

Spatial Effects of the Minimum Wage

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Introduction

- **Minimum wages** are usually set at the national level.
- However, **labor markets are local** and differ widely across space.

▶ Wages

▶ Unemployment Rates

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Question:

- How does a **national minimum wage** affect the **labor allocation across space**?
 - **Migration** decision can dampen/amplify aggregate effects on employment and wages

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This paper:

- Build a **spatial general equilibrium model** featuring:
 - Frictional labor markets (Search and Matching)
 - Frictional **migration** decisions (*Schmutz, Sidibé (2019)*)
 - A binding **national minimum wage** (*Flinn (2006)*)
- Estimate the effects of an increase of the Minimum Wage on labor market outcomes

This presentation

- **Key Mechanism:**
 - Low-productivity jobs disappear \Rightarrow \uparrow unemployment
 - Higher wages in productive regions attract migration
 - **Moving costs create a hold-up** – the minimum wage helps overcome it
- With location heterogeneity; a **Minimum Wage:**
 - Rises unemployment at a given location
 - **Reallocates** workers

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 - The minimum wage can **raise employment**
 - Migration boosts **average wages**

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- **10% Minimum Wage Increase:**

With Migration

Unemployment \uparrow 1.3%

Average wage \uparrow 0.7%

14% of wage growth due to reallocation

Without Migration

Unemployment \uparrow 4.5%

Wage bill declines

A Toy Model description

- Continuous time economy with 2 locations, $j \in \{1, 2\}$

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 - Production of the match = $A_j y$
 - Wage decided by a bargaining process over the surplus

Workers' Value Function

- Workers' reside either in 1 or 2
- Unemployed workers match randomly with local vacancies at a rate $\lambda(\theta_1)$

Value function while unemployed solves:

$$rU_1 = b_1 + \lambda(\theta_1) \int_0^{\infty} \max \{W_1(w) - U_1, 0\} dG_1(w|y)$$

Introducing frictional mobility

- Workers' reside either in 1 or 2
- Unemployed workers match randomly with local vacancies at a rate $\lambda(\theta_1)$ and with vacancies abroad at a rate $s_{12}\lambda(\theta_2)$

Value function while unemployed solves:

$$rU_1 = b_1 + \lambda(\theta_1) \int_0^\infty \max\{W_1(w) - U_1, 0\} dG_1(w|y) \\ + s_{12}\lambda(\theta_2) \int_0^\infty \max\{W_2(w) - U_1 - c_{12}, 0\} dG_2(w|y)$$

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Value function while unemployed solves:

$$rU_1(m) = b_1 + \lambda(\theta_1) \int_m^\infty \max\{W_1(w) - U_1(m), 0\} dG_1(w|y) \\ + s_{12}\lambda(\theta_2) \int_m^\infty \max\{W_2(w) - U_1(m) - c_{12}, 0\} dG_2(w|y)$$

The value of employment:

$$rW_1(y) = w + \delta_1(U_1 - W_1(y))$$

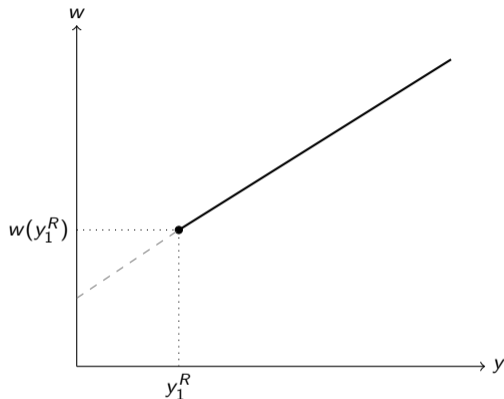
Effect of the minimum wage on local jobs

- Acceptance strategy of local offers given by y_1^R

$$W_1(y_1^R) = U_1$$

- Production of the worker: $A_1 y$
- Wage (Nash Bargaining):

$$w(y) = \alpha A_1 y + (1 - \alpha) A_1 y_1^R$$



Effects on unemployment:

$$\underbrace{(1 - u_1) L_1 \delta}_{\mathcal{E}_1 \text{ who lose their job}} = \underbrace{u_1 L_1 \lambda(\theta_1) \left(1 - F(y_1^R)\right)}_{\mathcal{U}_1 \text{ who find a job in 1}} + \underbrace{u_1 L_1 s_{12} \lambda(\theta_2) \left(1 - F(y_{12}^M)\right)}_{\mathcal{U}_1 \text{ who find a job in 2}}$$

