Unequal Lifespans and Redistribution

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Abstract

Inequality in life expectancy across individuals is large, and it shows a strong correlation with income. But does this critical dimension of inequality, extending beyond income and wealth, call for additional fiscal redistribution? In this paper, we explore how systematic differences in life expectancy and health influence optimal fiscal redistribution. We propose a parsimonious modeling framework that allows us to immediately point to the mechanisms that shape the optimal fiscal tax and transfer system when individuals differ in their life expectancy. Theoretically, we demonstrate that heterogeneity in life expectancy alone prompts a utilitarian government to redistribute from individuals with shorter to those with longer life expectancy. However, if we consider that health status may also impact on the ability to enjoy late life consumption, this redistribution can be reversed. We then develop and calibrate a quantitative life-cycle model with heterogeneous agents that differ in income and health to study optimal fiscal redistribution through the pension system.

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1 Introduction

Differences in life expectancy across individuals are substantial and they systematically correlate with socio-economics status. Within the "Opportunity Insights" project, Chetty et al. (2016) document a life expectancy gap of almost 15 years between the income-richest 1% and poorest 1% of individuals in the United States. Other studies have shown that the education-life-expectancy gap has risen significantly over time.[1] These alarming findings clearly indicate that economic inequality goes well beyond the income and wealth distribution. However, research on the optimal design of the fiscal tax and transfer system that incorporates differences in health and life expectancy is scarce.

In this paper, we ask how differences in life expectancy and health shape optimal fiscal redistribution. We first study a series of analytically tractable two-period models. These models shed light on the main mechanisms that determine the optimal direction and size of fiscal taxes and transfers in the presence of heterogeneity in life expectancy. Our starting point is a model with two types of households that can differ both in their labor earnings as well as in their probability to survive from young to old age. A benevolent government can observe the individual type and chooses lump-sum transfers between the two types so as to maximize utilitarian social welfare. Not surprisingly, but somewhat against the common notion of fairness, heterogeneity in life expectancy leads a utilitarian government to redistribute from the short-lived towards the long-lived. The intuition behind this finding, which Leroux and Ponthiere (2013) and Koehne (2023) have already pointed to, is simple: Let us consider a situation in which everybody earns the same wage, but some individuals randomly draw a short and others a long life expectancy. From the (ex ante) perspective of a utilitarian planner, drawing a long life expectancy is a "bad risk", as the household has to stretch the same amount of resources as a short-lived one over a longer time horizon. When the individual utility function exhibits curvature, this means that individuals with a long life expectancy have a higher marginal utility of consumption. A utilitarian government would want to insure this risk by giving transfers to the long-lived. This intuition carries over to a situation in which life expectancy correlates with labor earnings. Differences in earnings between the two types of households leads the government to redistribute from the rich towards the poor. Differences in life expectancy, however, will lead the government to redistribute from the short-lived towards the long-lived. Consequently, there are two opposing forces that will shape fiscal redistribution, with the general lesson that the government's transfer system turns out less progressive when differences in life expectancy are taken into account.

As a next step, we propose an augmented and parsimonious modeling framework that allows us to investigate the relation between life expectancy and fiscal

[1] Examples of this are Meara et al. (2008), Mackenbach et al. (2015), de Gelder et al. (2017), or Boháček et al. (2021).

redistribution more thoroughly. Our approach is inspired by a literature that investigates how utility depends on health, see for example Finkelstein et al. (2013), Viscusi (2019), Gyrd-Hansen (2017), or Bassoli (2023). Specifically, we assume that there is a common factor – which could be interpreted as health – that impacts on both the individual's survival probability as well as on how much an individual can enjoy later life consumption expenditure. Put simply, we propose individual preferences that can be represented by the functional form

$$U(c_1, c_2; h) = u\left[c_1\right] + \pi(h)u\Big[\kappa(h)c_2\Big]. \tag{1}$$

In this formulation, $u[\cdot]$ is an instantaneous utility function with the typical assumptions on curvature, $\pi(h)$ is the health-status-dependent survival probability and $\kappa(h)$ is the individual's ability to enjoy later life consumption expenditure. Using this framework, we derive conditions that determine the direction of optimal fiscal transfers between individuals of different health status. In general, optimal redistribution can go in either direction, meaning from individuals with good health towards those with bad health or vice versa. More specifically, we show this crucially depends on three factors: the curvature of instantaneous utility $u[\cdot]$ as measured by relative risk aversion, the elasticity of survival $\pi(h)$ with respect to health status h, and the elasticity of late-life consumption enjoyment $\kappa(h)$ with respect to health status. When risk aversion is greater than one, then a (relatively) small and positive elasticity of survival and a (relatively) large and positive elasticity of consumption enjoyment will lead to an optimal redistribution scheme that transfers resources from good-health and long-lived individuals to those with worse health and a shorter life expectancy. The intuition is again straightforward: On the one hand, the same logic as before still holds, meaning that higher life expectancy generally increases an individual's marginal utility of consumption. On the other hand, a positive elasticity of $\kappa(h)$ with respect to health status means that healthier individuals with a higher life expectancy are able to enjoy their late life consumption expenditure at a higher intensity. This immediately counteracts the first effect and therefore lowers marginal utility of consumption of the long-lived. If the second effect is larger than the first one, then marginal utility of consumption is higher for poor health individuals which directly shapes the direction of fiscal transfers.

Our model provides a general formulation for the trade-offs that shape a utilitarian government's desire to redistribute based on differences in life expectancy and health. Through the lens of our model, however, variations in the enjoyment of late-life consumption expenditure arise exogenously. Specifically, to generate a motive for redistribution towards individuals with poor health, we need the marginal utility of consumption expenditure in late life to increase as health deteriorates. To give this model a useful interpretation, we show that it is, under some assumptions, isomorphic to a model in which old individuals have to divide their resources between non-medical consumption and the consumption of medical goods and services. Such models have recently been used by Finkelstein et al. (2013) or Blundell et al. (2024) to understand late life savings behavior and the health-dependent utility from consumption expenditure. Most importantly, when thinking about the model this way, an important distinction arises: A negative shock to the individual health status depresses the marginal utility of non-medical consumption expenditure like in Finkelstein et al. (2013). Consequently, individuals will spend less on *non-medical consumption* and increase their purchase of medical goods and services. Yet, the marginal utility of *total consumption expenditure* (i.e. non-medical plus medical) rises upon a negative health shock. Therefore, negative health shocks constitute a risk that households would want to insure by precautionary savings. It is, hence, not surprising that the presence of health risk can generate an increase in old-age savings in our model, just like in De Nardi et al. (2010), French and Jones (2011), Kopecky and Koreshkova (2014), Lockwood (2018) or Ameriks et al. (2020).

Finally, we use our theory to construct a quantitative life-cycle model of income, health, and lifespan risk calibrated to the US economy. Our model features heterogeneous individuals who face consumption-savings decisions. Households supply labor inelastically, and their labor earnings are subject to both permanent and transitory shocks, which are education-specific. In addition to labor earnings shocks, households receive health shocks that can correlate with earnings shocks. We measure individual health based on a frailty index that quantifies the sum of an individual's health deficits, as suggested in Mitnitski et al. (2001) and Hosseini et al. (2022). The individuals' health deficits are also subject to education-specific permanent and transitory shocks. These deficits influence individuals through three major channels: the households' conditional survival probability is negatively correlated with health deficits, the households' utility is health-state-dependent, and labor earnings are negatively correlated with health deficits. To replicate the theoretical results of the two-period model, we allow for a pension reform, comparing the long-run welfare effects of switching from the US pension system to unconditional basic old-age income. As in the two-period model, fiscal redistribution is mitigated by differences in life expectancy if survival probability is positively correlated with labor earnings, making the reform less beneficial when different life expectancies are considered. However, if we additionally include health-state-dependence in the utility function, fiscal redistribution becomes more beneficial, and the positive welfare effect of the reform increases.

Literature Review Our paper relates to several strands of the literature. Only few papers are concerned with the question to which degree inequality in life expectancy shapes optimal fiscal redistribution. Leroux and Ponthiere (2013) were among the first to note that, if life expectancy differences are deterministic, a utilitarian government would want to redistribute towards long-lived individuals. They classify this result as counter intuitive, as common sense would suggest that short-lived individuals should be compensated for not being able to enjoy more years of life. They propose to use a compensation-constrained utilitarian welfare measure to overcome this putative paradox. More recently, Koehne (2023) studies a Mirrleesian life-cycle setting with unequal life expectancy. When life expectancy correlates with labor productivity, then the optimal tax schedule entails less redistribution than under common life spans. Pestieau and Ponthiere (2016) provide a thorough overview of research that incorporates life expectancy differences. Finally, Fleurbaey et al. (2016) and Jones and Li (2023) study the the optimal design of the pension system under the presence of differences in life spans. All these papers draw on the standard expected utility framework and consider differences in life spans detached from other factors that may relate to individual health. In contrast, we want to point to the importance of considering the relation between individual health, life expectancy and consumption possibilities or consumption enjoyment in shaping a government's desire to redistribution between individuals with different life spans.

Our paper also relates to research on the relation between health and the way individuals can enjoy consumption expenditure. So far, this literature has not reached a solid conclusion. While Viscusi and Evans (1990), Finkelstein et al. (2013), or Viscusi (2019) argue that the marginal utility of consumption falls upon negative health shocks, Lillard and Weiss (1997), Tengstam (2014), and Gyrd-Hansen (2017) find the opposite. Evans and Viscusi (1991) and Bassoli (2023) can not identify any significant effect of health on the marginal utility of consumption. To organize thoughts on these findings, our results show that it is important to distinguish between the marginal utility of non-health related consumption expenditure and the marginal utility of total consumption expenditure. Upon a negative health shock, the former will most likely fall, leading households to lower their non-health related consumption and to increase their health related spending. The marginal utility of total consumption expenditure, however, can well increase upon negative health shocks, leading such health shocks to be perceived as negative risks by households.

Our results are consistent with a larger number of papers that study the savings behavior of the elderly. Earlier papers, like De Nardi et al. (2010), French and Jones (2011), Kopecky and Koreshkova (2014), or Lockwood (2018) model risky health-related expenditure, such as medical co-payments, nursing home expenditure or other long-term care costs. Such expenditure risk creates a motive for late-life precautionary savings. Consequently, these models can rationalize the stylized fact that many households do not run down their savings quickly after they enter retirement. More recent work, like Ameriks et al. (2020) or Blundell et al. (2024), explicitly model changes in the utility upon the arrival of health shocks. Finally, Edwards (2008) and Yogo (2016) investigate optimal portfolio choice at retirement under individual health risks.

Another set of papers asks to which individuals have an incentive to invest into the extension of their life span. They argue that the value of life is an important determinant of health expenditure. Rosen (1988) was among the first to systematically investigate the value of changes in life expectancy. Hall and Jones (2007) argue in favor of augmenting the standard individual utility function by a constant term that indicates the absolute utility value of living for another period.

In doing so, they are able to explain a rising share of health spending over time in the US. Cordoba and Ripoll (2017) add to the discussion of the value of life by considering preferences of the Epstein-Zin type. Pashchenko and Porapakkarm (2022) and Bommier et al. (2023) analyze the relation between the value of life and individual portfolio choices. The discussion of the value of life and the desire of households to invest into their own house usually circles around the *sign* of the instantaneous utility function. Whenever individuals can generate a positive utility from living for another period, they may be inclined to invest into their health. Instead, as we show in this paper, the extent of optimal fiscal redistribution that is based on differences in life expectancy and health depends on the *curvature* of the instantaneous utility function as measured by risk aversion $R(c)$. One can even easily create cases in which the government wants to redistribute from individuals with a lower utility level and a lower first-period marginal utility of consumption towards individuals with a higher utility level and a higher first-period marginal utility of consumption.

Finally, our calibrated model also touches upon the importance of considering health deficits and frailty indices to understand the state of individual health beyond retirement. Mitnitski et al. (2001), Mitnitski et al. (2002) and Rockwood and Mitnitski (2006) have pointed to the importance of health deficits in explaining the ageing and mortality process. Abeliansky and Strulik (2018), Abeliansky and Strulik (2019) and Abeliansky and Strulik (2023) study health deficits across countries and across socio-demographic groups. Finally Hosseini et al. (2022) propose and estimate a stochastic process for frailty dynamics over the life-cycle accounting for mortality bias, while Foltyn and Olsson (2024) estimate an age-dependent health process and survival probabilities from self-reported health to display racial differences in life expectancy. In our calibrated model, we use a frailty index as proxy for individual health status, and therefore consider a direct relationship between health deficits, survival rates and the enjoyment of late life consumption.

Our paper is organized as follows. In Section 2, we study the economics of life span inequality and fiscal redistribution in analytically tractable, two period life-cycle models. We point to the most important aspects of life span inequality, health and the enjoyment of late-life consumption in shaping the government's desire to redistribute. Section 3 presents our quantitative model and Section 4 discusses its calibration. We present simulation results in 5. The final section concludes.

2 An Analytical Investigation

In this section, we want to investigate how inequality in lifespans shapes the desire of the government to redistribute. To this end, we study a simple two-period life-cycle framework in which individuals can differ in both their labor productivity and their life expectancy. The results in this section will immediately illustrate under which conditions differences in life expectancy lead to less or more fiscal redistribution. Therefore, they will guide our quantitative model development. All formal derivations can be found in Appendix A.

2.1 The standard expected utility framework

Let us consider two individuals $i = 1, 2$ that live for two periods.[2] In the first period they work, and in the second period they are retired. The individuals differ both in their labor earnings w_i in the first period as well as in their probability π_i to survive into the second period. There is a riskless, non-annuitized asset that allows agents to smooth their consumption across the two periods of life. For simplicity, we assume that the asset pays a return $r = 0$. Denoting by $c_{1,i}$ and $c_{2,i}$ individual consumption in the first and second period, respectively, we can write the intertemporal budget constraint as

$$c_{1,i} + c_{2,i} = w_i + T_i.$$

T_i denotes a lump-sum transfer the household may receive from the government, see below.

We assume that an individual's intertemporal preferences can be represented by a standard discounted, expected utility function à la von Neumann and Morgenstern (2004). We denote by $u(c)$ the instantaneous utility function with the usual properties $u'(c) > 0$ and $u''(c) < 0$. Consequently, each household chooses a consumption allocation over the life-cycle so as to maximize

$$U_i = \max_{c_{1,i}, c_{2,i}} \; u(c_{1,i}) + \pi_i u(c_{2,i}) \quad \text{s.t.} \quad c_{1,i} + c_{2,i} = w_i + T_i.$$

Again, for clarity, we let the individual discount factor be $\beta = 1$. Furthermore, we abstract from a bequest motive and therefore normalize utility in case of death to zero.[3] The solution to the maximization problem is characterized by the Euler equation

$$u'(c_{1,i}) = \pi_i u'(c_{2,i}). \tag{2}$$

Consequently, a consumption smoothing agent will strive to equate her marginal utility of consumption today with her expected marginal utility of consumption tomorrow.

[2] In what follows we use the words individual, household and agent synonymously.

[3] This normalization is a standard approach and has no impact on our results, see the discussion in Section 2.2 for additional remarks on the choice of the instantaneous utility function and its consequences.

Redistribution by a utilitarian government Now let us consider a benevolent government that wants to maximize utilitarian social welfare. We assume that the government can directly observe the type i of each individual. Hence, it can use individual specific lump-sum transfers T_i to implement its goal. The government's optimization problem reads

$$\max_{T_1,T_2} \; U_1 + U_2 \quad \text{s.t.} \quad T_1 + T_2 = 0.$$

The optimal choice of the government results in transfer $T_1 = -T_2$ that satisfy

$$u'(c_{1,1}) = u'(c_{1,2}).$$

Hence, the government will seek to equate the first-period marginal utility of consumption between the two agents.

We now want to understand how unequal life spans shape the government's desire for redistribution in this framework. Note that life expectancy at the moment the individual enters the economy is equal to $1 + \pi_i$. To pin down the size of the government transfers T_i, it is therefore vital to understand how consumption $c_{1,i}$ in the first period reacts to changes in π_i.

Proposition 1. *In the standard expected utility framework, we have*

$$\frac{dc_{1,i}}{d\pi_i}\frac{\pi_i}{c_{1,i}} = -\frac{1}{R(c_{1,i}) + R(c_{2,i}) \cdot \frac{c_{1,i}}{c_{2,i}}} < 0, \tag{3}$$

where $R(c) = -\frac{u''(c)c}{u'(c)} > 0$ *denotes relative risk aversion. Individuals with a higher life expectancy therefore ceteris paribus exhibit a larger first-period marginal utility of consumption.*

Proof: see Appendix A. □

In words, Proposition 1 means that the elasticity of first period consumption with respect to the survival probability π_i is negative. A longer-lived individual will therefore always choose a lower level of first-period consumption relative to a shorter-lived one.

Intuition This result has two important implications. First, in the absence of any differences in wages between the individuals, the government will redistribute away from the shorter-lived towards the longer-lived individual. Let us, without loss of generality, assume that $\pi_1 < \pi_2$. Without a government transfer, Proposition 1 tells us that we have $c_{1,1} > c_{1,2}$ and therefore $u'(c_{1,1}) < u'(c_{1,2})$. The government wants to undo this imbalance between marginal utilities of consumption by redistributing resources from individual 1 to individual 2, i.e. it will optimally choose $T_1 < 0 < T_2$. To understand this result intuitively, let us think of the insurance role of the government. Behind the veil of ignorance that is imposed by the utilitarian welfare criterion, both individuals are (ex ante) identical.

However, they face the risk of drawing different survival probabilities. From the perspective of insurance, a long life is a "bad risk", as the long-lived individual has to stretch the same amount of resources as a short-lived one over a longer period of time.[4] Hence, a benevolent government wants to insure this risk of having a long life and therefore provides a transfer to all individuals that face high survival probabilities.

Second, let us now think of a situation in which there were differences in wages but not in survival. Again, without loss of generality, we let $w_1 < w_2$. In such a situation, the government would obviously want to choose $T_1 > 0 > T_2$, i.e. it would redistribute from the income richer towards the income poorer individual. If we now add differences in survival and let survival be positively correlated with wages, i.e. $\pi_1 < \pi_2$, then lifespan inequality will mitigate fiscal redistribution. The government's desired redistribution scheme will, hence, be less progressive than under the absence of heterogeneous life spans.[5]

An example To illustrate these theoretical considerations, let us look at an example. We assume that the instantaneous utility function reads

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}. \tag{4}$$

In this case, the households will optimally choose the life-cycle consumption path

$$c_{1,i} = \frac{w_i + T_i}{1 + \pi_i^{\frac{1}{\gamma}}} \quad \text{and} \quad c_{2,i} = \frac{\pi_i^{\frac{1}{\gamma}} (w_i + T_i)}{1 + \pi_i^{\frac{1}{\gamma}}}.$$

The indirect utility function then reads

$$U_i = \frac{(w_i + T_i)^{1-\gamma}}{1-\gamma} \cdot \left(1 + \pi_i^{\frac{1}{\gamma}}\right)^{\gamma}.$$

Finally, the optimal transfer scheme of the government is determined from

$$T_1 = -T_2 = \frac{w_2 \left(1 + \pi_1^{\frac{1}{\gamma}}\right) - w_1 \left(1 + \pi_2^{\frac{1}{\gamma}}\right)}{1 + \pi_1^{\frac{1}{\gamma}} + 1 + \pi_2^{\frac{1}{\gamma}}}. \tag{5}$$

The formula for optimal fiscal redistribution in (5) illustrates the two determinants of fiscal transfers. The direction of fiscal redistribution – i.e. whether transfers are progressive or regressive in labor earnings – crucially depends on the wage difference between individuals as well as on the differences in survival and therefore life expectancy. A positive wage difference $w_2 - w_1$ directly increases the

[4] The same line of argument is the foundation of the famous annuitization results in Yaari (1965).

