When and what to learn in a changing world

César Barilla

Columbia → Oxford

EEA Congress

August 26th, 2025

How to adapt to a changing environment?

How to adapt to a changing environment?

- Example: Investment with uncertain & evolving returns distributions
  or: allocation of government funds, task attribution, technology choice, price setting, etc.

How to adapt to a changing environment?

- Example: Investment with uncertain & evolving returns distributions
  or: allocation of government funds, task attribution, technology choice, price setting, etc.

- Circumstances **change over time**
- Gathering & processing **information is costly**
  $\Rightarrow$ cannot constantly nor perfectly monitor $\Rightarrow$ act based on past knowledge

How to adapt to a changing environment?

- Example: Investment with uncertain & evolving returns distributions
  or: allocation of government funds, task attribution, technology choice, price setting, etc.

- Circumstances **change over time**

- Gathering & processing **information is costly**
  $\Rightarrow$ cannot constantly nor perfectly monitor $\Rightarrow$ act based on past knowledge

In settings with **infrequent** and **flexible** acquisition of new information:

$\longrightarrow$ **When** and **how** to update knowledge of the current state?

$\longrightarrow$ Effects of the **environment**?

- Dynamic model of **optimal information acquisition** about a changing state
  - → fixed cost of acquisition + variable cost of flexible content
  - → rich yet tractable setup that captures relation between timing and content

- Dynamic model of **optimal information acquisition** about a changing state
  - $\rightarrow$ fixed cost of acquisition + variable cost of flexible content
  - $\rightarrow$ rich yet tractable setup that captures relation between timing and content

$\star$ Results:

1. **Characterization of solutions**
   Recursive analysis $\Rightarrow$ simple geometric characterization of optimal policies
2. **Belief dynamics**
   Convergence to cyclical dynamics reduces long run problem to frequency & content
3. **Vanishing fixed costs**
   Convergence to optimal "wait-or-confirm" & closed form long run jump process
4. Applications
   Portfolio allocation and diversification; "confirmation bias" with a safe and a risky action

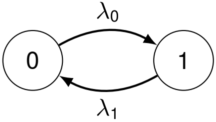
I. Model

States and actions

- Continuous time $t \geq 0$, single decision maker
- State of the world $\theta_t \in \{0, 1\}$ (unobserved)

States and actions

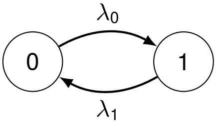
- Continuous time $t \geq 0$, single decision maker
- State of the world $\theta_t \in \{0, 1\}$ (unobserved)



$$\iff \begin{cases} \lambda & := \lambda_0 + \lambda_1 \text{ ("total volatility")} \\ \pi & := \dfrac{\lambda_0}{\lambda_0 + \lambda_1} \text{ (long run average prob.)} \end{cases}$$

- Continuous time $t \geq 0$, single decision maker

- State of the world $\theta_t \in \{0, 1\}$ (unobserved)



$$\iff \begin{cases} \lambda & := \lambda_0 + \lambda_1 \quad \text{("total volatility")} \\ \pi & := \dfrac{\lambda_0}{\lambda_0 + \lambda_1} \quad \text{(long run average prob.)} \end{cases}$$

- Actions and payoffs
  - DM chooses $a_t \in A$ for all t
  - Flow payoffs $u(a_t; \theta_t)$ (unobserved)

  $\Longrightarrow$ Convex and continuous indirect utility:

$$u(p) := \max_{a \in A} \Big(p \cdot u(a; 1) + (1 - p) \cdot u(a; 0) \Big) \ \text{ for } p \in [0, 1]$$

Information Acquisition

- DM chooses when to acquire information:
  - $\rightarrow$ Increasing sequence of times $\{\tau_i\}_{i\in\mathbb{N}}$
  - $\Rightarrow$ **Fixed cost** $\kappa > 0$ paid at each τ_i

- DM chooses when to acquire information:
  - $\rightarrow$ Increasing sequence of times $\{\tau_i\}_{i \in \mathbb{N}}$
  - $\Rightarrow$ **Fixed cost** $\kappa > 0$ paid at each τ_i
- DM chooses what to learn:
  - $\rightarrow$ Experiment about θ_{τ_i} at each τ_i

- DM chooses when to acquire information:
  - $\rightarrow$ Increasing sequence of times $\{\tau_i\}_{i \in \mathbb{N}}$
  - $\Rightarrow$ **Fixed cost** $\kappa > 0$ paid at each τ_i
- DM chooses what to learn:
  - $\rightarrow$ Distribution over posteriors $F_i \in \mathcal{B}(P_{\tau_i^-})$ at each τ_i
    $\mathcal{B}(p) := \{F \in \Delta\Delta(\Theta) | \int q dF(q) = p\}$

- DM chooses when to acquire information:
  - $\rightarrow$ Increasing sequence of times $\{\tau_i\}_{i \in \mathbb{N}}$
  - $\Rightarrow$ **Fixed cost** $\kappa > 0$ paid at each τ_i
- DM chooses what to learn:
  - $\rightarrow$ Distribution over posteriors $F_i \in \mathcal{B}(P_{\tau_i^-})$ at each τ_i
    $\mathcal{B}(p) := \{F \in \Delta\Delta(\Theta) | \int q dF(q) = p\}$
  - $\Rightarrow$ **Information cost** $C : \Delta\Delta(\Theta) \rightarrow \overline{\mathbb{R}}_+$

INFORMATION ACQUISITION

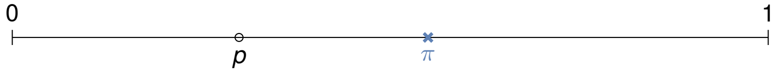
- DM chooses when to acquire information:
  - $\rightarrow$ Increasing sequence of times $\{\tau_i\}_{i\in\mathbb{N}}$
  - $\Rightarrow$ **Fixed cost** $\kappa > 0$ paid at each τ_i
- DM chooses what to learn:
  - $\rightarrow$ Distribution over posteriors $F_i \in \mathcal{B}(P_{\tau_i^-})$ at each τ_i
    $\mathcal{B}(p) := \left\{F \in \Delta\Delta(\Theta) \middle| \int q dF(q) = p\right\}$
  - $\Rightarrow$ **Information cost** $C : \Delta\Delta(\Theta) \to \overline{\mathbb{R}}_+$ uniformly posterior separable

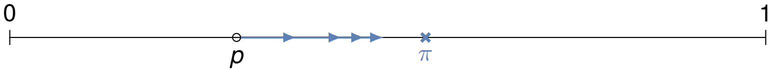
$$C(F) = \int (c(q) - c(p))\, dF(q) \text{ where } p = \int q dF(q)$$

with $c : \Delta(\Theta) \to \overline{\mathbb{R}}_+$ **convex** "measure of *certainty*"

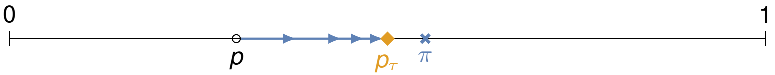
Caplin, Dean, Leahy (2022), Denti (2022), Frankel & Kamenica (2019)

Controlled beliefs dynamics

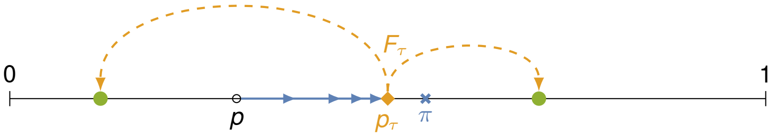




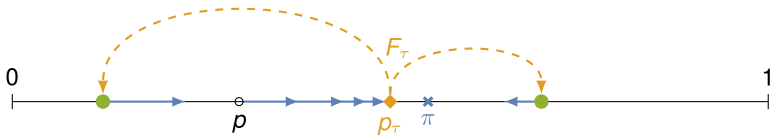
- In time intervals when *no information is acquired*: **Deterministic drift** ($\to \pi$)



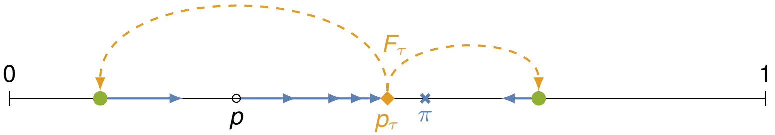
- In time intervals when *no information is acquired*: **Deterministic drift** $(\to \pi)$
- At times τ when *information is acquired*:



- In time intervals when *no information is acquired*: **Deterministic drift** $(\to \pi)$
- At times τ when *information is acquired*: **jumps stochastically to** •



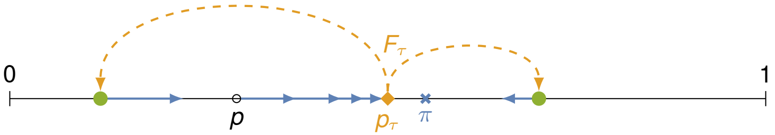
- In time intervals when *no information is acquired*: **Deterministic drift** ($\to \pi$)
- At times τ when *information is acquired*: **jumps stochastically to** ●



- In time intervals when *no information is acquired*: **Deterministic drift** $(\to \pi)$
- At times τ when *information is acquired*: **jumps stochastically to** •

⇒ **Problem:** $\underbrace{\max_{\{\tau_i, F_i\}_{i\geq 0}}}_{\text{dynamic info. acquisition}}$

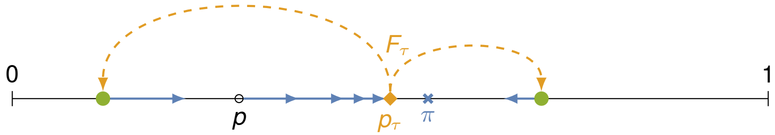
CONTROLLED BELIEFS DYNAMICS



- In time intervals when *no information is acquired*: **Deterministic drift** $(\to \pi)$
- At times τ when *information is acquired*: **jumps stochastically to •**

⇒ **Problem:**

$$\underbrace{\max_{\{\tau_i, F_i\}_{i\geq 0}}}_{\text{dynamic info. acquisition}} \quad \underbrace{\mathbb{E}\left[\int_0^\infty e^{-rt} u(P_t)dt\right]}_{\text{discounted flow utility}}$$

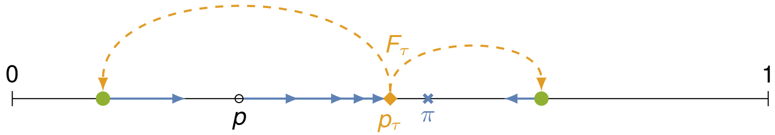


- In time intervals when *no information is acquired*: **Deterministic drift** ($\rightarrow \pi$)
- At times τ when *information is acquired*: **jumps stochastically to** •

$\Rightarrow$ **Problem:**

$$\underbrace{\max_{\{\tau_i, F_i\}_{i\geq 0}}}_{\text{dynamic info. acquisition}} \mathbb{E}\left[\underbrace{\int_0^\infty e^{-rt} u(P_t) dt}_{\text{discounted flow utility}} - \underbrace{\sum_{i\geq 0} e^{-r\tau_i}\Big(C(F_i) + \kappa\Big)}_{\text{discounted costs}}\right]$$

CONTROLLED BELIEFS DYNAMICS



- In time intervals when *no information is acquired*: **Deterministic drift** ($\to \pi$)
- At times τ when *information is acquired*: **jumps stochastically to •**

$$\Rightarrow \textbf{Problem: } v(p) = \underbrace{\max_{\substack{\{\tau_i, F_i\}_{i\geq 0} \\ F_0 \in \mathcal{B}(p)}}}_{\text{dynamic info. acquisition}} \mathbb{E}\left[\underbrace{\int_0^\infty e^{-rt} u(P_t) dt}_{\text{discounted flow utility}} - \underbrace{\sum_{i\geq 0} e^{-r\tau_i}\Big(C(F_i) + \kappa\Big)}_{\text{discounted costs}}\right]$$

