An experiment on a multi-period beauty contest game

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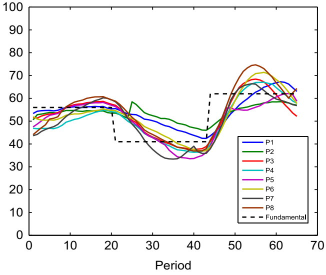
Background

- Expectation formation is the key ingredient of the modern macro- and financial economics.
- The (strong-form of) rational expectations model
  - can lead to erratic implications, e.g., the forward-guidance puzzle.
  - does not explain the results of laboratory experiments in the presence of strategic complementarity (positive feedback)

Result: Dynamics of T_t (Bao et al., 2012)

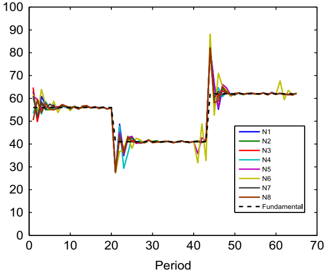
With complementarity

$$T_t = f_t + (20/21)(< x_t > - f_t) + \varepsilon_t$$



With substitutability

$$T_t = f_t - (20/21)(< x_t > - f_t) + \varepsilon_t$$



- $< x_t >$: the average forecast of T_t by participants
- f_t: fundamental. ε_t. noise
- see many others results summarized in Hommes (2021, JEL), Bao et al. (2021, JBEF)

Background

- Behavioral model proposed based on these experiments
  - Heuristic Switching Model (Anufriev and Hommes, 2012, AEJ-Micro)
    - “backward looking”
  - Unified model (Evans et al., 2025, AEJ-Macro)
    - Level-k based learning (with an adaptive learning as level-0)

Background

- Most existing learning to forecast experiments
  - do not elicit forecasts for multiple future periods
  - thus the evolution of the forecasts for the same future date cannot be studied, e.g.,
    - ★ how early do expectations (for the same future date) start to change before the known future shock?
    - ★ how do expectations (for the same future date) change in response to a sudden shock?
  - c.f., survey based research on expectation formation (e.g., Coibion et al. 2018 AER)

What do we do?

- Propose a learning-to-forecast (LtF) experiments motivated by Calvo (1983, JME) in which
  - participants submit forecasts for **multiple** future periods
  - all the submitted forecasts may influence prices (prices are endogenous)
  - all the submitted forecasts may count into participants' rewards (forecasts are all incentivized)

What do we do?

- In order to investigate
  - the evolution of the forecasts in response to an anticipated shock (forward lookingness)
  - the evolution of the forecasts in “normal” time
  - the effect of strategic environment: complementarity vs substitutability (sanity check)

What do we find?

- Evidence of forward lookingness of expectation formation
  - Forecasts react to an announcement
- Substantial heterogeneity in the way forecasts evolve
- Replicate the effect of strategic environment in our new framework
  - Deviation from REE larger under strategic complementarity than under strategic substitution
- Under strategic complement, participants extrapolate on trend (thus expectation can be de-anchored)

What's new about our multi-period forecasting?

- not just a longer horizon forecasting
  - Anufriev et al., (2022, JEBO): forecasting only outcome in $t+k$ period (submit one forecast)
  - Evans et al. (2022, JME): forecasting the **average** of multiple future periods (submit one forecast)
- all the submitted forecasts may influence prices
  - Colassante et al., (2020, JEE): forecasts beyond the single future period do not feedback into the price

What's new about our multi-period forecasting?