[5] A similar result was found by Koehne (2023).

progressivity of the government's transfer scheme. A positive survival difference $\pi_2 - \pi_1$ immediately lowers progressivity.

To make this statement even more precise, let us write wages and survival probabilities as

$$w_1 = \bar{w}(1-\sigma_w) \quad , \quad w_2 = \bar{w}(1+\sigma_w) \quad , \quad \pi_1 = \bar{\pi}(1-\sigma_\pi) \quad \text{and} \quad \pi_2 = \bar{\pi}(1+\sigma_\pi).$$

$\bar{w}\sigma_w$ and $\bar{\pi}\sigma_\pi$ therefore denote the standard deviation of wages and survival probabilities, respectively. We then immediately get

$$T_1 = \frac{\bar{w}(1+\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}\right] - \bar{w}(1-\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}\right]}{1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}+1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}}.$$

which leaves us with

$$\frac{\partial T_1}{\partial \sigma_w} = \bar{w} > 0 \quad \text{and} \quad \frac{\partial T_1}{\partial \sigma_\pi} = -\frac{2\bar{w}\bar{\pi}}{\gamma} \cdot \frac{\left(1+\pi_2^{\frac{1}{\gamma}}\right)\pi_1^{\frac{1}{\gamma}-1} + \left(1+\pi_1^{\frac{1}{\gamma}}\right)\pi_2^{\frac{1}{\gamma}-1}}{\left(1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}\right)^2} < 0.$$

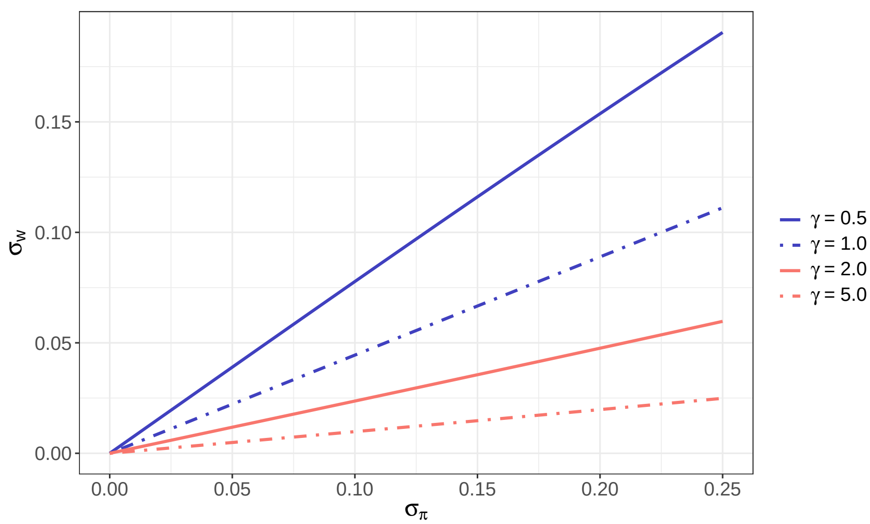
These derivatives tell us that the transfer to individual 1 – the individual with a lower wage and a lower life expectancy – increases as the wage dispersion in the population rises. In turn, the transfer unanimously falls as the inequality in life expectancy across the two types of individuals increases. Crucially, the relation between T_1 and σ_π depends on the curvature of the instantaneous utility function γ. In the utility function (4), γ denotes risk aversion and $\frac{1}{\gamma}$ denotes the intertemporal elasticity of substitution. Intuitively, a small intertemporal elasticity of substitution comes with a strong consumption smoothing motive for the household. As such, households will allocate consumption quite equally across the two periods of life, regardless of their life expectancy. Consequently, the elasticity of first-period consumption with respect to the survival probability (3) is small, and the government's desire to redistribute based on life expectancy shrinks.

Our results also suggest that there is a unique nexus between the dispersion in income and the dispersion in life expectancy across individuals that generate a transfer exactly equal to zero. In other words, there must be a series of combinations between σ_w and σ_π for which the governments desire to redistribute based on wages and the desire to redistribute based on survival exactly undo each other. In our setup, this relation is given by

$$\sigma_w = \frac{\bar{\pi}^{\frac{1}{\gamma}}\left[(1+\sigma_\pi)^{\frac{1}{\gamma}} - (1-\sigma_\pi)^{\frac{1}{\gamma}}\right]}{1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}+1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}}.$$

Figure 1 illustrates the relationship between σ_π and σ_w that leads to $T_1 = 0$. Ceteris paribus, a higher level of inequality in life expectancy across individuals leads to more regressive redistribution, meaning that the government wants T_1 to fall. To counteract this, the standard deviation of wages needs to increase as inequality in life expectancy rises. Hence, all the lines presented in Figure 1 are upward sloping. A higher level of risk aversion – meaning a lower intertemporal elasticity of substitution – dampens the government's desire to redistribute based on difference in survival. Consequently, the relationship between σ_π and σ_w that ensures zero redistribution becomes flatter as γ increases.

Figure 1: Nexus $\sigma_\pi - \sigma_w$ that generates no redistribution



Notes: For this figure we set $\bar{\pi} = 0.8$.

2.2 Relationship to the economics of the value of life

Our results relate to a quite recent strand of literature that investigates the value of life, among them Hall and Jones (2007) and Cordoba and Ripoll (2017). One of the fundamental questions asked in this literature is whether individuals would be willing to invest into their own health in order to extend their expected lifespan. Through the lens of our simple model, the answer to this question is quite easy. Taking the derivative of household utility with respect to π_i, we find that

$$\frac{dU_i}{d\pi_i} = u(c_{2,i}). \tag{6}$$

Hence, an individual would want to invest into extending her life if and only if $u(c_{2,i}) > 0$.

Hall and Jones (2007) now observe that the literature on quantitative life-cycle decision models typically employs an instantaneous utility function of the constant risk aversion type with a risk aversion parameter $\gamma > 1$, see equation (4). This immediately implies that $u(c_{2,i}) < 0$. In this case, individuals do not want to invest into their health to enjoy a longer life. Hall and Jones (2007) therefore advocate for adding a constant term $\bar{u}$ to the instantaneous utility function that indicates the value of living for an additional period. In particular, they propose to use

$$u(c) = \bar{u} + \frac{c^{1-\gamma}}{1-\gamma}.$$

Under $\gamma > 1$, individuals with a consumption level $\bar{c} > [(\gamma - 1)\bar{u}]^{\frac{1}{1-\gamma}}$ will, hence, enjoy positive instantaneous utility and therefore invest into their longevity. This feature is illustrated in more detail in Appendix A.

The argument in this paper is of entirely different nature. The extent of optimal fiscal redistribution that is based on differences in life expectancy solely depends on the *curvature* of the instantaneous utility function as measured by risk aversion $R(c)$, confer Proposition 1. It does in no case depend on the *sign* of the utility function. Even more so, in a world with constant relative risk aversion preferences, redistribution towards the long-lived is the largest for $\gamma < 1$. In this case, the government redistributes from individuals with a lower utility level and a lower first-period marginal utility of consumption towards individuals with a higher utility level and a higher first-period marginal utility of consumption. The same would be true for the preferences suggested by Hall and Jones (2007). This – rather absurd – property is an immediate consequence of the standard expected utility framework that only features heterogeneity in life expectancy.

2.3 An augmented utility framework

In this section, we want to investigate how we can extend the standard framework in order to incorporate a motive for redistribution towards the short-lived. Let us therefore assume that differences across individuals were due to a common factor h_i. We can think of h_i, for example, as denoting an individual's health, which directly influences survival $\pi_i = \pi(h_i)$. Furthermore, let us assume that individuals of different health status enjoy consumption at different intensities $\kappa(h_i)$. Our deviation from the standard expected utility model is therefore to augment the household's utility function to

$$U_i = u\left[c_{1,i}\right] + \pi(h_i) u\Big[\kappa(h_i) c_{2,i}\Big].$$

Note that the literature has provided ample evidence that individuals are able to enjoy consumption differently conditional on their health state, see for example Lillard and Weiss (1997), Finkelstein et al. (2013), or Kools and Knoef (2019). A further discussion of this assumption can be found in Section 2.4.

The first order condition of the household optimization problem then reads

$$u'[c_{1,i}] = \pi(h_i)\kappa(h_i) u'\Big[\kappa(h_i) c_{2,i}\Big]. \tag{7}$$

The solution to the intertemporal household problem is consequently shaped both by the survival probability $\pi(h_i)$ as well as the enjoyment of late-life consumption $\kappa(h_i)$. With the same argument as before, the government chooses its optimal transfer level such that both individuals attain the same first-period marginal utility of consumption. It is, hence, again vital to understand how first period consumption reacts to changes in health h_i.

Proposition 2. *In the augmented expected utility framework, in which health h_i shapes both survival as well as the enjoyment of consumption, we have*

$$\frac{dc_{1,i}}{dh_i}\frac{h_i}{c_{1,i}} = -\frac{\varepsilon_\pi(h_i) + \left[1 - R\Big(\kappa(h_i)c_{2,i}\Big)\right]\varepsilon_\kappa(h_i)}{R\Big(c_{1,i}\Big) + R\Big(\kappa(h_i)c_{2,i}\Big)\cdot\frac{c_{1,i}}{c_{2,i}}} \gtreqless 0. \tag{8}$$

$R(c) = -\frac{u''(c)c}{u'(c)} > 0$ *again is relative risk aversion. Furthermore,*

$$\varepsilon_\pi(h_i) = \pi'(h_i)\frac{h_i}{\pi(h_i)} \quad \textit{and} \quad \varepsilon_\kappa(h_i) = \kappa'(h_i)\frac{h_i}{\kappa(h_i)}$$

denote the elasticity of survival and the elasticity of consumption enjoyment with respect to health, respectively.

Proof: see Appendix A. □

Intuition This proposition has several implications: First, note that Proposition 1 is a special case of Proposition 2. When we set $\varepsilon_\pi(h_i) = 1$ and $\varepsilon_\kappa(h_i) = 0$, we can immediately recover the result that first-period consumption declines in the survival probability π_i and that the government wants to redistribute towards long-lived households. Second, in the augmented model with $\varepsilon_\kappa(h_i) \neq 0$, the relation between first-period consumption and health is not that clear cut anymore. In fact, whenever we have

$$\varepsilon_\pi(h_i) < \left[R\Big(\kappa(h_i)c_{2,i}\Big) - 1\right]\varepsilon_\kappa(h_i), \tag{9}$$

then first-period consumption rises with health (and therefore with survival). In such a situation, the government would want to redistribute from the long lived towards the short lived. Third, if we wanted to generate a case in which the government redistributes towards short-lived individuals, our choice of $\kappa(h_i)$ must depend on risk aversion $R(\kappa(h_i)c_{2,i})$. In particular, we would have to set

$$\kappa'(h_i)\begin{cases} < 0 & \text{if } R\Big(\kappa(h_i)c_{2,i}\Big) < 1 \text{ and} \\ > 0 & \text{if } R\Big(\kappa(h_i)c_{2,i}\Big) > 1. \end{cases}$$

This result is quite intuitive. Let us consider the case where $\kappa'(h_i) > 0$ and $R\Big(\kappa(h_i)c_{2,i}\Big) > 1$. A healthy individual hence is able to enjoy consumption in old age at a higher rate. As the individual's intertemporal elasticity of substitution

– the inverse of risk aversion – is small, the agent will try to smooth its effective consumption path $c_{1,i}$ and $\kappa(h_i)c_{2,i}$ over the life-cycle. This means that she would choose a lower level of second-period consumption expenditure to counteract the effect of a high $\kappa(h_i)$. As a result, first-period consumption rises and first-period marginal utility of consumption falls. The same argument holds, but vice versa, for the case of $\kappa'(h_i) < 0$ and $R\Big(\kappa(h_i)c_{2,i}\Big) < 1$.

An example Let us now augment the example chosen in Section 2.1 to illustrate the mechanism discussed so far. In particular, let us set

$$\pi(h_i) = \bar{\pi} h_i^{\epsilon_\pi} \quad \text{and} \quad \kappa(h_i) = \bar{\kappa} h_i^{\epsilon_\kappa}. \tag{10}$$

Both functions feature a constant elasticity, i.e.

$$\varepsilon_\pi(h_i) = \pi'(h_i)\frac{h_i}{\pi(h_i)} = \epsilon_\pi \quad \text{and} \quad \varepsilon_\kappa(h_i) = \kappa'(h_i)\frac{h_i}{\kappa(h_i)} = \epsilon_\kappa.$$

With risk aversion being constant, too, and equal to γ, we can consequently write

$$\frac{dc_{1,i}}{dh_i}\frac{h_i}{c_{1,i}} = -\frac{\epsilon_\pi + (1-\gamma)\epsilon_\kappa}{\gamma\left[1 + \frac{c_{1,i}}{c_{2,i}}\right]}.$$

For values $\gamma > 1$, the government would therefore want to redistribute towards the short-lived if and only if $\epsilon_\pi < (\gamma - 1)\epsilon_\kappa$. This can readily be seen from the optimal choice of life-cycle consumption

$$c_{1,i} = \frac{w_i + T_i}{1 + \pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}} \quad \text{and} \quad c_{2,i} = \frac{\pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}(w_i + T_i)}{1 + \pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}}.$$

The utility a household can attain now is

$$U_i = \frac{(w_i + T_i)^{1-\gamma}}{1-\gamma} \cdot \left(1 + \pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}\right)^\gamma = \frac{(w_i + T_i)^{1-\gamma}}{1-\gamma} \cdot \left(1 + \bar{a} h_i^{\frac{\epsilon_\pi - (\gamma-1)\epsilon_\kappa}{\gamma}}\right)^\gamma$$

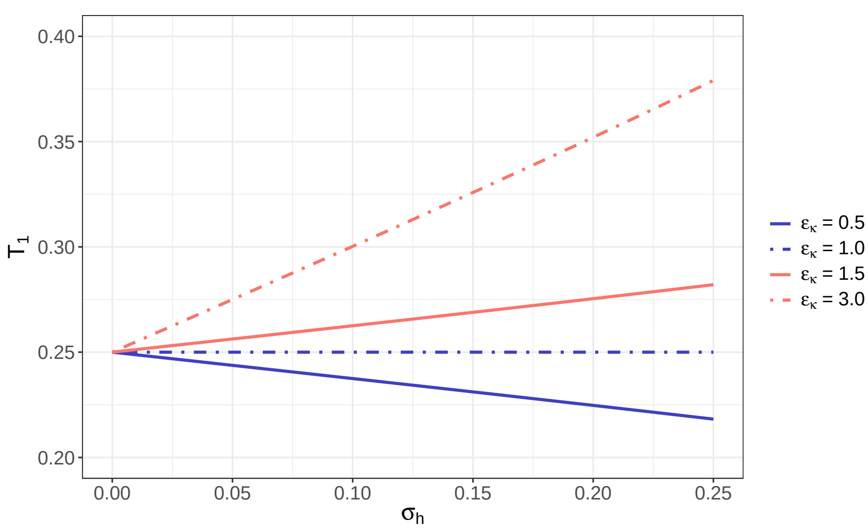
with $\bar{a} = \bar{\pi}^{\frac{1}{\gamma}} \bar{\kappa}^{\frac{1}{\gamma}-1}$. Finally, the optimal fiscal transfer level reads

$$T_1 = \frac{w_2\left(1 + \bar{a} h_1^{\frac{\epsilon_\pi - (\gamma-1)\epsilon_\kappa}{\gamma}}\right) - w_1\left(1 + \bar{a} h_2^{\frac{\epsilon_\pi - (\gamma-1)\epsilon_\kappa}{\gamma}}\right)}{1 + \bar{a} h_1^{\frac{\epsilon_\pi - (\gamma-1)\epsilon_\kappa}{\gamma}} + 1 + \bar{a} h_2^{\frac{\epsilon_\pi - (\gamma-1)\epsilon_\kappa}{\gamma}}}.$$

From this formulation of the government transfer, we can immediately see that the relation between ϵ_π and $(\gamma - 1)\epsilon_\kappa$ is the central determinant of the direction of fiscal redistribution. If $\epsilon_\pi < (\gamma - 1)\epsilon_\kappa$, then a poor health status of individual 1 will increase the size of the transfer towards this household beyond the government's motive for redistribution based on wages.

Figure 2 illustrates how fiscal redistribution depends on health differences across individuals and on the elasticities ϵ_π and ϵ_κ. We assume that $w_1 = 0.75$ and $w_2 = 1.25$. In a baseline scenario without differences in health ($\sigma_h = 0$), the government would therefore set $T_1 = 0.25$ in order to mitigate all differences in wage inequality across individuals. On top of inequality in wages, we let individuals exhibit health levels $h_1 = \bar{h}(1 - \sigma_h)$ and $h_2 = \bar{h}(1 + \sigma_h)$. We set $\bar{h} = 0.75$ and let σ_h vary continuously between values of 0 and 0.25. The solid blue line shows a situation in which $\epsilon_\pi > (\gamma - 1)\epsilon_\kappa$. In this case, the survival probability channel dominates and health inequality induces less income redistribution across individuals like in Section 2.1. The dashed blue line marks the case of $\epsilon_\pi = (\gamma - 1)\epsilon_\kappa$ in which health inequality does not generate any additional redistribution at all. The two red lines indicate situations in which $\epsilon_\pi < (\gamma - 1)\epsilon_\kappa$. The government, hence, increases its transfer level T_1 in response to a rise in health inequality, as the consumption enjoyment channel is the dominant one. This effect is more pronounced the larger is the elasticity ϵ_κ.

Figure 2: Transfer level as a function of health differences ($\epsilon_\pi = 1$)



Notes: For this figure we set $w_1 = 0.75$, $w_2 = 1.25$, $\gamma = 2$, $\bar{h} = 0.75$, $\bar{\kappa} = 0.75$, $\bar{\pi} = 0.75$, and $\epsilon_\pi = 1$. The figure shows the transfer level T_1 as a function of health difference for $h_1 = \bar{h}(1 - \sigma_h)$ and $h_2 = \bar{h}(1 + \sigma_h)$ as health levels.

2.4 Discussion of the augmented framework

Our formulation of the augmented model is quite general in that we only assume the enjoyment of consumption expenditure to move with the individual's health state. In order to give a suitable interpretation of this feature, we want to relate our model specification to a "standard" model of consumption and health expenditure.

A model of health expenditure Let us assume that, in the second period of life, the individual can choose to spend some of her resources n_2 on non-medical consumption and some on medical products and services m_2. Both n_2 and m_2 raise individual utility. In particular, we assume that the household maximizes

$$\max_{c_1,n_2,m_2} \; u[c_1] + \pi(h)u\left[(n_2)^{\alpha(h)}(m_2)^{1-\alpha(h)}\right] \quad \text{s.t.} \quad c_1 + n_2 + p_m m_2 = w + T, \tag{11}$$

where p_m denotes the price of medical products. $\alpha(h)$ is a utility weight that governs the importance of non-medical consumption and medical expenditure, respectively. It is safe to assume that $\alpha'(h) > 0$, meaning that healthy consumers derive a higher utility from non-medical consumption, whereas for sick consumers, medical expenditure is a more important component of the consumption bundle.