II. Characterization of solutions

Recursive characterization

- Recursive logic: solve for next time & experiment given continuation value v.

$$\sup_{\tau \geq 0} \int_0^{\tau} e^{-rt} u(p_t) dt + e^{-r\tau} \left[\sup_{F \in \mathcal{B}(p_\tau)} \left[\int_{\Delta(\Theta)} v(q) dF(q) - C(F) \right] - \kappa \right]$$

Recursive characterization

- Recursive logic: solve for next time & experiment given continuation value v.

$$\sup_{\tau \geq 0} \int_0^\tau e^{-rt} u(p_t) dt + e^{-r\tau} \left[\sup_{F \in \mathcal{B}(p_\tau)} \left[\int_{\Delta(\Theta)} v(q) dF(q) - C(F) \right] - \kappa \right]$$

$\Rightarrow$ Decomposition between **Content** & **Timing**
(static optimal information acquisition & deterministic optimal stopping)

Recursive characterization

- Recursive logic: solve for next time & experiment given continuation value v.

$$v(p) = \sup_{\tau \geq 0} \int_0^\tau e^{-rt} u(p_t) dt + e^{-r\tau} \sup_{F \in \mathcal{B}(p_\tau)} \left[\int_{\Delta(\Theta)} v(q) dF(q) - C(F) - \kappa \right]$$

$\Rightarrow$ Decomposition between **Content** & **Timing**
(static optimal information acquisition & deterministic optimal stopping)

$\Rightarrow$ Familiar Bellman equation **but:** *not a contraction.*

Recursive characterization

- Recursive logic: solve for next time & experiment given continuation value v.

$$v(p) = \sup_{\tau \geq 0} \int_0^{\tau} e^{-rt} u(p_t) dt + e^{-r\tau} \sup_{F \in \mathcal{B}(p_\tau)} \left[\int_{\Delta(\Theta)} v(q) dF(q) - C(F) - \kappa \right]$$

$\Rightarrow$ Decomposition between **Content** & **Timing**
(static optimal information acquisition & deterministic optimal stopping)

$\Rightarrow$ Familiar Bellman equation **but:** *not a contraction.*

⋆ **Theorem:** v is the unique solution to the Bellman equation.
(over a suitable domain of candidate value functions) ▷

Proof: Tarski-style fixed point on order-interval of a Riesz space + convexity
(Marinacci and Montrucchio, MOR 2019)

Optimal policies

- Optimal experiments

$$\mathcal{G}v(p) := \max_{F \in \mathcal{B}(p)} \int v dF - C(F)$$

$\Rightarrow$ Residual value of information $\mathcal{G}v(p) - v(p) \in [0, \kappa]$

thm. $\triangleright$

Optimal policies

- Optimal experiments via net value $v - c$

$$\mathcal{G}v(p) - c(p) = \max_{F \in \mathcal{B}(p)} \int [v - c]dF = \text{Cav}[v - c](p)$$

$\Rightarrow$ Residual value of information $\text{Cav}[v - c](p) - [v - c](p) \in [0, \kappa]$

$\Rightarrow$ Binary experiments spt. over nearest beliefs s.t. $\text{Cav}[v - c] = v - c$

Kamenica, Gentzkow (2011, 2014); Caplin, Dean, Leahy (2022)

thm. ▷

Optimal policies

- Optimal experiments via net value $v - c$

$$\mathcal{G}v(p) - c(p) = \max_{F \in \mathcal{B}(p)} \int [v - c] dF = \text{Cav}[v - c](p)$$

$\Rightarrow$ Residual value of information $\text{Cav}[v - c](p) - [v - c](p) \in [0, \kappa]$

$\Rightarrow$ Binary experiments spt. over nearest beliefs s.t. $\text{Cav}[v - c] = v - c$

Kamenica, Gentzkow (2011, 2014); Caplin, Dean, Leahy (2022)

- Optimal timing via net recursive form

$$[v - c](p) = \sup_{\tau \geq 0} \int_0^{\tau} e^{-rt} f(p_t) dt + e^{-r\tau} \left(\text{Cav}[v - c](p_\tau) - \kappa \right)$$

where $f(p) := u(p) - rc(p) + \lambda(\pi - p)c'(p)$

$\Rightarrow$ Acquire information when $v - c \leq \text{Cav}[v - c] - \kappa$

thm. ▷

Optimal policies

- Optimal experiments via net value w

$$\max_{F \in \mathcal{B}(p)} \int w dF = \text{Cav}[w](p)$$

$\Rightarrow$ Residual value of information $\text{Cav}[w] - w \in [0, \kappa]$

$\Rightarrow$ Binary experiments spt. over nearest beliefs s.t. $\boxed{\text{Cav}[w] - w = 0}$

- Optimal timing via net recursive form

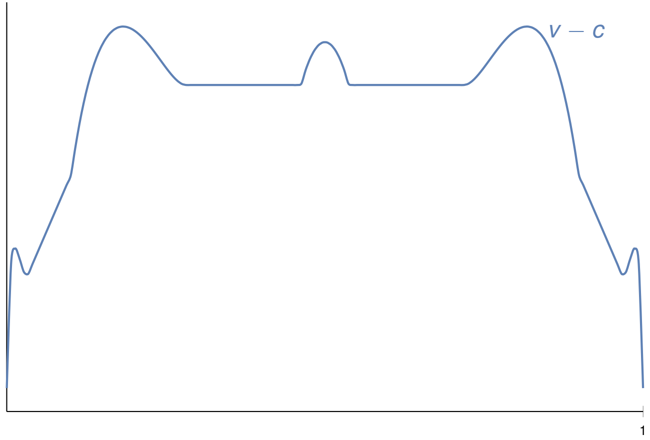
$$w(p) = \sup_{\tau \geq 0} \int_0^\tau e^{-rt} f(p_t) dt + e^{-r\tau} \Big(\text{Cav}[w](p_\tau) - \kappa \Big)$$

where $f(p) := u(p) - rc(p) + \lambda(\pi - p)c'(p)$

$\Rightarrow$ Acquire information when $\boxed{\text{Cav}[w] - w = \kappa}$

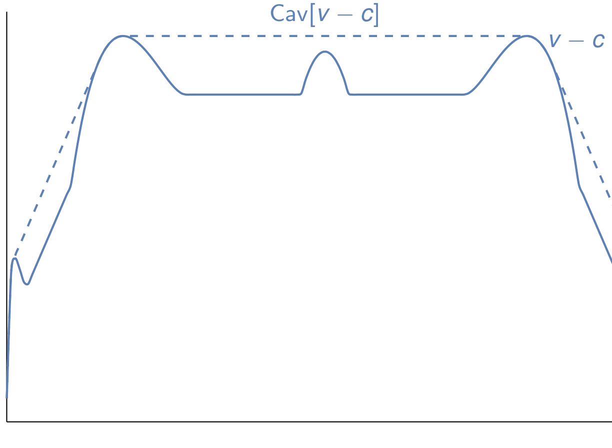
thm. ▷

Net Value and Optimal Information: Illustration



u ▷ thm. ▷

Net Value and Optimal Information: Illustration

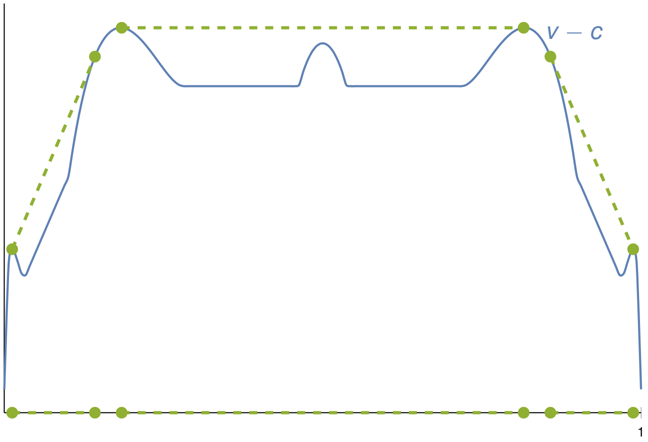


Gross value of information iff $\text{Cav}[v - c] > v - c$

(⇝ one-shot or residual value)

u ▷ thm. ▷

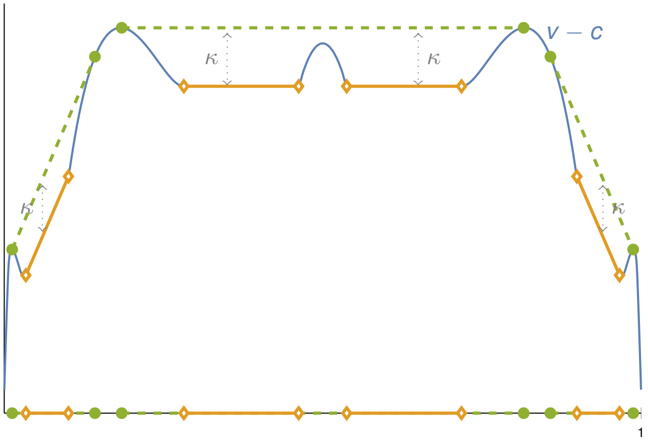
NET VALUE AND OPTIMAL INFORMATION: ILLUSTRATION



Optimal binary experiments supported on tangency points of Cav$[v - c]$ & $v - c$

u ▷ thm. ▷

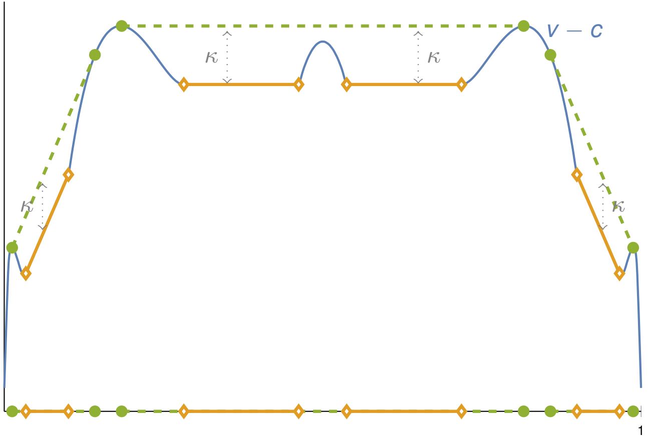
NET VALUE AND OPTIMAL INFORMATION: ILLUSTRATION



Information acquisition at $p \Longleftrightarrow \text{Cav}[v-c](p) - [v-c](p) = \kappa$

u ▷ thm. ▷

Net Value and Optimal Information: Illustration



Policy summarized by:

Collection of experiment intervals $\mathcal{E}$ & Info. Acquisition Region $\mathcal{I}$