- Simple!
  - Only one aggregate variable unlike more complex New Keynesian LtF experiments
    - Adam (2007, EJ)
    - Rholes and Petersen (2021, JEBO) and Petersen and Rholes (2022, JEDC)
    - Lustenhouwer and Salle (2025, EER)

Our framework

- based on the staggered price adjustment new Keynesian macro model of Calvo (1983 JME)
  - A model frequently used in new Keynesian macro model
- In each period, participants submit forecasts for multiple (5) future periods, e.g.,
  - In period 1: forecasts for periods 1, 2, 3, 4, 5
  - In period 2: forecasts for periods 2, 3, 4, 5, 6
- Accuracies of all the submitted forecasts can (potentially) determine participant's reward
- All the submitted forecasts can (potentially) determine the future prices

Model (Calvo 1983). Static case

- Consider monopolistic competition
- let p_i be the price chosen by firm i
- Demand for firm i is $D(p_i; P) \equiv (a - bp_i + cP)^+$
  - P is the average price of all the firms p_j.
  - $a > 0$, $b > 0$ and $c \in R$.
- with a constant marginal cost κ, the optimal price p_i^*

$$p_i^* = \underbrace{\frac{1}{2}\left(\kappa + \frac{a}{b}\right)}_{\equiv\alpha} + \underbrace{\frac{1}{2}\frac{c}{b}}_{\equiv\beta} P.$$

- the best response function

$$\tilde{p}(P) = \alpha + \beta P$$

- with homogeneous firms, $p_i^* = \frac{\alpha}{1-\beta}$ for all i when $\beta < 1$

Model (Calvo 1983). Dynamic game

- Indefinite horizon (game ends with a probability γ at the end of each date)
- Firm i maximizes the present value of the profit at each date, but is subject to a pricing friction
- Firm can change its price at date t with probability $1-\theta$
- if it has not changed the price for K periods, it can do so with probability 1.
- the problem for the firm is

$$\max_{p_t} \sum_{s=0}^{K-1} ((1-\gamma)\theta)^s (p_t - \kappa) D(p_t; P_{t+s}) \tag{1}$$

where P_{t+s} is the expectation about the average price in period $t+s$

Model (Calvo 1983). Dynamic game

- The solution is

$$p_t^* = \sum_{s=0}^{K-1} \frac{((1-\gamma)\theta)^s}{\sum_{l=0}^{K-1}((1-\gamma)\theta)^l} \underbrace{(\alpha + \beta P_{t+s})}_{=\tilde{p}(P_{t+s})}$$

- i.e., a weighted average of the static optimal price $\tilde{p}\,(P_{t+s})$ given the expected future prices.
- Thus, the optimal price is a function of $\mathbf{P}_t = (P_{t+s})_{s=0}^{K-1}$.
- forecasts for prices in period t to $t+K-1$ determine the price in period t
- if $\theta = 0$ (no pricing friction)

$$p_t^* = \alpha + \beta P_t$$

- Thus, back to the standard repeated beauty contest game

Experiment

- $N = 6$ participants (acting as a firm).
- in each period $t \in \{1, 2, ..., \}$
  - submit price forecasts for 5 future periods (including t)
  - let $f^i_{t,t+k}$ be the forecast for period $t + k$ price, submitted by i in period t ($k \in \{0, 1, 2, 3, 4\}$)
- Participants are rewarded based on the accuracy of their forecasts.
- For participant i in period t his reward is determined according to

$$100/(|F^i_t - P_t| + 1)$$

  where F^i_t is his "payoff-relevant" forecast for period t price, and P_t is the realized price in period t.
  - often used in LtF experiment (Adam 2007 EJ, Assenza et al. 2021 JME, Anufriev et al. 2022 JET)

"Payoff-relevant" forecast for period t price

- let v_t^i be the period in which i submitted his "payoff-relevant" forecast for period t
- for $t = 1$ (first period) $v_t^i = 1$ so that $F_1^i = f_{1,1}^i$
- for $t \geq 2$ (second period on)
  - with prob $1 - \theta$: new forecasts become payoff-relevant. $v_t^i = t$, $F_t^i = f_{t,t}^i$
  - with prob θ: old forecasts continue to be payoff-relevant. $v_t^i = v_{t-1}^i < t$, $F_t^i = f_{v_t^i,t}^i$
  - if a set of forecasts have been payoff-relevant for $K = 5$ consecutive periods, in the next period, the new forecasts become payoff-relevant with probability one.
- Capture the friction of price adjustment in Calvo
- All the entered forecasts **can** determine participants' reward