Given the assumptions about functional forms, it is now easy to show that the optimization problem in (11) is isomorphic to the model

$$\max_{c_1,c_2} \; u[c_1] + \pi(h)u\left[\kappa(h)c_2\right] \quad \text{s.t.} \quad c_1 + c_2 = w + T$$

with

$$\kappa(h) = \alpha(h)^{\alpha(h)}\left[1-\alpha(h)\right]^{1-\alpha(h)}(p_m)^{\alpha(h)-1},$$

see Appendix A.4 for details. Note that with this formulation we have

$$\kappa'(h)\begin{cases} > 0 & \text{for all } \alpha(h) > \frac{1}{1+p_m} \text{ and} \\ \leq 0 & \text{otherwise.} \end{cases}$$

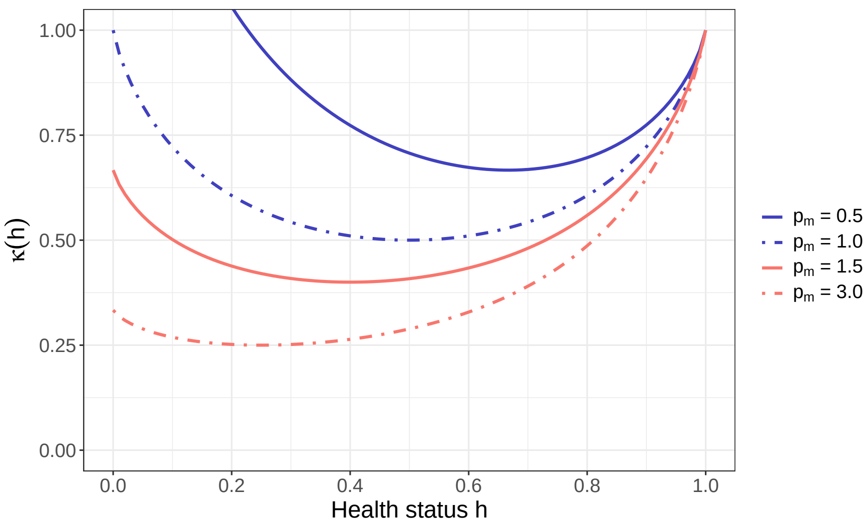
Figure 3 shows the evolution of $\kappa(h)$ across different health states h of an individual. Note that for this figure we simply assumed that we measure health h on a scale between 0 and 1, where $h = 1$ marks the most health person. Consequently, we let $\alpha(h) = h$. Regardless of our choice of the price of medical goods and services p_m, the individual consumption enjoyment is equal to 1 for the healthiest individual ($h = 1$). $\kappa(h)$ then declines as an individual's health status depletes. It does so until the value $\alpha(h) = h = \frac{1}{1+p_m}$. Consequently, the larger is the price of medical goods, the larger will be the interval on which the enjoyment of consumption declines with health status.

Note that in this model, there is a clear but important distinction between the marginal utility of non-medical consumption and the marginal utility of total consumption expenditure:

$$\begin{aligned} \text{MUN} &= \frac{\partial u\left[(n_2)^{\alpha(h)}(m_2)^{1-\alpha(h)}\right]}{\partial n_2} = \alpha(h)\cdot u'[\cdot]\cdot\frac{(n_2)^{\alpha(h)}(m_2)^{1-\alpha(h)}}{n_2} \text{ and} \\ \text{MUC} &= \kappa(h)u'\left[\kappa(h)c_2\right]. \end{aligned}$$

A negative health shock ceteris paribus decreases the marginal utility of non-medical consumption (MUN). It hence leads to a substitution away from non-medical consumption towards expenditure on health goods and services. As regards the marginal utility of total consumption expenditure (MUC), the results

Figure 3: Consumption enjoyment as a function of health status h



Notes: For this figure we set $\alpha(h) = h$.

are not that clear cut. In fact, as shown in Section 2.3, the marginal utility of total consumption expenditure may well rise in h when $\kappa'(h) > 0$ and $R(\kappa(h)c_2) > 1$. This distinction is important, as it will allow us to put structure on empirical results that investigate how the utility of consumption relates to an individuals health status.

Health risk and household savings How does health risk shape an individual's savings path? To answer this question, we have to slightly change the timing of health shocks in the model. Specifically, let us assume that all individuals are identical when making their consumption-savings decision in period one. After the consumption decision has been made, a health shock h_i realizes which determines both survival $\pi_i = \pi(h_i)$ into the next period as well as next period's consumption enjoyment $\kappa_i = \kappa(h_i)$. The individual then chooses consumption so as to maximize

$$U_i = \max_{c_1, c_2} \frac{c_1^{1-\gamma}}{1-\gamma} + E\left[\pi_i \frac{(\kappa_i c_2)^{1-\gamma}}{1-\gamma}\right] \quad \text{s.t.} \quad c_1 + c_2 = w + T.$$

There are again two health shocks, which are given by $h_1 = \bar{h}(1 - \sigma_h)$ and $h_2 = \bar{h}(1 + \sigma_h)$, which occur with equal probability. In Appendix A.4, we show that we can write the household's maximization problem as

$$U_i = \max_{c_1, c_2} \frac{c_1^{1-\gamma}}{1-\gamma} + \beta(\sigma_h) \frac{c_2^{1-\gamma}}{1-\gamma} \quad \text{s.t.} \quad c_1 + c_2 = w + T.$$

with an implicit discount factor

$$\beta(\sigma_h) := 0.5\bar{\pi}\bar{\kappa}^{1-\gamma}\bar{h}^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}\left[(1-\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}+(1+\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}\right]. \qquad (12)$$

How household savings react to changes in health risk then crucially depends on the sign of $\beta'(\sigma_h)$. Specifically, we can show that

$$\frac{ds_1}{d\sigma_h} = \frac{w+T}{\gamma}\cdot\frac{[\beta(\sigma_h)]^{\frac{1}{\gamma}-1}}{\left[1+[\beta(\sigma_h)]^{\frac{1}{\gamma}}\right]^2}\cdot\beta'(\sigma_h).$$

For the derivative of the implicit discount factor, we obtain

$$\beta'(\sigma_h) = 0.5\bar{\pi}\bar{\kappa}^{1-\gamma}\bar{h}^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}\cdot[\epsilon_\pi-(\gamma-1)\epsilon_\kappa] \left[(1+\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa-1}-(1-\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa-1}\right].$$

This result gives rise to two central conclusions. First, in a model without shocks to consumption enjoyment, we have $\epsilon_\pi = \bar{\kappa} = \bar{h} = 1$ and $\epsilon_\kappa = 0$. In this case, we immediately get $\beta(\sigma_h) = \bar{\pi}$ and therefore $\beta'(\sigma_h) = 0$. In this scenario, a mean preserving change in life expectancy risk does have no impact on the household's savings behavior. In fact, the household only cares about its mean life expectancy. Second, whenever $\epsilon_\pi < (\gamma-1)\epsilon_\kappa$, we immediately get $\beta'(\sigma_h) > 0$. A higher health risk consequently will lead household's to increase their savings in order to build a buffer stock that allows them to cope with shocks to life expectancy and consumption enjoyment. Our model can, hence, rationalize substantial household savings at old age.

3 A Quantitative Simulation Model

The previous theoretical considerations have indicated that the impact of longevity and health disparities on the fiscal desire for redistribution is ambiguous. Consequently, determining whether observed health differences give rise to increased redistribution is a quantitative issue. In this section, we therefore develop a life-cycle simulation model to quantify the benefits of old-age income redistribution under various assumptions regarding health, survival, and the ability to enjoy consumption.

3.1 Demographics and Health

Demographics and education We study a life-cycle of households that begins at age 25 and lasts at most until age 95. We denote a household's current age by j. At the beginning of life, households stochastically draw an education level s. They can either be high-school graduates ($s = 0$) or have completed a college degree ($s = 1$). The probability of drawing each education level is given by $\phi_s(s)$. Education influences the individual's[6] life-cycle labor earnings profile, health dynamics, and desire to leave bequests.

Health dynamics We use a frailty index $f_j \in [0,1]$ to measure individual health. This index indicates the proportion of potential health deficits an individual has. Originally from gerontology, frailty indices are now well-established measures of health in economics (Dalgaard and Strulik, 2014; Hosseini et al., 2022). Importantly, frailty is a key predictor of survival. Since the frailty index is normalized between zero and one, we define health as $h_j = 1 - f_j$.

We follow Hosseini et al. (2022) in formulating life-cycle health dynamics. Upon entering the economy, all individuals draw a fixed effect $\alpha_h \sim N(0, \sigma^2_{\alpha_h,s})$ that indicates their general degree of healthiness. Individuals can either be in perfect health ($h_j = 1$), i.e., they have zero frailty, or they can accumulate health deficits ($h_j < 1$). We denote by $\varsigma_{j,s}$ the age-specific share of individuals in perfect health. At the beginning of life, a share $\varsigma_{25,s}$ consequently starts with $h_{25} = 1$. The remainder of the population features a health level of $h_{25} = 1 - \exp(\alpha_h + \mu_{25,s})$. The starting point of health deficit accumulation, hence, depends on the individual fixed effect α_h as well as on an age- and education-specific mean $\mu_{j,s}$.

Over time, health evolves according to a two-step stochastic process. The *extensive margin* process indicates whether individuals with $h_j = 1$ remain in perfect health, or whether they start accumulating health deficits. Specifically, if $h_j = 1$, we have:

$$h_{j+1} = \begin{cases} 1 & \text{with probability } \psi^h_{j+1,s} \\ 1 - \exp(\alpha_h + \mu_{j+1,s}) & \text{with probability } 1 - \psi^h_{j+1,s} \end{cases} \tag{13}$$

The age- and education-specific probability $\psi^h_{j+1,s} = \frac{\varsigma_{j+1,s}}{\varsigma_{j,s}}$ determines how many individuals remain without frailty over time. Once individuals leave the state of perfect health, they (again) start accumulating health deficits at a level depending on their fixed effect α_h and an age- and education-specific mean $\mu_{j+1,s}$. Over time, their health evolves according to the auto-regressive *intensive margin* process:

$$\begin{aligned} h_{j+1} = 1 - \exp\left(\alpha_h + \mu_{j+1,s} + \chi_{j+1}\right), \\ \text{with} \quad \chi_{j+1} = \rho_s \chi_j + \epsilon_{j+1}, \quad \epsilon_{j+1} \sim N(0, \sigma^2_{\epsilon,s}). \end{aligned} \tag{14}$$

Survival Survival from one period to the next is uncertain. We denote the conditional survival probability from age j to age $j+1$ by $\pi(j+1, s, h_{j+1})$. This probability depends on education s, age $j+1$, and health h_{j+1}.[7]

[6] We use the terms household, individual, and agent synonymously.

[7] Note that our timing convention assumes the health transition occurs before survival.

3.2 Labor earnings and the budget constraint

During their working life, households inelastically supply one unit of labor. Labor earnings follow an age- and education-specific life-cycle profile $e_{j,s}$. Furthermore, each household draws a fixed effect $\alpha_w \sim N(0, \sigma^2_{\alpha_w,s})$ that permanently shifts their labor earnings profile up or down. There are two components of earnings risk: first, households receive persistent earnings shocks η_j that follow an AR(1)-process:

$$\eta_{j+1} = \varrho_s \eta_j + \varepsilon_j \quad \text{with} \quad \eta_{25} = 0 \quad \text{and} \quad \varepsilon_j \sim N(0, \sigma^2_{\varepsilon,s}) \tag{15}$$

Second, labor productivity is related to individual health h_j, which is itself a risky process. More precisely, we define individual labor earnings as

$$\log(y_j) = \log(e_{j,s}) + \alpha_w + \xi_{1,j,s} \times \eta_j + \xi_{0,s} \times \Big(h_j - \bar{h}_{j,s}\Big) \tag{16}$$

Here, $\bar{h}_{j,s}$ represents the mean frailty of all households of age j and education s, and $\xi_{0,s}$ indicates the sensitivity of earnings with respect to health. $\xi_{1,j,s}$ is a normalizing factor that keeps the variance of the earnings process unchanged, see the discussion in Section 4. Note that our formulation ensures that health does not influence average earnings, but only shapes the distribution of earnings across individuals of a given age and education.

We denote the conditional working probability of households from age j to age $j+1$ by $\omega(j+1, s, h_{j+1})$. This probability depends on age $j+1$, education s, and health h_{j+1}.[8] Households can only retire during ages $J_{r,1}$ to $J_{r,2}$. At this point, they stop working and begin receiving pension benefits p, which are calculated based on a household's lifetime earnings history, see Section 4 for details. Once a household retires, they cannot go back to work in any subsequent period.

Households can save in a risk-free asset a_j that yields interest payments r, but are not allowed to borrow. Furthermore, they pay progressive taxes $T(\cdot)$ on taxable earnings y_{tax}. Their budget constraint consequently reads

$$(1+r)a_j + y_j + p_j - T(y_{tax}) = c_j + a_{j+1}. \tag{17}$$

3.3 Preferences

Households have preferences over stochastic streams of consumption $c_j \geq 0$. These preferences are represented by the discounted expected utility function:

$$U_0 = \mathbb{E}_0\Bigg[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{k=1}^{j-1} \pi(k, s, h_k)\right) \Big[\pi(j, s, h_j) u\Big(\kappa(h_j) c_j\Big) + \Big(1 - \pi(j, s, h_j)\Big) v(a_j)\Big]\Bigg].$$

[8]Note that our timing convention assumes the health transition occurs before retirement transition.

Households discount the future with a time discount factor β and form expectations regarding their survival, earnings, and health risks. If they survive, households derive instantaneous utility $u\Big(\kappa(h_j)c_j\Big)$ from current consumption. Like in Section 2, their ability to enjoy consumption $\kappa(h_j)$ depends on health, but also on education s. In the event of death, agents experience a (warm-glow) motive for leaving bequests $v(a_j)$.

3.4 The dynamic optimization problem

The current and next-period states of a household are described by the state vectors

$$\mathcal{X} = (j, s, \alpha_h, h, \alpha_w, \eta, a) \quad \text{and} \quad \mathcal{X}^+ = (j+1, s, \alpha_h, h^+, \alpha_w, \eta^+, a^+).$$

In every period, the household's dynamic optimization problem reads

$$V(\mathcal{X}) = \max_{c, a^+ \geq 0} \; u\Big(\kappa(h)c\Big) + \beta\mathbb{E}\Big[\pi(j+1, s, h^+)V(\mathcal{X}^+) + \Big(1 - \pi(j+1, s, h^+)\Big)\, v(a^+)\Big]$$

subject to the budget constraint (17) and the stochastic processes (13) and (14) for health h and (15) for earnings shocks η. The solution to the household problem is a value function $V(\mathcal{X})$ as well as policy functions $c(\mathcal{X}), a^+(\mathcal{X})$ that all depend on the household's current state $\mathcal{X}$.

4 Calibration

In this section, we discuss our choice of parameters. We take as many parameters as possible directly from the data or from other studies that provide credible estimates. We then calibrate the remaining parameters so as to match certain empirical targets.

4.1 Demographics and education

The United States Census Bureau (2010) provides data on the educational attainment of the male US population aged 25 and older. According to this data, a share $\phi(s = 0) = 0.559$ of the population are high-school graduates or have some college education, but no completed degree. Consequently, $\phi(s = 1) = 0.441$ of the population are college graduates.[9] For the calculation of retirement deductions, we use the normal retirement age of $J_r = 66$, which corresponds to the normal retirement age of the cohorts 1943-54, see United States Social Security

[9] Note that we exclude high-school dropouts from our analysis.

(2024). The first of these cohorts turned 66 in 2009, and the last one in 2020. Additionally, we set the lower limit for retirement at age $J_{r,1} = 62$ and the upper limit at $J_{r,2} = 70$.

4.2 Health dynamics

Our parameter estimates of the health process are based on Hosseini et al. (2022), who have estimated frailty dynamics using the 1994-2018 waves of the Health and Retirement Study (HRS). We mostly use their estimates for men with either a high school diploma or a college degree. Our description of health dynamics will therefore be brief. All parameter choices are summarized in Table 1. We assume that the share of individuals with perfect health evolves over the life-cycle according to

$$\varsigma_{j,s} = \Phi\left(b_{0,s} + b_{1,s} \times j + b_{2,s} \times j^2\right).$$

Φ denotes the cumulative distribution function of the standard normal distribution.[10] Mean frailty of all individuals that already have some health deficits is

$$\mu_{j,s} = d_{0,s} + d_{1,s} \times j_n + d_{2,s} \times j_n^2 + d_{3,s} \times j_n^3 + d_{4,s} \times j_n^4 \quad \text{with} \quad j_n = \frac{j - 25}{100}.$$

Finally, the relation between age, health and survival is

$$\pi(j, s, h) = \sqrt{1 - \Phi(z)} \text{ with } z = g_{0,s} + g_{1,s} \times h + g_{2,s} \times h^2 + g_{3,s} \times j + g_{4,s} \times j^2.$$

Table 1: Parameters of health dynamics

Share with perfect health			Health shocks		
	$s=0$	$s=1$		$s=0$	$s=1$
$b_{0,s}$	-0.661	-0.258	ρ_s	1.001	0.963
$b_{1,s}$	4.92×10^{-4}	4.92×10^{-4}	$\sigma^2_{\epsilon,s}$	0.015	0.026
$b_{2,s}$	2.98×10^{-4}	2.98×10^{-4}	$\sigma^2_{\alpha_h,s}$	0.145	0.129
Median health over life-cycle			**Health and survival**		
	$s=0$	$s=1$		$s=0$	$s=1$
$d_{0,s}$	-2.609	-2.810	$g_{0,s}$	-0.236	0.710
$d_{1,s}$	1.330	0.726	$g_{1,s}$	-1.598	-1.222
$d_{2,s}$	0.820	2.320	$g_{2,s}$	-0.821	-1.338
$d_{3,s}$	-2.909	-0.803	$g_{3,s}$	-0.025	-0.054
$d_{4,s}$	11.427	3.495	$g_{4,s}$	4.28×10^{-4}	6.32×10^{-4}

[10] Note that although the results in Hosseini et al. (2022) show a positive coefficient for being a high school graduate, using their replication files this coefficient should actually be negative. Thus, the constant we use in the calculation of the share high school graduates with perfect health differs to their paper.

Note that the HRS is a biannual survey. We therefore convert probabilities to a one-year horizon by taking the square-root. On top, we adapt the constants $g_{0,s}$ to match the conditional life expectancy at age 25 of 76.86 years calculated for US men from the Human Mortality Database (2024) if individuals had a maximum age of 95.

