

# Mandatory vs. Voluntary a priori Investment in Information Acquisition in Procurement Auctions

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## Abstract

Procurement auctions are essential for securing goods or services at competitive prices, yet participants often face high uncertainty regarding future costs. This uncertainty is mitigated by investment in information acquisition, that has to be carried out by the auction winner anyway, either before or after the auction. In the mandatory setting, participants are required to invest in information acquisition prior to the auctions. In contrast, the voluntary setting allows participants to decide whether to invest before the auction or to participate without prior investment, accepting uncertainty and investing after winning the auction. This paper complements the existing literature by providing a comprehensive analytical comparison between mandatory and voluntary settings. We develop a theoretical model that characterizes five types of symmetric equilibria and compare the settings in terms of participation, efficiency (expected welfare) and expected profits. We show that the voluntary setting yields weakly higher expected participation and weakly higher expected welfare. It also leads to weakly higher expected profits of participants under realistic conditions. While the auctioneer's expected profit can favor either setting depending on the parameters, the voluntary setting shows an advantage when information costs are high. Overall, the analytical comparison generally favors the voluntary setting, especially under high information costs. By providing theoretical insights into the comparisons, this study contributes to the broader discussion on auction design and procurement policy.

**Keywords:** Procurement auctions, information acquisition, sunk costs, efficiency

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# 1 Introduction

Procurement auctions play an essential role in securing goods or services at competitive prices. One of the key challenges is the uncertainty bidders face regarding future costs of fulfilling the contract they have won. For example, this is prominent in auctions for renewable energy, where the auctions that distribute support to renewable energy projects can be classified as procurement auctions (Kreiss et al., 2017). Measures such as site and environmental investigations, project specifications and permits mitigate the uncertainty. In any case, the auction winners must carry out these costly measures before project implementation. The question is whether these measures will be carried out before or after the auction. Further examples can be found in industrial and public procurement.

It is a common practice that many auctions mandate these measures as prequalification, requiring potential bidders to invest in information acquisition prior to the auctions as an essential part of project preparation (Matthäus, 2020). Such a setting is denoted as the mandatory setting. While the mandatory setting reduces uncertainty, it imposes sunk costs on participants and may exclude interested bidders, because potential bidders will only participate if the expected auction payoff exceeds the mandatory investment costs. This leads to reduced participation, lower competition level, and an inefficient outcome (Samuelson, 1985). In contrast to the mandatory setting, the alternative voluntary setting allows participants to decide whether to invest prior to the auction or to participate without prior investment. Participants who decide not to invest prior to the auction have to accept uncertainty and invest after winning the auction. In such a case, no additional information will be released prior to the auction. Consequently, participants face a trade-off between eliminating uncertainty at sunk costs and accepting uncertainty without sunk costs.

In this paper, we develop a theoretical model and identify the symmetric equilibria in both the mandatory and voluntary settings. Our analysis complements the existing literature on the voluntary setting and the comparison between the two settings. In the comparison, we consider reserve prices in general, efficient reserve prices, and (for the auctioneer) optimal reserve prices. The different equilibria derived from both settings are compared in terms of participation, a priori investment levels, profits of participants and the auctioneer, and efficiency. [xxx?] [The voluntary setting is characterized by mixed equilibria, which include probability of investing in uncertainty-reducing measures before the auction and after the auction. Thus, the positive effects on efficiency and bidders' profits resulting from the voluntary setting by allowing investments after the auction require investments to be made before the auction as well.](#)

The mandatory and the voluntary settings serve as theoretical benchmarks designed to illustrate key concepts and to clarify the fundamental differences between them. Neither setting fully reflects the reality. In practice, even after investment in information acquisition, uncertainty regarding future costs cannot be entirely eliminated; similarly, it is unrealistic for

the auctioneer to require no project preparation at all - some degree of preparation, such as obtaining permits or defining project specifications, is necessary in nearly all auctions. Nevertheless, by abstracting from real-world complexities and focusing on the simplified theoretical benchmarks, the model allows an analytical comparison between these two settings and helps to better understand the challenges in auction designs. Despite simplification, features of both mandatory and voluntary settings are typical in procurement auctions, particularly in auctions used to allocate support for renewable energy projects (Kreiss et al., 2017).

In Germany, for example, firms planning to participate in an auction for renewable energy support with their photovoltaic projects can decide the degree of preparation they want to perform before the auction (Bundestag, 2017). While the winning firms have to carry out investments related to site and environmental investigations, project specifications, and permitting before project implementation, these investments are not mandatory prequalifications prior to the auction. This reflects the voluntary setting. A similar framework applies to auctions for offshore wind projects (Bundesministerium der Justiz, 2023). On the contrary, an example for the mandatory setting can be found in auctions for onshore wind projects, that require higher a priori investment than in the cases above due to physical prequalification (Bundestag, 2017). The feature of the mandatory setting can also be observed in the context of industrial procurement auctions. For example, in the case of a large order for a newly developed product, such as an electronic or mechanical component for a new car series, suppliers build prototypes prior to production (e.g. Elverum and Welo, 2016). These prototypes represent investments in information acquisition. A purchasing company may request prototypes from all potential suppliers as a prequalification, allowing for testing and evaluation prior to the auction. In public procurement, prequalifications, such as feasibility studies or certifications that must be completed before the contract execution, are often required prior to the auction (e.g. Qiao and Cummings, 2003).

The remainder of the paper is structured as follows. Section 2 reviews the relevant literature. Section 3 introduces the theoretical model, followed by an analysis of its equilibria in Section 4. Section 5 compares the mandatory and voluntary settings. Model extensions and discussion are provided in Section 6. Finally, Section 7 concludes.

## 2 Literature

The existing literature can be broadly divided into two interrelated strands. The first strand examines auctions with participation costs, while the second investigates auctions with information acquisition before or during the auction. Both strands focus on how potential bidders decide whether to enter an auction and how they choose to gather information – and ultimately how these decisions affect bidding strategies, auction outcomes, and overall efficiency.

In the early work on auctions with participation costs, Samuelson (1985) and McAfee and McMillan (1987) formalize the role of participation costs, showing that costly entry can reduce competition and that the optimal reserve price equals the auctioneer’s valuation. Their analyses confine themselves to pure strategy equilibria, where only bidders with valuations above a certain threshold (cutoff type) participate. Similar settings are investigated in Tan and Yilankaya (2006) and Celik and Yilankaya (2009), who also study asymmetric equilibria. In contrast, Levin and Smith (1994) introduces mixed entry strategies, where potential bidders enter the auction with a certain probability — the so-called randomized participation. They also extend the well-known revenue-equivalence theorem, which is confirmed by Menezes and Monteiro (2000). Jehiel and Lamy (2015) also allows randomized participation and studies discrimination in auction participation between incumbents and new entrants. Our model is similar to theirs for new entrants. More recently, Gillen et al. (2017) introduces empirical perspectives on participation costs.

Complementing this literature, a second strand focuses on the cost of information acquisition. Unlike participation costs, which serve solely as entry barriers, information acquisition costs help potential bidders learn their true valuation and thus influence their decision-making. Stegeman (1996) extends the classical participation cost models by jointly considering participation costs and information acquisition in a single-unit independent private values setting, showing that the second-price auction induces efficient information acquisition. Bergemann and Välimäki (2002) studies information acquisition prior to mechanism selection, proving that the VCG mechanism provides optimal incentives for ex-ante learning and ex-post efficiency in every private value setting. Following their definition of efficiency given certain information, our model yields similar results. The main difference with our model is that bidders only invest in information acquisition before the auction by deciding how much to invest. In our model, information acquisition is part of the total investment that must be made by the auction winner in any case. The question is when, before or after the auction.

Further, Schweizer and Szech (2017) introduces dispersion criteria to classify circumstances under which revenue incentives dominate welfare incentives in information release. More recent papers compare different auction formats: Compte and Jehiel (2007) considers a model where bidders are either fully informed or completely uninformed and shows that ascending auctions encourage more information acquisition and yield higher revenue for the auctioneer than static formats. Similarly, when information acquisition costs are low, Gretschko and Wambach (2014) shows that dynamic auction formats outperform static auction formats in terms of efficiency. The most recent paper, Gretschko and Simon (2024), analyzes a setting where participants can covertly acquire information over time. They show that the dynamic pivot mechanism provides the right incentives to follow the information acquisition recommendations and to report truthfully, and therefore can implement any first-best information acquisition and allocation rule.

Together, these papers provide a comprehensive foundation for analyzing how participation costs and information acquisition influence auction design, strategic behavior, and efficiency. Based on these insights, our paper complements the existing literature on the voluntary setting and the comparison between mandatory and voluntary settings.

### 3 Model

We consider a single-unit procurement auction with  $N$  potential bidders (firms), where  $N$  is finite and  $N > 2$ . The lowest bid wins and the second lowest bid determines the price (Vickrey, 1961). The choice of a second-price auction is well established in the related literature (see e.g., Ganuza and Penalva, 2010; Schweizer and Szech, 2017). The auctioneer has a maximum willingness to pay of  $x_0$ ,  $x_0 > \underline{x}$  and sets a reserve price  $r$ ,  $\underline{x} < r \leq x_0$ . If the auction results in a winning bidder,  $x_0$  also represents the value of the fulfilled procurement to the auctioneer.

All firms are risk-neutral expected-utility maximizers and a priori symmetric. Each firm  $i$  has private cost  $x_i$ , which is a priori unknown to the firm, and  $x_i$  is the realization of the random variable  $X_i, i \in \{1, \dots, N\}$ . The  $X_i$  are i.i.d. with the distribution function  $F$  and the density function  $f$  on the support  $[\underline{x}, \bar{x}]$ . The realization of  $X_i$  is known only after an investment  $c$  in information acquisition. The investment cost  $c$  is the same for all firms, which is sensible as all firms with similar products or projects face similar preparations.<sup>1</sup> All assumptions above are common knowledge. Since  $c$  will be incurred for the auction winner in any case, the firm's individual cost of producing the products or implementing the project is  $x_i + c$ . A similar approach can be found in Jehiel and Lamy (2015), which corresponds to our mandatory setting, while our model extends their model to include the voluntary setting and full participation.

In the mandatory setting, the auctioneer requires participants to invest in information acquisition before participating in the auction. That is, all participants are investors who have already learned their true costs  $x_i$  when submitting their bids and  $c$  is sunk cost. Therefore, all participants will bid truthfully in the second-price auction if  $x_i \leq r$  or will not bid if  $x_i > r$ .

In the voluntary setting, it is up to the participants whether or not they invest in information acquisition prior to the auction. However, the auction winner must undertake this investment at the latest after the auction. If a participant chooses to invest  $c$  to learn its own cost, it is referred to as an investor. Alternatively, if a participant decides not to invest, no additional information is revealed other than the common knowledge, and it is referred to as

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<sup>1</sup>It is mostly unrealistic to invest partially in information acquisition by only conducting a lower level of project preparation, as it is unreasonable to prepare only half a prototype, or half a permit. See Section 6.4 for model discussion. Moreover,  $c$  can also be interpreted as the expected value of the symmetric random variable  $C$ , allowing different realization of the investment cost. Nevertheless, this extension does not change the results of our model below, because for the investment decisions, only the expected investment cost  $c$  is relevant; after the investment is made, the realized  $C$  is sunk cost and thus is not considered in future decisions.

a non-investor. In the auction, an investor will bid truthfully if  $x_i \leq r$  or will not bid if  $x_i > r$ , as in the mandatory setting, while a non-investor will bid  $\mathbb{E}[X_i] + c$  as the dominant strategy in terms of expected profit if  $\mathbb{E}[X_i] + c \leq r$  or will not bid if  $\mathbb{E}[X_i] + c > r$  (Ehrhart et al., 2015). Since the firms are a priori symmetric, we omit the index  $i$  and use  $\mathbb{E}[X_i] =: \mathbb{E}[X]$  for the ease of simplicity.

## 4 Equilibria and properties

This section presents symmetric equilibria of the model and their properties. The key findings for both settings are summarized in the following two propositions, which characterize the strategic behavior and the expected results of the firms. These propositions are further supported by several lemmas that analyze equilibrium properties and aid in their proofs. For completeness, derivations and proofs can be found in the appendix.

The equilibria differ in the probability of participation and the probability of investor and non-investor behavior. If firms in the mandatory setting or in the voluntary setting participate only as investors, their participation probability is denoted by  $q$ . If firms in the voluntary setting participate with a strategy with mixed investment behavior, and  $q_1$  and  $q_2$  denote the probabilities of investor and non-investor behavior, respectively, where  $q_1 + q_2 \leq 1$  is the participation probability. If firms in the voluntary setting participate only as non-investors, their participation probability is denoted by  $q'$ .

Three relevant thresholds characterize the equilibria below and are given by

$$c_{max}(r) := \int_{\underline{x}}^r (r - t)f(t)ds = \int_{\underline{x}}^r F(t)ds,$$

$$c_{min}^m(r) := \int_{\underline{x}}^r F(t)(1 - F_{(1,N-1)}(t))dt,$$

$$c_{min}^v(r) := \int_{\underline{x}}^r F(t)(1 - F_{(1,N-1)}(t))dt - \int_{\mathbb{E}[X]+c}^r (1 - F_{(1,N-1)}(t))dt, \text{ where } r \geq \mathbb{E}[X] + c.$$

Depending on  $r$ ,  $c_{max}(r)$  denotes the highest value of  $c$  for which an equilibrium with participation exists, while  $c_{min}^m(r)$  and  $c_{min}^v(r)$  denote the lowest values of  $c$  such that a slight increase in  $c$  would prevent full participation and investment in the mandatory and voluntary setting, respectively.

**Lemma 1.** *For any distribution, it holds that  $0 < c_{min}^m(r) < c_{max}(r)$ , and additionally  $0 < c_{min}^v(r) \leq c_{min}^m(r)$ , if  $c_{min}^v(r)$  exists.*

Given Lemma 1, we have the following propositions.

**Proposition 1.** *In the mandatory setting, the following applies:*

- (i) For all  $c \in [0, c_{min}^m(r)]$ , there exists a unique equilibrium, where  $q = 1$  (i.e. full participation). The expected profit of each participant is weakly positive.
- (ii) For all  $c \in (c_{min}^m(r), c_{max}(r))$ , there exists a unique equilibrium, which is characterized by a unique  $q \in (0, 1)$  (i.e. randomized participation), determined by

$$\pi^m = \sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^r F(t)(1 - F(t))^n dt = c. \quad (1)$$

The expected profit of each participant is zero.

- (iii) For all  $c \geq c_{max}(r)$ , there exists a unique equilibrium, where  $q = 0$  (i.e. no participation).

In the mandatory setting, if the investment cost  $c$  is very low (depending on  $r$ ) or even zero, it is worthwhile for all firms to participate and invest, leading to a positive expected profit.<sup>2</sup> As  $c$  increases towards the threshold  $c_{min}^m(r)$ , the expected profit decreases. At the threshold, firms are indifferent between participating or not, as their expected profit is zero. If  $c$  increases further, full participation would result in negative profits, making it infeasible. However, an equilibrium exists in the form of randomized participation, where all firms participate with probability  $q \in (0, 1)$ , so that their expected profit is zero and thus all firms remain indifferent between participating or not. As  $c$  approaches the upper threshold  $c_{max}(r)$ , the participation probability  $q$  decreases to zero. Beyond this point, participation is no longer worthwhile.

For the cases of our main focus, where  $c \in (c_{min}^m(r), c_{max}(r))$ , and equilibria with randomized participation exist, we have the following lemmas (proofs can be found in Appendix A).

**Lemma 2.** *It holds that  $\frac{\partial \pi^m}{\partial r} > 0$ ,  $\frac{\partial \pi^m}{\partial q} < 0$ , and  $\frac{\partial \pi^m}{\partial N} < 0$ .*

Lemma 2 states that the expected payoff increases in  $r$  and decreases in  $q$  and  $N$ . Therefore, to satisfy the equilibrium condition (1), we have the following:  $\frac{dq}{dr} > 0$ ,  $\frac{dq}{dc} < 0$ ,  $\frac{dq}{dN} < 0$ . That is, a higher reserve price, lower information costs, and less competition increase participation.

**Lemma 3.** *Given  $N$ ,  $c$ , and  $r$ , for a distribution  $G$  that is mean-preserving contraction of  $F$  with support  $[\underline{y}, \bar{y}]$  and  $\frac{\mathbb{E}[Y]-y}{\mathbb{E}[X]-\underline{x}} = \frac{\bar{y}-\mathbb{E}[Y]}{\bar{x}-\mathbb{E}[X]}$ , the probability of participation corresponding to  $q$  is less than  $q$ .*

Since  $G$  is a mean-preserving contraction of  $F$ , whose distribution is proportionally contracted to a narrower support, for the random variable  $Y$  with distribution  $G$ ,  $\mathbb{E}[Y] = \mathbb{E}[X]$

<sup>2</sup>In this section, as we focus on the strategic behavior of the firms given  $c$  and  $r$ , expected profit refers specifically to the expected profit of the participants. In the following sections, where the auctioneer's profit is also considered, we will clearly distinguish between the expected profit of the participants and that of the auctioneer.

applies. Lemma 3 implies that, the wider the distribution, the higher  $q$  will be. This is because, given the same conditions, including the same information costs, the information gain from wider distribution is higher, which favors investment.