An example of "payoff-relevant" forecasts

	Forecast Periods k								
Period t	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	F_t^i
$t=1$	**10**	**11**	~~12~~	~~12~~	~~12~~	–	–	–	**10**
$t=2$	–	~~12~~	~~11~~	~~12~~	~~13~~	~~13~~	–	–	**11**
$t=3$	–	–	**9**	~~10~~	~~11~~	~~10~~	~~10~~	–	**9**
$t=4$	–	–	–	**13**	12	12	10	10	**13**

Price determination

$$P_t = \frac{1}{N} \sum_i \pi_t^i \qquad (2)$$

where

$$\pi_t^i = \begin{cases} \sum_{j=0}^{K-1} \frac{((1-\gamma)\theta)^j}{\sum_{l=0}^{K-1}((1-\gamma)\theta)^l} \left(\alpha + \beta f_{t,t+j}^i \right) & \text{if } v_t^i = t \\ \pi_{t-1}^i & \text{otherwise} \end{cases} \qquad (3)$$

- All the forecasts may feedback into the price

Parameterization

- N = 6 (number of participants in a group)
- K = 5 (number of forecasts to be submitted per period)
- θ = 0.5 (probability of the new set of forecasts becoming payoff-relevant in every period)
- γ = 5% (termination probability at the end of each period)
- B = 20 (Length of the block)
  - random number sequence pre-determined so that a game ends between 30-35 periods (partly to ensure that participants earn enough)
  - An experimental session ended after the first game (as it took more than 30 minutes for it to be completed)
- α and β to be varied across treatment and also within treatment (shock)

Treatments: 3 by 2 design

- In all the treatment:
  - participants are informed of current value of α and β
- Positive vs Negative feedback ($\beta \in \{0.9, -0.9, -1.8\}$)
- Shock to α known vs unknown
  - Known: change in α is pre-announced (2 periods before the shock)
  - Unknown: not pre-announced (announced only at the realization of the shock)

Shocks

- Implemented as a change α (once in a block of 20 periods).
- β is constant through out the game
- Steady state price $p^* = \alpha/(1-\beta)$ is given by

$$p^* = \begin{cases} 65 & t < 14 \\ 85 & 14 \leq t < 28 \\ 110 & 29 \leq t \end{cases} \tag{4}$$

Experiment

- between April - May 2023.
- oTree (Chen et al., 2016)
- online with Zoom (participants camera off, microphone off), anonymized (subject ID)
- Instruction movie broadcasted + slides can be checked during the quiz.
- Questions are asked privately via zoom chat.
- Experimenter's camera on, and microphone on when necessary.
- 30 participants recruited for a session.
  - But show-up rate vary
  - mostly conducted with 24 participants. 30 participants in 1 session
- A total of 294 participants

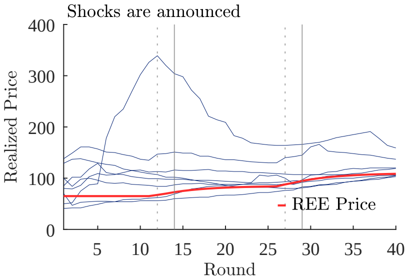
Results

Number of groups

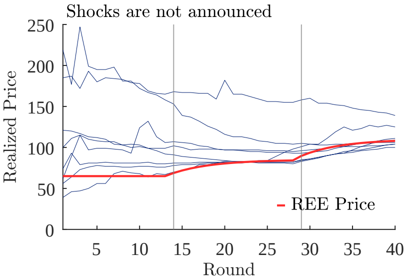
Treatments	$\beta = 0.9$	$\beta = -0.9$	$\beta = -1.8$
With Announcement	8	8	8
Without Announcement	8	8	7