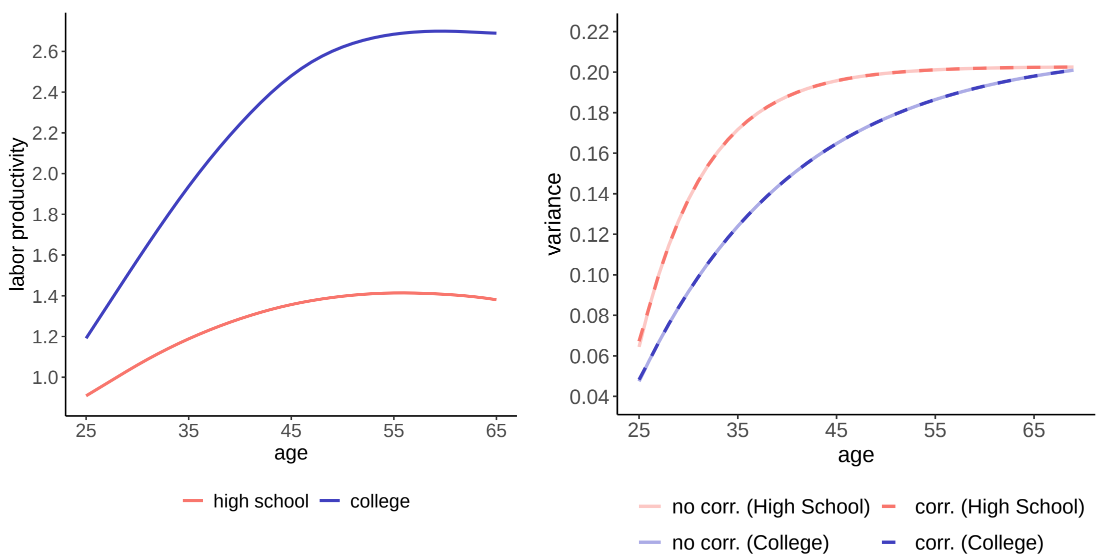
4.3 Labor earnings

Our baseline estimates for life-cycle earnings dynamics come from Krueger and Ludwig (2016). The left panel of Figure 4 shows median life-cycle earnings profiles $e_{j,s}$ for high-school and college graduates, respectively.

Figure 4: Life-cycle earnings dynamics

(a) Median earnings profiles $e_{j,s}$

(b) Variance of earnings



The parameters of the baseline earnings shock process η_j are given in the first three rows of Table 2.

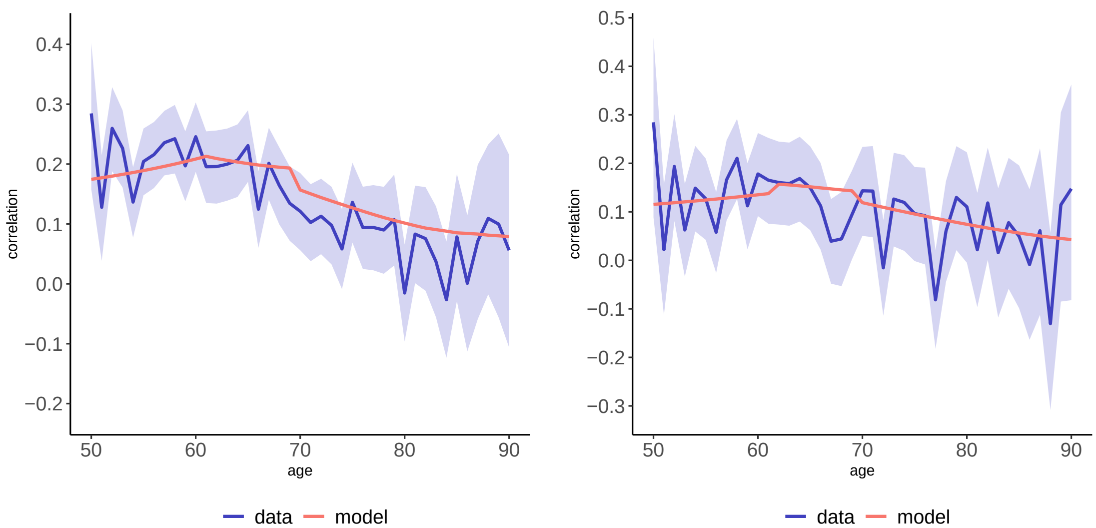
Table 2: Parameters of earnings dynamics

	$s = 0$	$s = 1$
ϱ_s	0.928	0.969
$\sigma^2_{\eta,s}$	0.019	0.010
$\sigma^2_{\alpha_w,s}$	0.064	0.047
$\xi_{0,s}$	1.300	0.900

Health and earnings Note, however, that we cannot use the earnings shock process off the shelf, as health and earnings are correlated. The blue line in Figure 5 illustrates the empirical correlation between health (measured by one minus the frailty index) and labor income (earnings plus social security benefits, pensions, and annuities) across different age groups from the 2004-2018 waves of the HRS.[11] The shaded area represents the 95% confidence interval. The correlation between health and labor income is positive, ranging from approximately 0.10 to 0.30 for working-age cohorts of both education levels. After age 66 – the normal retirement age – the correlation decreases with age and approaches zero for very old individuals.

Figure 5: Correlation between health and labor income

(a) High School (b) College



Notes: Correlation between health (measured as 1 minus the frailty index) and labor income (earnings + social security payments + pensions and annuities) conditional on age. Shaded areas represent 95% confidence bands.

We calibrate the education-specific sensitivities $\xi_{0,s}$ of earnings with respect to health in (16) to match these empirical targets. This leads to values of $\xi_{0,0} = 1.25$ for high school graduates and $\xi_{0,1} = 0.80$ for college graduates. The red lines in Figure 5 illustrate the predicted correlation of the model between health and labor income. The model-generated correlations lie well within the empirical confidence bounds. Note that the retirement interval starts at age 62 and ends at 70. Thus, at age 70, all individuals receive social security benefits which, owing to the progressive nature of the pension formula, lowers the correlation between

[11] Specifically, we combined the RAND HRS Longitudinal File 2020 (V2) with the RAND HRS CAMS File 2021 (V1), resulting in a representative dataset for U.S. citizens aged 50 and older, see Appendix B for details on data construction and handling. This dataset includes detailed information on their health, income distribution, and consumption spending. The frailty index calculation follows the method used in Hosseini et al. (2022).

health and income. A positive value of $\xi_{0,s}$ leads not only to correlation between health and earnings. Ceteris paribus it also increases the earnings variance. We hence have to correct for the effect of health on earnings so as to keep the variance of earnings constant relative to a scenario without a health-earnings correlation. To this end, we lower the importance of the exogenous earnings shocks η_j by multiplying them with a factor

$$\xi_{1,j,s} = \sqrt{1 - \min\left[\xi_{0,s}^2 \times \frac{\mathrm{Var}(h_j|s)}{\mathrm{Var}(\eta_j|s)}, 1\right]},$$

see (16). This ensures that the age-specific variance of earnings remains unchanged relative to the empirical estimates. The right panel of Figure 4 shows the age-specific earnings variance for a scenario without health-earnings correlation and for the scenario with positive $\xi_{0,s}$.

Table 3: Parameters of retirement dynamics

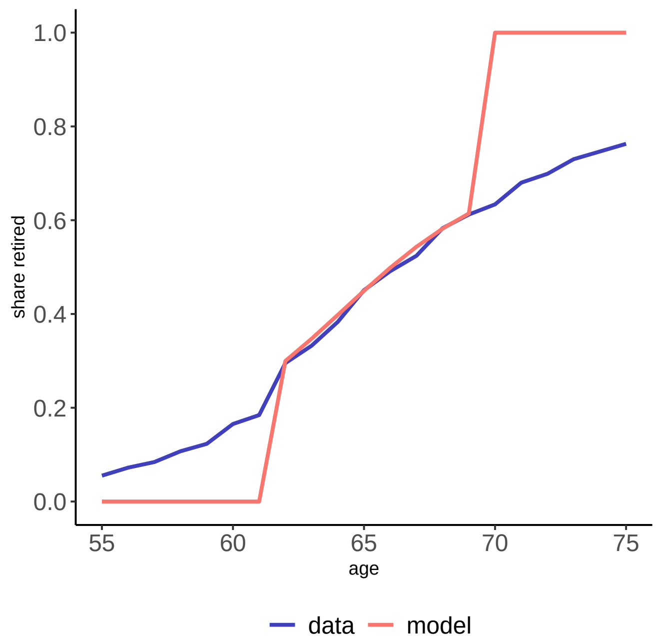
Probability at age $J_{r,1}$			Probability for subsequent ages		
	$s=0$	$s=1$		$s=0$	$s=1$
$m_{0,s}$	1.190	0.768	$f_{0,s}$	−52.572	84.839
$m_{1,s}$	−1.873	−1.708	$f_{1,s}$	6.839	7.925
			$f_{2,s}$	−4.890	−5.403
			$f_{3,s}$	1.495	2.437
			$f_{4,s}$	−0.011	−0.018

Retirement age We derive the parameters of the conditional working probability using again the 1994-2018 waves of the HRS. Once an individual is retired in the model, they can no longer go back to work. All parameters are summarized in Table 3. The probabilities are age-, health- and education-specific and follow the following form, where Φ again denotes the cumulative distribution function of the standard normal distribution:

$$\omega_j = \begin{cases} 1 & \text{if } j < J_{r,1} \\ 1 - \Phi\left(m_{0,s} + m_{1,s} \times h\right) & \text{if } j = J_{r,1} \\ \sqrt{1 - \Phi\left(f_{0,s} + f_{1,s} \times h + f_{2,s} \times h^2 + f_{3,s} \times j + f_{4,s} \times j^2\right)} & \text{if } J_{r,1} < j < J_{r,2} \\ 0 & \text{if } j \geq J_{r,2} \end{cases}$$

Thus, before the household reaches age $J_{r,1}$ they will continue working with certainty, and at age $J_{r,2}$ they will retire with certainty and stay retired. The probability of working at age $J_{r,1}$ is estimated using the unconditional probability of being retired at that age, while the probability of working for subsequent ages is estimated using the probability of being retired at age $j+1$, while not being retired at age j. Note again that the HRS is a biannual survey. Thus, we convert the probabilities to a one-year horizon by taking the square-root. A comparison between the share of retired households in the HRS and our model can be seen in Figure 6.

Figure 6: Share retired households



Notes: Retirement share from the HRS using individuals, who have at least a High School degree and consider themselves fully retired.

Pensions Once a household enters retirement, it receives a pension that depends on an index of her labor earnings history. We define a households actual retirement age as j_r. We calculate the index based on the household's education level s, as well as the last realizations of the labor earnings shock η_{j_r-1} and personal health h_{j_r-1} just before retirement:[12]

$$y_{index}\left(s, \eta_{j_r-1}, h_{j_r-1}\right) = \mathbb{E}\left[\frac{\sum_{j=1}^{J_{r,1}-1} \min[y_j, y_{max}]}{J_{r,1}-1} \middle| s, \eta_{j_r-1}, h_{j_r-1}\right].$$

Note that, in line with US Social Security law, individuals receive pension claims for their individual earnings up to an annual threshold of $y_{max} = 2.56\bar{y}$, where $\bar{y}$ stands for average earnings of the entire workforce.[13] We apply the statutory US

[12]This approach keeps the computational time of the model feasible, since it allows us to omit an extra continuous state variable that tracks a household's earnings history.

[13]The annual threshold level is calculated by dividing the contribution and benefit base for 2010 by the average wage index for 2010.

pension formula to this earnings index:

$$p = \begin{cases} 0.9 \times y_{index} & \text{if } y_{index} \leq b_1 \\ 0.9 \times b_1 + 0.32 \times (y_{index} - b_1) & \text{if } b_1 < y_{index} \leq b_2 \\ 0.9 \times b_1 + 0.32 \times (b_2 - b_1) + 0.15 \times (y_{index} - b_2) & \text{if } b_2 < y_{index} \end{cases}$$

The pension formula is progressive in that the replacement rates change at certain bend points. We set these bend points to their status quo values of $b_1 = 0.22\bar{y}$ and $b_2 = 1.33\bar{y}$. Additionally, we apply deductions to social security benefits for early retirement before the normal retirement age J_r and benefits for late retirement following the US Social Security law.

Taxes The government taxes income from labor earnings and social security payments according to a progressive tax schedule $T(\cdot)$. Specifically, we define taxable income as the sum of labor earnings and taxable social security benefits, i.e. $y_{tax} = y_j + p_{tax}$. In line with current US tax law, the taxation of social security benefits depends on "combined income", which in our model is equal to $0.5p_j$, or 50% of the pension benefit. Specifically, we define taxable social security benefits as

$$p_{tax} = \begin{cases} 0 & \text{if } 0.5p_j \leq b_3 \\ 0.5 \times (0.5p_j - b_3) & \text{if } b_3 < 0.5p_j \leq b_4 \\ 0.5 \times (b_4 - b_3) + 0.85 \times (0.5p_j - b_4) & \text{if } b_4 < 0.5p_j \end{cases}$$

with $b_3 = 0.60\bar{y}$ and $b_4 = 0.82\bar{y}$. We then apply the statutory US marginal tax rate formula for single tax payers to taxable income y_{tax}, see Figure 7.[14]

4.4 Preferences

In our model, we assume the instantaneous utility function follows a constant relative risk aversion (CRRA) form. Specifically, we define it as:

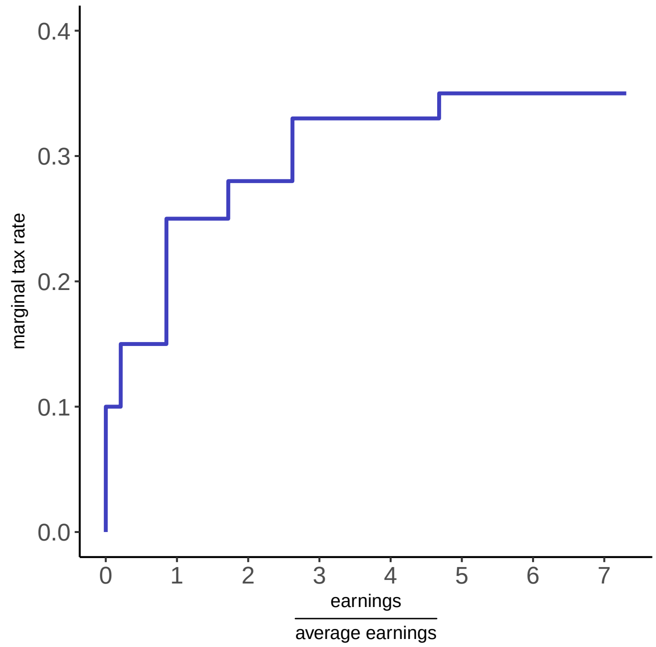
$$u\Big(\kappa(h_j)c\Big) = \frac{\Big(\kappa(h_j)c\Big)^{1-\gamma}}{1-\gamma},$$

where γ represents the coefficient of relative risk aversion. We set $\gamma = 4$, which is at the higher end of typical values used in the heterogeneous agent macroeconomics literature, but at the lower end of values that indicate a significant preference for redistribution.[15] We use an interest rate of $r = 0.01$, aligning with

[14] Note that we specifically use the 2010 marginal tax rate formula in our analysis.

[15] In this model, γ serves a dual purpose: it determines both the coefficient of relative risk aversion and, through its inverse, the intertemporal elasticity of substitution. Estimates for the latter usually range from 1 to 3, while risk aversion can be quite high, often exceeding 10 when inferred from individual financial decisions, as discussed in Vissing-Jørgensen and Attanasio (2003).

Figure 7: Marginal tax rate schedule



estimates of the 10-year real rate from Federal Reserve Bank of Cleveland (2024) for the pre-Covid-19 period of 2018-2019. Finally, we set the time discount factor to $\beta = 0.97$.

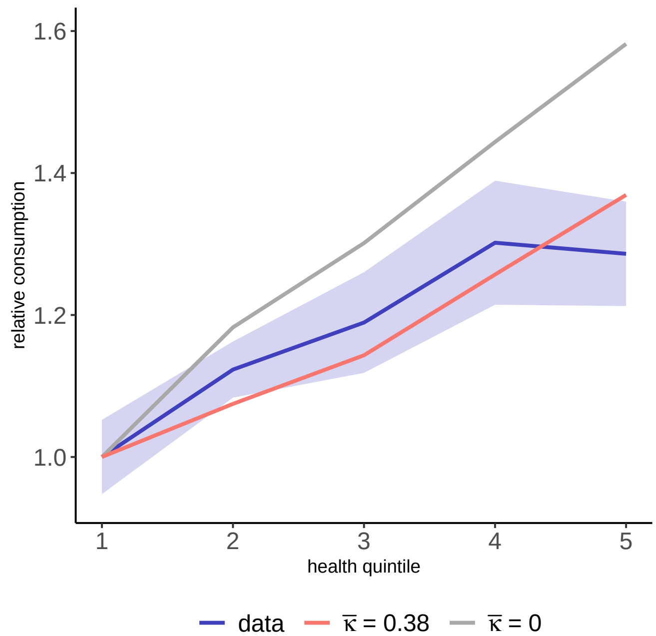
As in equation (10) of our analytical investigation, we assume a simple relationship between health and the ability to consume:

$$\kappa(h) = h^{\bar{\kappa}}$$

To determine the elasticities of consumption enjoyment with respect to health, we use data on the correlation between health (measured by one minus the frailty index) and consumption. More specific, we choose $\bar{\kappa}$ to match the correlation between health and consumption for all male individuals in the HRS with at least a High School degree aged 50 to 90 years old. From the data we get a correlation of 0.154 which we mach with $\bar{\kappa} = 0.38$ to get a correlation of 0.155 in our model. Additionally, we can show how this effects relative consumption between health quintiles. In Figure 8, we plot the average consumption of each health quintile relative to this first health quintile using HRS data in blue. The shaded area represents the 95% confidence interval. The gray line does the same for our model if we set $\bar{\kappa} = 0$ while the red line shows the result for our model if we set $\bar{\kappa} = 0.38$.[16] Thus, while $\bar{\kappa}$ decreases the correlation between consumption and health, it also flattens the relative consumption between health quintiles.

[16] Appendix B details the calculation and presents the raw correlation graphs for each age for High school and College graduates separately.

Figure 8: Relative quintile consumption



Notes: Health quintiles are calculated using the HRS and their average consumption is divided by average consumption of the first quintile.

Lastly, we assume that bequests are luxury goods. Following De Nardi (2004), we use the following functional form for warm-glow bequest giving:

$$v(a) = \theta_0 \times \frac{(\theta_1 + a)^{1-\gamma}}{1-\gamma}.$$

Here, θ_0 measures the general strength of the bequest motive, while θ_1 determines the extent to which bequests are considered luxury goods. We calibrate these values to target the consumption-to-wealth ratio of individuals aged 50 to 90. Specifically, θ_0 is chosen to match the consumption-wealth ratio of the entire population in this age group, and θ_1 is set to match this ratio for the wealthiest 10 to 30 percent of households. Table 4 summarizes our parameter choices, targets, and model-generated counterparts.

Table 4: Parameters of the bequest function

		Target		
	Value	Data	Model	
θ_0	27,000	0.149	0.148	cons.-wealth ratio (all)
θ_1	11	0.120	0.120	cons.-wealth ratio (70^{th} to 90^{th} percentile)

5 Simulation Results

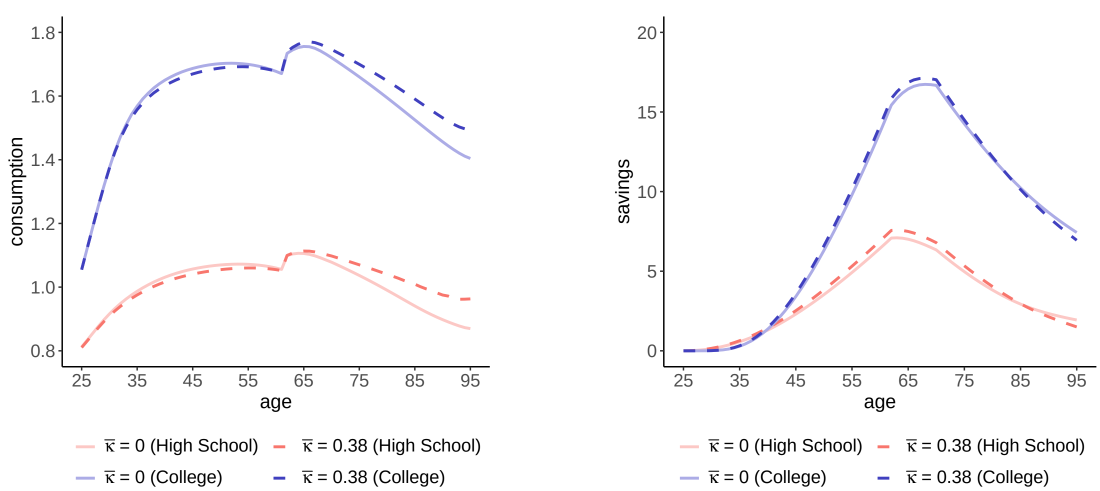
In this section, we examine the impact of incorporating health dependent utility from consumption expenditure on the simulation results. We begin by analyzing its effect on life-cycle dynamics within the simulation model. Next, we introduce a flat pension formula as a counterfactual to the US Social Security system and compare the resulting welfare effects across different specifications.

