_0}(v < p, buy)}{X_0(buy)} \geq \frac{\mathbb{P}_Y(v < p, buy)}{Y(buy)} = \mathbb{P}_Y(v < p \mid buy),$$

because $\mathbb{P}_{X_0}(v < p, buy) = \mathbb{P}_Y(v < p, buy)$ and $Y(buy) \geq X_0(buy)$, and we have

$$E_{X_0}(p - v \mid buy, v < p) = E_Y(p - v \mid buy, v < p),$$

so the RHS of (BnS) is lower for $X = Y$ than for $X = X_0$.

The arguments for showing that a buyer who receives the *drop* signal will not want to deviate are similar. For a buyer receiving signal *drop*, $E_Y(v \mid drop)$ is a convex combination of $E_{X_0}(v \mid drop)$ and $E_{X_0}(v \mid search, v < p)$, which are both lower than p, so (DnB) is also satisfied for $X = Y$. Finally, consider the buyer's incentive to search instead of dropping when he receives the *drop* signal, i.e., condition (DnS). We have

$$\mathbb{P}_{X_0}(v \geq p \mid drop) E_{X_0}(v - p \mid drop, v \geq p) \geq \mathbb{P}_Y(v \geq p \mid drop) E_Y(v - p \mid drop, v \geq p),$$

because $\mathbb{P}_{X_0}(v \geq p \mid drop) \geq \mathbb{P}_Y(v \geq p \mid drop)$ and $E_{X_0}(v - p \mid drop, v \geq p) = E_Y(v - p \mid drop, v \geq p)$, so the RHS of (DnS) is lower for $X = Y$ than for $X = X_0$.

Next, consider some incentive compatible disclosure policy Y such that $Y(search) = 0$. Let $\tilde{v}$ be the unique solution to $1 - G(\tilde{v}) = Y(buy)$, and define a threshold disclosure policy Z such that $Z(buy \mid v) = 1$ if $v \geq \tilde{v}$ and $Z(drop \mid v) = 1$ if $v < \tilde{v}$. There is no information acquisition with either Y or Z, and they both yield the same probability of purchase.

Clearly, we have $E_Y(v \mid buy) \leq E_Z(v \mid buy)$ and $E_Y(v \mid drop) \geq E_Z(v \mid drop)$, so the incentive constraints (BnD) and (DnB) are satisfied for $X = Z$. Consider next the constraint (BnS). It is obviously satisfied if $p \leq \tilde{v}$, so let $p > \tilde{v}$. We have

$$E_Z(v \mid buy, v < p) \geq E_Y(v \mid buy, v < p),$$

and

$$\mathbb{P}_Z(v < p \mid buy) \leq \mathbb{P}_Y(v < p \mid buy),$$

so the RHS of (BnS) is smaller with $X = Z$ than with $X = Y$. Similarly, consider the constraint (DnS). It is obviously satisfied if $p \geq \tilde{v}$, so let $p < \tilde{v}$. We have

$$E_Z(v - p \mid drop, v \geq p) \leq E_Y(v - p \mid drop, v \geq p),$$

and

$$\mathbb{P}_Z(v \geq p \mid drop) \leq \mathbb{P}_Y(v \geq p \mid drop),$$

so the RHS of (DnS) is smaller with $X = Z$ than with $X = Y$.

Given the disclosure policy Z above, a firm charging $p \leq \tilde{v}$ could increase p slightly above $\tilde{v}$ without violating any incentive compatibility constraint and hence make more profit. So we must have $p > \tilde{v}$.

A.2 Additional results and proofs for Section 4

Proof of Lemma 4.1. (i) Along the Δ_b curve p is strictly increasing in $\tilde{v}$ because the LHS of (Δ_b) is $E(v \mid v \geq \tilde{v}) - p$, which strictly decreasing in p and strictly increasing in $\tilde{v}$. The fact that $p = \mu$ for $\tilde{v} = 0$ and $p = \bar{v}$ for $\tilde{v} = \bar{v}$ directly follows from (Δ_b).