**Proposition 2.** *In the voluntary setting, the following applies:*

(i) *If  $r < \mathbb{E}[X] + c$ , the equilibria and their conditions are the same as in the mandatory setting.*

(ii) *If  $r = \mathbb{E}[X] + c$ , it holds that:*

*For all  $0 \leq c \leq c_{min}^v(r) = c_{min}^m(r)$ , there exists a unique equilibrium, where  $q = 1$  (i.e. full participation and investment). The expected profit of each participant is weakly positive.*

*For all  $c \in (c_{min}^m(r), \bar{x} - \mathbb{E}[X])$ , there exists two types of equilibria: The first type is an equilibrium, where  $q \in (0, 1)$  (i.e. randomized participation and investment), determined by*

$$\sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^r F(t)(1 - F(t))^n dt - c = 0. \quad (2)$$

*The expected profit of each participant is zero. The second type are equilibria, where  $q_1 = q \in (0, 1)$  as determined by Equation (2) and  $q_2 \in (0, 1 - q_1]$  arbitrarily (i.e. randomized participation and mixed investment behavior). The expected profit of each participant is zero.*

*For all  $c \geq \bar{x} - \mathbb{E}[X]$ , there exists equilibria, where  $q' \in (0, 1]$  arbitrarily (i.e. randomized participation without investment). The expected profit of each participant is zero.*

(iii) *If  $r > \mathbb{E}[X] + c$ , it holds that:*

*For all  $0 \leq c \leq c_{min}^v(r)$ , there exists a unique equilibrium, where  $q = 1$ . The expected profit of each participant is strictly positive.*

*For all  $c \in (c_{min}^v(r), \bar{x} - \mathbb{E}[X])$ , there exists a unique equilibrium, where  $q_1 \in (0, 1)$  and  $q_2 = 1 - q_1$  (i.e. full participation and mixed investment behavior), and  $q_1$  is determined by*

$$\Delta\pi^v = \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1 - F(t))^{n_1} dt - P(N - 1, q_1) \int_{\mathbb{E}[X]+c}^r (1 - F(t))^N dt - c = 0. \quad (3)$$

*The expected profit of each participant is strictly positive.*

*For all  $c \geq \bar{x} - \mathbb{E}[X]$ , there exists a unique equilibrium, where  $q' = 1$  (i.e. full participation without investment). The expected profit of each bidder is zero.*

In the voluntary setting, the equilibria depend on the relationship between  $\mathbb{E}[X] + c$  and  $r$ . Since the bid of a non-investor is  $\mathbb{E}[X] + c$ , firms may participate as non-investors in the

auction only if  $r \geq \mathbb{E}[X] + c$ . If  $r < \mathbb{E}[X] + c$ , firms do not participate as non-investors and thus the equilibria and their conditions are the same as in the mandatory setting.

In case  $r = \mathbb{E}[X] + c$ , the participation of non-investors is arbitrary and in addition to the participation of investors in the mandatory setting, as long as the mandatory setting does not yield full participation. That is, for each  $c > c_{min}^m(r)$ , where  $q \in [0, 1]$  in the mandatory setting, there exist equilibria in mixed investment behavior with  $q_1 = q$  and any  $q_2 \in [0, 1 - q]$  in the voluntary setting. The reason is that  $q_2$  has no additional effect on the expected profit of both the investors and the auctioneer than the reserve price  $r$  and the non-investors' expected profit is zero. This is a special case exactly because of the equality  $r = \mathbb{E}[X] + c$ .

In case  $r > \mathbb{E}[X] + c$ , the non-investors will always expect a positive profit, unless all potential bidders participate as non-investors. Therefore, all firms have an incentive to participate, either as an investor with probability  $q_1$  or as a non-investor with probability  $q_2 = 1 - q_1$ . If  $c$  is very low or even zero, it is worthwhile for all firms to participate and invest ( $q = 1$ ). As  $c$  increases, the expected profit decreases. In contrast to the mandatory setting, the expected profit of the non-investors catches the expected profit of the investors before it falls to zero. That is, at a lower threshold of  $c$  than in the mandatory case ( $c_{min}^v \leq c_{min}^m$ ), not all participants will invest. From here on, the equilibrium strategy changes to a mixed investment behavior with  $q_2 = 1 - q_1 > 0$ . Note that in this equilibrium, the coexistence of investors and non-investors is complementary: both types rely on the existence of each other to sustain the equilibrium. As  $c$  further increases, the advantage of non-investors over investors increases because the value of the information gain remains unchanged but the information cost increases. As a result, as  $c$  increases from the threshold  $c_{min}^v$  to the threshold  $\bar{x} - \mathbb{E}[X]$ ,  $q_1$  decreases from 1 to 0, while  $q_2$  increases from 0 to 1. Beyond the threshold  $\bar{x} - \mathbb{E}[X]$ , there are only non-investors ( $q' = 1$ ) and thus the the expected profit is zero. If  $q' < 1$ , the expected profit would be strictly positive, so that all firms have an incentive to participate, which raises the participation probability to 1 and lowers the expected profit to zero. The equilibria at low  $c$  with  $q = 1$  and high  $c$  with  $q' = 1$  can be seen as a continuous extension of the equilibria in mixed investment behavior at the two extremes  $q_1 = 1$  and  $q_2 = 1$ .

For the cases of our main focus with  $r > \mathbb{E}[X] + c$  and  $c \in (c_{min}^v(r), \bar{x} - \mathbb{E}[X])$  so that the equilibrium in mixed investment behavior exists (with  $q_1 + q_2 = 1$ ), we have the following lemmas (proofs can be found in Appendix B).

**Lemma 4.** *Let  $\Delta\pi^v = \pi^{vi} - c - \pi^{vn}$ , it holds that  $\frac{\partial\Delta\pi^v}{\partial r} < 0$ ,  $\frac{\partial\Delta\pi^v}{\partial c} < 0$ ,  $\frac{\partial\Delta\pi^v}{\partial q_1} < 0$ , and  $\frac{\partial\Delta\pi^v}{\partial N} < 0$ .*

The term  $\Delta\pi^v$  is the difference in expected profits between investors ( $\pi^{vi} - c$ ) and non-investors ( $\pi^{vn}$ ) in the voluntary setting. Lemma 4 states that  $\Delta\pi^v$  decreases in  $r$ ,  $c$ ,  $q_1$ , and  $N$ . To satisfy the equilibrium condition (3), given any two of three variables  $r$ ,  $c$ , and  $N$ , the probability of investment  $q_1$  decreases in the third variable:  $\frac{dq_1}{dr} < 0$ ,  $\frac{dq_1}{dc} < 0$ ,  $\frac{dq_1}{dN} < 0$ . Both

investors and non-investors benefit from a higher  $r$ , but non-investors benefit more. Similarly, while increased competition harms both groups, investors are more adversely affected. Additionally, investors incur the cost  $c$ , whereas non-investors do not. As a result, a higher  $r$ ,  $c$  or  $N$  increases the relative advantage of non-investors over investors, leading to a lower  $q_1$ . Overall, an increase in  $r$ ,  $c$ , or  $N$  speaks in favor of non-investment and thus shifts the equilibrium in mixed investment behavior towards more non-investors.

**Lemma 5.** *Given  $N$ ,  $c$ , and  $r$ , for a distribution  $G$  that is mean-preserving contraction of  $F$  with support  $[\underline{y}, \bar{y}]$  and  $\frac{\mathbb{E}[Y]-\underline{y}}{\mathbb{E}[X]-\underline{x}} = \frac{\bar{y}-\mathbb{E}[Y]}{\bar{x}-\mathbb{E}[X]}$ , the probability of participation corresponding to  $q_1$  is less than  $q_1$ .*

Analogous to Lemma 3, Lemma 5 implies that, a wider distribution of information is in favor of the investment and thus leads to a higher  $q_1$ .

In total, our analysis identifies five types of symmetric equilibria depending on  $c$  and  $r$ , which are classified and labeled below:

- $E_0$ : No participation,  $q = 0$
- $E_1^f$ : Full participation, all firms participate and invest  $c$ ,  $q = 1$
- $E_1^r$ : Randomized participation, all firms participate and invest  $c$  with probability  $q \in (0, 1)$
- $E_2$ : All firms participate without investment with probability  $q' \in (0, 1]$
- $E_{mix}$ : Mixed investment behavior, all firms participate and invest  $c$  with probability  $q_1 \in (0, 1)$  and participate without investment with  $q_2 \in (0, 1)$  where  $q_1 + q_2 \leq 1$

These equilibria are summarized and illustrated in Figure 1, Table 1, and Table 2.

Condition	Equilibrium	Characteristics
$0 \leq c \leq c_{min}^m(r)$	$E_1^f$	$q = 1, \pi^m \geq 0$
$c_{min}^m(r) < c < c_{max}(r)$	$E_1^r$	$q \in (0, 1), \pi^m = 0$
$c \geq c_{max}(r)$	$E_0$	No participation

Table 1: Equilibria in the mandatory setting

## 5 Comparison between mandatory and voluntary settings

In this section, we compare the mandatory and voluntary settings under three reserve price regimes: reserve prices in general (identical for both settings), the efficient reserve price and

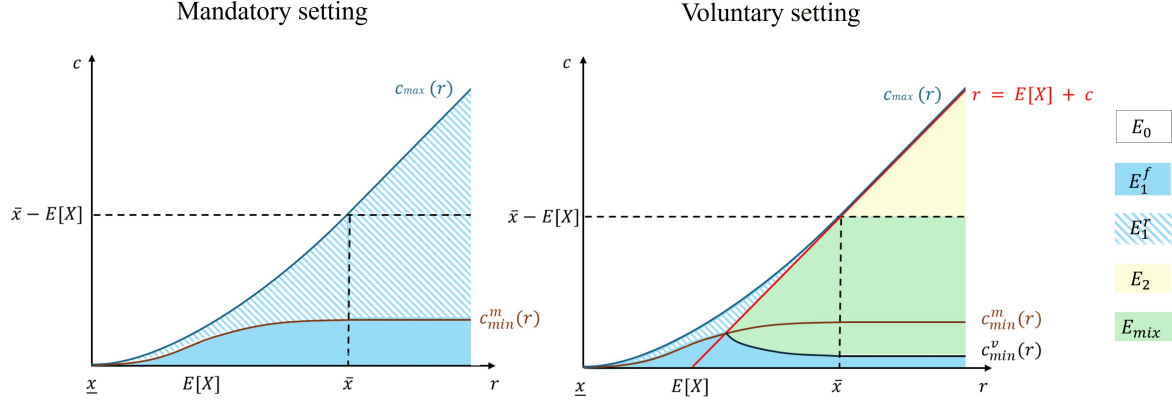


Figure 1: Equilibria depending on  $c$  and  $r$  in mandatory and voluntary settings

	Condition	Equilibrium	Characteristics
$r < \mathbb{E}[X] + c$	$0 \leq c \leq c_{min}^m(r)$	$E_1^f$	$q = 1, \pi^{vi} \geq 0$
	$c_{min}^m(r) < c < c_{max}(r)$	$E_1^r$	$q \in (0, 1), \pi^{vi} = 0$
	$c \geq c_{max}(r)$	$E_0$	No participation
$r = \mathbb{E}[X] + c$	$0 \leq c \leq c_{min}^m(r) = c_{min}^v(r)$	$E_1^f$	$q = 1, \pi^{vi} \geq 0$
	$c_{min}^m(r) < c < \bar{x} - \mathbb{E}[X]$	$E_1^r$ & $E_{mix}$	$q_1 \in (0, 1), q_2 \in [0, 1 - q_1],$ $\pi^{vi} = \pi^{vn} = 0$
	$c \geq \bar{x} - \mathbb{E}[X]$	$E_2$	$\pi^{vn} = 0$
$r > \mathbb{E}[X] + c$	$0 \leq c \leq c_{min}^v(r)$	$E_1^f$	$q = 1, \pi^{vi} > 0$
	$c_{min}^v(r) < c < \bar{x} - \mathbb{E}[X]$	$E_{mix}$	$q_1, q_2 \in (0, 1), q_1 + q_2 = 1,$ $\pi^{vi} = \pi^{vn} > 0$
	$c \geq \bar{x} - \mathbb{E}[X]$	$E_2$	$q_2 = 1, \pi^{vn} = 0$

Table 2: Equilibria in the voluntary setting

the (for the auctioneer) optimal reserve prices. The comparison focuses on participation, a priori investment levels, profits of participants and auctioneer, and efficiency (expected welfare). To avoid redundancy, we restrict attention to cases in which the two settings yield different equilibria, i.e., when  $r \geq \mathbb{E}[X] + c$  and  $c > c_{min}^v(r)$ . Since all comparisons involve weak orderings, the results below naturally also apply to cases where the two settings yield the same equilibrium.

## 5.1 Comparison under reserve price in general

First, consider the regime in which both settings share the same but unspecified reserve price. Then, the following results hold. The formal analysis is provided in Appendix C.

**Proposition 3.** *For any pair of  $c$  and  $r$  with, the total participation probability in the voluntary setting is weakly higher than that in the mandatory setting, while the probability of a priori investment is reversed.*

That is, although the expected number of investors in the voluntary setting is weakly lower than in the mandatory setting, the total expected number of participants is higher in the voluntary setting due to the presence of non-investors.

**Proposition 4.** *For any pair of  $c$  and  $r$ , participants always expect weakly higher profits in the voluntary setting than in the mandatory setting.*

That is, given the same auction environment, all firms will weakly prefer the voluntary setting over the mandatory one. Although the voluntary setting offers participants an additional degree of freedom in their decision, it is uncertain that they will benefit from it. Thus, this result is nontrivial. Since the model is game-theoretic rather than decision-theoretic, having more strategic options does not necessarily imply better outcomes. For example, the introduction of an additional strategy to the players' strategy set in a game, which induces a new equilibrium similar to a prisoner's dilemma, leads to strictly lower profits for the players despite their increased degree of freedom.

In the following subsections, we introduce the auctioneer as a designer who can use the reserve price to serve its own objective: either by setting the efficient reserve price to maximize expected welfare, or by setting the optimal reserve price to maximize its own expected profit. Note that the reserve price may differ between the mandatory and voluntary settings.

## 5.2 Comparison under efficient reserve price

A second-price auction with  $r = x_0$  implements the (interim) efficient mechanism in both settings. This follows from the characteristics of the sunk costs  $c$  in the mandatory setting and for the investors in the voluntary setting. Since these costs are incurred prior to the

auction, they are not welfare-relevant at the time of the auction. Even immediately after the auction - in the voluntary setting before the winning bidder invests, if this has not already occurred - the result is still efficient given the available information. A similar statement can be found in Bergemann and Välimäki (2002).

Given  $c$  and  $x_0$ , since the efficient reserve price is the same in both setting, Proposition 3 and 4 applies under the efficient reserve price. That is, the voluntary setting yields weakly higher expected participation, weakly lower expected investment level, and weakly higher expected profits of the participants than the mandatory setting.

To compare the two settings from the auctioneer's perspective, we use expected welfare as the measure of efficiency. Welfare consists of two parts: the value the auctioneer receives from either fulfilled or unfulfilled procurement, and the associated total costs. If the auction results in a winning bidder, the value of the fulfilled procurement to the auctioneer is  $x_0$ , and the costs incurred include  $x_{winner} + c$  and the investment costs of the remaining participants. If there is no participation, the value of the unfulfilled procurement to the auctioneer is 0, and no costs are incurred. If there are participants but no bids are submitted (due to higher realized private costs than the reserve price), the value of the unfulfilled procurement to the auctioneer is 0, while all participants bear their investment costs.

**Proposition 5.** *Given  $r = x_0$ , the voluntary setting always equals or exceeds the mandatory setting in terms of expected welfare.*

On the boundary  $x_0 = r = \mathbb{E}[X] + c$ , the expected welfare is the same in the mandatory and voluntary settings, as the participation of the non-investors does not raise the welfare gain. Within  $E_{mix}$  and  $E_2$ , the difference in expected welfare between voluntary and mandatory settings increases in  $c$  if  $c \in (c_{min}^v(x_0), c_{min}^m(x_0))$ , decreases in  $c$  if  $c > c_{min}^m(x_0)$  and weakly increases in  $x_0$ . That is, the voluntary setting has its maximum advantage in expected welfare over the mandatory setting if  $x_0 \geq \bar{x}$  and  $c = c_{min}^m(x_0)$ . See Appendix D for the proof of Proposition 5.

### 5.3 Comparison under optimal reserve price

To compare the two settings under their respective optimal reserve prices, we first determine the optimal reserve price as a function of  $x_0$  given  $c$ . Consider first the locally optimal reserve price for each equilibrium with participation summarized in Table 3, under the assumption that the equilibrium type remains unchanged at the locally optimal reserve price.

In  $E_1^f$ , due to full participation, the classical optimal reserve price applies, i.e.  $r^* = x_0 - \frac{F(r^*)}{f(r^*)}$ , which is independent of  $N$  (Myerson, 1981). Define  $\gamma_m(r^*) = r^* + \frac{F(r^*)}{f(r^*)} = x_0$ . Assume that  $\frac{F(x)}{f(x)}$  is increasing in  $x$ , which is a standard assumption satisfied by most common distributions. Then  $\frac{dr^*}{dx_0} > 0$ , and the shading term  $\frac{F(r^*)}{f(r^*)}$  increases in  $x_0$  and is strictly positive

Table 3: Locally optimal reserve prices  $r^*$  in different equilibria

Equilibrium	Optimal reserve price $r^*$
$E_1^f$	$r^* = x_0 - \frac{F(r^*)}{f(r^*)}$ (Myerson, 1981)
$E_1^r$	$r^* = x_0$ (Jehiel and Lamy, 2015)
$E_2$	Any $r^* \in [\bar{x}, x_0]$ is optimal, in particular $r^* = x_0$ is efficient
$E_{mix}$	$r^* \leq x_0 - \frac{F(r^*)}{f(r^*)}$

whenever  $x_0 > \underline{x}$ . In  $E_1^r$ , due to randomized participation ( $q \in (0, 1)$ ) and zero expected payoff, Jehiel and Lamy (2015) show that the efficient reserve price is also optimal, i.e.,  $r^* = x_0$ . In the special case  $x_0 = \mathbb{E}[X] + c$ , we have  $r^* = x_0$ , since the additional participation of the non-investors has no effect on the auctioneer's expected profit. If  $x_0 > \mathbb{E}[X] + c$ : In  $E_2$ , since  $q' = 1$ , the price is not determined by the reserve price, so any reserve price between  $\bar{x}$  and  $x_0$  is optimal. In particular, the efficient reserve price  $r^* = x_0$  is also an optimal reserve price. In  $E_{mix}$ , where  $q_1 + q_2 = 1$ , full participation occurs. Nevertheless, the presence of non-investors weakens competition compared to  $E_1^f$ , leading to a lower optimal reserve price, i.e.,  $r^* \leq x_0 - \frac{F(r^*)}{f(r^*)}$  (see Appendix E). Let  $\gamma_v(r^*) = x_0$ , we have  $\frac{dr^*}{dx_0} > 0$ .