- average duration: 1h30m (recruited for 2h)
- average payment: 2480 JPY ($\approx$ 18 USD)
  - including 500 JPY show-up fee. 1pt = 2 JPY
  - 1778 JPY in $\beta = -1.8$
  - 2806 JPY in $\beta = -0.9$
  - 2844 JPY in $\beta = 0.9$

with annoucement

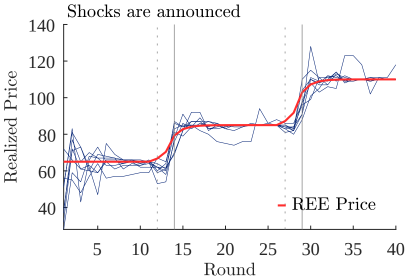


without announcement



Price Dynamics: $\beta = -0.9$

with annoucement



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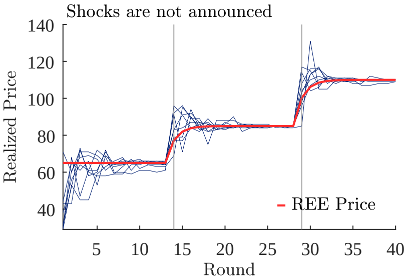
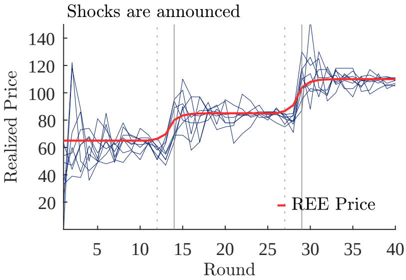


with annoucement



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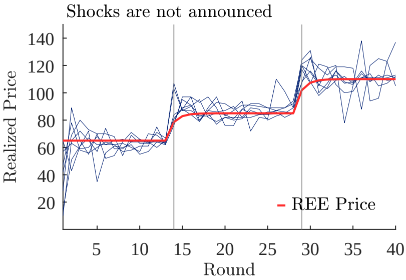
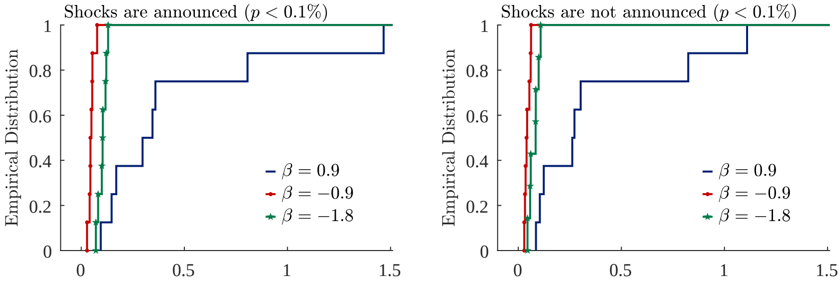


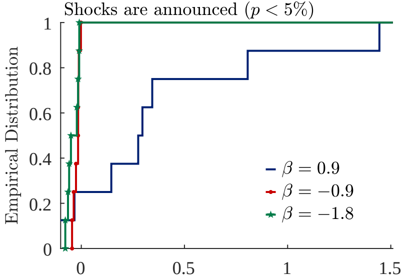
Deviation from REE. $RAD_i = \frac{1}{T}\sum_t \frac{|p_{i,t} - REE_t|}{REE_t}$ (Stockel et al., 2010 EXEX)

with annoucement | without announcement

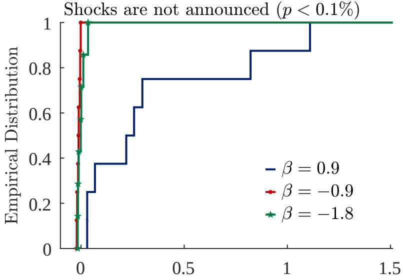


Deviation from REE. $RD_i = \frac{1}{T}\sum_t \frac{p_{i,t} - REE_t}{REE_t}$ (Stockel et al., 2010 EXEX)