(ii) To show the properties of the Σ_b curve we first show that $\phi(\tilde{v}, p) = \frac{\int_{\tilde{v}}^{p} (p-v)g(v)\,dv}{1-G(\tilde{v})}$ is strictly increasing in p and strictly decreasing in $\tilde{v}$ for $p \in [0, \bar{v}]$ and $\tilde{v} \leq p$. By integrating by parts, we have

$$\phi(\tilde{v}, p) = \frac{\int_{\tilde{v}}^{p} G(v) - G(\tilde{v})\,dv}{1 - G(\tilde{v})}.$$

Hence,

$$\frac{\partial \phi}{\partial p} = \frac{G(p) - G(\tilde{v})}{1 - G(\tilde{v})} > 0, \tag{7}$$

$$\frac{\partial \phi}{\partial \tilde{v}} = -\frac{g(\tilde{v})}{(1 - G(\tilde{v}))^2} \int_{\tilde{v}}^{p} 1 - G(v)\,dv < 0. \tag{8}$$

Hence, along the Σ_b curve, p is strictly increasing in $\tilde{v}$, and Σ_b shifts upward if s increases.

The fact that $p = \mu$ for $\tilde{v} = 0$ if $s = \bar{s}$ directly follows from the definition of $\bar{s}$.

Finally, for $p = \bar{v}$, (Σ_b) is $\frac{\int_{\tilde{v}}^{\bar{v}} (\bar{v}-v)g(v)\,dv}{1-G(\tilde{v})} = s$, i.e.,

$$E(v \mid v \geq \tilde{v}) = \bar{v} - s.$$

As $\tilde{v}$ increases from 0 to $\bar{v}$, the left hand side increases strictly and continuously from μ to $\bar{v}$. For $s \in (0, \bar{s}]$, the right hand side is in $(\mu, \bar{v})$ because, from Equation (1) and using $\mu = \frac{\int_0^{\mu} v dG(v)}{1-G(0)}$, we have $\bar{s} = \frac{\mathbb{P}\{v \geq \mu\}}{\mathbb{P}\{v \geq 0\}} E(v - \mu \mid v \geq \mu) < \bar{v} - \mu$. Hence, the equality holds for some unique $\tilde{v} \in (0, \bar{v})$. ■

Before proving the next lemmas we prove the following result.

Claim 1 If at $(\tilde{v}^*, p^*)$ we have $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$, then $p_M < p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$.

Proof of Claim 1. The slope of the iso-profit curve at some $(\tilde{v}, p)$ is given by

$$p'_{ISO}(\tilde{v}) = \frac{g(\tilde{v})p}{1 - G(\tilde{v})}. \tag{9}$$

Using (7) and (8) in the proof of Lemma 4.1, the slope of the Σ_b curve at $(\tilde{v}, p)$ is given by

$$p'_{\Sigma}(\tilde{v}) = -\frac{\frac{\partial \phi}{\partial \tilde{v}}}{\frac{\partial \phi}{\partial p}} = \frac{g(\tilde{v})}{1 - G(\tilde{v})} \frac{\int_{\tilde{v}}^{p} 1 - G(v)\, dv}{G(p) - G(\tilde{v})}. \tag{10}$$

If $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$, then

$$p^* = \frac{\int_{\tilde{v}^*}^{p^*} 1 - G(v)\, dv}{G(p^*) - G(\tilde{v}^*)}, \tag{11}$$

which can be rewritten as

$$\int_{\tilde{v}^*}^{p^*} 1 - G(v) - p^* g(v)\, dv = \int_{\tilde{v}^*}^{p^*} \left(\frac{1 - G(v)}{g(v)} - p^* \right) g(v)\, dv = 0. \tag{12}$$

Because $\frac{g(\cdot)}{1-G(\cdot)}$ is increasing and $p^* > \tilde{v}^*$, we have $\frac{1-G(p^*)}{g(p^*)} < \frac{1-G(v)}{g(v)}$ for $v \in [\tilde{v}^*, p^*]$, so

$$\int_{\tilde{v}^*}^{p^*} \left(\frac{1 - G(p^*)}{g(p^*)} - p^* \right) g(v)\, dv < \int_{\tilde{v}^*}^{p^*} \left(\frac{1 - G(v)}{g(v)} - p^* \right) g(v)\, dv = 0.$$

This inequality implies

$$\frac{1 - G(p^*)}{g(p^*)} < p^*, \tag{13}$$

and therefore, from the increasing hazard rate, $p^* > p_M = \frac{1-G(p_M)}{g(p_M)}$.