All locally optimal reserve prices are monotonically increasing in  $x_0$  and this monotonicity should also not be violated across the equilibria; otherwise more firms will be excluded at a higher  $x_0$ , which cannot be optimal.

In the next step, we consider the globally optimal reserve price  $r_m^*(x_0)$  and  $r_v^*(x_0)$  given  $c$ . According to the above argument, within an equilibrium, an increase in  $x_0$  leads to an increase in both  $r_m^*(x_0)$  and  $r_v^*(x_0)$ . At the intersection point where the equilibrium will change to a different one (from the left equilibrium to the right equilibrium in Figure 1), the globally optimal reserve price should be the maximum of the locally optimal reserve price at the intersection point (in the left equilibrium) and the locally optimal reserve price on the right side of it (in the right equilibrium) due to the monotonicity. Note that  $x_0 > (\gamma_m)^{-1}(x_0) \geq (\gamma_v)^{-1}(x_0)$ , so for each  $c$ , the direct equilibrium change from a lower  $x_0$  (left) to a higher  $x_0$  (right) always leads to a lower locally optimal reserve price. That is, the globally optimal reserve price should first remain unchanged in the locally one at the intersection point in the left equilibrium, until being caught but the locally optimal reserve price in the right equilibrium. For example, in the voluntary setting with  $c_{min}^m(\bar{x}) \leq c < \bar{x} - \mathbb{E}[X]$ , at the intersection point  $x_0 = \mathbb{E}[X] + c$  we have  $r_v^*(x_0) = \mathbb{E}[X] + c$  and for  $\mathbb{E}[X] + c < x_0 \leq \gamma_v(\bar{x})$  we

have  $r_v^*(x_0) = \max\{\mathbb{E}[X] + c, (\gamma_v)^{-1}(x_0)\}$ , that is

$$r_v^*(x_0) = \begin{cases} \mathbb{E}[X] + c, & \text{if } \mathbb{E}[X] + c < x_0 \leq \gamma_v(\mathbb{E}[X] + c) \\ (\gamma_v)^{-1}(x_0), & \text{if } \gamma_v(\mathbb{E}[X] + c) < x_0 \leq \gamma_v(\bar{x}) \end{cases}$$

as illustrated in Figure 2.<sup>3</sup>

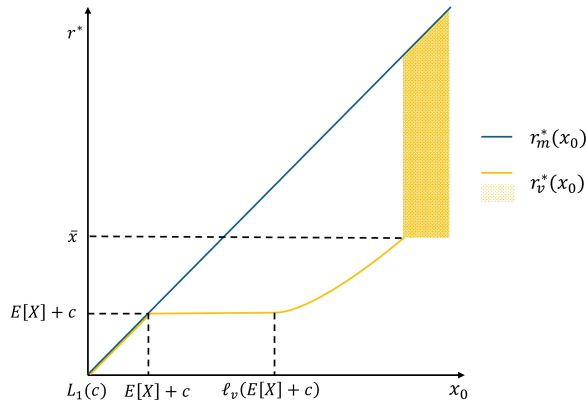


Figure 2: Exemplary optimal reserve price

Under the different optimal reserve prices derived above, Proposition 3 still holds, provided that  $q_2 = 1 - q_1$  in case  $r = \mathbb{E}[X] + c$ .

However, Proposition 4 no longer applies in general. Given  $x_0 > \mathbb{E}[X] + c$ , the different optimal reserve prices in two settings can lead to different equilibria, making participants' expected profits not directly comparable. Beside the effect that participants do not necessarily benefit from the increased degree of freedom, the auctioneer can also exploit the increased flexibility by setting different reserve prices. Thus, without further analysis, no general conclusion can be drawn on participants' expected profits.

**Proposition 6.** *Given  $r_m^*(x_0)$  and  $r_v^*(x_0)$ , for  $c \geq c_{min}^m(\bar{x})$ , participants expect weakly higher profits in the voluntary setting than in the mandatory setting.*

Proposition 6 states that given the optimal reserve price, as long as the information cost  $c$  is so high that potential bidders are excluded in the mandatory setting, participants expect weakly higher profits in the voluntary setting. We consider this case to be relevant in practice, as the main motivation for the voluntary setting lies in the fact that the sunk information cost in the mandatory setting excludes potential bidders. In the contrast, with a lower  $c$ , where  $E_1^f$  exists in the mandatory setting and  $E_{mix}$  in the voluntary setting, the mandatory setting yields higher expected profits for the participants at some  $x_0$  (see Appendix E).

<sup>3</sup>Note that  $\gamma_v(\mathbb{E}[X] + c)$  can be very large. For example, under uniform distribution,  $\gamma_v(\mathbb{E}[X] + c) > 2\mathbb{E}[X] + 2c > \bar{x}$ .

**Proposition 7.** *Given  $r_m^*(x_0)$  and  $r_v^*(x_0)$ , either the voluntary or the mandatory setting can yield a higher expected profit for the auctioneer, depending on  $c$ ,  $x_0$ , and  $F$ .*

Finally, we compare the expected profit of the auctioneer. Given the optimal reserve price, either setting can be preferred, depending on  $c$  and  $x_0$ . In general, a high information cost  $c$  tends to favor the voluntary setting. We have shown that in the equilibrium  $E_2$ , or when  $c$  approaches  $\bar{x} - \mathbb{E}[X]$  within the equilibrium  $E_{mix}$ , the auctioneer expects higher profit in the voluntary setting. Conversely, when  $c$  approaches  $c_{min}^m(r)$  within the equilibrium  $E_{mix}$ , the auctioneer expects higher profit in the mandatory setting (see Appendix E).

## 6 Model extensions and discussion

Our model contributes to the broader discussion of auction design and procurement policy by comparing two theoretical benchmarks, but it abstracts from certain practical complexities that warrant extension and discussion.

### 6.1 Risk aversion

While the model assumes risk-neutral firms, it can be extended to incorporate risk aversion, which is often a more realistic assumption in practice (e.g. Salm, 2018; Abate et al., 2022). This extension provides a more nuanced understanding of the difference between mandatory and voluntary settings and allows for a broader representation of the real world.

In a risk-averse setting, the weakly dominant bidding strategy in a second-price auction remains unchanged. Investors will truthfully bid their known private costs, and non-investors will bid their certainty equivalent of their uncertain total private costs (Ehrhart et al., 2015). Specifically, a non-investor  $i$  will bid  $CE_{X_i} + c$ , where  $CE_{X_i}$  is the certainty equivalent of the uncertain private cost  $X_i$ . The non-investor is exactly indifferent between winning the auction with this bid and not winning the auction. Since risk aversion implies  $CE_{X_i} > \mathbb{E}[X_i]$ , the non-investor demands a risk premium  $CE_{X_i} - \mathbb{E}[X_i]$ . Assuming symmetric and equally risk-averse firms, all firms have the same certainty equivalent  $CE_X$ , which shifts the relevant reserve price threshold for non-investors from  $\mathbb{E}[X] + c$  to  $CE_X + c$ .

Risk aversion has a different effect on participation in the two settings. Since non-investors can price their risk into their bids while investors cannot, risk aversion reduces the participation probability of investors. This leads to a lower participation probability in the mandatory setting and a lower share of investors in the voluntary setting, because the perceived risk of incurring high costs outweighs the potential benefits of investing in information while incurring sunk costs.

Compared to the risk-neutral setting, illustrated in Figure 1, risk aversion shifts the key threshold quantitatively, but leaves the structure of the equilibria unchanged. In particular,

the red threshold, which marks the lower bound of the reserve price where non-investors may exist in the equilibrium, shifts parallel to the right by the risk premium. The thresholds  $c_{max}(r)$ ,  $c_{min}^m(r)$ , and  $c_{min}^v(r)$  move downward.

At first glance, it may seem counterintuitive that non-investors, who face greater uncertainty while bidding, would participate relatively more than investors under risk aversion. However, the sunk cost dynamic changes the picture: investors who have already incurred these costs face a greater risk of losing their investment if they do not win the auction. This makes them more vulnerable to the risk. In contrast, non-investors can better hedge against risk through their certainty equivalents and thus improve their position relative to investors in a risk-averse setting.

In summary, risk aversion causes non-investors to participate at higher reserve prices relative to risk neutrality, but then with a relative advantage over investors due to their ability to account for risk with their bids.

## 6.2 Central a priori investment in information acquisition by the auctioneer

[Work in progress]

## 6.3 No post-auction investment costs for non-investors upon winning

[Work in progress]

## 6.4 Partial a priori investment in information acquisition

As mentioned above, it is generally unrealistic to only conduct only part of the project preparation. However, in case of multiple preparation steps or measures (such as obtaining different types of permits), it may be possible to complete a subset of project preparation, i.e., to make partial a priori investment, which can result in partial information acquisition.

The mixed investment behavior with  $q_1 + q_2 = 1$  in our model can be alternatively interpreted as a pure strategy, where  $q_1 \in [0, 1]$  denotes the investment level instead of the investment probability. That is, all firms make a partial a prior investment of size  $q_1c$  in information acquisition, and learn their true costs with probability  $q_1$ , while no information is revealed with probability  $q_2$ .

A possible extension for future research would be to explicitly model partial information revelation resulting from partial investment. In such a model, a partial investment  $q_1c$  could reveal an imprecise private cost  $\tilde{x}_i(q_1)$ , while the remaining uncertainty persists in the form of a noise term denoted by  $Z_i(q_1)$ .  $Z_i(q_1)$ , depending on the investment level  $q_1$ , is distributed on a narrower support  $[-\bar{z}(q_1), \bar{z}(q_1)]$  than the original cost distribution  $X_1$ , with  $\mathbb{E}[Z_i] = 0$ . This extension captures the intuitive idea that a higher level of investment reduces, but do

not necessarily eliminates, cost uncertainty. This corresponds to the approach of Bergemann and Välimäki (2002).

## 6.5 Limitations

A key limitation of the model is the assumption that the auction outcome is binding. In practice, however, this is often not the case – especially for non-investors in our model, who may commit to a project based on expected rather than realized costs. After winning the auction, these bidders may face unforeseen challenges such as unfavorable geological or meteorological conditions, technical complications, ecological constraints, or delays in permitting processes. These factors can significantly increase project costs or create barriers to implementation. If such ex-post costs significantly exceed initial expectations, this may lead to a decision not to proceed with the project despite the sunk costs already incurred.

This phenomenon has been observed in several procurement auctions. For example, in the German onshore wind auction, less than half of the so-called citizen energy projects, which had lower prequalifications for the auction than the projects of other energy companies, were ultimately realized (Liñeiro and Müsgens, 2023), indicating the discrepancy between auction results and project realization.

In addition to prequalification requirements, other commonly discussed instruments to mitigate the risk of non-realization include penalties and financial securities (Kreiss et al., 2017). Both instruments are intended to strengthen commitment by increasing the cost of strategic default. However, bidders incorporate potential penalties when bidding, and thus the weakly dominant strategy of truthful bidding in second-price auctions no longer holds. Moreover, the effectiveness of penalties is limited in practice. Penalties may go unpaid if the winning firm declares bankruptcy. High financial security requirements can act as entry barriers, potentially excluding financially weaker but otherwise efficient participants.

Further discussion of the measures against the risk of non-realization and their effect on equilibrium outcomes is beyond the scope of this paper, but left for future research.

## 7 Conclusion

This paper complements the existing literature by providing a comprehensive analytical comparison between mandatory and voluntary a priori investment in information acquisition in procurement auctions. We develop a theoretical model that characterizes five types of symmetric equilibria across mandatory and voluntary settings and compare them in terms of participation, efficiency (expected welfare), and expected profits.

Our main results reveal several advantages of the voluntary setting. It yields weakly higher expected participation and weakly higher expected welfare compared to the mandatory

setting. It also leads to weakly higher expected profits of participants, when the information costs are so high that in the mandatory setting potential bidders are excluded – a situation common in many real-world procurement auctions. The auctioneer may also benefit from the voluntary setting, but not to the same extent as the participants. While the auctioneer’s expected profit can favor either setting depending on the parameters, the voluntary setting shows an advantage when information costs are high. High information costs are particularly realistic in public procurement auctions, such as renewable energy auctions, where participants must invest in costly technical, environmental, and permitting assessments.

Several variations of the model are discussed. First, the model can be extended to incorporate risk aversion, a more realistic assumption about firms. In the risk-averse case, investors reduce their participation probability due to sunk costs and the risk of not winning the auction, while non-investors can price their risk through bids based on certainty equivalents. As a result, compared to the risk-neutral case, risk-averse non-investors tend to participate at higher reserve prices, but with a relative advantage over risk-averse investors due to their ability to account for uncertainty. Second, the mixed investment behavior can be alternatively interpreted as partial investment, leading to probabilistic information revelation. A potential extension for future research is to explicitly model partial information revelation, allowing for a reduction but not complete elimination of cost uncertainty.

While the analytical comparison generally favors the voluntary setting, especially under high information costs, these results are based on theoretical benchmarks. A key limitation of our model lies in the assumption that the auction outcome is binding, i.e. the auction winner will certainly realize the project. In practice, however, the risk of non-realization is a concern, particularly for non-investors, when ex-post realized costs exceed expectations. We leave this for future research.

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## Appendix

### A Proof of Proposition 1 for the mandatory setting

In the mandatory setting, a firm has to invest  $c$  and will learn the private costs  $x_i$  before participating in the auction. Thus,  $c$  is sunk costs and the participant will bid truthfully in the auction,  $\beta(x_i) = x_i$  if  $x_i < r$ , or not bid in the auction if  $x_i \geq r$ . Given the participation of the considered firm and there are  $n, n \in [0, N]$  other participants, the expected payoff of the considered firm is:

$$\pi(x_i, n) = \begin{cases} r - x_i, & \text{if } n = 0, \\ \int_{x_i}^r (t - x_i) f_{(1,n)}(t) dt + (1 - F_{(1,n)}(r))(r - x_i) = \int_{x_i}^r (1 - F(t))^n dt, & \text{if } n \geq 1. \end{cases}$$

Let  $q$  denote the symmetric participation probability of each firm, i.e., the probability that a firm will invest  $c$ . If  $c$  is sufficiently small (for example  $c = 0$ ), all firms will invest and participate (full participation), i.e.,  $q = 1$  and  $n = N - 1$ . The expected payoff of each participant is

$$\begin{aligned} \pi_i &= \int_{\underline{x}}^r \pi(s, N - 1) f(s) ds \\ &= \int_{\underline{x}}^r \left[ \int_s^r (t - s) f_{(1,N-1)}(t) dt + (1 - F_{(1,N-1)}(r))(r - s) \right] f(s) ds \\ &= \int_{\underline{x}}^r F(t)(1 - F_{(1,N-1)}(t)) dt \end{aligned}$$

which increases in  $r$  for  $r \leq \bar{x}$  and remains unchanged for  $r > \bar{x}$ . The upper bound of  $c$  in case of full participation is determined by the expected profit of zero before the investment decision, i.e., the expected payoff from the auction is equal to the sunk costs  $c$ , and thus before learning  $x_i$ :

$$\pi_i = \int_{\underline{x}}^r F(t)(1 - F_{(1,N-1)}(t)) dt = c$$

If  $c$  is still higher, then the expected profit under full participation is negative, leading to  $q = 0$ . However, an equilibrium in mixed strategy may exist. That is, each firm participates with probability  $q, q \in (0, 1)$ . When such an equilibrium exists, it is called an equilibrium with randomized participation or endogenous entry (Jehiel and Lamy, 2015). The upper bound of  $c$  for full participation is then also the lower bound of  $c$  for endogenous participation. Define this limit value as  $c_{min}^m(r)$ ,

$$c_{min}^m(r) := \int_{\underline{x}}^r F(t)(1 - F_{(1,N-1)}(t))dt, \quad (\text{A.4})$$

$$\frac{dc_{min}^m(r)}{dr} = \begin{cases} F(r)(1 - F(r))^{N-1}, & \text{if } r < \bar{x}, \\ 0, & \text{if } r \geq \bar{x}. \end{cases}$$

Give the participation probability  $q \in (0, 1)$ , an equilibrium exists if all firms are indifferent between participating and not participating, i.e., the expected payoff is  $c$ :

$$\begin{aligned} \pi^m &= \sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^r \pi(s, n) f(s) ds = c \\ \Leftrightarrow c &= P(0, q) \int_{\underline{x}}^r (r - s) f(s) ds + \sum_{n=1}^{N-1} P(n, q) \int_{\underline{x}}^r \left[ \int_s^r (t - s) f_{(1,n)}(t) dt + (1 - F_{(1,n)}(r))(r - s) \right] f(s) ds \\ \Leftrightarrow c &= P(0, q) \int_{\underline{x}}^r F(s) ds + \sum_{n=1}^{N-1} P(n, q) \int_{\underline{x}}^r F(t)(1 - F_{(1,n)}(t)) dt \\ \Leftrightarrow c &= \sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^r F(t)(1 - F(t))^n dt \end{aligned} \quad (\text{A.5})$$

where  $P(n, q) = \binom{N-1}{n} q^n (1 - q)^{N-n-1}$ .