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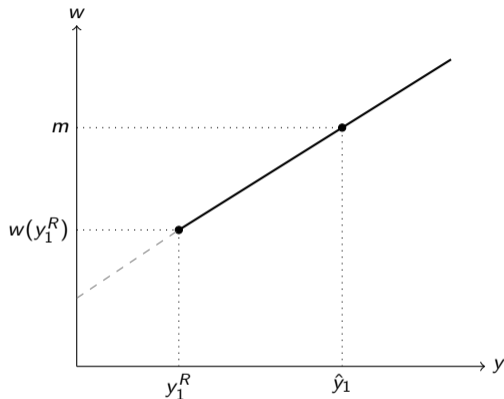
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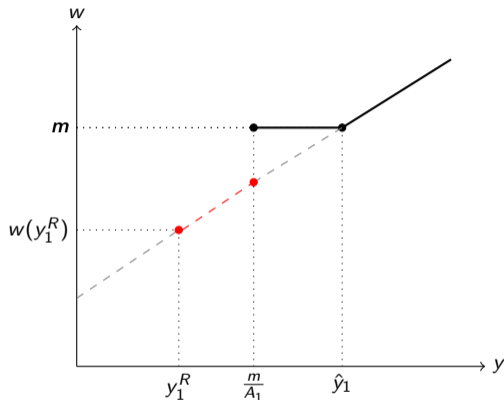
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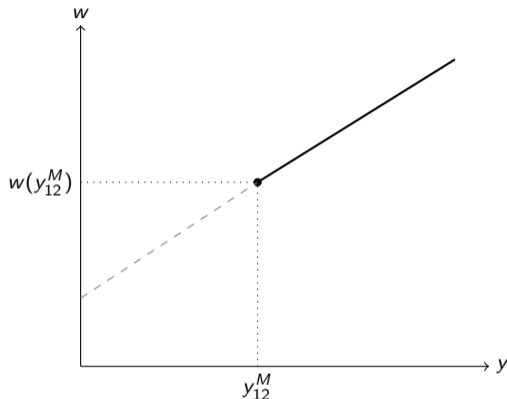
Effect of the minimum wage on migration

- Acceptance strategy of offers from **other locations** given by y_{12}^M :

$$W_2(y_{12}^M) - c_{12} = U_1$$

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$$w_2(y) = \alpha A_2y + (1 - \alpha)A_2y_2^R$$



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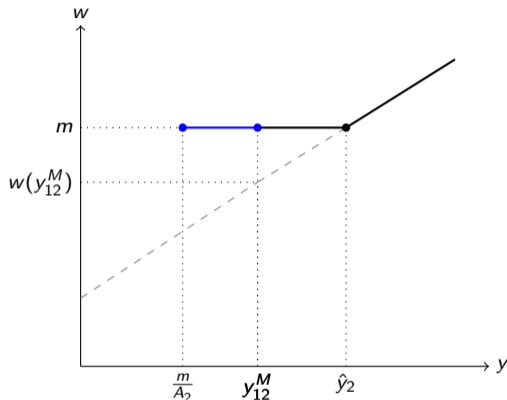
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Estimation

French Data: DADS Panel 2009-2016

- 12 regions (continental France)
- Set of national parameters: $\{r, k, \alpha, \eta, m, F(\gamma, \beta)\}$
- Set of location specific parameters: $\{\delta_j, b_j, A_j, h_j\}$
- Spatial parameters: $\{c_{jk}, s_{jk}\}$

Estimation:

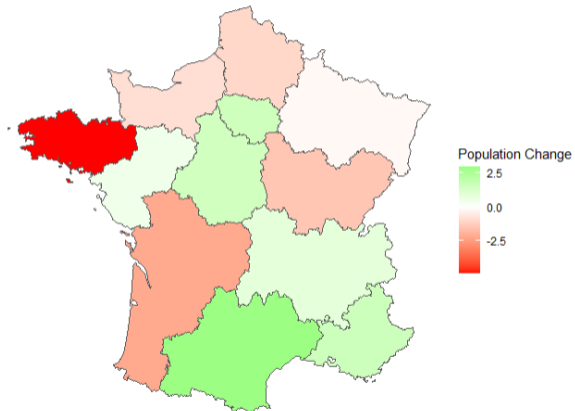
- Calibrate most parameters with the direct empirical counterpart
- GMM estimation
- $c_{jk} \rightarrow$ Differences between locals and migrants average wages
- $s_{jk} \rightarrow$ Capture labor flows across locations