To prove that $p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$, we use (11) and the fact that $\frac{1-G(v)}{g(v)}$ is decreasing in v:

$$p^* = \frac{\int_{\tilde{v}^*}^{p^*} \frac{(1-G(v))}{g(v)} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} < \frac{\int_{\tilde{v}^*}^{p^*} \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} = \frac{1 - G(\tilde{v}^*)}{g(\tilde{v}^*)} \frac{\int_{\tilde{v}^*}^{p^*} g(v)\, dv}{G(p^*) - G(\tilde{v}^*)} = \frac{1 - G(\tilde{v}^*)}{g(\tilde{v}^*)}. \tag{14}$$

■

Proof of Lemma 4.2. The proof proceeds in two steps. First we show that first-order conditions have only one solution. Second, we show that $\tilde{v}^* > 0$ so that, if there is a corner solution, then we have $p^* = \bar{v}$.

Step 1. To show that there is only one solution to first-order conditions we first show that whenever $p'_{ISO}(\tilde{v}) = p'_{\Sigma}(\tilde{v})$ then the difference $p'_{ISO} - p'_{\Sigma}$ is strictly increasing in the neighborhood of $\tilde{v}$ along the Σ_b curve. This in turn implies that $p'_{ISO} - p'_{\Sigma}$ can only switch sign from being negative to being positive and hence switches sign only once. This ensures that there is a unique solution which is either interior, in which case $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$, at $\tilde{v}^* = \underline{v}$ if $p'_{ISO}(\underline{v}) \geq p'_{\Sigma}(\underline{v})$ or $\tilde{v}^*$ is such that $p^* = \bar{v}$, which would require $p'_{ISO}(\tilde{v}^*) \leq p'_{\Sigma}(\tilde{v}^*)$. Therefore there is a unique point $(\tilde{v}^*, p^*)$ satisfying the FOC.

From Equations (9) and (10) we have

$$p'_{ISO}(\tilde{v}) - p'_{\Sigma}(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})}\left(p - \frac{\int_{\tilde{v}}^{p} 1-G(v)\,dv}{G(p)-G(\tilde{v})}\right).$$

Because $\frac{g(\tilde{v})}{1-G(\tilde{v})}$ is positive and strictly increasing, it suffices to show that $f(\tilde{v}) := p - \frac{\int_{\tilde{v}}^{p} 1-G(v)\,dv}{G(p)-G(\tilde{v})}$ is strictly increasing in $\tilde{v}$ at whenever $f'(\tilde{v}) = 0$. We have

$$f'(\tilde{v}) = p'_{\Sigma}(\tilde{v}) - \frac{d}{d\tilde{v}}\frac{\int_{\tilde{v}}^{p} 1-G(v)\,dv}{G(p)-G(\tilde{v})},$$

so, using (10), we get:

$$f'(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})}\frac{\int_{\tilde{v}}^{p} 1-G(v)\,dv}{G(p)-G(\tilde{v})} - \frac{d}{d\tilde{v}}\frac{\int_{\tilde{v}}^{p} 1-G(v)\,dv}{G(p)-G(\tilde{v})}.$$

$$= \frac{g(\tilde{v})}{1-G(\tilde{v})}\frac{N}{D} - \frac{N'D - ND'}{D^2},$$

where $N = \int_{\tilde{v}}^{p} 1-G(v)\,dv$, $D = G(p)-G(\tilde{v})$, $N' = p'_{\Sigma}(\tilde{v})(1-G(p)) - (1-G(\tilde{v}))$ and $D' = p'_{\Sigma}(\tilde{v})g(p) - g(\tilde{v})$.