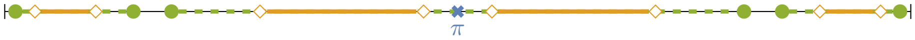
with $\mathcal{I} \subset \mathcal{E}$

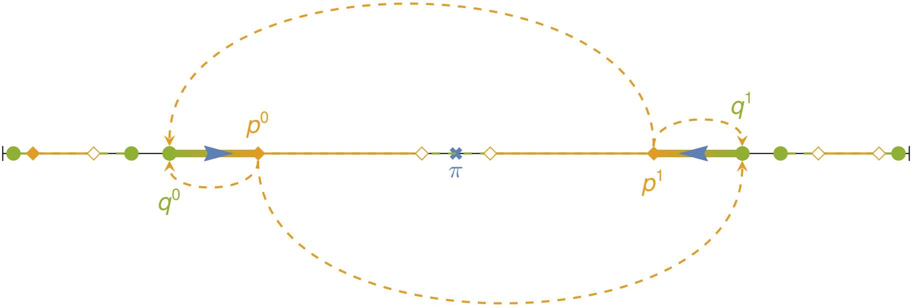
u ▷ thm. ▷

III. Dynamics of Information Acquisition

Cyclical Information Acquisition

Illustration



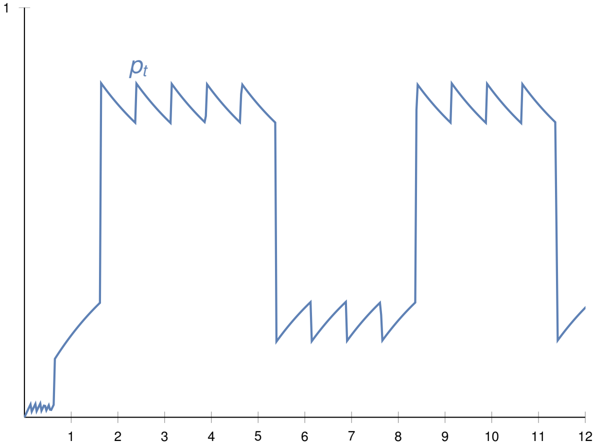


$\longrightarrow$ Belief cycle: $\left((q^0, q^1), (p^0, p^1), (\tau^0, \tau^1)\right)$

where τ^i s.t. $q^i_{\tau^i} = p^i \iff \tau^i = \frac{1}{\lambda} \log \frac{q^i - \pi}{p^i - \pi}$

Belief sample paths

Illustration



Sample path for **beliefs**

⋆ **THEOREM:** There exists an a.s. finite time such that either:

(A) Learning stops

(B) Informations acquisition follows a cycle

⋆ **THEOREM:** There exists an a.s. finite time such that either:

(A) Learning stops

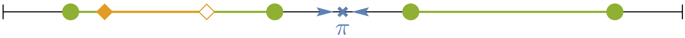
(B) Informations acquisition follows a cycle

- Intuition:
  - When belief jump inwards they "never return"
  - Keep getting closer to π
  - Must fall in a cycle or stop
- *Formally:* Kolmogorov's 0-1 law $\rightsquigarrow$ (not) eventually entering "cycle" region is a tail event

When does learning stop?

prop. ▷ ex. ▷
skip to takeaways ▷

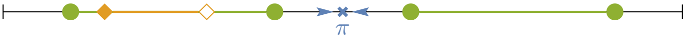
1. No profitable experiment at π



prop. ▷ ex. ▷
skip to takeaways ▷

When does learning stop?

1. No profitable experiment at π

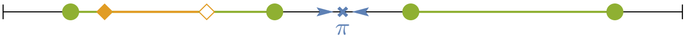


2. Information not sustainable around π

prop. ▷ ex. ▷
skip to takeaways ▷

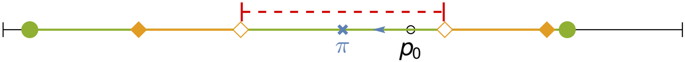
When does learning stop?

1. No profitable experiment at π



2. Information not sustainable around π

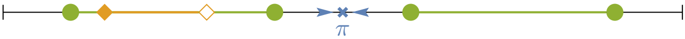
3. "Learning Trap"



prop. ▷ ex. ▷
skip to takeaways ▷

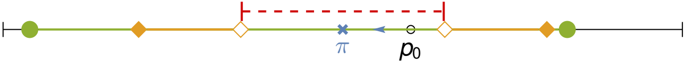
When does learning stop?

1. No profitable experiment at $\pi \Rightarrow$ learning stops $\forall p_0$



2. Information not sustainable around $\pi \Rightarrow$ learning stops $\forall p_0$

3. "Learning Trap" $\Rightarrow$ path dependency



⚠ A "trap" can only occur with both:

- fixed cost $\kappa > 0$ high enough
- variable cost $c > 0$ convex enough.

prop. ▷ ex. ▷
skip to takeaways ▷

Long Run Dynamics: Takeaways

- Many "static" intuitions break in general but hold *in the long run*

    e.g. *"no information acquisition that a.s. does not lead to changing action"* ▷

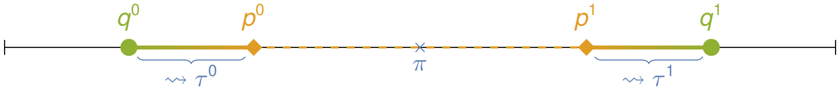
Long Run Dynamics: Takeaways

- Many "static" intuitions break in general but hold *in the long run*

  e.g. *"no information acquisition that a.s. does not lead to changing action"*

- Everything reduces to characterizing **belief cycles**

$\Longrightarrow$ A simple approach to compare "informativeness" of dynamic process



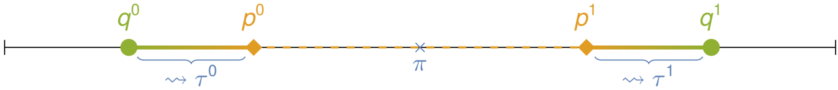
Long Run Dynamics: Takeaways

- Many "static" intuitions break in general <u>but</u> hold *in the long run*

  e.g. *"no information acquisition that a.s. does not lead to changing action"* ▷

- Everything reduces to characterizing **belief cycles**

⟹ A simple approach to compare "informativeness" of dynamic process ▷

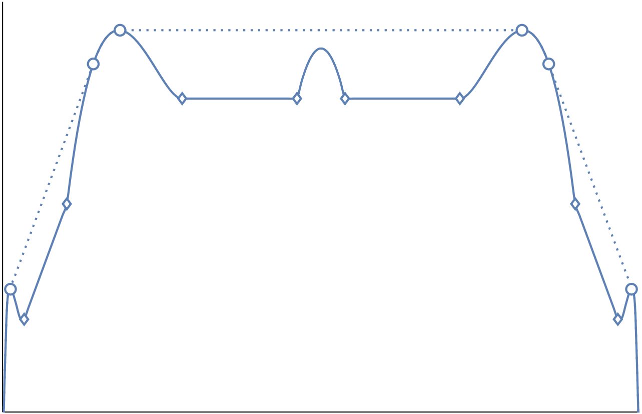


→ *In the paper:*

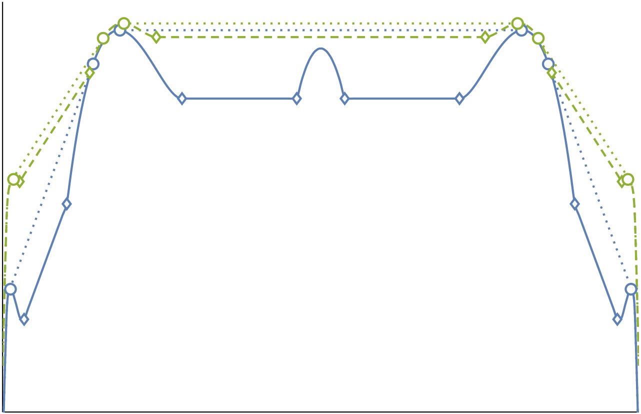
- ∗ Closed form expression for long run payoffs & stationary problem ▷
- ∗ Frequency of information acquisition non-monotonic in volatility ▷
- ∗ Short run info. acquisition amounts to accelerating or delaying convergence
- ∗ Characterization of long run distribution of beliefs

IV. Vanishing fixed costs

Decreasing and Vanishing Fixed Costs: Intuition



Decreasing and Vanishing Fixed Costs: Intuition



Decreasing fixed costs ⇒ *lower residual value of information* $\text{Cav}[v - c] - [v - c]$

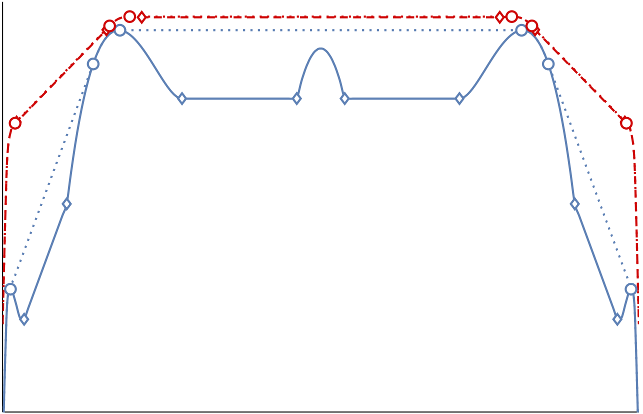
≈ "less waiting necessary if profitable experiment"

≈ "threshold & target beliefs get closer to one another"

⇝ less likely to jump to furthest belief

overview ▷ 0 ▷ 1 ▷ 2 ▷ 3 ▷

DECREASING AND VANISHING FIXED COSTS: INTUITION



Vanishing fixed costs ⇒ *no residual value of information* $\text{Cav}[v-c]-[v-c]$

no waiting necessary if profitable experiment

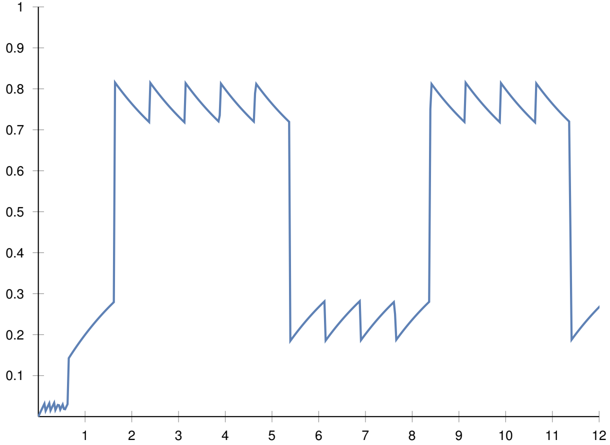
threshold & target beliefs converge to one another