Counterfactual

- What are the effects of an **increase of 10%** on the minimum wage on...?
 - Unemployment
 - Wages
 - Population shares
 - Welfare
- Assessing the impact of migration:
 - **With endogenous migration** (benchmark)
 - **Without migration**

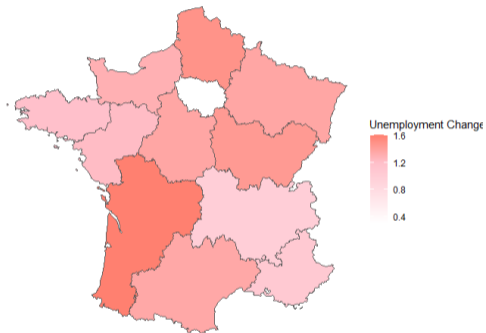
Effects of Δm (Population)

With Migration



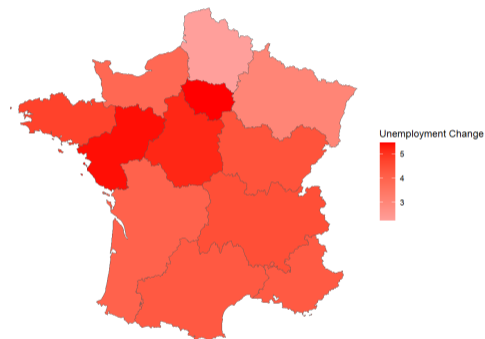
Effects of Δm (Unemployment)

With Migration



National Unemployment: 9,22% \rightarrow 9,47%
Change in unemployment: \uparrow 0.25pp

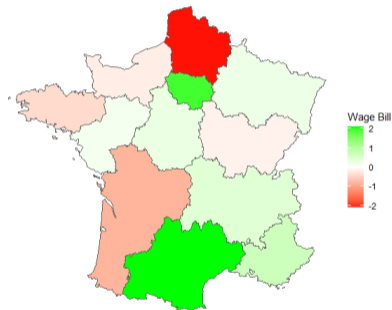
Without Migration



National Unemployment: 9,22% \rightarrow 9,64%
Change in unemployment: \uparrow 0.42pp

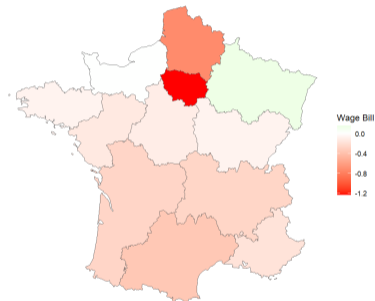
Effects of Δm (Wage Bill)

With Migration



↑ wage bill:
Wage Effect: 86%
Reallocation Effect: 14%

Without Migration



Change in wage bill: ↓ 0.46%

Conclusions

- Multiregional spatial general equilibrium model with migration dynamics
- A positive minimum wage **induce migration** from high unemployment/low wages to low unemployment/high wages
- Results:
 - **Reallocation** mitigates the negative employment effects of the Minimum wage
 - Induced migration to higher wage locations increases the average wage

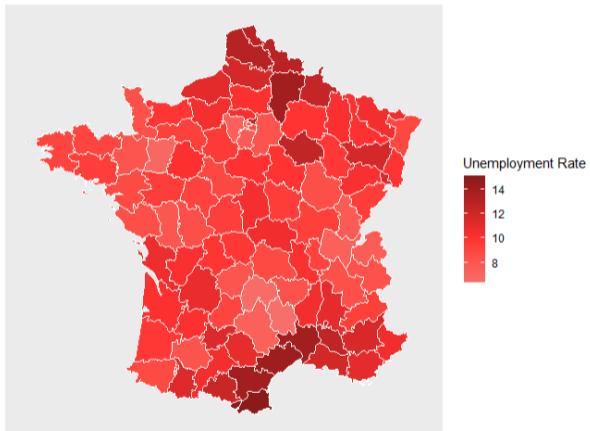
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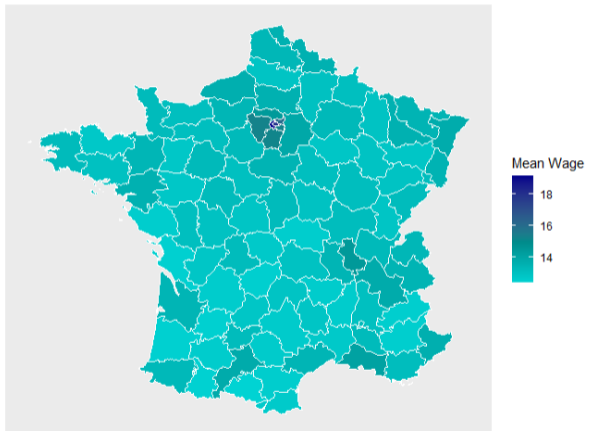
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Unemployment Rates