Firms do not have an incentive to deviate from this equilibrium, because no investment means no participation, which also leads to a zero expected profit.

To prove the uniqueness of  $q$  in this equilibrium, we first need to prove Lemma 2.

*Proof of Lemma 2.* We have  $\frac{\partial \pi^m}{\partial r} = \sum_{n=0}^{N-1} P(n, q) \pi(r, n) f(r) > 0$ .

To proof that  $\frac{\partial \pi^m}{\partial q} < 0$ , we first show that  $\pi(s, n)$  decreases in  $n$ :

$$\pi(s, n) = \begin{cases} r - s, & \text{if } n = 0, \\ \int_s^r (1 - F(t))^n dt, & \text{if } n \geq 1. \end{cases}$$

As  $\int_s^r (1 - F(t))^n dt$  decreases in  $n$  and  $\int_s^r (1 - F(t)) dt < r - s$ ,  $\pi(s, n)$  decreases in  $n$ . Thus,

$\int_{\underline{x}}^r \pi(s, n)f(s)ds$  also decreases in  $n$ . Consider the expected payoff in  $q$

$$\pi^m(q) = \sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^r \pi(s, n)f(s)ds.$$

For  $n = 0$ ,  $P(0, q) = (1 - q)^{N-1}$  decreases in  $q, \forall q \in (0, 1]$ . For  $n \in [1, N - 2]$ ,  $P(n, q) = \binom{N-1}{n} q^n (1 - q)^{N-n-1}$ ,  $\frac{\partial P(n, q)}{\partial q} = \binom{N-1}{n} q^{n-1} (1 - q)^{N-n-2} [n - (N - 1)q]$ , so  $P(n, q)$  increases in  $q$  if  $q \in [0, \frac{n}{N-1})$  and decreases in  $q$  if  $q \in (\frac{n}{N-1}, 1], \forall q \in (0, 1]$ ; For  $n = N - 1, P(N - 1, q) = q^{N-1}$  increases in  $q, \forall q \in (0, 1]$ .

Consider  $q$  and  $q + \Delta q$ , where  $\Delta q > 0$  is negligibly small. As a result, if  $q$  and  $q + \Delta q$  belongs to  $[\frac{i-1}{N-1}, \frac{i}{N-1}], \forall i \in [1, N - 1]$ ,  $P(n, q)$  with  $n \in [0, i - 1]$  decreases in  $q$ , and  $P(n, q)$  with  $n \in [i, N - 1]$  increases in  $q$ . So we have  $P(n, q + \Delta q) - P(n, q) < 0$  if  $n \in [0, i - 1]$  and  $P(n, q + \Delta q) - P(n, q) > 0$  if  $n \in [i, N - 1]$ . As  $\sum_{n=0}^{N-1} P(n, q) = 1 = \sum_{n=0}^{N-1} P(n, q + \Delta q)$ , it holds that

$$\sum_{n=i}^{N-1} [P(n, q + \Delta q) - P(n, q)] = - \sum_{n=0}^{i-1} [P(n, q + \Delta q) - P(n, q)] = \Delta P > 0$$

Now, compare  $\pi^m(q + \Delta q)$  with  $\pi^m(q)$  by calculating  $\Delta \pi^m = \pi^m(q + \Delta q) - \pi^m(q)$ ,

$$\begin{aligned} & \Delta \pi^m \\ &= \sum_{n=0}^{N-1} P(n, q + \Delta q) \int_{\underline{x}}^r \pi(s, n)f(s)ds - \sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^r \pi(s, n)f(s)ds \\ &= \sum_{n=0}^{N-1} [P(n, q + \Delta q) - P(n, q)] \int_{\underline{x}}^r \pi(s, n)f(s)ds \\ &= \sum_{n=0}^{i-1} [P(n, q + \Delta q) - P(n, q)] \int_{\underline{x}}^r \pi(s, n)f(s)ds + \sum_{n=i}^{N-1} [P(n, q + \Delta q) - P(n, q)] \int_{\underline{x}}^r \pi(s, n)f(s)ds \\ &\leq \sum_{n=0}^{i-1} [P(n, q + \Delta q) - P(n, q)] \int_{\underline{x}}^r \pi(s, i - 1)f(s)ds + \sum_{n=i}^{N-1} [P(n, q + \Delta q) - P(n, q)] \int_{\underline{x}}^r \pi(s, i)f(s)ds \\ &= - \Delta P \int_{\underline{x}}^r \pi(s, i - 1)f(s)ds + \Delta P \int_{\underline{x}}^r \pi(s, i)f(s)ds \\ &= \Delta P \int_{\underline{x}}^r (\pi(s, i) - \pi(s, i - 1))f(s)ds \\ &< 0. \end{aligned}$$

Similarly, as  $N$  increases, the probability density shifts towards higher  $n$ , leading to a decrease in  $\pi^m$ .  $\square$

As  $\frac{\partial \pi^m}{\partial q} < 0$ , there is a unique  $q$  such that  $\pi^m = 0$ . If  $q \rightarrow 1, c \rightarrow c_{min}^m(r)$ . But  $c$  cannot be arbitrarily large. If  $c$  increases,  $q$  must decrease so that  $\pi^m = 0$  holds. The upper bound on  $c$

under randomized participation is that it is just worthwhile for one single firm to participate:

$$\int_{\underline{x}}^r (r-s)f(s)ds = \int_{\underline{x}}^r F(s)ds = c := c_{max}(r) \quad (\text{A.6})$$

As  $c$  approaches  $c_{max}(r)$ ,  $q \rightarrow 0$ ,  $P(0, q) \rightarrow 1$  and  $\sum_{n=1}^{N-1} P(n, q) \rightarrow 0$ , so that the equilibrium condition (A.5) could just still be fulfilled. That is,  $c_{max}$  is an open cap for  $c$  so that an equilibrium under randomized participation exists. Comparing Equation (A.6) with (A.4),  $\forall r > \underline{x}$ , it is obvious that  $c_{max} > c_{min}^m(r) > 0$ , which proves the first part of Lemma 1.

*Proof of Lemma 3.* Let  $\frac{\mathbb{E}[Y]-\underline{y}}{\mathbb{E}[X]-\underline{x}} = \frac{\bar{y}-\mathbb{E}[Y]}{\bar{x}-\mathbb{E}[X]} = \alpha \in (0, 1)$ , since  $\mathbb{E}[X] = \mathbb{E}[Y]$  we have  $\underline{y} = (1-\alpha)\mathbb{E}[X] + \alpha\underline{x}$ ,  $\bar{y} = \alpha\bar{x} + (1-\alpha)\mathbb{E}[X]$ , and  $G(s) = F(\frac{s-\underline{y}}{\alpha} + \underline{x})$ .

Following Lemma 2, given  $N$ ,  $c$ , and  $r$ , to prove that the participation probability is higher with  $F$  than with  $G$ , it is sufficient to show that  $\forall n, 0 \leq n \leq N-1$ ,

$$\begin{aligned} & \int_{\underline{x}}^r F(t)(1-F(t))^n dt \geq \int_{\underline{y}}^r G(s)(1-G(s))^n ds \\ \Leftrightarrow & \int_{\underline{x}}^r F(t)(1-F(t))^n dt \geq \int_{\underline{y}}^r F(\frac{s-\underline{y}}{\alpha} + \underline{x})(1-F(\frac{s-\underline{y}}{\alpha} + \underline{x}))^n ds \\ \Leftrightarrow & \int_{\underline{x}}^r F(t)(1-F(t))^n dt \geq \alpha \int_{\underline{x}}^{\frac{r-(1-\alpha)\mathbb{E}[X]}{\alpha}} F(t)(1-F(t))^n dt \end{aligned} \quad (\text{A.7})$$

If  $r \leq \mathbb{E}[X]$ , then  $\frac{r-(1-\alpha)\mathbb{E}[X]}{\alpha} \leq r$  and Inequality A.7 applies. Otherwise, the right side of Inequality A.7 has its maximum if  $r \geq \bar{y}$  while the left side increases in  $r$  for all  $r \leq \bar{x}$ . Since  $\bar{x} > \bar{y}$ , a sufficient condition for Inequality A.7 is

$$\begin{aligned} & \int_{\underline{x}}^{\bar{y}} F(t)(1-F(t))^n dt \geq \alpha \int_{\underline{x}}^{\frac{\bar{y}-(1-\alpha)\mathbb{E}[X]}{\alpha}} F(t)(1-F(t))^n dt \\ \Leftrightarrow & (1-\alpha) \int_{\underline{x}}^{\bar{y}} F(t)(1-F(t))^n dt \geq \alpha \int_{\bar{y}}^{\bar{x}} F(t)(1-F(t))^n dt \end{aligned}$$

If  $n = 0$ , we have

$$\begin{aligned}
& (1 - \alpha) \int_{\underline{x}}^{\bar{y}} F(t) dt \\
&= (1 - \alpha) \int_{\underline{x}}^{\bar{x}} F(t) dt - (1 - \alpha) \int_{\bar{y}}^{\bar{x}} F(t) dt \\
&= (1 - \alpha)(\bar{x} - \mathbb{E}[X]) - (1 - \alpha) \int_{\bar{y}}^{\bar{x}} F(t) dt \\
&= (\bar{x} - \bar{y}) - \int_{\bar{y}}^{\bar{x}} F(t) dt + \alpha \int_{\bar{y}}^{\bar{x}} F(t) dt \\
&> \alpha \int_{\bar{y}}^{\bar{x}} F(t) dt
\end{aligned}$$

If  $n \geq 1$ , since  $F(t)(1 - F(t))^n$  increases if  $F(t) < \frac{1}{n+1}$  and decreases if  $F(t) > \frac{1}{n+1}$ ,  $F(t)(1 - F(t))^n$  decreases in  $t$  for  $t \geq \mathbb{E}[X]$ . Thus we have

$$\begin{aligned}
& (1 - \alpha) \int_{\underline{x}}^{\bar{y}} F(t)(1 - F(t))^n dt \\
&> (1 - \alpha) \int_{\mathbb{E}[X]}^{\bar{y}} F(t)(1 - F(t))^n dt \\
&\geq (1 - \alpha)(\bar{y} - \mathbb{E}[X])F(\bar{y})(1 - F(\bar{y}))^n \\
&= \alpha(\bar{x} - \bar{y})F(\bar{y})(1 - F(\bar{y}))^n \\
&> \alpha \int_{\bar{y}}^{\bar{x}} F(t)(1 - F(t))^n dt
\end{aligned}$$

□

## B Proof of Proposition 2 for the voluntary setting

In the voluntary setting, each participant can either invest  $c$  and then learn the private costs  $x_i$ , or not invest and does not acquire any information other than the distribution.

### B.1 Case $r = \mathbb{E}[X] + c$

In this case, all four equilibria with participation can exist. Note that the non-investors always have an expected profit of zero  $\pi^{vn} = 0$ .

**Equilibrium  $E_1^f$**  For  $c \in [0, c_{min}^m(r)]$ , the equilibrium  $E_1^f$  exists because the participants expect a positive profit and therefore have no incentive to deviate from an investor to a non-investor, leading to an expected profit of zero.

**Equilibrium  $E_1^r$**  Since  $\forall r \geq \bar{x}, c = r - \mathbb{E}[X] = c_{max}(r)$ , and  $c_{max}(r)$  is an open cap for  $E_1^r$ ,  $E_1^r$  exists for  $c \in (c_{min}^m(r), \bar{x} - \mathbb{E}[X])$ . The expected profit is zero, and  $q$  is the same as in the mandatory setting given  $r = \mathbb{E}[X] + c$ , which is determined by

$$\pi^{vi} = \sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^n dt = c. \quad (\text{B.8})$$

Analogous to  $E_1^f$ , the participants do not have an incentive to deviate.

**Equilibrium  $E_2$**  All firms choose not to invest but participate with  $q' \in (0, 1]$  and  $\pi^{vn} = 0$  is an equilibrium if the one-sided deviation from non-investor to investor does not worth, i.e.,

$$\int_{\underline{x}}^{\mathbb{E}[X]+c} (\mathbb{E}[X] + c - s)f(s)ds - c \leq 0 \Leftrightarrow c \geq \bar{x} - \mathbb{E}[X]. \quad (\text{B.9})$$

This is because the difference on the left side decreases in  $c$  for  $c \in [0, \bar{x} - \mathbb{E}[X]]$  and remains unchanged for  $c > \bar{x} - \mathbb{E}[X]$ . At the point  $c = \bar{x} - \mathbb{E}[X]$ , the difference is zero. Note that  $\bar{x} - \mathbb{E}[X] > c_{min}^m(r)$ :

$$c_{min}^m(r) = \int_{\underline{x}}^r F(t)(1-F(t))^{N-1} dt \leq \int_{\underline{x}}^{\bar{x}} F(t)(1-F(t))^{N-1} dt < \int_{\underline{x}}^{\bar{x}} F(t) dt = \bar{x} - \mathbb{E}[X].$$

As a result, for  $c \geq \bar{x} - \mathbb{E}[X]$ , on the line  $r = \mathbb{E}[X] + c$  as well as  $c_{max}(\mathbb{E}[X] + c)$ ,  $E_2$  exists in the voluntary setting, while no equilibrium exists in the mandatory setting.

**Equilibrium  $E_{mix}$**  All firms choose to invest  $c$  with the probability  $q_1$ , not to invest but participate with the probability  $q_2$ , and not participate with the probability  $1 - q_1 - q_2$ , where  $q_1 \in (0, 1), q_2 \in (0, 1), q_1 + q_2 \leq 1$ . The equilibrium  $E_{mix}$  exists if the expected profit of the investor is equal to the expected profit of the non-investor  $\pi^{vi} - c = \pi^{vn}$ , given that other firms follow the strategy above. As  $\pi^{vn} = 0$ ,  $E_{mix}$  requires only  $\pi^{vi} = c$ .

Let  $n_1$  denote the number of investors and  $n_2$  the number of non-investors, the corresponding probability given  $q_1$  and  $q_2$  is

$$\begin{aligned} P(n_1, n_2, q_1, q_2) &= \binom{N-1}{n_1+n_2} \binom{n_1+n_2}{n_1} q_1^{n_1} q_2^{n_2} (1-q_1-q_2)^{N-n_1-n_2-1} \\ &= \binom{N-1}{n_1} \binom{N-n_1-1}{n_2} q_1^{n_1} q_2^{n_2} (1-q_1-q_2)^{N-n_1-n_2-1} \end{aligned}$$

Then we have

$$\begin{aligned}
& \pi^{vi} \\
= & \sum_{n_2 \in [0, N-1]} P(0, n_2, q_1, q_2) \int_{\underline{x}}^{\mathbb{E}[X]+c} (\mathbb{E}[X] + c - s) f(s) ds \\
& + \sum_{\substack{n_1 \in [1, N-1] \\ n_2 \in [0, N-1-n_1]}} P(n_1, n_2, q_1, q_2) \int_{\underline{x}}^{\mathbb{E}[X]+c} \left[ \int_s^{\mathbb{E}[X]+c} (t-s) f_{(1, n_1)}(t) dt + (1 - F_{(1, n_1)}(\mathbb{E}[X] + c)) (\mathbb{E}[X] + c - s) \right] f(s) ds \\
= & \sum_{n_2 \in [0, N-1]} P(0, n_2, q_1, q_2) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(s) ds + \sum_{n_1=1}^{N-1} \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t) (1 - F_{(1, n_1)}(t)) dt \sum_{n_2=0}^{N-1-n_1} P(n_1, n_2, q_1, q_2)
\end{aligned}$$

where

$$\sum_{n_2 \in [0, N-1]} P(0, n_2, q_1, q_2) = \sum_{n_2 \in [0, N-1]} \binom{N-1}{n_2} q_2^{n_2} (1 - q_1 - q_2)^{N-n_2-1} = (1 - q_1)^{N-1} = P(0, q_1)$$

and

$$\begin{aligned}
\sum_{n_2=0}^{N-1-n_1} P(n_1, n_2, q_1, q_2) &= \sum_{n_2=0}^{N-1-n_1} \binom{N-1}{n_1} \binom{N-n_1-1}{n_2} q_1^{n_1} q_2^{n_2} (1 - q_1 - q_2)^{N-n_1-n_2-1} \\
&= \binom{N-1}{n_1} q_1^{n_1} \sum_{n_2=0}^{N-1-n_1} \binom{N-n_1-1}{n_2} q_2^{n_2} (1 - q_1 - q_2)^{N-n_1-n_2-1} \\
&= \binom{N-1}{n_1} q_1^{n_1} (1 - q_1)^{N-n_1-1} \\
&= P(n_1, q_1)
\end{aligned}$$

Thus,

$$\begin{aligned}
\pi^{vi} &= c \\
\Leftrightarrow c &= \sum_{n=0}^{N-1} P(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t) (1 - F(t))^n dt
\end{aligned}$$

This equilibrium condition corresponds to Equation (B.8) in  $E_1^r$  and to Equation (A.5) with  $r = \mathbb{E}[X] + c$  in the mandatory case. We have shown that there exists a unique  $q$  that fulfills the condition. Therefore, here we have  $q_1 = q$ .