⇒ infinitesimal prob. to jump to furthest belief

Example: Decreasing Fixed Costs

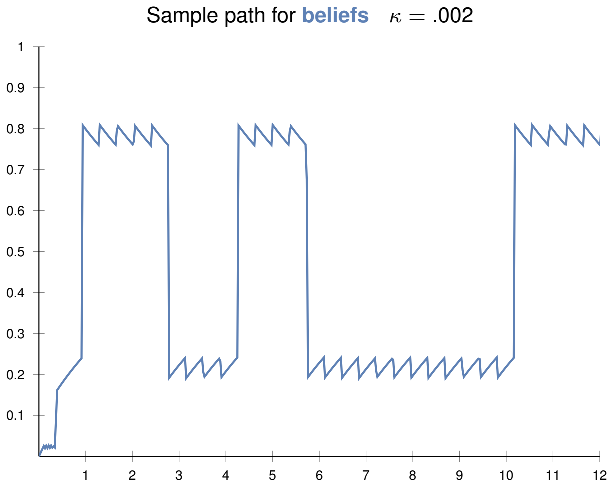
Belief paths as fixed costs vanish

Sample path for **beliefs** $\kappa = .01$



Example: Decreasing Fixed Costs

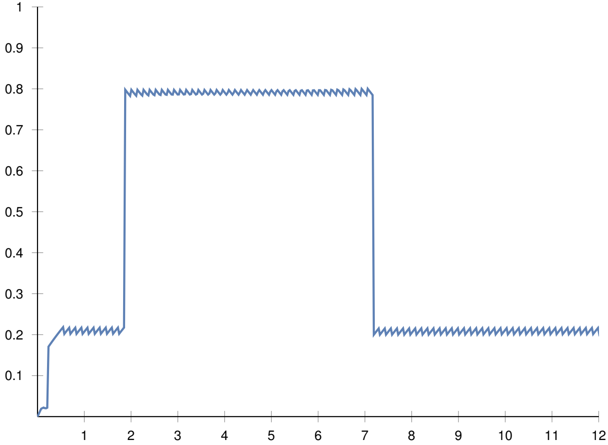
Belief paths as fixed costs vanish



EXAMPLE: DECREASING FIXED COSTS

BELIEF PATHS AS FIXED COSTS VANISH

Sample path for **beliefs** $\kappa = .0001$

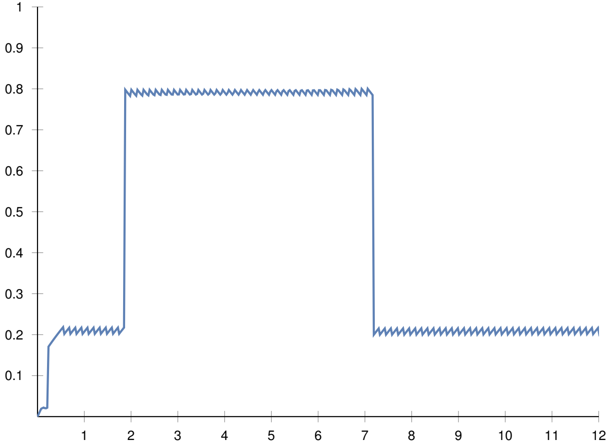


overview ▷ 0 ▷ 1 ▷ 2 ▷

Example: Decreasing Fixed Costs

Belief paths as fixed costs vanish

Sample path for **beliefs** $\kappa = .0001$



$\Longrightarrow$ In the limit, lumpy adjustments of beliefs **and actions**

overview ▷ 0 ▷ 1 ▷ 2 ▷

VANISHING FIXED COSTS: OVERVIEW

- Difficulties beyond intuition
  - Interval structure: no monotonicity & hard to "keep track" of intervals
  - Change in $v \not\Leftrightarrow$ change in value of info & thresholds
  - "$\kappa = 0$"? $\rightsquigarrow$ class of policies not closed, costs not defined, non-uniqueness in limit

Vanishing Fixed Costs: Overview

- Difficulties beyond intuition
  - Interval structure: no monotonicity & hard to "keep track" of intervals
  - Change in $v \not\Leftrightarrow$ change in value of info & thresholds
  - "$\kappa = 0$"? $\rightsquigarrow$ class of policies not closed, costs not defined, non-uniqueness in limit

▷ Preliminaries:

(i) Rewrite the problem explicitly over distribution of belief process
measure over càdlàg paths $+$ compensated martingale condition

(ii) Extend objective function continuously over entire space (with $\kappa \geq 0$)

(iii) Limit of solutions as $\kappa \downarrow 0$ is a solution of the limit problem (despite discontinuity) ▷

VANISHING FIXED COSTS: OVERVIEW

- Difficulties beyond intuition
  - Interval structure: no monotonicity & hard to "keep track" of intervals
  - Change in $v \not\Leftrightarrow$ change in value of info & thresholds
  - "$\kappa = 0$"? $\rightsquigarrow$ class of policies not closed, costs not defined, non-uniqueness in limit

▷ Preliminaries:

(i) Rewrite the problem explicitly over distribution of belief process
measure over càdlàg paths + compensated martingale condition

(ii) Extend objective function continuously over entire space (with $\kappa \geq 0$)

(iii) Limit of solutions as $\kappa \downarrow 0$ is a solution of the limit problem (despite discontinuity) ▷

⋆ Results:

1. Optimality of "wait-or-confirm" policies in the limit problem ▷
   & optimal policies as $\kappa \downarrow 0$ must converge to this class ▷

   ≈ limit of recursive analysis **&** simplified non-recursive form

2. Explicit characterization of optimal policy in the long run when $\kappa = 0$ ▷

   in terms of the concave envelope of "virtual net flow payoff" ▷

conclusion ▷

LIMIT PROBLEM

- Rewrite the problem over belief processes:
  - feasible belief processes (equipped with Skorohod topology)

$$\mathbb{B}(p) := \Big\{ (P_t)_{t\geq 0} \text{ càdlàg process in } [0,1] \;\Big|\; \mathbb{E}[P_0] = p, \; \forall t, s \geq 0, \mathbb{E}[P_{s+t}|P_s] = e^{-\lambda t} P_s + (1 - e^{-\lambda t})\pi \text{ a.s.} \Big\}$$

$$\mathbb{B}_d(p) := \text{ subset of } \mathbb{B}(p) \text{ generated by discrete information acquisition}$$

◁

Limit problem

- Rewrite the problem over belief processes:
  - feasible belief processes (equipped with Skorohod topology)

$$\mathbb{B}(p) := \Big\{ (P_t)_{t\geq 0} \text{ càdlàg process in } [0,1] \;\Big|\; \mathbb{E}[P_0] = p,$$
$$\forall t, s \geq 0, \mathbb{E}[P_{s+t}|P_s] = e^{-\lambda t} P_s + (1 - e^{-\lambda t})\pi \text{ a.s.} \Big\}$$
$$\mathbb{B}_d(p) := \text{ subset of } \mathbb{B}(p) \text{ generated by discrete information acquisition}$$

  - rewrite objective in terms of measure in $\mathbb{B}_d$
    payoffs and costs *linear* in measure, unif. continuous over dense subset of Polish space

◁ details ▷

Limit problem

- Rewrite the problem over belief processes:
  - feasible belief processes (equipped with Skorohod topology)

$$\mathbb{B}(p) := \left\{ (P_t)_{t\geq 0} \text{ càdlàg process in } [0,1] \;\middle|\; \mathbb{E}[P_0] = p, \right.$$
$$\left. \forall t, s \geq 0, \mathbb{E}[P_{s+t}|P_s] = e^{-\lambda t} P_s + (1 - e^{-\lambda t})\pi \text{ a.s.} \right\}$$
$$\mathbb{B}_d(p) := \text{ subset of } \mathbb{B}(p) \text{ generated by discrete information acquisition}$$

  - rewrite objective in terms of measure in $\mathbb{B}_d$
    payoffs and costs *linear* in measure, unif. continuous over dense subset of Polish space

⇒ unique continuous extension of cost over whole space
extends generator-based costs: Zhong '22, Hébert & Woodford '23, Georgiadis-Harris '24

◁ details ▷

Limit problem

- Rewrite the problem over belief processes:
  - feasible belief processes (equipped with Skorohod topology)

$$\mathbb{B}(p) := \Big\{ (P_t)_{t\geq 0} \text{ càdlàg process in } [0,1] \;\Big|\; \mathbb{E}[P_0] = p,$$
$$\forall t, s \geq 0, \mathbb{E}[P_{s+t}|P_s] = e^{-\lambda t} P_s + (1 - e^{-\lambda t})\pi \text{ a.s.} \Big\}$$

$$\mathbb{B}_d(p) := \text{ subset of } \mathbb{B}(p) \text{ generated by discrete information acquisition}$$

  - rewrite objective in terms of measure in $\mathbb{B}_d$
    payoffs and costs *linear* in measure, unif. continuous over dense subset of Polish space

  ⇒ unique continuous extension of cost over whole space
  extends generator-based costs: Zhong '22, Hébert & Woodford '23, Georgiadis-Harris '24

- Solutions over $\mathbb{B}$ and $\mathbb{B}_d$ coincide exactly ($\kappa > 0$) or approximately ($\kappa = 0$)

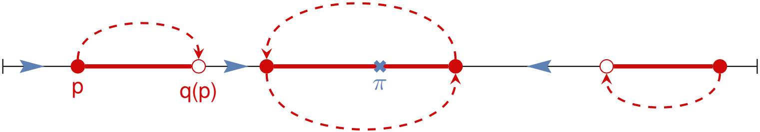
- Limit of solutions (as $\kappa \downarrow 0$) is solution of the limit ($\kappa = 0$)
  Uses epi-convergence since cost not continuous in κ at 0 (limits order problem)

▷

◁ details ▷

Wait-or-confirm policies

- **Define:** $P \sim \text{WoC}_{p_0}[\mathcal{I}]$ "wait-or-confirm" belief process such that:
  - beliefs drift until they hit the boundary of interval in $\mathcal{I}$
  - then stay at boundary point until random jump to closest point not in $\mathcal{I}$
  - $P_0 = p_0$ if $p_0 \notin \mathcal{I}$, otherwise immediate jump to closest points not in $\mathcal{I}$



⟶ simple class of policy which always either "waits" or "confirms"

⟶ jump rates pinned down by compensated martingale constraint:

$$\lambda \left| \frac{\pi - p}{\pi - q(p)} \right|$$

◁ thm. ▷ formal def. ▷

Optimality and convergence

⋆ **Theorem:** w_0 is concave and:

1. *(optimality)* $P \sim \mathrm{WoC}_p\left[\mathrm{int}L_0\right]$ is optimal, where:

$$L_0 := \left\{ p \in [0,1] \;\middle|\; w_0 \text{ locally linear} \right\};$$

2. *(convergence)* Assume $P^\kappa \to P$, then:

$$P \sim \mathrm{WoC}_p\left[\liminf_{\kappa \downarrow 0} \mathcal{I}_\kappa\right]$$

3. *(relation and local uniqueness)* for all p s.t. w_0 is *stly* concave (in the dir. of π), it is uniquely optimal not to acquire information; further:

$$\liminf_{\kappa \downarrow 0} \mathcal{I}_\kappa \subset \mathrm{int}L_0$$

- Poisson structure reminiscent of Zhong '22, Hébert & Woodford '23, Georgiadis-Harris '24
- Natural selection when non-uniqueness in the limit (extreme case: f linear)

THE VIRTUAL FLOW PAYOFF

- For any $\kappa \geq 0$, let value v_κ & *net* value $w_\kappa := v_\kappa - c$
- "Virtual flow payoff":

$$f(p) := u(p) - rc(p) + \lambda(\pi - p)c'(p)$$

◁ overview ◁ f ▷

THE VIRTUAL FLOW PAYOFF

- For any $\kappa \geq 0$, let value v_κ & *net* value $w_\kappa := v_\kappa - c$
- "Virtual flow payoff":

$$f(p) := u(p) - rc(p) + \lambda(\pi - p)c'(p)$$

LEMMA:

$$w_0(p) = \sup_{P \in \mathbb{B}(p)} \mathbb{E}\left[\int_0^\infty e^{-rt} f(P_t)dt\right]$$

◁ overview ◁ f ▷

THE VIRTUAL FLOW PAYOFF

- For any $\kappa \geq 0$, let value v_κ & *net* value $w_\kappa := v_\kappa - c$
- "Virtual flow payoff":

$$f(p) := u(p) - rc(p) + \lambda(\pi - p)c'(p)$$

LEMMA:

$$w_0(p) = \sup_{P \in \mathbb{B}(p)} \mathbb{E}\left[\int_0^\infty e^{-rt} f(P_t)dt\right]$$

- Proof: decompose $\kappa > 0$ problem, rewrite normalizing term $e^{-rt}c(P_{t-})$ as integral, then use epi-continuity of modified cost and density. (only a useful trick for $\kappa = 0$)

◁ overview ◁ f ▷

Explicit long run characterization

⋆ **Theorem:** In the information acquisition problem with no fixed costs:

EXPLICIT LONG RUN CHARACTERIZATION

⋆ **THEOREM:** In the information acquisition problem with no fixed costs:

- If $\text{Cav}[f](\pi) = f(\pi)$: optimal to not acquire any information in the long run

  uniquely so if f is not locally affine at π.