Average Wages



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- **Employment effects of the Minimum Wage**

Card and Krueger (1994); Flinn (2006); Cengiz et al. (2019); Neumark, Salas, and Wascher (2014); Giupponi et al. (2024).

- **Effects of the Minimum Wage**

Dustmann et al. (2022); Meghir, Narita, and Robin (2015); Lawson, Lelarge, and Spanos (2023); Monras (2019); Todd and Zhang (2022).

- **Spatial Search Models**

Kline and Moretti (2013); Heise and Porzio (2022); Kuhn, Manovskii, and Qiu (2021); Bilal (2021).

- **Migration**

Bryan and Morten (2019); Kennan and Walker (2011); Schmutz and Sidibé (2019).

Mobility patters

Table: Number and share of transitions in the same location or in a different location

	Region		Département		Zones Urbaines	
	Same	Different	Same	Different	Same	Different
UE Total	998.653	58.256	935.081	121.828	915.488	141.421
UE Share	94.5	5.5	88.47	11.53	86.62	13.38
EE Total	681.973	25.961	632.425	75.509	621.378	86.556
EE Share	96.33	3.67	89.33	10.67	87.77	12.23

Moving costs

$$\ln(\pi_{odt}) = \beta \ln(\text{dist}_{odt}) + \delta_{dt} + \delta_{ot} + \epsilon_{odt}$$

- $\pi_{odt} \equiv$ Share of workers from o in d
- $\delta_{jt} \equiv$ Destination/Origin-year fixed effects

Dependent Variable	$\log \pi_{odt}$
Log distance	-0.19 (0.001)***

▶ Back

$$\ln(\text{wage}_{it}) = \alpha_{it} + \beta \mathbb{1}(\text{migr}_{it}) + \delta_{dt} + \delta_{ot} + \epsilon_{odt}$$

Dependent Variable	log wage _{it}	log wage _{it}
Migrant	0.11 (0.0009)***	0.02 (0.0006)***
Origin x Year FE	Yes	Yes
Destination x Year FE	Yes	Yes
Individual controls	No	Yes

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Wage Migrants vs. Locals

Figure: Wage Comparisons Between Migrants and Locals Across Different Productive Locations

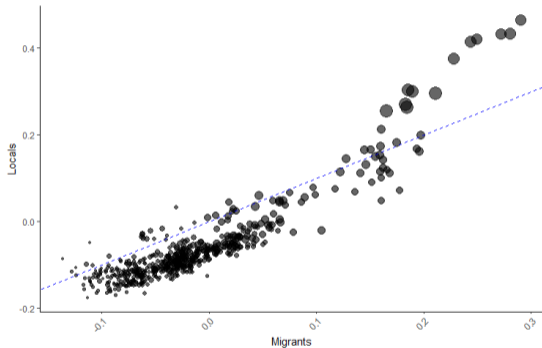


Figure: Wages of Migrants vs. Locals Across Locations

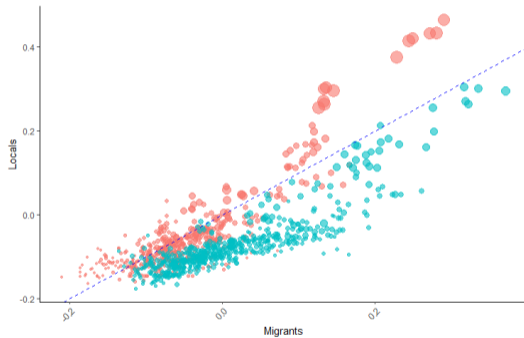


Figure: Wage Differentials by Origin Productivity Level

Source: Panel tous salaries 2009-2016

More MW migrants to productive locations

$$\ln(\pi_{dt}^{MW}) = \beta \ln(\bar{wage}_{dt}) + \delta_t + \epsilon_{dt}$$

Dependent Variable	π_{dt}^{MW}	π_{dt}^{MW}
Average Wage	-0.027 (0.002)***	0.13 (0.006)***
Sample	Full	Migrants
Year FE	Yes	Yes