with annoucement

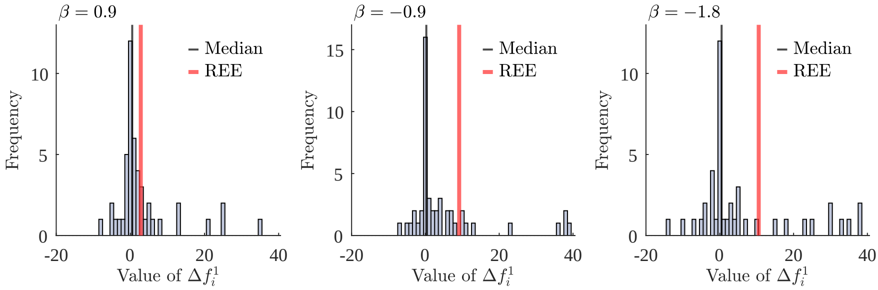


without announcement



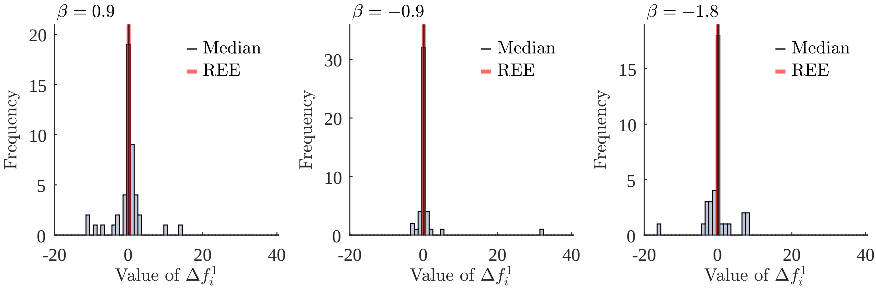
Forward-lookingness (response to annoucement)

$$\Delta f_1^i = \underbrace{\frac{1}{2}\sum_{t=12}^{13}\left(f_{t,14}^i - f_{t,13}^i\right)}_{\text{after annoucement}} - \underbrace{\frac{1}{2}\sum_{t=10}^{11}\left(f_{t,14}^i - f_{t,13}^i\right)}_{\text{before announcement}}$$



- forward looking forecast adjustment observed!

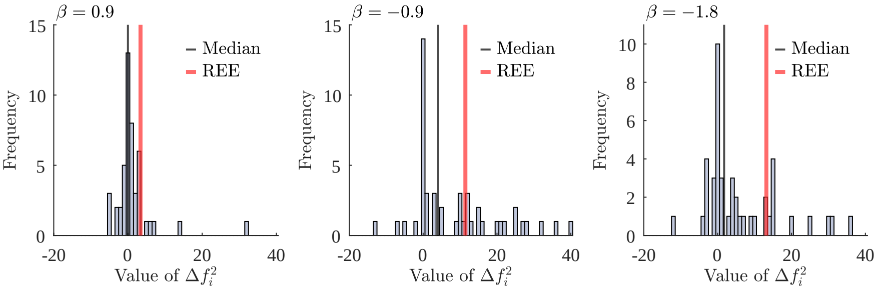
Δf_1^i without annoucement



- Mostly around zero!

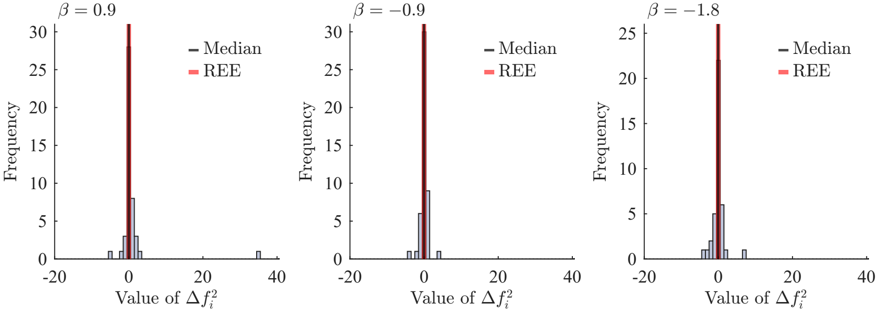
Forward-lookingness (response to annoucement) 2

$$\Delta f_2^i = \underbrace{\frac{1}{2}\sum_{t=27}^{28}\left(f_{t,29}^i - f_{t,28}^i\right)}_{\text{after annoucement}} - \underbrace{\frac{1}{2}\sum_{t=25}^{26}\left(f_{t,29}^i - f_{t,28}^i\right)}_{\text{before announcement}}$$