If $f(\tilde{v}) = 0$, then $p = \frac{N}{D}$ and $p'_{\Sigma}(\tilde{v}) = p'_{ISO}(\tilde{v}) = \frac{g(\tilde{v})p}{1-G(\tilde{v})}$ so $f'(\tilde{v})$ can be rewritten as follows:

$$f'(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})}p - \frac{N' - pD'}{D}$$

$$= \frac{g(\tilde{v})}{1-G(\tilde{v})}p - \frac{p'_{\Sigma}(\tilde{v})(1-G(p)) - (1-G(\tilde{v})) - p(p'_{\Sigma}(\tilde{v})g(p) - g(\tilde{v}))}{G(p)-G(\tilde{v})}$$

$$= \frac{g(\tilde{v})}{1-G(\tilde{v})}p - \frac{\frac{g(\tilde{v})p}{1-G(\tilde{v})}(1-G(p)) - (1-G(\tilde{v})) - p(\frac{g(\tilde{v})p}{1-G(\tilde{v})}g(p) - g(\tilde{v}))}{G(p)-G(\tilde{v})}$$

$$= \frac{1}{G(p)-G(\tilde{v})}\left(\frac{g(\tilde{v})p(G(p)-G(\tilde{v}))}{1-G(\tilde{v})} - \frac{g(\tilde{v})p(1-G(p))}{1-G(\tilde{v})} + (1-G(\tilde{v})) + \frac{pg(\tilde{v})pg(p)}{1-G(\tilde{v})} - pg(\tilde{v})\right)$$

$$= \frac{1}{G(p)-G(\tilde{v})}\left(\frac{pg(\tilde{v})}{1-G(\tilde{v})}\Big[G(p)-G(\tilde{v})-(1-G(p))+pg(p)-(1-G(\tilde{v}))\Big] + (1-G(\tilde{v}))\right)$$

$$= \frac{1}{G(p)-G(\tilde{v})}\left(\frac{pg(\tilde{v})g(p)}{1-G(\tilde{v})}\Big[p-2\frac{1-G(p)}{g(p)}\Big] + (1-G(\tilde{v}))\right).$$

Using the properties that $G(p)-G(\tilde{v}) \geq 0$ (because $p \geq \tilde{v}$) and $p \geq \frac{1-G(p)}{g(p)}$ (because $p \geq p_M$) a sufficient condition for $f'(\tilde{v}) > 0$ is

$$p < \frac{1-G(\tilde{v})}{g(\tilde{v})}\frac{1-G(\tilde{v})}{1-G(p)}.$$

$p \geq \tilde{v}$ also implies $\frac{1-G(\tilde{v})}{1-G(p)} \geq 1$ so a sufficient condition for $f'(\tilde{v}) > 0$ is $p < \frac{1-G(\tilde{v})}{g(\tilde{v})}$, which follows from Claim 1.

Step 2. We show that $\tilde{v}^* > 0$. From our analysis above, it suffices to show that for $s \leq \bar{s}$, $p'_{ISO}(0) < p'_{\Sigma}(0)$. Indeed this implies that there is no $\tilde{v} \leq 0$ at which $p'_{ISO}(\tilde{v}) = p'_{\Sigma}(\tilde{v})$, which in turn implies that we cannot have $p'_{ISO}(\tilde{v}) \geq p'_{\Sigma}(\tilde{v})$ at $\tilde{v} = \underline{v}$, so first-order conditions never hold for $\tilde{v} \leq 0$.

Let $p'_{\Delta}(\tilde{v})$ be the slope of the Δ_b curve at $(\tilde{v}, p)$. We can write $p'_{\Delta}(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})}h(\tilde{v})$ and $p'_{\Sigma}(\tilde{v}) = \frac{g(\tilde{v})}{1-G(\tilde{v})}h(p)$, where

$$h(x) = \frac{\int_{\tilde{v}}^{x} 1-G(v)dv}{G(x)-G(\tilde{v})},$$

and

$$h'(x) = \frac{[1-G(x)][G(x)-G(\tilde{v})] - g(x)\int_{\tilde{v}}^{x} 1-G(v)dv}{[G(x)-G(\tilde{v})]^2}.$$