◁ overview ◁ f

⋆ **THEOREM:** In the information acquisition problem with no fixed costs:

- If $\text{Cav}[f](\pi) = f(\pi)$: optimal to not acquire any information in the long run
  uniquely so if f is not locally affine at π.
- If $\text{Cav}[f] > f$ in interval (q_0^f, q_1^f) around π (s.t. $f = \text{Cav}[f]$ at q_0^f, q_1^f):

◁ overview ◁ f

Explicit long run characterization

⋆ **Theorem:** In the information acquisition problem with no fixed costs:

- If $\text{Cav}[f](\pi) = f(\pi)$: optimal to not acquire any information in the long run

  uniquely so if f is not locally affine at π.

- If $\text{Cav}[f] > f$ in interval (q_0^f, q_1^f) around π (s.t. $f = \text{Cav}[f]$ at q_0^f, q_1^f):

$$\forall p \in [q_0^f, q_1^f], \;\; w_0(p) = \int_0^\infty e^{-rt}\, \text{Cav}[f]\big(e^{-\lambda t} p + (1 - e^{-\lambda t})\pi\big)\, dt$$

◁ overview ◁ f

EXPLICIT LONG RUN CHARACTERIZATION

⋆ **THEOREM:** In the information acquisition problem with no fixed costs:

- If $\text{Cav}[f](\pi) = f(\pi)$: optimal to not acquire any information in the long run
  uniquely so if f is not locally affine at π.
- If $\text{Cav}[f] > f$ in interval (q_0^f, q_1^f) around π (s.t. $f = \text{Cav}[f]$ at q_0^f, q_1^f):

$$\forall p \in [q_0^f, q_1^f], \quad w_0(p) = \int_0^\infty e^{-rt}\, \text{Cav}[f]\big(e^{-\lambda t} p + (1 - e^{-\lambda t})\pi\big)\, dt$$

  and the belief process which consists in:

  - Jumping from q_0^f to q_1^f at rate $\lambda \frac{\pi - q_0^f}{q_1^f - q_0^f}$
  - Jumping from q_1^f to q_0^f at rate $\lambda \frac{q_1^f - \pi}{q_1^f - q_0^f}$
    & jumping immediately to $\{q_0^f, q_1^f\}$ from any $p \in (q_0^f, q_1^f)$

  is **uniquely optimal**.

- **Costly info. acquisition with changing states**

  Dynamic rational inattention: Sims (2003); Steiner, Stewart, Matějka (2017); Maćkowiak, Matějka, Wiederholt (2018); Afrouzi, Yang (2021)

  Inattentiveness: Reis (2006)

  Experimental: Khaw, Stevens, Woodford (2017)

- **Learning from actions in a changing world**

  Bandits w/ exogenously evolving arms: Whittle (1988); Che, Kim and Mierendorff (2022)

  Social learning w/ changing states: Ottaviani, Moscarini, Smith (1998); Dasaratha, Golub, Hak (2023); Lévy, Pęski, Vieille (2024)

- **Applications**

  Info. acquisition in finance: Van Nieuwerburgh, Veldkamp (2010); Sicherman et al. (2016)

- **Info. acquisition with persistent states / stopping**

  Wald (1947); Che, Mierendorff (2019); Zhong (2022); Hébert, Woodford (2023); Georgiadis-Harris (2024)

This paper:

- ▷ Infrequency + flexibility
- ▷ More general costs & environment
- ▷ Info. acquisition dynamics
- ▷ Costly info. acquisition
- ▷ Forward-looking
- ▷ Dynamics of diversification
- ▷ Info. driven "biases"
- ▷ Changing states
- ▷ Repeated decisions

Conclusion

- **Rich yet tractable model** of infrequent info. acquisition about a changing state.

  $\Longrightarrow$ simple characterization of solutions and dynamics

Conclusion

- **Rich yet tractable model** of infrequent info. acquisition about a changing state.
  $\Longrightarrow$ simple characterization of solutions and dynamics

- More questions & applications
  - e.g. asymmetries in updating after good vs bad news
    model can produce rational behavior $\approx$ short run confirmation bias example ▷
  - e.g. adding a safe action can create traps and path dependency

CONCLUSION

- **Rich yet tractable model** of infrequent info. acquisition about a changing state.
  $\Longrightarrow$ simple characterization of solutions and dynamics

- More questions & applications
  - e.g. asymmetries in updating after good vs bad news
    model can produce rational behavior $\approx$ short run confirmation bias example ▷
  - e.g. adding a safe action can create traps and path dependency

- Generalizability
  - Fixed point methodology extends to arbitrary costs and N states
  - Characterization with UPS costs extends to N states
  - Long run behavior generalizes *in some cases*; other richer behavior can appear

CONCLUSION

- **Rich yet tractable model** of infrequent info. acquisition about a changing state.
  $\Longrightarrow$ simple characterization of solutions and dynamics

- More questions & applications
  - e.g. asymmetries in updating after good vs bad news
    model can produce rational behavior $\approx$ short run confirmation bias example ▷
  - e.g. adding a safe action can create traps and path dependency

- Generalizability
  - Fixed point methodology extends to arbitrary costs and N states
  - Characterization with UPS costs extends to N states
  - Long run behavior generalizes *in some cases*; other richer behavior can appear

- Many natural extensions
  Strategic settings, design questions, distributional aspects, switching costs,...

◁ lit.

Thank you !

DEFINITION:: let $\overline{v}$ and $\underline{v}$ the value from perfect costless observation of the state and from never getting any information, from initial belief $p \in \Delta(\Theta)$

$$\overline{v}(p) := \int_0^\infty e^{-rt}\mathbb{E}_{\theta\sim p_t}\left[\max_a u(a,\theta)\right]dt,$$

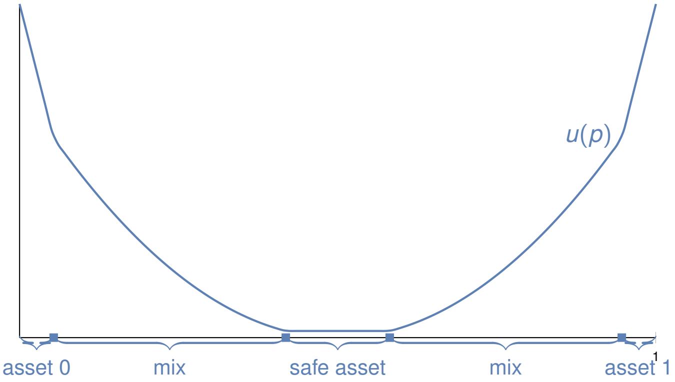
$$\underline{v}(p) := \int_0^\infty e^{-rt}u(p_t)dt.$$

The set of candidate value functions $\mathbb{V}$ is the set of real-valued bounded measurable functions on $\Delta(\Theta)$ which are pointwise between $\underline{v}$ and $\overline{v}$.

◁ go back

Running example: portfolio allocation

- Risk averse investor with unit budget
- 2 risky assets + 1 safe asset
- θ_t indicates which asset is best to hold; changes at rate $\lambda/2 = \lambda_0 = \lambda_1$ ($\Rightarrow \pi = 1/2$)



◁ back

OPTIMAL INFORMATION ACQUISITION

- **Define** the gross value of information operator:

$$\mathcal{G}v(p) := \text{Cav}[v-c](p) - [v-c](p)$$

⋆ **THEOREM:** The following policy is optimal:

- Updating when the gross value of information equals the fixed cost

$$\tau^*(p) := \inf\left\{ t \geq 0 \,\middle|\, \mathcal{G}v(p_t) = \kappa \right\}$$

- Binary experiments supported over closest points s.t. no gross value of info.

$$\text{spt}(F_p^*) = \{q^0, q^1\} \text{ with } q^0 \leq p \leq q^1 \text{ and } \mathcal{G}v(q^i) = 0$$

$\Longrightarrow$ Optimal policy described by:

$$\mathcal{E} := \{p | \mathcal{G}v(p) > 0\} \;\&\; \mathcal{I} := \{p | \mathcal{G}v(p) = \kappa\}$$

◁ back

When does learning stop?

∗ **Proposition:** under optimal dynamics:

(i) Learning stops in finite time only if $\pi \notin \mathcal{I}$

(ii) If $\pi \notin \Gamma$, information is acquired at most a finite number of times.

(iii) If $\pi \in \Gamma \setminus \mathcal{I}$, either:

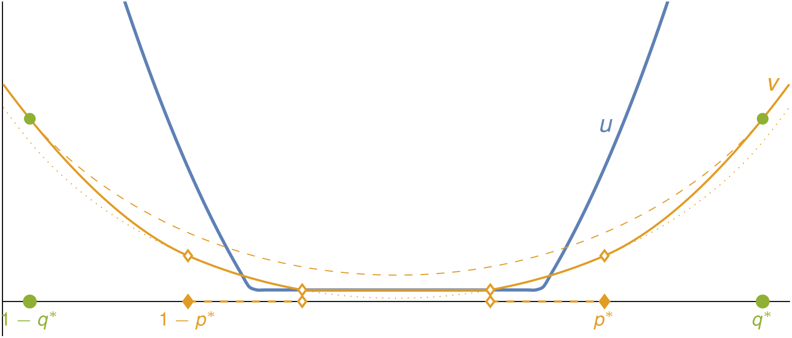
a. information acquisition is acquired at most a finite number of times for all priors

b. there exists $(\underline{p}, \overline{p})$ s.t. no information is ever acquired for $p_0 \in (\underline{p}, \overline{p})$ but any $p_0 \notin (\underline{p}, \overline{p})$ leads to a belief cycle in the long run.

◁ back

Example: Cyclical diversification

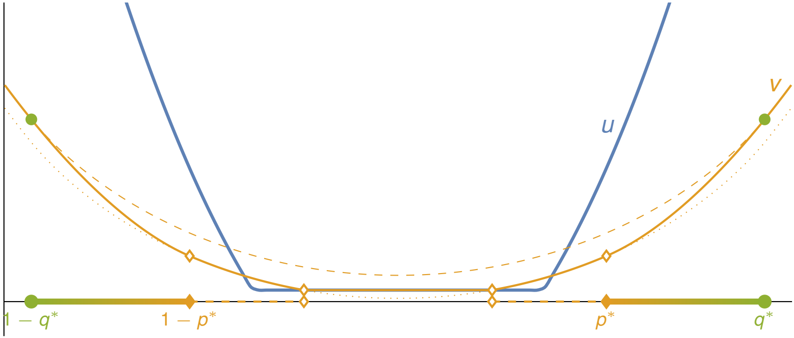
Optimal cycle



◁ back ◁ to takeaways

Example: Cyclical diversification

Optimal cycle

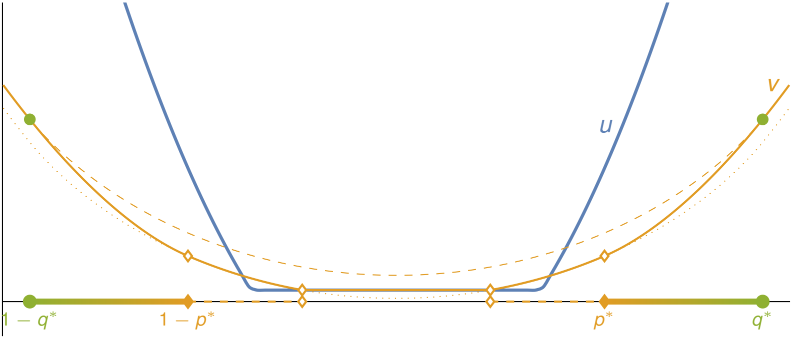


- Continuous rebalancing & increased diversification as become more uncertain

◁ back ◁ to takeaways

Example: Cyclical diversification

Optimal cycle

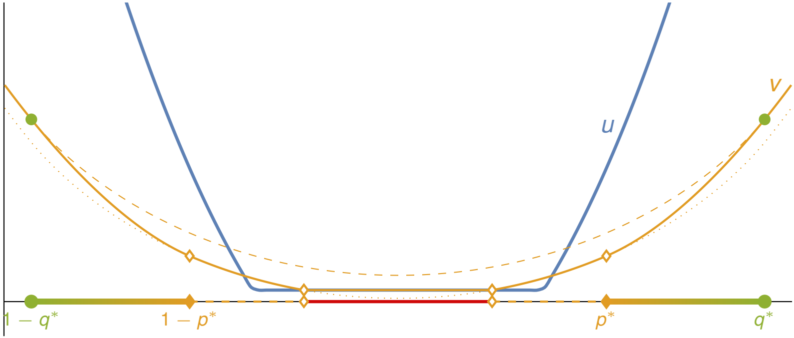


- Continuous rebalancing & increased diversification as become more uncertain
- Interrupted by periodic jumps to more extreme portfolio when information is acquired

◁ back ◁ to takeaways

EXAMPLE: CYCLICAL DIVERSIFICATION

OPTIMAL CYCLE

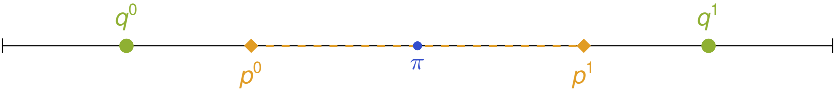


- Continuous rebalancing & increased diversification as become more uncertain
- Interrupted by periodic jumps to more extreme portfolio when information is acquired
- ...Unless **initial beliefs too uninformed** $\Rightarrow$ only buy the safe asset, never acquire info.