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Matching Function

$$\hat{u}_1 = u_1 L_1 + s_{21} u_2 L_2$$

$$M(\mathcal{V}_1, \hat{u}_1) = \hat{u}_1^\eta \mathcal{V}_1^{(1-\eta)}$$

Job Finding probability:

$$\lambda(\theta_1) = \frac{M(\mathcal{V}_1, \hat{u}_1)}{\hat{u}_1} = \left(\frac{u_1 L_1 + s_{21} u_2 L_2}{\mathcal{V}_1} \right)^{(\eta-1)}$$

Vacancy Filling probability:

$$q(\theta_1) = \frac{M(\mathcal{V}_1, \hat{u}_1)}{\mathcal{V}_1} = \left(\frac{u_1 L_1 + s_{21} u_2 L_2}{\mathcal{V}_1} \right)^\eta$$

with $\lambda(\theta_1) = \theta_1 q(\theta_1)$.

Strategies

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- Firms: post **vacancy** in 1 if $V_1 > 0$ ▶ Firms' value functions

Free-entry condition

Stationary Equilibrium

- An acceptance strategy of **local** jobs given by y_j^R
- A mobility strategy given by the acceptance strategy of **distant jobs** given by y_{jk}^M
- The number of **vacancies** in each region is given by ν_j
- Stationary distribution of **unemployment**: u_j ▶ Unemployment
- Stationary distribution of **population**: L_j ▶ Population
- **Spatial equilibrium** condition holds: $U_j \geq U_k - c_{jk}$

▶ Back

Firms' value functions

The value of a vacancy in j :

$$rV_1 = -k + \frac{q(\theta_1)}{u_1 L_1 + s_{21} u_2 L_2} \left[u_1 L_1 \int_0^\infty \max \{J_1(y) - V_1, 0\} dF(y) + s_{21} u_2 L_2 \int_0^\infty \max \{J_1(y) - V_1, 0\} dF(y) \right]$$

The value of a job in j :

$$rJ_1(y) = A_1 y - w + \delta(V_1 - J_1(y))$$

Free-Entry Condition: $V_1 = 0$.

Stationary Equilibrium

The stationary distribution of **unemployment** is given by:

Inflow into u_1 = Outflow from u_1

$$\underbrace{(1 - u_1)L_1\delta}_{\mathcal{E}_1 \text{ who lose their job}} = \underbrace{u_1 L_1 \lambda(\theta_1) (1 - F(y_1^R))}_{\mathcal{U}_1 \text{ who find a job in 1}} + \underbrace{u_1 L_1 s_{12} \lambda(\theta_2) (1 - F(y_{12}^M))}_{\mathcal{U}_1 \text{ who find a job in 2}}$$

The stationary distribution of **population** is given by:

Inflow into 1 = Outflow from 1

$$\underbrace{u_2 L_2 s_{21} \lambda(\theta_1) (1 - F(y_{21}^M))}_{\mathcal{U}_2 \text{ who find a job in 1}} = \underbrace{u_1 L_1 s_{12} \lambda(\theta_2) (1 - F(y_{12}^M))}_{\mathcal{U}_1 \text{ who find a job in 2}}$$

Minimum Wage on employment and migration

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Identification

$$w_{it} = x_{it}\beta + \gamma_t + A_j + \epsilon_{it},$$

- $x_{it} \equiv$ Individual characteristics
- $\gamma_t \equiv$ Time fixed effects
- $A_j \equiv$ Region fixed effects

Given A_j and $\mathbb{E}[w_j|\mathcal{U}_j\mathcal{E}_j]$, y_j^R is given by:

$$\mathbb{E}[w_j|\mathcal{U}_j\mathcal{E}_j] = A_j \left(\alpha \mathbb{E}[y|y \geq y_j^R] + (1 - \alpha)y_j^R \right)$$

Given y_j^R and $\mathbb{E}[w_j|\mathcal{U}_k\mathcal{E}_j]$, y_k^M is given by:

$$\mathbb{E}[w_j|\mathcal{U}_k\mathcal{E}_j] = A_j \left(\alpha \mathbb{E}[y|y \geq y_k^M] + (1 - \alpha)y_j^R \right)$$