Standard arguments show that the increasing hazard rate property of G implies that the integral is bounded below by $\frac{[1-G(x)][G(x)-G(\tilde{v})]}{g(x)}$ so $h'(x) < 0$. It follows that at any crossing point between Σ_b and Δ_b, $p'_{\Delta}(\tilde{v}) < p'_{\Sigma}(\tilde{v})$. Now consider $\tilde{v} = 0$. The two curves cross at $\tilde{v} = 0$ and $p = \mu$. Furthermore, at that point, profit is maximized subject to the constraint Δ alone so we have $p'_{ISO}(\tilde{v}) = p'_{\Delta}(\tilde{v}) < p'_{\Sigma}(\tilde{v})$, which proves the result for $s = \bar{s}$. Now because h is decreasing, $p'_{\Sigma}(\tilde{v})$ becomes larger for prices $p < \mu$ whereas $p'_{ISO}(\tilde{v}) = \frac{pg(0)}{1-G(0)}$ is lower. Furthermore, for $s < \bar{s}$, (Σ) binds at $p < \mu$ when $\tilde{v} = 0$, so we have $p'_{ISO}(\tilde{v}) < p'_{\Sigma}(\tilde{v})$ when $\tilde{v} = 0$. ■

Proof of Lemma 4.3. From the proof of Claim 1, the first-order condition $p'_{ISO}(\tilde{v}^*) = p'_{\Sigma}(\tilde{v}^*)$ can be rewritten as (12). Differentiating with respect to s yields, after simplification:

$$\frac{dp^*}{ds} = \frac{1 - G(\tilde{v}^*) - p^* g(\tilde{v}^*)}{1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*)} \frac{d\tilde{v}^*}{ds}. \tag{15}$$

Note that the denominator is negative,

$$1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*) < 0, \tag{16}$$

because $1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*) = 1 - G(p^*) - p^* g(p^*) + G(\tilde{v}^*) - G(p^*)$, $1 - G(p^*) - p^* g(p^*) < 0$ and $G(\tilde{v}^*) - G(p^*) < 0$.

Next, we differentiate the binding constraint (Σ_b), which gives

$$\frac{d\phi}{ds} = \frac{d\tilde{v}^*}{ds} \frac{\partial \phi}{\partial \tilde{v}^*} + \frac{dp^*}{ds} \frac{\partial \phi}{\partial p^*} = 1.$$

Using (7) and (8) we get:

$$\frac{dp^*}{ds} = \frac{1 - G(\tilde{v}^*)}{G(p^*) - G(\tilde{v}^*)} + \frac{g(\tilde{v}^*) \int_{\tilde{v}^*}^{p^*} (1 - G(v))\, dv}{(G(p^*) - G(\tilde{v}^*))(1 - G(\tilde{v}^*))} \frac{d\tilde{v}^*}{ds}. \tag{17}$$

Combining (15) and (17) we get:

$$\frac{d\tilde{v}^*}{ds} \left(\frac{1 - G(\tilde{v}^*) - p^* g(\tilde{v}^*)}{1 - 2G(p^*) + G(\tilde{v}^*) - p^* g(p^*)} - \frac{g(\tilde{v}^*) \int_{\tilde{v}^*}^{p^*} (1 - G(v))\, dv}{(G(p^*) - G(\tilde{v}^*))(1 - G(\tilde{v}^*))} \right) = \frac{1 - G(\tilde{v}^*)}{G(p^*) - G(\tilde{v}^*)}. \tag{18}$$

Observe that the RHS of (18) is positive, the second fraction of the LHS is positive, and the denominator of the first fraction of the LHS is negative as observed in (16). To show that $\frac{d\tilde{v}^*}{ds} < 0$ it remains to show that the numerator of the first fraction of the LHS of (18) is positive, which follows from $p^* < \frac{1 - G(\tilde{v}^*)}{g(\tilde{v}^*)}$ (Claim 1). We conclude that $\frac{d\tilde{v}^*}{ds} < 0$, and hence from Equations (15) and (16) that $\frac{dp^*}{ds} > 0$.

A.3 Proofs for Section 5

Proof of Proposition 4. Arguments for items 1 and 3 are provided in the text. So, assume $\min\{\varphi(\pi_M), \gamma(p_M)\} < s < \min\{\varphi(E(v)), \gamma(0)\}$. To establish 2a, assume $\varphi(\pi_M) < \gamma(p_M)$. Then, $s \leq \varphi(E(v))$ implies that the buyer buys at price $\varphi^{-1}(s)$ with corresponding profit $\varphi^{-1}(s) > \pi_M$ because $s > \varphi(\pi_M)$. Hence, charging $\gamma^{-1}(s)$ to induce search would yield a lower profit.