This means that for each  $c \in (c_{min}^m(r), \bar{x} - \mathbb{E}[X])$  where  $E_1^r$  with  $q$  exists, there exists an  $E_{mix}$  with  $q_1 = q$  and any  $q_2 \in (0, 1 - q]$ . The reason is that  $q_2$  has no additional influence on the expected payoff of the investors as the reserve price  $r$ . This is a special case exactly because of the equality  $r = \mathbb{E}[X] + c$ .

## B.2 Case $r > \mathbb{E}[X] + c$

In this case, several equilibria can exist.

**Equilibrium  $E_2$**  If all firms choosing to participate ( $q' = 1$ ), then  $\pi^{vn} = 0$ . This conducts an equilibrium if the following condition is fulfilled (see also Inequation (B.9)):

$$\begin{aligned}\pi^{vi} - c &= \int_{\underline{x}}^{\mathbb{E}[X]+c} (\mathbb{E}[X] + c - s)f(s)ds - c \leq \pi^{vn} = 0 \\ \Leftrightarrow c &\geq \bar{x} - \mathbb{E}[X]\end{aligned}$$

Note that if  $q' \in (0, 1)$ , the expected payoff will be strictly positive so that all firms have an incentive to participate, which increases the participation probability to 1, which again lowers the expected payoff to zero. That is, the equilibrium  $E_2$  with  $q' = 1$  exists for  $c \geq \bar{x} - \mathbb{E}[X]$  and  $\pi^{vn} = 0$ .

**Equilibrium  $E_{mix}$**  Note that a non-investor expects a strictly positive profit unless all other potential bidders also participate but do not invest. Thus,  $E_1^r$  (with  $q \in (0, 1)$  and  $\pi^{vi} = 0$ ) can not exist, because then each bidder has the incentive to deviate from a investor to a non-investor. However, there may exist  $E_{mix}$  with strictly positive payoff, if  $\pi^{vi} - c = \pi^{vn}$ , where  $q_1 \in (0, 1), q_2 \in (0, 1), q_1 + q_2 = 1, n_1 + n_2 = N - 1$ .

$$P(n_1, n_2, q_1, q_2) = \binom{N-1}{n_1} q_1^{n_1} (1 - q_1)^{N-n_1-1} = P(n_1, q_1) \quad (\text{B.10})$$

$$\begin{aligned}\pi^{vi} &= P(N-1, q_1) \int_{\underline{x}}^r \left[ \int_s^r (t-s)f_{(1, N-1)}(t)dt + (1 - F_{(1, N-1)}(r))(r-s) \right] f(s)ds \\ &\quad + P(0, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} (\mathbb{E}[X] + c - s)f(s)ds \\ &\quad + \sum_{n_1=1}^{N-2} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} \left[ \int_s^{\mathbb{E}[X]+c} (t-s)f_{(1, n_1)}(t)dt + (1 - F_{(1, n_1)}(\mathbb{E}[X] + c))(\mathbb{E}[X] + c - s) \right] f(s)ds \\ &= P(N-1, q_1) \int_{\underline{x}}^r F(t)(1 - F_{(1, N-1)}(t))dt + P(0, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(s)ds \\ &\quad + \sum_{n_1=1}^{N-2} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1 - F_{(1, n_1)}(t))dt\end{aligned}$$

$$\begin{aligned}\pi^{vn} &= P(N-1, q_1) \left[ \int_{\mathbb{E}[X]+c}^r (t - \mathbb{E}[X] - c) f_{(1, N-1)}(t) dt + (1 - F_{(1, N-1)}(r))(r - \mathbb{E}[X] - c) \right] \\ &= P(N-1, q_1) \int_{\mathbb{E}[X]+c}^r (1 - F_{(1, N-1)}(t)) dt > 0\end{aligned}$$

$$\begin{aligned}\Delta\pi^v &= \pi^{vi} - c - \pi^{vn} = 0 \\ \Leftrightarrow 0 &= P(N-1, q_1) \int_{\underline{x}}^r F(t)(1 - F_{(1, N-1)}(t)) dt + P(0, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(s) ds \\ &\quad + \sum_{n_1=1}^{N-2} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1 - F_{(1, n_1)}(t)) dt - c - P(N-1, q_1) \int_{\mathbb{E}[X]+c}^r (1 - F_{(1, N-1)}(t)) dt \\ \Leftrightarrow \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1 - F(t))^{n_1} dt - P(N-1, q_1) \int_{\mathbb{E}[X]+c}^r (1 - F(t))^N dt - c &= 0 \quad (\text{B.11})\end{aligned}$$

$\Delta\pi^v > 0$  means that investment is preferred to non-investment.  $\Delta\pi^v < 0$  means that non-investment is preferred to investment. Thus, a higher  $\Delta\pi^v$  speaks in favor of investment. Since  $\pi^{vi} > 0 = \pi^m$ , given the same  $c$  and  $r$ , the participation probability  $q_1$  must be smaller than the participation probability  $q$  in the mandatory setting.

*Proof of Lemma 4.* We have  $\frac{\partial \Delta\pi^v}{\partial r} = -P(N-1, q_1)(1 - F(r))^N < 0$  and

$$\begin{aligned}&\frac{\partial \Delta\pi^v}{\partial c} \\ &= \sum_{n_1=0}^{N-1} P(n_1, q_1) F(\mathbb{E}[X] + c)(1 - F(\mathbb{E}[X] + c))^{n_1} dt + P(N-1, q_1)(1 - F(\mathbb{E}[X] + c))^N dt - 1 \\ &= \sum_{n_1=0}^{N-2} P(n_1, q_1) F(\mathbb{E}[X] + c)(1 - F(\mathbb{E}[X] + c))^{n_1} dt + P(N-1, q_1)(1 - F(\mathbb{E}[X] + c))^{N-1} - \sum_{n_1=0}^{N-1} P(n_1, q_1) \\ &< 0.\end{aligned}$$

Analogous to the proof of Lemma 2, the first term in Equation (B.11) decreases in  $q_1$ , and the second term decreases in  $q_1$  obviously, thus,  $\frac{\partial \Delta\pi^v}{\partial q_1} < 0$ . Similarly, as  $N$  increases, the probability density shifts towards higher  $n_1$ , leading to a decrease in  $\Delta\pi^v$ .

Therefore,  $\frac{dq_1}{dr} < 0$ ,  $\frac{dq_1}{dc} < 0$ , and  $\frac{dq_1}{dN} < 0$ . That is, the higher  $r$ ,  $c$ , or  $N$  is, the lower is  $\Delta\pi^v$  and thus in favor of non-investment and a lower  $q_1$ .  $\square$

As  $\frac{\partial \Delta\pi^v}{\partial q_1} < 0$ , there exists a unique  $q_1$  that satisfies Equation (B.11). As a result, the equilibrium  $E_{mix}$  yields  $q_1$  determined by Equation (B.11),  $q_2 = 1 - q_1$  for  $c \in (c_{min}^v(r), \bar{x} - \mathbb{E}[X])$ , and  $\pi^{vn} = \pi^{vi} > 0$ .

Due to the monotonicity of  $\Delta\pi^v$  in  $q_1$ , the upper and lower limits of  $c$  can be determined by  $q \Rightarrow 0$  and  $q \Rightarrow 1$ , respectively. If  $q_1 \rightarrow 0$ ,  $c \rightarrow \bar{x} - \mathbb{E}[X] = c_{max}(r)$ ; If  $q_1 \rightarrow 1$ ,

$$c \rightarrow \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1 - F_{(1,N-1)}(t))dt - \int_{\mathbb{E}[X]+c}^r (1 - F_{(1,N)}(t))dt \quad (\text{B.12})$$

$$= \int_{\underline{x}}^r F(t)(1 - F_{(1,N-1)}(t))dt - \int_{\mathbb{E}[X]+c}^r (1 - F_{(1,N-1)}(t))dt := c_{min}^v(r) \quad (\text{B.13})$$

*Proof of Lemma 1.* It is obvious that  $c_{min}^v(r) < c_{min}^m(r) < c_{max}(r)$  for  $r > \mathbb{E}[X] + c$ , and  $c_{min}^v(\mathbb{E}[X] + c) = c_{min}^m(\mathbb{E}[X] + c)$ . Since  $\frac{\partial c_{min}^v(r)}{\partial r} = -(1 - F(r))^N \leq 0$ ,  $c_{min}^v(r)$  decreases in  $r$  for  $r \leq \bar{x}$  and is constant for  $r > \bar{x}$ , so  $c_{min}^v(r)$  has its minimum at  $r = \bar{x}$ . To prove  $c_{min}^v(r) > 0$ , it is sufficient to show  $c_{min}^v(\bar{x}) > 0$ . Let

$$I(c) = \int_{\underline{x}}^{\bar{x}} F(t)(1 - F_{(1,N-1)}(t))dt - \int_{\mathbb{E}[X]+c}^{\bar{x}} (1 - F_{(1,N-1)}(t))dt - c.$$

Since  $\frac{dI(c)}{dc} = (1 - F(\mathbb{E}[X] + c))^{N-1} - 1 < 0$ ,  $I(c)$  decreases in  $c$ . We have  $I(c_{min}^m(\bar{x})) = -\int_{\mathbb{E}[X]+c_{min}^m(\bar{x})}^{\bar{x}} (1 - F_{(1,N-1)}(t))dt < 0$  and  $I(0) = \int_{\underline{x}}^{\bar{x}} F(t)(1 - F_{(1,N-1)}(t))dt - \int_{\mathbb{E}[X]}^{\bar{x}} (1 - F_{(1,N-1)}(t))dt$ . If  $I(0) > 0$ , we can conclude that  $0 < c_{min}^v(\bar{x}) < c_{min}^m(\bar{x})$ .

The first term in  $I(0)$ ,  $\int_{\underline{x}}^{\bar{x}} F(t)(1 - F_{(1,N-1)}(t))dt$ , corresponds to the a priori (i.e., before bidders learn their private costs) expected profit of a bidder in a second-price auction without a reserve price, assuming all bidders bid their true private costs after learning them. The second term,  $\int_{\mathbb{E}[X]}^{\bar{x}} (1 - F_{(1,N-1)}(t))dt$ , corresponds to the a priori expected profit of a bidder who bids a fixed value equal to  $\mathbb{E}[X]$  in a second-price auction without a reserve price, while all other bidders bid their true private costs after learning them. Since truthful bidding is the unique weakly dominant strategy in a second-price auction without a reserve price, the expected profit from bidding one's true private cost is strictly higher than that from bidding  $\mathbb{E}[X]$ , regardless of the actual private cost realization. As a result, the a priori expected profit is also strictly higher in the truthful bidding case, implying that  $I(0)$  is strictly positive.  $\square$

This result is plausible because the advantage of investors over non-investors is strictly

positive at  $c = 0$ . Besides, we have

$$\begin{aligned}
\frac{\partial I(0)}{\partial N} &= \int_{\underline{x}}^{\bar{x}} F(t)(1 - F_{(1,N-1)}(t)) \ln(1 - F(t)) dt - \int_{\mathbb{E}[X]}^{\bar{x}} (1 - F_{(1,N-1)}(t)) \ln(1 - F(t)) dt \\
&= \int_{\underline{x}}^{\mathbb{E}[X]} F(t)(1 - F(t))^{N-1} \ln(1 - F(t)) dt - \int_{\mathbb{E}[X]}^{\bar{x}} (1 - F(t))^N \ln(1 - F(t)) dt \\
&< \int_{\underline{x}}^{\mathbb{E}[X]} F(t)(1 - F(t))^{N-1} \ln(1 - F(\mathbb{E}[X])) dt - \int_{\mathbb{E}[X]}^{\bar{x}} (1 - F(t))^N \ln(1 - F(\mathbb{E}[X])) dt \\
&= \ln\left(\frac{1}{2}\right) I(0) < 0
\end{aligned}$$

The higher  $N$ , the lower is  $I(0)$ , and thus the lower is  $c_{min}^v(\bar{x})$ . In other words, with more competition, full participation requires a lower  $c$ . For instance, under uniform distribution, we have  $F(x) = x$ ,  $I(0) = \frac{1}{N}(\frac{1}{N+1} - (\frac{1}{2})^N)$ , which is strictly positive for  $N \geq 2$  and decreases in  $N$ .

*Proof of Lemma 5.* Let  $\frac{\mathbb{E}[Y]-y}{\mathbb{E}[X]-\underline{x}} = \frac{\bar{y}-\mathbb{E}[Y]}{\bar{x}-\mathbb{E}[X]} = \alpha \in (0, 1)$ , since  $\mathbb{E}[X] = \mathbb{E}[Y]$  we have  $\underline{y} = (1 - \alpha)\mathbb{E}[X] + \alpha\underline{x}$ ,  $\bar{y} = \alpha\bar{x} + (1 - \alpha)\mathbb{E}[X]$ , and  $G(s) = F(\frac{s-\underline{y}}{\alpha} + \underline{x})$ .

If  $\mathbb{E}[X] + c \geq \bar{y}$ , there will only be non-investors in the equilibrium, so the investment probability is zero with  $G$ , while  $q_1 > 0$  with  $F$ .

If  $\mathbb{E}[X] + c \geq \bar{y}$ , with both  $F$  and  $G$ , the investment probability is positive. Following Lemma 2 and 4, given  $N$ ,  $c$ , and  $r$ , to prove that under  $G$  the probability of participation corresponding  $q_1$  is less than  $q_1$ , it is sufficient to show that

$$\int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1 - F(t))^{N-1} dt - \int_{\mathbb{E}[X]+c}^r (1 - F(t))^N dt \geq \int_{\underline{y}}^{\mathbb{E}[X]+c} G(s)(1 - G(s))^{N-1} ds - \int_{\mathbb{E}[X]+c}^r (1 - G(s))^N ds$$

Analogous to Lemma 3, since  $r > \mathbb{E}[X] + c$  and  $\frac{\partial \Delta \pi^v}{\partial r} < 0$ , a sufficient condition is to show that the above inequality applies if  $r = \bar{x}$ , i.e.,

$$\begin{aligned}
& \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{N-1} dt - \int_{\mathbb{E}[X]+c}^{\bar{x}} (1-F(t))^N dt \geq \int_{\underline{y}}^{\mathbb{E}[X]+c} G(s)(1-G(s))^{N-1} ds - \int_{\mathbb{E}[X]+c}^{\bar{x}} (1-G(s))^N ds \\
\Leftrightarrow & \int_{\underline{x}}^{\bar{x}} F(t)(1-F(t))^{N-1} dt - \int_{\mathbb{E}[X]+c}^{\bar{x}} (1-F(t))^{N-1} dt \geq \int_{\underline{y}}^{\bar{y}} G(s)(1-G(s))^{N-1} ds - \int_{\mathbb{E}[X]+c}^{\bar{y}} (1-G(s))^{N-1} ds \\
\Leftrightarrow & \int_{\underline{x}}^{\bar{x}} F(t)(1-F(t))^{N-1} dt - \int_{\mathbb{E}[X]+c}^{\bar{x}} (1-F(t))^{N-1} dt \geq \alpha \int_{\underline{x}}^{\bar{x}} F(t)(1-F(t))^{N-1} dt - \alpha \int_{\mathbb{E}[X]+\frac{c}{\alpha}}^{\bar{x}} (1-F(t))^{N-1} dt \\
\Leftrightarrow & (1-\alpha) \left[ \int_{\underline{x}}^{\bar{x}} F(t)(1-F(t))^{N-1} dt - \int_{\mathbb{E}[X]+c_{min}^v(\bar{x})}^{\bar{x}} (1-F(t))^{N-1} dt \right] + \int_{\mathbb{E}[X]+c_{min}^v(\bar{x})}^{\mathbb{E}[X]+c} (1-F(t))^{N-1} dt \\
& - \alpha \int_{\mathbb{E}[X]+\frac{c}{\alpha}}^{\mathbb{E}[X]+c_{min}^v(\bar{x})} (1-F(t))^{N-1} dt \geq 0 \\
\Leftrightarrow & (1-\alpha)c_{min}^v(\bar{x}) + (1-\alpha) \int_{\mathbb{E}[X]+c_{min}^v(\bar{x})}^{\mathbb{E}[X]+c} (1-F(t))^{N-1} dt - \alpha \int_{\mathbb{E}[X]+c}^{\mathbb{E}[X]+\frac{c}{\alpha}} (1-F(t))^{N-1} dt \geq 0 \\
\Leftarrow & (1-\alpha)c_{min}^v(\bar{x}) + (1-\alpha)(c-c_{min}^v(\bar{x}))(1-F(\mathbb{E}[X]+c))^{N-1} - \alpha\left(\frac{c}{\alpha}-c\right)(1-F(\mathbb{E}[X]+c))^{N-1} \geq 0 \\
\Leftrightarrow & (1-\alpha)c_{min}^v(\bar{x})(1-(1-F(\mathbb{E}[X]+c))^{N-1}) \geq 0
\end{aligned}$$

□

**Equilibrium**  $E_1^f$  As  $c$  approaches zero, the advantage of investors against investors is so large that all firms will participate and invest, so  $E_1^f$  exists. The upper bound of  $c$  for  $E_1^f$  is determined by the scenario that if all  $N-1$  other firms participate and invest, the considered firm will participate and is indifferent between investing and not investing.

$$\begin{aligned}
\pi^{vi} &= \int_{\underline{x}}^r \left[ \int_s^r (t-s)f_{(1,N-1)}(t)dt + (1-F_{(1,N-1)}(r))(r-s) \right] f(s)ds \\
&= \int_{\underline{x}}^r F(t)(1-F_{(1,N-1)}(t))dt
\end{aligned}$$

$$\begin{aligned}
\pi^{vn} &= \int_{\mathbb{E}[X]+c}^r (t-\mathbb{E}[X]-c)f_{(1,N-1)}(t)dt + (1-F_{(1,N-1)}(r))(r-\mathbb{E}[X]-c) \\
&= \int_{\mathbb{E}[X]+c}^r (1-F_{(1,N-1)}(t))dt
\end{aligned}$$

$$\pi^{vi} = \pi^{vn} - c \Leftrightarrow c = \int_{\underline{x}}^r F(t)(1-F_{(1,N-1)}(t))dt - \int_{\mathbb{E}[X]+c}^r (1-F_{(1,N-1)}(t))dt = c_{min}^v(r)$$

As a result, the equilibrium  $E_1^f$  exists for  $c \in [0, c_{min}^v(r)]$ .