Identification

Given y_j^R and y_{jk}^M , estimate c_{jk} such that:

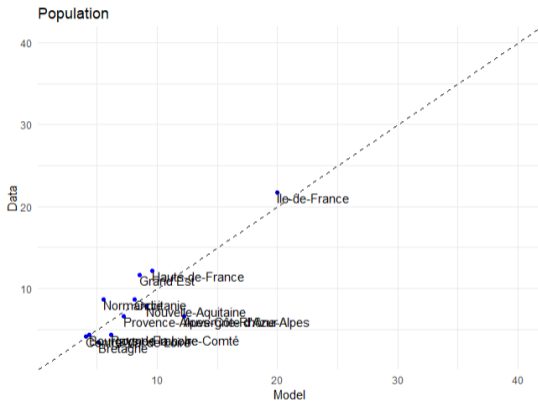
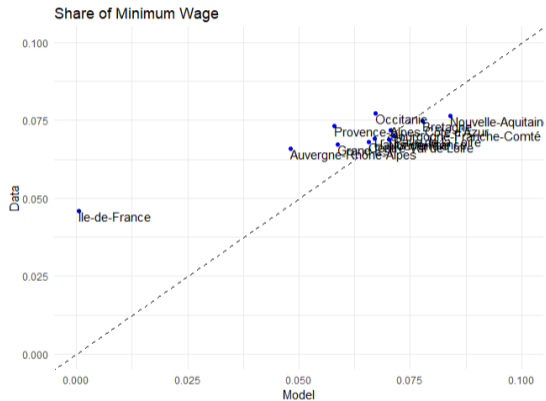
$$W_j(y_j^R) = W_k(y_{jk}^M) - c_j$$

$$\frac{\mathcal{U}_j \mathcal{E}_j}{\mathcal{U}_k \mathcal{E}_j} = \frac{u_j L_j f(\theta_j) (1 - F(y_j^R))}{s_k u_k L_k f(\theta_j) (1 - F(y_{jk}^M))}$$

Parameters

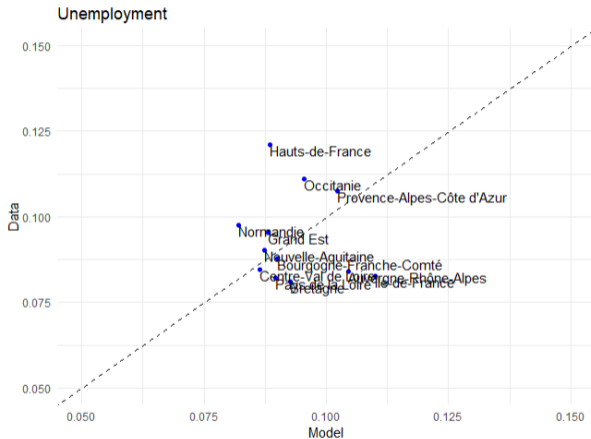
Parameter	Name	Moment
r	Interest Rate	0.04
k	Cost of a vacancy	Average unemployment
α	Workers bargaining power	0.5
η	Matching elasticity	Hosios condition
m	Minimum wage	2011 minimum wage
δ_j	Job destruction rate in j	Match job separations
A_j	Location specific productivity	Match regression estimates
b_j	Home production	Average wage of locals
h_j	Living Costs	INSEE-Filosofi
c_{jk}	Moving costs	Average wage of movers $j \rightarrow k$
s_{jk}	Spatial search friction	Transitions $j \rightarrow k$

Model fit of untargeted moments



► Unemployment Rates

Model Fit: Unemployment Rates



Welfare

The **welfare** in location j is given by:

$$\begin{aligned}\Omega_j = & \text{Average welfare of local workers} \\ & + \text{Average welfare of migrants} \\ & + \text{Average welfare of unemployed workers} \\ & + \text{Average earnings from filled jobs}\end{aligned}$$

The **total welfare** in this economy will be $\sum_{j \in \mathcal{J}} \Omega_j = \Omega$