5.1 Life-cycle dynamics

To better understand how the health-dependence of the utility function influences individual decision-making within our quantitative model, we present the life-cycle dynamics in Figure 9. The figure illustrates life-cycle consumption and wealth as education-specific averages, comparing scenarios without health dependency to those with health dependency, calibrated using the respective values of $\bar{\kappa}$. Note first that there exists some strong non-smoothness at the first possible retirement age $J_{r,1} = 62$. This happens since the initial probability of retiring is about 30% and hits individuals randomly. This is an extreme shock, since individuals who are affected cannot go back to work and receive the much lower pension payments instead, which are further affected by deductions. Thus, individuals decrease consumption before $J_{r,1}$ in anticipation of this shock and all individuals not affected by the shock increase their consumption afterwards.

Figure 9: Life-cycle dynamics conditional on $\bar{\kappa}$

(a) Consumption (b) Wealth



As discussed in Section 2.3, the inclusion of $\kappa(h)$ causes a shift in consumption toward later periods of life. This shift occurs because health is negatively correlated with age, as determined by the parametrization of the frailty index, and because health and consumption are substitutes. Consequently, the decline in average health during old age requires higher consumption to compensate, leading to a reallocation of consumption from younger to older ages. This effect is directly connected to $\kappa(h)$, i.e. the larger $\bar{\kappa}$ the more consumption will be shifted toward later periods.

To finance consumption in old age, individuals must accumulate more wealth during their early years, see Figure 9b. In addition, both high school and college graduates leave less wealth at very advanced ages. This occurs because the inclusion of health dependency increases the importance of old age consumption while reducing the relative importance of bequests. As a result, the marginal propensity to leave bequests decreases with declining health, and the bequest threshold increases. Consequently, individuals allocate less wealth for bequests and instead prioritize consumption during their lifetime. For a more detailed discussion of bequest motives and a comparison to related studies, see Appendix B.6.

5.2 Welfare effects of flat pension reform

In this section, we examine the theoretical results derived in Sections 2.1 and 2.3 within the framework of our quantitative model. Specifically, we analyze the redistributive effects arising from differences in life expectancy and health-state-dependent utility by assessing the long-run welfare implications of a pension reform.

As a baseline, we consider the statutory US pension formula described in Section 4.3, which is inherently progressive. For the counterfactual scenario, we replace this progressive formula with a flat pension system, characterized by an unconditional basic old-age income. To ensure comparability, the reform is designed to be budget neutral, meaning the total pension expenditures remain constant before and after the reform. In this framework, each individual receives an unconditional pension equivalent to 44% of the average earnings.

The details of both the status quo and counterfactual pension payment schemes are illustrated in Figure 10. The figure highlights the redistributive nature of the flat pension reform: individuals earning below the average income receive higher pension payments, whereas those with above-average earnings experience reduced pension benefits. This shift underscores the stronger redistributive effect of the unconditional basic old-age income compared to the original US statutory pension formula.

Since we aim to examine the effects of differences in survival probability and the health-state-dependency of utility on optimal redistribution, we calculate the long-run welfare effects in the form of the consumption equivalence variation for three major scenarios and compare the welfare effects:

1. There is only heterogeneity in earnings, derived from the labor earnings process defined in Equations (15) and (16). The survival probability is

age-specific but equal for all individual types within each age group. It is calculated by averaging survival rates in the economy for each age.

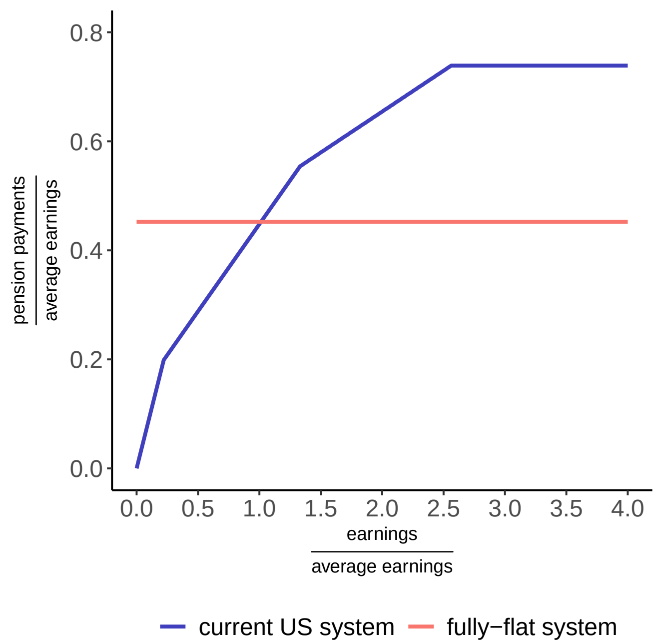
2. There is heterogeneity in both earnings and survival probabilities. We differentiate between two versions of survival differences:

   (a) Survival probabilities are education-specific, with no differences within education groups. These are calculated by averaging survival probabilities for each age within each education group.

   (b) Survival differences exist both between and within educational groups. These probabilities are determined by the age-, education-, and health-specific survival probability function $\pi(j, s, h)$.

3. There is heterogeneity in earnings and survival probabilities both between and within education groups, and additionally, utility is health-dependent. This is the complete model described in Sections 3 and 4.

Figure 10: Pension payment schedule



The welfare effects of these scenarios are presented in Table 5. Note that in all scenarios, the welfare effect of a more redistributive pension scheme is positive. However, this should be interpreted cautiously since labor supply is inelastic, meaning the pension reform does not lead to labor supply distortions. Additionally, the reform is only examined in the long run, so potential welfare costs during the transition phase are not considered. Both of these factors are omitted since our primary interest is not the benefit of the reform itself but rather how the different scenarios compare to each other.

Table 5: Welfare effect of pension reform (CEV in %)

		bequest motive		no bequest motive	
Scenario		CEV	Δ	CEV	Δ
1.	earnings	3.57		4.14	
2a.	earnings + educ. specific survival	3.45	-0.12	4.04	-0.10
2b.	earnings + survival	3.27	-0.30	3.85	-0.29
3.	earnings + survival + consumption	4.00	0.43	4.58	0.44

The results correspond with the arguments presented in the theoretical two-period model. Using scenario 1 as a baseline, we observe that the addition of differences in survival probabilities in scenarios 2a and 2b leads to a decrease in the welfare gain from the reform. This is because a utilitarian government would prefer to redistribute towards longer-lived individuals if there were no heterogeneity in earnings. Thus, differences in life expectancy mitigate fiscal income redistribution. This effect is more pronounced in scenario 2b due to greater heterogeneity in survival probabilities.

Furthermore, when examining scenario 3 and comparing it to scenario 2b, it becomes apparent that the inclusion of health-state-dependence in the utility function can counteract the effect of differences in life expectancy. Notably, the welfare gain through the reform is even greater in scenario 3 than in scenario 1. This suggests that the addition of health-state-dependence can lead to additional fiscal redistribution being optimal.

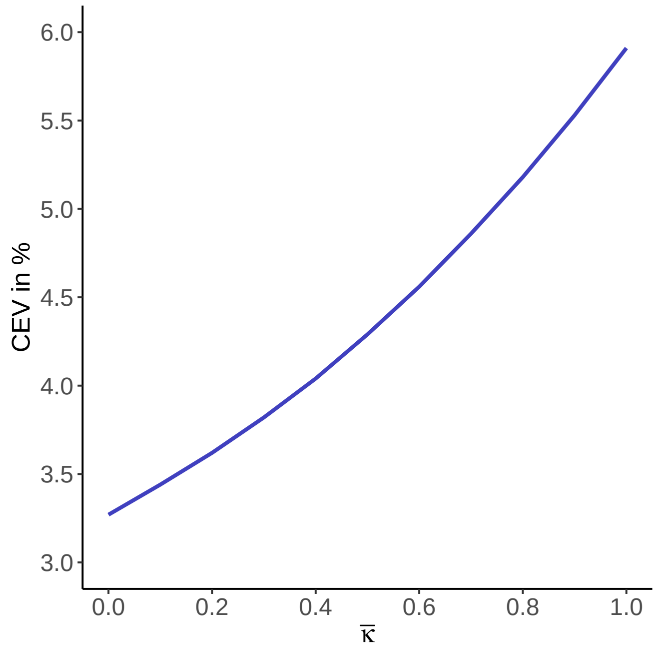
The same results can be observed if we exclude bequest motives from the quantitative model while keeping the remaining calibration the same. The addition of survival heterogeneity still leads to a decrease in welfare benefits from the pension reform, whereas the inclusion of health-state-dependence increases the welfare gain.

Furthermore, in Figure 11, we plot the Consumption Equivalence Variation (CEV) for different values of $\bar{\kappa}$. The figure illustrates how the welfare gains from the reform increase with the importance of health in the utility function.

5.3 Robustness check of the welfare effects

To check the robustness of the welfare results with respect to our choice of relative risk aversion γ and the time preference parameter β, we use different values of the parameters. For this, we first set $\gamma = 3$ and secondly $\gamma = 5$, while keeping $\beta = 0.97$. We let all other parameters unchanged and only recalibrate the elasticity of consumption enjoyment $\bar{\kappa}$ and the parameters of the bequest function θ_0 and θ_1

Figure 11: Welfare effect of pension reform conditional on $\bar{\kappa}_s$



Notes: $\bar{\kappa}$ is equalized for both high school and college graduates.

equivalently to how we did in the main model. In a next step we do the same for β. We first set $\beta = 0.96$ and secondly $\beta = 0.98$, while keeping $\gamma = 4$ unchanged. Again we do not change the other parameters and only recalibrate the elasticity of consumption enjoyment $\bar{\kappa}$ and the parameters of the bequest function θ_0 and θ_1 equivalently to how we did in the main model.

The results of the welfare analysis can be found in Table 6. While the they are somewhat quantitatively different from the results of our main model calibration, they are qualitatively the same. Thus, while changing relative risk aversion and time preference might have a quantitative effect, the effects we expected from our simple two period theoretical model can still be observed.

6 Conclusion

In this paper, we investigate the intricate relationship between life expectancy, health, and optimal fiscal redistribution. Our findings highlight the significant impact of life expectancy on redistribution policies. Specifically, we demonstrate that a utilitarian government would typically redistribute resources from individuals with shorter life expectancies to those with longer ones, due to the higher marginal utility of consumption for the latter group. In a setting where life expectancy is positively correlated with income, this would mitigate fiscal redistribution.

Table 6: Robustness of the welfare effects (CEV in %)

Scenario		bequest motive		no bequest motive	
		CEV	Δ	CEV	Δ
$\gamma = 3$					
1.	earnings	2.80		3.60	
2a.	earnings + educ. specific survival	2.75	−0.05	3.58	−0.02
2b.	earnings + survival	2.66	−0.14	3.47	−0.13
3.	earnings + survival + consumption	3.33	0.53	4.20	0.60
$\gamma = 5$					
1.	earnings	3.91		4.18	
2a.	earnings + educ. specific survival	3.73	−0.18	4.01	−0.17
2b.	earnings + survival	3.49	−0.42	3.76	−0.42
3.	earnings + survival + consumption	4.05	0.14	4.32	0.14
$\beta = 0.96$					
1.	earnings	2.66		3.20	
2a.	earnings + educ. specific survival	2.58	−0.12	3.13	−0.07
2b.	earnings + survival	2.46	−0.20	2.99	−0.21
3.	earnings + survival + consumption	3.34	0.68	3.92	0.72
$\beta = 0.98$					
1.	earnings	4.69		5.20	
2a.	earnings + educ. specific survival	4.53	−0.16	5.07	−0.13
2b.	earnings + survival	4.29	−0.40	4.83	−0.37
3.	earnings + survival + consumption	4.76	0.07	5.28	0.08

However, our analysis goes further by incorporating health-state-dependent utility into the model. With this addition, the ability to enjoy consumption varies with health status, which can reverse the direction of redistribution. In scenarios where healthier individuals can enjoy consumption more, the optimal policy may involve redistributing from healthier, longer-lived individuals to those with poorer health and shorter life expectancy if the relative risk aversion is sufficiently large. Furthermore, in scenarios where health and income are positively correlated, this could yield additional fiscal redistribution.

Lastly, we develop and calibrate a quantitative life-cycle model that integrates income, health, and lifespan risks, using data from the US economy. This model allows us to examine the complex interactions between health shocks, earnings shocks, and fiscal policy. Our analysis of a pension reform, switching from the current US pension system to an unconditional basic old-age income system, shows that differences in survival probabilities reduce the welfare gains from the reform.

However, when health-state-dependence is included in the utility function, the welfare gains increase, suggesting that accounting for health in redistribution policies can lead to more equitable outcomes.

These findings have important policy implications. Policymakers should consider both life expectancy and health status when designing fiscal policies. Redistribution policies that account for health-state-dependence can better address the needs of individuals with varying health and life expectancy, leading to more effective outcomes.

However, there are some aspects that we do not take into account in our analysis. A direct interpretation of the pension reform is not possible because, on the one hand, we assume an inelastic labor supply, meaning that there are no labor distortions due to the extremely progressive pension reform. On the other hand, we omit the transition phase between the two pension systems. In a follow-up paper, it would be interesting to include both factors and additionally solve the problem in a general equilibrium framework. This would allow us to determine the optimal amount of redistribution in a quantitative model and compare the results between a model with and without health-state-dependence.

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Unequal Lifespans and Redistribution

Appendix for Online Publication

Fabian Kindermann and Sebastian Kunz

A An Analytical Investigation: Calculations

In this appendix, we present detailed calculations for the theoretical analysis presented in Section 2.

A.1 The standard expected utility framework

The household optimization problem The household maximizes expected discounted utility

$$U_i = \max_{c_{1,i}, c_{2,i}} \; u(c_{1,i}) + \pi_i u(c_{2,i}) \quad \text{s.t.} \quad c_{1,i} + c_{2,i} = w_i + T_i.$$

The Lagrangean for this optimization problem reads

$$\mathcal{L}_i = u(c_{1,i}) + \pi_i u(c_{2,i}) + \lambda_i \left[w_i + T_i - c_{1,i} - c_{2,i} \right]. \tag{18}$$

The first order conditions of this problem are

$$u'(c_{1,i}) = \lambda_i \quad \text{and} \quad \pi_i u'(c_{2,i}) = \lambda_i.$$

They combine to the Euler equation

$$u'(c_{1,i}) = \pi_i u'(c_{2,i}). \tag{19}$$

The government's optimization problem The government maximizes utilitarian welfare subject to the constraint that total transfers between household 1 and household 2 need to add up to zero, i.e.

$$\max_{T_1, T_2} \; U_1 + U_2 \quad \text{s.t.} \quad T_1 + T_2 = 0.$$

The solution to the government's optimization problem then is

$$\frac{dU_1}{dT_1} - \frac{dU_2}{dT_2} = 0 \quad \text{or} \quad \frac{dU_1}{dT_1} = \frac{dU_2}{dT_2}.$$

Using the envelope theorem and the Lagrangean in (18), we have

$$\frac{dU_i}{dT_i} = \frac{\mathcal{L}_i}{dT_i} = \lambda_i = u'(c_{1,i}).$$

The last equality results directly from the first order conditions. Consequently, the government will optimally choose $T_1 = -T_2$ such that

$$u'(c_{1,1}) = u'(c_{1,2}).$$

How first period consumption reacts to changes in π_i In order to understand the government's desire for redistribution between the two types $i = 1, 2$, we have to investigate how first period consumption moves with π. To this end, we totally differentiate the first order condition (19) of the household:

$$\begin{aligned}
& & u''(c_{1,i})dc_{1,i} &= \pi_i u''(c_{2,i})dc_{2,i} + u'(c_{2,i})d\pi_i \\
\Rightarrow & & u''(c_{1,i})c_{1,i} \cdot \frac{dc_{1,i}}{c_{1,i}} &= \pi_i u''(c_{2,i})c_{2,i} \cdot \frac{dc_{2,i}}{c_{2,i}} + \pi_i u'(c_{2,i}) \cdot \frac{d\pi_i}{\pi_i} \\
\Rightarrow & & \frac{u''(c_{1,i})}{\pi_i u'(c_{2,i})}c_{1,i} \cdot \frac{dc_{1,i}}{c_{1,i}} &= \frac{u''(c_{2,i})}{u'(c_{2,i})}c_{2,i} \cdot \frac{dc_{2,i}}{c_{2,i}} + \frac{d\pi_i}{\pi_i} \\
\Rightarrow & & \underbrace{\frac{u''(c_{1,i})}{u'(c_{1,i})}c_{1,i}}_{=-R(c_{1,i})} \cdot \frac{dc_{1,i}}{c_{1,i}} &= \underbrace{\frac{u''(c_{2,i})}{u'(c_{2,i})}c_{2,i}}_{=-R(c_{2,i})} \cdot \frac{-dc_{1,i}}{c_{2,i}} + \frac{d\pi_i}{\pi_i} \\
\Rightarrow & -R(c_{1,i}) \cdot \frac{dc_{1,i}}{c_{1,i}} - R(c_{2,i}) \cdot \frac{dc_{1,i}}{c_{2,i}} & &= \frac{d\pi_i}{\pi_i} \\
\Rightarrow & -\left[R(c_{1,i}) + R(c_{2,i}) \cdot \frac{c_{1,i}}{c_{2,i}}\right] \frac{dc_{1,i}}{c_{1,i}} & &= \frac{d\pi_i}{\pi_i} \\
\Rightarrow & & \frac{dc_{1,i}}{d\pi_i}\frac{\pi_i}{c_{1,i}} &= -\frac{1}{R(c_{1,i}) + R(c_{2,i}) \cdot \frac{c_{1,i}}{c_{2,i}}} < 0.
\end{aligned}$$

In this derivation, we use the fact that $dc_{2,i} = -dc_{1,i}$, which immediately follows from the household's budget constraint. $R(c)$ denotes relative risk aversion at the consumption level c. The above derivation shows that the elasticity of period one consumption with respect to the survival probability π_i is strictly negative. Longer-lived individuals will therefore have a lower level $c_{1,i}$. This increases the desire of the government to redistribute towards individuals i that feature a higher life expectancy.