◁ back ◁ to takeaways

Compare long run dynamics $\longrightarrow$ more or less information acquisition?

Fix belief cycle $\Upsilon = (q^i, p^i, \tau^i)$

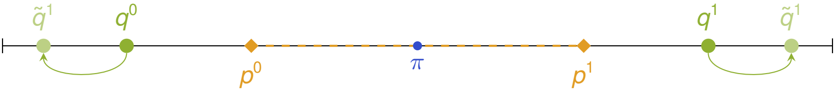


◁ back to takeaways

COMPARING BELIEF CYCLES

Compare long run dynamics $\longrightarrow$ more or less information acquisition?

Fix belief cycle $\Upsilon = (q^i, p^i, \tau^i)$



- $\tilde{\Upsilon} = (\tilde{q}^i, \tilde{p}^i, \tilde{\tau}^i)$ has **more informative experiments** than Υ if:

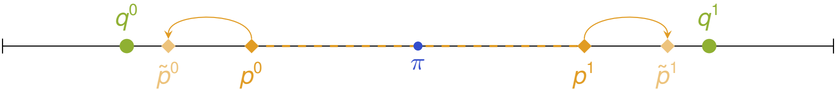
$$|\tilde{q}^i - \pi| \geq |q^i - \pi| \text{ for } i = 0, 1$$

◁ back to takeaways

Comparing belief cycles

Compare long run dynamics $\longrightarrow$ more or less information acquisition?

Fix belief cycle $\Upsilon = (q^i, p^i, \tau^i)$



- $\tilde{\Upsilon} = (\tilde{q}^i, \tilde{p}^i, \tilde{\tau}^i)$ has **more informative experiments** than Υ if:

$$|\tilde{q}^i - \pi| \geq |q^i - \pi| \text{ for } i = 0, 1$$

- $\tilde{\Upsilon} = (\tilde{q}^i, \tilde{p}^i, \tilde{\tau}^i)$ has **higher certainty thresholds for updates** than Υ if:

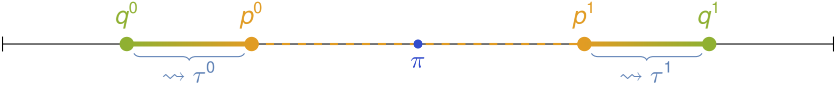
$$|\tilde{p}^i - \pi| \geq |p^i - \pi| \text{ for } i = 0, 1$$

◁ back to takeaways

Comparing belief cycles

Compare long run dynamics $\longrightarrow$ more or less information acquisition?

Fix belief cycle $\Upsilon = (q^i, p^i, \tau^i)$



- $\tilde{\Upsilon} = (\tilde{q}^i, \tilde{p}^i, \tilde{\tau}^i)$ has **more informative experiments** than Υ if:

$$|\tilde{q}^i - \pi| \geq |q^i - \pi| \text{ for } i = 0, 1$$

- $\tilde{\Upsilon} = (\tilde{q}^i, \tilde{p}^i, \tilde{\tau}^i)$ has **higher certainty thresholds for updates** than Υ if:

$$|\tilde{p}^i - \pi| \geq |p^i - \pi| \text{ for } i = 0, 1$$

- $\tilde{\Upsilon} = (\tilde{q}^i, \tilde{p}^i, \tilde{\tau}^i)$ has **more frequent information acquisition** than Υ if:

$$\tilde{\tau}^i \leq \tau^i \text{ for } i = 0, 1$$

◁ back to takeaways

Effect of volatility

$*$ **Proposition:** In any symmetric problem, the frequency (τ^{-1}) of information acquisition is **non-monotonic** in the volatility λ of the environment:

- It is increasing for persistent enough states (λ close to 0)
- It is decreasing for volatile enough states (λ high enough)

◁ back to takeaways

Effect of volatility

∗ **Proposition:** In any symmetric problem, the frequency (τ^{-1}) of information acquisition is **non-monotonic** in the volatility λ of the environment:

- It is increasing for persistent enough states (λ close to 0)
- It is decreasing for volatile enough states (λ high enough)

- Intuition:
  - Close enough to persistence, informational effects dominate
  - For high enough volatility, transience dominates

- *Often* single-peaked in examples
  but precise conditions difficult to obtain

◁ back to takeaways

Effect of volatility – Example

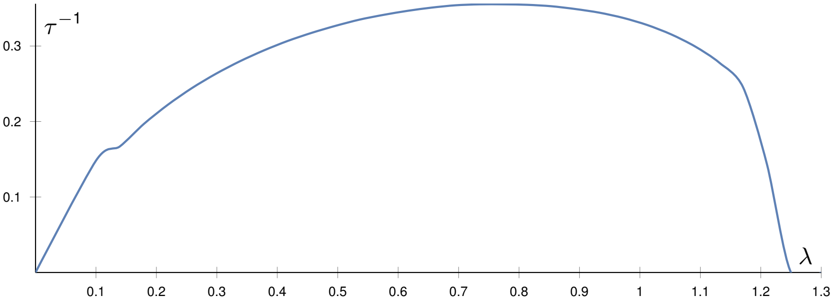


Figure: **Frequency of information acquisition as a function of volatility**

$u(p) = \max\{p, 1-p\}$, $r = 1$, $\kappa = .05$, entropy costs

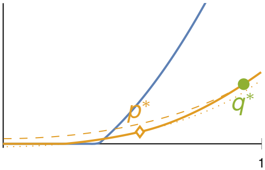
◁ back to takeaways

wiggly example ▷

payoffs & quality ▷

Optimization over cyclical behavior

- Problem reduces to optimization directly over cycles
  (closed form expressions for payoffs)



- Simplest in the class of **symmetric problems**
  (i.e invariant to relabeling states)

- Long run payoffs:

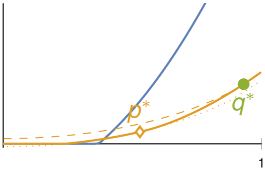
$$v_S(\Upsilon) := (1 - e^{-r\tau})^{-1} \left(\int_0^{\tau} e^{-rt} u(q_t) dt - e^{-r\tau} \big(c(q) - c(p) + \kappa \big) \right)$$

where $\Upsilon = (q, p, \tau)$

◁ takeaways ◁ λ

Optimization over cyclical behavior

- Problem reduces to optimization directly over cycles
  (closed form expressions for payoffs)



- Simplest in the class of **symmetric problems**
  (i.e invariant to relabeling states)

- Long run payoffs:

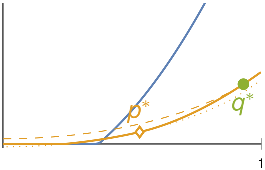
$$v_S(\Upsilon) := (1 - e^{-r\tau})^{-1} \left(\int_0^{\tau} e^{-rt} u(q_t) dt - e^{-r\tau} \left(c(q) - c(p) + \kappa \right) \right)$$

where $\Upsilon = (q, p, \tau)$

◁ takeaways ◁ λ

Optimization over cyclical behavior

- Problem reduces to optimization directly over cycles
  (closed form expressions for payoffs)



- Simplest in the class of **symmetric problems**
  (i.e invariant to relabeling states)

- Long run payoffs:

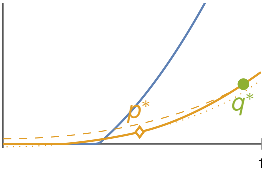
$$v_S(\Upsilon) := (1 - e^{-r\tau})^{-1} \left(\int_0^\tau e^{-rt} u(q_t) dt - e^{-r\tau} \left(c(q) - c(p) + \kappa \right) \right)$$

where $\Upsilon = (q, p, \tau)$

◁ takeaways ◁ λ

Optimization over cyclical behavior

- Problem reduces to optimization directly over cycles
  (closed form expressions for payoffs)



- Simplest in the class of **symmetric problems**
  (i.e invariant to relabeling states)

- Long run payoffs:

$$v_S(\Upsilon) := (1 - e^{-r\tau})^{-1} \left(\int_0^{\tau} e^{-rt} u(q_t) dt - e^{-r\tau} \left(c(q) - c(p) + \kappa \right) \right)$$

where $\Upsilon = (q, p, \tau)$

- Stationary problem:

$$v(\pi) = \max \left\{ u(\pi), \max_{\Upsilon} v_S(\Upsilon) - \left(c(q) - c(\pi) + \kappa \right) \right\}$$

◁ takeaways ◁ λ

Optimization over belief processes

- Identify belief process P in $\mathbb{B}$ with its distribution μ over space of càdlàg paths.
- Payoffs: $\mathfrak{U}(\mu) := \mathbb{E}_{P\sim\mu}\left[\int_0^\infty e^{-rt}u(P_t)dt\right]$
- Costs **with discrete info. acquisition** $\mu \in \mathbb{B}_d$:

$$\mathfrak{C}(\mu) := \mathbb{E}_{P\sim\mu}\left[\sum_{t|P_t\neq P_{t-}} e^{-rt}\big(c(P_t) - c(P_{t-}) + \kappa\big)\right]$$

$\rightsquigarrow$ belief process in $\mathbb{B}_d$: gen. by $\{\tau_i, F_i\}$, *all* info. acquisition at discontinuities

Claim: $\mathfrak{C}$ unif. continuous over $\mathbb{B}_d$, hence unique continuous extension over $\mathbb{B}$

If μ admits infinitesimal generator $\mathcal{A}$ and $\kappa = 0$: $\mathfrak{C}(\mu) = \mathbb{E}\left[\int_0^\infty \big(\mathcal{A}c(P_t) - \lambda(\pi - P_t)c'(P_t)\big)dt\right]$

- Problem(s):

$$\sup_{\mu\in\mathbb{B}}\left[\mathfrak{U}(\mu) - \mathfrak{C}(\mu)\right] = \sup_{\mu\in\mathbb{B}_d}\left[\mathfrak{U}(\mu) - \mathfrak{C}(\mu)\right]$$

attained in $\mathbb{B}_d$ if $\kappa > 0$.