To prove 2b suppose $\gamma(p_M) < \varphi(\pi_M)$ so $s > \gamma(p_M)$ and hence $\gamma^{-1}(s) < p_M$. The first bullet point is covered in the text. To establish the other two bullet points note that, if price is $\gamma^{-1}(s)$, profit is $\gamma^{-1}(s)[1 - G(\gamma^{-1}(s))]$ which is π_M for $s = \gamma(p_M)$ and, because monopoly profit is strictly increasing on $[0, p_M]$ and γ^{-1} is strictly decreasing, it strictly decreases to $E(v)[1 - G(E(v))]$ as s increases from $\gamma(p_M)$ to $\varphi(E(v))$ (where we have used (5) to substitute $\varphi(E(v)) = \gamma(E(v))$). If price is $\varphi^{-1}(s)$, so the buyer purchases the product without search, the corresponding profit is $\varphi^{-1}(s)$ which strictly increases as s increases from $\gamma(p_M)$ to $\varphi(E(v))$. For s close to $\gamma(p_M)$, the corresponding profit is strictly less than π_M and hence, strictly less than the profit at price $\gamma^{-1}(s)$, because $\gamma(p_M) < \varphi(\pi_M)$. It is $E(v)$ for $s = \varphi(E(v))$ which is larger than profit at $\gamma^{-1}(\varphi(E(v)))$. It follows that there exists a unique $\hat{s} \in [\gamma(p_M), \varphi(E(v))]$ at which the two profits cross, so (6) holds, and profit is larger at $\gamma^{-1}(s)$ if and only if $s \in (\gamma(p_M), \hat{s})$. ■

Proof of Proposition 5. To show that $\bar{s} \leq \varphi(E(v))$, for every $x \in [\underline{v}, \bar{v}]$ define $\nu(x) = E(v \mid v > x)$. Consider now $V(x) = \frac{\int_x^{\nu(x)} (\nu(x) - v) g(v) dv}{1 - G(x)}$ so $\bar{s} = \frac{\int_0^{\mu} (\mu - v) g(v)\, dv}{1 - G(0)} = V(0)$ and $\varphi(E(v)) = \int_{\underline{v}}^{E(v)} (E(v) - v) g(v) dv = V(\underline{v})$. We show that $V' < 0$. It has the sign of

$$\big[[G(\nu(x)) - G(x)]\nu'(x) - (\nu(x) - x) g(x)\big][1 - G(x)] + g(x) \int_x^{\nu(x)} (\nu(x) - v) g(v) dv.$$

Substituting for $\nu'(x) = \frac{g(x)}{(1 - G(x))^2} \int_x^{\bar{v}} 1 - G(v) dv$ and integrating by parts, V' has the sign of

$$\left(\frac{G(\nu(x)) - G(x)}{1 - G(x)} - 1 \right) \int_x^{\nu(x)} 1 - G(v) dv + \frac{G(\nu(x)) - G(x)}{1 - G(x)} \int_{\nu(x)}^{\bar{v}} 1 - G(v) dv.$$

Increasing hazard rate for G implies $\frac{1 - G(v)}{g(v)} \geq \frac{1 - G(\nu(x))}{g(\nu(x))}$ for $v \leq \nu(x)$ and $\frac{1 - G(v)}{g(v)} \leq \frac{1 - G(\nu(x))}{g(\nu(x))}$ for $v \geq \nu(x)$, so an upper bound of the above term is

$$\left(\frac{G(\nu(x)) - G(x)}{1 - G(x)} - 1 \right) \int_x^{\nu(x)} \frac{1 - G(\nu(x))}{g(\nu(x))} g(v) dv + \frac{G(\nu(x)) - G(x)}{1 - G(x)} \int_{\nu(x)}^{\bar{v}} \frac{1 - G(\nu(x))}{g(\nu(x))} g(v) dv,$$

which simplifies to zero. The second part of the proposition follows from Proposition 4. ■

Proof of Proposition 6. Clearly, for $\underline{v}$ low enough, we have $E(v) \leq 0$, so $\varphi(\pi_M) > \gamma(p_M)$, and

Proposition 4 implies that under no disclosure consumer welfare is 0 for every $s \geq \gamma(p_M)$. Proposition 2 implies that under optimal disclosure consumer welfare is strictly positive for every $s < s_1$. Therefore, if $\gamma(p_M) < s_1$, consumer welfare is strictly higher under optimal disclosure for every s in $[\gamma(p_M), s_1)$ and for every s slightly below $\gamma(p_M)$.