An example Let us assume that instantaneous preferences were represented by the utility function

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Then from the first order condition we obtain

$$c_{1,i}^{-\gamma} = \pi_i c_{2,i}^{-\gamma} \quad \Leftrightarrow \quad c_{2,i} = \pi_i^{\frac{1}{\gamma}} c_{1,i}.$$

Plugging this into the budget constraint, we immediately get

$$c_{1,i} = \frac{w_i + T_i}{1 + \pi_i^{\frac{1}{\gamma}}} \quad \text{and} \quad c_{2,i} = \frac{\pi_i^{\frac{1}{\gamma}} (w_i + T_i)}{1 + \pi_i^{\frac{1}{\gamma}}}$$

This yields the indirect utility function

$$\begin{aligned} U_i &= \frac{c_{1,i}^{1-\gamma}}{1-\gamma} + \pi_i \frac{c_{2,i}^{1-\gamma}}{1-\gamma} = \frac{\left(\frac{w_i+T_i}{1+\pi_i^{\frac{1}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} + \pi_i \frac{\left(\frac{\pi_i^{\frac{1}{\gamma}}(w_i+T_i)}{1+\pi_i^{\frac{1}{\gamma}}}\right)^{1-\gamma}}{1-\gamma} \\ &= \frac{(w_i+T_i)^{1-\gamma}}{1-\gamma} \cdot \frac{1 + \pi_i \cdot \pi_i^{\frac{1}{\gamma}(1-\gamma)}}{\left(1+\pi_i^{\frac{1}{\gamma}}\right)^{1-\gamma}} = \frac{(w_i+T_i)^{1-\gamma}}{1-\gamma} \cdot \left(1+\pi_i^{\frac{1}{\gamma}}\right)^{\gamma}. \end{aligned}$$

The government then maximizes

$$\max_{T_1} \; \frac{(w_1+T_1)^{1-\gamma}}{1-\gamma} \cdot \left(1+\pi_1^{\frac{1}{\gamma}}\right)^{\gamma} + \frac{(w_2-T_1)^{1-\gamma}}{1-\gamma} \cdot \left(1+\pi_2^{\frac{1}{\gamma}}\right)^{\gamma}.$$

The optimal government transfer can then be calculated from

$$\begin{aligned} & (w_1+T_1)^{-\gamma}\left(1+\pi_1^{\frac{1}{\gamma}}\right)^{\gamma} = (w_2-T_1)^{-\gamma}\left(1+\pi_2^{\frac{1}{\gamma}}\right)^{\gamma} \\ \Leftrightarrow \quad & (w_1+T_1)^{-1}\left(1+\pi_1^{\frac{1}{\gamma}}\right) = (w_2-T_1)^{-1}\left(1+\pi_2^{\frac{1}{\gamma}}\right) \\ \Leftrightarrow \quad & (w_2-T_1)\left(1+\pi_1^{\frac{1}{\gamma}}\right) = (w_1+T_1)\left(1+\pi_2^{\frac{1}{\gamma}}\right) \\ \Leftrightarrow \quad & T_1\left(1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}\right) = w_2\left(1+\pi_1^{\frac{1}{\gamma}}\right) - w_1\left(1+\pi_2^{\frac{1}{\gamma}}\right) \\ \Leftrightarrow \quad & T_1 = \frac{w_2\left(1+\pi_1^{\frac{1}{\gamma}}\right) - w_1\left(1+\pi_2^{\frac{1}{\gamma}}\right)}{1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}}. \end{aligned}$$

We can clearly see that the governments desire to redistribute increases in the wage difference between household 2 and household 1. Differences in life expectancy, however, mitigate the governments desire for redistribution.

Now let us write

$$w_1 = \bar{w}(1-\sigma_w) \quad , \quad w_2 = \bar{w}(1+\sigma_w) \quad , \quad \pi_1 = \bar{\pi}(1-\sigma_\pi) \quad \text{and} \quad \pi_2 = \bar{\pi}(1+\sigma_\pi).$$

Then $\bar{w}\sigma_w$ is the standard deviation of the wage inequality and $\bar{\pi}\sigma_\pi$ the standard deviation of inequality in survival probabilities. With this notation, we have

$$T_1 = \frac{\bar{w}(1+\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}\right] - \bar{w}(1-\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}\right]}{1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}+1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}}.$$

Taking the partial derivative with respect to σ_w we get

$$\frac{\partial T_1}{\partial \sigma_w} = \frac{\bar{w}\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}\right]+\bar{w}\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}\right]}{1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}+1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}} = \bar{w} > 0.$$

In addition, we have

$$\begin{aligned}
\frac{\partial T_1}{\partial \sigma_\pi} =& -\frac{\left(1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}\right)\left[\bar{w}(1+\sigma_w)\frac{1}{\gamma}\bar{\pi}\pi_1^{\frac{1}{\gamma}-1}+\bar{w}(1-\sigma_w)\frac{1}{\gamma}\bar{\pi}\pi_2^{\frac{1}{\gamma}-1}\right]}{\left(1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}\right)^2} \\
&-\frac{\left[\bar{w}(1+\sigma_w)\left(1+\pi_1^{\frac{1}{\gamma}}\right)-\bar{w}(1-\sigma_w)\left(1+\pi_2^{\frac{1}{\gamma}}\right)\right]\left(-\frac{1}{\gamma}\bar{\pi}\pi_1^{\frac{1}{\gamma}-1}+\frac{1}{\gamma}\bar{\pi}\pi_2^{\frac{1}{\gamma}-1}\right)}{\left(1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}\right)^2} \\
=& \frac{\bar{w}\bar{\pi}}{\gamma}\cdot\Bigg[-\left(1+\pi_1^{\frac{1}{\gamma}}\right)(1+\sigma_w)\pi_1^{\frac{1}{\gamma}-1}-\left(1+\pi_2^{\frac{1}{\gamma}}\right)(1+\sigma_w)\pi_1^{\frac{1}{\gamma}-1} \\
&\quad -\left(1+\pi_1^{\frac{1}{\gamma}}\right)(1-\sigma_w)\pi_2^{\frac{1}{\gamma}-1}-\left(1+\pi_2^{\frac{1}{\gamma}}\right)(1-\sigma_w)\pi_2^{\frac{1}{\gamma}-1} \\
&\quad +\left(1+\pi_1^{\frac{1}{\gamma}}\right)(1+\sigma_w)\pi_1^{\frac{1}{\gamma}-1}-\left(1+\pi_1^{\frac{1}{\gamma}}\right)(1+\sigma_w)\pi_2^{\frac{1}{\gamma}-1} \\
&\quad -\left(1+\pi_2^{\frac{1}{\gamma}}\right)(1-\sigma_w)\pi_1^{\frac{1}{\gamma}-1}+\left(1+\pi_2^{\frac{1}{\gamma}}\right)(1-\sigma_w)\pi_2^{\frac{1}{\gamma}-1}\Bigg]\cdot\frac{1}{\left(1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}\right)^2} \\
=& -\frac{2\bar{w}\bar{\pi}}{\gamma}\cdot\frac{\left(1+\pi_2^{\frac{1}{\gamma}}\right)\pi_1^{\frac{1}{\gamma}-1}+\left(1+\pi_1^{\frac{1}{\gamma}}\right)\pi_2^{\frac{1}{\gamma}-1}}{\left(1+\pi_1^{\frac{1}{\gamma}}+1+\pi_2^{\frac{1}{\gamma}}\right)^2} < 0.
\end{aligned}$$

Finally, we can calculate the nexus between σ_w and σ_π that leads to a zero transfer level from

$$T_1 = \frac{\bar{w}(1+\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}\right]-\bar{w}(1-\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}\right]}{1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}+1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}} \stackrel{!}{=} 0$$

$$\begin{aligned}
\Leftrightarrow&\quad (1+\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}\right] = (1-\sigma_w)\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}\right] \\
\Leftrightarrow&\quad \sigma_w\left[1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}+1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}\right] = \bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}-\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}} \\
\Leftrightarrow&\quad \sigma_w = \frac{\bar{\pi}^{\frac{1}{\gamma}}\left[(1+\sigma_\pi)^{\frac{1}{\gamma}}-(1-\sigma_\pi)^{\frac{1}{\gamma}}\right]}{1+\bar{\pi}^{\frac{1}{\gamma}}(1-\sigma_\pi)^{\frac{1}{\gamma}}+1+\bar{\pi}^{\frac{1}{\gamma}}(1+\sigma_\pi)^{\frac{1}{\gamma}}}.
\end{aligned}$$

A.2 Relationship to the economics of the value of life

To investigate whether an individual would be willing to invest in the extension of her life, let us derive U_i with respect to π_i. The total derivative of U_i reads

$$\begin{aligned} dU_i &= u'(c_{1,i})dc_{1,i} + \pi_i u'(c_{2,i})dc_{2,i} + u(c_{2,i})d\pi_i \\ &= \left[u'(c_{1,i}) - \pi_i u'(c_{2,i})\right] dc_{1,i} + u(c_{2,i})d\pi_i = u(c_{2,i})d\pi_i \end{aligned}$$

In this derivation we used the fact that the budget constraint implies $dc_{1,i} = -dc_{2,i}$ as well as the Euler equation (19) of the household.

For an instantaneous utility function of

$$u(c) = \bar{u} + \frac{c^{1-\gamma}}{1-\gamma}$$

as proposed in Hall and Jones (2007), there is a unique utility level $\bar{c}$ that ensures that $u(c) = 0$. We can calculate this from

$$\begin{aligned} & \bar{u} + \frac{\bar{c}^{1-\gamma}}{1-\gamma} \stackrel{!}{=} 0 \\ \Leftrightarrow \quad & \bar{c}^{1-\gamma} = -(1-\gamma)\bar{u} \\ \Leftrightarrow \quad & \bar{c} = \Big[(\gamma-1)\bar{u}\Big]^{\frac{1}{1-\gamma}}. \end{aligned}$$

Figure A1 illustrates this property for different choices of γ. When $\gamma < 1$, we don't need to add any value to the instantaneous utility function of the households, as $u(c)$ is already a positive number. However, in the case of $\gamma > 1$, we add a baseline utility level of $\bar{u} = 4$. This ensures that instantaneous utility is positive for $c > 0.25$ and $c > 0.5$ for $\gamma = 2$ and $\gamma = 5$, respectively.

A.3 An augmented model

The household optimization problem Let us now investigate an augmented model in which both survival $\pi(h_i)$ and the ability to consume $\kappa(h_i)$ depend on a common factor h_i. The household optimization problem reads

$$U_i = \max_{c_{1,i}, c_{2,i}} \; u\left[c_{1,i}\right] + \pi(h_i)u\Big[\kappa(h_i)c_{2,i}\Big] \quad \text{s.t.} \quad c_{1,i} + c_{2,i} = w_i + T_i.$$

The Lagrangean of the household optimization problem is

$$\mathcal{L}_i = u[c_{1,i}] + \pi(h_i)u\Big[\kappa(h_i)c_{2,i}\Big] + \lambda_i\left[w_i + T_i - c_{1,i} - c_{2,i}\right]. \tag{20}$$

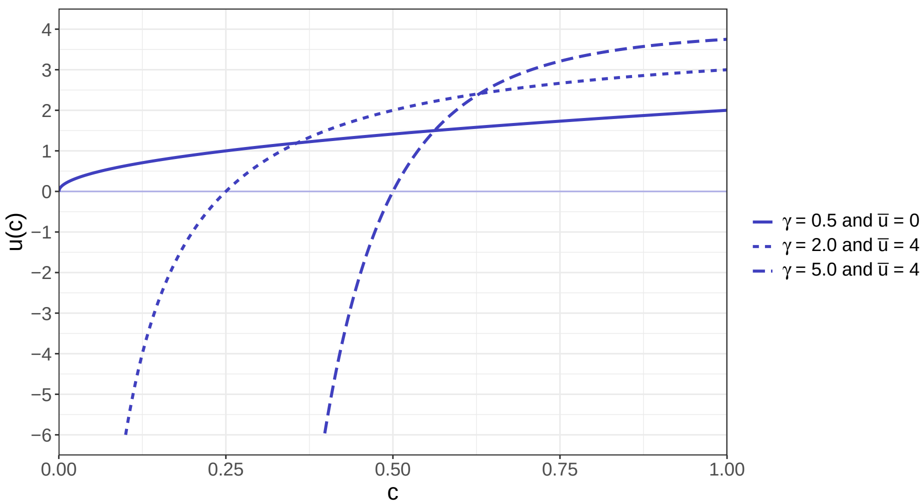
The first order conditions read

$$u'[c_{1,i}] = \lambda_i \quad \text{and} \quad \pi(h_i)\kappa(h_i)u'\Big[\kappa(h_i)c_{2,i}\Big] = \lambda_i.$$

This leads us to the Euler equation

$$u'[c_{1,i}] = \pi(h_i)\kappa(h_i)u'\Big[\kappa(h_i)c_{2,i}\Big]. \tag{21}$$

Figure A1: Instantaneous utility and the value of life



How first period consumption reacts to changes in h_i In order to understand how first period consumption changes with h_i, we again have to totally differentiate the household's Euler equation (21). We use the abbreviations $\pi_i = \pi(h_i)$ and $\kappa_i = \kappa(h_i)$ for notational purposes.

$$\begin{aligned}
&\pi_i\kappa_i u''[\kappa_i c_{2,i}]\kappa_i dc_{2,i} + \kappa_i u'[\kappa_i c_{2,i}]\pi'(h_i)dh_i \\
&\qquad + \pi_i u'[\kappa_i c_{2,i}]\kappa'(h_i)dh_i + \pi_i\kappa_i u''[\kappa_i c_{2,i}]c_{2,i}\kappa'(h_i)dh_i = u''[c_{1,i}]dc_{1,i} \\
\Rightarrow\quad &\pi_i\kappa_i u''[\kappa_i c_{2,i}]\kappa_i c_{2,i}\frac{dc_{2,i}}{c_{2,i}} + \pi_i\kappa_i u'[\kappa_i c_{2,i}]\cdot\underbrace{\pi'(h_i)\frac{h_i}{\pi_i}}_{=:\varepsilon_\pi(h_i)}\cdot\frac{dh_i}{h_i} \\
&\qquad + \pi_i\kappa_i u'[\kappa_i c_{2,i}]\cdot\underbrace{\kappa'(h_i)\frac{h_i}{\kappa_i}}_{=:\varepsilon_\kappa(h_i)}\cdot\frac{dh_i}{h_i} + \pi_i\kappa_i u''[\kappa_i c_{2,i}]\kappa_i c_{2,i}\cdot\underbrace{\kappa'(h_i)\frac{h_i}{\kappa_i}}_{=:\varepsilon_\kappa(h_i)}\cdot\frac{dh_i}{h_i} \\
&\qquad = u''[c_{1,i}]c_{1,i}\cdot\frac{dc_{1,i}}{c_{1,i}} \\
\Rightarrow\quad &\underbrace{\frac{u''[\kappa_i c_{2,i}]}{u'[\kappa_i c_{2,i}]}\kappa_i c_{2,i}}_{=-R(\kappa_i c_{2,i})}\underbrace{\frac{dc_{2,i}}{c_{2,i}}}_{=-\frac{c_{1,i}}{c_{2,i}}\frac{dc_{1,i}}{c_{1,i}}} + \varepsilon_\pi(h_i)\frac{dh_i}{h_i} + \varepsilon_\kappa(h_i)\frac{dh_i}{h_i} \\
&\qquad + \underbrace{\frac{u''[\kappa_i c_{2,i}]}{u'[\kappa_i c_{2,i}]}\kappa_i c_{2,i}}_{=-R(\kappa_i c_{2,i})}\varepsilon_\kappa(h_i)\frac{dh_i}{h_i} = \underbrace{\frac{u''[c_{1,i}]}{u'[c_{1,i}]}c_{1,i}}_{=-R(c_{1,i})}\cdot\frac{dc_{1,i}}{c_{1,i}} \\
\Rightarrow\quad &-\left[R(c_{1,i}) + R(\kappa_i c_{2,i})\cdot\frac{c_{1,i}}{c_{2,i}}\right]\frac{dc_{1,i}}{c_{1,i}} = \Big[\varepsilon_\pi(h_i) + \Big(1 - R(\kappa_i c_{2,i})\Big)\varepsilon_\kappa(h_i)\Big]\frac{dh_i}{h_i}
\end{aligned}$$

$$\Rightarrow \qquad \frac{dc_{1,i}}{dh_i}\frac{h_i}{c_{1,i}} = -\frac{\varepsilon_\pi(h_i) + [1 - R(\kappa_i c_{2,i})]\,\varepsilon_\kappa(h_i)}{R(c_{1,i}) + R(\kappa_i c_{2,i}) \cdot \frac{c_{1,i}}{c_{2,i}}} \gtrless 0$$

The elasticity of first period consumption with respect to changes in h_i can now be positive or negative. It is a composite of the elasticity of $\pi(\cdot)$ with respect to h_i and the elasticity of $\kappa(\cdot)$ with respect to h_i. Whenever risk aversion (and therefore curvature of the utility function) is greater than a value of 1, the fact that marginal utility of consumption increases with health leads consumption to drop as h_i increases. Hence, we can have both rising and falling consumption in health.