- forward looking forecast adjustment observed

Δf_2^i without annoucement

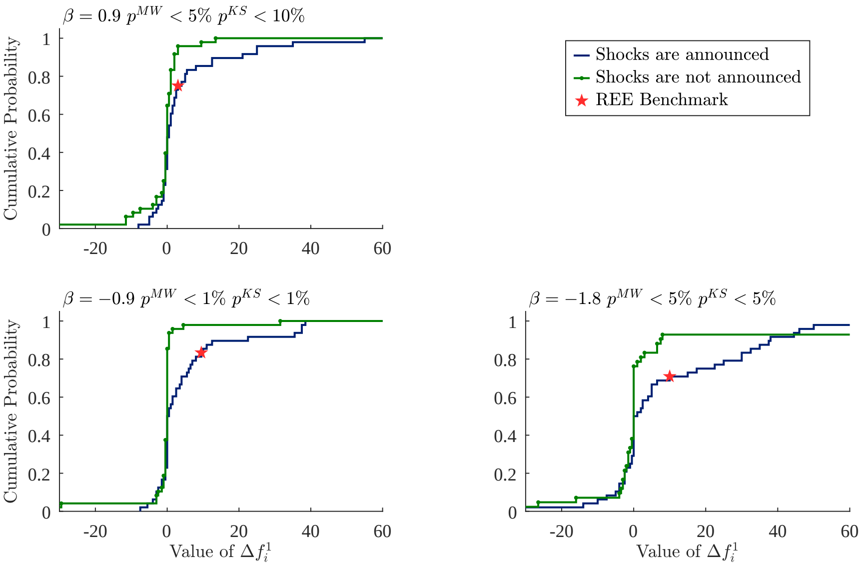


- Mostly around zero!

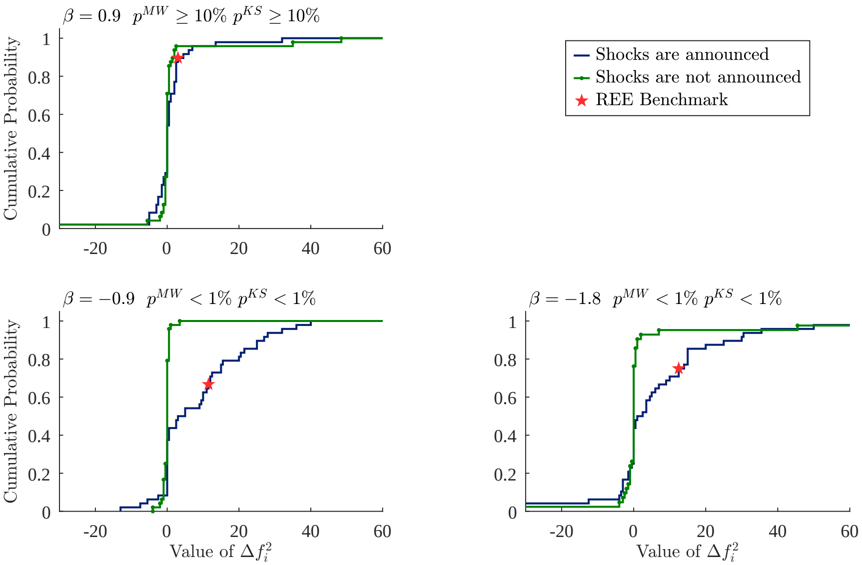
Heterogeneity

First Announcement	Overreaction	Underreaction	
		No Revisions	Other Types
$\beta = 0.9$	27%	17%	56%
$\beta = -0.9$	19%	27%	54%
$\beta = -1.8$	30%	21%	49%
Second Announcement	**Overreaction**	**Underreaction**	
		No Revisions	Other Types
$\beta = 0.9$	13%	25%	62%
$\beta = -0.9$	44%	29%	57%
$\beta = -1.8$	29%	19%	57%