## C Proof for the comparison under reserve price in general

*Proof of Proposition 3.* To compare the expected participation and investment level in both settings, the following three cases are considered:

In case  $r = \mathbb{E}[X] + c$ , the voluntary setting allows the participation of non-investors in addition to the investors in the mandatory setting, leading to a higher expected participation, while the expected a priori investment level remains the same. That is,  $0 \leq q_1 = q \leq q_1 + q_2 \leq 1$  if  $c < \bar{x} - \mathbb{E}[X]$ , and  $0 = q \leq q' = 1$  if  $c \geq \bar{x} - \mathbb{E}[X]$ .

In case  $r > \mathbb{E}[X] + c$  and  $c \in (c_{min}^v(r), c_{min}^m(r)]$ , both settings lead to full participation. While in the mandatory setting all participants invest, in the voluntary setting the participants invest with the probability  $q_1$ . That is,  $0 < q_1 < q = q_1 + q_2 = 1$ .

In case  $r > \mathbb{E}[X] + c$  and  $c > c_{min}^m(r)$ , the voluntary setting leads to full participation ( $N$  participants) while the mandatory setting leads to randomized participation with  $Nq < N$  expected participants. That is,  $0 < q_1 < q < q_1 + q_2 = 1$  if  $c < \bar{x} - \mathbb{E}[X]$ , and  $0 < q < q' = 1$  if  $c \geq \bar{x} - \mathbb{E}[X]$ .  $\square$

*Proof of Proposition 4.* In the symmetric equilibria in both settings, participant's expected profits are positive in  $E_1^f$  and  $E_{mix}$ , and zero in  $E_1^r$  and  $E_2$ .

In case  $r > \mathbb{E}[X] + c$  and  $c \in (c_{min}^v(r), c_{min}^m(r))$ , both  $E_1^f$  in the mandatory setting and  $E_{mix}$  in the voluntary setting result in full participation and  $q > q_1$ . Although total participation is identical in both settings, a higher share of investors lowers the expected value of the first-order statistic, i.e. the expected lowest cost among investors, while the expected costs of non-investors remain fixed at  $\mathbb{E}[X] + c$ . Thus, the increased share of investors leads to more intense competition within the auction. As a result, investors in the mandatory setting expect lower expected profits compared to the voluntary setting.  $\square$

## D Proof of Proposition 5 for the welfare comparison

A SA with  $r = x_0$  maximizes the expected welfare and implements the (interim) efficient mechanism. In the following, we compare the expected welfare under  $r = x_0$  in both settings and differentiate the areas marked in Figure 3.

### D.1 Area 1 with $r < \mathbb{E}[X] + c$ and $c > c_{min}^m(r)$

The equilibrium  $E_1^r$  is the same in both mandatory and voluntary settings, so the expected welfare is also the same.

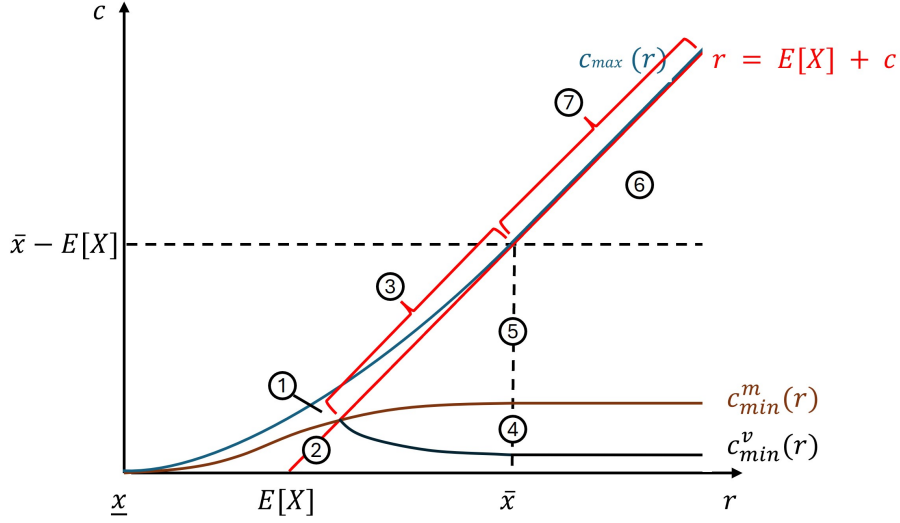


Figure 3: Division and numbering of the relevant areas

**D.2 Area 2 with  $c \leq c_{min}^m(r)$  if  $r \leq \mathbb{E}[X] + c$  and  $c \leq c_{min}^v(r)$  if  $r > \mathbb{E}[X] + c$**

The equilibrium  $E_1^f$  is the same in both mandatory and voluntary settings, so the expected welfare is also the same.

**D.3 Area 3 with  $r = x_0 = \mathbb{E}[X] + c$  and  $c_{min}^v(r) = c_{min}^m(r) < c < \bar{x} - \mathbb{E}[X]$**

In the mandatory setting, the equilibrium  $E_1^r$  yields the following expected welfare

$$W^m = \sum_{n=1}^N P_A(n, q) \left[ \int_{\underline{x}}^{x_0} (x_0 - t) f_{(1,n)}(t) dt - n \cdot c \right] = \sum_{n=1}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(1,n)}(t) dt - Nqc$$

where  $nP_A(n, q) = NqP(n-1, q)$  and  $\sum_{n=1}^N nP_A(n, q) = Nq \sum_{n=0}^{N-1} P(n, q) = Nq$ .

In the equilibria in the voluntary setting,  $q_1 = q$  and  $q_2 \in [0, 1 - q]$ . If  $q_2 = 0$ , the equilibrium  $E_1^r$  is the same as in the mandatory case, thus yields the same expected welfare.

If  $q_2 \in (0, 1 - q)$ , the equilibrium  $E_{mix}$  yields the expected welfare

$$\begin{aligned}
W^v &= \sum_{n_1=1}^N P_A(n_1, 0, q_1, q_2) \left[ \int_{\underline{x}}^{x_0} (x_0 - t) f_{(1, n_1)}(t) dt - n_1 c \right] \\
&\quad + \sum_{n_2=1}^N P_A(0, n_2, q_1, q_2) (x_0 - \mathbb{E}[X] - c) \\
&\quad + \sum_{\substack{n_1 \in [1, N-1] \\ n_2 \in [1, N-n_1]}} P_A(n_1, 0, q_1, q_2) \left[ \int_{\underline{x}}^{x_0} (x_0 - t) f_{(1, n_1)}(t) dt - n_1 c + (1 - F_{(1, n_1)}(x_0))(x_0 - \mathbb{E}[X] - c) \right] \\
&= \sum_{n_1=1}^N \sum_{n_2=0}^{N-n_1} P_A(n_1, n_2, q_1, q_2) \left[ \int_{\underline{x}}^{x_0} (x_0 - t) f_{(1, n_1)}(t) dt - n_1 c \right] \\
&= \sum_{n_1=1}^N P_A(n_1, q_1) \left[ \int_{\underline{x}}^{x_0} (x_0 - t) f_{(1, n_1)}(t) dt - n_1 c \right] = W^m
\end{aligned}$$

where  $P_A(n_1, n_2, q_1, q_2) = \binom{N}{n_1} \binom{N-n_1}{n_2} q_1^{n_1} q_2^{n_2} (1-q_1-q_2)^{N-n_1-n_2}$  and  $\sum_{n_2=0}^{N-n_1} P_A(n_1, n_2, q_1, q_2) = P_A(n_1, q_1) = \binom{N}{n_1} q_1^n (1-q)^{N-n}$ , analog to  $E_{mix}$  in case 2 in the voluntary setting.

As a result, regardless of  $q_2$ , the expected welfare is always the same in mandatory and voluntary settings.

#### D.4 Area 4 with $r = x_0 > \mathbb{E}[X] + c$ and $c \in (c_{min}^v(r), c_{min}^m(r))$

In the mandatory setting,  $E_1^f$  yields the expected welfare

$$W^m = \int_{\underline{x}}^{x_0} (x_0 - t) f_{(1, N)}(t) dt - N \cdot c = \int_{\underline{x}}^{x_0} F_{(1, N)}(t) dt - Nc.$$

In the voluntary setting,  $E_{mix}$  with  $q_1 \in (0, 1)$  and  $q_2 = 1 - q_1$  yields the expected welfare

$$\begin{aligned}
W^v &= P_A(N, q_1) \int_{\underline{x}}^{x_0} (x_0 - t) f_{(1, N)}(t) dt - P_A(N, q_1) Nc + P_A(0, q_1) (x_0 - \mathbb{E}[X] - c) \\
&\quad + \sum_{n_1=1}^{N-1} P_A(n_1, q_1) \left[ \int_{\underline{x}}^{\mathbb{E}[X]+c} (x_0 - t) f_{(1, n_1)}(t) dt - n_1 c + (1 - F_{(1, n_1)}(\mathbb{E}[X] + c))(x_0 - \mathbb{E}[X] - c) \right] \\
&= P_A(N, q_1) \int_{\underline{x}}^{x_0} F_{(1, N)}(t) dt + \sum_{n_1=1}^{N-1} P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1, n_1)}(t) dt \\
&\quad + (1 - P_A(N, q_1))(x_0 - \mathbb{E}[X] - c) - Nq_1c
\end{aligned}$$

where

$$c = \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t) (1 - F(t))^{n_1} dt - P(N-1, q_1) \int_{\mathbb{E}[X]+c}^r (1 - F(t))^N dt$$

$$\begin{aligned}
Nq_1c &= \sum_{n_1=0}^{N-1} Nq_1P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt - Nq_1P(N-1, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (1-F(t))^N dt \\
&= \sum_{n=1}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} nF(t)(1-F(t))^{n-1} dt - P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} N(1-F(t))^N dt
\end{aligned}$$

The welfare difference is

$$\begin{aligned}
\Delta W &= W^v - W^m \\
&= P_A(N, q_1) \int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt + \sum_{n_1=1}^{N-1} P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1,n_1)}(t) dt + (1 - P_A(N, q_1))(x_0 - \mathbb{E}[X] - c) \\
&\quad - \int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt + Nc - \sum_{n=1}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} nF(t)(1-F(t))^{n-1} dt \\
&\quad + P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} N(1-F(t))^N dt \\
&= P_A(N, q_1) \int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt - P_A(N, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1,N)}(t) dt + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt \\
&\quad + (1 - P_A(N, q_1))(x_0 - \mathbb{E}[X] - c) - \int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt + Nc + P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} N(1-F(t))^N dt
\end{aligned}$$

If  $x_0 \geq \bar{x}$ ,

$$\begin{aligned}
\Delta W &= P_A(N, q_1) \int_{\underline{x}}^{\bar{x}} F_{(1,N)}(t) dt - P_A(N, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1,N)}(t) dt + \sum_{n=1}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt \\
&\quad + (1 - P_A(N, q_1))(\bar{x} - \mathbb{E}[X] - c) - \int_{\underline{x}}^{\bar{x}} F_{(1,N)}(t) dt + Nc + P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} N(1-F(t))^N dt
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Delta W}{\partial c} &= -P_A(N, q_1)F_{(1,N)}(\mathbb{E}[X] + c) + \sum_{n=1}^N P_A(n, q_1)F_{(2,n)}(\mathbb{E}[X] + c) - (1 - P_A(N, q_1)) + N \\
&\quad - P_A(N, q_1)N(1 - F(\mathbb{E}[X] + c))^N \\
&\geq -P_A(N, q_1)F_{(1,N)}(\mathbb{E}[X] + c) - (1 - P_A(N, q_1)) + N - P_A(N, q_1)N(1 - F(\mathbb{E}[X] + c))^N \\
&= -q_1^N(1 - (1 - F(\mathbb{E}[X] + c))^N) - 1 + q_1^N + N - q_1^N N(1 - F(\mathbb{E}[X] + c))^N \\
&= q_1^N(1 - F(\mathbb{E}[X] + c))^N - 1 + N - q_1^N N(1 - F(\mathbb{E}[X] + c))^N \\
&= (N - 1)(1 - q_1^N(1 - F(\mathbb{E}[X] + c))^N) \\
&\geq 0
\end{aligned}$$

Thus,  $\Delta W$  has its minimum when  $c = c_{min}^v(\bar{x})$ . Since  $q_1 = 1$  when  $c = c_{min}^v(\bar{x})$ ,  $\Delta W = 0$ . Therefore,  $\Delta W \geq 0$  if  $x_0 \geq \bar{x}$ .

If  $x_0 < \bar{x}$ , we have

$$\begin{aligned}
& \Delta W(x_0, c, q_1) \\
&= P_A(N, q_1) \int_{\underline{x}}^{\bar{x}} F_{(1,N)}(t) dt - P_A(N, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1,N)}(t) dt + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt \\
&\quad + (1 - P_A(N, q_1))(\bar{x} - \mathbb{E}[X] - c) - \int_{\underline{x}}^{\bar{x}} F_{(1,N)}(t) dt + Nc + P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} N(1 - F(t))^N dt \\
&\quad - (1 + P_A(N, q_1)(N - 1)) \int_{x_0}^{\bar{x}} (1 - F(t))^N dt \\
&= P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} (N - 1)(1 - F(t))^N dt + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt + (\bar{x} - \mathbb{E}[X] - c) \\
&\quad + Nc - \int_{\underline{x}}^{\bar{x}} F_{(1,N)}(t) dt - (1 + P_A(N, q_1)(N - 1)) \int_{x_0}^{\bar{x}} (1 - F(t))^N dt
\end{aligned}$$

subject to

$$\Delta \pi^v(x_0, c, q_1) = \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1 - F(t))^{n_1} dt - P(N - 1, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} (1 - F(t))^N dt - c = 0$$

To find the minimum of  $\Delta W$ , we have the following minimizing problem with Lagrange-multiplicand  $\lambda$  and the Karush-Kuhn-Tucker (KKT) conditions with factors  $\mu_1 \geq 0$  and  $\mu_2 \geq 0$  for  $q_1 \in (0, 1]$ :

$$\begin{aligned}
& \min_{x_0, c, q_1} L(x_0, c, q_1, \lambda, \mu_1, \mu_2) := \Delta W(x_0, c, q_1) + \lambda \Delta \pi^v(x_0, c, q_1) + \mu_1(-q_1) + \mu_2(q_1 - 1) \\
& \text{s.t. } \frac{\partial L}{\partial x_0} = (1 + P_A(N, q_1)(N - 1))(1 - F(x_0))^N - \lambda P(N - 1, q_1)(1 - F(x_0))^N = 0 \tag{D.14}
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial L}{\partial c} = (N - 1)(1 - P_A(N, q_1)(1 - F(\mathbb{E}[X] + c))^N) + \sum_{n=2}^N P_A(n, q_1) F_{(2,n)}(\mathbb{E}[X] + c) \\
& \quad + \lambda \left[ \sum_{n_1=0}^{N-1} P(n_1, q_1) F(\mathbb{E}[X] + c)(1 - F(\mathbb{E}[X] + c))^{n_1} + P(N - 1, q_1)(1 - F(\mathbb{E}[X] + c))^N - 1 \right] = 0 \tag{D.15}
\end{aligned}$$

$$\frac{\partial L}{\partial q_1} = \frac{\partial \Delta W}{\partial q_1} + \lambda \frac{\partial \Delta \pi^v}{\partial q_1} - \mu_1 + \mu_2 = 0 \tag{D.16}$$

$$\Delta \pi^v(x_0, c, q_1) = 0$$

$$-q_1 \leq 0$$

$$q_1 - 1 \leq 0$$

$$\mu_1(-q_1) = 0$$

$$\mu_2(q_1 - 1) = 0$$

Constraint (D.14) leads to  $x_0 = \bar{x}$  or  $\lambda = (N - 1)q_1 + \frac{1}{q_1^{N-1}}$ , note than in this case  $q_1 > 0$ .  $x_0 = \bar{x}$  is a boundary case, which has already be considered above (in case  $x_0 \geq \bar{x}$ ) and we have shown that  $\Delta W \geq 0$  if  $x_0 = \bar{x}$ . Now, let's substitute  $\lambda = (N - 1)q_1 + \frac{1}{q_1^{N-1}}$  in constraint