◁ back

Convergence of Solutions

- Definition: Let (X, d) a metric space and a sequence of functionals $f_n : X \to \overline{\mathbb{R}}$. Say that f_n epi-converges to $f : X \to \overline{\mathbb{R}}$ if for every $x \in X$
  - (i) For any x_n s.t. $x_n \to x$, $f(x) \leq \liminf_n f_n(x_n)$
  - (ii) There exists $x_n \to x$ such that $f(x) \geq \limsup_n f_n(x_n)$

  Denote $f_n \xrightarrow[n\to\infty]{\text{epi}} f$.

- Proposition: If $f_n \xrightarrow[n\to\infty]{\text{epi}} f$ then for any sequence $x_n \in \arg\min f_n$:

$$x_n \to x \implies x \in \arg\min f$$

- Lemma:

$$\mathfrak{C}_\kappa \xrightarrow[\kappa\to 0]{\text{epi}} \mathfrak{C}_0$$

◁ back

WAIT-OR-CONFIRM POLICIES: FORMAL DEFINITION

- Let $\mathcal{I} \subset [0,1]$ an open set, $p \in [0,1]$
- divide boundary $\partial\mathcal{I}$ into:

$$\text{"in-boundary"} \quad \partial_\pi^{\text{in}}\mathcal{I} := \{p \in \partial\mathcal{I} \mid \exists\varepsilon > 0, b_\pi(p,\varepsilon) \subset \mathcal{I}\},$$

$$\text{"out-boundary"} \quad \partial_\pi^{\text{out}}\mathcal{I} := \partial\mathcal{I} \setminus \partial_\pi^{\text{in}}\mathcal{I} = \{p \in \partial\mathcal{I} \mid \forall\varepsilon > 0, \exists q \in b_\pi(p,\varepsilon) \setminus \mathcal{I}\};$$

where $b_\pi(p,\varepsilon)$ denotes the (open) "π-neighborhood" of size ε at p:

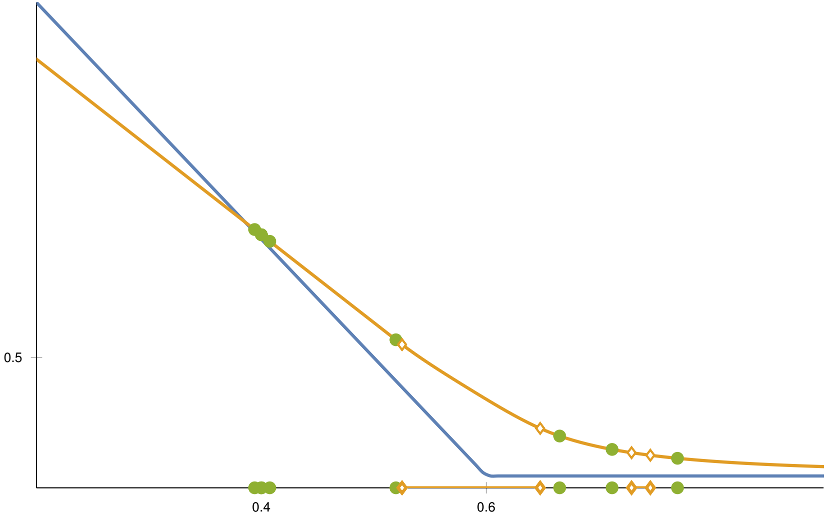
$$b_\pi(p,\varepsilon) := \begin{cases} (p, p+\varepsilon) & \text{if } p < \pi \\ (p-\varepsilon, p) & \text{if } p > \pi \end{cases}.$$

- Denote $\text{WoC}_p[\mathcal{I}]$ the distribution of the belief process $P \in \mathbb{B}(p)$ such that:
  - (Initial jump) If $p \in \mathcal{I}$, P_0 is distributed according to the only binary experiment in $\mathcal{B}(p)$ supported over the two closest points from p not in $\mathcal{I}$, otherwise $P_0 = p$ a.s.
  - (Waiting beliefs) At all $p \in \mathcal{I}^c \cup \partial_\pi^{\text{out}}\mathcal{I}$, P evolves according to no information acquisition, i.e it drifts deterministically with $dP_t = \lambda(\pi - P_t)dt$
  - (Confirmation beliefs) At all $p \in \partial_\pi^{\text{in}}\mathcal{I}$, P stays at p (confirming) until an exponentially distributed time at which it jumps to the closest belief $q(p)$ in the direction of π that is not in $\mathcal{I}$.

The belief process P is called a wait-or-confirm (information acquisition) policy with initial belief p, and $\mathcal{I}$ is called its (instantaneous) information acquisition region.

◁ back

SHORT VS. LONG RUN – STATIC VS. DYNAMIC INTUITIONS



$$u(p) = \max\{1 - p, .4\},\ \lambda = .9,\ \pi = .4,\ \kappa = .00003,\ c(p) \propto \text{entropy}(p) + 2(p - .5)^2 - \cos\left(\frac{12(p-.5)\pi}{36\pi^2}\right)$$

◁ go back

Effect of the fixed cost – Example

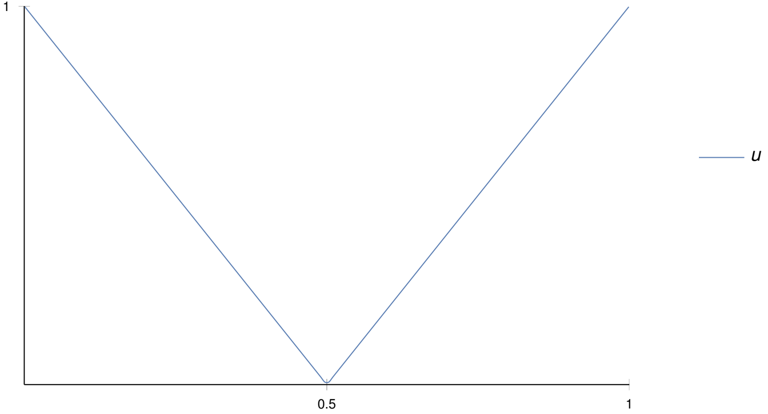


Figure: **Illustration of comparative statics in the fixed cost**

$u(p) = 2\max\{p, 1-p\}$, $r = 1$, $\lambda = .5$, entropy costs, $\kappa = .05$, $\tilde{\kappa} = .001$

◁ go back

to portfolio example ▷

EFFECT OF THE FIXED COST – EXAMPLE

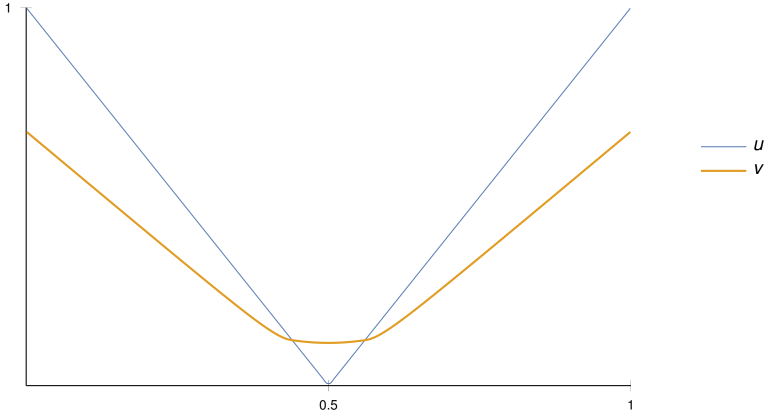


Figure: **Illustration of comparative statics in the fixed cost**

$u(p) = 2\max\{p, 1-p\}$, $r = 1$, $\lambda = .5$, entropy costs, $\kappa = .05$, $\tilde{\kappa} = .001$

◁ go back

to portfolio example ▷

Effect of the fixed cost – Example

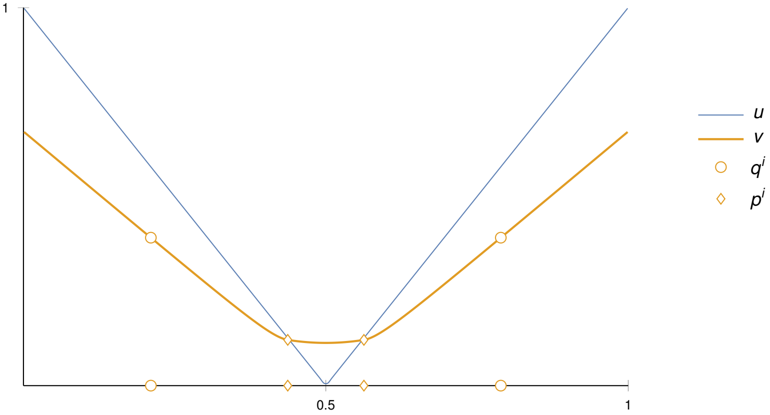


Figure: **Illustration of comparative statics in the fixed cost**

$u(p) = 2\max\{p, 1-p\}$, $r = 1$, $\lambda = .5$, entropy costs, $\kappa = .05$, $\tilde{\kappa} = .001$

◁ go back

to portfolio example ▷

Effect of the fixed cost – Example

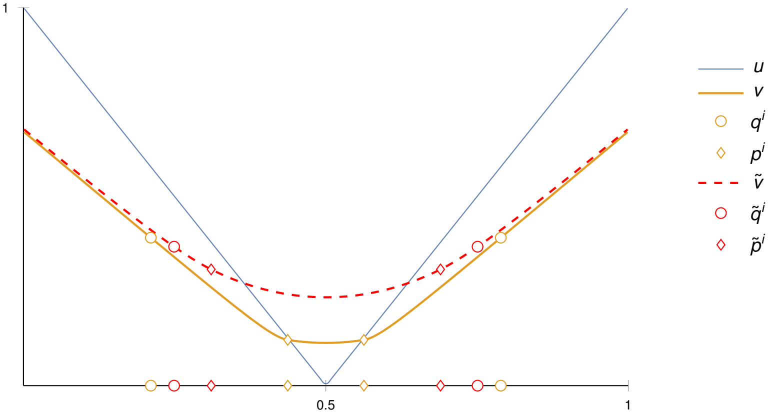


Figure: **Illustration of comparative statics in the fixed cost**

$u(p) = 2\max\{p, 1-p\}$, $r = 1$, $\lambda = .5$, entropy costs, $\kappa = .05$, $\tilde{\kappa} = .001$

◁ go back

to portfolio example ▷

Effect of the fixed cost – Example

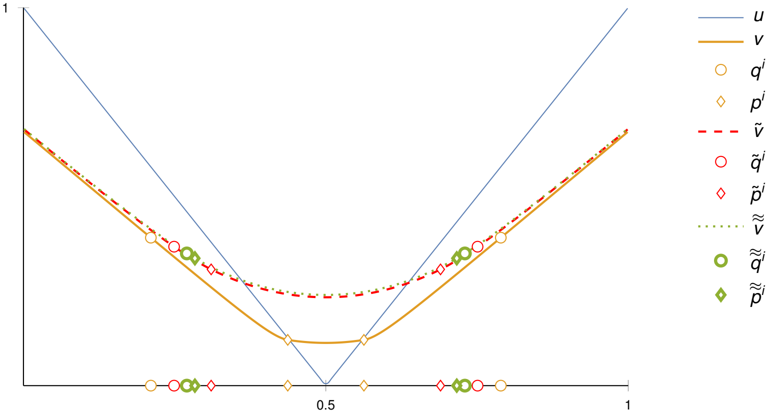


Figure: **Illustration of comparative statics in the fixed cost**

$u(p) = 2\max\{p, 1-p\}$, $r = 1$, $\lambda = .5$, entropy costs, $\kappa = .05$, $\tilde{\kappa} = .001$

◁ go back

to portfolio example ▷

Effect of volatility – Example

$\longrightarrow$ How about quality?