▸ Welfare Function

Welfare function

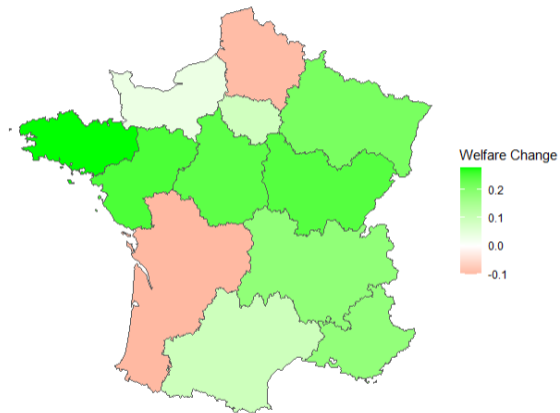
Define the number of migrants from location k to location j to be:

$$\mathcal{M}_{kj} = s_{kj} f(\theta_j) u_k L_k \left(1 - F \left(\max \left\{ \mathbb{1}(y_{kj}^M > \hat{y}_j) y_{kj}^M, \frac{m}{A_j} \right\} \right) \right)$$

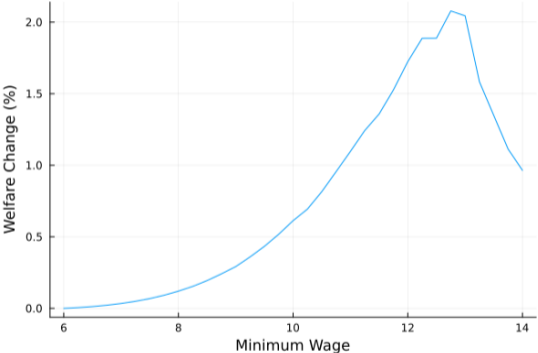
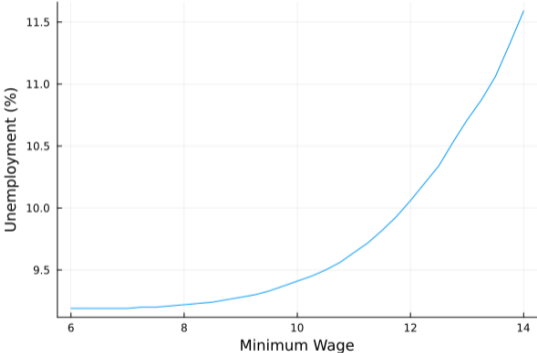
The welfare of region j looks like this:

$$\begin{aligned} \Omega_j = & \bar{W}_{jj}((1 - u_j)L_j - \sum_{k \in \mathcal{J}_{-j}} \mathcal{M}_{kj}) + U_j u_j L_j + \sum_{k \in \mathcal{J}_{-j}} \mathcal{M}_{kj}(\bar{W}_{kj} - c_{kj}) \\ & + \bar{J}_{jj}((1 - u_j)L_j - \sum_{k \in \mathcal{J}_{-j}} \mathcal{M}_{kj}) + \sum_{k \in \mathcal{J}_{-j}} \mathcal{M}_{kj} \bar{J}_{kj} \end{aligned}$$

Effects of Δm (Welfare)



Minimum wage effects



Decomposition

Unemployment rate: $u = \sum_j L_j u_j$

$$\Delta u = \sum_j \underbrace{\left(\frac{L_j + L'_j}{2} \right) (u'_j - u_j)}_{\text{within-region employment effect}} + \sum_j \underbrace{\left(\frac{u_j + u'_j}{2} \right) (L'_j - L_j)}_{\text{reallocation effect}}$$

Wage Bill : $WB = \sum_j \mathcal{E}_j \cdot \mathbb{E}(w_j)$

$$\Delta WB = \sum_j \underbrace{\left(\frac{\mathcal{E}_j + \mathcal{E}'_j}{2} \right) (\mathbb{E}(w_j)' - \mathbb{E}(w_j))}_{\text{wage effect}} + \sum_j \underbrace{\left(\frac{\mathbb{E}(w_j) + \mathbb{E}(w_j)'}{2} \right) (\mathcal{E}'_j - \mathcal{E}_j)}_{\text{reallocation effect}}$$

$$\begin{aligned} Y = & \bar{y}_A(1 - u_A)L_A + bu_AL_A - k\theta_A(u_AL_A + u_BL_B) \\ & + \bar{y}_B(1 - u_B)L_B + bu_BL_B - k\theta_B(u_BL_B + u_AL_A) \\ & - c_{AB}u_AL_A\theta_Bq(\theta_B)(1 - F(y_{AB}^M)) - c_{BA}u_BL_B\theta_Aq(\theta_A)(1 - F(y_{BA}^M)) \end{aligned}$$

$$SW = u_AL_AU_A + u_B(1 - L_B)U_B$$