An example Let us assume that survival $\pi(h_i)$ and the ability to consume $\kappa(h_i)$ are represented by the following functions:

$$\pi_i = \pi(h_i) = \bar{\pi} h_i^{\epsilon_\pi} \quad \text{and} \quad \kappa_i = \kappa(h_i) = \bar{\kappa} h_i^{\epsilon_\kappa}.$$

It is then straightforward to see that both functions have constant elasticities:

$$\varepsilon_\pi(h_i) = \pi'(h_i)\frac{h_i}{\pi(h_i)} = \epsilon_\pi \quad \text{and} \quad \varepsilon_\kappa(h_i) = \kappa'(h_i)\frac{h_i}{\kappa(h_i)} = \epsilon_\kappa.$$

Plugging this into the elasticity of first period consumption with respect to changes in h_i, we get

$$\frac{dc_{1,i}}{dh_i}\frac{h_i}{c_{1,i}} = -\frac{\epsilon_\pi + (1-\gamma)\epsilon_\kappa}{\gamma\left[1 + \frac{c_{1,i}}{c_{2,i}}\right]}.$$

Let us again assume that instantaneous preferences were represented by the utility function

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

Then from the first order condition we obtain

$$c_{1,i}^{-\gamma} = \pi_i \kappa_i^{1-\gamma} c_{2,i}^{-\gamma} \quad \Leftrightarrow \quad c_{2,i} = \pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1} c_{1,i}.$$

Plugging this into the budget constraint, we immediately get

$$c_{1,i} = \frac{w_i + T_i}{1 + \pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}} \quad \text{and} \quad c_{2,i} = \frac{\pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}(w_i + T_i)}{1 + \pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}}$$

This yields the indirect utility function

$$U_i = \frac{\left(\frac{w_i+T_i}{1+\pi_i^{\frac{1}{\gamma}}\kappa_i^{\frac{1}{\gamma}-1}}\right)^{1-\gamma}}{1-\gamma} + \pi_i \frac{\left(\kappa_i \frac{\pi_i^{\frac{1}{\gamma}}\kappa_i^{\frac{1}{\gamma}-1}(w_i+T_i)}{1+\pi_i^{\frac{1}{\gamma}}\kappa_i^{\frac{1}{\gamma}-1}}\right)^{1-\gamma}}{1-\gamma}$$

$$= \frac{(w_i+T_i)^{1-\gamma}}{1-\gamma} \cdot \frac{1+\pi_i \cdot \left(\pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}}\right)^{1-\gamma}}{\left(1+\pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}\right)^{1-\gamma}} = \frac{(w_i+T_i)^{1-\gamma}}{1-\gamma} \cdot \left(1+\pi_i^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}\right)^{\gamma}.$$

The government then maximizes

$$\max_{T_1} \; \frac{(w_1+T_1)^{1-\gamma}}{1-\gamma} \cdot \left(1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1}\right)^{\gamma} + \frac{(w_2-T_1)^{1-\gamma}}{1-\gamma} \cdot \left(1+\pi_2^{\frac{1}{\gamma}} \kappa_2^{\frac{1}{\gamma}-1}\right)^{\gamma}.$$

The optimal government transfer can then be calculated from

$$\begin{aligned}
& (w_1+T_1)^{-\gamma} \left(1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1}\right)^{\gamma} = (w_2-T_1)^{-\gamma} \left(1+\pi_2^{\frac{1}{\gamma}} \kappa_2^{\frac{1}{\gamma}-1}\right)^{\gamma} \\
\Leftrightarrow \quad & (w_1+T_1)^{-1} \left(1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1}\right) = (w_2-T_1)^{-1} \left(1+\pi_2^{\frac{1}{\gamma}} \kappa_2^{\frac{1}{\gamma}-1}\right) \\
\Leftrightarrow \quad & (w_2-T_1) \left(1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1}\right) = (w_1+T_1) \left(1+\pi_2^{\frac{1}{\gamma}} \kappa_2^{\frac{1}{\gamma}-1}\right) \\
\Leftrightarrow \quad & T_1 \left(1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1} + 1+\pi_2^{\frac{1}{\gamma}} \kappa_2^{\frac{1}{\gamma}-1}\right) = w_2 \left(1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1}\right) - w_1 \left(1+\pi_2^{\frac{1}{\gamma}} \kappa_i^{\frac{1}{\gamma}-1}\right) \\
\Leftrightarrow \quad & T_1 = \frac{w_2 \left(1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1}\right) - w_1 \left(1+\pi_2^{\frac{1}{\gamma}} \kappa_2^{\frac{1}{\gamma}-1}\right)}{1+\pi_1^{\frac{1}{\gamma}} \kappa_1^{\frac{1}{\gamma}-1} + 1+\pi_2^{\frac{1}{\gamma}} \kappa_2^{\frac{1}{\gamma}-1}}.
\end{aligned}$$

Now using the definitions of π_i and κ_i, we can write

$$\begin{aligned}
T_1 &= \frac{w_2 \left(1+\bar{\pi}^{\frac{1}{\gamma}} \bar{\kappa}^{\frac{1}{\gamma}-1} h_1^{\frac{\epsilon_\pi}{\gamma}} h_1^{\epsilon_\kappa\left(\frac{1}{\gamma}-1\right)}\right) - w_1 \left(1+\bar{\pi}^{\frac{1}{\gamma}} \bar{\kappa}^{\frac{1}{\gamma}-1} h_2^{\frac{\epsilon_\pi}{\gamma}} h_2^{\epsilon_\kappa\left(\frac{1}{\gamma}-1\right)}\right)}{1+\bar{\pi}^{\frac{1}{\gamma}} \bar{\kappa}^{\frac{1}{\gamma}-1} h_1^{\frac{\epsilon_\pi}{\gamma}} h_1^{\epsilon_\kappa\left(\frac{1}{\gamma}-1\right)} + 1+\bar{\pi}^{\frac{1}{\gamma}} \bar{\kappa}^{\frac{1}{\gamma}-1} h_2^{\frac{\epsilon_\pi}{\gamma}} h_2^{\epsilon_\kappa\left(\frac{1}{\gamma}-1\right)}} \\
&= \frac{w_2 \left(1+\bar{a} h_1^{\frac{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}{\gamma}}\right) - w_1 \left(1+\bar{a} h_2^{\frac{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}{\gamma}}\right)}{1+\bar{a} h_1^{\frac{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}{\gamma}} + 1+\bar{a} h_2^{\frac{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}{\gamma}}}
\end{aligned}$$

where we used $\bar{a} = \bar{\pi}^{\frac{1}{\gamma}} \bar{\kappa}^{\frac{1}{\gamma}-1}$. Again we can clearly see that the governments desire to redistribute rises in the wage difference between household 2 and household 1 and that the differences in life expectancy mitigate the governments desire for redistribution. However, the addition of letting the ability to consume $\kappa(h_i)$ depend on h_i increases the governments desire to redistribute whenever $\epsilon_\pi < (\gamma-1)\epsilon_\kappa$.

A.4 Discussion of the augmented framework

A model of health expenditure Let us assume that in the second period of life, the household has to divide its total consumption expenditure c_2 into some non-medical consumption n_2 as well as medical expenditure m_2. Medical expenditure has a price p_m. The household's optimization problem reads

$$\max_{c_1,n_2,m_2} \quad u[c_1] + \pi(h)u\left[(n_2)^{\alpha(h)}(m_2)^{1-\alpha(h)}\right] \quad \text{s.t.} \quad c_1 + n_2 + p_m m_2 = w + T,$$

where we assume that in the second period, non-medical consumption and medical expenditure aggregate according to a Cobb-Douglas function. The second-period first-order conditions of this problem read

$$\pi(h)u'[\cdot]\alpha(h)\frac{(n_2)^{\alpha(h)}(m_2)^{1-\alpha(h)}}{n_2} = \lambda \quad \text{and}$$
$$\pi(h)u'[\cdot]\left(1-\alpha(h)\right)\frac{(n_2)^{\alpha(h)}(m_2)^{1-\alpha(h)}}{m_2} = \lambda p_m.$$

Together, these conditions translate into the intratemporal relation between n_2 and m_2:

$$m_2 = \frac{1-\alpha(h)}{p_m\alpha(h)} \cdot n_2 \quad \text{and} \quad c_2 = n_2 + p_m m_2 = \frac{n_2}{\alpha(h)},$$

where c_2 denotes an individual's total consumption expenditure on both non-medical consumption as well as health goods. With this result, we can immediately write

$$\begin{aligned}(n_2)^{\alpha(h)}(m_2)^{1-\alpha(h)} &= \left(\frac{1-\alpha(h)}{p_m\alpha(h)}\right)^{1-\alpha(h)} n_2 \\ &= \left(\frac{1-\alpha(h)}{p_m\alpha(h)}\right)^{1-\alpha(h)} \alpha(h)c_2 \\ &= \underbrace{\alpha(h)^{\alpha(h)}\left[1-\alpha(h)\right]^{1-\alpha(h)}(p_m)^{\alpha(h)-1}}_{=:\kappa(h)} c_2.\end{aligned}$$

The household's optimization problem is consequently isomorphic to the problem

$$\max_{c_1,c_2} \quad u[c_1] + \pi(h)u\left[\kappa(h)c_2\right] \quad \text{s.t.} \quad c_1 + c_2 = w + T$$

with $\kappa(h) = \alpha(h)^{\alpha(h)}\left[1-\alpha(h)\right]^{1-\alpha(h)}(p_m)^{\alpha(h)-1}$.

We can now investigate the properties of $\kappa(h)$, in particular how κ_h changes with h. For simplicity, we take the derivative

$$\begin{aligned}\frac{d\log(\kappa(h))}{dh} &= \frac{d\left[\alpha(h)\log(\alpha(h)) + [1-\alpha(h)]\log(1-\alpha(h)) + [\alpha(h)-1]\log(p_m)\right]}{dh} \\ &= \alpha'(h)\log(\alpha(h)) + \frac{\alpha(h)}{\alpha(h)}\alpha'(h) \\ &\qquad - \alpha'(h)\log(1-\alpha(h)) - \frac{1-\alpha(h)}{1-\alpha(h)}\alpha'(h) + \alpha'(h)\log(p_m)\end{aligned}$$

$$= \alpha'(h)\left[\log(\alpha(h)) - \log(1-\alpha(h)) + \log(p_m)\right].$$

Under the assumption that the importance of consuming health goods declines with health status $\alpha'(h) > 0$, we then have

$$\kappa'(h) \begin{cases} > 0 & \text{for all } \alpha(h) > \frac{1}{1+p_m} \text{ and} \\ \leq 0 & \text{otherwise.} \end{cases}$$

Health risk and household savings The household maximizes

$$U_i = \max_{c_1,c_2} \frac{c_1^{1-\gamma}}{1-\gamma} + E\left[\pi_i \frac{(\kappa_i c_2)^{1-\gamma}}{1-\gamma}\right] \quad \text{s.t.} \quad c_1 + c_2 = w + T.$$

There are two health shocks, $h_1 = \bar{h}(1-\sigma_h)$ and $h_2 = \bar{h}(1+\sigma_h)$ that both occur with equal probability. We can hence write the household's utility function as

$$\begin{aligned}
&\frac{c_1^{1-\gamma}}{1-\gamma} + E\left[\pi_i \frac{(\kappa_i c_2)^{1-\gamma}}{1-\gamma}\right] \\
&\quad = \frac{c_1^{1-\gamma}}{1-\gamma} + 0.5\pi_1 \frac{(\kappa_1 c_2)^{1-\gamma}}{1-\gamma} + 0.5\pi_2 \frac{(\kappa_2 c_2)^{1-\gamma}}{1-\gamma} \\
&\quad = \frac{c_1^{1-\gamma}}{1-\gamma} + \left[0.5\pi_1\kappa_1^{1-\gamma} + 0.5\pi_2\kappa_2^{1-\gamma}\right]\frac{c_2^{1-\gamma}}{1-\gamma} \\
&\quad = \frac{c_1^{1-\gamma}}{1-\gamma} + \Big[0.5\bar{\pi}\bar{\kappa}^{1-\gamma}\bar{h}^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}(1-\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa} \\
&\qquad\qquad + 0.5\bar{\pi}\bar{\kappa}^{1-\gamma}\bar{h}^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}(1+\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}\Big]\frac{c_2^{1-\gamma}}{1-\gamma}.
\end{aligned}$$

Defining the implicit discount factor as

$$\beta(\sigma_h) := 0.5\bar{\pi}\bar{\kappa}^{1-\gamma}\bar{h}^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}\left[(1-\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa} + (1+\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa}\right],$$

we can immediately write the household optimization problem as

$$U_i = \max_{c_1,c_2} \frac{c_1^{1-\gamma}}{1-\gamma} + \beta(\sigma_h)\frac{c_2^{1-\gamma}}{1-\gamma} \quad \text{s.t.} \quad c_1 + c_2 = w + T.$$

The first order condition of the problem then reads

$$c_1^{-\gamma} = \beta(\sigma_j)c_2^{-\gamma} \quad \rightarrow \quad c_2 = [\beta(\sigma_h)]^{\frac{1}{\gamma}} c_1.$$

Plugging this into the first order condition yields

$$c_1 = \frac{w+T}{1+[\beta(\sigma_h)]^{\frac{1}{\gamma}}} \quad \text{and} \quad s_1 = w + T - c_1 = \frac{[\beta(\sigma_h)]^{\frac{1}{\gamma}}(w+T)}{1+[\beta(\sigma_h)]^{\frac{1}{\gamma}}}.$$

We consequently get

$$\frac{ds_1}{d\sigma_h} = (w+T) \cdot \frac{\frac{1}{\gamma}\left[1+[\beta(\sigma_h)]^{\frac{1}{\gamma}}\right][\beta(\sigma_h)]^{\frac{1}{\gamma}-1}\beta'(\sigma_h) - \frac{1}{\gamma}[\beta(\sigma_h)]^{\frac{1}{\gamma}}[\beta(\sigma_h)]^{\frac{1}{\gamma}-1}\beta'(\sigma_h)}{\left[1+[\beta(\sigma_h)]^{\frac{1}{\gamma}}\right]^2}$$

$$= \frac{w+T}{\gamma} \cdot \frac{[\beta(\sigma_h)]^{\frac{1}{\gamma}-1}}{\left[1+[\beta(\sigma_h)]^{\frac{1}{\gamma}}\right]^2} \cdot \beta'(\sigma_h).$$

The sign of this derivative consequently depends on the sign of $\beta'(\sigma_h)$. Using the definition of $\beta(\sigma_h)$, we have

$$\beta'(\sigma_h) = 0.5\bar{\pi}\bar{\kappa}^{1-\gamma}\bar{h}^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa} \cdot [\epsilon_\pi - (\gamma-1)\epsilon_\kappa] \left[(1+\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa-1} - (1-\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa-1}\right].$$

For $\epsilon_\pi < (\gamma-1)\epsilon_\kappa$, we then immediately have

$$\epsilon_\pi - (\gamma-1)\epsilon_\kappa < 0 \quad \text{and} \quad (1+\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa-1} - (1-\sigma_h)^{\epsilon_\pi-(\gamma-1)\epsilon_\kappa-1} < 0.$$

This means that $\beta'(\sigma_h) > 0$, and consequently savings increase in an individual's health risk.

Note that in the case without shocks to consumption enjoyment we have $\epsilon_\pi = 1$ and $\epsilon_\kappa = 0$, as well as $\bar{\kappa} = 1$ and $\bar{h}$. In this case, we immediately get $\beta = \bar{\pi}$, which means that $\beta'(\sigma_h) = 0$. In this case, health risk has no impact on the household's savings profile.

B Calibration

In this appendix, we present details of the data work for the calibration of the quantitative model.

B.1 Details on the HRS and main variables used

The RAND HRS Longitudinal File 2020 (V2) consists of 15 waves conducted biennially from 1992 to 2020, while the RAND HRS CAMS File 2021 (V1) comprises 11 waves conducted in the off-years of the HRS from 2001 to 2021. To combine the CAMS data with the HRS longitudinal data, the waves in the CAMS data are assigned the wave value from the previous year's HRS wave and then combined. Thus, the RAND HRS CAMS File 2021 (V1) starts in wave 5 and continues up to wave 15. The central variables used for calibration from the RAND HRS Longitudinal File 2020 (V2) and the RAND HRS CAMS File 2021 (V1) are listed in Table B1 and Table B2, respectively.

Survival status Each wave the respondent's interview status is recorded as a sign of their mortality status. The variable `RwIWSTAT` can have seven different values: *Inapplicable*, *respondence and alive*, *non-respondence and alive*, *non-respondence and died this wave*, *non-respondence and died previous wave*, *non-respondence and dropped from sample* and *non-respondence with unclear survival.* Following Hosseini et al. (2022), we construct an indicator variable that takes the value 1 if `RwIWSTAT` is *non-respondence and died this wave*, and 0 otherwise. By shifting this indicator variable for each individual to one year prior, we can determine whether the respondents died during the interval between waves.

Education Since we are interested in the differences between individuals with a high school diploma and those with a college degree, we use the variable `RAEDUC` to differentiate. This variable consists of five categories: *Less than high school*, *GED*, *high school graduate*, *some college*, and *college and above.* For the analysis, we drop all respondents with less than a high school degree and group categories 2 to 4 together as those with a high school degree or equivalent. In the remaining sample, only 26.43% of individuals have a college degree or higher, compared to the 44.1% reported by United States Census Bureau (2010), which we use to calibrate the initial demographic distribution in the quantitative model. This discrepancy is due to the fact that the average age in the HRS is much older than in the CENSUS data, which includes all individuals aged at least 25 years.

Table B1: Variables from the RAND HRS Longitudinal File 2020 (V2)

Variable	**Description**	**RAND HRS variable**
Age	Respondent's age in years at beginning of the interview	`RwAGEY_B`
Sex	Respondent's sex, recoded as male indicator variable	`RAGENDER`
Education	Educational attainment, recoded into two groups	`RAEDUC`
Survival status	Respondent's survival to the current wave, recoded from interview status	`RwIWSTAT`
Household size	Number of people living in the household	`HwHHRES`
Earnings	Respondent's and spouses individual earnings	`RwIEARN`, `SwIEARN`
Social security	Respondent's and spouses individual retirement social security income	`RwSRET`, `SwSRET`
Pension income	Respondent's and spouses individual income from employer pension or annuity	`RwIPENA`, `SwIPENA`
Retirement	Whether Respondent considers themselves retired	`RwSAYRET`
SSDI	Whether Respondent receives SSDI in the current wave	`RwSSDI`
Wealth	Households total non-housing wealth	`HwATOTN`
Weight	Respondent level analysis weight for full sample	`RwWTRESP`
Stratum ID	Stratum identifier	`RAESTRAT`
PSU ID	Primary stage unit identifier	`RAEHSAMP`

Table B2: Variables from the RAND HRS CAMS File 2021 (V1)

Variable	**Description**	**RAND HRS CAMS variable**
Consumption	Total household consumption	HwCCTOT
Weight	Respondent level analysis weight for CAMS sample	HwWGTR

Consumption As a measure of consumption, we use total household consumption as constructed in the variable HwCCTOT. This includes all durable consumption, housing consumption, transportation consumption, and nondurable spending of a household.[17] The RAND HRS CAMS File 2021 (V1) (2024) generates durable, housing, and transportation consumption from their respective spending amounts by approximating their per-period 'usage.' Since consumption is reported in nominal dollars for each wave, we use the consumer price index from the Federal Reserve Bank of Minneapolis (2024) to adjust the values to 2010 dollars. Additionally, we approximate individual consumption by dividing total household consumption by the number of people living in each household.

Wealth As a measure of wealth, we use the net value of all non-housing wealth defined in HwATOTN. This is calculated as the sum of the *net value of real estate (excluding primary residence)*, *net value of vehicles*, *net value of businesses*, *net value of IRA or Keogh accounts*, *net value of stocks/mutual funds/investment trusts*, *value of checking/savings/money market accounts*, *value of CD/government savings bonds/T-bills*, *net value of bonds/bond funds*, and *net value of all other savings*, minus the *value of other debt*. Since wealth is reported in nominal dollars and calculated for the full household, we use the consumer price index from the Federal Reserve Bank of Minneapolis (2024) to convert it to 2010 dollars and divide it by household size to obtain per respondent wealth.

Income source We use three main income sources for the calibration: *Individual earnings*, *social security income* and *pension income*. For the analysis, we include both the respondent's and the spouse's components, if applicable. Individual earnings consist of the individual's wage/salary income, bonuses, overtime pay, commissions, tips, second job or military reserve earnings, and professional practice or trade income. Social security income additionally includes spouse or widow benefits. Pension income is calculated as the sum of all employer pensions and annuities. Since again income variables are reported in nominal dollars, we use the consumer price index from the Federal Reserve Bank of Minneapolis (2024) to adjust the values to 2010 USD. Additionally to allow for a better comparison to wealth and consumption we sum up both the respondent's and their spouses income and divide it by household size for our measures of individual income.

[17] A more thorough differentiation can be found in the RAND CAMS codeplan.

Retirement We us the variable RwSAYRET to define whether an individual counts as retired or not. For our calculation individuals only count as retired if the consider themselves as fully retired. Individuals who answer that they consider themselves either "not retired" or "partly retired" count as working individuals.

Analysis weight Following the notion of the HRS when merging two datasets, we always use the weights of the more restricted datasets. Thus, for all calculations without consumption we use the respondent level analysis weight RwWTRESP from the RAND HRS Longitudinal File 2020 (V2) (2024) and whenever we have to merge data and we are interested in consumption we use the respondent level analysis weight HwCWGTR from the RAND HRS CAMS File 2021 (V1) (2024). We normalize both weights so that the total weight of each wave is equalized.

B.2 Frailty index and survival probability

For the calculation of the frailty index, we follow Hosseini et al. (2022). We use waves 2-15 since too many variables are missing in the first wave. The variables used are listed in Table B3. Most of the variables are already in a form that allows us to easily calculate a non-weighted frailty index. Additionally, we calculate an indicator variable from RwBMI to measure whether the respondent had a self-reported BMI score of 30 or higher, and we renormalize the cognitive impairment score. The original cognitive impairment variable reports a score from 0 to 35, indicating how many mental tasks can be performed successfully. To fit the framework of a frailty index, we divide the score by 35 and invert it. Thus, a respondent who could perform all tasks perfectly now has a score of 0, while a respondent who could not perform a single task has a score of 1.