EFFECT OF VOLATILITY – EXAMPLE

⟶ How about quality? *Often* decreasing in simple examples (but not always...)

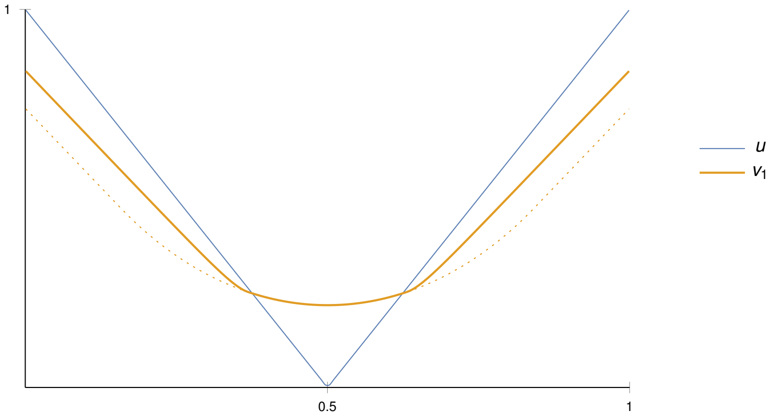


Figure: **Illustration of comparative statics in the volatility**

$u(p) = \max\{p, 1-p\}$, $r = 1$, $\kappa = .05$, entropy costs, $\lambda_1 = .2$, $\lambda_2 = .8$ $\lambda_3 = 1.1$

◁ back

Effect of volatility – Example

⟶ How about quality? *Often* decreasing in simple examples (but not always...)

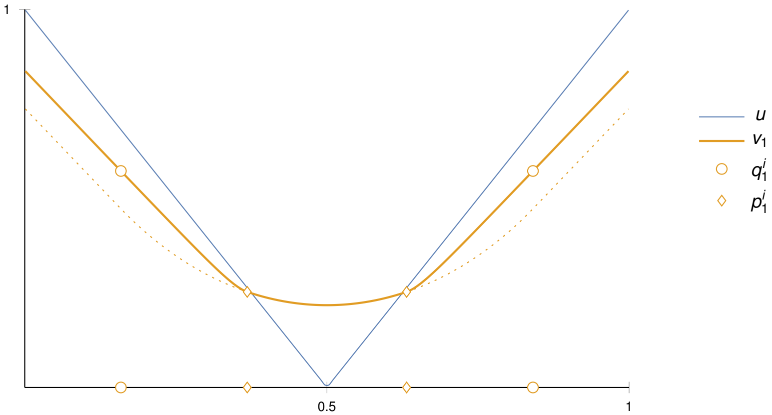


Figure: **Illustration of comparative statics in the volatility**

$u(p) = \max\{p, 1-p\}$, $r = 1$, $\kappa = .05$, entropy costs, $\lambda_1 = .2$, $\lambda_2 = .8$ $\lambda_3 = 1.1$

◁ back

Effect of volatility – Example

⟶ How about quality? *Often* decreasing in simple examples (but not always...)

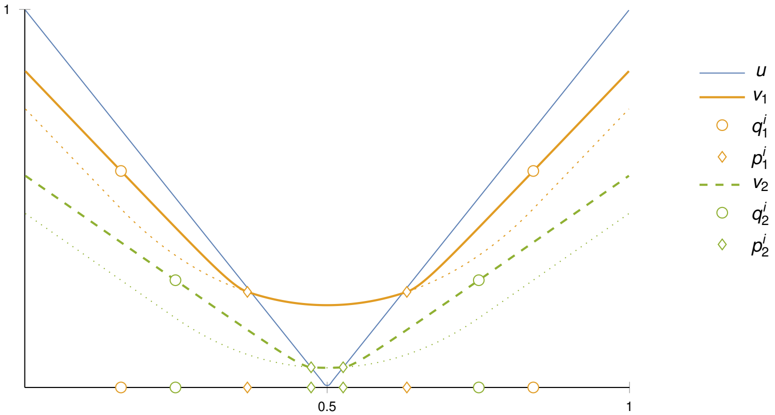


Figure: **Illustration of comparative statics in the volatility**

$u(p) = \max\{p, 1-p\}$, $r = 1$, $\kappa = .05$, entropy costs, $\lambda_1 = .2$, $\lambda_2 = .8$ $\lambda_3 = 1.1$

◁ back

Effect of volatility – Example

⟶ How about quality? *Often* decreasing in simple examples (but not always...)

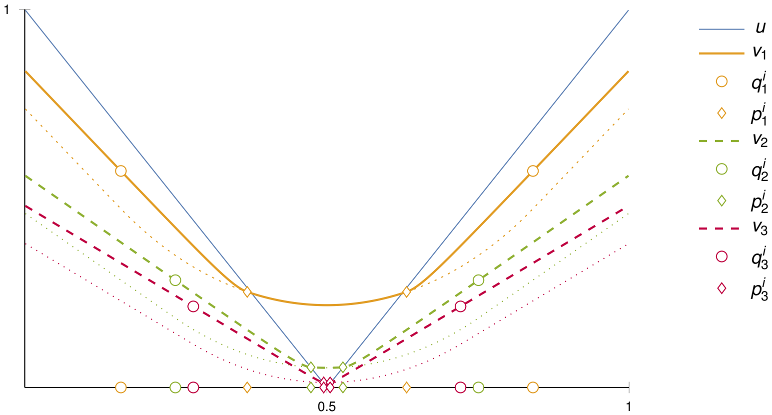


Figure: **Illustration of comparative statics in the volatility**

$u(p) = \max\{p, 1-p\}$, $r = 1$, $\kappa = .05$, entropy costs, $\lambda_1 = .2$, $\lambda_2 = .8$ $\lambda_3 = 1.1$

◁ back

EFFECT OF VOLATILITY – EXAMPLE

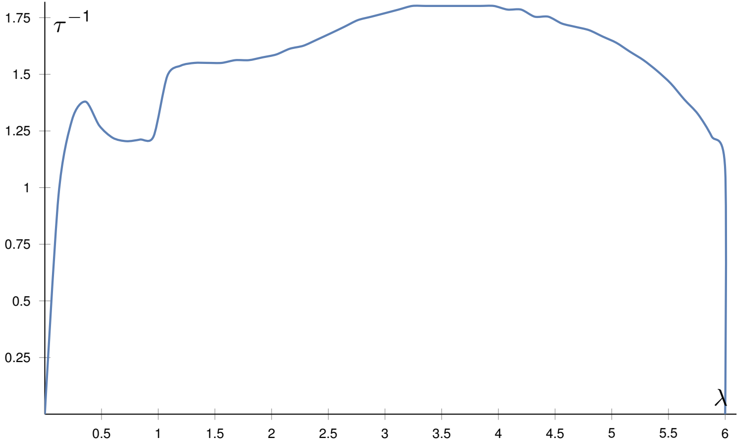


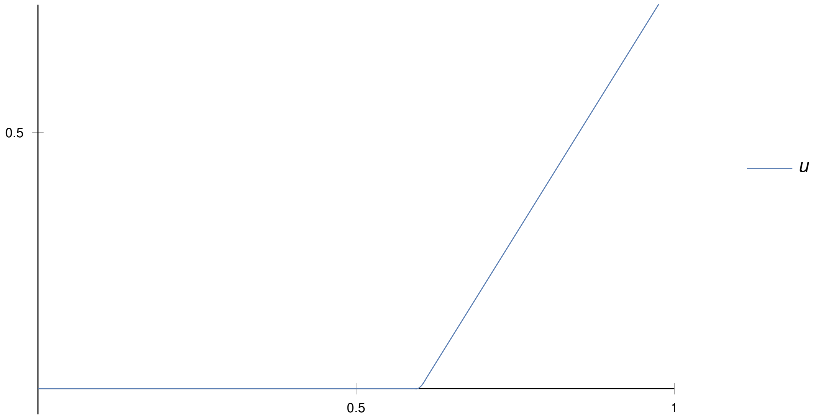
Figure: **Frequency of information acquisition as a function of volatility**

$$u(p) = \max\{p, 1-p\},\ r = 1,\ \kappa = .05,\ c(p) = \frac{(x-.5)^2}{2} + \frac{1-\cos(16\pi(x-.5)^2)}{256\pi^2}$$

◁ go back

"Confirmation bias" in asymmetric problems

A simple example with a safe and a risky action



$u(p) = \max\{0, 2(p - .6)\}$, $\lambda = 1$, $\pi = .5$, entropy cost,

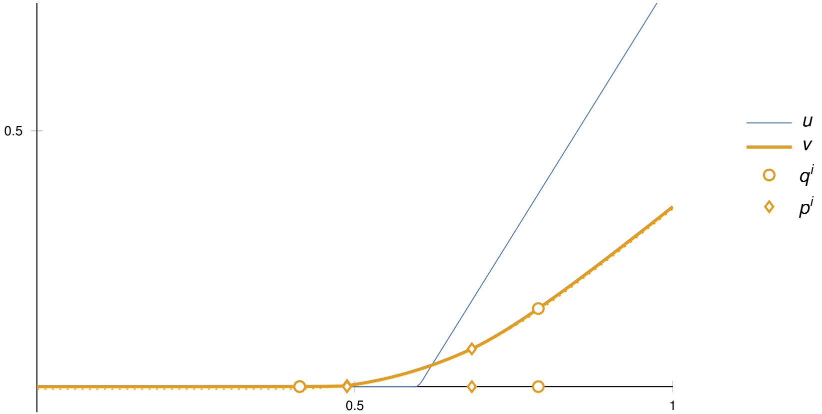
Only asymmetry $\rightarrow$ indifference point

◁ conclusion

"Confirmation bias" in asymmetric problems

A simple example with a safe and a risky action

$\tau^0 = 1.95, \tau^1 = 0.5$



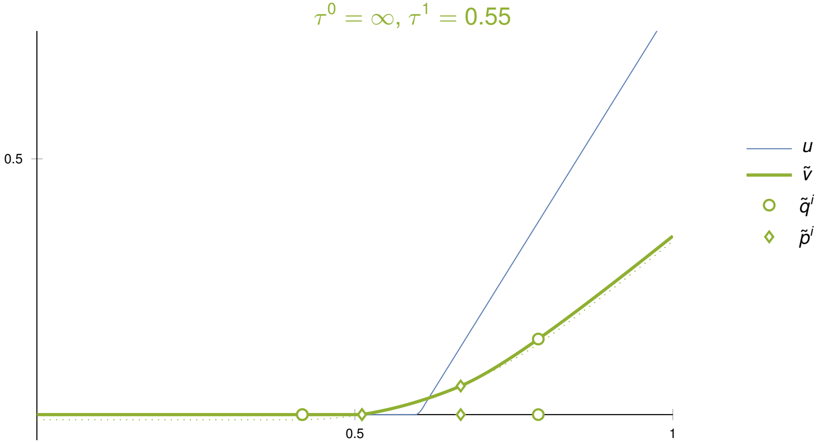
$u(p) = \max\{0, 2(p - .6)\}$, $\lambda = 1$, $\pi = .5$, entropy cost, $\kappa = .005$

Only asymmetry $\rightarrow$ indifference point

More frequent updating after good news than bad news

"Confirmation bias" in asymmetric problems

A simple example with a safe and a risky action



$u(p) = \max\{0, 2(p - .6)\}$, $\lambda = 1$, $\pi = .5$, entropy cost, $\tilde{\kappa} = .01$

Only asymmetry $\rightarrow$ indifference point

For high enough κ, never acquire info. after the first bad news

< conclusion