Distribution of Δf_1^i with and without announcement



Distribution of Δf_2^i with and without announcement



Individual Expectation formation. A model estimation

- Consider two types of expectation formation
  - Adaptive heuristic

$$f^i_{t,t+j} = P_{t-1} + \omega^i_j \left(f_{t-1,t-1} - P_{t-1}\right).$$

  - Trend-following

$$f^i_{t,t+j} = P_{t-1} + \chi^i_j \left(P_{t-1} - P_{t-2}\right),$$

- Nesting the two

$$f^i_{t,t+j} = \alpha^i_j + \gamma^i_j P_{t-1} + \omega^i_j \left(f_{t-1,t-1} - P_{t-1}\right) + \chi^i_j \left(P_{t-1} - P_{t-2}\right) + \varepsilon^i_{t,t+j}$$

- estimate it in “normal” time for each participant
  - periods associated with shock and their announcement excluded

Classification of individual

$$f^i_{t,t+j} = \alpha^i_j + \gamma^i_j P_{t-1} + \omega^i_j \left(f_{t-1,t-1} - P_{t-1}\right) + \chi^i_j \left(P_{t-1} - P_{t-2}\right) + \varepsilon^i_{t,t+j}$$

- Check if
  - γ^i_j significant at 10%?
  - ω^i_j significant at 10%?
  - χ^i_j significant at 10%?
- if answers are (Yes, No, No) (1-0-0): Rely mostly on P_{t-1}
- if answers are (Yes, Yes, Yes) (1-1-1): Rely both on adaptive heuristics and trend-following
- if answers are (Yes, Yes, No) (1-1-0): Rely on adaptive heuristics
- if answers are (Yes, No, Yes) (1-0-1): Rely on trend-following

Classification

Treatment	Horizon	Type							
		1-0-0	1-1-1	1-1-0	1-0-1	0-1-0	0-1-1	0-0-1	0-0-0
$\beta = 0.9$	$j = 0$	29%	38%	15%	15%	0%	1%	0%	3%
	$j = 1$	24%	41%	16%	16%	0%	1%	1%	2%
	$j = 2$	28%	39%	14%	14%	1%	2%	1%	2%
	$j = 3$	27%	42%	15%	11%	0%	1%	1%	3%
	$j = 4$	30%	35%	14%	14%	0%	4%	1%	2%
$\beta = -0.9$	$j = 0$	34%	25%	26%	11%	1%	1%	1%	0%
	$j = 1$	38%	18%	27%	14%	1%	0%	2%	1%
	$j = 2$	32%	22%	29%	12%	1%	0%	2%	1%
	$j = 3$	36%	23%	24%	12%	1%	0%	2%	1%
	$j = 4$	40%	22%	22%	10%	1%	0%	2%	3%
$\beta = -1.8$	$j = 0$	27%	14%	30%	18%	2%	0%	1%	8%
	$j = 1$	30%	16%	33%	13%	0%	0%	0%	8%
	$j = 2$	29%	19%	27%	14%	3%	1%	0%	7%
	$j = 3$	32%	19%	27%	12%	1%	0%	1%	8%
	$j = 4$	32%	19%	24%	14%	1%	0%	1%	8%