Using the 34 indicator variables and the newly constructed cognitive impairment score, we calculate the frailty index for each observation by summing the scores and dividing by the number of applicable variables. We exclude all observations for which fewer than 25 of the frailty variables are reported. Additionally, we generate a health variable which is defined as 1 minus the frailty index. We present box plots of the resulting frailty index separately for each education group and age in Figure B1.

For the survival probability function, we first regress the survival status variable on health, health^2, age, age^2, sex, and controls for wave status, separately for high school graduates and college graduates, using a Probit model. The results can be found in Table B4.

We use the coefficients for health, health^2, age, and age^2 directly in the quantitative model for the parameters of the survival probability function $g_{1,s}$, $g_{2,s}$, $g_{3,s}$, and $g_{4,s}$. For $g_{0,s}$, we combine the gender coefficient, the constant, and an additional fixed effect to accurately specify the conditional life expectancy. To achieve this, we calculate the conditional life expectancy for men at age 25, assuming a maximum age of 95, using data from the Human Mortality Database (2024). This

Table B3: Variables used for frailty index

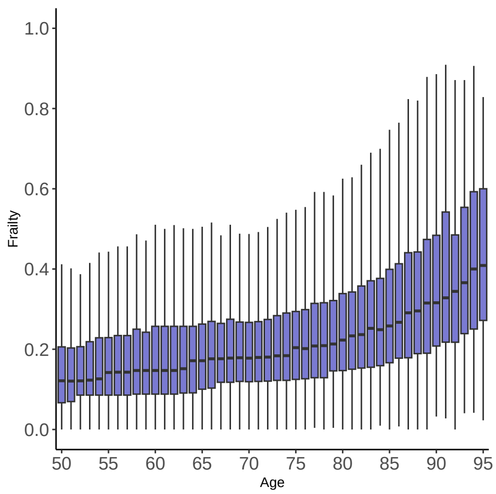
Variable	**Description**	**RAND HRS variable**
Any difficulty with (ADL/IADLs)		
Walking across a room	Yes=1, No=0	`RwWALKRA`
Walking several blocks	Yes=1, No=0	`RwWALKSA`
Dressing themself	Yes=1, No=0	`RwDRESSA`
Bathing/showering themself	Yes=1, No=0	`RwBATHA`
Eating	Yes=1, No=0	`RwEATA`
Getting in and out of bed	Yes=1, No=0	`RwBEDA`
Getting up from a chair	Yes=1, No=0	`RwCHAIRA`
Using the toilet	Yes=1, No=0	`RwTOILTA`
Using a map	Yes=1, No=0	`RwMAPA`
Using a telephone	Yes=1, No=0	`RwPHONEA`
Managing money	Yes=1, No=0	`RwMONEYA`
Taking medication	Yes=1, No=0	`RwMEDSA`
Shopping for groceries	Yes=1, No=0	`RwSHOPA`
Preparing a hot meal	Yes=1, No=0	`RwMEALSA`
Climbing one flight of stairs	Yes=1, No=0	`RwCLIM1A`
Stooping, kneeling or crouching	Yes=1, No=0	`RwSTOOPA`
Lifting/carrying 10 lbs	Yes=1, No=0	`RwLIFTA`
Picking up a dime	Yes=1, No=0	`RwDIMEA`
Reaching their arms above shoulder level	Yes=1, No=0	`RwRMSA`
Pushing or pulling large objects	Yes=1, No=0	`RwPUSHA`
Ever had the following conditions		
High blood pressure	Yes=1, No=0	`RwHIBPE`
Diabetes	Yes=1, No=0	`RwDIABE`
Cancer	Yes=1, No=0	`RwCANCRE`
Lung disease	Yes=1, No=0	`RwLUNGE`
Heart problems	Yes=1, No=0	`RwHEARTE`
Stroke	Yes=1, No=0	`RwSTROKE`
Psychological problems	Yes=1, No=0	`RwPSYCHE`
Arthritis	Yes=1, No=0	`RwARTHRE`
Overnight hospital stay in reference period	Yes=1, No=0	`RwHOSP`
Overnight nursing home stay in reference period	Yes=1, No=0	`RwNRSHOM`
Doctor visit in reference period	Yes=1, No=0	`RwDOCTOR`
Home health care in reference period	Yes=1, No=0	`RwHOMCAR`
Ever smoked cigarettes	Yes=1, No=0	`RwSMOKEV`
BMI ≥ 30	Yes=1, No=0	`RwBMI`
Cognitive impairment score	$\in [0, 1]$	`RwCOGTOT`, `RwCOGTOTP`

Notes: In waves 1 and 2A the reference period are the 12 months prior. In the remaining waves the reference period is either the 2 years prior or since the last interview.

calculation yields a conditional male life expectancy of 76.86 years. To align our model with the data, we need to add a fixed effect of -0.2042 to the constant for both education types. This adjustment results in a conditional life expectancy of

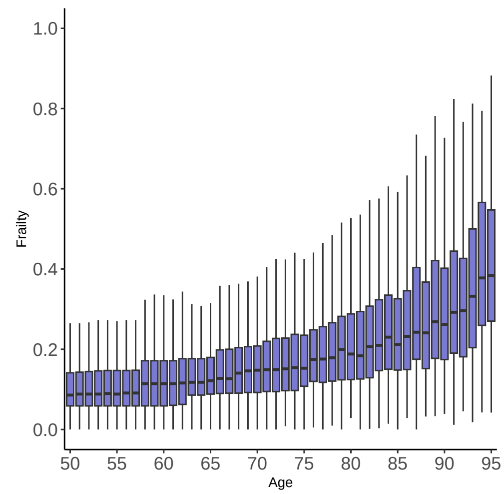
Figure B1: Box plots for frailty conditional on age

(a) High School



Notes: Outliers are excluded from depiction.

(b) College



Notes: Outliers are excluded from depiction.

Table B4: Mortality probit

	High School	College
health	-1.616	-1.220
	(0.320)	(0.766)
health^2	-0.809	-1.330
	(0.254)	(0.569)
age	-0.031	-0.060
	(0.010)	(0.020)
age^2	4.64×10^{-4}	6.79×10^{-4}
	(7.05×10^{-5})	(1.37×10^{-4})
male	0.307	0.271
	(0.019)	(0.032)
cons	-0.144	0.860
	(0.373)	(0.719)
wave fixed effect	included	included
observations	144,318	50,957

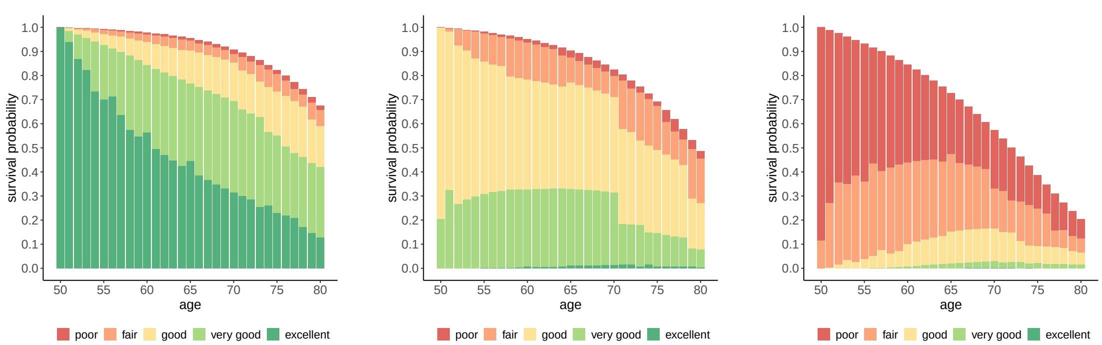
Notes: Robust standard errors in parentheses.

75.32 years for high school graduates and 78.82 years for college graduates. The 3.50-year difference in conditional life expectancy is comparable to the findings of Foltyn and Olsson (2024), who report a difference of 3.1 years for non-black men and 1.7 years for black men.

As a further comparison to Foltyn and Olsson (2024), we impute self-reported health states in our model and predict both the unconditional survival probability and the health state distribution from age 50 to 80, conditional on initial health status at age 50. To do this, we first obtain the self-reported health distribution for each age separately using the variable RwSHLT. This variable allows for five different reported states: *Excellent*, *very good*, *good*, *fair*, and *poor*. Using the cumulative distribution, we construct cutoffs for the frailty index and assign the corresponding self-reported health value. For example, at age 50, 21.31% of respondents reported having *excellent* health, so we assign the 21.31% of individuals in our model with the lowest frailty index an *excellent* health status. We repeat this process for all health states and ages. We then use the newly created health groups with *excellent*, *good*, and *fair* health to calculate their survival probabilities and show their probability of belonging to one of the five health groups. The survival probabilities reported in Figure B2 are similar to those of Foltyn and Olsson (2024). However, there seems to be a higher persistence of health states in our model.

Figure B2: Predicted health and survival probabilities

(a) Excellent health (b) Good health (c) Poor health



B.3 The retirement probability

For the estimation of the probability of working in at age 62 and the subsequent conditional probabilities, we again use waves 2-15 of the HRS. However, since our model does not allow for early retirement before age 62, we exclude all individuals who ever received Social Security Disability Insurance (SSDI), so that the share of individuals who retire before age 62 is decreased. We then run two separate probit regressions. The first probit regression estimates the unconditional probability for individuals of being retired at age 62, which generates the coefficients for the working probability at age 62, see Table B5. The second probit regression estimates the conditional probability for men of retiring in age $j + 2$ when they are not retired at age j, see Table B6.[18] We define the probit regression such that individuals who retired once cannot go back to work.

Table B5: Retirement probability at age 62

	High School	College
health	−1.873	−1.708
	(0.292)	(0.462)
male	−0.045	−0.230
	(0.050)	(0.075)
cons	−1.354	1.118
	(0.236)	(0.430)
wave fixed effect	included	included
observations	4,013	1,675

Notes: Robust standard errors in parentheses.

Table B6: Conditional retirement probability

	High School	College
health	6.839	7.925
	(1.403)	(3.848)
health^2	−4.890	−5.403
	(0.918)	(2.384)
age	1.495	2.437
	(0.369)	(0.737)
age^2	−0.011	−0.018
	(0.003)	(0.006)
male	0.096	−0.107
	(0.045)	(0.062)
cons	−52.368	−84.432
	(12.320)	(24.174)
wave fixed effect	included	included
observations	15.905	6.048

Notes: Standard errors in parentheses.

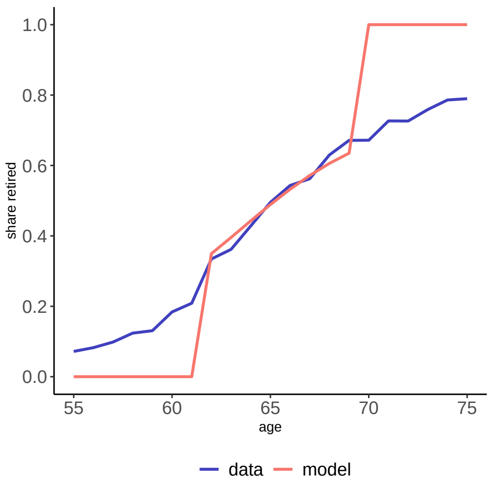
Since the health distribution and by that the survival probability of our model is not perfectly aligned with the HRS data, we need to add a fixed effect of -0.12 to perfectly estimate the probability of being retired at age 62 for all men with at least a High School degree. We do the same for the conditional probability for the subsequent years, which warrants a fixed value of -0.30. The respective sum

[18] The 2-year gap is a result of the biannual nature fo the HRS.

of the constant, the male dummy and the fixed effect is then defined as $m_{0,s}$ and $f_{0,s}$. The result of the calibration of the retirement probability for all individuals can be seen in Figure 6, while the results for High School and College graduates separately are depicted in Figure B3.

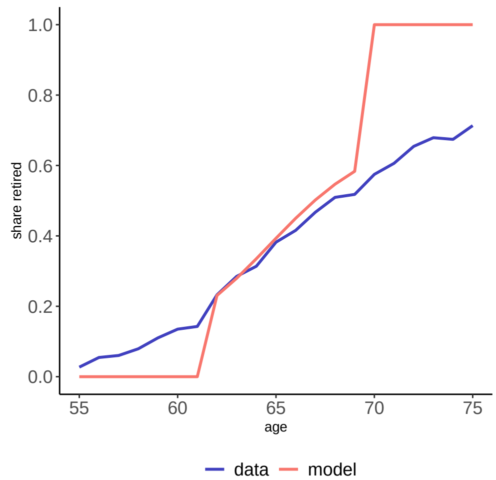
Figure B3: Share retired households by educational attainment

(a) High School



Notes: Retirement share from the HRS using individuals with a High School degree who consider themselves fully retired.

(b) College



Notes: Retirement share from the HRS using individuals with a College degree who consider themselves fully retired.

B.4 The correlation between health and income

For the correlation between health and labor income, we use the health variable derived from the frailty index and a combination of the three described income sources. To maintain comparability with the correlation between health and consumption, we only use waves 7-14 of the HRS. We define labor income as the sum of *individual earnings*, *social security*, and *pension income*. We calculate the respective correlation for men of ages 50-90 and for each education type separately. To exclude any outliers, we omit individuals with negative non-housing wealth and the top 1% of observations in the income, consumption, and non-housing wealth distribution. The correlation is then calculated using the normalized respondent-level analysis weights `RwWTRESP`, and the 95% confidence bounds are approximated using Fisher's r-to-z transformation. We then calibrate $\xi_{0,s}$ so that the correlation in our model matches the correlation in the data. The results are shown in Figures 5a and 5b.

B.5 The correlation between health and consumption

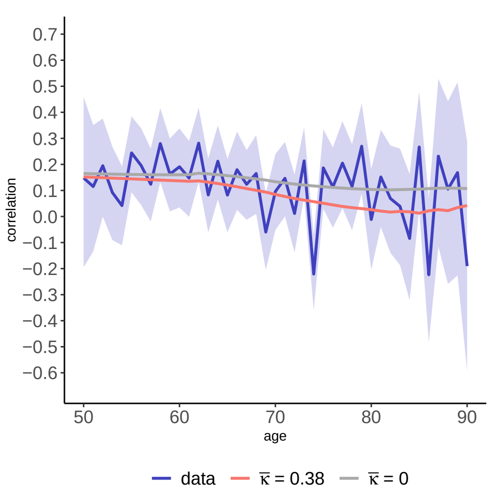
For the initial correlation between health and consumption, we proceed similarly to the method used for income. As mentioned, we only use waves 7-14. This is because *total durable consumption*, *total transportation consumption*, *total housing consumption*, and by extension *total household consumption*, are not included for wave 15. Additionally, significant changes to the questionnaire after wave 6 make it advisable to exclude waves 5 and 6 from cross-wave inspections. Again we exclude outliers by omitting individuals with negative non-housing wealth and the top 1% of observations in the income, consumption, and non-housing wealth distribution.

To calculate the correlation between health and consumption, we use the newly constructed variable of individual consumption. We then calculate the correlation between health and individual consumption for ages 50-90 for all men with at least a High School degree. For this, we use the normalized respondent-level analysis weight `HwWGTR` from RAND HRS CAMS File 2021 (V1) (2024).

Additionally, we calculate the age- and education-specific correlation between health and consumption. We again use the normalized respondent-level analysis weight `HwWGTR` from RAND HRS CAMS File 2021 (V1) (2024), and now approximate the 95% confidence bounds using Fisher's r-to-z transformation. The results are shown in Figure B4, where we also include the correlation from the quantitative model. Again the gray line shows the model results if we assume $\bar{\kappa} = 0$ while the red line depicts the case $\bar{\kappa} = 0.38$.

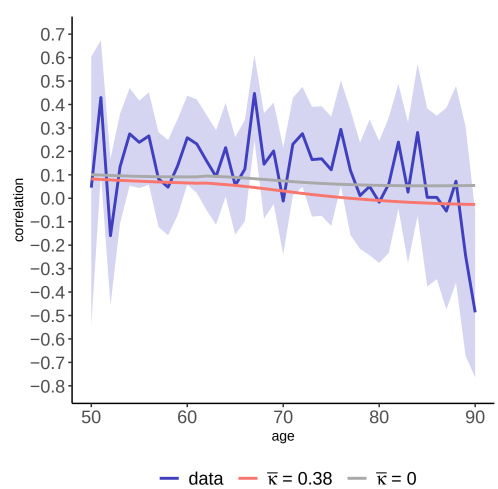
Figure B4: Correlation between consumption and health

(a) High School



Notes: Correlation between consumption and health measured as 1 minus the frailty index conditional on age.

(b) College



Notes: Correlation between income and health measured as 1 minus the frailty index conditional on age.

B.6 The consumption-to-wealth ratio

To calculate the consumption-to-wealth ratio, we first exclude respondents with negative non-housing wealth, as the model only accommodates non-negative wealth. Additionally, we omit the top 1% of observations in the income, consumption, and non-housing wealth distribution. Using the remaining data, we calculate total consumption and wealth for men aged 50-90, weighted by the normalized variable `HwWGTR`, and then compute the ratio. For the consumption-to-wealth ratio of the 70th to 90th percentile, we follow a similar process after determining the percentiles. These consumption-to-wealth ratios are then used to calibrate both θ_0, which measures the general strength of the bequest motive, and θ_1, which indicates the extent to which bequests are considered a luxury good.

Since the parameters of the bequest function cannot be directly compared with other studies that use slightly different bequest functions, we need to calculate both the marginal propensity to bequest (MPB) and the bequest threshold. Pashchenko and Porapakkarm (2024) describe in their appendix how the bequest parameters can be used to do so, by assuming an individual has only one period left to live with certainty. Following their approach, we calculate the MPB and the bequest threshold in 2010 dollars for several studies, with the results displayed in Table B7.

Table B7: MPB and bequest threshold across other studies

Study	**MPB**	**Threshold (in $2010)**
De Nardi (2004)	0.950	27,000
De Nardi et al. (2010)		
bad health	0.893	34,517
good health	0.904	30,554
Ameriks et al. (2011)	0.979	8,213
De Nardi et al. (2016)	0.786	3,962
Lockwood (2018)	0.958	17,969
Pashchenko and Porapakkarm (2024)	0.946	7,969

The elasticity of consumption enjoyment $\bar{\kappa}$ affects utility and by extension the MPB and bequest threshold. We only allow for a minimum health state of $h_j = 0.3$, as only 1.6% of HRS respondents have worse health in the quantitative model. Therefore, Table B8 only depicts results for health states of 0.3 or higher. Both the MPB and bequest threshold in our model are comparable to those in other studies. The inverse effect of health on the MPB and the bequest threshold, compared to De Nardi et al. (2010), is due to the specification of utility derived from health.

Furthermore, our model shows that for both high school and college graduates, the marginal propensity to bequeath decreases with deteriorating health, while the bequest threshold increases. This occurs because, in the model, health and consumption are substitutes. Consequently, individuals with poorer health substitute lower health levels with higher consumption, which gains relative importance. As a result, the significance of bequests diminishes compared to consumption, leading to a lower MPB and a higher bequest threshold. A similar result can be seen in De Nardi et al. (2010).

Table B8: MPB and bequest threshold conditional on health

Health	MPB	Threshold (in $2010)
0.3	0.900	27,334
0.4	0.907	25,183
0.5	0.913	23,631
0.6	0.917	22,434
0.7	0.920	21,470
0.8	0.923	20,668
0.9	0.925	19,986
1.0	0.927	19,395