(D.15), which can be simplified to

$$\begin{aligned}
& \frac{\partial L}{\partial c} \\
&= N - (N-1)q_1^N(1 - F(\mathbb{E}[X] + c))^N - (1 - q_1F(\mathbb{E}[X] + c))^N - Nq_1F(\mathbb{E}[X] + c)(1 - q_1F(\mathbb{E}[X] + c))^{N-1} \\
&\quad + \lambda \left[ F(\mathbb{E}[X] + c)(1 - q_1F(\mathbb{E}[X] + c))^{N-1} + q_1^{N-1}(1 - F(\mathbb{E}[X] + c))^{N-1} - 1 \right] \\
&= N - (1 - q_1F(\mathbb{E}[X] + c))^{N-1} + F(\mathbb{E}[X] + c)\left(\frac{1}{q_1} - F(\mathbb{E}[X] + c)\right)^{N-1} + (1 - F(\mathbb{E}[X] + c))^N - (N-1)q_1 - \frac{1}{q_1^{N-1}}
\end{aligned}$$

where

$$\begin{aligned}
& \sum_{n_1=0}^{N-1} P(n_1, q_1)(1 - T)^{n_1} = \sum_{n_1=0}^{N-1} \binom{N-1}{n_1} (q_1(1 - T))^{n_1} (1 - q_1)^{N-n_1-1} \\
&= (q_1(1 - T) + 1 - q_1)^{N-1} = (1 - q_1T)^{N-1}
\end{aligned}$$

$$\begin{aligned}
& \sum_{n=0}^N P_A(n, q)(1 - T)^n = (1 - qT)^N \\
&\Rightarrow \sum_{n=0}^N \binom{N}{n} q^{n-1}(1 - q)^{N-n-1}(n - Nq)(1 - T)^n = -NT(1 - qT)^{N-1} \\
&\Rightarrow \sum_{n=0}^N \binom{N}{n} q^{n-1}(1 - q)^{N-n-1}n(1 - T)^n = N \sum_{n=0}^N \binom{N}{n} q^n(1 - q)^{N-n-1}(1 - T)^n - NT(1 - qT)^{N-1} \\
&\Rightarrow \sum_{n=0}^N \binom{N}{n} q^n(1 - q)^{N-n}n(1 - T)^n = q(1 - q)\left[\frac{N}{1 - q}(1 - qT)^N - NT(1 - qT)^{N-1}\right] \\
&\Rightarrow \sum_{n=0}^N \binom{N}{n} q^n(1 - q)^{N-n}nT(1 - T)^{n-1} = Nq(1 - qT)^{N-1}T
\end{aligned}$$

$$\begin{aligned}
& \sum_{n=2}^N P_A(n, q_1)F_{(2,n)}(\mathbb{E}[X] + c) \\
&= \sum_{n=0}^N P_A(n, q_1)F_{(2,n)}(\mathbb{E}[X] + c) \\
&= \sum_{n=0}^N P_A(n, q_1)(1 - (1 - F(\mathbb{E}[X] + c)))^N - NF(\mathbb{E}[X] + c)(1 - F(\mathbb{E}[X] + c))^{N-1} \\
&= 1 - (1 - q_1F(\mathbb{E}[X] + c))^N - Nq_1F(\mathbb{E}[X] + c)(1 - q_1F(\mathbb{E}[X] + c))^{N-1}
\end{aligned}$$

Deviate  $\frac{\partial L}{\partial c}$  by  $q_1$ , it holds that

$$\frac{\partial^2 L}{\partial c \partial q_1} = (N-1)\left(\frac{1}{q_1^N} - 1\right)(1 - F(\mathbb{E}[X] + c)(1 - q_1F(\mathbb{E}[X] + c))^{N-1}) \geq 0.$$

So  $\frac{\partial L}{\partial c}$  has its maximum at  $q_1 = 1$ , which is 0. Given  $q_1 = 1$ , the minimizing problem has the following KKT points:  $q_1 = 1, \lambda = N, \mu_1 = 0, c = c_{min}^v(x_0)$ ,  $\mu_2$  depends on  $x_0$  according to Constraint (D.16) and  $x_0$  arbitrary. These points are on the lower boundary. Thus, there exists no internal minimum or maximum. Next, we compare all  $\Delta W$  on the boundary: If  $x_0 = \bar{x}$ , as mentions above,  $\Delta W \geq 0$  and increases in  $c$ ; If  $x_0 = \int_{\underline{x}}^{x_0} F(t)(1 - F(t))^{N-1} dt, \Delta W = 0$ ; If  $q_1 = 1$ , that is  $c = c_{min}^v(x_0), \Delta W = 0$ ; If  $q_1$  is minimal, that is  $x_0 = \bar{x}$  and  $c = c_{min}^m(\bar{x})$  according to Lemma 4,  $\Delta W > 0$ ; If  $c = c_{min}^v(x_0), \Delta W = 0$ ; If  $c = c_{min}^m(x_0) := \tilde{c}$ , substitute  $c$  in  $\Delta W$  and we have

$$\begin{aligned} \frac{\partial \Delta W}{\partial x_0} &= (1 + P_A(N, q_1)(N - 1))(1 - F(x_0))^N + \sum_{n_1=2}^N P_A(n_1, q_1) F_{(2, n_1)}(\mathbb{E}[X] + \tilde{c}) F(x_0)(1 - F(x_0))^{N-1} \\ &\quad + (N - 1)(1 - P_A(N, q_1)(1 - F(\mathbb{E}[X] + \tilde{c}))^N) F(x_0)(1 - F(x_0))^{N-1} \\ &\geq 0 \end{aligned}$$

Thus,  $c = c_{min}^m(x_0), \Delta W$  has its minimum zero when  $x_0 = \mathbb{E}[X] + c$  and its maximum at  $x_0 = \bar{x}$ .

In sum,  $\Delta W$  is always nonnegative, has its minimum at  $c = c_{min}^v(x_0)$ , increases in  $x_0$  and  $c$  monotoneously, and has its maximum at  $c = c_{min}^m(\bar{x})$  and  $x_0 = \bar{x}$ .

## D.5 Area 5 with $r = x_0 > \mathbb{E}[X] + c$ and $c \in [c_{min}^m(r), \bar{x} - \mathbb{E}[X])$

In the mandatory setting,  $E_1^r$  yields the expected welfare

$$W^m = \sum_{n=1}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(1, n)}(t) dt - Nqc = \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2, n)}(t) dt$$

Where

$$c = P(0, q) \int_{\underline{x}}^{x_0} F(s) ds + \sum_{n=1}^{N-1} P(n, q) \int_{\underline{x}}^{x_0} F(t)(1 - F_{(1, n)}(t)) dt$$

In the voluntary setting,  $E_{mix}$  with  $q_1 \in (0, 1)$  and  $q_2 = 1 - q_1$  yields the expected welfare

$$\begin{aligned} W^v &= P_A(N, q_1) \int_{\underline{x}}^{x_0} F_{(1, N)}(t) dt + \sum_{n_1=1}^{N-1} P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1, n_1)}(t) dt \\ &\quad + (1 - P_A(N, q_1))(x_0 - \mathbb{E}[X] - c) - Nq_1c \end{aligned}$$

where

$$\begin{aligned}
c &= \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt - P(N-1, q_1) \int_{\mathbb{E}[X]+c}^r (1-F(t))^N dt \\
Nq_1c &= \sum_{n_1=0}^{N-1} Nq_1P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt - Nq_1P(N-1, q_1) \int_{\mathbb{E}[X]+c}^r (1-F(t))^N dt \\
&= \sum_{n=1}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} nF(t)(1-F(t))^{n-1} dt - P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} N(1-F(t))^N dt
\end{aligned}$$

The welfare difference is

$$\begin{aligned}
\Delta W &= W^v - W^m \\
&= P_A(N, q_1) \int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt + \sum_{n_1=1}^{N-1} P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1,n_1)}(t) dt + (1 - P_A(N, q_1))(x_0 - \mathbb{E}[X] - c) \\
&\quad \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt - \sum_{n=1}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} nF(t)(1-F(t))^{n-1} dt \\
&\quad + P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} N(1-F(t))^N dt \\
&= P_A(N, q_1) \int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt - P_A(N, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(1,N)}(t) dt + \sum_{n_1=2}^N P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n_1)}(t) dt \\
&\quad + (1 - P_A(N, q_1))(x_0 - \mathbb{E}[X] - c) - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt + P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} N(1-F(t))^N dt \\
&= P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (N-1)(1-F(t))^N dt + \sum_{n_1=2}^N P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n_1)}(t) dt \\
&\quad + (x_0 - \mathbb{E}[X] - c) - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt
\end{aligned}$$

If  $x_0 \geq \bar{x}$

$$\begin{aligned}
\Delta W &= P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} (N-1)(1-F(t))^N dt + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt \\
&\quad + (\bar{x} - \mathbb{E}[X] - c) - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{\bar{x}} F_{(2,n)}(t) dt + (P_A(0, q) + P_A(1, q))(x_0 - \bar{x}) \\
&\geq P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} (N-1)(1-F(t))^N dt + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt \\
&\quad + (\bar{x} - \mathbb{E}[X] - c) - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{\bar{x}} F_{(2,n)}(t) dt \\
&:= \underline{\Delta W}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \Delta W}{\partial c} &= -P_A(N, q_1)(N-1)(1-F(\mathbb{E}[X]+c))^N + \sum_{n=2}^N P_A(n, q_1)F_{(2,n)}(\mathbb{E}[X]+c) - 1 \\
&\leq -P_A(N, q_1)(N-1)(1-F(\mathbb{E}[X]+c))^N + 1 - P_A(0, q_1) - P_A(1, q_1) - 1 \\
&\leq 0
\end{aligned}$$

Thus,  $\underline{\Delta W}$  has its minimum when  $c = \bar{x} - \mathbb{E}[X]$ . Since  $q_1 = q = 0$  when  $c = \bar{x} - \mathbb{E}[X]$ ,  $\underline{\Delta W} = \bar{x} - \mathbb{E}[X] - c = 0$ . Therefore,  $\Delta W \geq \underline{\Delta W} = 0$  and hence the expected welfare in the voluntary setting is always larger than or equal to that in the mandatory setting.

If  $x_0 < \bar{x}$ ,

$$\begin{aligned}
\Delta W &= P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (N-1)(1-F(t))^N dt + \sum_{n_1=2}^N P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n_1)}(t) dt \\
&\quad + (x_0 - \mathbb{E}[X] - c) - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt \\
&> P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (N-1)(1-F(t))^N dt + \sum_{n_1=2}^N P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n_1)}(t) dt \\
&\quad + (x_0 - \mathbb{E}[X] - c) - \int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt + Nc \\
&:= \Delta W'
\end{aligned}$$

Analogous to the proof of Lemma 2, since  $F_{(2,n_1)}$  increases in  $n_1$ ,  $P_A(n_1, q_1)$  increases in  $q$  if  $q \in [0, \frac{n}{N})$  and decreases in  $q$  if  $q \in (\frac{n}{N}, 1]$ , it can be shown that  $\sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt$  increases in  $q$ . Thus, the maximum is at  $q = 1$ , which is  $\int_{\underline{x}}^{x_0} F_{(1,N)}(t) dt - Nc$ . In Area 5 with  $x_0 < \bar{x}$ ,  $q \in (0, 1)$ , therefore  $\Delta W > \Delta W'$ .

Note that  $\Delta W'$  corresponds to  $\Delta W$  in Area 4. To find the minimum of  $\Delta W'$ , the same minimizing problem as in Section D.4 should be solved. As shown above, the minimizing problem has the following KKT points:  $q_1 = 1, \lambda = N, \mu_1 = 0, c = c_{min}^v(x_0)$ ,  $\mu_2$  depends on  $x_0$  according to Constraint (D.16) and  $x_0$  arbitrary. However, these points are not in Area 5. Thus, there exists no internal minimum or maximum. Next, we compare all  $\Delta W$  on the boundary: If  $x_0 = \bar{x}$ , as mentions above,  $\Delta W = \Delta W' \geq 0$  and  $\Delta W'$  decreases in  $c$ ; If  $x_0 = \int_{\underline{x}}^{x_0} F(t)(1-F(t))^{N-1} dt$ ,  $\Delta W = \Delta W' = 0$ ; If  $q_1 = 1$ ,  $\Delta W = \Delta W' = 0$ ; If  $q_1 = 0$ ,  $\Delta W = 0$  (see Section D.7); If  $c = c_{min}^m(x_0) := \tilde{c}$ ,  $\Delta W = \Delta W' \geq 0$  and  $\Delta W'$  increases in  $x_0$  (see Section D.4); If  $c = \bar{x} - \mathbb{E}[X]$ ,  $\Delta W = 0$  (see Section D.7).

In sum,  $\Delta W$  is always nonnegative, has its minimum at  $x_0 = \mathbb{E}[X] + c$ , increases in  $x_0$  and decreases in  $c$  monotoneously, and has its maximum at  $c = c_{min}^m(\bar{x})$  and  $x_0 = \bar{x}$ .

**D.6 Area 6 with  $r = x_0 > \mathbb{E}[X] + c > \bar{x}$  and  $c \in [\bar{x} - \mathbb{E}[X], x_0 - \mathbb{E}[X])$**

In the mandatory setting,  $E_1^r$  yields the expected welfare

$$\begin{aligned} W^m &= \sum_{n=1}^N P_A(n, q) \left[ \int_{\underline{x}}^{\bar{x}} (x_0 - t) f_{(1,n)}(t) dt - n \cdot c \right] \\ &= \sum_{n=1}^N P_A(n, q) \left[ \int_{\underline{x}}^{\bar{x}} (x_0 - t) f_{(1,n)}(t) dt \right] - Nqc \\ &= \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt \end{aligned}$$

Where

$$c = P(0, q) \int_{\underline{x}}^{x_0} F(s) ds + \sum_{n=1}^{N-1} P(n, q) \int_{\underline{x}}^{\bar{x}} F(t) (1 - F_{(1,n)}(t)) dt$$

In the voluntary setting,  $E_2$  yields the expected welfare

$$W^v = x_0 - \mathbb{E}[X] - c > 0 \tag{D.17}$$

The welfare difference is

$$\begin{aligned} \Delta W &= W^v - W^m \\ &= x_0 - \mathbb{E}[X] - c - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt \\ &= x_0 - \mathbb{E}[X] - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt - P(0, q) \int_{\underline{x}}^{x_0} F(s) ds - \sum_{n=1}^{N-1} P(n, q) \int_{\underline{x}}^{\bar{x}} F(t) (1 - F_{(1,n)}(t)) dt \end{aligned}$$

$$\frac{\partial \Delta W}{\partial x_0} = P_A(0, q) + P_A(1, q) - P(0, q) = (N-1)q(1-q)^{N-1} \geq 0$$

Thus,  $\Delta W$  has its minimum at  $x_0 = \mathbb{E}[X] + c$ , where  $q = 0$  and  $\Delta W = 0$ . As a result,  $\Delta W \geq 0$  and the expected welfare in the voluntary setting is always larger than or equal to that in the mandatory setting.

**D.7 Area 7 with  $r = x_0 = \mathbb{E}[X] + c$  and  $c \geq \bar{x} - \mathbb{E}[X]$**

In the mandatory setting, no equilibrium exists and thus  $W^m = 0$ .

In the voluntary setting,  $E_2$  yields the expected welfare

$$W^v = x_0 - \mathbb{E}[X] - c = 0 = W^m$$

So the expected welfare is always the same in mandatory and voluntary settings.

## E Comparison of the expected profits

**Locally optimal reserve price** In  $E_{mix}$ , the coexistence of investors and non-investors induces a contracted distribution function  $F'$  of the (expected) private cost, where  $F' \succ_{SSD} F$  and  $F' = q_1 F + (1 - q_1)\delta_{\mathbb{E}[X]+c}$ , with  $\delta_{\mathbb{E}[X]+c}$  denoting a point mass at  $\mathbb{E}[X] + c$ . For any  $r > \mathbb{E}[X] + c$ ,  $F'(r) = q_1 F(r) + (1 - q_1)$ ,  $f'(r) = q_1 f(r)$ , which implies  $\frac{F'(r)}{f'(r)} = \frac{q_1 F(r) + (1 - q_1)}{q_1 f(r)} = \frac{F(r)}{f(r)} + \frac{1 - q_1}{q_1 f(r)} \geq \frac{F(r)}{f(r)}$ . The equality holds if and only if  $q_1 = 1$ . Let  $\gamma_v(r) := \gamma_{F'}(r)$ , which is increasing in  $r$ . Then  $\gamma_v(r) \geq \gamma_F(r) = \gamma_m(r)$ . According to Myerson (1981),  $r^* = \gamma^{-1}(x_0)$ , and therefore in  $E_{mix}$ ,  $r^* = \gamma_v^{-1}(x_0) \leq \gamma_m^{-1}(x_0)$ , where  $\gamma_m^{-1}(x_0)$  is the optimal reserve price in  $E^f$ . Consequently,  $r^* \leq x_0 - \frac{F(r^*)}{f(r^*)}$ .

*Proof of Proposition 6.* From the optimal reserve price we learn that  $(r = x_0, c)$  and  $(r^*, c)$  always belong to the same equilibrium (including the boundary). In area 1,3,6,7, the participants expect a zero profit in both settings. In Area 2,  $r_m^*(x_0) = r_v^*(x_0)$ , participants expect the same profit which is strictly positive if  $r_m^*(x_0) = r_v^*(x_0) > \{x \mid c_{min}^m(x) = c\} := L_2(c) \Rightarrow x_0 > \gamma_m(L_2(c))$ . Under uniform distribution, this means that  $x_0 > 2L_2(c)$ . In area 5,  $\mathbb{E}[\pi_i^{vn}] = \mathbb{E}[\pi_i^{vi}] - c \geq \mathbb{E}[\pi_i^m] - c = 0$ , where the difference is strictly positive if  $r_v^*(x_0) > \mathbb{E}[X] + c \Rightarrow x_0 > \gamma_v(\mathbb{E}[X] + c)$ .

In area 4, however, participants can expect higher profits in the mandatory setting:  $\mathbb{E}[\pi_i^m] - c > \mathbb{E}[\pi_i^{vn}] = \mathbb{E}[\pi_i^{vi}] - c = 0$  if  $r_m^*(x_0) > L_2(c) = r_v^*(x_0) \Rightarrow \gamma_m(\mathbb{E}[X] + c) < x_0 \leq \gamma_v(\mathbb{E}[X] + c)$ .  $\square$

*Proof of Proposition 7.* For the auctioneer: We only consider the relevant cases where equilibrium exists in both settings, namely when  $x_0 > L_1(c)$ , where  $L_1(c) := \{x \mid c_{max}(x) = \int_{\underline{x}}^x F(s)ds = c\}$ . In the special case  $x_0 = \mathbb{E}[X] + c$  and  $c \geq \bar{x} - \mathbb{E}[X]$ , where an equilibrium only exists in the voluntary setting ( $E_2$ ),  $r^* = x_0$  and thus  $\pi_0 = 0$  as in the mandatory setting where no equilibrium exists.