- Most participants rely on P_{t-1}.
- Substantial heterogeneity

Conditional mean ($p < 0.1$) of estimated coefficient

Treatment	Coefficients	Horizon j $j = 0$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$\beta = 0.9$	γ_j^i	1.02 (0.06)	1.04 (0.08)	1.05 (0.10)	1.11 (0.13)	1.19 (0.18)
$\beta = -0.9$	γ_j^i	0.98 (0.05)	0.96 (0.06)	0.95 (0.06)	0.95 (0.07)	0.94 (0.08)
$\beta = -1.8$	γ_j^i	0.96 (0.06)	0.90 (0.08)	0.89 (0.09)	0.97 (0.10)	0.90 (0.11)
$\beta = 0.9$	ω_j^i	0.42 (0.11)	0.52 (0.15)	0.58 (0.21)	0.74 (0.25)	0.63 (0.32)
$\beta = -0.9$	ω_j^i	0.34 (0.10)	0.38 (0.13)	0.46 (0.18)	0.50 (0.17)	0.56 (0.19)
$\beta = -1.8$	ω_j^i	0.36 (0.11)	0.36 (0.13)	0.72 (0.34)	0.22 (0.19)	0.28 (0.15)
$\beta = 0.9$	χ_j^i	0.53 (0.18)	0.75 (0.25)	1.23 (0.31)	1.74 (0.52)	1.95 (0.46)
$\beta = -0.9$	χ_j^i	0.45 (0.22)	0.45 (0.22)	0.58 (0.26)	0.77 (0.27)	0.90 (0.30)
$\beta = -1.8$	χ_j^i	-0.17 (0.16)	0.03 (0.16)	0.47 (0.42)	-0.03 (0.21)	-0.06 (0.26)

$$f_{t,t+j}^i = \alpha_j^i + \gamma_j^i P_{t-1} + \omega_j^i \left(f_{t-1,t-1} - P_{t-1} \right) + \chi_j^i \left(P_{t-1} - P_{t-2} \right) + \varepsilon_{t,t+j}^i$$

- Effect of trend (χ_j) increasing with forecasting horizon for $\beta = 0.9$!

A different look.

$$f^i_{t,t+j} - f^i_{t,t} = \alpha^i + \mathrm{FE}_j + \gamma^i P_{t-1} + \omega^i \left(f_{t-1,t-1} - P_{t-1}\right) + \chi^i \left(P_{t-1} - P_{t-2}\right)$$
$$+\tilde{\gamma}^i j P_{t-1} + \tilde{\omega}^i j \left(f_{t-1,t-1} - P_{t-1}\right) + \tilde{\chi}^i j \left(P_{t-1} - P_{t-2}\right)$$

- How does the effect of P_{t-1}, $\left(f_{t-1,t-1} - P_{t-1}\right)$, and $\left(P_{t-1} - P_{t-2}\right)$ change with horizon?

Conditional mean ($p < 0.1$) of $\tilde{\gamma}$, $\tilde{\omega}$, $\tilde{\chi}$

Treatment	Term	Mean Estimate (Std)	$p < 10\%$
$\beta = 0.9$	$\tilde{\gamma}^i$	0.05 (0.01)	60%
$\beta = -0.9$	$\tilde{\gamma}^i$	-0.03 (0.01)	58%
$\beta = -1.8$	$\tilde{\gamma}^i$	-0.02 (0.02)	56%
$\beta = 0.9$	$\tilde{\omega}^i$	0.06 (0.03)	53%
$\beta = -0.9$	$\tilde{\omega}^i$	0.05 (0.02)	47%
$\beta = -1.8$	$\tilde{\omega}^i$	-0.03 (0.01)	38%
$\beta = 0.9$	$\tilde{\chi}^i$	0.43 (0.07)	**54%**
$\beta = -0.9$	$\tilde{\chi}^i$	0.11 (0.02)	44%
$\beta = -1.8$	$\tilde{\chi}^i$	0.01 (0.03)	43%

- Expect the trend to continue, thus, expectation can be de-anchored!

Summary

- Presented a novel framework for a LtF experiment with a multi-period forecasting elicitation based on Calvo (1983)
- Evidence of forward-lookingness in response to pre-announced change in FV
- Substantial heterogeneity in the way expectations are formed.
- With strategic complementarity, participants expect the trend to continue!
- The mode of expectation formation is endogenous to the economic environment!

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