To show that $\gamma(p_M) < s_1$ for $\underline{v}$ sufficiently low, let us first consider the price y such that (Δ) is binding for the disclosure threshold $\tilde{v} = p_M$. Then,

$$\int_{p_M}^{\bar{v}} (v - y)\, dv = 0 \iff \int_{p_M}^{y} (v - y)\, dv + \int_{y}^{\bar{v}} (v - y)\, dv = 0$$

$$\iff \int_{p_M}^{y} (y - v)\, dv = \int_{y}^{\bar{v}} (v - y)\, dv \iff \phi(p_M, y) = \frac{\gamma(y)}{1 - G(p_M)}.$$

Let $(\tilde{v}^*, p^*)$ be the seller's optimum for search cost s_1. Because (Σ) is binding at s_1, we have $s_1 = \phi(\tilde{v}^*, p^*)$. At s_1, we also have (see Claim 1) $\frac{1-G(p_M)}{g(p_M)} = p_M < p^* < \frac{1-G(\tilde{v}^*)}{g(\tilde{v}^*)}$. By the increasing hazard rate property, we have $p_M > \tilde{v}^*$. This implies $y > p^*$ because $(\tilde{v}^*, p^*)$ and (p_M, y) belong to Δ_b, and Δ_b is increasing. Hence, because $\phi(\tilde{v}, p)$ is decreasing in $\tilde{v}$ and increasing in p (see the proof of Lemma 4.1), we get

$$s_1 = \phi(\tilde{v}^*, p^*) > \phi(p_M, y) = \frac{\gamma(y)}{1 - G(p_M)}.$$

To show that $s_1 > \gamma(p_M)$, it suffices to show that $\frac{\gamma(y)}{1-G(p_M)} > \gamma(p_M)$, i.e.,

$$\frac{\gamma(y)}{\gamma(p_M)} \frac{1}{1 - G(p_M)} > 1. \tag{19}$$

Next, we show that

$$\frac{d}{dp_M} \frac{\gamma(y)}{\gamma(p_M)} > 0, \tag{20}$$

which is equivalent to $\frac{dy}{dp_M} < \frac{\frac{\gamma'(p_M)}{\gamma(p_M)}}{\frac{\gamma'(y)}{\gamma(y)}}$. Standard arguments using the increasing hazard rate show that $\frac{\gamma'(p)}{\gamma(p)}$ is decreasing in p. Hence, because $y = E(v \mid v \geq p_M) > p_M$, we have $\frac{\frac{\gamma'(p_M)}{\gamma(p_M)}}{\frac{\gamma'(y)}{\gamma(y)}} > 1$. Finally, we show $\frac{dy}{dp_M} < 1$. Because (p_M, y) binds (Δ), we have $\int_{p_M}^{\bar{v}} (v - y)\, dv = 0$. Integrating by parts and simplifying yields

$$y = p_M - \frac{\gamma(p_M)}{\gamma'(p_M)}.$$

Because $-\frac{\gamma(p_M)}{\gamma'(p_M)}$ is decreasing, we get $\frac{dy}{dp_M} < 1$, and therefore (19).

To conclude the proof of the proposition, note that decreasing $\underline{v}$ is equivalent to increasing the marginal cost, i.e., increasing p_M, while keeping the distribution G fixed so that $\frac{1}{1-G(p_M)}$ becomes arbitrarily large. Therefore, by (20), $\frac{\gamma(y)}{\gamma(p_M)}\frac{1}{1-G(p_M)}$ becomes larger than 1, proving (19) and therefore $s_1 > \gamma(p_M)$. ∎

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