**In case**  $c \geq \bar{x} - \mathbb{E}[X]$ , we have  $x_0 > L_1(c) \geq L_1(\bar{x} - \mathbb{E}[X]) = \bar{x}$ .

$$r_m^*(x_0) = x_0$$

$$r_v^*(x_0) \in [\bar{x}, x_0]$$

$$\pi_0^m = \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{\bar{x}} (x_0 - t) f_{(2,n)}(t) dt = \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt$$

$$\pi_0^v = x_0 - \mathbb{E}[X] - c$$

$$\Delta \pi_0 = \pi_0^v - \pi_0^m = x_0 - \mathbb{E}[X] - c - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt$$

which is equal to  $\Delta W$  in Area 6 (see Section D.6) and thus

$$\frac{\partial \Delta \pi_0}{\partial x_0} = \frac{\partial \Delta W}{\partial x_0} = P_A(0, q) + P_A(1, q) - P(0, q) \geq 0$$

Therefore, given a  $c$ ,  $\Delta \pi_0$  has its minimum at  $x_0 = \mathbb{E}[X] + c$ , where  $q = 0$  and thus  $\pi_0^m = \pi_0^v = 0$ . As a result,  $\pi_0^v \geq \pi_0^m$ .

**In case**  $c_{min}^m(\bar{x}) \leq c < \bar{x} - \mathbb{E}[X]$ , the optimal reserve prices are given by

$$r_m^*(x_0) = x_0$$

$$r_v^*(x_0) = \begin{cases} x_0, & \text{if } x_0 \leq \mathbb{E}[X] + c \\ \mathbb{E}[X] + c, & \text{if } \mathbb{E}[X] + c < x_0 \leq \gamma_v(\mathbb{E}[X] + c) \\ (\gamma_v)^{-1}(x_0), & \text{if } \gamma_v(\mathbb{E}[X] + c) < x_0 \leq \gamma_v(\bar{x}) \\ r_v^*(x_0) \in [\bar{x}, x_0], & \text{if } x_0 > \gamma_v(\bar{x}) \end{cases}$$

For  $x_0 \leq \mathbb{E}[X] + c$ , both settings yield the same optimal reserve price and the same equilibrium. In the following, we focus on the case when  $x_0 > \mathbb{E}[X] + c$  and assume that if  $r_v = \mathbb{E}[X] + c$ ,  $q_2 = 1 - q_1$ . The expected profit of the auctioneer in the mandatory setting is

$$\pi_0^m = \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt = \sum_{n=1}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(1,n)}(t) dt - Nqc$$

where  $c = \sum_{n=0}^{N-1} P(n, q) \int_{\underline{x}}^{x_0} F(t)(1 - F(t))^n dt$ .

For a given reserve price  $r_v$ , the expected profit of the auctioneer in the voluntary setting is denoted  $\pi_0^v(r_v)$ . We first consider  $r_v^1 = \mathbb{E}[X] + c$ :

$$\begin{aligned} \pi_0^v(r_v^1) &= (P_A(0, q_1) + P_A(1, q_1))(x_0 - \mathbb{E}[X] - c) \\ &\quad + \sum_{n=2}^{N-1} P_A(n, q_1) \left[ \int_{\underline{x}}^{\mathbb{E}[X]+c} (x_0 - t) f_{(2,n)}(t) dt + (1 - F_{(2,n)}(\mathbb{E}[X] + c))(x_0 - \mathbb{E}[X] - c) \right] \\ &\quad + P_A(N, q_1) \left[ \int_{\underline{x}}^{\mathbb{E}[X]+c} (x_0 - t) f_{(2,N)}(t) dt + (F_{(1,N)}(\mathbb{E}[X] + c) - F_{(2,N)}(\mathbb{E}[X] + c))(x_0 - \mathbb{E}[X] - c) \right] \\ &= [1 - P_A(N, q_1)(1 - F_{(1,N)}(\mathbb{E}[X] + c))](x_0 - \mathbb{E}[X] - c) + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt \end{aligned}$$

where  $c = \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt$ ; Similarly, for  $r_v^2 = x_0$ :

$$\begin{aligned}
\pi_0^v(r_v^2) &= (P_A(0, q_1) + P_A(1, q_1))(x_0 - \mathbb{E}[X] - c) \\
&+ \sum_{n_1 \in [2, N-2]} P_A(n_1, q_1) \left[ (1 - F_{(2, n_1)}(\mathbb{E}[X] + c))(x_0 - \mathbb{E}[X] - c) + \int_{\underline{x}}^{\mathbb{E}[X]+c} (x_0 - t) f_{(2, n_1)}(t) dt \right] \\
&+ P_A(N-1, q_1) \left[ \int_{\mathbb{E}[X]+c}^{x_0} (x_0 - t) f_{(1, N-1)}(t) dt + \right. \\
&\quad \left. (F_{(1, N-1)}(\mathbb{E}[X] + c) - F_{(2, N-1)}(\mathbb{E}[X] + c))(x_0 - \mathbb{E}[X] - c) + \int_{\underline{x}}^{\mathbb{E}[X]+c} (x_0 - t) f_{(2, N-1)}(t) dt \right] \\
&+ P_A(N, q_1) \left[ \int_{\underline{x}}^{x_0} (x_0 - t) f_{(2, N)}(t) dt \right] \\
&= (1 - P_A(N-1, q_1) - P_A(N, q_1))(x_0 - \mathbb{E}[X] - c) + \sum_{n_1 \in [2, N]} P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2, n_1)}(t) dt \\
&+ P_A(N-1, q_1) \int_{\mathbb{E}[X]+c}^{x_0} F_{(1, N-1)}(t) dt + P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} F_{(2, N)}(t) dt \\
&= (x_0 - \mathbb{E}[X] - c) + \sum_{n_1 \in [2, N]} P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2, n_1)}(t) dt \\
&\quad - P_A(N-1, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (1 - F(t))^{N-1} dt - P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (1 - F(t))^N + NF(t)(1 - F(t))^{N-1} dt
\end{aligned}$$

where  $c = \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt - P(N-1, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (1-F(t))^N dt$ .

To compare  $\pi_0^v(r_v^1)$  with  $\pi_0^v(r_v^2)$ , consider the two extreme cases of  $c$ .

If  $c \rightarrow \bar{x} - \mathbb{E}[X]$ ,  $q_1 \rightarrow 0$  under both reserve prices, so  $\pi_0^v(r_v^1) \rightarrow x_0 - \mathbb{E}[X] - c$ , and  $\pi_0^v(r_v^2) \rightarrow x_0 - \mathbb{E}[X] - c$ . That is, only non-investors with bids  $\mathbb{E}[X] + c$  participate. Thus, the optimal reserve price  $r_v^*(x_0)$  remains at the level of  $\mathbb{E}[X] + c$  and  $\gamma_v(\mathbb{E}[X] + c) \rightarrow \infty$ .

If  $c \rightarrow c_{min}^m(\bar{x})$ , the difference in  $q_1$  between  $r_v^1$  and  $r_v^2$  increases in  $x_0$  and has its maximum at  $x_0 \geq \bar{x}$ . For  $x_0 \geq \bar{x}$ , any  $r_v^2 \geq \bar{x}$  never sets the price and thus has the same effect on  $q_1$  as  $r_v = \bar{x}$ . Consider now the case  $x_0 \geq \bar{x}$  and let  $r_v = \bar{x}$  without loss of generality. Then we have

$$\begin{aligned}
\pi_0^v(r_v^2 \geq \bar{x}) &= \pi_0^v(r_v = \bar{x}) \\
&= (x_0 - \mathbb{E}[X] - c) + \sum_{n_1 \in [2, N]} P_A(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2, n_1)}(t) dt \\
&\quad - P_A(N-1, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} (1 - F(t))^{N-1} dt - P_A(N, q_1) \int_{\mathbb{E}[X]+c}^{\bar{x}} (1 - F(t))^{N-1} (1 + (N-1)F(t)) dt.
\end{aligned}$$

Since  $\frac{d\pi_0^v(\mathbb{E}[X]+c)}{dx_0} = 1 - P_A(N, q_1)(1 - F_{(1, N)}(\mathbb{E}[X] + c)) < 1 = \frac{d\pi_0^v(x_0)}{dx_0}$ ,  $\pi_0^v(r_v^2)$  increases more than  $\pi_0^v(r_v^1)$  as  $x_0$  increases. As a result,  $\pi_0^v(r_v^2)$  surpasses  $\pi_0^v(r_v^1)$  for sufficiently large  $x_0$ , making the optimal reserve price increase beyond  $r_v^1 = \mathbb{E}[X] + c$ . This is consistent with the  $r_v^*(x_0)$ , where the reserve price increases after  $x_0$  reaches a  $\gamma_v(\mathbb{E}[X] + c)$ .

Comparing the equilibrium conditions  $c = \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt$  under  $r_v^1$

and  $c = \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt - P(N-1, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (1-F(t))^N dt$  under  $r_v^2$ , we observe that the term  $P(N-1, q_1) \int_{\mathbb{E}[X]+c}^{x_0} (1-F(t))^N dt$  decreases in  $c$  and converges to 0 as  $c \rightarrow \bar{x} - \mathbb{E}[X]$ , making the corresponding values of  $q_1$  identical in both settings. For smaller  $c$ , this term becomes larger, resulting in a greater difference in  $q_1$  between  $r_v^1$  and  $r_v^2$ . This implies a larger relative advantage of  $r_v^2$  over  $r_v^1$ . Consequently, when  $c$  is large and  $x_0$  is (relatively) small, the optimal reserve price remains  $\mathbb{E}[X] + c$ , whereas for small  $c$  and (relatively) large  $x_0$ , the optimal reserve price rises from  $\mathbb{E}[X] + c$  to  $\bar{x}$ .

After characterizing  $r_v^*(x_0)$ , we compare the profit difference  $\Delta\pi_0 = \pi_0^v - \pi_0^m$ .

Consider first the case where  $c \rightarrow \bar{x} - \mathbb{E}[X]$  and  $x_0 < \infty$ , then  $r_v^*(x_0) = \mathbb{E}[X] + c$ ,

$$\begin{aligned} \Delta\pi_0 &= \pi_0^v(\mathbb{E}[X] + c) - \pi_0^m \\ &= (1 - P_A(N, q_1)(1 - F_{(1,N)}(\mathbb{E}[X] + c))(x_0 - \mathbb{E}[X] - c) + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F_{(2,n)}(t) dt \\ &\quad - \sum_{n=2}^N P_A(n, q) \int_{\underline{x}}^{x_0} F_{(2,n)}(t) dt \end{aligned}$$

where  $q_1 \rightarrow 0$ ,  $q \rightarrow 0$  and thus  $\Delta\pi_0 \rightarrow x_0 - \mathbb{E}[X] - c > 0$ .

Consider then the following specific case: Suppose  $X_i, i \in \{1, \dots, N\}$  are uniform distributed on the interval  $[\underline{x}, \bar{x}] = [0, 1]$ ,  $c = c_{min}^m(\bar{x}) = \frac{1}{N(N+1)}$ , and  $x_0 \in [\bar{x}, \gamma_v(\mathbb{E}[X] + c)]$ . We have  $r_m^*(x_0) = x_0$  and  $\gamma_v(\mathbb{E}[X] + c) \geq \gamma_m(\mathbb{E}[X] + c) = \mathbb{E}[X] + c + \frac{F(\mathbb{E}[X]+c)}{f(\mathbb{E}[X]+c)} = 2(\mathbb{E}[X] + c) = 1 + \frac{2}{N(N+1)} > \bar{x}$ , implying  $r_v^*(x_0) = \mathbb{E}[X] + c = \frac{1}{2} + \frac{1}{N(N+1)}$ . Under  $r_m^*(x_0) = x_0 > \bar{x}$ ,  $q = 1$  and  $\pi_0^m = \frac{N-1}{N+1} + (x_0 - 1)$ . Under  $r_v^*(x_0) = \frac{1}{2} + \frac{1}{N(N+1)}$ ,  $q_1$  is determined by

$$\begin{aligned} c &= \sum_{n_1=0}^{N-1} P(n_1, q_1) \int_{\underline{x}}^{\mathbb{E}[X]+c} F(t)(1-F(t))^{n_1} dt \\ \Leftrightarrow c &= \sum_{n_1=0}^{N-1} P(n_1, q_1) \left[ -\frac{(\mathbb{E}[X] + c)(1 - \mathbb{E}[X] - c)^{n+1}}{n+1} - \frac{(1 - \mathbb{E}[X] - c)^{n+2}}{(n+1)(n+2)} + \frac{1}{(n+1)(n+2)} \right] \\ \Leftrightarrow c &= -\frac{(\mathbb{E}[X] + c)(1 - q_1(\mathbb{E}[X] + c))^N}{Nq_1} + \frac{(\mathbb{E}[X] + c)(1 - q_1)^N}{Nq_1} \\ &\quad - \frac{(1 - q_1(\mathbb{E}[X] + c))^{N+1}}{N(N+1)q_1^2} + \frac{(1 - q_1)^{N+1}}{N(N+1)q_1^2} + \frac{(N+1)q_1(1 - \mathbb{E}[X] - c)(1 - q_1)^N}{N(N+1)q_1^2} \\ &\quad + \frac{1 - (1 - q_1)^{N+1} - (N+1)q_1(1 - q_1)^N}{N(N+1)q_1^2} \\ \Leftrightarrow \frac{1}{N(N+1)} &= \frac{1 - (1 - q_1(\frac{1}{2} + \frac{1}{N(N+1)}))^{N+1}}{N(N+1)q_1^2} - \frac{(\frac{1}{2} + \frac{1}{N(N+1)})(1 - q_1(\frac{1}{2} + \frac{1}{N(N+1)}))^N}{Nq_1} \\ \Rightarrow 1 - q_1^2 - (1 - (\frac{1}{2} + \frac{1}{N(N+1)})q_1)^N (1 + N(\frac{1}{2} + \frac{1}{N(N+1)})q_1) &= 0 \end{aligned}$$

and

$$\begin{aligned}
\pi_0^v &= [1 - P_A(N, q_1)(1 - F_{(1,N)}(\mathbb{E}[X] + c))](x_0 - \mathbb{E}[X] - c) + \sum_{n=2}^N P_A(n, q_1) \int_{\underline{x}}^{\mathbb{E}[X] + c} F_{(2,n)}(t) dt \\
&= [1 - q_1^N (1 - F(\mathbb{E}[X] + c))^N](x_0 - \mathbb{E}[X] - c) \\
&\quad + \sum_{n=2}^N P_A(n, q_1) \left[ \mathbb{E}[X] + c + (\mathbb{E}[X] + c)(1 - \mathbb{E}[X] - c)^N - \frac{2}{N+1} + \frac{2(1 - \mathbb{E}[X] - c)^{N+1}}{N+1} \right] \\
&= [1 - q_1^N (1 - F(\mathbb{E}[X] + c))^N](x_0 - \mathbb{E}[X] - c) + \mathbb{E}[X] + c + (\mathbb{E}[X] + c)(1 - q_1(\mathbb{E}[X] + c))^N \\
&\quad - \frac{2 - 2(1 - q_1)^{N+1}}{(N+1)q_1} + \frac{2(1 - q_1(\mathbb{E}[X] + c))^{N+1} - 2(1 - q_1)^{N+1}}{(N+1)q_1} \\
&= 1 - q_1^N \left( \frac{1}{2} - \frac{1}{N(N+1)} \right)^{N+1} + \left( \frac{1}{2} + \frac{1}{N(N+1)} \right) \left( 1 - \left( \frac{1}{2} + \frac{1}{N(N+1)} \right) q_1 \right)^N \\
&\quad + \frac{2}{q_1(N+1)} \left[ \left( 1 - \left( \frac{1}{2} + \frac{1}{N(N+1)} \right) q_1 \right)^{N+1} - 1 \right] + (x_0 - 1) \left( 1 - q_1^N \left( \frac{1}{2} - \frac{1}{N(N+1)} \right)^N \right)
\end{aligned}$$

With different  $N$ , the numerical values of  $\pi_0^m$  and  $\pi_0^v$  are calculated in the following table.

$N$	$c$	$\pi_0^m$	$q_1$	$\pi_0^v$	$\Delta\pi_0$
3	$\frac{1}{12}$	$0.5 + (x_0 - 1)$	0.7938	$0.4977 + 0.9638(x_0 - 1)$	$-0.0023 - 0.0362(x_0 - 1)$
4	$\frac{1}{20}$	$0.6 + x_0 - 1$	0.8965	$0.5932 + 0.9735(x_0 - 1)$	$-0.0068 - 0.0265(x_0 - 1)$
5	$\frac{1}{30}$	$0.67 + (x_0 - 1)$	0.9459	$0.6609 + 0.9832(x_0 - 1)$	$-0.0057 - 0.0168(x_0 - 1)$

Table E.1: Examples of numeric values of  $\pi_0^m$  and  $\pi_0^v$  under uniform distribution

As shown in Table E.1,  $\Delta\pi_0 < 0$  at  $x_0 \in [\bar{x}, \gamma_v(\mathbb{E}[X] + c)]$  and  $c = c_{min}^m(\bar{x})$ .

Overall, we find both examples with  $\Delta\pi_0 > 0$  and  $\Delta\pi_0 < 0$ . Particularly with high  $c$ , the expected profit of auctioneer favors the voluntary setting. □